TABLE F.1 FORMULAS FOR UNIT CONVERSIONS*

Name, Symbol, Dimensions			Conversion Formula
Length	L	L	1 m = 3.281 ft = 1.094 yd = 39.37 in = km/1000 = 10^6 μm 1 ft = 0.3048 m = 12 in = mile/5280 = km/3281 1 mm = m/1000 = in/25.4 = 39.37 mil = 1000 μm = 10^7 Å
Speed	V	L/T	1 m/s = 3.600 km/hr = 3.281 ft/s = 2.237 mph = 1.944 knots 1 ft/s = 0.3048 m/s = 0.6818 mph = 1.097 km/hr = 0.5925 knots
Mass	m	M	1 kg = 2.205 lbm = 1000 g = slug/14.59 = (metric ton or tonne or Mg)/1000 1 lbm = lbf·s²/(32.17ft) = kg/2.205 = slug/32.17 = 453.6 g = 16 oz = 7000 grains = short ton/2000 = metric ton (tonne)/2205
Density	ρ	M/L^3	1000 kg/m³ = 62.43 lbm/ft³ = 1.940 slug/ft³ = 8.345 lbm/gal (US)
Force	F	ML/T^2	1 lbf = 4.448 N = 32.17 lbm·ft/s² 1 N = kg·m/s² = 0.2248 lbf = 10^5 dyne
Pressure	P	M/LT^2	1 Pa = N/m² = kg/m · s² = 10^{-5} bar = 1.450 × 10^{-4} lbf/in² = inch H_2O/249.1 = 0.007501 torr = 10.00 dyne/cm² 1 atm = 101.3 kPa = 2116 psf = 1.013 bar = 14.70 lbf/in² = 33.90 ft of water = 29.92 in of mercury = 10.33 m of water = 760 mm of mercury = 760 torr 1 psi = atm/14.70 = 6.895 kPa = 27.68 in H_2O = 51.71 torr
Volume	V	L^3	1 m³ = 35.31 ft³ = 1000 L = 264.2 U.S. gal 1 ft³ = 0.02832 m³ = 28.32 L = 7.481 U.S. gal = acre-ft/43,560 1 U.S. gal = 231 in³ = barrel (petroleum)/42 = 4 U.S. quarts = 8 U.S. pints = 3.785 L = 0.003785 m³
Volume Flow Rate (Discharge)	Q	L^3/T	1 m³/s = 35.31 ft³/s = 2119 cfm = 264.2 gal (US)/s = 15850 gal (US)/m 1 cfs = 1 ft³/s = 28.32 L/s = 7.481 gal (US)/s = 448.8 gal (US)/m
Mass Flow Rate	$\dot{m}$	M/T	1 kg/s = 2.205 lbm/s = 0.06852 slug/s
Energy and Work	E, W	ML^2/T^2	1 J = kg·m²/s² = N·m = W·s = volt·coulomb = 0.7376 ft·lbf = 9.478 × 10^{-4} Btu = 0.2388 cal = 10^7 erg = kWh/3.600 × 10^6
Power	$P, \dot{E}, \dot{W}$	ML^2/T^3	1 W = J/s = N·m/s = kg·m²/s³ = 1.341 × 10^{-3} hp = 0.7376 ft · lbf/s = 1.0 volt-ampere = 0.2388 cal/s = 9.478 × 10^{-4} Btu/s 1 hp = 0.7457 kW = 550 ft·lbf/s = 33,000 ft·lbf/min = 2544 Btu/h
Angular Speed	ω	T^{-1}	1.0 rad/s = 9.549 rpm = 0.1591 rev/s
Viscosity	μ	M/LT	1 Pa·s = kg/m·s = N·s/m² = 10 poise = 0.02089 lbf·s/ft² = 0.6720 lbm/ft·s
Kinematic Viscosity	ν	L^2/T	1 m²/s = 10.76 ft²/s = 10^6 cSt
Temperature	T	Θ	K = °C + 273.15 = °R/1.8 °C = (°F − 32)/1.8 °R = °F + 459.67 = 1.8 K °F = 1.8°C + 32

* A useful online reference is www.onlineconversion.com

Chapter One
PROPERTIES OF FLUIDS

Any characteristic of a system is called a property. Some familiar properties are pressure P, temperature T, volume V, and mass m. The list can be extended to include less familiar ones such as viscosity, thermal conductivity, modulus of elasticity, thermal expansion coefficient, electric resistively, and even velocity and elevation.

DENSITY AND SPECIFIC GRAVITY

Density is defined as mass per unit volume (Fig.1). That is

$$\rho = \frac{m}{V} \qquad (\text{kg/m}^3)$$

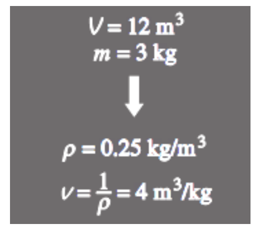


FIG.1
Density is mass per unit volume; specific volume is volume per unit mass

The reciprocal of density is the specific volume v, which is defined as volume per unit mass. That is, $v = V/m = 1/\rho$. For a differential volume element of mass δm and volume density δV, can be expressed as $\rho = \delta m/\delta V$.

Sometimes the density of a substance is given relative to the density of a well-known substance. Then it is called specific gravity, or relative density, and is defined as the ratio of the density of a substance to the density of some standard substance at a specified temperature (usually water at 4°C, for which $\rho_{H_2O} = 1000\ \text{kg/m}^3$). That is,

Specific gravity: $$SG = \frac{\rho}{\rho_{H_2O}}$$

The weight of a unit volume of a substance is called specific weight and is expressed as

Specific weight: $$\gamma_s = \rho g \qquad (\text{N/m}^3)$$

where *g* is the gravitational acceleration.

Specific gravities of some substances at 0°C

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013

Density of Ideal Gases

Any equation that relates the pressure, temperature, and density (or specific volume) of a substance is called an equation of state. The simplest and best-known equation of state for substances in the gas phase is the ideal-gas equation of state, expressed as

$$Pv = RT \quad \text{or} \quad P = \rho RT$$

where P is the absolute pressure, v is the specific volume, T is the thermodynamic (absolute) temperature, ρ is the density, and *R* is the gas constant. The gas constant *R* is different for each gas and is determined from $R = R_u /M$, where R_u is the universal gas constant whose value is Ru = 8.314 kJ/kmol·K = 1.986 Btu/lbmol·R, and M is the molar mass (also called molecular weight) of the gas.

The thermodynamic temperature scale in the SI is the Kelvin scale, and the temperature unit on this scale is the Kelvin, designated by K. In the English system, it is the Rankin scale, and the temperature unit on this scale is the Rankin, R. Various temperature scales are related to each other by

$$T(\text{K}) = T(^\circ\text{C}) + 273.15$$
$$T(\text{R}) = T(^\circ\text{F}) + 459.67$$

ideal-gas equation of state, or simply the ideal-gas relation, and a gas that obeys this relation is called an ideal gas. For an ideal gas of volume V, mass m, and number of moles N= m/M, the ideal-gas equation of state can also be written as PV = mRT or $PV = NR_uT$. For a fixed mass m, writing the ideal-gas relation twice and simplifying, the properties of an ideal gas at two different states are related to each other by $P_1V_1/T_1 = P_2V_2/T_2$. An ideal gas is a hypothetical substance that obeys the relation Pv = RT. It has been experimentally observed that the ideal-gas relation closely approximates the P-v-T behavior of real gases at low densities. At low pressures and high temperatures, the density of a gas decreases and the gas behaves like an ideal gas.

Ex1 Determine the density, specific gravity, and mass of the air in a room whose dimensions are 4 m *5 m * 6 m at 100 kPa and 25°C (Fig.)

Solution The density, specific gravity, and mass of the air in a room are to be determined.

Assumptions At specified conditions, air can be treated as an ideal gas.

Properties The gas constant of air is $R = 0.287$ kPa · m³/kg · K.

Analysis The density of air is determined from the ideal-gas relation $P = \rho RT$ to be

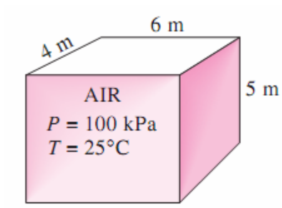
$$\rho = \frac{P}{RT} = \frac{100 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(25 + 273) \text{ K}} = \mathbf{1.17 \text{ kg/m}^3}$$

Then the specific gravity of air becomes

$$\text{SG} = \frac{\rho}{\rho_{H_2O}} = \frac{1.17 \text{ kg/m}^3}{1000 \text{ kg/m}^3} = \mathbf{0.00117}$$

Finally, the volume and the mass of air in the room are

$$V = (4 \text{ m})(5 \text{ m})(6 \text{ m}) = 120 \text{ m}^3$$
$$m = \rho V = (1.17 \text{ kg/m}^3)(120 \text{ m}^3) = \mathbf{140 \text{ kg}}$$



COEFFICIENT OF COMPRESSIBILITY

We know from experience that the volume (or density) of a fluid changes with a change in its temperature or pressure. Fluids usually expand as they are heated or depressurized and contract as they are cooled or pressurized. But the amount of volume change is different for different fluids, and we need to define properties that relate volume changes to the changes in pressure and temperature. Two such properties are the bulk modulus of elasticity ***k*** and the coefficient of volume expansion β

It is a common observation that a fluid contracts when more pressure is applied on it and expands when the pressure acting on it is reduced (Fig.2). That is, fluids act like elastic solids with respect to pressure. Therefore, in an analogous manner to Young's modulus of elasticity for solids, it is appropriate to define a coefficient of compressibility ***k*** (also called the bulk modulus of compressibility or bulk modulus of elasticity) for fluids as

$$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T \qquad \text{(Pa)}$$

It can also be expressed approximately in terms of finite changes as

$$\kappa \cong -\frac{\Delta P}{\Delta v/v} \cong \frac{\Delta P}{\Delta \rho/\rho} \qquad (T = \text{constant})$$

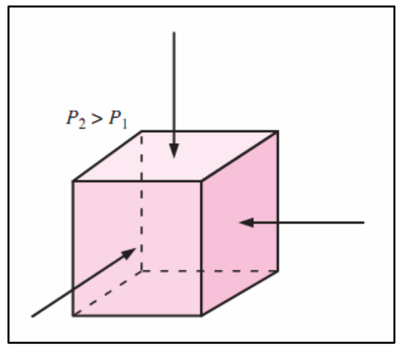


FIGURE 2
Fluids, like solids, compress when the applied pressure is increased from P_1 to P_2

Noting that $\Delta v/v$ or $\Delta\rho/\rho$ is dimensionless, k must have the dimension of pressure (Pa or psi). Also, the coefficient of compressibility represents the change in pressure corresponding to a fractional change in volume or density of the fluid while the temperature remains constant. Then it follows that the coefficient of compressibility of a truly incompressible substance (v = constant) is infinity. Also, differentiating $\rho = 1/v$ gives $d\rho = -dv/v^2$,which can be rearranged as

$$\frac{d\rho}{\rho} = -\frac{dv}{v}$$

That is, the fractional changes in the specific volume and the density of a fluid ***are equal in magnitude but opposite in sign***.

For an ideal gas, $P = \rho RT$ and $(\partial P/\partial \rho)_T = RT = P/\rho$, and thus

$$\kappa_{\text{ideal gas}} = P \qquad \text{(Pa)}$$

Therefore, the coefficient of compressibility of an ideal gas is equal to its absolute pressure, and the coefficient of compressibility of the gas increases with increasing pressure. Substituting k = P into the definition of the coefficient of compressibility and rearranging gives

$$\textit{Ideal gas:} \qquad \frac{\Delta\rho}{\rho} = \frac{\Delta P}{P} \qquad (T = \text{constant})$$

Therefore, the percent increase of density of an ideal gas during isothermal compression is equal to the percent increase in pressure.

For air at 1 atm pressure, $\kappa = P = 1$ atm and a decrease of 1 percent in volume ($\Delta V/V = -0.01$) corresponds to an increase of $\Delta P = 0.01$ atm in pressure. But for air at 1000 atm, $\kappa = 1000$ atm and a decrease of 1 percent in volume corresponds to an increase of $\Delta P = 10$ atm in pressure. Therefore, a small fractional change in the volume of a gas can cause a large change in pressure at very high pressures.

The inverse of the coefficient of compressibility is called the isothermal compressibility α and is expressed as

$$\alpha = \frac{1}{\kappa} = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T \qquad (1/\text{Pa})$$

The isothermal compressibility of a fluid represents the fractional change in volume or density corresponding to a unit change in pressure.

Coefficient of Volume Expansion

The density of a fluid, in general, depends more strongly on temperature than it does on pressure, and the variation of density with temperature is responsible for numerous natural phenomena such as winds, currents in oceans, rise of plumes in chimneys, the operation of hot-air balloons, heat transfer by natural convection, and even the rise of hot air and thus the phrase “heat rises”. The property that provides that information is the coefficient of volume expansion (or volume expansively) β, defined as (Fig.3).

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (1/\text{K})$$

It can also be expressed approximately in terms of finite changes as

$$\beta \approx \frac{\Delta v/v}{\Delta T} = -\frac{\Delta \rho/\rho}{\Delta T} \qquad (\text{at constant } P)$$

A large value of β for a fluid means a large change in density with temperature, and the product $\beta\ \Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change of ΔT at constant pressure. It can be shown easily that the volume expansion coefficient of an ideal gas ($P = \rho RT$) at a temperature, T is equivalent to the inverse of the temperature:

$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\text{K})$$

where T is the absolute temperature.

In the study of natural convection currents, the condition of the main fluid body that surrounds the finite hot or cold regions is indicated by the subscript "infinity" to serve as a reminder that this is the value at a distance where the presence of the hot or cold region is not felt. In such cases, the volume expansion coefficient can be expressed approximately as

$$\beta \approx -\frac{(\rho_\infty - \rho)/\rho}{T_\infty - T} \qquad \text{or} \qquad \rho_\infty - \rho = \rho\beta(T - T_\infty)$$

The combined effects of pressure and temperature changes on the volume change of a fluid can be determined by taking the specific volume to be a function of T and P. Differentiating $v = v(T, P)$ and using the definitions of the compression and expansion coefficients α and β give

$$dv = \left(\frac{\partial v}{\partial T}\right)_P dT + \left(\frac{\partial v}{\partial P}\right)_T dP = (\beta\, dT - \alpha\, dP)v$$

Then the fractional change in volume (or density) due to changes in pressure and temperature can be expressed approximately as

$$\frac{\Delta v}{v} = -\frac{\Delta\rho}{\rho} \cong \beta\,\Delta T - \alpha\,\Delta P$$

Ex.2

Consider water initially at 20°C and 1 atm. Determine the final density of water (*a*) if it is heated to 50°C at a constant pressure of 1 atm, and (*b*) if it is compressed to 100-atm pressure at a constant temperature of 20°C. Take the isothermal compressibility of water to be $\alpha = 4.80 \times 10^{-5}\ \text{atm}^{-1}$.

Properties The density of water at 20°C and 1 atm pressure is $\rho_1 = 998.0\ \text{kg/m}^3$. The coefficient of volume expansion at the average temperature of (20 + 50)/2 = 35°C is $\beta = 0.337 \times 10^{-3}\ \text{K}^{-1}$. The isothermal compressibility of water is given to be $\alpha = 4.80 \times 10^{-5}\ \text{atm}^{-1}$.

Analysis When differential quantities are replaced by differences and the properties α and β are assumed to be constant, the change in density in terms of the changes in pressure and temperature is expressed approximately as

$$\Delta\rho = \alpha\rho\,\Delta P - \beta\rho\,\Delta T$$

(*a*) The change in density due to the change of temperature from 20°C to 50°C at constant pressure is

$$\Delta\rho = -\beta\rho\,\Delta T = -(0.337 \times 10^{-3}\ \text{K}^{-1})(998\ \text{kg/m}^3)(50 - 20)\ \text{K}$$
$$= -10.0\ \text{kg/m}^3$$

Noting that $\Delta\rho = \rho_2 - \rho_1$, the density of water at 50°C and 1 atm is

$$\rho_2 = \rho_1 + \Delta\rho = 998.0 + (-10.0) = \mathbf{988.0\ kg/m^3}$$

which is almost identical to the listed value of 988.1 kg/m^3 at 50°C in Table A–3. This is mostly due to β varying with temperature almost linearly,

(*b*) The change in density due to a change of pressure from 1 atm to 100 atm at constant temperature is

$$\Delta\rho = \alpha\rho\,\Delta P = (4.80 \times 10^{-5}\ \text{atm}^{-1})(998\ \text{kg/m}^3)(100 - 1)\ \text{atm} = 4.7\ \text{kg/m}^3$$

Then the density of water at 100 atm and 20°C becomes

$$\rho_2 = \rho_1 + \Delta\rho = 998.0 + 4.7 = \mathbf{1002.7\ kg/m^3}$$

VISCOSITY

When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion. To move a table on the floor, for example, we have to apply a force to the table in the horizontal direction large enough to overcome the friction force. The magnitude of the force needed to move the table depends on the friction coefficient between the table and the floor. Moving in oil would be even more difficult, as can be observed by the slower downward motion of a glass ball dropped in a tube filled with oil. It appears that there is a property that represents the internal resistance of a fluid to motion or the "fluidity," and that property is the viscosity. The force a flowing fluid exerts on a body in the flow direction is called the drag force, and the magnitude of this force depends, in part, on viscosity (Fig. 3).

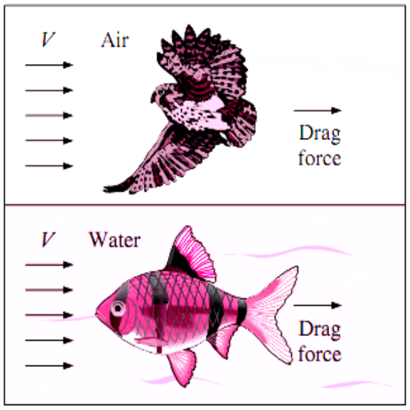
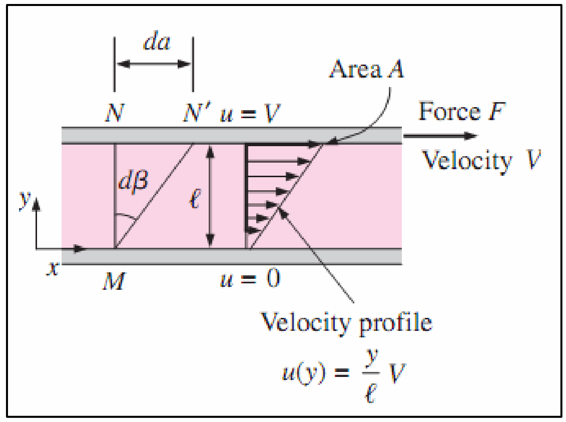


FIG.3
A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity.

To obtain a relation for viscosity, consider a fluid layer between two very large parallel plates (or equivalently, two parallel plates immersed in a large body of a fluid) separated by a distance ℓ (Fig. 4). Now a constant parallel force F is applied to the upper plate while the lower plate is held fixed. After the initial transients, it is observed that the upper plate moves continuously under the influence of this force at a constant velocity V.

FIGURE 4
The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity



The fluid in contact with the upper plate sticks to the plate surface and moves with it at the same velocity, and the shear stress τ acting on this fluid layer is

$$\tau = \frac{F}{A}$$

The fluid in contact with the lower plate assumes the velocity of that plate, which is zero (again because of the no-slip condition). In steady laminar flow, the fluid velocity between the plates varies linearly between 0 and V, and thus the velocity profile and the velocity gradient are

$$u(y) = \frac{y}{\ell}V \qquad \text{and} \qquad \frac{du}{dy} = \frac{V}{\ell}$$

where y is the vertical distance from the lower plate.

During a differential time interval dt, the sides of fluid particles along a vertical line MN rotate through a differential angle $d\beta$ while the upper plate moves a differential distance $da = V\,dt$. The angular displacement or deformation (or shear strain) can be expressed as

$$d\beta \approx \tan\beta = \frac{da}{\ell} = \frac{V\,dt}{\ell} = \frac{du}{dy}dt$$

Rearranging, the rate of deformation under the influence of shear stress τ becomes

$$\frac{d\beta}{dt} = \frac{du}{dy}$$

Thus we conclude that the rate of deformation of a fluid element is equivalent to the velocity gradient du/dy. Further, it can be verified experimentally that for most fluids the rate of deformation (and thus the velocity gradient) is directly proportional to the shear stress τ,

$$\tau \propto \frac{d\beta}{dt} \qquad \text{or} \qquad \tau \propto \frac{du}{dy}$$

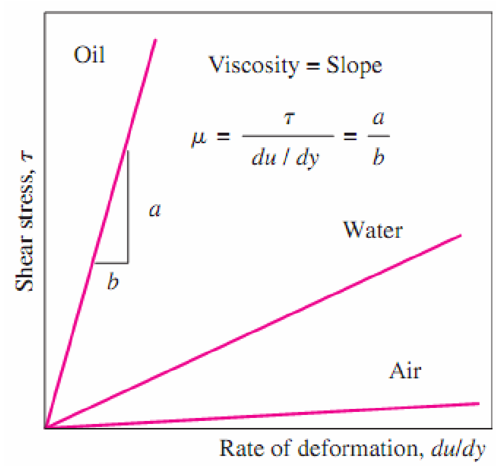
Fluids for which the rate of deformation is proportional to the shear stress are called Newtonian fluids after Sir Isaac Newton, who expressed it first in 1687. Most common fluids such as water, air, gasoline, and oils are Newtonian fluids. Blood and liquid plastics are examples of non-Newtonian fluids. In one-dimensional shear flow of Newtonian fluids, shear stress can be expressed by the linear relationship

Shear stress: $$\tau = \mu \frac{du}{dy} \qquad (\text{N/m}^2)$$

where the constant of proportionality μ is called the coefficient of viscosity or the absolute viscosity of the fluid, whose unit is kg/m · s,

A plot of shear stress versus the rate of deformation (velocity gradient) for a Newtonian fluid is a straight line whose slope is the viscosity of the fluid, as shown in Fig. 5. Note that viscosity is independent of the rate of deformation.

FIGURE 5
The rate of deformation (velocity gradient) of a Newtonian fluid is proportional to shear stress, and the constant of proportionality is the viscosity.



The shear force acting on a Newtonian fluid layer (or, by Newton's third law, the force acting on the plate) is

Shear force: $$F = \tau A = \mu A \frac{du}{dy} \qquad (\text{N})$$

where again A is the contact area between the plate and the fluid. Then the force F required to move the upper plate in Fig. 4 at a constant velocity of V while the lower plate remains stationary is

$$F = \mu A \frac{V}{\ell} \qquad (\text{N})$$

For non-Newtonian fluids, the relationship between shear stress and rate of deformation is not linear, as shown in Fig. 6. The slope of the curve on the τ versus du/dy chart is referred to as the apparent viscosity of the fluid. Fluids for which the apparent viscosity increases with the rate of deformation (such as solutions with suspended starch or sand) are referred to as dilatants or shear thickening fluids, and those that exhibit the opposite behavior (the fluid becoming less viscous as it is sheared harder, such as some paints, polymer solutions, and fluids with suspended particles) are referred to as pseudoplastic or shear thinning fluids. Some materials such as toothpaste can resist a finite shear stress and thus behave as a solid, but deform continuously when the shear stress exceeds the yield stress and thus behave as a fluid. Such materials are referred to as Bingham plastics after E. C. Bingham, who did pioneering work on fluid viscosity for the U.S. National Bureau of Standards in the early twentieth century.

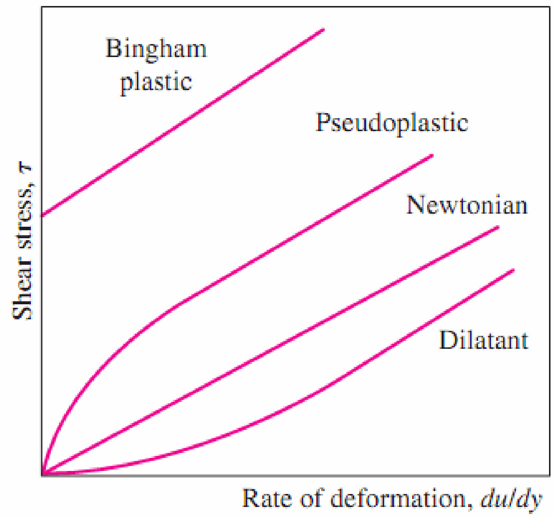


FIGURE 6
Variation of shear stress with the rate of deformation for Newtonian and non-Newtonian fluids (the slope of a curve at a point is the apparent viscosity of the fluid at that point)

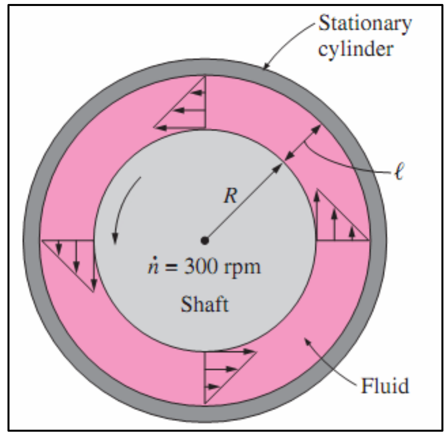
In fluid mechanics and heat transfer, the ratio of dynamic viscosity to density appears frequently. For convenience, this ratio is given the name kinematic viscosity ν and is expressed as $\nu = \mu/\rho$.

Consider a fluid layer of thickness ℓ within a small gap between two concentric cylinders, such as the thin layer of oil in a journal bearing. The gap
between the cylinders can be modeled as two parallel flat plates separated by a fluid. Noting that torque is $\boldsymbol{T = FR}$ (force times the moment arm, which is the radius R of the inner cylinder in this case), the tangential velocity is $V = \omega R$ (angular velocity times the radius), and taking the wetted surface area of the inner cylinder to be $A = 2\pi RL$ by disregarding the shear stress acting on the two ends of the inner cylinder, torque can be expressed as

$$\mathrm{T} = FR = \mu\frac{2\pi R^3\omega L}{\ell} = \mu\frac{4\pi^2 R^3\dot{n}L}{\ell}$$

where L is the length of the cylinder and $\dot{n}$ is the number of revolutions per unit time, which is usually expressed in rpm (revolutions per minute). Note that the angular distance traveled during one rotation is 2π rad, and thus the relation between the angular velocity in rad/min and the rpm is $\omega = 2\pi\dot{n}$.

Ex.3 The viscosity of a fluid is to be measured by a viscometer constructed of two 40-cm-long concentric cylinders (Fig.). The outer diameter of the inner cylinder is 12 cm, and the gap between the two cylinders is 0.15 cm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be 1.8 N . m. Determine the viscosity of the fluid.



SOLUTION The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

$$\mu = \frac{T\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(1.8\ \text{N}\cdot\text{m})(0.0015\ \text{m})}{4\pi^2(0.06\ \text{m})^3(300/60\ 1/\text{s})(0.4\ \text{m})} = \mathbf{0.158\ N \cdot s/m^2}$$

SURFACE TENSION AND CAPILLARY EFFECT

The magnitude of this force per unit length is called surface tension σ_S and is usually expressed in the unit N/m (or lbf /ft in English units). This effect is also called surface energy and is expressed in the equivalent unit of N . m/m^2 or J/m^2. In this case, σ_S represents the stretching work that needs to be done to increase the surface area of the liquid by a unit amount. Consider a liquid film (such as the film of a soap bubble) suspended on a U-shaped wire frame with a movable side (Fig.7). Normally, the liquid film tends to pull the movable wire inward in order to minimize its surface area. A force F needs to be applied on the movable wire in the opposite direction to balance this pulling effect. The thin film in the device has two surfaces (the top and bottom surfaces) exposed to air, and thus the length along which the tension acts in this case is *2b*. Then a force balance on the movable wire gives $F = 2\,b\,\sigma_S$, and thus the surface tension can be expressed as

$$\sigma_s = \frac{F}{2b}$$

Note that for $b = 0.5\ m$, the force F measured (in N) is simply the surface tension in N/m. An apparatus of this kind with sufficient precision can be used to measure the surface tension of various fluids. In the U-shaped wire, the force F remains constant as the movable wire is pulled to stretch the film and increase its surface area. When the movable wire is pulled a distance Δx, the surface area increases by $\Delta A = 2b\ \Delta x$, and the work done W during this stretching process is

$$W = \text{Force} \times \text{Distance} = F\,\Delta x = 2b\sigma_s\,\Delta x = \sigma_s\,\Delta A$$

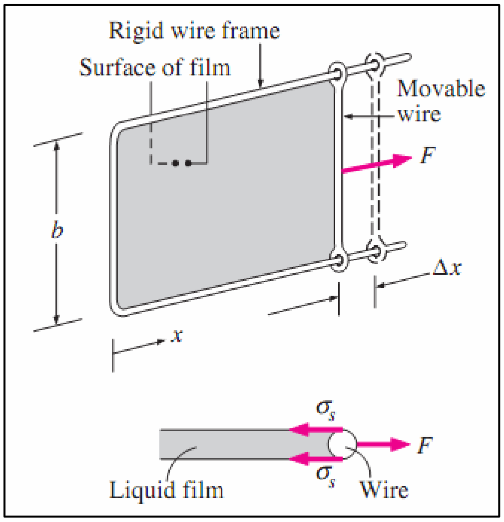


FIGURE 7
Stretching a liquid film with a U-shaped wire, and the forces acting on the movable wire of length b.

Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

Fluid	Surface Tension σ_s, N/m*
Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028

* Multiply by 0.06852 to convert to lbf/ft.

A curved interface indicates a pressure difference (or "pressure jump") across the interface with pressure being higher on the concave side. The excess pressure ΔP inside a droplet or bubble above the atmospheric pressure, for example, can be determined by considering the free-body diagram of half a droplet or bubble (Fig. 8). Noting that surface tension acts along the circumference and the pressure acts on the area, horizontal force balances for the droplet and the bubble give

Droplet: $(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{droplet}} \rightarrow \Delta P_{\text{droplet}} = P_i - P_o = \dfrac{2\sigma_s}{R}$

Bubble: $2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{bubble}} \rightarrow \Delta P_{\text{bubble}} = P_i - P_o = \dfrac{4\sigma_s}{R}$

where P_i and P_o are the pressures inside and outside the droplet or bubble, respectively. When the droplet or bubble is in the atmosphere, P_o is simply atmospheric pressure. The factor ***2*** in the force balance for the bubble is due to the bubble consisting of a film with two surfaces (inner and outer surfaces) and thus two circumferences in the cross section.

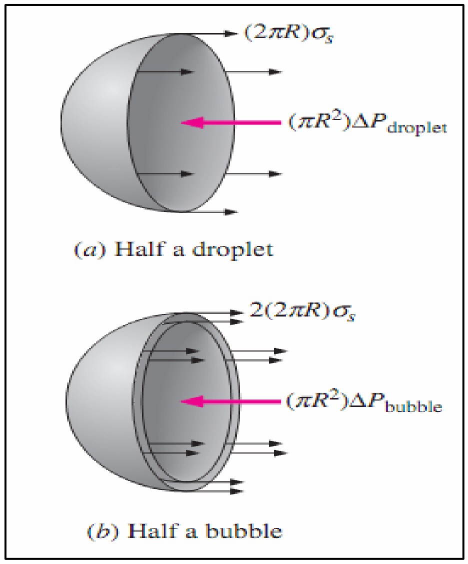


FIG.8
The free-body diagram of half a droplet and half a bubble.

The excess pressure in a droplet (or bubble) also can be determined by considering a differential increase in the radius of the droplet due to the addition of a differential amount of mass and interpreting the surface tension as the increase in the surface energy per unit area. Then the increase in the surface energy of the droplet during this differential expansion process becomes

$$\delta W_{\text{surface}} = \sigma_s\, dA = \sigma_s\, d(4\pi R^2) = 8\pi R \sigma_s\, dR$$

The expansion work done during this differential process is determined by multiplying the force by distance to obtain

$$\delta W_{\text{expansion}} = \text{Force} \times \text{Distance} = F\, dR = (\Delta P A)\, dR = 4\pi R^2\, \Delta P\, dR$$

Equating the two expressions above gives $\Delta P_{\text{droplet}} = 2\sigma_s / R$

Capillary Effect

Another interesting consequence of surface tension is the capillary effect, which is the rise or fall of a liquid in a small-diameter tube inserted into the liquid.

The relative magnitudes of these forces determine whether a liquid wets a solid surface or not. Obviously, the water molecules are more strongly attracted to the glass molecules than they are to other water molecules, and thus water tends to rise along the glass surface. The opposite occurs for mercury, which causes the liquid surface near the glass wall to be suppressed (Fig.9). The phenomenon of capillary effect can be explained microscopically by considering ***cohesive forces*** (the forces between like molecules, such as water and water) and ***adhesive forces*** (the forces between unlike molecules, such as water and glass).

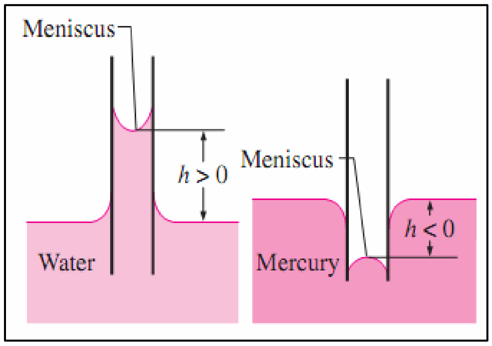


FIG.9
The capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.

The magnitude of the capillary rise in a circular tube can be determined from a force balance on the cylindrical liquid column of height h in the tube (Fig.10). The bottom of the liquid column is at the same level as the free surface of the reservoir, and thus the pressure there must be atmospheric pressure. This balances the atmospheric pressure acting at the top surface, and thus these two effects cancel each other. The weight of the liquid column is approximately

$$W = mg = \rho V g = \rho g(\pi R^2 h)$$

Equating the vertical component of the surface tension force to the weight gives

$$W = F_{\text{surface}} \rightarrow \rho g(\pi R^2 h) = 2\pi R \sigma_s \cos\phi$$

Solving for h gives the capillary rise to be

Capillary rise: $$h = \frac{2\sigma_s}{\rho g R}\cos\phi \qquad (R = \text{constant})$$

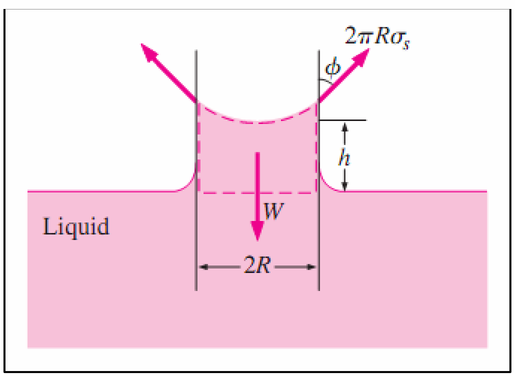


FIGURE 10
The forces acting on a liquid column that has risen in a tube due to the capillary effect.

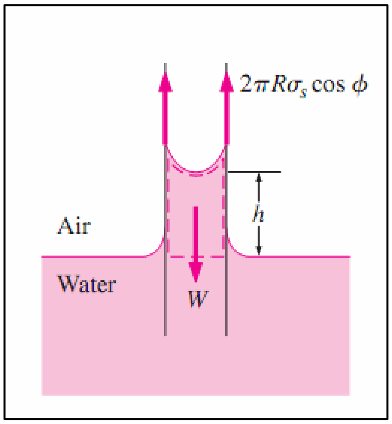
This relation is also valid for non wetting liquids (such as mercury in glass) and gives the capillary drop. In this case $\phi > 90°$ and thus $\cos\phi < 0,$ which makes ***h*** negative. Therefore, a negative value of capillary rise corresponds to a capillary drop (Fig. 9). For Clear Glass Tube is $\phi = 0^o$.

Ex.4 A 0.6-mm-diameter glass tube is inserted into water at 20°C in a cup. Determine the capillary rise of water in the tube (Fig.)

Properties The surface tension of water at 20°C is 0.073 N/m (Table 2–3). The contact angle of water with glass is 0° (from preceding text). We take the density of liquid water to be 1000 kg/m³.

$$h = \frac{2\sigma_s}{\rho g R}\cos\phi = \frac{2(0.073\ \text{N/m})}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.3\times10^{-3}\text{m})}(\cos 0°)\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right)$$

$$= 0.050\ \text{m} = \mathbf{5.0\ cm}$$



Chapter Two

Fluid Statics

PRESSURE

Pressure is defined as a normal force exerted by a fluid per unit area. We speak of pressure only when we deal with a gas or a liquid.
The pressure unit pascal is too small for pressures encountered in practice. Therefore, its multiples kilopascal (1 kPa = 10^3 Pa) and megapascaln(1 MPa = 10^6 Pa) are commonly used.
Three other pressure units commonly used in practice, especially in Europe, are bar, standard atmosphere, and kilogram-force per square centimeter:

$$1\text{ bar} = 10^5\text{ Pa} = 0.1\text{ MPa} = 100\text{ kPa}$$

$$1\text{ atm} = 101{,}325\text{ Pa} = 101.325\text{ kPa} = 1.01325\text{ bars}$$

$$\begin{aligned} 1\text{ kgf/cm}^2 &= 9.807\text{ N/cm}^2 = 9.807 \times 10^4\text{ N/m}^2 = 9.807 \times 10^4\text{ Pa} \\ &= 0.9807\text{ bar} \\ &= 0.9679\text{ atm} \end{aligned}$$

The actual pressure at a given position is called the ***absolute pressure***, and it is measured relative to absolute vacuum (i.e., absolute zero pressure). Most pressure-measuring devices, however, are calibrated to read zero in the atmosphere (Fig.1), and so they indicate the difference between the absolute pressure and the local atmospheric pressure. This difference is called the ***gage pressure***. Pressures below atmospheric pressure are called ***vacuum pressures*** and are measured by vacuum gages that indicate the difference between the atmospheric pressure and the absolute pressure. Absolute, gage, and vacuum pressures are all positive quantities and are relate to each other by

$$P_{gage} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$

FIG.1
Some basic pressure gages.

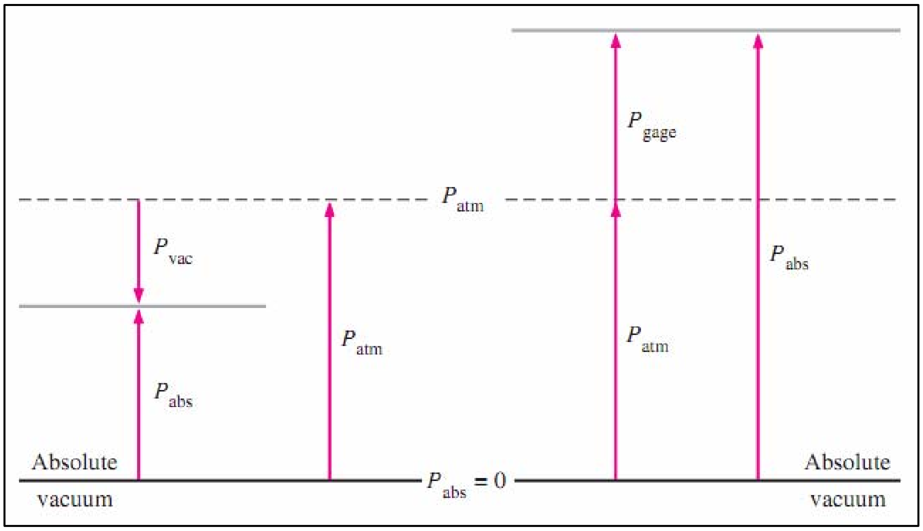


FIG.2 Absolute, gage, and vacuum pressures.

This is illustrated in Fig.2.
Like other pressure gages, the gage used to measure the air pressure in an automobile tire reads the gage pressure. Therefore, the common reading of 32 psi (2.25 kgf/cm^2) indicates a pressure of 32 psi above the atmospheric pressure. At a location where the atmospheric pressure is 14.3 psi, for example, the absolute pressure in the tire is 32 + 14.3 = 46.3 psi.
In thermodynamic relations and tables, absolute pressure is almost always used. Throughout this text, the pressure P will denote absolute pressure unless specified otherwise. Often the letters "a" (for absolute pressure) and "g" (for gage pressure) are added to pressure units (such as psia and psig) to clarify what is meant.
Ex.1 A vacuum gage connected to a chamber reads 5.8 psi at a location where the atmospheric pressure is 14.5 psi. Determine the absolute pressure in the chamber

SOLUTION The gage pressure of a vacuum chamber is given. The absolute pressure in the chamber is to be determined.
Analysis The absolute pressure is easily determined from

$$P_{abs} = P_{atm} - P_{vac} = 14.5 - 5.8 = 8.7 \text{ psi}$$

PASCAL'S LAW FOR PRESSURE AT A POINT

By considering the equilibrium of a small fluid element in the form of a triangular prism surrounding a point in the fluid (Fig.3), a relationship can be established between the pressures ***px*** in the ***x*** direction, ***py*** in the ***y*** direction and ***ps*** normal to any plane inclined at any angle ***θ*** to the horizontal at this point.

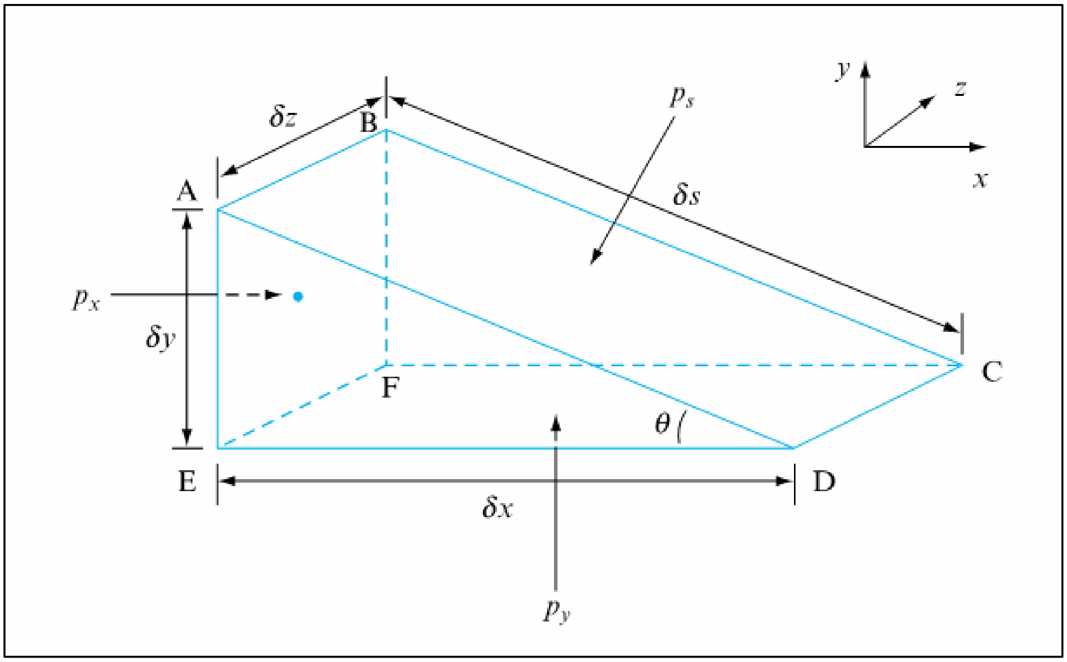


FIG.3 Equality of pressure in all directions at a point

If the fluid is at rest, ***px*** will act at right angles to the plane ***ABFE***, ***py*** at right angles to ***CDEF*** and ***ps*** at right angles to ***ABCD***. Since the fluid is at rest, there will be no shearing forces on the faces of the element and the element will not be accelerating. The sum of the forces in any direction must, therefore, be zero.

Considering the x direction

$$\text{Force due to } p_x = p_x \times \text{Area ABFE} = p_x \delta y \delta z;$$

$$\text{Component of force due to } p_s = -(p_s \times \text{Area ABCD}) \sin\theta$$

$$= -p_s \delta s \delta z \frac{\delta y}{\delta s} = -p_s \delta y \delta z$$

(since $\sin\theta = \delta y/\delta s$). As p_y has no compound in the x direction, the element will be in equilibrium if

$$p_x \delta y \delta z + (-p_s \delta y \delta z) = 0,$$

$$p_x = p_s. \qquad 1$$

Similarly, in the y direction,

$$\text{Force due to } p_y = p_y \times \text{Area CDEF} = p_y \delta x \delta z;$$

$$\text{Component of force due to } p_s = -(p_s \times \text{Area ABCD}) \cos\theta$$

$$= -p_s \delta s \delta z \frac{\delta x}{\delta s} = -p_s \delta x \delta z$$

(since $\cos\theta = \delta x/\delta s$).

$$\text{Weight of element} = -\text{Specific weight} \times \text{Volume}$$
$$= -\rho g \times \tfrac{1}{2}\,\delta x \delta y \delta z.$$

As p_x has no component in the y direction, the element will be in equilibrium if

$$p_y \delta x \delta z + (-p_s \delta x \delta z) + (-\rho g \times \tfrac{1}{2}\,\delta x \delta y \delta z) = 0.$$

Since δx, δy and δz are all very small quantities, $\delta x \delta y \delta z$ is negligible in comparison with the other two terms, and the equation reduces to

$$p_y = p_s. \qquad 2$$

Thus, from equations (1) and (2),

$$p_s = p_x = p_y.$$

VARIATION OF PRESSURE VERTICALLY IN A FLUID UNDER GRAVITY

Fig.4 shows an element of fluid consisting of a vertical column of constant cross-sectional area A and totally surrounded by the same fluid of mass density ρ. Suppose that the pressure is p_1 on the underside at level z_1 and p_2 on the top at level z_2. Since the fluid is at rest the element must be in equilibrium and the sum of all the vertical forces must be zero. The forces acting are:

Force due to p_1 on area A acting up $= p_1 A$,

Force due to p_2 on area A acting down $= p_2 A$,

Force due to the weight of the element $= mg$

$= \text{Mass density} \times g \times \text{Volume} = \rho g A(z_2 - z_1)$.

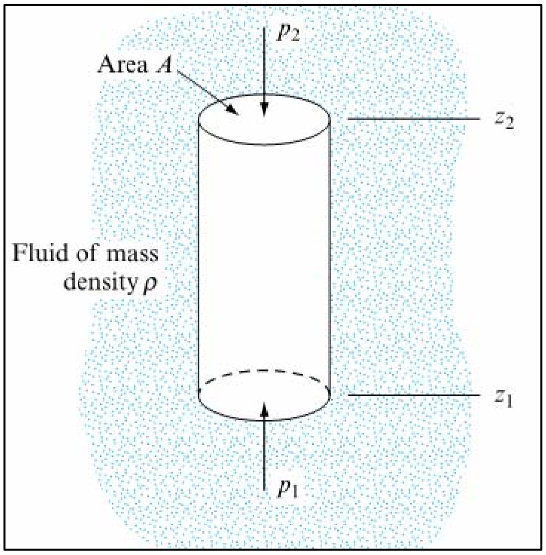


FIGURE 4
Vertical variation of pressure

Since the fluid is at rest, there can be no shear forces and, therefore, no vertical forces act on the side of the element due to the surrounding fluid. Taking upward forces as positive and equating the algebraic sum of the forces acting to zero,

$$p_1 A - p_2 A - \rho g A(z_2 - z_1) = 0,$$

Then

$$p_2 - p_1 = -\rho g(z_2 - z_1).$$

Thus, in any fluid under gravitational attraction, pressure decreases with increase of height ***z***.

EX.2 A diver descends from the surface of the sea to a depth of 30 m. What would be the pressure under which the diver would be working above that at the surface assuming that the density of sea water is 1025 kg m^{-3} and remains constant.

Solution

Taking sea level as datum, $z_1 = 0$. Since z_2 is lower then z_1 the value of z_2 is −30 m. Substituting these values and putting $\rho = 1025$ kg m^{-3}

$$\text{Increase of pressure} = p_2 - p_1$$
$$= -1025 \times 9.81(-30 - 0) = \mathbf{301.7 \times 10^3\ N\ m^{-2}}.$$

EQUALITY OF PRESSURE AT THE SAME LEVEL IN A STATIC FLUID

If P and Q are two points at the same level in a fluid at rest (Fig.5), a horizontal prism of fluid of constant cross-sectional area A will be in equilibrium. The forces acting on this element horizontally are p1A at P and p2A at Q. Since the fluid is at rest, there will be no horizontal shear stresses on the sides of the element. For static equilibrium the sum of the horizontal forces must be zero:

$$p_1 A = p_2 A$$

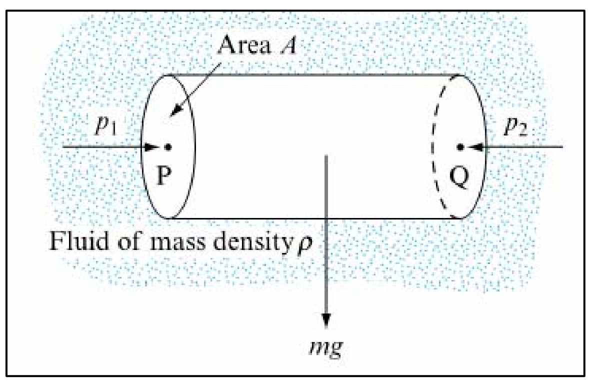


FIG.5
Equality of pressures at the same level

$$p_1 = p_2.$$

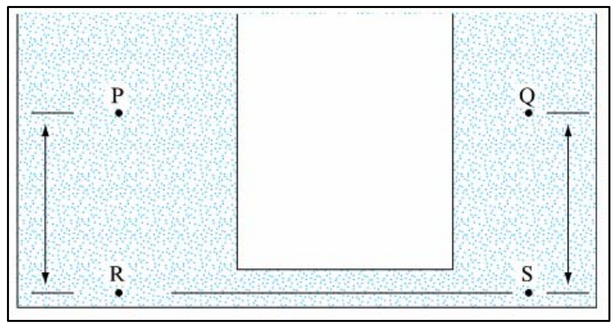
Thus, the pressure at any two points at the same level in a body of fluid at rest will be the same.

In mathematical terms, if (x, y) is the horizontal plane

$$\frac{\partial p}{\partial x} = 0 \quad \text{and} \quad \frac{\partial p}{\partial y} = 0$$

partial derivatives are used because pressure p could vary in three directions. Pressures at the same level will be equal even though there is no direct horizontal path between P and Q, provided that P and Q are in the same continuous body of fluid. Thus, in Fig.6, P and Q are connected by a horizontal pipe, R and S being two points at the same level at the entrance and exit to the pipe. If the pressure is p_P at P, p_Q at Q, p_R at R and p_S at S, then, since R and S are at the same level,

FIG. 6
Equality of pressures in a continuous body of fluid



$$p_R = p_S;$$

also $p_R = p_P + \rho g z$ and $p_S = p_Q + \rho g z$. 1

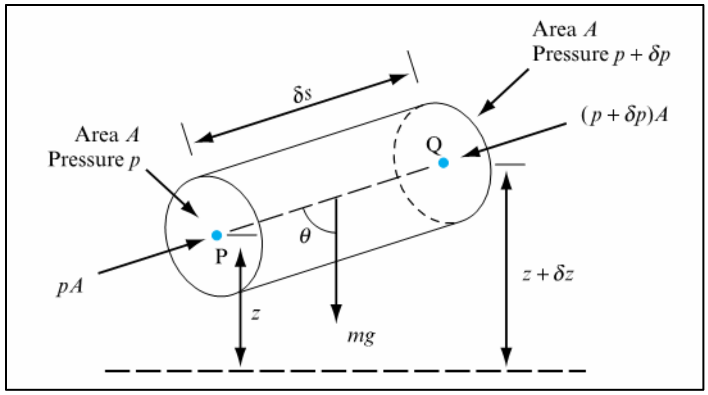
Substituting in equation 1

$$p_P + \rho g z = p_Q + \rho g z, \quad p_P = p_Q.$$

GENERAL EQUATION FOR THE VARIATION OF PRESSURE DUE TO GRAVITY FROM POINT TO POINT IN A STATIC FLUID

Let p be the pressure acting on the end P of an element of fluid of constant cross-sectional area A and $p + \delta p$ be the pressure at the other end Q (Fig.7). The axis of the element is inclined at an angle θ to the vertical, the height of P above a horizontal datum is z and that of Q is $z + \delta z$. The forces acting on the element are:

FIG.7
Variation of pressure in a stationary fluid



pA	acting at right angles to the end face at P along the axis of the element,
$(p + \delta p)A$	acting at Q along the axis in the opposite direction;
mg	the weight of the element, due to gravity, acting vertically down

= Mass density × Volume × Gravitational acceleration
$= \rho \times A\delta s \times g$.

There are also forces due to the surrounding fluid acting normal to the sides of the element, since the fluid is at rest, and, therefore, perpendicular to its axis PQ. For equilibrium of the element PQ, the resultant of these forces in any direction must be zero. Resolving along the axis PQ,

$$pA - (p + \delta p)A - \rho g A\delta s \cos\theta = 0, \quad \delta p = -\rho g \delta s \cos\theta,$$

or, in differential form,

$$\frac{dp}{ds} = -\rho g \cos\theta.$$

In the general three-dimensional case, s is a vector with components in the x, y and z directions. Taking the (x, y) plane as horizontal, if the axis of the element is also horizontal, θ = 90° and

$$\left(\frac{dp}{ds}\right)_{\theta=90^\circ} = \frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$$

If the axis of the element is in the vertical z direction, θ = 0° and

$$\left(\frac{dp}{ds}\right)_{\theta=0^\circ} = \frac{\partial p}{\partial z} = -\rho g$$

and, since ∂p/∂x = ∂p/∂y = 0, the partial derivative ∂p/∂z can be replaced by the total differential dp/dz, giving

$$\frac{dp}{dz} = -\rho g$$

Also, considering any two horizontal planes a vertical distance z apart,

Pressure at all points on lower plane $= p$,

Pressure at all points on upper plane $= p + z\dfrac{\partial p}{\partial z}$,

$$\text{Difference of pressure} = z\frac{\partial p}{\partial z}.$$

Since the planes are horizontal, the pressure must be constant over each plane; therefore, $\partial p/\partial z$ cannot vary horizontally.This implies that ρg shall be constant and, therefore, for equilibrium, the density ρ must be constant over any horizontal plane

Thus, the conditions for equilibrium under gravity are:

1. The pressure at all points on a horizontal plane must be the same.
2. The density at all points on a horizontal plane must be the same.
3. The change of pressure with elevation is given by $\mathrm{d}p/\mathrm{d}z = -\rho g$.

The actual pressure variation with elevation is found by integrating equation

$$\mathrm{d}p = -\int \rho g \, \mathrm{d}z \quad \text{or} \quad p_2 - p_1 = -\int_{z_1}^{z_2} \rho g \, \mathrm{d}z,$$

PRESSURE AND HEAD

In a fluid of constant density, dp/dz = −ρg can be integrated immediately to give

$$p = -\rho g z + \text{constant}$$

In a liquid, the pressure p at any depth z, measured downwards from the free surface so that z = −h (Fig.8), will be $p = \rho g h + \text{constant}$
and, since the pressure at the free surface will normally be atmospheric pressure p_{atm},

$$p = \rho g h + p_{\text{atm}}$$

It is often convenient to take atmospheric pressure as a datum. Pressures measured above atmospheric pressure are known as *gauge pressures*.

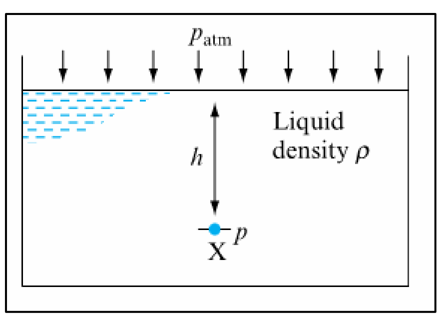


FIG.8
Pressure and head

Since atmospheric pressure varies with atmospheric conditions, a perfect vacuum is taken as the absolute standard of pressure. Pressures measured above perfect vacuum are called absolute pressures:

Absolute pressure = Gauge pressure + Atmospheric pressure

Taking p_{atm} as zero
Then
$p = \rho gh$

EX.3 A cylinder contains a fluid at a gauge pressure of 350 kN m^{-2} . Express this pressure in terms of a head of (a) water (ρH_2O = 1000 kg m^{-3}), (b) mercury (relative density 13.6).
What would be the absolute pressure in the cylinder if the atmospheric pressure is 101.3kNm^{-2}
Solution
From equation , head, $h = p/\rho g$

(a) Putting $p = 350 \times 10^3$ N m^{-2}, $\rho = \rho_{H_2O} = 1000$ kg m^{-3},

$$\text{Equivalent head of water} = \frac{350 \times 10^3}{10^3 \times 9.81} = \mathbf{35.68\ m}$$

(b) For mercury $\rho_{Hg} = \sigma\rho_{H_2O} = 13.6 \times 1000$ kg m^{-3},

$$\text{Equivalent head of water} = \frac{350 \times 10^3}{1.36 \times 10^3 \times 9.81} = \mathbf{2.62\ m},$$

Absolute pressure = Gauge pressure + Atmospheric pressure

$$= 350 + 101.3 = \mathbf{451.3\ kN\ m^{-2}}.$$

PRESSURE MEASUREMENT BY MANOMETER

The relationship between pressure and head is utilized for pressure measurement in the manometer or liquid gauge. The simplest form is the pressure tube or piezometer shown in Fig.1, consisting of a single vertical tube, open at the top, inserted into a pipe or vessel containing liquid under pressure which rises in the tube to a height depending on the pressure. If the top of the tube is open to the atmosphere, the pressure measured is 'gauge' pressure:

Pressure at A = Pressure due to column of liquid of height h_1

$$p_A = \rho g h_1.$$

Similarly,

Pressure at B = $p_B = \rho g h_2$.

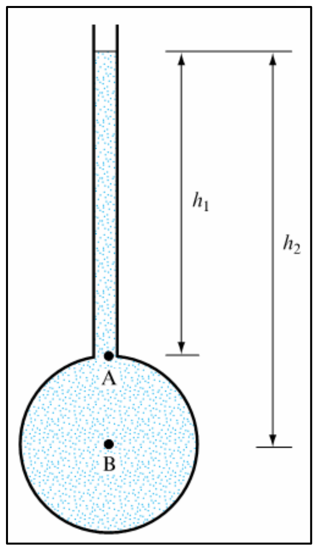


FIG.
Pressure tube or piezometer

EX.1 What is the maximum gauge pressure of water that can be measured by means of a piezometer tube 2 m high? (Mass density of water ρH2O = 10^3kg m^{-3})

Solution

Since p = ρgh for maximum pressure, put $\rho = \rho H_2O = 10^3$ and h = 2 m, giving

Maximum pressure, $p = 10^3 \times 9.81 \times 2 = \mathbf{19.62 \times 10^3\ N\ m^{-2}}$

The U-tube gauge, shown in Fig. 2, can be used to measure the pressure of either liquids or gases. The bottom of the U-tube is filled with a manometric liquid Q which is of greater density ρ_{man} and is immiscible with the fluid P, liquid or gas, of density ρ, whose pressure is to be measured. If B is the level of the interface in the left-hand limb and C is a point at the same level in the right-hand limb

Pressure p_B at B = Pressure p_C at C.

For the left-hand limb,

p_B = Pressure p_A at A + Pressure due to depth h_1 of fluid P

$= p_A + \rho g h_1.$

For the right-hand limb,

p_C = Pressure p_D at D + Pressure due to depth h_2 of liquid Q.

But p_D = Atmospheric pressure = Zero gauge pressure,

and so $p_C = 0 + \rho_{man} g h_2.$

Since $p_B = p_C$,

$$p_A + \rho g h_1 = \rho_{man} g h_2$$

$$p_A = \rho_{man} g h_2 - \rho g h_1$$

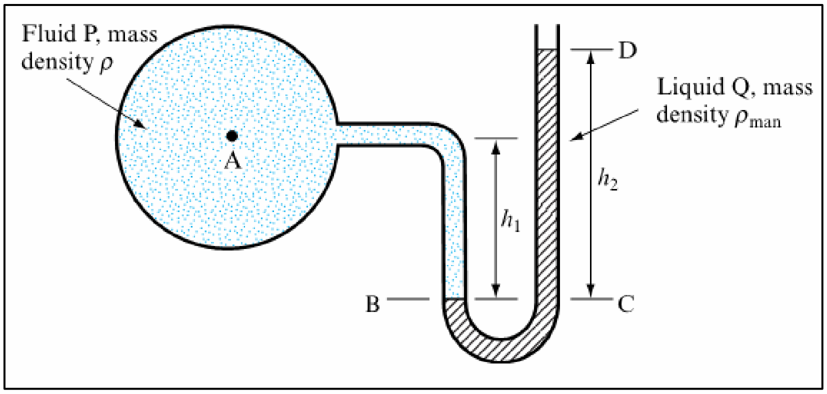


FIG. 2
U-tube manometer

EX.2 A U-tube manometer similar to that shown in Fig. 2. is used to measure the gauge pressure of a fluid P of density $\rho = 800\ kg\ m^{-3}$. If the density of the liquid Q is $13.6 \times 10^3\ kg\ m^{-3}$, what will be the gauge pressure at A if (a) h1 = 0.5 m and D is 0.9 m above BC, (b) h1 = 0.1 m and D is 0.2 m below BC

Solution

(a) $\rho_{man} = 13.6 \times 10^3\ kg\ m^{-3}$, $\rho = 0.8 \times 10^3\ kg\ m^{-3}$, h1 = 0.5 m,
h2 = 0.9 m

$p_A = 13.6 \times 10^3 \times 9.81 \times 0.9 - 0.8 \times 10^3 \times 9.81 \times 0.5$

$= \mathbf{116.15 \times 10^3\ N\ m^{-2}}.$

(b) Putting h1 = 0.1 m and h2 = −0.2 m, since D is below BC:

$$p_A = 13.6 \times 10^3 \times 9.81 \times (-0.2) - 0.8 \times 10^3 \times 9.81 \times 0.1$$
$$= \mathbf{-27.45 \times 10^3\ N\ m^{-2}},$$

the negative sign indicating that p_A is below atmospheric pressure

In Fig. 3, a U-tube gauge is arranged to measure the pressure difference between two points in a pipeline. As in the previous case, the principle involved in calculating the pressure difference is that the pressure at the same level CD in the two limbs must be the same, since the fluid in the bottom of the U-tube is at rest. For the left-hand limb

$$p_C = p_A + \rho g a.$$

For the right-hand limb

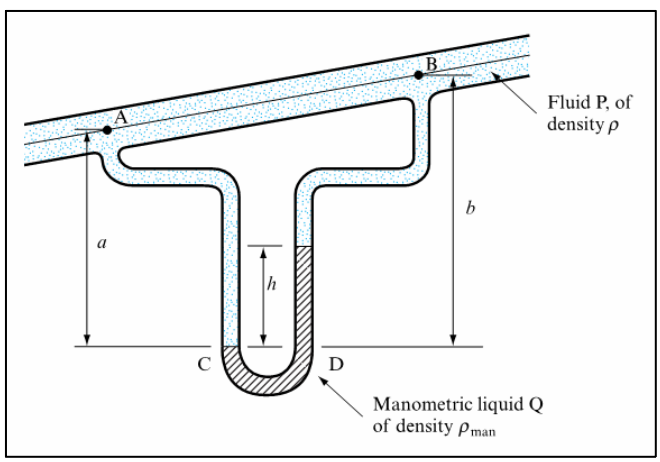
$$p_D = p_B + \rho g(b - h) + \rho_{man} g h.$$

Since $p_C = p_D$,

$$p_A + \rho g a = p_B + \rho g(b - h) + \rho_{man} g h,$$

$$\text{Pressure difference} = p_A - p_B = \rho g(b - a) + hg(\rho_{man} - \rho)$$

FIG.3
Measurement of pressure difference



EX.3 A U-tube manometer is arranged, as shown in Fig.3, to measure the pressure difference between two points A and B in a pipeline conveying water of density $\rho = \rho H_2O = 10^3 kg\ m^{-3}$. The density of the manometric liquid Q is 13.6×10^3 kg m^{-3}, and point B is 0.3 m higher than point A. Calculate the pressure difference when h = 0.7 m.

Solution

$\rho = 10^3$ kg m^{-3} , $\rho_{man} = 13.6 \times 10^3$ kg m^{-3} , (b − a) = 0.3 m and h = 0.7 m.

$$\text{Pressure difference} = p_A - p_B$$
$$= 10^3 \times 9.81 \times 0.3 + 0.7 \times 9.81(13.6 - 1) \times 10^3 \text{ N m}^{-2}$$
$$= \mathbf{89.467 \times 10^3 \text{ N m}^{-2}}.$$

The inverted U-tube shown in Fig.4 is used for measuring pressure differences in liquids. The top of the U-tube is filled with a fluid, frequently air, which is less dense than that connected to the instrument. Since the fluid in the top is at rest, pressures at level XX will be the same in both limbs.

For the left-hand limb,

$$p_{XX} = p_A - \rho g a - \rho_{man} g h.$$

For the right-hand limb,

$$p_{XX} = p_B - \rho g(b + h).$$

Thus $\quad p_B - p_A = \rho g(b - a) + gh(\rho - \rho_{man})$

or, if A and B are at the same level,

$$p_B - p_A = (\rho - \rho_{man})gh.$$

If the top of the tube is filled with air ρ_{man} is negligible compared with ρ and pB − pA = ρgh. On the other hand, if the liquid in the top of the tube is chosen so that ρ_{man} is very nearly equal to ρ, and provided that the liquids do not mix, the result will be a very sensitive manometer giving a large value of h for a small pressure difference.

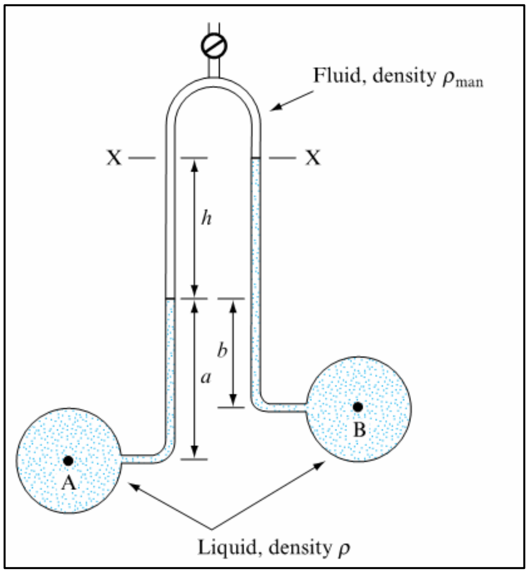


FIG.4
Inverted U-tube manomete

EX.4 An inverted U-tube of the form shown in Fig. 4 is used to measure the pressure difference between two points A and B in an inclined pipeline through which water is flowing ($\rho H2O = 10^3$ kg m^{-3}). The difference of level h = 0.3 m, a = 0.25 m and b = 0.15 m. Calculate the pressure difference pB − pA if the top of the manometer is filled with (a) air, (b) oil of relative density 0.8.

Solution

In either case, the pressure at XX will be the same in both limbs, so that

$$p_{XX} = p_A - \rho g a - \rho_{man} g h = p_B - \rho g(b + h),$$

$$p_B - p_A = \rho g(b - a) + gh(\rho - \rho_{man}).$$

(a) If the top is filled with air ρ_{man} is negligible compared with ρ. Therefore,

$$p_B - p_A = \rho g(b - a) + \rho g h = \rho g(b - a + h).$$

Putting $\rho = \rho_{H_2O} = 10^3$ kg m^{-3}, $b = 0.15$ m, $a = 0.25$ m, $h = 0.3$ m:

$$p_B - p_A = 10^3 \times 9.81(0.15 - 0.25 + 0.3)$$
$$= \mathbf{1.962 \times 10^3\ N\ m^{-2}}.$$

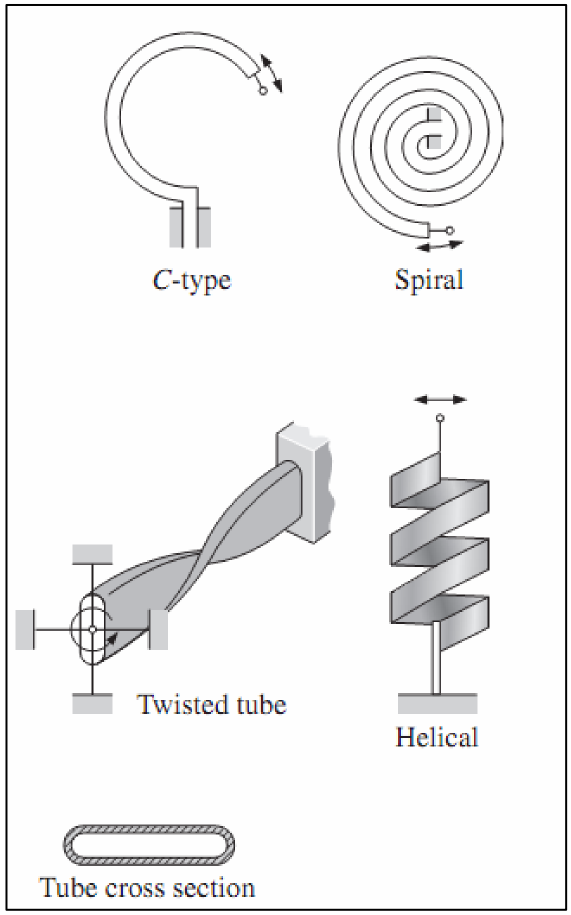
(b) If the top is filled with oil of relative density 0.8, $\rho_{man} = 0.8\rho_{H_2O} = 0.8 \times 10^3$ kg m^{-3}

$$p_B - p_A = \rho g(b - a) + gh(\rho - \rho_{man})$$
$$= 10^3 \times 9.81(0.15 - 0.25) + 9.81 \times 0.3 \times 10^3(1 - 0.8)\ \text{N m}^{-2}$$
$$= 10^3 \times 9.81(-0.1 + 0.06) = \mathbf{-392.4\ N\ m^{-2}}.$$

Other Pressure Measurement Devices

Another type of commonly used mechanical pressure measurement device is the Bourdon tube, named after the French engineer and inventor Eugene Bourdon (1808–1884), which consists of a hollow metal tube bent like a hook whose end is closed and connected to a dial indicator needle (Fig. 5). When the tube is open to the atmosphere, the tube is undeflected, and the needle on the dial at this state is calibrated to read zero (gage pressure). When the fluid inside the tube is pressurized, the tube stretches and moves the needle in proportion to the pressure applied. Electronics have made their way into every aspect of life, including pressure measurement devices. Modern pressure sensors, called pressure transducers, use various techniques to convert the pressure effect to an electrical effect such as a change in voltage, resistance, or capacitance. Pressure transducers are smaller and faster, and they can be more sensitive, reliable, and precise than their mechanical counterparts. They can measure pressures from less than a millionth of 1 atm to several thousands of atm. A wide variety of pressure transducers is available to measure gage, absolute, and differential pressures in a wide range of applications. Gage pressure transducers use the atmospheric pressure as a reference by venting the back side of the pressure-sensing diaphragm to the atmosphere, and they give a zero signal output at atmospheric pressure regardless of altitude. The absolute pressure transducers are calibrated to have a zero signal output at full vacuum. Differential pressure transducers measure the pressure difference between two locations directly instead of using two pressure transducers and taking their difference.

FIG.5
Various types of Bourdon tubes used to measure pressure.



EFFECT OF VERTICAL ACCELERATION

If the acceleration is vertical, the free surface will remain horizontal. Considering a vertical prism of cross-sectional area A (Fig.6), subject to an upward acceleration a, then at depth h below the surface, where the pressure is p,

$$\text{Upward accelerating force, } F = \text{Force due to } p - \text{Weight of prism}$$
$$= pA - \rho g h A.$$

By Newton's second law,

$$F = \text{Mass of prism} \times \text{Acceleration} = \rho h A \times a.$$

Therefore,

$$pA - \rho g h A = \rho h A a,$$
$$p = \rho g h(1 + a/g).$$

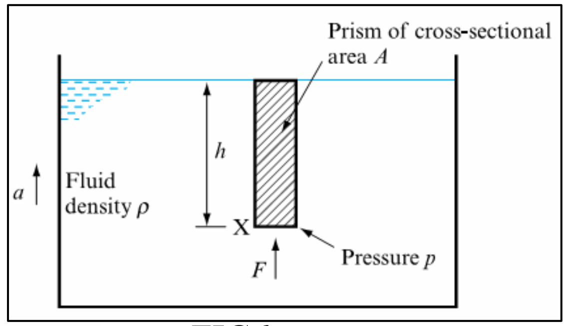


FIG6
Effect of vertical Acceleration

HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES

A plate exposed to a liquid, such as a gate valve in a dam, the wall of a liquid storage tank, or the hull of a ship at rest, is subjected to fluid pressure distributed over its surface. On a plane surface, the hydrostatic forces form a system of parallel forces, and we often need to determine the magnitude of the force and its point of application, which is called the center of pressure. In most cases, the other side of the plate is open to the atmosphere (such as the dry side of a gate), and thus atmospheric pressure acts on both sides of the plate, yielding a zero resultant. In such cases, it is convenient to subtract atmospheric pressure and work with the gage pressure only (Fig.7).

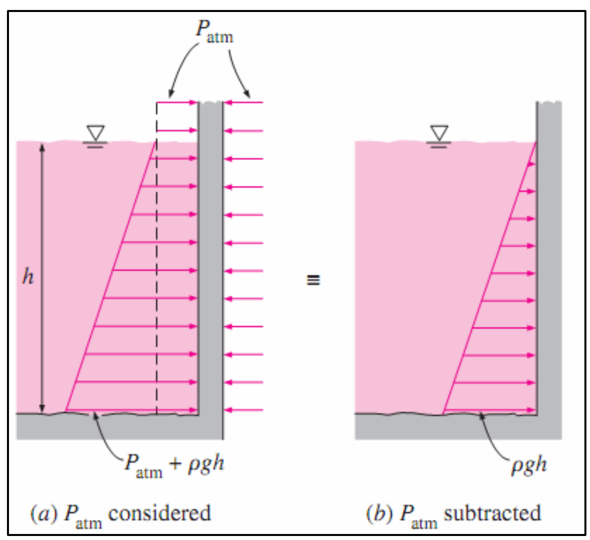


FIG.7
When analyzing hydrostatic forces on submerged surfaces, the atmospheric pressure can be subtracted for simplicity when it acts on both sides of the structure.

Consider the top surface of a flat plate of arbitrary shape completely submerged in a liquid, as shown in Fig.8 together with its top view. The plane of this surface (normal to the page) intersects the horizontal free surface with an angle u,and we take the line of intersection to be the x-axis. The absolute pressure above the liquid is P_0,which is the local atmospheric pressure P_{atm} if the liquid is open to the atmosphere (but P_0 may be different than P_{atm} if the space above the liquid is evacuated or pressurized). Then the absolute pressure at any point on the plate is: $P = P_0 + \rho gh = P_0 + \rho gy \sin\theta$

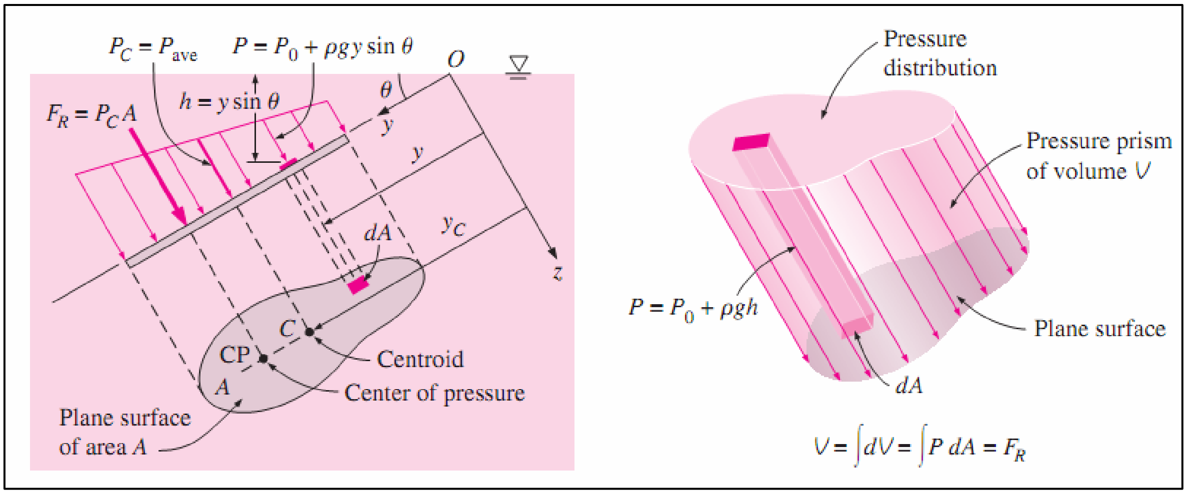


FIG.8 Hydrostatic force on an inclined plane surface completely submerged in a liquid

where h is the vertical distance of the point from the free surface and y is the distance of the point from the x-axis (from point O in Fig.8). The resultant hydrostatic force F_R acting on the surface is determined by integrating the force *P dA* acting on a differential area *dA* over the entire surface area

$$F_R = \int_A P\,dA = \int_A (P_0 + \rho gy \sin\theta)\,dA = P_0 A + \rho g \sin\theta \int_A y\,dA$$

But the *first moment of area* $\int_A y\,dA$ is related to the *y*-coordinate of the centroid (or center) of the surface by

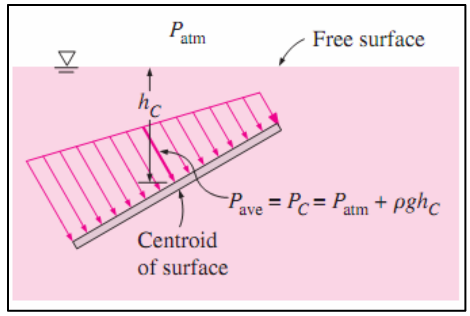
$$y_C = \frac{1}{A}\int_A y\,dA$$

Substituting,

$$F_R = (P_0 + \rho g y_C \sin\theta)A = (P_0 + \rho g h_C)A = P_C A = P_{ave} A$$

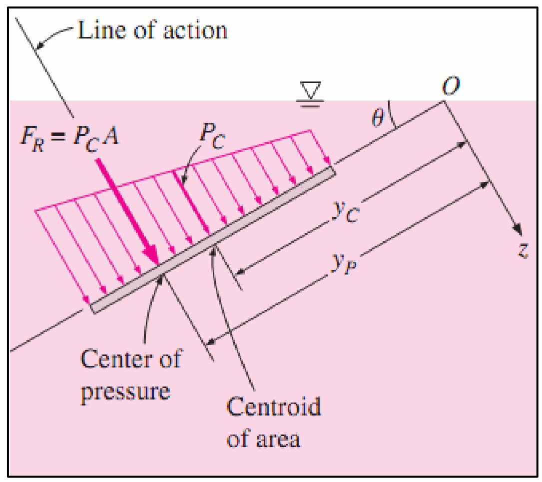
where $P_C = P_0 = \rho g h_C$ is the pressure at the centroid of the surface, which is equivalent to the average pressure on the surface, and $h_C = y_C \sin\theta$ is the vertical distance of the centroid from the free surface of the liquid (Fig.9).

FIG.9
The pressure at the centroid of a surface is equivalent to the average pressure on the surface.



Thus it can conclude that:
The magnitude of the resultant force acting on a plane surface of a completely submerged plate in a homogeneous (constant density) fluid is equal to the product of the pressure PC at the centroid of the surface and the area A of the surface (Fig.10).

FIG.10
The resultant force acting on a plane surface is equal to the product of the pressure at the centroid of the surface and the surface area, and its line of action passes through the center of pressure.



The pressure P_0 is usually atmospheric pressure, which can be ignored in most cases since it acts on both sides of the plate. When this is not the case, a practical way of accounting for the contribution of P_0 to the resultant force is simply to add an equivalent depth $h_{equiv} = P_0 / \rho g$ *to* h_C; that is, to assume the presence of an additional liquid layer of thickness h_{equiv} on top of the liquid with absolute vacuum above.
Next we need to determine the line of action of the resultant force F_R. Two parallel force systems are equivalent if they have the same magnitude and the same moment about any point. The line of action of the resultant hydrostatic force, in general, does not pass through the centroid of the surface—it lies underneath where the pressure is higher. The point of intersection of the line of action of the resultant force and the surface is the center of pressure. The vertical location of the line of action is determined by equating the moment of the resultant force to the moment of the distributed pressure force about the x-axis. It gives

$$y_P F_R = \int_A yP \, dA = \int_A y(P_0 + \rho g y \sin\theta) \, dA = P_0 \int_A y \, dA + \rho g \sin\theta \int_A y^2 \, dA$$

or

$$y_P F_R = P_0 y_C A + \rho g \sin\theta \, I_{xx,O}$$

where y_P is the distance of the center of pressure from the x-axis (point O in Fig. 10) and

$$I_{xx,\,O} = \int_A y^2\, dA \text{ is the } \textit{second moment of area}$$

(also called the area moment of inertia) about the x-axis.

The second moments of area are widely available for common shapes in engineering handbooks, but they are usually given about the axes passing through the centroid of the area. Fortunately, the second moments of area about two parallel axes are related to each other by the parallel axis theorem, which in this case is expressed as

$$I_{xx,\,O} = I_{xx,\,C} + y_C^2 A$$

where $I_{xx,\,C}$ is the second moment of area about the x-axis passing through the centroid of the area and y_C (the y-coordinate of the centroid) is the distance between the two parallel axes. Substituting the FR relation and the $I_{xx,\,O}$,and solving for y_P gives

$$y_P = y_C + \frac{I_{xx,\,C}}{[y_C + P_0/(\rho g \sin\theta)]A}$$

For $P_0 = 0$, which is usually the case when the atmospheric pressure is ignored, it simplifies to

$$y_P = y_C + \frac{I_{xx,\,C}}{y_C A}$$

Knowing yP,the vertical distance of the center of pressure from the free surface is determined from $h_P = y_P \sin\,\theta$.

The $I_{xx,\,C}$ values for some common areas are given in Fig.11.

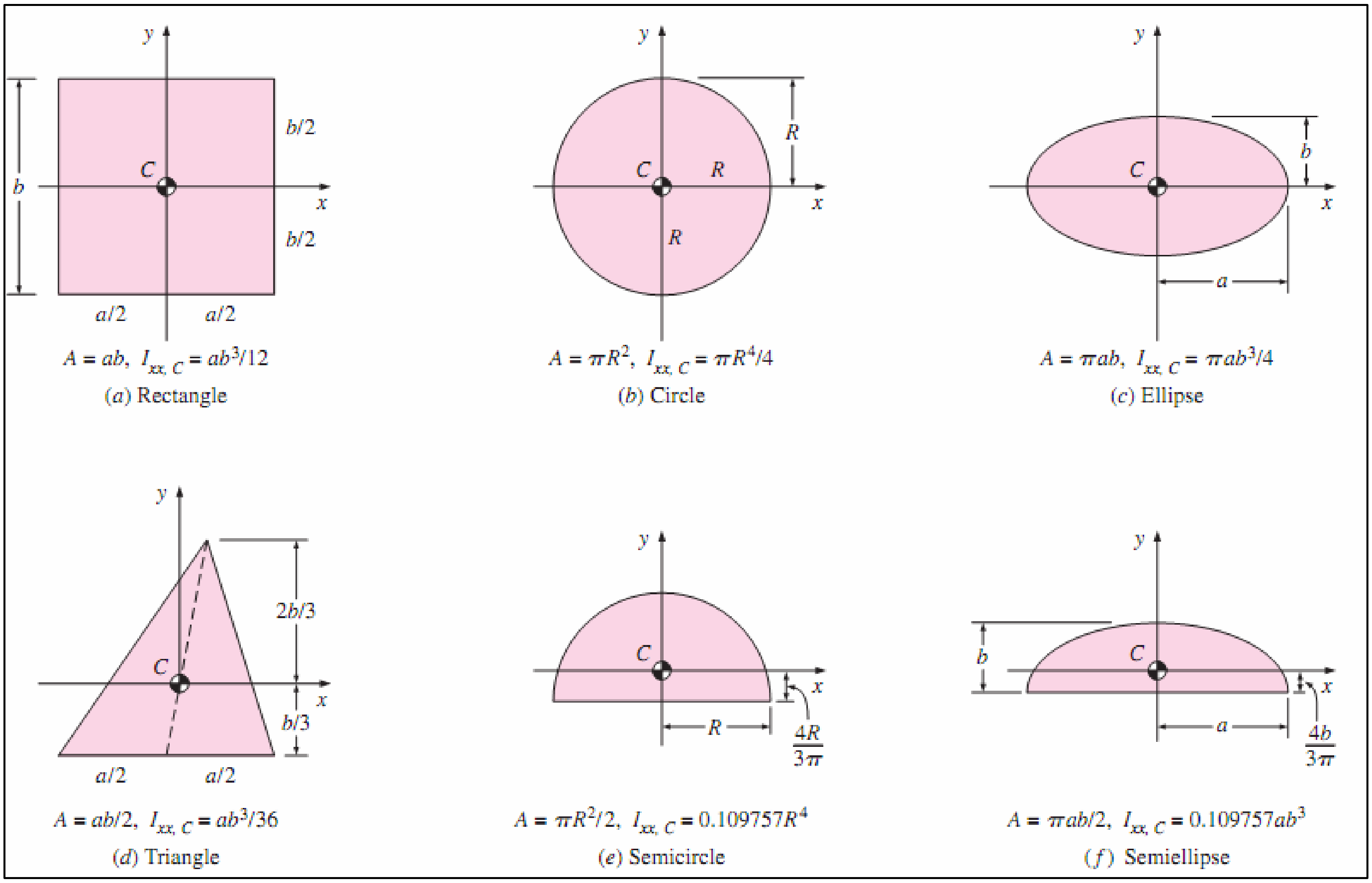
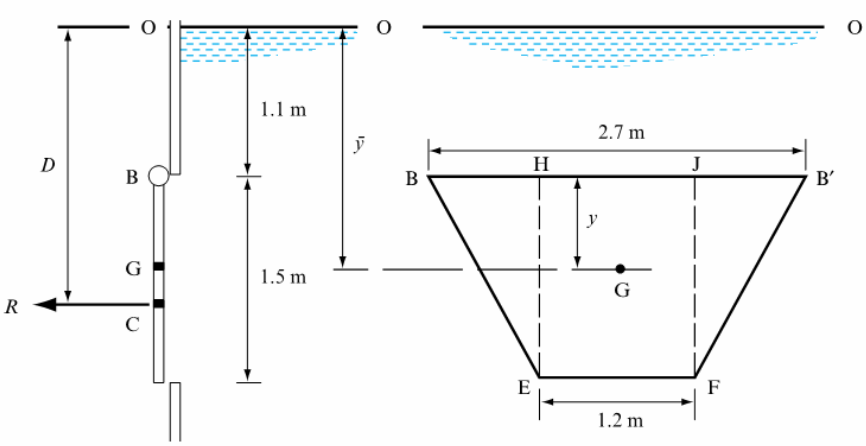


FIG.11 The centroid and the centroidal moments of inertia for some common geometries.

Ex.5 A trapezoidal opening in the vertical wall of a tank is closed by a flat plate which is hinged at its upper edge (Fig.). The plate is symmetrical about its centerline and is 1.5 m deep. Its upper edge is 2.7 m long and its lower edge is 1.2 m long. The free surface of the water in the tank stands 1.1 m above the upper edge of the plate. Calculate the moment about the hinge line required to keep the plate closed.



Area of plate, $A = \frac{1}{2}(2.7 + 1.2) \times 1.5 = 2.925\ \text{m}^2$

To find the position of the centroid G, take moments of area about BB′, putting the vertical distance GB = y:

$$A \times y = \text{Moment of areas BHE and FJB}' + \text{Moment of EFJH}$$
$$= 2 \times (\tfrac{1}{2} \times 1.5 \times 0.75) \times 0.5 + (1.2 \times 1.5) \times 0.75.$$
$$2.925y = 0.5625 + 1.35 = 1.9125,$$
$$y = 0.654\ \text{m}.$$

Depth to the centre of pressure,

$$\bar{y} = y + \text{OB} = 0.654 + 1.1 = 1.754\ \text{m}.$$

Substituting in

$$\text{Resultant force, } R = 10^3 \times 9.81 \times 2.925 \times 1.754 = \mathbf{50.33\ kN}.$$

From equation

$$\text{Depth to centre of pressure C, } D = \sin^2\phi(I_O/A\bar{y}).$$

Using the parallel axis rule for second moments of area,

$$I_O = \text{Second moment of EFJH about O} + \text{Second moment of BEH and B}'\text{FJ about O}$$
$$= \left(\frac{1.2 \times 1.5^3}{12} + 1.2 \times 1.5 \times 1.85^2\right) + \left(\frac{1.5 \times 1.5^3}{36} + 1.5 \times 0.75 \times 1.6^2\right)\ \text{m}^4$$
$$= 9.5186\ \text{m}^4.$$

As the wall is vertical, $\sin\phi = 1$; therefore,

$$\text{Depth to centre of pressure, } D = \frac{9.5186}{2.925 \times 1.754} = 1.8553\ \text{m}.$$

$$\text{Moment about hinge} = R \times \text{BC} = 50.33(1.8553 - 1.1)$$
$$= \mathbf{38.01\ kN\,m}.$$

FORCE ON A CURVED SURFACE DUE TO HYDROSTATIC PRESSURE

If a surface is curved, the forces produced by fluid pressure on the small elements making up the area will not be parallel and, therefore, must be combined vectorially. It is convenient to calculate the horizontal and vertical components of the resultant force. This can be done in three dimensions, but the following analysis is for a surface curved in one plane only.

In Fig.1(a) and (b), AB is the immersed surface and R_h and R_v are the horizontal and vertical components of the resultant force R of the liquid on one side of the surface. In Fig. 1(a) the liquid lies above the immersed surface, while in Fig. 1(b) it acts below the surface. In Fig.. 1(a), if ACE is a vertical plane through A, and BC is a horizontal plane, then, since element ACB is in equilibrium, the resultant force P on AC must equal the horizontal component R_h of the force exerted by the fluid on AB because there are no other horizontal forces acting. But AC is the projection of AB on a vertical plane; therefore.

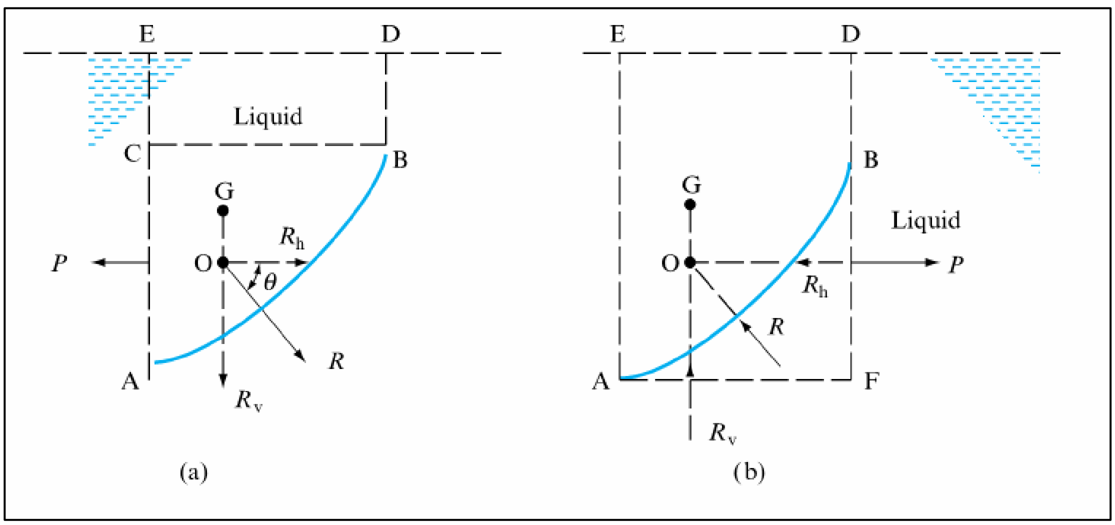


FIG.1 Hydrostatic force on a curved surface

Horizontal component R_h = Resultant force on the projection of AB on a vertical plane.

Also, for equilibrium, P and R_h must act in the same straight line; therefore, the horizontal component R_h acts through the centre of pressure of the projection of AB on a vertical plane. Similarly, in Fig. 1(b), element ABF is in equilibrium, and so the horizontal component R_h is equal to the resultant force on the projection BF of the curved surface AB on a vertical plane, and acts through the centre of pressure of this projection. In Fig. 1(a), the vertical component R_v will be entirely due to the weight of the fluid in the area ABDE lying vertically above AB. There are no other vertical forces, since there can be no shear forces on AE and BD because the fluid is at rest. Thus, Vertical component, R_v = Weight of fluid vertically above AB,

and will act vertically downwards through the centre of gravity G of ABDE.
In Fig.1(b), if the surface AB were removed and the space ABDE filled with the liquid, this liquid would be in equilibrium under its own weight and the vertical force on the boundary AB. Therefore,

Vertical component, R_v = Weight of the volume of the same fluid which would lie vertically above AB,

and will act vertically upwards through the centre of gravity G of this imaginary volume of fluid. n the case of closed vessels under pressure, a free surface does not exist, but an imaginary free surface can be substituted at a level p ρg above a point at which the pressure p is known, ρ being the mass density of the actual fluid. The resultant force R is found by combining the components vectorially. In the general case, the components in three directions may not meet at a point and, therefore, cannot be represented by a single force. However, in Fig. 1, if the surface is of uniform width perpendicular to the diagram, R_h and R_v will intersect at O. Thus,

$$\text{Resultant force, } R = \sqrt{(R_h^2 + R_v^2)},$$

and acts through O at an angle θ given by $\tan\theta = R_v/R_h$.
In the special case of a cylindrical surface, all the forces on each small element of area acting normal to the surface will be radial and will pass through the centre of curvature O (Fig. 2). The resultant force R must, therefore, also pass through the centre of curvature O.

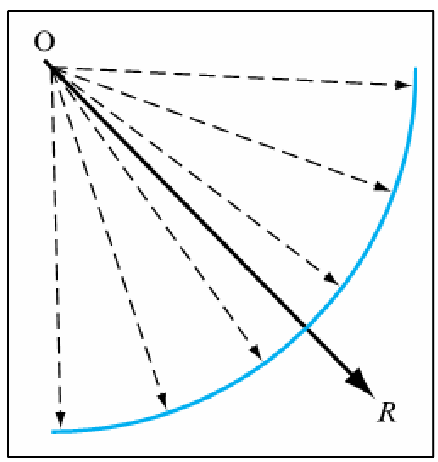
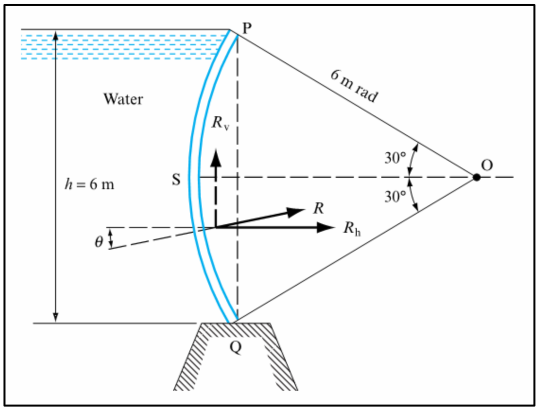


FIG. 2
Resultant force on a cylindrical surface

EX.1 A sluice gate is in the form of a circular arc of radius 6m as shown in Fig. Calculate the magnitude and direction of the resultant force on the gate, and the location with respect to O of a point on its line of action.
Solution
Since the water reaches the top of the gate



Depth of water, $h = 2 \times 6 \sin 30° = 6\,\text{m}$,

Horizontal component of force on gate = R_h per unit length

= Resultant force on PQ per unit length

$= \rho g \times h \times h/2 = \rho g h^2/2$

$= (10^3 \times 9.81 \times 36)/2\ \text{N m}^{-1} = 176.58\ \text{kN m}^{-1}$,

Vertical component of force on gate = R_v per unit length

= Weight of water displaced by segment PSQ

= (Sector OPSQ − ΔOPQ)ρg

$= [(60/360) \times \pi \times 6^2 - 6 \sin 30° \times 6 \cos 30°] \times 10^3 \times 9.81\ \text{N m}^{-1}$

$= 32.00\ \text{kN m}^{-1}$,

Resultant force on gate, $R = \sqrt{(R_h^2 + R_v^2)}$

$= \sqrt{(176.58^2 + 32.00^2)} = \mathbf{179.46\ kN\ m^{-1}}$.

If R is inclined at an angle θ to the horizontal,

$\tan \theta = R_v/R_h = 32.00/176.58 = 0.181\,22$

$\theta =$ **10.27° to the horizontal**.

Since the surface of the gate is cylindrical, the resultant force R **must pass through O**.

Chapter Two
FLUID FLOW

The motion of a fluid is usually extremely complex. The study of a fluid at rest, or in relative equilibrium, was simplified by the absence of shear forces, but when a fluid flows over a solid surface or other boundary, whether stationary or moving, the velocity of the fluid in contact with the boundary must be the same as that of the boundary, and a velocity gradient is created at right angles to the boundary.

UNIFORM FLOW AND STEADY FLOW

Flow is described as ***uniform*** if the velocity at a given instant is the same in magnitude and direction at every point in the fluid. If, at the given instant, the velocity changes from point to point, the flow is described as ***non-uniform***.

A steady flow is one in which the velocity, pressure and cross-section of the stream may vary from point to point but do not change with time. If, at a given point, conditions do change with time, the flow is described as unsteady .

There are, therefore, four possible types of flow:

1. Steady uniform flow. Conditions do not change with position or time. The velocity and cross-sectional area of the stream of fluid are the same at each cross-section: for example, flow of a liquid through a pipe of uniform bore running completely full at constant velocity.

2. Steady non-uniform flow. Conditions change from point to point but not with time. The velocity and cross-sectional area of the stream may vary from cross-section to cross-section, but, for each cross-section, they will not vary with time: for example, flow of a liquid at a constant rate through a tapering pipe running completely full.

3. Unsteady uniform flow. At a given instant of time the velocity at every point is the same, but this velocity will change with time: for example, accelerating flow of a liquid through a pipe of uniform bore running full, such as would occur when a pump is started up.

4. Unsteady non-uniform flow. The cross-sectional area and velocity vary from point to point and also change with time: for example, a wave travelling along a channel.

Streamlines

Streamlines are useful as indicators of the instantaneous direction of fluid motion throughout the flow field. For example, regions of recirculating flow and separation of a fluid off of a solid wall are easily identified by the streamline pattern. Streamlines cannot be directly observed experimentally except in steady flow fields, in which they are coincident with pathlines and streaklines. As show in Fig.1,

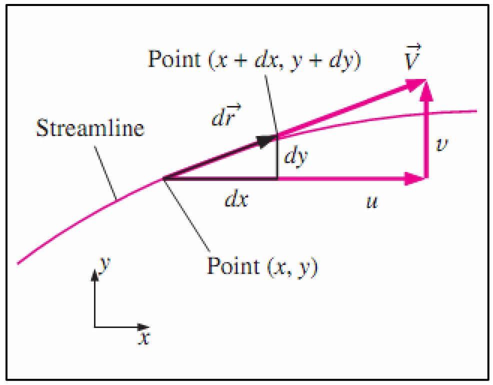
Consider an infinitesimal arc length $d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$

must be parallel to the local velocity vector $\vec{V} = u\vec{i} + v\vec{j} + w\vec{k}$

Hence

Equation for a streamline: $$\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$

FIG.1
For two-dimensional flow in the *xy*-plane, arc length $d\vec{r} = (dx, dy)$ along a *streamline* is everywhere tangent to the local instantaneous velocity vector $\vec{V} = (u, v)$.

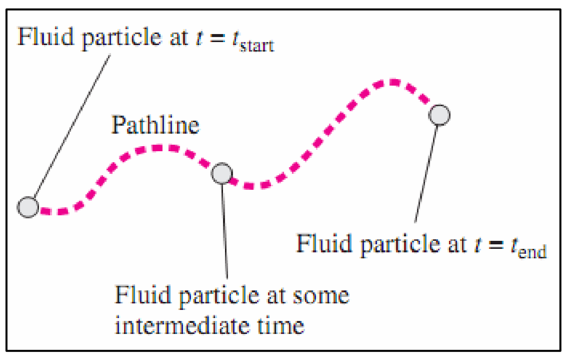


Streamline in the xy-plane: $$\left(\frac{dy}{dx}\right)_{\text{along a streamline}} = \frac{v}{u}$$

Pathlines

A pathline is the actual path traveled by an individual fluid particle over some time period. Pathlines are the easiest of the flow patterns to understand. A pathline is a Lagrangian concept in that we simply follow the path of an individual fluid particle as it moves around in the flow field (Fig.2).

FIG.2
A pathline is formed by following the actual path of a fluid particle.



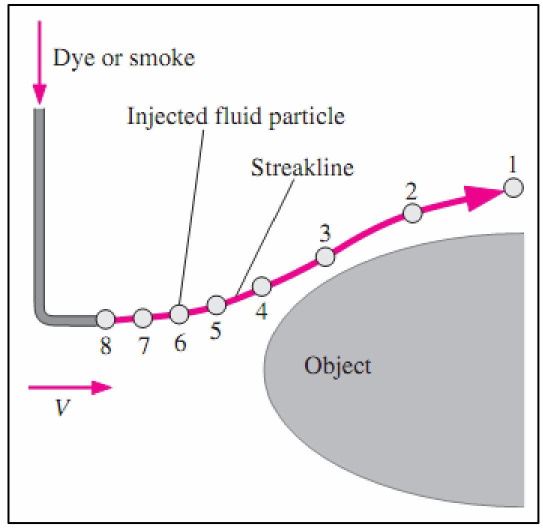
Streaklines

A streakline is the locus of fluid particles that have passed sequentially through a prescribed point in the flow.

Streaklines are the most common flow pattern generated in a physical experiment. If you insert a small tube into a flow and introduce a continuous stream of tracer fluid (dye in a water flow or smoke in an airflow), the observed pattern is a streakline as show in Fig.3.

The tracer being injected into a free-stream flow containing an object, such as the nose of a wing. The circles represent individual injected tracer fluid particles, released at a uniform time interval.

FIG.3
A streakline is formed by continuous introduction of dye or smoke from a point in the flow. Labeled tracer particles (1 through 8) were introduced sequentially.



ACCELERATION OF A FLUID PARTICLE

The forces acting on a particle are related to the resultant acceleration δv/δt of the particle by Newton's second law:

$$\text{Acceleration in the direction of flow, } a = \frac{dv}{dt} = \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t}.$$

To denote that the derivative dv/dt is obtained by following the motion of a single particle, it is written Dv/Dt, and since $ds/dt = v$,

$$a = \frac{Dv}{Dt} = v\frac{\partial v}{\partial s} + \frac{\partial v}{\partial t}.$$

For steady flow, $\partial v/\partial t = 0$, while for uniform flow, $\partial v/\partial s = 0$.
In general, the motion of a fluid particle is three-dimensional and its velocity and acceleration can be expressed in terms of three mutually perpendicular components. Thus, if v_x, v_y and v_z are the components of the velocity in the ***x, y*** and ***z*** directions, respectively, and a_x, a_y and a_z the corresponding components of acceleration, the velocity field is described by

$$v_x = v_x(x, y, z, t), \quad v_y = v_y(x, y, z, t), \quad v_z = v_z(x, y, z, t),$$

and the velocity **v** at any point is given by

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k},$$

where i, j and k are the unit vectors in the x, y and z directions.
The change of the component velocities in each direction as a particle moves in a fluid can now be calculated. Thus, in the x direction,

$$\delta v_x = \frac{\partial v_x}{\partial x}(\delta x) + \frac{\partial v_x}{\partial y}(\delta y) + \frac{\partial v_x}{\partial z}(\delta z) + \frac{\partial v_x}{\partial t}(\delta t),$$

and the acceleration in the x direction, in the limit as $\delta t \to 0$, will be

$$a_x = \frac{\mathrm{D}v_x}{\mathrm{D}t} = \frac{\partial v_x}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial v_x}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\partial v_x}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}t} + \frac{\partial v_x}{\partial t}$$

or, since $\mathrm{d}x/\mathrm{d}t = v_x$, $\mathrm{d}y/\mathrm{d}t = v_y$, $\mathrm{d}z/\mathrm{d}t = v_z$,

$$a_x = \frac{\mathrm{D}v_x}{\mathrm{D}t} = v_x\frac{\partial v_x}{\partial x} + v_y\frac{\partial v_x}{\partial y} + v_z\frac{\partial v_x}{\partial z} + \frac{\partial v_x}{\partial t}.$$

Similarly,

$$a_y = \frac{\mathrm{D}v_y}{\mathrm{D}t} = v_x\frac{\partial v_y}{\partial x} + v_y\frac{\partial v_y}{\partial y} + v_z\frac{\partial v_y}{\partial z} + \frac{\partial v_y}{\partial t},$$

$$a_z = \frac{\mathrm{D}v_z}{\mathrm{D}t} = v_x\frac{\partial v_z}{\partial x} + v_y\frac{\partial v_z}{\partial y} + v_z\frac{\partial v_z}{\partial z} + \frac{\partial v_z}{\partial t}.$$

CONTINUITY OF FLOW

Except in nuclear processes, matter is neither created nor destroyed. This principle of conservation of mass can be applied to a flowing fluid. Considering any fixed region in the flow (Fig.4) constituting a control volume,

Mass of fluid entering per unit time = Mass of fluid leaving per unit time + Increase of mass of fluid in the control volume per unit time.

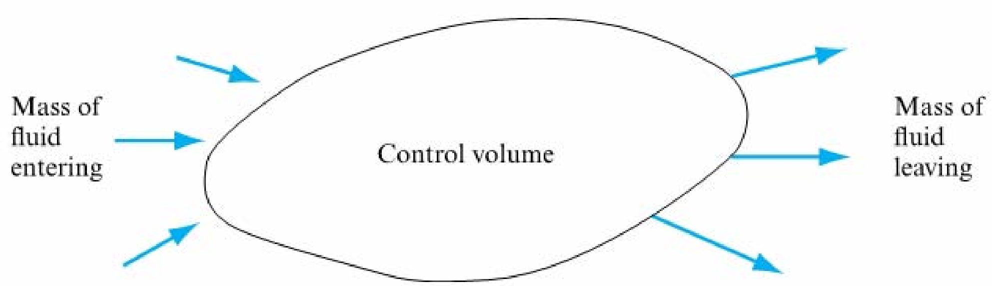


FIG. 4. Continuity of flow

For steady flow, the mass of fluid in the control volume remains constant as show in Fig. 5 and the relation reduces to

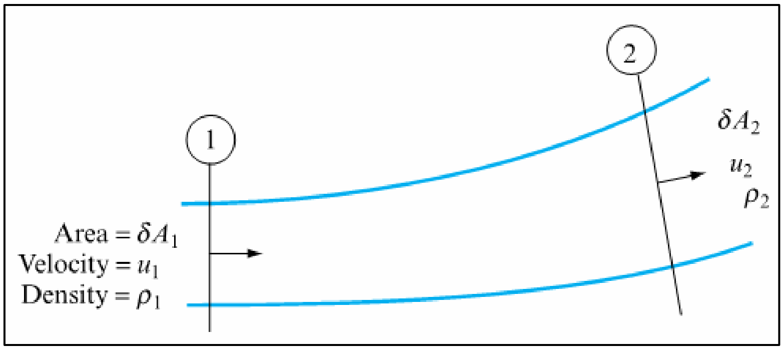
$$\text{Mass of fluid entering per unit time} = \text{Mass of fluid leaving per unit time.}$$

Suppose that at section 1 the area of the streamtube is δA_1, the velocity of the fluid u_1 and its density ρ_1, while at section 2 the corresponding values are δA_2, u_2 and ρ_2; then

$$\text{Mass entering per unit time at } 1 = \rho_1 \delta A_1 u_1,$$

$$\text{Mass leaving per unit time at } 2 = \rho_2 \delta A_2 u_2.$$

FIG.5
Continuous flow through a streamtub



Then, for steady flow,

$$\rho_1 \delta A_1 u_1 = \rho_2 \delta A_2 u_2 = \text{Constant}$$

For the flow of a real fluid through a pipe or other conduit, the velocity will vary from wall to wall. However, using the mean velocity $\bar{u}$, the equation of continuity for steady flow can be written as

$$\rho_1 A_1 \bar{u}_1 = \rho_2 A_2 \bar{u}_2 = \dot{m}$$

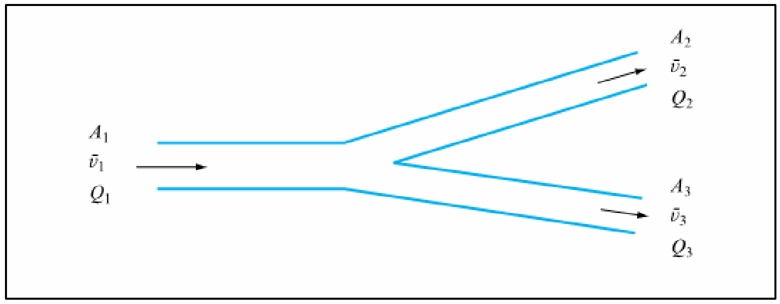
$$A_1 \bar{u}_1 = A_2 \bar{u}_2 = Q$$

The continuity equation can also be applied to determine the relation between the flows into and out of a junction. In Fig. 6, for steady conditions,

$$\text{Total inflow to junction} = \text{Total outflow from junction,}$$

$$\rho_1 Q_1 = \rho_2 Q_2 + \rho_3 Q_3.$$

FIG.6
Applications of the continuity equation



For an incompressible fluid, $\rho_1 = \rho_2 = \rho_3$ so that

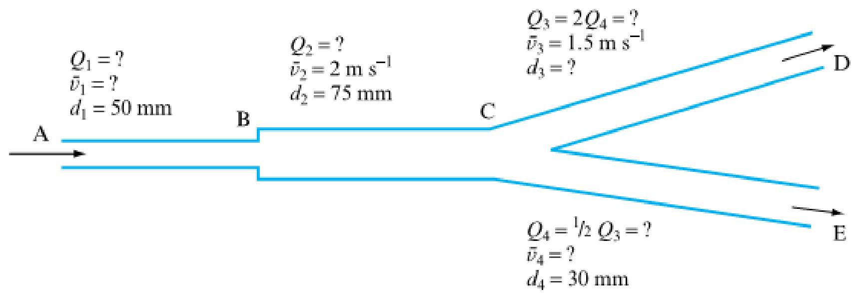
$$Q_1 = Q_2 + Q_3$$

or $$A_1\bar{v}_1 = A_2\bar{v}_2 + A_3\bar{v}_3.$$

In general, if we consider flow towards the junction as positive and flow away from the junction as negative, then for steady flow at any junction the algebraic sum of all the mass flows must be zero

$$\sum \rho Q = 0$$

EX.1Water flows from A to D and E through the series pipeline shown in Fig. Given the pipe diameters, velocities and flow rates below, complete the tabular data for this system.



PIPE	DIAMETER (mm)	FLOW RATE ($m^3 s^{-1}$)	VELOCITY ($m s^{-1}$)
AB	$d_1 = 50$	$Q_1 = ?$	$\bar{v}_1 = ?$
BC	$d_2 = 75$	$Q_2 = ?$	$\bar{v}_2 = 2.0$
CD	$d_3 = ?$	$Q_3 = 2Q_4$	$\bar{v}_3 = 1.5$
DE	$d_4 = 30$	$Q_4 = 0.5Q_3$	$\bar{v}_4 = ?$

Solution

Adding area A = $(22/7)d^2/4$ to the data table and noting that Q = AC and that $Q_1 = Q_2 = (Q_3 + Q_4) = 1.5Q_3$ allows the table to be completed

DIAMETER (mm)	AREA (m^2)	FLOW RATE ($m^3 s^{-1}$)	VELOCITY ($m s^{-1}$)
$d_1 = 50$	1.9643×10^{-3}	$Q_1 = Q_2 = 8.839 \times 10^{-3}$	$\bar{v}_1 = \bar{v}_2 A_2/A_1$ $= 2.0 \times 4.4196/1.9643$ $= 4.27$
$d_2 = 75$	4.4196×10^{-3}	$Q_2 = 2.0 \times 4.4196 \times 10^{-3}$ $= 8.839 \times 10^{-3}$	$\bar{v}_2 = 2.0$
$d_3 = [Q_3/(\bar{v}_3 \pi/4)]^{0.5}$ $= (5.893 \times 10^{-3}/1.5 \times 0.786)^{0.5}$ $= 0.707$		$Q_3 = Q_2/1.5$ $= 5.893 \times 10^{-3}$	$\bar{v}_3 = 1.5$
$d_4 = 30$	0.707×10^{-3}	$Q_4 = 0.5Q_3$ $= 0.5 \times 5.893 \times 10^{-3}$ $= 2.947 \times 10^{-3}$	$\bar{v}_4 = Q_4/A_4$ $= 2.947/0.7071$ $= 4.17$

(The calculation route is as follows: calculate areas where possible and then Q_2 and hence Q_1 and $\bar{v}_1$. From Q_2 calculate Q_3 and Q_4 and hence $\bar{v}_4$. Calculate d_3 from Q_3 and $\bar{v}_3$.)

Angular Velocity and Vorticity

Visualize afluidflow as the motion of acollection of fluid particles that deform and rotate as they travel along. At some instant in time, we could think of all the particles that make up the flow as being little cubes. If the cubessimply deform anddonot rotate, we refer to the flow, or aregion of the flow, as an irrotational flow.Such flows are of particular interest in our study of fluids; they exist in tornados away from the ‘‘eye’’ and in the flow away from the surfaces of airfoils and automobiles. Consider the rectangular face of an infinitesimal volume shown in Fig.7. The angular velocity Ω_z about the z-axis is the average of the angular velocity of segments AB and AC,counter-clockwise taken as positive:

$$\Omega_z = \frac{\Omega_{AB} + \Omega_{AC}}{2} = \frac{1}{2}\left[\frac{v_B - v_A}{dx} + \frac{-(u_C - u_A)}{dy}\right]$$

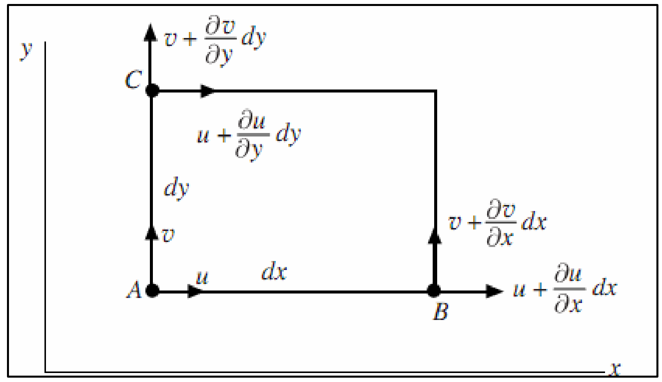
$$= \frac{1}{2}\left[\frac{\frac{\partial v}{\partial x}dx}{dx} - \frac{\frac{\partial u}{\partial y}dy}{dy}\right] = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

If we select the other faces, we would find

$$\Omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) \qquad \Omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

These three components of the angular velocity components represent the rate at which a fluid particle rotates about each of the coordinate axes.

Fig. 7The rectangular face of afluid element



The *vorticity vector* **ω** is defined as twice the angular velocity vector: **ω** = 2**Ω**. The vorticity components are

$$\omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \qquad \omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \qquad \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

The vorticity components in cylindrical coordinates are listed in Table.1

Table 1 The Material Derivative, Acceleration, and Vorticity in Rectangular, Cylindrical, and Spherical Coordinate

Material derivative

Rectangular

$$\frac{D}{Dt} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} + \frac{\partial}{\partial t}$$

Cylindrical

$$\frac{D}{Dt} = v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z} + \frac{\partial}{\partial t}$$

Spherical

$$\frac{D}{Dt} = v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial}{\partial \phi} + \frac{\partial}{\partial t}$$

Acceleration

Rectangular

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \qquad a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \qquad a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

Cylindrical

$$a_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r} + \frac{\partial v_r}{\partial t} \qquad a_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z} + \frac{v_r v_\theta}{r} + \frac{\partial v_\theta}{\partial t}$$

$$a_z = v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z} + \frac{\partial v_z}{\partial t}$$

Spherical

$$a_r = v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_r}{\partial \phi} - \frac{v_\theta^2 + v_\phi^2}{r} + \frac{\partial v_r}{\partial t} \qquad a_\theta = v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\theta}{\partial \phi} + \frac{v_r v_\theta - v_\phi^2\cot\theta}{r} + \frac{\partial v_\theta}{\partial t}$$

$$a_\phi = v_r\frac{\partial v_\phi}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta} + \frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi} + \frac{v_r v_\phi + v_\theta v_\phi\cot\theta}{r} + \frac{\partial v_\phi}{\partial t}$$

Vorticity

Rectangular

$$\omega_x = \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \qquad \omega_y = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \qquad \omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

Cylindrical

$$\omega_r = \frac{1}{r}\frac{\partial v_z}{\partial \theta} - \frac{\partial v_\theta}{\partial z} \qquad \omega_\theta = \frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \qquad \omega_z = \frac{1}{r}\frac{\partial (r v_\theta)}{\partial r} - \frac{1}{r}\frac{\partial v_r}{\partial \theta}$$

CONTINUITY EQUATIONS FOR THREE-DIMENSIONAL FLOW USING CARTESIAN COORDINATES

All the basic differential equations can be derived by considering either an elemental control volume or an elemental system. Here we choose an infinitesimal fixed control volume (***dx, dy, dz***), as in Fig.8. The flow through each side of the element is approximately one dimensional, and so the appropriate mass-conservation relation to use here is:

$$\int_{CV}\frac{\partial \rho}{\partial t}\,d\mathcal{V} + \sum_i (\rho_i A_i V_i)_{out} - \sum_i (\rho_i A_i V_i)_{in} = 0$$

The element is so small that the volume integral simply reduces to a differential term

$$\int_{CV}\frac{\partial \rho}{\partial t}\,d\mathcal{V} \approx \frac{\partial \rho}{\partial t}\,dx\,dy\,dz$$

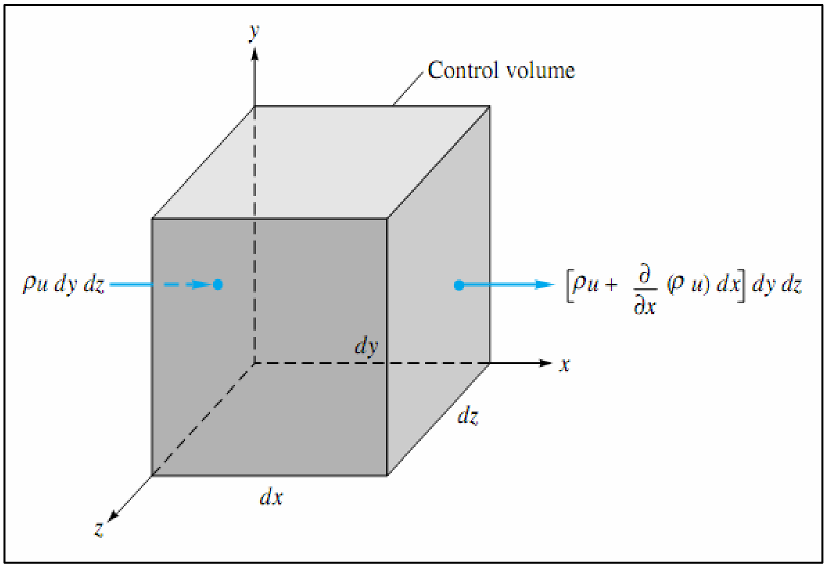


Fig.8 Elemental cartesian fixed control volume showing the inlet and outlet mass flows on the x faces.

Fig.8, shows only the mass flows on the x or left and right faces. The flows on the y (bottom and top) and the z (back and front) faces have been omitted to avoid cluttering up the drawing. We can list all these six flows as follows:

Face	Inlet mass flow	Outlet mass flow
x	$\rho u\, dy\, dz$	$\left[\rho u + \frac{\partial}{\partial x}(\rho u)\, dx\right] dy\, dz$
y	$\rho v\, dx\, dz$	$\left[\rho v + \frac{\partial}{\partial y}(\rho v)\, dy\right] dx\, dz$
z	$\rho w\, dx\, dy$	$\left[\rho w + \frac{\partial}{\partial z}(\rho w)\, dz\right] dx\, dy$

Introduce these terms into:

$$\frac{\partial \rho}{\partial t}\, dx\, dy\, dz + \frac{\partial}{\partial x}(\rho u)\, dx\, dy\, dz + \frac{\partial}{\partial y}(\rho v)\, dx\, dy\, dz + \frac{\partial}{\partial z}(\rho w)\, dx\, dy\, dz = 0$$

The element volume cancels out of all terms, leaving a partial differential equation involving the derivatives of density and velocity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

The vector-gradient operator

$$\nabla = \mathbf{i}\frac{\partial}{\partial x} + \mathbf{j}\frac{\partial}{\partial y} + \mathbf{k}\frac{\partial}{\partial z}$$

enables us to rewrite the equation of continuity in a compact form, not that it helps much in finding a solution. The last three terms are equivalent to the divergence of the vector ρV

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) \equiv \nabla \cdot (\rho \mathbf{V})$$

so that the compact form of the continuity relation is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

Cylindrical Polar Coordinates

The most common alternative to the cartesian system is the cylindrical polar coordinate system, sketched in Fig.9. An arbitrary point P is defined by a distance z along the axis, a radial distance r from the axis, and a rotation angle about the axis. The three independent velocity components are an axial velocity v_z, a radial velocity v_r, and a circumferential velocity v_θ

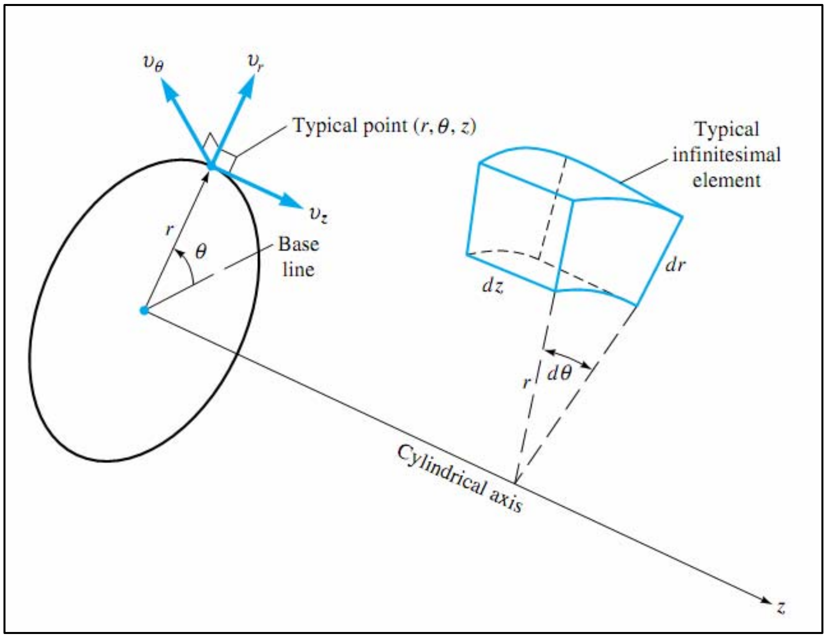


Fig.9.Definition sketch for the cylindrical coordinate system

The divergence of any vector function $\mathbf{A}(r, \theta, z, t)$ is found by making the transformation of coordinates

$$r = (x^2 + y^2)^{1/2} \qquad \theta = \tan^{-1}\frac{y}{x} \qquad z = z$$

and the result is given here

$$\nabla \cdot \mathbf{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(A_\theta) + \frac{\partial}{\partial z}(A_z)$$

The general continuity equation in cylindrical polar coordinates is thus

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Steady Compressible Flow

If the flow is steady, $\partial/\partial t = 0$ and all properties are functions of position only

Cartesian: $$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Cylindrical: $$\frac{1}{r}\frac{\partial}{\partial r}(r\rho v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0$$

Incompressible Flow

A special case which affords great simplification is incompressible flow,

$$\nabla \cdot \mathbf{V} = 0$$

Cartesian: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Cylindrical: $$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(v_z) = 0$$

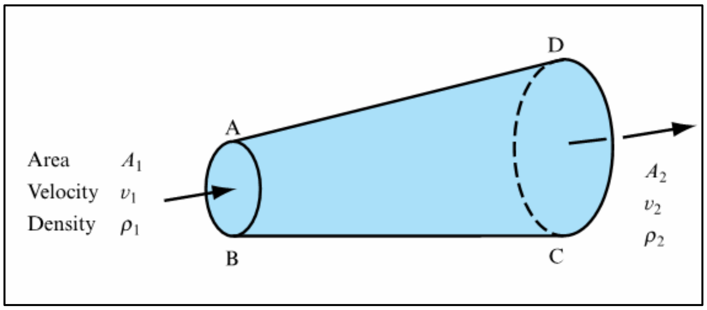
MOMENTUM AND FLUID FLOW

In mechanics, the momentum of a particle or object is defined as the product of its mass m and its velocity *v*: $\text{Momentum} = mv$

The particles of a fluid stream will possess momentum, and, whenever the velocity of the stream is changed in magnitude or direction, there will be a corresponding change in the momentum of the fluid particles.

To determine the rate of change of momentum in a fluid stream consider a control volume ABCD (Fig.1). As the fluid flow is assumed to be steady and non-uniform in nature the continuity of mass flow across the control volume may be expressed as

FIG.1
Momentum in a flowing fluid



i.e. there is no storage within the control volume and A is the fluid mass flow

The rate at which momentum exits the control volume across boundary CD may be defined as

$$\rho_2 A_2 v_2 v_2.$$

Similarly the rate at which momentum enters the control volume across AB may be expressed as

$$\rho_1 A_1 v_1 v_1.$$

Thus the rate of change of momentum across the control volume may be seen to be

$$\rho_2 A_2 v_2 v_2 - \rho_1 A_1 v_1 v_1$$

or, from the continuity of mass flow equation,

$$\rho_1 A_1 v_1 (v_2 - v_1) = \dot{m}(v_2 - v_1)$$

$$= \text{Mass flow per unit time} \times \text{Change of velocity.}$$

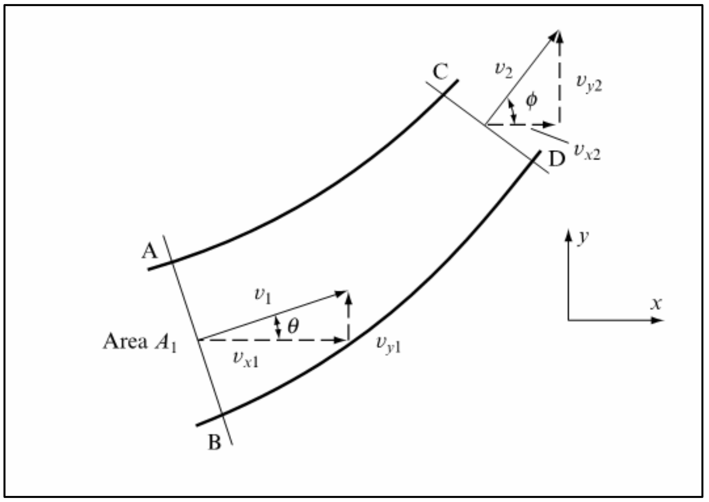
Note that this is the *increase* of momentum per unit time in the direction of motion, and according to Newton's second law will be caused by a force *F*, such that

$$F = \dot{m}(v_2 - v_1)$$

MOMENTUM EQUATION FOR TWO- AND THREE-DIMENSIONAL FLOW ALONG A STREAMLINE

Fig.2 shows a two-dimensional problem in which v1 makes an angle θ with the x axis, while v2 makes a corresponding angle φ. Since both momentum and force are vector quantities, they can be resolved into components in the x and y directions. Thus, if Fx and Fy are the components of the resultant force on the element of fluid ABCD,

FIG.2
Momentum equation for two-dimensional flow



$$F_x = \text{Rate of change of momentum of fluid in } x \text{ direction}$$
$$= \text{Mass per unit time} \times \text{Change of velocity in } x \text{ direction}$$
$$= \dot{m}(v_2 \cos\phi - v_1 \cos\theta) = \dot{m}(v_{x2} - v_{x1}).$$

Similarly,

$$F_y = \dot{m}(v_2 \sin\phi - v_1 \sin\theta) = \dot{m}(v_{y2} - v_{y1}).$$

These components can be combined to give the resultant force,

$$F = \sqrt{(F_x^2 + F_y^2)}.$$

Again, the force exerted by the fluid on the surroundings will be equal and opposite.For three-dimensional flow, the same method can be used, but the fluid will also have component velocities v_{z1} and v_{z2} in the z direction and the corresponding rate of change of momentum in this direction will require a force:-

$$F_z = \dot{m}(v_{z2} - v_{z1})$$

For any control volume, the total force F which acts upon it in a given direction will be made up of three component forces:

F_1 = Force exerted *in the given direction* on the fluid in the control volume by any *solid body* within the control volume or coinciding with the boundaries of the control volume.

F_2 = Force exerted *in the given direction* on the fluid in the control volume by *body forces such as gravity.*

F_3 = Force exerted *in the given direction* on the fluid in the control volume by the fluid outside the control volume.

Thus,

$$F = F_1 + F_2 + F_3 = \dot{m}(v_{out} - v_{in})$$

When a real fluid flows past a solid boundary, shear stresses are developed and the velocity is no longer uniform over the cross-section. In a pipe, for example, the velocity will vary from zero at the wall to a maximum at the centre. The momentum per unit time for the whole flow can be found by summing the momentum per unit time through each element of the cross-section, provided that these are sufficiently small for the velocity perpendicular to each element to be taken as uniform. Thus, if the velocity perpendicular to the element is u and the area of the element is δA,

Mass passing through element in unit time $= \rho \delta A \times u$,

Momentum per unit time passing through element $=$ Mass per unit time $\times$ Velocity

$$= \rho \delta A u \times u = \rho u^2 \delta A,$$

Total momentum per unit time passing whole cross-section $= \int \rho u^2 \, dA.$

To evaluate this integral, the velocity distribution must be known.

If we consider turbulent flow through a pipe of radius R (Fig.3), the velocity u at any distance y from the pipe wall is given approximately by Prandtl's one-seventh power law:

$$u = u_{max}(y/R)^{1/7}$$

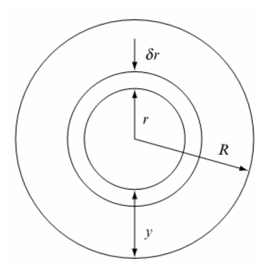


FIG.3 Calculation of Momentum correction factor

the maximum velocity, u_{max}, occurring at the centre of the pipe. Since the velocity is constant at any radius r = R − y, it is convenient to take the element of area δA as an annulus of radius r and width δr,

$$\delta A = 2\pi r \delta r$$

for the whole cross-section

$$\text{Total momentum per unit time} = \int_0^R \rho u^2 \, dA$$

$$= \int_0^R \rho u_{max}^2 (y/R)^{2/7} 2\pi r \, dr = \left(\frac{2\pi\rho}{R^{2/7}}\right) u_{max}^2 \int_0^R y^{2/7} r \, dr$$

Since r = R − y, dr = −dy, and so, substituting for r and dr and changing the limits (because y = 0 when r = R)

$$\text{Total momentum per unit time} = \frac{2\pi\rho u_{max}^2}{R^{2/7}} \int_R^0 y^{2/7}(R-y)(-dy)$$

$$= \frac{2\pi\rho u_{max}^2}{R^{2/7}} \int_R^0 (y^{9/7} - Ry^{2/7}) \, dy = \frac{2\pi\rho u_{max}^2}{R^{2/7}} \left(\frac{7}{16} y^{16/7} - \frac{7}{9} R y^{9/7}\right)_R^0$$

$$= \frac{2\pi\rho u_{max}^2}{R^{2/7}} R^{16/7} \left(\frac{7}{9} - \frac{7}{16}\right) = \frac{49}{72} \pi \rho R^2 u_{max}^2.$$

In practice, it is usually more convenient to use the mean velocity B instead of the maximum velocity u_{max}:

$$\text{Mean velocity, } \bar{u} = \frac{\text{Total volume per unit time passing section}}{\text{Total area of cross-section}}$$

$$= \frac{1}{\pi R^2} \int_0^R u \delta A.$$

Putting $u = u_{max}(y/R)^{1/7}$ and $\delta A = 2\pi r \delta r$,

$$\bar{u} = \frac{1}{\pi R^2} \int_0^R u_{max} \left(\frac{y}{R}\right)^{1/7} 2\pi r \, dr = \frac{2u_{max}}{R^{15/7}} \int_0^R y^{1/7} r \, dr.$$

Putting $r = R - y$, $dr = -dy$, and changing the limits,

$$\bar{u} = \frac{2u_{max}}{R^{15/7}} \int_R^0 y^{1/7}(R-y)(-dy) = \frac{2u_{max}}{R^{15/7}} \int_R^0 (y^{8/7} - Ry^{1/7}) \, dy$$

$$= \frac{2u_{max}}{R^{15/7}} \left(\frac{7}{15} y^{15/7} - \frac{7}{8} R y^{8/7}\right)_R^0 = \frac{49}{60} u_{max},$$

Ex.1 Water flows through a pipeline 60 m long at a velocity of 1.8 m s^{-1} when the pressure difference between the inlet and outlet ends is 25 kN m^{-2}. What increase of pressure difference is required to accelerate the water in the pipe at the rate of 0.02 m s^{-2}? Neglect elasticity effects.

Solution

Let A = cross-sectional area of the pipe, l = length of pipe, ρ = mass density of water, a = acceleration of water, δp = increase in pressure at inlet required to produce acceleration a.

As this is not a steady flow problem, consider a control mass comprising the whole of the water in the pipe. By Newton's second law,

$$\begin{matrix}\text{Force due to } \delta p \text{ in} \\ \text{direction of motion}\end{matrix} = \begin{matrix}\text{Rate of change of momentum of water in} \\ \text{the whole pipe}\end{matrix}$$

$$= \text{Mass of water in pipe} \times \text{Acceleration}, \qquad \text{(I)}$$

$$\text{Force due to } \delta p = \text{Cross-sectional area} \times \delta p = A\delta p,$$

$$\text{Mass of water in pipe} = \text{Mass density} \times \text{Volume} = \rho A l.$$

Substituting in (I),

$$A\delta p = \rho A l a,$$

$$\delta p = \rho l a = 10^3 \times 60 \times 0.02 \text{ N m}^{-2}$$

$$= \mathbf{1.2\ kN\ m^{-2}}.$$

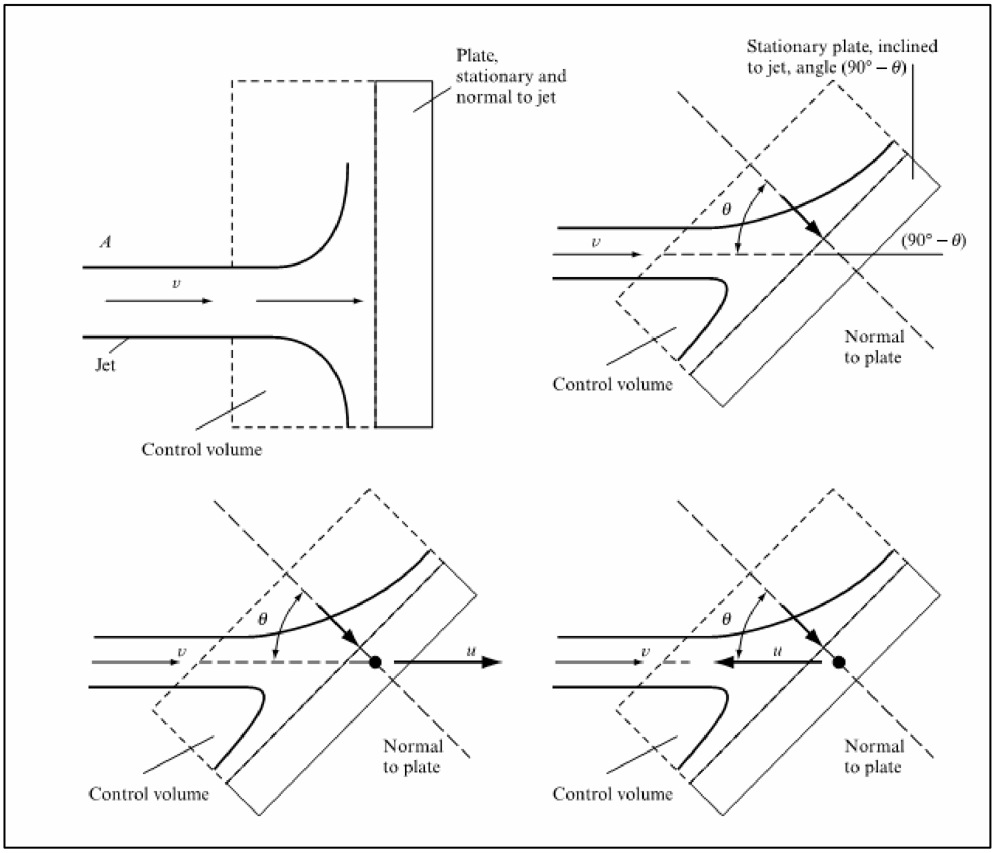
FORCE EXERTED BY A JET STRIKING A FLAT PLATE

Consider a jet of fluid striking a flat plate that may be perpendicular or inclined to the direction of the jet, or indeed may be moving in the initial direction of the jet (Fig.4).

A control volume encapsulating the approaching jet and the plate may be established, this control volume being fixed relative to the plate and therefore moving with it. It is helpful to consider components of the velocity and force vectors perpendicular and parallel to the surface of the plate. In each of the cases illustrated the impingement of the jet on the plate surface reduces the jet velocity component normal to the plate surface to zero. In general terms the jet velocity thus destroyed may be expressed as

$$V_{normal} = (v - u)\cos\theta.$$

FIG.4
Force exerted on a flat plate



The mass flow entering the control volume is also affected by the superposition of a velocity equal and opposite to the plate velocity and may be expressed as

$$\dot{m} = \rho A(v - u)\cos\theta,$$

which reduces to

$$\dot{m} = \rho A v$$

Thus the rate of change of momentum normal to the plate surface is given by

$$\text{dMomentum}/\text{d}t = \rho A(v - u)(v - u)\cos\theta.$$

Clearly this expression reduces to

$$\text{dMomentum}/\text{d}t = \rho A v^2 \cos\theta$$

if the plate is stationary, and

$$\text{dMomentum}/\text{d}t = \rho A v^2$$

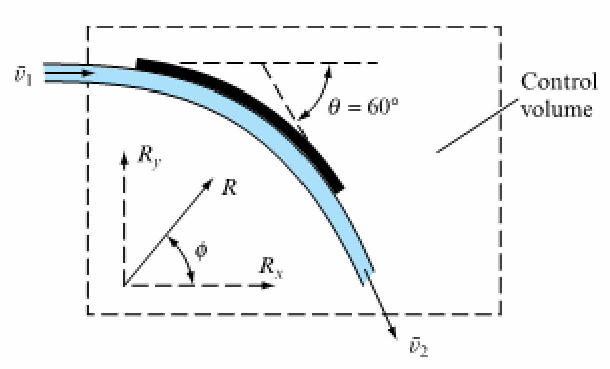
There will therefore be a force exerted upon the plate equal to the rate of momentum destroyed normal to the plate, given in the general case by an expression of the form

$$\text{Force normal to plate} = \rho A(v - u)(v - u)\cos\theta.$$

FORCE DUE TO THE DEFLECTION OF A JET BY A CURVED VANE

Both velocity and momentum are vector quantities and, therefore, even if the magnitude of the velocity remains unchanged, a change in direction of a stream of fluid will give rise to a change of momentum. If the stream is deflected by a curved vane (Fig.5), entering and leaving tangentially without impact, a force will be exerted between the fluid and the surface of the vane to cause this change of momentum.

FIG.5
Force exerted on a curved vane



Ex.2 A jet of water from a nozzle is deflected through an angle θ = 60° from its original direction by a curved vane which it enters tangentially (see Fig.5) without shock with a mean velocity $\boldsymbol{v_1}$ of 30 m s^{-1} and leaves with a mean velocity $\boldsymbol{v_2}$ of 25 m s^{-1}. If the discharge A from the nozzle is 0.8 kg s^{-1}, calculate the magnitude and direction of the resultant force on the vane if the vane is stationary.

Solution

The control volume will be as shown in Fig. 5. The resultant force R *exerted by the fluid* on the vane is found by determining the component forces R_x and R_y in the x and y directions, as shown.

$$R_x = -F_1 = F_2 + F_3 - \dot{m}(v_{out} - v_{in})_x.$$

Neglecting force F_2 due to gravity and assuming that for a free jet the pressure is constant everywhere, so that $F_3 = 0$,

$$R_x = \dot{m}(v_{in} - v_{out})_x, \qquad \text{(I)}$$

and, similarly,

$$R_y = \dot{m}(v_{in} - v_{out})_y. \qquad \text{(II)}$$

Since the nozzle and vane are fixed relative to each other,

$$\begin{matrix}\text{Mass per unit} & & \text{Mass per unit}\\ \text{time entering} & = \dot{m} = & \text{time leaving}\\ \text{control volume} & & \text{nozzle.}\end{matrix}$$

In the x direction,

$$v_{in} = \text{Component of } \bar{v}_1 \text{ in } x \text{ direction} = \bar{v}_1,$$

$$v_{out} = \text{Component of } \bar{v}_2 \text{ in } x \text{ direction} = \bar{v}_2 \cos\theta.$$

Substituting in (I),

$$R_x = \dot{m}(\bar{v}_1 - \bar{v}_2 \cos\theta). \qquad \text{(III)}$$

Putting $\dot{m} = 0.8$ kg s^{-1}, $\bar{v}_1 = 30$ m s^{-1}, $\bar{v}_2 = 25$ m s^{-1}, $\theta = 60°$,

$$R_x = 0.8(30 - 25\cos 60°) = 14 \text{ N}.$$

In the y direction,

$$v_{in} = \text{Component of } \bar{v}_1 \text{ in } y \text{ direction} = 0,$$

$$v_{out} = \text{Component of } \bar{v}_2 \text{ in } y \text{ direction} = \bar{v}_2 \sin\theta.$$

Thus, from (II),

$$R_y = \dot{m}\bar{v}_2 \sin\theta. \qquad \text{(IV)}$$

Putting in the numerical values,

$$R_y = 0.8 \times 25 \sin 60° = 17.32 \text{ N}.$$

Combining the rectangular components R_x and R_y,

$$\text{Resultant force exerted by fluid on vane, } R = \sqrt{(R_x^2 - R_y^2)}$$

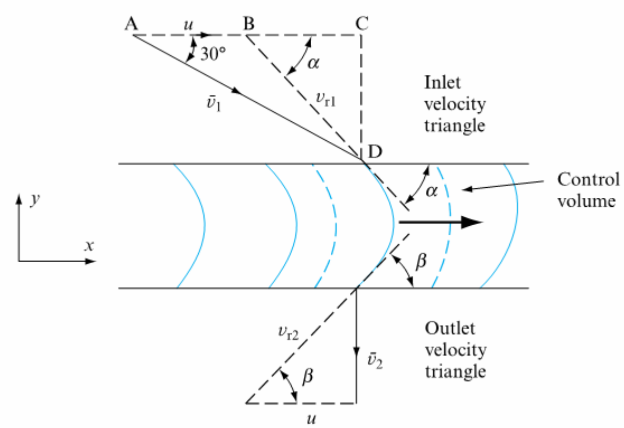
$$= \sqrt{(14^2 + 17.32^2)} = \mathbf{22.27 \text{ N}}.$$

This resultant force R will be inclined to the x direction at an angle $\phi = \tan^{-1}(R_y/R_x) = \tan^{-1}(17.32/14) =$ **51°3′**.

FORCE EXERTED WHEN A JET IS DEFLECTED BY A MOVING CURVED VANE

If a jet of fluid is to be deflected by a moving curved vane without impact at the inlet to the vane, the relation between the direction of the jet and the tangent to the curve of the vane at inlet must be such that the relative velocity of the fluid at inlet is tangential to the vane. The force in the direction of motion of the vane will be equal to the rate of change of momentum of the fluid in the direction of motion, i.e. the mass deflected per second multiplied by the change of velocity in that direction.

EX.2 A jet of water 100 mm in diameter leaves a nozzle with a mean velocity $\overline{v_1}$ of 36 m s^{-1} (Fig.) and is deflected by a series of vanes moving with a velocity u of 15 m s^{-1} in a direction at 30° to the direction of the jet, so that it leaves the vane with an absolute mean velocity $\overline{v_2}$ which is at right angles to the direction of motion of the vane. Owing to friction, the velocity of the fluid relative to the vane at outlet $\overline{vr_2}$ is equal to 0.85 of the relative velocity $\overline{vr_1}$ at inlet. Calculate (a) the inlet angle α and outlet angle β of the vane which will permit the fluid to enter and leave the moving vane tangentially without shock, and (b) the force exerted on the series of vanes in the direction of motion u.



(a) To determine the inlet angle α, consider the inlet velocity triangle. The velocity of the fluid relative to the vane at inlet, $\bar{v}_{r1}$, must be tangential to the vane and make an angle α with the direction of motion,

$$\tan \alpha = CD/BC = \bar{v}_1 \sin 30°/(\bar{v}_1 \cos 30° - u).$$

Putting $\bar{v}_1 = 36$ m s^{-1} and $u = 15$ m s^{-1},

$$\tan \alpha = 36 \times 0.5/(36 \times 0.866 - 15) = 1.113,$$

$$\alpha = \mathbf{48°3'}.$$

To determine the outlet angle β, if $\bar{v}_2$ has no component in the direction of motion, the outlet velocity triangle is right angled, $\cos \beta = u/\bar{v}_{r2}$, but $\bar{v}_{r2} = 0.85\bar{v}_{r1}$ and, from the inlet triangle,

$$\bar{v}_{r1} = CD/\sin \alpha = \bar{v}_1 \sin 30°/\sin \alpha.$$

Therefore

$$\cos\beta = \frac{u\sin\alpha}{0.85v_1\sin 30°} = \frac{15\times 0.744}{0.85\times 36\times 0.5} = 0.729,$$

$$\beta = \mathbf{43°11'}.$$

(b) Since the jet strikes a series of vanes, perhaps mounted on the periphery of a wheel, so that as each vane moves on its place is taken by the next in the series, the average length of the jet does not alter and the whole flow from the nozzle of diameter d is deflected by the vanes.

$$R_x = \dot{m}(v_{in} - v_{out})_x$$

$$\begin{matrix}\text{Mass per unit time}\\ \text{entering control}\\ \text{volume}\end{matrix} = \dot{m} = \begin{matrix}\text{Mass per unit time}\\ \text{leaving nozzle}\end{matrix}$$

$$= \rho(\pi/4)d^2\bar{v}_1,$$

$$v_{in} = \text{Component of } \bar{v}_1 \text{ in } x \text{ direction} = \bar{v}_1 \cos 30°,$$

$$v_{out} = \text{Component of } \bar{v}_2 \text{ in } x \text{ direction} = \bar{v}_2 \cos 90° = 0.$$

Substituting in (I),

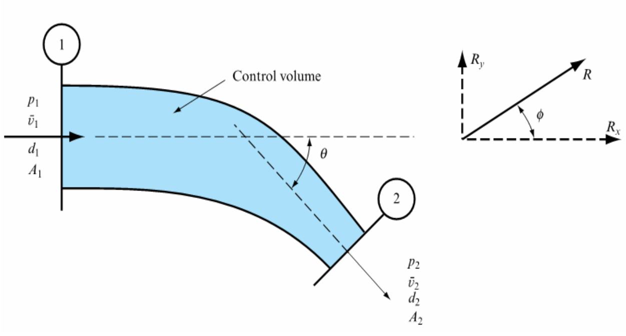
$$\begin{matrix}\text{Force on vanes in}\\ \text{direction of motion}\end{matrix} = R_x = \rho(\pi/4)d^2\bar{v}_1 \times \bar{v}_1 \cos 30°.$$

Putting in the numerical values,

$$\begin{matrix}\text{Force on vanes in}\\ \text{direction of motion}\end{matrix} = 1000 \times (\pi/4)(0.1)^2 \times 36 \times 36 \times 0.866\ \text{N} = \mathbf{8816\ N}.$$

FORCE EXERTED ON PIPE BENDS AND CLOSED CONDUITS

EX.3 A pipe bend tapers from a diameter of d1 of 500 mm at inlet (see Fig.) to a diameter of d2 of 250 mm at outlet and turns the flow through an angle θ of 45°. Measurements of pressure at inlet and outlet show that the pressure p1 at inlet is 40 kN m^{-2} and the pressure p2 at outlet is 23 kN m^{-2} . If the pipe is conveying oil which has a density ρ of 850 kg m^{-3} , calculate the magnitude and direction of the resultant force on the bend when the oil is flowing at the rate of 0.45 $m^3\ s^{-1}$. The bend is in a horizontal plane.



Mass per unit time entering control volume = ρQ.

in the x direction:

$$(F_1 + F_3)_x = \dot{m}(v_{\text{out}} - v_{\text{in}})_x$$

and, since $R_x = -(F_1)_x$,

$$R_x = (F_3)_x - \dot{m}(v_{\text{out}} - v_{\text{in}})_x.$$

Now $(F_3)_x = p_1A_1 - p_2A_2 \cos\theta$,

$$v_{\text{out}} = \text{Component of } \bar{v}_2 \text{ in } x \text{ direction} = \bar{v}_2 \cos\theta,$$

$$v_{\text{in}} = \text{Component of } \bar{v}_1 \text{ in } x \text{ direction} = \bar{v}_1.$$

Substituting in (I),

$$R_x = p_1A_1 - p_2A_2 \cos\theta - \rho Q(\bar{v}_2 \cos\theta - \bar{v}_1).$$

Resolving in the y direction,

$$(F_1 + F_3)_y = \dot{m}(v_{\text{out}} - v_{\text{in}})_y$$

and, since $R_y = -(F_1)_y$,

$$R_y = (F_3)_y - \dot{m}(v_{\text{out}} - v_{\text{in}})_y.$$

Now, $(F_3)_y = 0 + p_2A_2 \sin\theta$,

$$v_{\text{out}} = \text{Component of } \bar{v}_2 \text{ in } y \text{ direction} = -\bar{v}_2 \sin\theta,$$

$$v_{\text{in}} = \text{Component of } \bar{v}_1 \text{ in } y \text{ direction} = 0.$$

Substituting in (III),

$$R_y = p_2A_2 \sin\theta + \rho Q\bar{v}_2 \sin\theta.$$

For the given problem,

$$A_1 = (\pi/4)d_1^2 = (\pi/4)(0.5)^2 = 0.196\ 35\ \text{m}^2,$$

$$A_2 = (\pi/4)d_2^2 = (\pi/4)(0.25)^2 = 0.049\ 09\ \text{m}^2,$$

$$Q = 0.45\ \text{m}^3\ \text{s}^{-1},$$

$$\bar{v}_1 = Q/A_1 = 0.45/0.196\,35 = 2.292\ \text{m s}^{-1},$$

$$\bar{v}_2 = Q/A_2 = 0.45/0.049\,09 = 9.167\ \text{m s}^{-1}.$$

Putting $\rho = 850$ kg m^{-3}, $\theta = 45°$, $p_1 = 40$ kN m^{-2}, $p_2 = 23$ kN m^{-2}, and substituting in equation (II),

$$\begin{aligned} R_x &= 40 \times 10^3 \times 0.196\,35 - 23 \times 10^3 \times 0.049\,09 \cos 45° \\ &\quad - 850 \times 0.45(9.167 \cos 45° - 2.292)\ \text{N} \\ &= 10^3(7.855 - 0.798 - 1.603)\ \text{N} \\ &= \mathbf{5.454 \times 10^3\ N}. \end{aligned}$$

Substituting in equation (IV),

$$\begin{aligned} R_y &= 23 \times 10^3 \times 0.049\,09 \sin 45° + 850 \times 0.45 \times 9.167 \sin 45°\ \text{N} \\ &= 10^3(0.798 + 2.479)\ \text{N} \\ &= \mathbf{3.277 \times 10^3\ N}. \end{aligned}$$

Combining the x and y components,

$$\begin{aligned} \text{Resultant force on bend, } R &= \sqrt{(R_x^2 + R_y^2)} \\ &= \sqrt{(5.454^2 + 3.277^2)}\ \text{kN} \\ &= \mathbf{6.362\ kN}. \end{aligned}$$

The inclination of R to the x direction is given by

$$\phi = \tan^{-1}(R_y/R_x) = \tan^{-1}(3.277/5.454) = \mathbf{31°}.$$

MECHANICAL ENERGY OF A FLOWING FLUID

An element of fluid, as shown in Fig.1, will possess potential energy due to its height z above datum and kinetic energy due to its velocity v, in the same way as any other object. For an element of weight mg,

$$\text{Potential energy} = mgz,$$

$$\text{Potential energy per unit weight} = z,$$

$$\text{Kinetic energy} = \tfrac{1}{2}mv^2,$$

$$\text{Kinetic energy per unit weight} = v^2/2g.$$

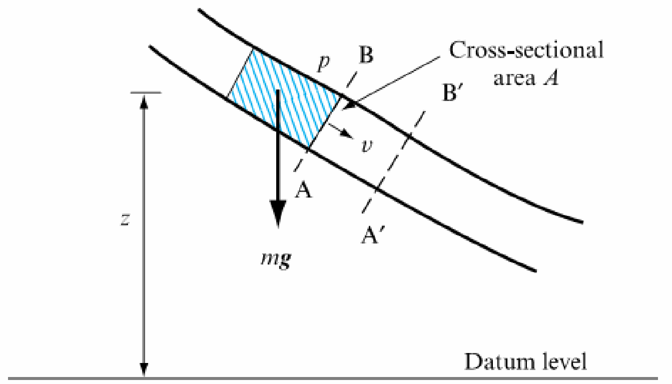


FIG.1
Energy of a flowing fluid

A steadily flowing stream of fluid can also do work because of its pressure. At any given cross-section, the pressure generates a force and, as the fluid flows, this cross-section will move forward and so work will be done. If the pressure at a section AB is p and the area of the cross-section is A,

After a weight mg of fluid has flowed along the streamtube, section AB will have moved to A′B′:

$$\text{Volume passing AB} = mg/\rho g = m/\rho.$$

Therefore,

$$\text{Distance AA}' = m/\rho A,$$

$$\text{Work done} = \text{Force} \times \text{Distance AA}'$$

$$= pA \times m/\rho A,$$

$$\text{Work done per unit weight} = p/\rho g.$$

Thus, Bernoulli's equation states that, for steady flow of a frictionless fluid along a streamline, the total energy per unit weight remains constant from point to point although its division between the three forms of energy may vary:

Pressure energy per unit weight	+	Kinetic energy per unit weight	+	Potential energy per unit weight	=	Total energy per unit weight	=	constant,

$$p/\rho g \times v^2/2g + z = H.$$

Each of these terms has the dimension of a length, or head, and they are often referred to as the pressure head $p/\rho g$, the velocity head $v^2/2g$, the potential head z and the total head H. Between any two points, suffixes 1 and 2, on a streamline,

$$\frac{p_1}{\rho_1 g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho_2 g} + \frac{v_2^2}{2g} + z_2$$

or

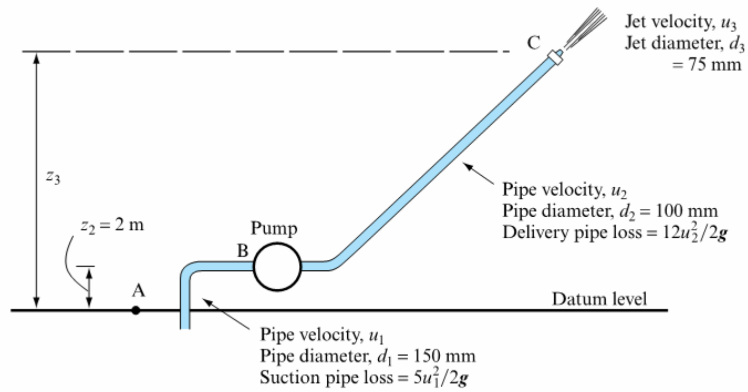
Total energy per unit weight at 1 = Total energy per unit weight at 2,

Energy could have been supplied by introducing a pump; equally, energy could have been lost by doing work against friction or in a machine such as a turbine. Bernoulli's equation can be expanded to include these conditions, giving

Total energy per unit weight at 1	=	Total energy per unit weight at 2	+	Loss per unit weight	+	Work done per unit weight	−	Energy supplied per unit weight

$$\frac{p_1}{\rho_1 g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho_2 g} + \frac{v_2^2}{2g} + z_2 + h + w - q.$$

EX.1 A fire engine pump develops a head of 50 m, i.e. it increases the energy per unit weight of the water passing through it by 50 N m N^{-1} . The pump draws water from a sump at A (Fig.) through a 150 mm diameter pipe in which there is a loss of energy per unit weight due to friction h1 = 5 $u_1^2/2g$ varying with the mean velocity u1 in the pipe, and discharges it through a 75 mm nozzle at C, 30 m above the pump, at the end of a 100 mm diameter delivery pipe in which there is a loss of energy per unit weight h2 = 12 $u_2^2/2g$. Calculate (a) the velocity of the jet issuing from the nozzle at C and (b) the pressure in the suction pipe at the inlet to the pump at B.



(a) We can apply Bernouilli's equation between two points, one of which will be C, since we wish to determine the jet velocity u3, and the other a point at which the conditions are known, such as a point A on the free surface of the sump where the pressure will be atmospheric, so that pA = 0, the velocity vA will be zero if the sump is large, and A can be taken as the datum level so that zA = 0.

Then,

$$\begin{matrix}\text{Total energy}\\ \text{per unit}\\ \text{weight at A}\end{matrix} = \begin{matrix}\text{Total energy}\\ \text{per unit}\\ \text{weight at C}\end{matrix} + \begin{matrix}\text{Loss in}\\ \text{inlet}\\ \text{pipe}\end{matrix} - \begin{matrix}\text{Energy per}\\ \text{unit weight}\\ \text{supplied by}\\ \text{pump}\end{matrix} + \begin{matrix}\text{Loss in}\\ \text{discharge}\\ \text{pipe,}\end{matrix} \qquad \text{(I)}$$

$$\begin{matrix}\text{Total energy}\\ \text{per unit}\\ \text{weight at A}\end{matrix} = \frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = 0,$$

$$\begin{matrix}\text{Total energy}\\ \text{per unit}\\ \text{weight at C}\end{matrix} = \frac{p_C}{\rho g} + \frac{u_3^2}{2g} + z_3,$$

$$p_C = \text{Atmospheric pressure} = 0,$$

$$z_3 = 30 + 2 = 32 \text{ m}.$$

Therefore,

$$\begin{matrix}\text{Total energy}\\ \text{per unit}\\ \text{weight at C}\end{matrix} = 0 + u_3^2/2g + 32 = u_3^2/2g + 32 \text{ m}.$$

Loss in inlet pipe, $h_1 = 5u_1^2/2g$,

Energy per unit weight supplied by pump = 50 m,

Loss in delivery pipe, $h_2 = 12u_2^2/2g$.

Substituting in (I),

$$0 = (u_3^2/2g + 32) + 5u_1^2/2g - 50 + 12u_2^2/2g,$$

$$u_3^2 + 5u_1^2 + 12u_2^2 = 2g \times 18. \qquad \text{(II)}$$

From the continuity of flow equation,

$$(\pi/4)d_1^2 u_1 = (\pi/4)d_2^2 u_2 = (\pi/4)d_3^2 u_3;$$

therefore,

$$u_1 = \left(\frac{d_3}{d_1}\right)^2 u_3 = \left(\frac{75}{150}\right)^2 u_3 = \frac{1}{4}u_3,$$

$$u_2 = \left(\frac{d_3}{d_2}\right)^2 u_3 = \left(\frac{75}{100}\right)^2 u_3 = \frac{9}{16}u_3.$$

Substituting in equation (II),

$$u_3^2[1 + 5 \times (\tfrac{1}{4})^2 + 12 \times (\tfrac{9}{16})^2] = 2g \times 18,$$

$$5.109u_3^2 = 2g \times 18$$

$$u_3 = \mathbf{8.314\ m\ s^{-1}}.$$

(b) If p_B is the pressure in the suction pipe at the pump inlet, applying Bernoulli's equation to A and B,

Total energy per unit weight at A = Total energy per unit weight at B + Loss in inlet pipe,

$$0 = (p_B/\rho g + u_1^2/2g + z_2) + 5u_1^2/2g,$$

$$p_B/\rho g = -z_2 - 6u_1^2/2g,$$

$$z_2 = 2\text{ m},\ u_1 = \tfrac{1}{4}u_3 = 8.314/4 = 2.079\text{ m s}^{-1},$$

$$p_B/\rho g = -(2 + 6 \times 2.079^2/2g) = -(2 + 1.32) = -3.32\text{ m},$$

$$p_B = -1000 \times 9.81 \times 3.32 = \mathbf{32.569\ kN\ m^{-2}\ below\ atmospheric\ pressure}.$$

STEADY FLOW ENERGY EQUATION

It is possible to develop an energy equation for the steady flow of a fluid from the principle of conservation of energy, which states:

For any mass system, the net energy supplied to the system equals the increase of energy of the system plus the energy leaving the system.

Thus, if ΔE is the increase of energy of the system, ΔQ is the energy supplied to the system and ΔW the energy leaving the system, then, considering the energy balance for the system,

$$\Delta E = \Delta Q - \Delta W.$$

The energy of a mass of fluid will have the following forms:

1. internal energy due to the activity of the molecules of the fluid forming the mass;
2. kinetic energy due to the velocity of the mass of fluid itself;
3. potential energy due to the mass of fluid being at a height above the datum level and acted upon by gravity.

For example, q may be in the form of heat energy, while w might take the form of mechanical work as shown in Fig.2.

Energy entering at AA in unit time, E_1 = Kinetic energy + Potential energy + Internal energy $= \dot{m}(\frac{1}{2}v_1^2 + gz_1 + e_1)$,

Energy leaving at BB in unit time, E_2 $= \dot{m}(\frac{1}{2}v_2^2 + gz_2 + e_2)$.

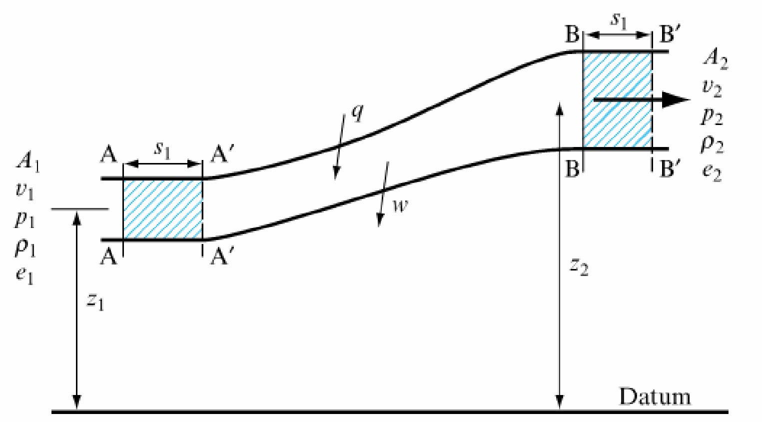


FIG.2
Steady flow energy equation

Therefore

$$\Delta E = E_2 - E_1 = \dot{m}[\tfrac{1}{2}(v_2^2 - v_1^2) + g(z_2 - z_1) + (e_2 - e_1)].$$

This change of energy has occurred because energy has entered and left the fluid between AA and BB. Also, work is done on the fluid in the control volume between the two sections AA and BB by the fluid entering at AA and by the fluid in the control volume as it leaves at BB.

Energy entering per unit time between AA and BB $= \dot{m}q$,

Energy leaving per unit time between AA and BB $= \dot{m}w$.

As the fluid flows, work will be done by the fluid entering at AA since a force $p_1 A_1$ is exerted on the cross-section by the pressure p_1 and, in unit time t, the fluid which was at AA will move a distance s_1 to A′A′:

Work done in unit time on the fluid at AA $= p_1 A_1 s_1 / t$.

But,

Mass passing per unit time, $\dot{m} = \rho_1 A_1 s_1 / t$;

therefore,

$A_1 s_1 = \dot{m}/\rho_1$,

Work done per unit time on the fluid at AA $= p_1 \dot{m}/\rho_1$.

Similarly,

Change of energy of the system, ΔE = Work done on fluid at AA
− Work done by fluid at BB
+ Energy entering between AA and BB
− Energy leaving between AA and BB

$$= \dot{m}p_1/\rho_1 - \dot{m}p_2/\rho_2 + \dot{m}q - \dot{m}w$$

$$= \dot{m}(q - w + p_1/\rho_1 - p_2/\rho_2).$$

Comparing equations to obtain on

$$\tfrac{1}{2}(v_2^2 - v_1^2) + g(z_2 - z_1) + (e_2 - e_1) = p_1/\rho_1 - p_2/\rho_2 + q - w.$$

Thus,

$$gz_1 + \tfrac{1}{2}v_1^2 + (p_1/\rho_1 + e_1) + q - w = gz_2 + \tfrac{1}{2}v_2^2 + (p_2/\rho_2 + e_2).$$

The terms $(p_1/\rho_1 + e_1)$ and $(p_2/\rho_2 + e_2)$ can be replaced by the enthalpies H_1 and H_2, giving

$$gz_1 + \tfrac{1}{2}v_1^2 + H_1 + q - w = gz_2 + \tfrac{1}{2}v_2^2 + H_2.$$

APPLICATIONS OF THE STEADY FLOW ENERGY EQUATION

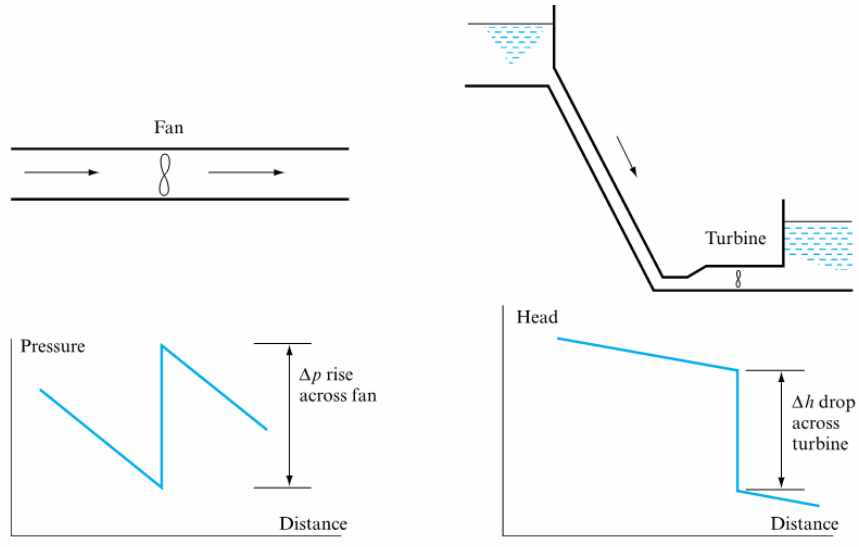
Applying the steady flow energy equation to a wide range of fluid flow conditions. Reference to the control volume AA′BB′ in Fig.2 allows the steady flow energy equation to be recast, from equation for a constant density, i.e. incompressible flow, as

$$p_1 + \tfrac{1}{2}\rho v^2 + \rho g z_1 + \rho q - \rho w = p_2 + \tfrac{1}{2}\rho v^2 + \rho g z_2 + \rho \Delta e$$

The terms ρq and ρw may be identified, for example in Fig.3, as the pressure rise across a pump or fan maintaining flow through a pipe or duct, or the pressure drop across a turbine. The term ρΔe represents an energy 'loss' due to frictional or separation losses between the boundaries of the control volume.

Again the value of ρΔe will depend upon the flow rate in the system and on the fluid and conduit parameters; appropriate expressions defining these energy transfers.

FIG.3
Energy addition or extraction at rotodynamic machines



Chapter Three
Flow in Conduits

Laminar Flow and Turbulent Flow

Flow in a conduit is classified as being either laminar or turbulent, depending on the magnitude of the Reynolds number. The original research involved visualizing flow in a glass tube as shown in Fig. 1a. Reynolds (1) in the 1880s injected dye into the center of the tube and observed the following:

• When the velocity was low, the streak of dye flowed down the tube with little expansion, as shown in Fig.1b. However, if the water in the tank was disturbed, the streak would shift about in the tube.

• If velocity was increased, at some point in the tube, the dye would all at once mix with the water as shown in Fig.1c.

• When the dye exhibited rapid mixing (Fig.1c), illumination with an electric spark revealed eddies in the mixed fluid as shown in Fig.1d.

The flow regimes shown in Fig. 10.1 are laminar flow (Fig. 10.1b) and turbulent flow (Figs.1c and 1d). Reynolds showed that the onset of turbulence was related to a π-group that is now called the Reynolds number in honor of Reynolds' pioneering work. Reynolds discovered that if the fluid in the upstream reservoir was not completely still or if the pipe had some vibrations, then the change from laminar to turbulent flow occurred at Re 2100 However, if conditions were ideal, it was possible to reach a much higher Reynolds number before the flow became turbulent. Reynolds also found that, when going from high velocity to low velocity, the change back to laminar flow occurred at Re 2000. Based on Reynolds' experiments, engineers use guidelines to establish whether or not flow in a conduit will be laminar or turbulent. The guidelines used in this text are as follows:

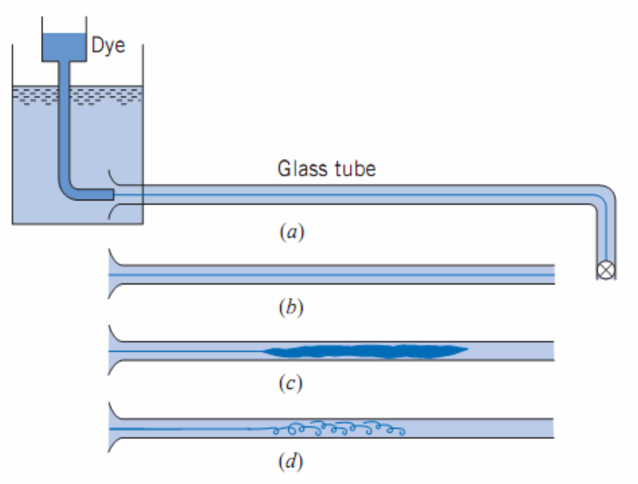
$\mathrm{Re} \leq 2000$	laminar flow
$2000 \leq \mathrm{Re} \leq 3000$	unpredictable
$\mathrm{Re} \geq 3000$	turbulent flow

There are several equations for calculating Reynolds number in a pipe.

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{\rho VD}{\mu} = \frac{4Q}{\pi D\nu} = \frac{4\dot{m}}{\pi D\mu}$$

Figure 1

Reynolds' experiment.
(a) Apparatus.
(b) Laminar flow of dye in tube.
(c) Turbulent flow of dye in tube.
(d) Eddies in turbulent flow.



Developing Flow and Fully Developed Flow

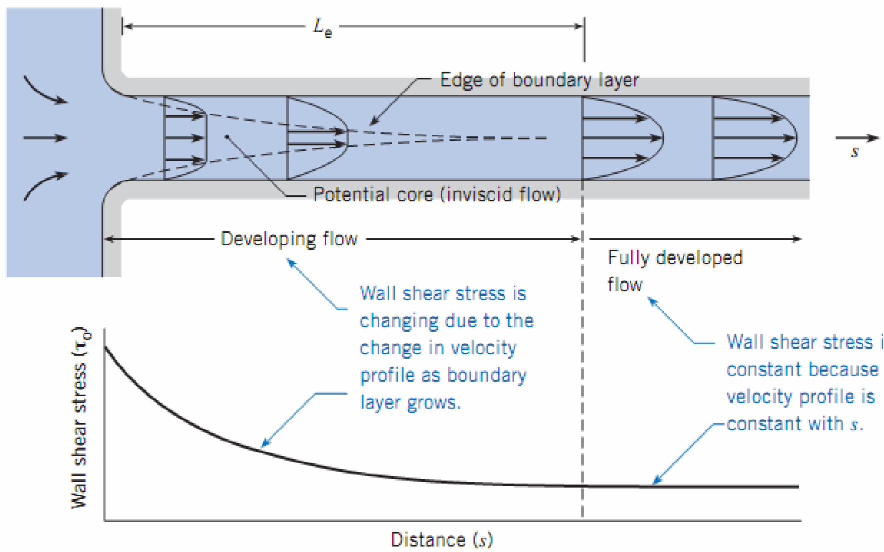
Flow in a conduit is classified as being developing flow or fully developed flow. For example, consider laminar fluid entering a pipe from a reservoir as shown in Fig. 2. As the fluid moves down the pipe, the velocity distribution changes in the streamwise direction as viscous effects cause the plug-type profile to gradually change into a parabolic profile. This region of changing velocity profile is called ***developing flow***. After the parabolic distribution is achieved, the flow profile remains unchanged in the streamwise direction, and flow is called ***fully developed flow***. The distance required for flow to develop is called the ***entrance length(L_e).*** This length depends on the shear stress that acts on the pipe wall. For laminar flow, the wall shear-stress distribution is shown in Fig. 2. Near the pipe entrance, the radial velocity gradient (change in velocity with distance from the wall) is high, so the shear stress is large. As the velocity profile progresses to a parabolic shape, the velocity gradient and the wall shear stress decrease until a constant value is achieved. The entry length is defined as the distance at which the shear stress reaches to within 2% of the fully developed value. Correlations for entry length are:

$$\frac{L_e}{D} = 0.05\,\text{Re} \qquad (\text{laminar flow: Re} \le 2000)$$

$$\frac{L_e}{D} = 50 \qquad (\text{turbulent flow: Re} \ge 3000)$$

Figure 2

In developing flow, the wall shear stress is changing. In fully developed flow, the wall shear stress is constant.

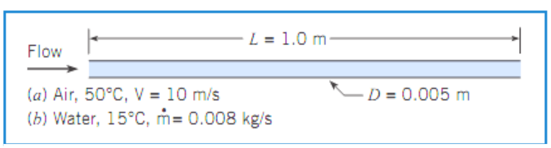


Ex.1 Consider fluid flowing in a round tube of length 1 m and diameter 5 mm. Classify the flow as laminar or turbulent and calculate the entrance length for
(a) air (50°C) with a speed of 12 m s and
(b) water (15°C) with a mass flow rate of m = 8 g/s.

Ans. **Properties:**

1. Air (50°C), Table A.3, $\nu = 1.79 \times 10^{-5}\ \text{m}^2/\text{s}$.
2. Water (15°C), Table A.5, $\mu = 1.14 \times 10^{-3}\ \text{N} \cdot \text{s/m}^2$.

Sketch:



a. Air

$$\text{Re} = \frac{VD}{\nu} = \frac{(12\ \text{m/s})(0.005\text{m})}{1.79 \times 10^{-5}\ \text{m}^2/\text{s}} = 3350$$

Since Re > 3000, the $\boxed{\text{flow is turbulent.}}$

$$L_e = 50D = 50(0.005\ \text{m}) = \boxed{0.25\ \text{m}}$$

b. Water

$$\text{Re} = \frac{4\dot{m}}{\pi D \mu} = \frac{4(0.008\ \text{kg/s})}{\pi(0.005\text{m})(1.14 \times 10^{-3}\ \text{N} \cdot \text{s/m}^2)}$$

$$= 1787$$

Since Re < 2000 , the $\boxed{\text{flow is laminar.}}$

$$L_e = 0.05\text{Re}D = 0.05(1787)(0.005\text{m}) = \boxed{0.447\ \text{m}}$$

Derivation of the Darcy-Weisbach Equation

To derive the Darcy-Weisbach equation, start with the situation shown in Fig.3. Assume fully developed and steady flow in a round tube of constant diameter D. Situate a cylindrical control volume of diameter D and length inside the pipe. Define a coordinate system with an axial coordinate in the streamwise direction (s direction) and a radial coordinate in the r direction.

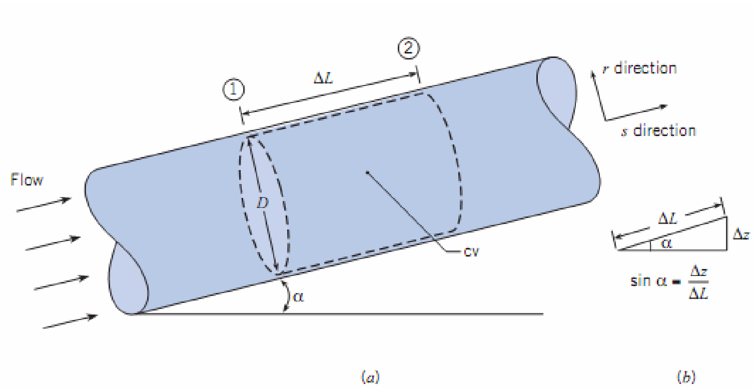
$$\sum \mathbf{F} = \frac{d}{dt}\int_{cv} v\rho d\Psi + \int_{cs} v\rho \mathbf{V} \cdot d\mathbf{A}$$

(Net forces) = (Momentum accumulation rate) + (Net efflux of momentum)

Select the streamwise direction and analyze each of the three terms in Eq. (above). The net efflux of momentum is zero because the velocity distribution at section 2 is identical to the velocity distribution at section 1.

Figure.3

Initial situation for the derivation of the Darcy-Weisbach equation.



The momentum accumulation term is also zero because the flow is steady. This Eq. is simplifies to Forces are shown in Fig. 4. Summing of forces in the streamwise direction gives

$$F_{\text{pressure}} + F_{\text{shear}} + F_{\text{weight}} = 0$$

$$(p_1 - p_2)\left(\frac{\pi D^2}{4}\right) - \tau_0(\pi D \Delta L) - \gamma\left[\left(\frac{\pi D^2}{4}\right)\Delta L\right]\sin\alpha = 0$$

Figure 3b shows that $\sin\alpha = (\Delta z / \Delta L)$

$$(p_1 + \gamma z_1) - (p_2 + \gamma z_2) = \frac{4\Delta L \tau_0}{D}$$

Next, apply the energy equation to the control volume shown in Fig. 4. Recognize that hp = ht = 0, V1 = V2, and $\alpha_1 = \alpha_2$. Thus, the energy equation reduces to

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + h_L$$

$$(p_1 + \gamma z_1) - (p_2 + \gamma z_2) = \gamma h_L$$

and replace ΔL by L. Also, introduce a new symbol h_f to represent head loss in pipe.

$$h_f = \begin{pmatrix}\text{head loss}\\ \text{in a pipe}\end{pmatrix} = \frac{4L\tau_0}{D\gamma}$$

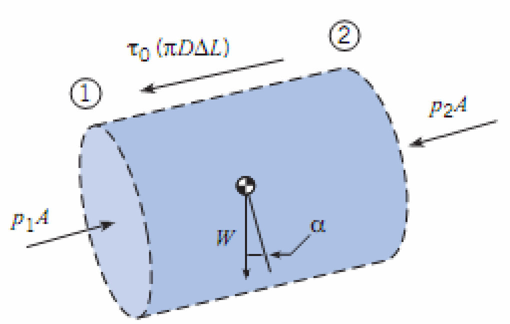
Rearrange the right side

$$h_f = \left(\frac{L}{D}\right)\left\{\frac{4\tau_0}{\rho V^2/2}\right\}\left\{\frac{\rho V^2/2}{\gamma}\right\} = \left\{\frac{4\tau_0}{\rho V^2/2}\right\}\left(\frac{L}{D}\right)\left\{\frac{V^2}{2g}\right\}$$

Define a new π-group called the *friction factor* f that gives the ratio of wall shear stress (τ_o) to kinetic pressure ($\rho V^2/2$):

$$f \equiv \frac{(4\cdot\tau_0)}{(\rho V^2/2)} \approx \frac{\text{shear stress acting at the wall}}{\text{kinetic pressure}}$$

Figure 4
Force diagram



Darcy friction factor, Darcy-Weisbach friction factor, and the resistance coefficient. There is also another coefficient called the Fanning friction factor, often used by chemical engineers, which is related to the Darcy-Weisbach friction factor by a factor of 4.

$$f_{\text{Darcy}} = 4f_{\text{Fanning}}$$

This text uses only the Darcy-Weisbach friction factor. Combining Eqs. Above to gives the Darcey-Weisbach equation:

$$h_f = f\frac{L}{D}\frac{V^2}{2g}$$

Stress Distributions in Pipe Flow

In pipe flow the pressure acting on a plane that is normal to the direction of flow is hydrostatic. This means that the pressure distribution varies linearly as shown in Fig.5. The reason that the pressure distribution is hydrostatic.

To derive an equation for the shear-stress variation, consider flow of a Newtonian fluid in a round tube that is inclined at an angle α with respect to the horizontal as shown in Fig.6. Assume that the flow is fully developed, steady, and laminar. Define a cylindrical control volume of length and radius r.

Apply the momentum equation in the s direction. The net momentum efflux is zero because the flow is fully developed; that is, the velocity distribution at the inlet is the same as the velocity distribution at the exit. The momentum accumulation is also zero because the flow is steady.

Fig.5

For fully developed flow in a pipe, the pressure distribution on an area normal to streamlines is hydrostatic.

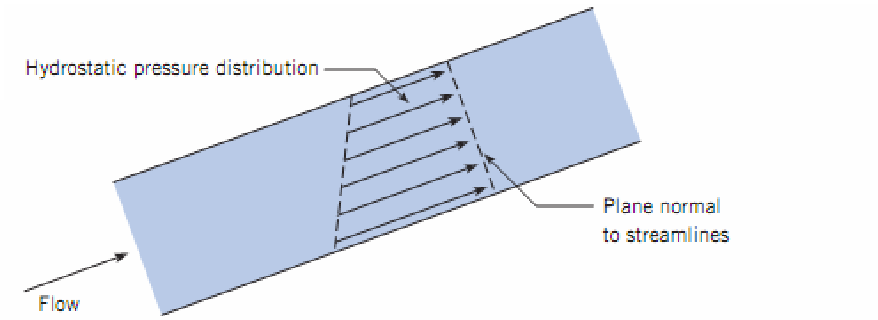
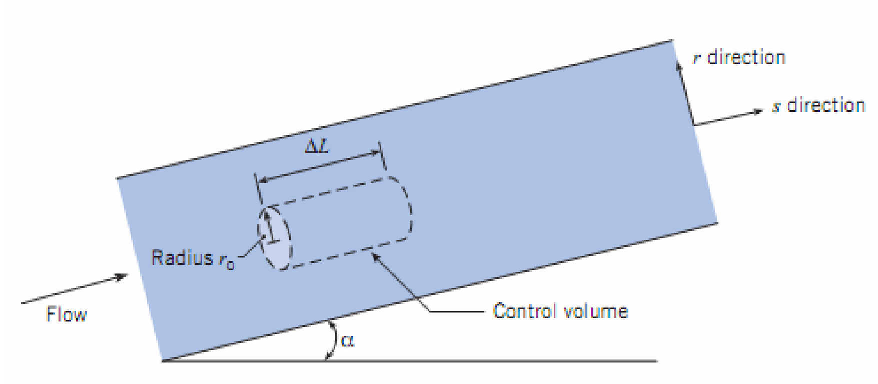


Fig.6

Sketch for derivation of an equation for shear stress.



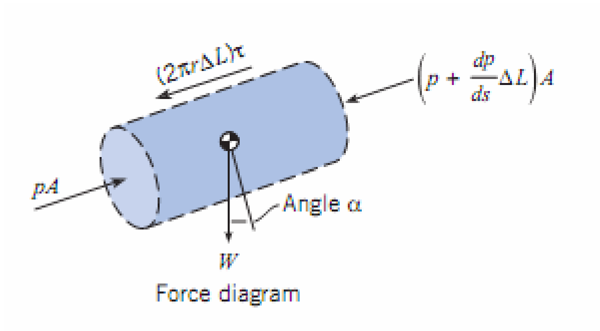
The momentum equation simplifies to force equilibrium:-

$$\sum F_s = F_{\text{pressure}} + F_{\text{weight}} + F_{\text{shear}} = 0$$

Analyze each term in of above Eq. using the force diagram shown in Fig.7:

$$pA - \left(p + \frac{dp}{ds}\Delta L\right)A - W\sin\alpha - \tau(2\pi r)\Delta L = 0$$

Figure 7
Force diagram corresponding to the control volume defined in Fig.5

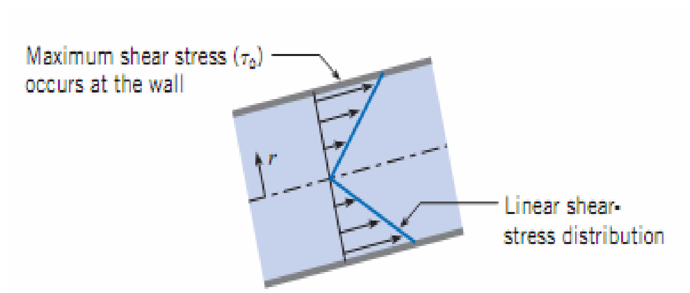


Let $W = \gamma A\Delta L$ and $\sin\alpha = \Delta z/\Delta L$ let as shown in Fig. 3b. Next divided above Eq. by $A\Delta L$

$$\tau = \frac{r}{2}\left[-\frac{d}{ds}(p+\gamma z)\right]$$

Equation above shows that the shear-stress distribution varies linearly with r as shown in Fig.8. Notice that the shear stress is zero at the centerline, it reaches a maximum value of at the wall, and the variation is linear in between. This linear shear stress variation applies to both laminar and turbulent flow.

Figure 8
In fully developed flow (laminar or turbulent),
the shear-stress distribution on an area that is normal to streamlines is linear.



1-Laminar Flow in a Round Tube

Laminar flow is a flow regime in which fluid motion is smooth, the flow occurs in layers (laminar), and the mixing between layers occurs by molecular diffusion, a process that is much slower than turbulent mixing. According to Eq., laminar flow occurs when Laminar flow in a round tube is called Poiseuille flow or Hagen-Poiseuille flow in honor of pioneering researchers who studied low-speed flows in the 1840s.

Velocity Profile

To derive an equation for the velocity profile in laminar flow, begin by relating stress to rate-of-strain

$$\tau = \mu \frac{dV}{dy}$$

where y is the distance from the pipe wall. Change variables by letting $y = r_0 - r$, where r_0 is pipe radius and r is the radial coordinate. Next, use the chain rule of calculus:

$$\tau = \mu\left(\frac{dV}{dy}\right) = \mu\left(\frac{dV}{dr}\right)\left(\frac{dr}{dy}\right) = -\left(\mu\frac{dV}{dr}\right)$$

Substitute Eq. ($\tau = \frac{r}{2}\left[-\frac{d}{ds}(p+\gamma z)\right]$) into Eq. (above)

$$-\left(\frac{2\mu}{r}\right)\left(\frac{dV}{dr}\right) = \frac{d}{ds}(p+\gamma z)$$

In Eq. (above), the left side of the equation is a function of radius r, and the right side is a function of axial location s. This can be true if and only if each side of Eq. this is equal to a constant. Thus,

$$\text{constant} = \frac{d}{ds}(p+\gamma z) = \left(\frac{\Delta(p+\gamma z)}{\Delta L}\right) = \left(\frac{\gamma \Delta h}{\Delta L}\right)$$

where Δh is the change in piezometric head over a length ΔL of conduit.

$$\frac{dV}{dr} = -\left(\frac{r}{2\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right)$$

By Integrated :-

$$V = -\left(\frac{r^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) + C$$

To evaluate the constant of integration C in Eq. (above), apply the no-slip condition, which states that the velocity of the fluid at the wall is zero. Thus,

$$V(r = r_0) = 0$$

$$0 = -\frac{r_0^2}{4\mu}\left(\frac{\gamma \Delta h}{\Delta L}\right) + C$$

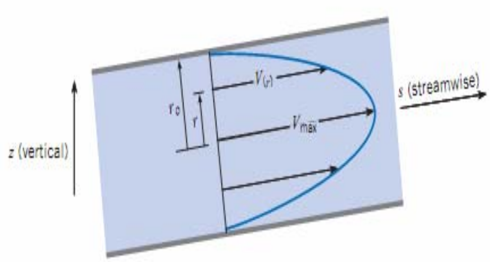
Solve for C and substitute the result

$$V = \frac{r_0^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p+\gamma z)\right] = -\left(\frac{r_0^2 - r^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right)$$

The maximum velocity occurs at $r = r_0$:

$$V_{max} = -\left(\frac{r_0^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right)$$

$$V(r) = -\left(\frac{r_0^2 - r^2}{4\mu}\right)\left(\frac{\gamma \Delta h}{\Delta L}\right) = V_{max}\left(1 - \left(\frac{r}{r_0}\right)^2\right)$$



Discharge and Mean Velocity V

To derive an equation for discharge Q, introduce the velocity profile

$$Q = \int V\,dA$$

$$= -\int_0^{r_0} \frac{(r_0^2 - r^2)}{4\mu}\left(\frac{\gamma \Delta h}{\Delta L}\right)(2\pi r\,dr)$$

Integrate Eq.

$$Q = -\left(\frac{\pi}{4\mu}\right)\left(\frac{\gamma\Delta h}{\Delta L}\right)\frac{(r^2 - r_0^2)^2}{2}\Bigg|_0^{r_0} = -\left(\frac{\pi r_0^4}{8\mu}\right)\left(\frac{\gamma\Delta h}{\Delta L}\right)$$

To derive an equation for mean velocity, apply $Q = \overline{V}A$

$$\overline{V} = -\left(\frac{r_0^2}{8\mu}\right)\left(\frac{\gamma\Delta h}{\Delta L}\right)$$

The final result is an equation for mean velocity in a round tube.

$$\overline{V} = -\left(\frac{D^2}{32\mu}\right)\left(\frac{\gamma\Delta h}{\Delta L}\right) = \frac{V_{max}}{2}$$

Head Loss and Friction Factor *f*

To derive an equation for head loss in a round tube, assume fully developed flow in the pipe shown in Fig. 9. Apply the energy equation from section 1 to 2 and simplify to give

$$\left(\frac{p_1}{\gamma} + z_1\right) = \left(\frac{p_2}{\gamma} + z_2\right) + h_L$$

Let $h_L = h_f$ and then

$$\left(\frac{p_1}{\gamma} + z_1\right) = \left(\frac{p_2}{\gamma} + z_2\right) + h_f$$

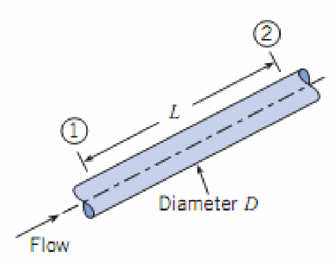


Fig. 9

$$\overline{V} = -\left(\frac{D^2}{32\mu}\right)\left(\frac{1}{\gamma}\right)\left(\frac{\Delta h}{\Delta L}\right) = -\left(\frac{\gamma D^2}{32\mu}\right)\frac{\left(\frac{p_2}{\gamma} + z_2\right) - \left(\frac{p_1}{\gamma} + z_1\right)}{\Delta L}$$

Reorganize Eq. and replace ΔL with L.

$$\left(\frac{p_1}{\gamma} + z_1\right) = \left(\frac{p_2}{\gamma} + z_2\right) + \frac{32\mu \overline{V} L}{D^2}$$

Then, gives an equation for head loss in a pipe $h_f = \dfrac{32\mu L\overline{V}}{\gamma D^2}$

Key assumptions on above Eq. are (a) laminar flow, (b) fully developed flow, (c) steady flow, and (d) Newtonian fluid.
To derive an equation for the friction factor f, combine Eq. (h_f) with the Darcy-Weisbach equation:-

$$h_f = \frac{32\mu L V}{\gamma D^2} = f\frac{L}{D}\frac{V^2}{2g}$$

$$\text{or } f = \left(\frac{32\mu L V}{\gamma D^2}\right)\left(\frac{D}{L}\right)\left(\frac{2g}{V^2}\right) = \frac{64\mu}{\rho D V} = \frac{64}{\text{Re}}$$

This Equation shows that the friction factor for laminar flow depends only on Reynolds number.

Ex.2 Oil (S 0.85) with a kinematic viscosity of $6 * 10^{-4}$ m^2/sec flows in a 15 cm pipe at a rate of 0.020 m^3/sec. What is the head loss per 100 m length of pipe?

Solution

1. Mean velocity

$$V = \frac{Q}{A} = \frac{0.020 \text{ m}^3/\text{s}}{(\pi D^2)/4} = \frac{0.020 \text{ m}^3/\text{s}}{\pi((0.15 \text{ m})^2/4)} = 1.13 \text{ m/s}$$

2. Reynolds number

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.13 \text{ m/s})(0.15 \text{ m})}{6 \times 10^{-4} \text{ m}^2/\text{s}} = 283$$

3. Since Re < 2000, the flow is laminar.
4. Head loss (laminar flow).

$$h_f = \frac{32\mu L V}{\gamma D^2} = \frac{32\rho\nu L V}{\rho g D^2} = \frac{32\nu L V}{g D^2}$$

$$= \frac{32(6 \times 10^{-4} \text{ m}^2/\text{s})(100 \text{ m})(1.13 \text{ m/s})}{(9.81 \text{ m/s}^2)(0.15 \text{ m})^2} = \boxed{9.83 \text{ m}}$$

Or

$$f = \frac{64}{\text{Re}} = \frac{64}{283} = 0.226$$

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = 0.226\left(\frac{100 \text{ m}}{0.15 \text{ m}}\right)\left(\frac{(1.13 \text{ m/s})^2}{2 \times 9.81 \text{ m/s}^2}\right)$$

$$= 9.83 \text{ m}$$

2-LAMINAR FLOWBETWEENPARALLEL PLATES

The element of fluid shown in Fig.10 can be considered a control volume into and from which the fluid flows or it can be considered amass of fluid at a particular moment. Considering it to be an instantaneous mass of fluid that is not accelerating in this steady, developed flow, Newton's second law takes the form

$$\sum F_x = 0 \quad \text{or} \quad p\,dy - (p+dp)dy + \tau dx - (\tau + d\tau)dx + \gamma\,dx\,dy \sin\theta = 0$$

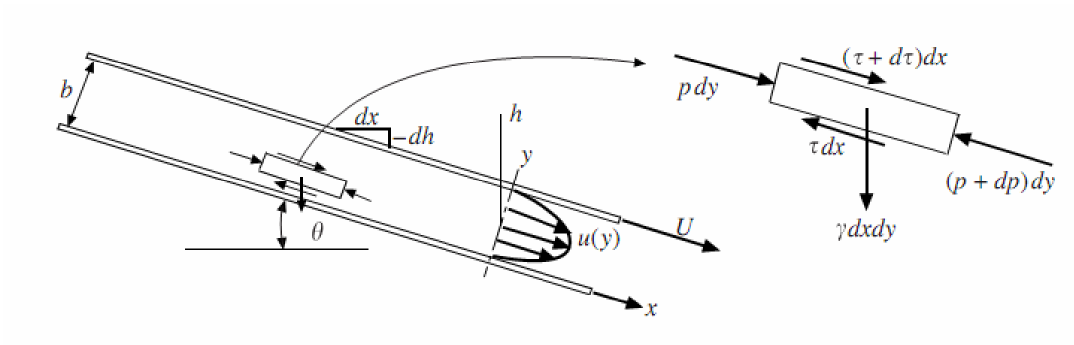


Figure 10 Steady, developed flow between parallel plates.

where τ is the shear on the wall of the element and γ is the specific weight of the fluid. We have assumed a unit length into the paper (in the z-direction). To simplify, divide by *dx dy* and use $dh = -\sin\theta\, dx$ with h measured in the vertical direction:

$$\frac{d\tau}{dy} = \frac{d}{dx}(p + \gamma h)$$

For this laminar flow, the shear stress is related to the velocity gradient by $\tau = \mu\, du/dy$, so that it is becomes

$$\mu \frac{d^2u}{dy^2} = \frac{d}{dx}(p + \gamma h)$$

The left-hand side is a function of y only for this developed flow (we assume a wide channel with an aspect ratio in excess of 8) and the right-handside is a function of x only. So, we can integrate twice on y to obtain

$$u(y) = \frac{1}{2\mu}\frac{d(p+\gamma h)}{dx} y^2 + Ay + B$$

Using the boundary conditions $u(0) = 0$ and $u(b) = U$, the constants of integration are evaluated and a parabolic profile results

$$u(y) = \frac{1}{2\mu}\frac{d(p+\gamma h)}{dx}(y^2 - by) + \frac{U}{b}y$$

If the plates are horizontal and $U = 0$, the velocity profile simplifies to

$$u(y) = \frac{\Delta p}{2\mu L}(by - y^2)$$

Let $d(p+\gamma h)/dx = -\Delta p/L$

The horizontal plates where Δp is the pressure drop, a positive quantity.
If the flow is due only to the top plate moving ,with zero pressure gradient , it is a Couette flow so that $u(y)=U\,y/b$ .If both plates are stationary and the flow is due only to a pressure gradient ,it is a Poiseuille flow.

Mean Velocity V

Let us consider several quantities of interest for the case of two fixed plates with U = 0. The first quantity of interest in the flow is the average velocity V. The average velocity is, assuming unit width of the plates

$$V = \frac{1}{b \times 1}\int u(y)dy$$

$$= \frac{\Delta p}{2b\mu L}\int_0^b (by - y^2)dr = \frac{\Delta p}{2b\mu L}\left[b\frac{b^2}{2} - \frac{b^3}{3}\right] = \frac{b^2\,\Delta p}{12\mu L}$$

The maximum velocity occurs at $y = b/2$ and is

$$u_{max} = \frac{\Delta p}{2\mu L}\left(\frac{b^2}{2} - \frac{b^2}{4}\right) = \frac{b^2\,\Delta p}{8\mu L} = \frac{2}{3}V$$

The pressure drop

$$\Delta p = \frac{12\mu L V}{b^2}$$

The shear stress at either wall can be found by considering a free body of length L in the channel. For a horizontal channel, the pressure force balances the shear force:

$$(b \times 1)\Delta p = 2(L \times 1)\tau_0 \qquad \therefore\ \tau_0 = \frac{b\,\Delta p}{2L}$$

In terms of the friction factor f, defined by

$$f = \frac{\tau_0}{\frac{1}{8}\rho V^2}$$

the head loss for the horizontal channel is

$$h_L = \frac{\Delta p}{\gamma} = f\frac{L}{2b}\frac{V^2}{2g}$$

Several of the above equations can be combined to find

$$f = \frac{48}{\text{Re}}$$

where $\text{Re} = bV/\nu$.

- ***If interest is in a horizontal channel flow where the top plate is moving and there is no pressure gradient, then the velocity profile would be the linear profile***

$$u(y) = \frac{U}{b}y$$

EX.3 The thin layer of rain at 20 °C flows down a parking lot at a relatively constant depth of 4mm.The area is 40 m wide with aslope of 8 cm over 60 m of length. Estimate (a)the flow rate, (b)shear at the surface, (c)the Reynolds number, and the velocity at the surface.

Solution: (*a*) The velocity profile can be assumed to be one-half of the profile a laminar flow. The average velocity would remain as given by

$$V = \frac{b^2\gamma h}{12\mu L}$$

where Δp has been replaced with γh. The flow rate is

$$Q = AV = bw\frac{b^2\gamma h}{12\mu L} = 0.004 \times 40\frac{0.004^2 \times 9810 \times 0.08}{12 \times 10^{-3} \times 60} = 2.80 \times 10^{-3}\ \text{m}^3/\text{s}$$

(*b*) The shear stress acts only at the solid wall, so Eq. (7.42) would provide

$$\tau_0 = \frac{b\gamma h}{L} = \frac{0.004 \times 9810 \times 0.08}{60} = 0.0523\ \text{Pa}$$

(*c*) The Reynolds number is

$$\text{Re} = \frac{bV}{\nu} = \frac{0.004}{10^{-6}} \times \frac{0.004^2 \times 9810 \times 0.08}{12 \times 10^{-3} \times 60} = 697$$

The Reynolds number is below 1500, so the assumption of laminar flow is acceptable.

Losses in Pipe Flow

The head loss is of considerable interest in pipe flows. $h_L = f\frac{L}{D}\frac{V^2}{2g}$ or $h_L = \frac{\Delta p}{\gamma} + z_2 - z_1$

So, once the friction factor is known, the head loss and pressure drop can be determined. The friction factor depends on a number of properties of the fluid and the pipe:

$$f = f(\rho, \mu, V, D, e)$$

Where the roughness height *e* accounts for the turbulence generated by the roughness elements.

$$f = f\left(\frac{e}{D}, \frac{VD\rho}{\mu}\right)$$

where e/D is termed the *relative roughness.*

Experimental data has been collected and presented in the form of the ***Moody diagram.***

	e (ft)	e (mm)
Riveted steel	~ 0.01	3
Concrete	~ 0.001-0.01	0.3-3
Wood	~ 0.001	0.3
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Drawn tubing	0.000005	0.0015

EX.1 A pressure drop of 500 kPa is measured over 200 m of a horizontal length of 8-cm-diameter cast iron pipe transporting water at 20 C. Estimate the flow rate using (a) the Moody diagram.

Solution: (a) The relative roughness

$$\frac{e}{D} = \frac{0.26}{80} = 0.00325$$

Assuming a completely turbulent flow, the friction factor from Chart $f = 0.026$. The head loss is

$$h_L = \frac{\Delta p}{\gamma} = \frac{500\,000}{9800} = 51 \text{ m}$$

The average velocity

$$V = \sqrt{\frac{2gDh_L}{fL}} = \sqrt{\frac{2 \times 9.8 \times 0.08 \times 51}{0.026 \times 200}} = 3.92 \text{ m/s}$$

We must check the Reynolds number to make sure the flow is completely turbulent, and it is

$$\text{Re} = \frac{VD}{\nu} = \frac{3.92 \times 0.08}{10^{-6}} = 3.14 \times 10^5$$

This is just acceptable and requires no iteration to improve the friction factor. So, the flow rate is

$$Q = AV = \pi \times 0.04^2 \times 3.92 = 0.0197 \text{ m}^3/\text{s}$$

Losses in Noncircular Conduits

To determine the head loss in a relatively ''open'' noncircular conduit, we use the hydraulic radius R, defined as

$$R = \frac{A}{P}$$

where A is the cross-sectional area and P is the wetted perimeter, the perimeter of the conduit that is in contact with the fluid. The Reynolds number, relative roughness, and head loss are respectively

$$\text{Re} = \frac{4VR}{\nu} \qquad \begin{matrix}\text{relative}\\ \text{roughness}\end{matrix} = \frac{e}{4R} \qquad h_L = f\frac{L}{4R}\frac{V^2}{2g}$$

A rectangular area should have an aspect ratio ,4. This method should not be used with shapes like an annulus

Minor Losses

The preceding losses were for the developed flow in long conduits. Most piping systems, however, include sudden changes such as elbows, valves, inlets, etc., that add additional losses to the system.

These losses are called minor losses that may, in fact, add up to exceed the head loss found in the preceding sections. These minor losses are expressed in terms of a loss coefficient K ,defined for most devices by

$$h_L = K\frac{V^2}{2g}$$

There often equate the losses in adevice to an ***equivalent length*** (L_e) of pipe, i.e.,

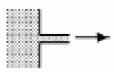
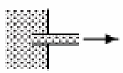
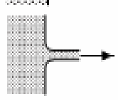
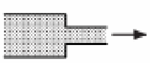
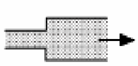
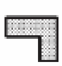
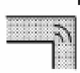
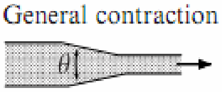
$$h_L = K\frac{V^2}{2g} = f\frac{L_e}{D}\frac{V^2}{2g}$$

This provides the relationship

$$L_e = K\frac{D}{f}$$

A last comment relating to minor losses is in order : if the pipe is quite long, >1000 diameters, the minor losses are usually neglected. For lengths as short as 100 diameters, the minor losses usually exceed the frictional losses. For intermediate lengths, the minor losses should be included.

Minor Loss Coefficients K for Selected Devices

Type of fitting	Screwed			Flanged		
Diameter	2.5 cm	5 cm	10 cm	5 cm	10 cm	20 cm
Globe value (fully open)	8.2	6.9	5.7	8.5	6.0	5.8
(half open)	20	17	14	21	15	14
(one-quarter open)	57	48	40	60	42	41
Angle valve (fully open)	4.7	2.0	1.0	2.4	2.0	2.0
Swing check valve (fully open)	2.9	2.1	2.0	2.0	2.0	2.0
Gate valve (fully open)	0.24	0.16	0.11	0.35	0.16	0.07
Return bend	1.5	0.95	0.64	0.35	0.30	0.25
Tee (branch)	1.8	1.4	1.1	0.80	0.64	0.58
Tee (line)	0.9	0.9	0.9	0.19	0.14	0.10
Standard elbow	1.5	0.95	0.64	0.39	0.30	0.26
Long sweep elbow	0.72	0.41	0.23	0.30	0.19	0.15
45° elbow	0.32	0.30	0.29			
Square-edged entrance 			0.5			
Reentrant entrance 			0.8			
Well-rounded entrance 			0.03			
Pipe exit			1.0			
	Area ratio					
Sudden contraction† 	2:1		0.25			
	5:1		0.41			
	10:1		0.46			
	Area ratio A/A_0					
Orifice plate 	1.5:1		0.85			
	2:1		3.4			
	4:1		29			
	≥6:1		$2.78\left(\frac{A}{A_0}-0.6\right)^2$			
Sudden enlargement‡ 			$\left(1-\frac{A_1}{A_2}\right)^2$			
90° miter bend (without vanes) 			1.1			
(with vanes) 			0.2			
General contraction 	(30° included anlge)		0.02			
	(70° included angle)		0.07			

EX.1 A1.5-cm-diameter, 20-m-long plastic pipe transports water from a pressurized 400-kPa tank out a free open end located 3m above the water surface in the tank. There are three elbows in the water line and a square-edged inlet from the tank. Estimate the flow rate?

Solution: The energy equation is applied between the tank and the faucet exit:

$$0 = \frac{V_2^2 - V_1^2}{2g} + \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + h_L$$

where

$$h_L = \left(f\frac{L}{D} + 3K_{\text{elbow}} + K_{\text{entrance}}\right)\frac{V^2}{2g}$$

Assume that the pipe has $e/D = 0$ and that $\text{Re} \cong 2 \times 10^5$ so that the Moody diagram yields $f = 0.016$. The energy equation yields

$$0 = \frac{V_2^2}{2 \times 9.8} - \frac{400\,000}{9800} + 3 + \left(0.016 \times \frac{20}{0.015} + 3 \times 1.6 + 0.5\right)\frac{V^2}{2 \times 9.8} \qquad \therefore\ V = 5.18\ \text{m/s}$$

The Reynolds number is then $\text{Re} = 5.18 \times 0.15/10^{-6} = 7.8 \times 10^4$. Try $f = 0.018$. Then

$$0 = \frac{V_2^2}{2 \times 9.8} - \frac{400\,000}{9800} + 3 + \left(0.018 \times \frac{20}{0.015} + 3 \times 1.6 + 0.5\right)\frac{V^2}{2 \times 9.8} \qquad \therefore\ V = 4.95\ \text{m/s}$$

Thus $\text{Re} = 4.95 \times 0.15/10^{-6} = 7.4 \times 10^4$. This is close enough so use $V = 5.0$ m/s. The flow rate is

$$Q = AV = \pi \times 0.0075^2 \times 5 = 8.8 \times 10^{-4}\ \text{m}^3/\text{s}$$

TURBULENT FLOW IN PIPES

Turbulent flow is characterized by random and rapid fluctuations of swirling regions of fluid, called ***eddies***, throughout the flow. These fluctuations provide an additional mechanism for momentum and energy transfer. In laminar flow, fluid particles flow in an orderly manner along pathlines, and momentum and energy are transferred across streamlines by molecular diffusion. In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer. As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients (Fig.1).

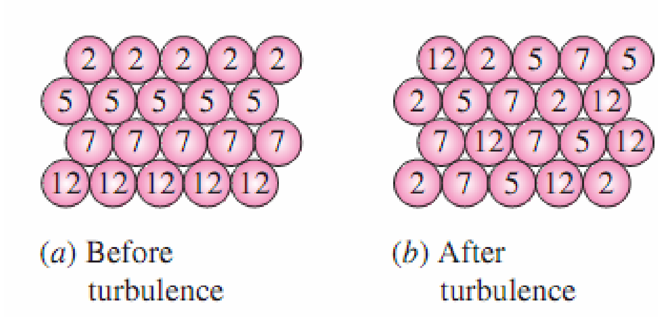


FIG.1

The intense mixing in turbulent flow brings fluid particles at different momentums into close contact and thus enhances momentum transfer

Even when the average flow is steady, the eddy motion in turbulent flow causes significant fluctuations in the values of velocity, temperature, pressure, and even density (in compressible flow). Fig.2 shows the variation of the instantaneous velocity component u with time at a specified location, as can be measured with a hot-wire anemometer probe or other sensitive device.

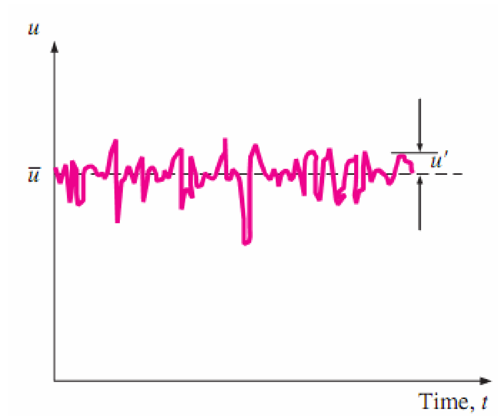


FIG.2

Fluctuations of the velocity component u with time at a specified location in turbulent flow

The velocity fluctuate about an average value, which suggests that the velocity can be expressed as the sum of an average value $\bar{u}$ and a fluctuating component u'

$$u = \bar{u} + u'$$

This is also the case for other properties such as the velocity component v in the y-direction, and thus $v = \bar{v} + v'$, $P = \bar{P} + P'$, and $T = \bar{T} + T'$. The average value of a property at some location is determined by averaging it over a time interval that is sufficiently large so that the time average levels off to a constant.

Turbulent Shear Stress

Then the turbulent shear stress can be expressed as $\tau_{turb} = -\rho\overline{u'v'} = \mu_t \frac{\partial \bar{u}}{\partial y}$

where μ_t is the eddy viscosity or turbulent viscosity, which accounts for momentum transport by turbulent eddies. Then the total shear stress can be expressed conveniently as

$$\tau_{total} = (\mu + \mu_t)\frac{\partial \bar{u}}{\partial y} = \rho(\nu + \nu_t)\frac{\partial \bar{u}}{\partial y}$$

where $\nu_t = \mu_t/\rho$ is the **kinematic eddy viscosity** or **kinematic turbulent viscosity** (also called the *eddy diffusivity of momentum*).

Considering that velocity changes from zero to nearly the core region value across a layer that is sometimes no thicker than a hair (almost like a step function), we would expect the velocity profile in this layer to be very nearly linear, and experiments confirm that. Then the velocity gradient in the viscous sublayer remains nearly constant at du/dy = u/y,and the wall shear stress can be expressed as

$$\tau_w = \mu \frac{u}{y} = \rho\nu \frac{u}{y} \qquad \text{or} \qquad \frac{\tau_w}{\rho} = \frac{\nu u}{y}$$

where y is the distance from the wall (note that y = R - r for a circular pipe). The quantity τ_w / ρ is frequently encountered in the analysis of turbulent velocity profiles. The square root of τ_w / ρ has the dimensions of velocity, and thus it is convenient to view it as a fictitious velocity called the ***friction velocity*** expressed as $u_* = \sqrt{\tau_w/\rho}$. , sub. in equ. Above to obtain on

$$\textit{Viscous sublayer:} \qquad \frac{u}{u_*} = \frac{yu_*}{\nu}$$

This equation is known as the law of the wall, and it is found to satisfactorily correlate with experimental data for smooth surfaces for $0 \le yu_*/\nu \le 5$ .Therefore, the thickness of the viscous sublayer is roughly

$$\textit{Thickness of viscous sublayer:} \qquad y = \delta_{\text{sublayer}} = \frac{5\nu}{u_*} = \frac{25\nu}{u_\delta}$$

The quantity ν/u_* has dimensions of length and is called the viscous length; it is used to nondimensionalize the distance y from the surface. In boundary layer analysis, it is convenient to work with nondimensionalized distance and nondimensionalized velocity defined as

$$\textit{Nondimensionalized variables:} \qquad y^+ = \frac{yu_*}{\nu} \qquad \text{and} \qquad u^+ = \frac{u}{u_*}$$

Then, the eq. $\frac{u}{u_*} = \frac{yu_*}{\nu}$ Become

$$\textit{Normalized law of the wall:} \qquad u^+ = y^+$$

Dimensional analysis indicates and the experiments confirm that the velocity in the overlap layer is proportional to the logarithm of distance, and the velocity profile can be expressed as

$$\textit{The logarithmic law:} \qquad \frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{yu_*}{\nu} + B$$

where k and B are constants whose values are determined experimentally to be about 0.40 and 5.0, respectively. Substituting the values of the constants, the velocity profile is determined to be

$$\textit{Overlap layer:} \qquad \frac{u}{u_*} = 2.5 \ln \frac{yu_*}{\nu} + 5.0 \quad \text{or} \quad u^+ = 2.5 \ln y^+ + 5.0$$

A good approximation for the outer turbulent layer of pipe flow can be obtained by evaluating the constant B in Eq. abve from the requirement that maximum velocity in a pipe occurs at the centerline where r = 0. Solving for B from Eq. by setting y = R - r = R and u = u_{max} ,and substitute ing it back into Eq. together with k = 0.4 gives

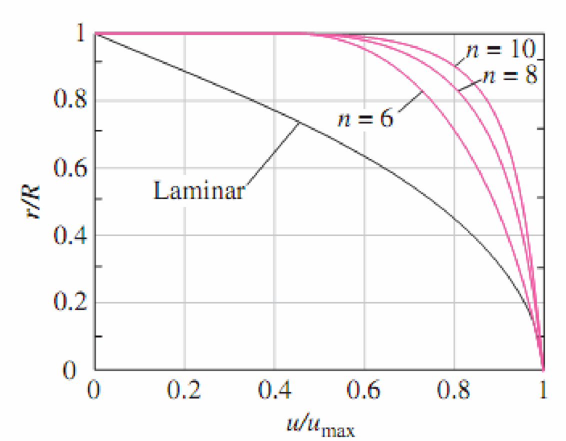
$$\text{Outer turbulent layer:} \qquad \frac{u_{max} - u}{u_*} = 2.5 \ln \frac{R}{R - r}$$

Numerous other empirical velocity profiles exist for turbulent pipe flow. Among those, the simplest and the best known is the power-law velocity profile expressed as

$$\text{Power-law velocity profile:} \qquad \frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{1/n} \quad \text{or} \quad \frac{u}{u_{max}} = \left(1 - \frac{r}{R}\right)^{1/n}$$

where the exponent n is a constant whose value depends on the Reynolds number. The value of n increases with increasing Reynolds number. The value n = 7 generally approximates many flows in practice, giving rise to the term one-seventh power-law velocity profile. Various power - law velocity profiles are shown in Fig.1 for n = 6, 8, and 10 together with the velocity profile for fully developed laminar flow for comparison. Note that the turbulent velocity profile is fuller than the laminar one, and it becomes more flat as n (and thus the Reynolds number) increases. Also note that the power-law profile cannot be used to calculate wall shear stress since it gives a velocity gradient of infinity there, and it fails to give zero slope at the centerline. But these regions of discrepancy constitute a small portion of flow, and the power-law profile gives highly accurate results for turbulent flow through a pipe.

FIG.1
Power-law velocity profiles for fully developed turbulent flow in a pipe for different exponents, and its comparison with the laminar velocity profile.



Ex.1 Assuming the following velocity distribution in a circular pipe $u = u_{max}(1 - r/R)^{1/7}$ where u_{max} is the maximum velocity, calculate (a) the ratio between the mean velocity and the maximum velocity, (b) the radius at which the actual velocity is equal to the mean velocity.

Solution

(a) The elementary discharge through an annulus dr is given by

$$dQ = 2\pi r u \, dr$$
$$= 2\pi u_{max}(1 - r/R)^{1/7} \, dr,$$

and discharge through the pipe by

$$Q = 2\pi u_{max} \int_0^R r(1 - r/R)^{1/7} \, dr.$$

Let $1 - r/R = x$; then

$$\frac{dx}{dr} = -\frac{1}{R} \quad \text{and} \quad dr = -R\,dx$$

so that $R - r = xR$, when $r = 0$, $x = 1$,

$r = R - xR = R(1 - x)$, when $r = R$, $x = 0$.

Therefore, substituting,

$$Q = 2\pi u_{max} \int_1^0 R(1-x)x^{1/7}(-R \, dx) = 2\pi R^2 u_{max} \int_0^1 (1-x)x^{1/7} \, dx$$
$$= 2\pi R^2 u_{max} \left(\frac{7}{8} x^{8/7} - \frac{7}{15} x^{15/7} \right)_0^1$$
$$= 2\pi R^2 u_{max} \left(\frac{7}{8} - \frac{7}{15} \right) = 2\pi R^2 u_{max} \left(\frac{105 - 56}{120} \right) = \pi R^2 u_{max} \frac{49}{60},$$

and $\bar{u} = Q/\pi R^2 = \pi R^2 u_{max} \frac{49}{60} / \pi R^2 = \frac{49}{60} u_{max}$,

with the result that

$$\bar{u}/u_{max} = \mathbf{49/60}.$$

(b) $u = \bar{u} = 49 u_{max}/60 = u_{max}(1 - r/R)^{1/7}$.

Therefore,

$$(49/60)^7 = 1 - r/R$$

and $r/R = 1 - (49/60)^7 = 1 - 0.242 = 0.758.$

Hence, $r = \mathbf{0.758R}$.

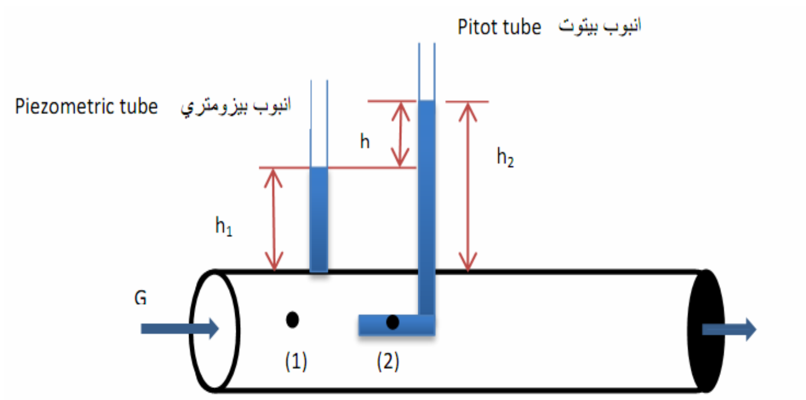
FLOW MEASURING INSTRUMENTS

Many types of instruments are available for flow measurement; some register the flow rate directly, whilst others measure the velocity, then the flow rate is calculated. Most of the instruments are based on Bernoulli principles, hence a restriction in the flow channel is used to cause a pressure drop accompanied by an increase in velocity. The flow rate is a function of pressure drop and therefore we can convert the reading of a manometer to flow rate. It will also study special types of instruments used for specific flow measurements, as well as those used for open channel flow.

Measurement of velocity by Pitot- Tube

A Pitot tube consists of a hollow-right angled tube positioned so that the open end is opposes the liquid flow direction, see Fig.1. The liquid rises in the tube depending on the velocity at that point.

Fig.1 Pitot tub



If a piezometric tube is fixed at the pipe wall, the height of the liquid column will provide the value of the static pressure. Applying Bernoulli's equation at points (1) and (2), we obtain:

$$\frac{P_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

The fluid at point (2) is stagnant, thus:

$$u_2 = 0$$

$$z_1 = z_2$$

Therefore equation

$$\frac{P_1}{\rho g} + \frac{u_1^2}{2g} = \frac{P_2}{\rho g}$$

The pressure P1 is the static pressure in the pipe. It is equivalent to the static pressure head h1 which is read from the peizometeric tube. At point (2), the height h2 is the total static head and the kinetic head. The kinetic energy at (2) is converted to an extra head.

P2 is called the impact pressure. Since

$$P = \rho g h$$

thus

$$h_1 + \frac{u_1^2}{2g} = h_2$$

$$h_2 - h_2 = \frac{u_1^2}{2g}$$

We can therefore compute the velocity by:

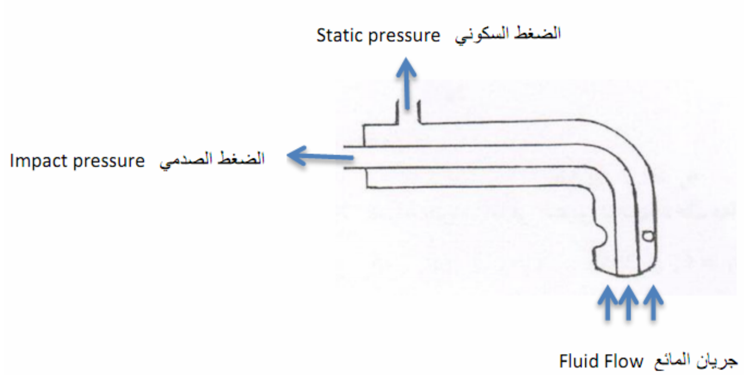
$$u_1 = \sqrt{2g\,\Delta h}$$

Equation above provides theoretical velocity values; to get realistic values, we introduce the instrument coefficient Cv and the equation will change to:

$$u_1 = C_v \sqrt{2g\,\Delta h} \;=\; C_v \sqrt{\frac{2\,(P_2 - P_1)}{\rho}}$$

A Pitot-static tube can also be used to replace the arrangement of Fig .2. It consists of two concentric tubes; the inner one measures the impact pressure whilst the outer tube gives the static pressure through the holes at the end.

Fig.2 Pitot-static tube



Ex.1 A Pitot-Static tube is used to measure the flow velocity of turpentine flowing in a pipeline. If the manometer reading is 12 cm of mercury, calculate the velocity at that point in the pipeline. Density of turpentine is 860 kg/m^3 and that of mercury is 13600 kg/m^3.
Solution

Given:

$\Delta h = 12\ cm = 0.12\ m$, $C_v = 1$, $\rho_T = 860\ kg/m^3$, $\rho_{Hg} = 13600\ kg/m^3$

$$u_1 = C_v \sqrt{\frac{2\,(P_2 - P_1)}{\rho}}$$

$$P_2 - P_1 = h_{Hg}\, g\, (\rho_{Hg} - \rho_T)$$

$$P_2 - P_1 = 0.12 \times 9.81 \times (13600 - 860)$$

$$= 14997.528\ Pa$$

$$u = \sqrt{\frac{2 \times 14997.528}{860}}$$

$$u = \underline{5.905\ m/s}$$

Orifice meter

The orifice meter consist of a flat plate with a sharp-edged hole placed concentrically inside a pipe, as shown in Fig .3

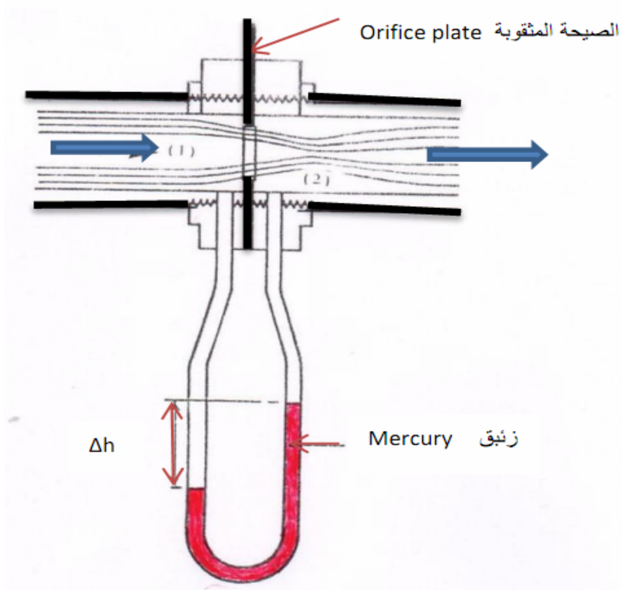


Fig .3Orifice meter

The orifice is machined carefully to provide a sharp-edged hole. The diameter of the plate is specified according to international standards. The pressure difference across the plate is measured by a manometer. To determine the flow rate, we apply the energy equation:

$$\frac{\Delta u^2}{2\alpha} + g\,\Delta z + \int_1^2 v\,dP + W_s + L_f = 0$$

In the case of incompressible fluids, the integral will be: $\int_1^2 v\,dP = v\,(P_2 - P_1)$
The energy equation can be simplified by assuming

$$z_1 = z_2$$

$$W_s = 0$$

$$L_f = 0$$

Then

$$\frac{u_2^2}{2\,\alpha_2} - \frac{u_1^2}{2\,\alpha_1} = v\,(P_1 - P_2)$$

The continuity equation is:

$$\dot{m}_1 = \dot{m}_2$$

$$u_1\,A_1\,\rho_1 = u_2\,A_2\,\rho_2$$

For an incompressible fluid:

$$\rho_1 = \rho_2$$

Thus:

$$u_1 = u_2\,(A_2/A_1)$$

Then

$$\frac{u_2^2}{2\,\alpha_2} - (1 - \frac{\alpha_2\,A_2^2}{\alpha_1\,A_1^2}) = v\,(P_1 - P_2)$$

and

$$u_2 = \sqrt{\frac{2\,\alpha_2\,v\,(P_1 - P_2)}{1 - \frac{\alpha_2}{\alpha_1}(\frac{A_2}{A_1})^2}}$$

However:

$$\dot{m} = u_2 A_2 \rho_2 = u_2 A_2/v_2$$

Therefore

$$\dot{m} = \frac{A_2}{v_2}\sqrt{\frac{2\,\alpha_2\, v\,(P_1 - P_2)}{1 - \frac{\alpha_2}{\alpha_1}\left(\frac{A_2}{A_1}\right)^2}}$$

It can be noted that the flow cross section area is reduced from A_1 in section (1) to the orifice area A_0 and then to A_2 in the vena contracta. The contraction coefficient C_c is defined in terms of the area of the orifice and vena contracta as follows:

and $$C_c = A_2/A_0$$

$$\dot{m} = \frac{C_c A_0}{v}\sqrt{\frac{2\,\alpha_2\, v\,(P_1 - P_2)}{1 - \frac{\alpha_2}{\alpha_1}\left(C_c\frac{A_0}{A_1}\right)^2}}$$

The specific volume v is considered constant because the fluid is incompressible. Now, substituting the discharge coefficient C_D for friction losses in the meter, and for the coefficient of contraction Cc including α_1 and α_2, it can write obtain the following format:

$$\dot{m} = \frac{C_D A_0}{v}\sqrt{\frac{2\, v\,(P_1 - P_2)}{1 - \left(\frac{A_0}{A_1}\right)^2}}$$

If we consider the orifice area A_0 small, compared to the area of the pipe A_1, the following expression becomes: $\left[1 - \left(\frac{A_0}{A_1}\right)^2\right]^{1/2} \rightarrow 1$

$$\dot{m} = \frac{C_D A_0}{v}\sqrt{2\, v\,(P_1 - P_2)}$$

In terms of density:

$$\dot{m} = C_D\, A_0\, \rho\sqrt{2\,\frac{(P_1 - P_2)}{\rho}}$$

In terms of the liquid head which is equivalent to the pressure difference $(P_1 - P_2)$:

$$\dot{m} = C_D\, A_0\, \rho\sqrt{2\, g\, \Delta h}$$

Venturi Meter

The venturi meter, Fig.4, is an instrument for measuring flow rate by using measurements of pressure across a converging-diverging flow passage. The main advantage of the venturi meter as compared to the orifice meter is that the head loss for a venturi meter is much smaller.The lower head loss results from streamlining the flow passage, as shown in Fig.3. Such streamlining eliminates any jet contraction beyond the smallest flow section. Consequently, the coefficient of contraction has a value of unity, and the venturi equation is

$$Q = \frac{A_t C_d}{\sqrt{1-(A_t/A_p)^2}}\sqrt{2g(h_p - h_t)}$$

$$Q = KA_t\sqrt{2g\Delta h}$$

where At is the throat area and h is the difference in piezometric head between the venturi entrance (pipe) and the throat. Note that the venturi equation is the same as the orifice equation. However, K for the venturi meter approaches unity at high values of the Reynolds number and small dD ratios. This trend can be seen in Fig.5, where values of K for the venturi meter are plotted along with similar data for the orifice.

Figure 4
Typical venturi meter.

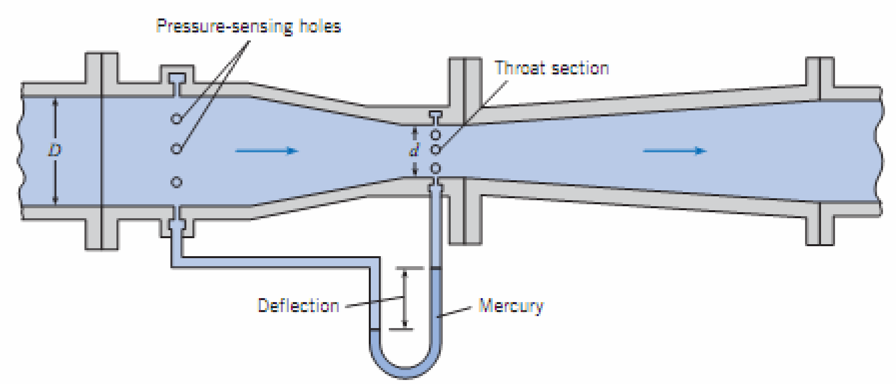
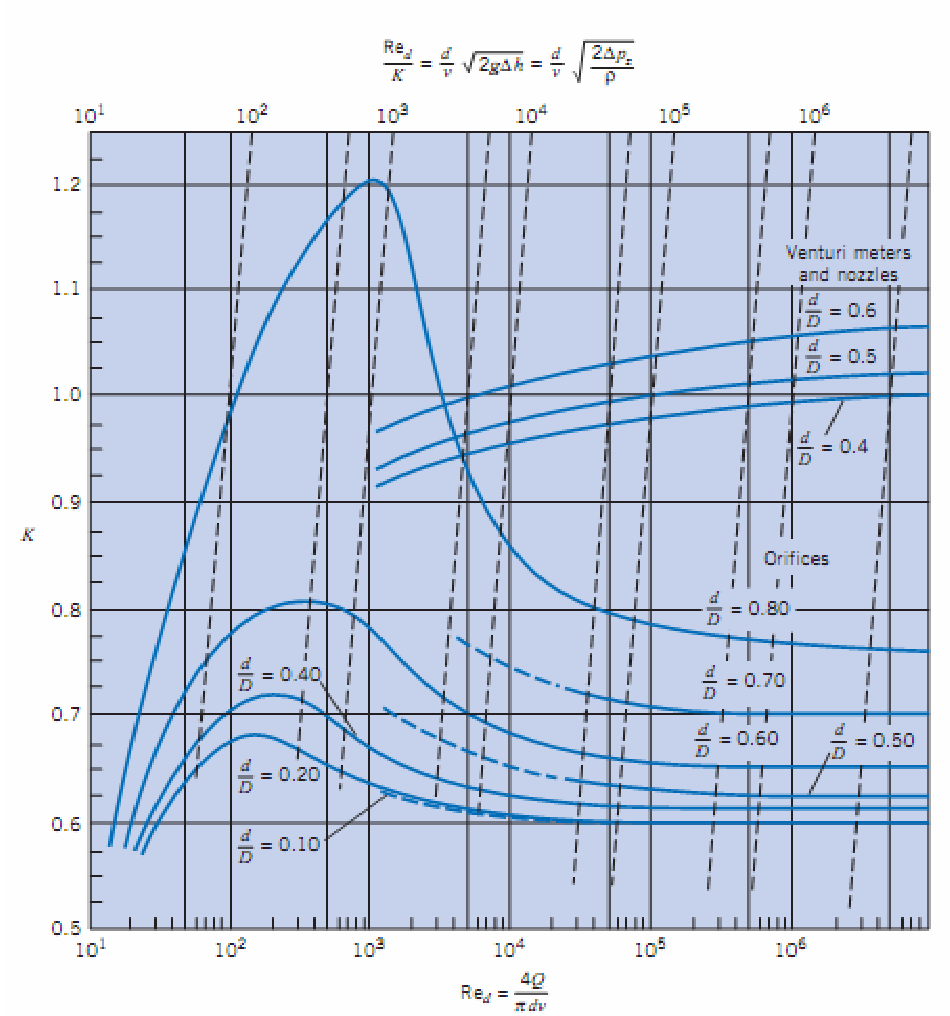


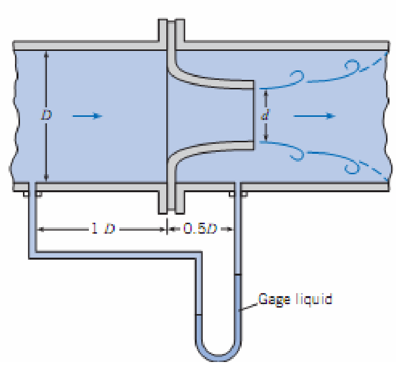
Figure 5
Flow coefficient K and R_{ed}/K versus the Reynolds number for orifices, nozzles, and venturi meters



Flow Nozzles

The flow nozzle, Fig.6, is an instrument for measuring flow rate by using the pressure drop across a nozzle that is typically placed inside a conduit. Similar to an orifice meter. As compared to an orifice meter, the flow nozzle is better in flows that cause wear (e.g., particle-laden flow). The reason is that erosion of an orifice will produce more change in the pressure-drop versus flow-rate relationship. Both the flow nozzle and orifice meter will produce about the same overall head loss.

Figure 6
Typical flow nozzle



Ex.2 The pressure difference between the taps of a horizontal venturi meter carrying water is 35 kPa. If d= 20 cm and D =40 cm, what is the discharge of water at 10°C?

Solution

1. Change in piezometric head

$$\Delta h = \frac{\Delta p}{\gamma} + \Delta z = \frac{\Delta p}{\gamma} + 0 = \frac{35{,}000\ \text{N/m}^2}{9810\ \text{N/m}^3} = 3.57\ \text{m of water}$$

2. Flow coefficient
   - Calculate (Re_d/K):

$$\frac{\text{Re}_d}{K} = \frac{d\sqrt{2g\Delta h}}{\nu} = \frac{0.20\sqrt{2(9.81)(3.57)}}{1.31(10^{-6})} = 1.28 \times 10^6$$

   - From Fig. find that $K = 1.02$.

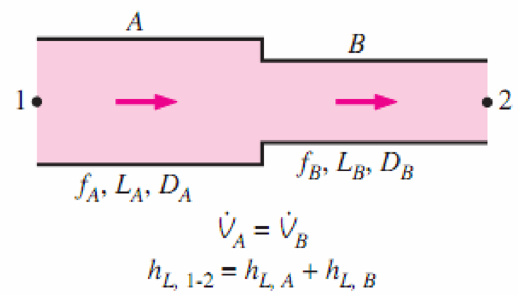
3. Discharge

$$Q = 1.02A_2\sqrt{2g\Delta h}$$
$$= 1.02(0.785)(0.20^2)\sqrt{2(9.81)(3.57)} = \boxed{0.268\ \text{m}^3/\text{s}}$$

PIPING NETWORKS AND PUMP SELECTION

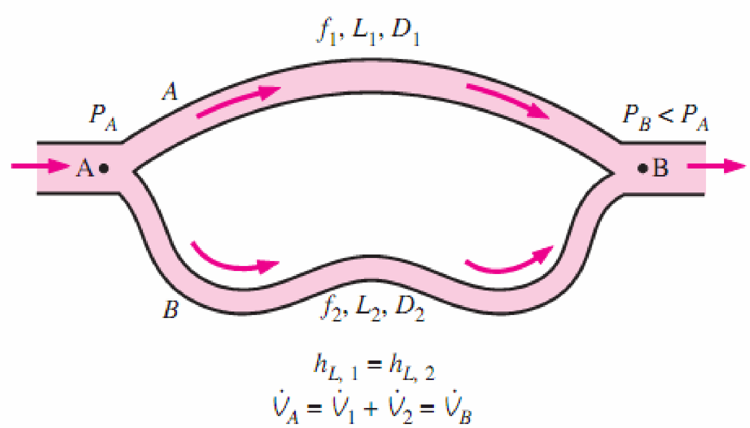
Most piping systems encountered in practice such as the water distribution systems in cities or commercial or residential establishments involve numerous parallel and series connections as well as several sources (supply of fluid into the system) and loads (discharges of fluid from the system) (Fig.1). The engineering objective in such projects is to design a piping system that will deliver the specified flow rates at specified pressures reliably at minimum total (initial plus operating and maintenance) cost. Once the layout of the system is prepared, the determination of the pipe diameters and the pressures throughout the system, while remaining within the budget constraints, typically requires solving the system repeatedly until the optimal solution is reached.

FIG.1
For pipes in ***series***, the flow rate is the same in each pipe, and the total head loss is the sum of the head losses in individual pipes



Piping systems typically involve several pipes connected to each other in series and/or in parallel, as shown in Figs.1 and 2. When the pipes are connected in series, the flow rate through the entire system remains constant regardless of the diameters of the individual pipes in the system.

FIG.2
For pipes in parallel, the head loss is the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes.



For a pipe that branches out into two (or more) parallel pipes and then rejoins at a junction downstream, the total flow rate is the sum of the flow rates in the individual pipes. The pressure drop (or head loss) in each individual pipe connected in parallel must be the same since ($\Delta P = P_A - P_B$) and the junction pressures P_A and P_B are the same for all the individual pipes. For a system of two parallel pipes 1 and 2 between junctions A and B with negligible minor losses, this can be expressed as:-

$$h_{L,1} = h_{L,2} \quad \rightarrow \quad f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$$

Then the ratio of the average velocities and the flow rates in the two parallel pipes become

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2} \quad \text{and} \quad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,1}V_1}{A_{c,2}V_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2}$$

The analysis of piping networks, no matter how complex they are, is based on two simple principles:

1. Conservation of mass throughout the system must be satisfied. This is done by requiring the total flow into a junction to be equal to the total flow out of the junction for all junctions in the system. Also, the flow rate must remain constant in pipes connected in series regardless of the changes in diameters.
2. Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions. This is because pressure is a point function and it cannot have two values at a specified point. In practice this rule is used by requiring that the algebraic sum of head losses in a loop (for all loops) be equal to zero. (A head loss is taken to be positive for flow in the clockwise direction and negative for flow in the counterclockwise direction.)

Piping Systems with Pumps and Turbines

When a piping system involves a pump and/or turbine, the steady-flow energy equation on a unit-mass basis can be expressed as:-

$$\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 + w_{\text{pump},u} = \frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2 + w_{\text{turbine},e} + gh_L$$

It can also be expressed in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump},u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine},e} + h_L$$

where $h_{pump,u}=W_{pump,u}/g$ is the useful pump head delivered to the fluid, $h_{turbine,e}=W_{turbine,e}/g$ is the turbine head extracted from the fluid, α is the kinetic energy correction factor whose value is nearly 1 for most (turbulent) flows encountered in practice, and ***hL*** is the total head loss in piping (including the minor losses if they are significant) between points 1 and 2. ***The pump head is zero if the piping system does not involve a pump or a fan, the turbine head is zero if the system does not involve a turbine, and both are zero if the system does not involve any mechanical work-producing or work-consuming devices.***

Many practical piping systems involve a pump to move a fluid from one reservoir to another. Taking points 1 and 2 to be at the free surfaces of the reservoirs, the energy equation in this case reduces for the useful pump head required to (Fig.3)

$$h_{pump,u} = (z_2 - z_1) + h_L$$

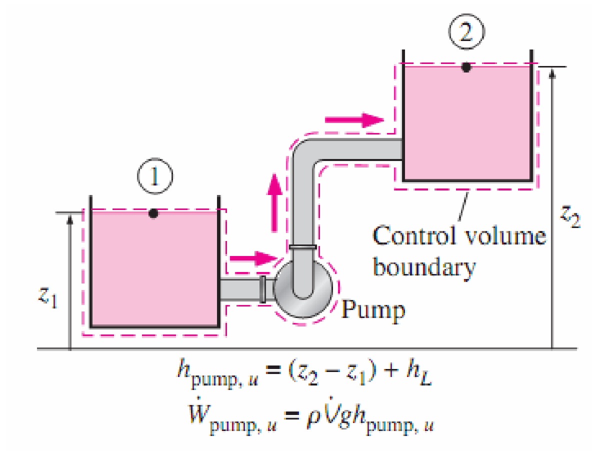


FIG.3
When a pump moves a fluid from one reservoir to another, the useful pump head requirement is equal to the elevation difference between the two reservoirs plus the head loss.

Once the useful pump head is known, the mechanical power that needs to be delivered by the pump to the fluid and the electric power consumed by the motor of the pump for a specified flow rate are determined from:-

$$\dot{W}_{pump,\,shaft} = \frac{\rho \dot{V} g h_{pump,u}}{\eta_{pump}} \quad \text{and} \quad \dot{W}_{elect} = \frac{\rho \dot{V} g h_{pump,u}}{\eta_{pump-motor}}$$

The head loss of a piping system increases (usually quadratically) with the flow rate. A plot of required useful pump head $h_{pump,u}$ as a function of flow rate is called the system (or **demand**) curve. The head produced by a pump is not a constant either. Both the pump head and the pump efficiency vary with the flow rate, and pump manufacturers supply this variation in tabular or graphical form, as shown in Fig.4.

These experimentally determined $h_{pump,u}$ and $\eta_{pump,u}$ versus $\dot{\forall}$ curves are called characteristic (or supply or performance) curves. Note that the flow rate of a pump increases as the required head decreases. The intersection point of the pump head curve with the vertical axis typically represents the maximum head the pump can provide, while the intersection point with the horizontal axis indicates the maximum flow rate (called the free delivery) that the pump can supply.

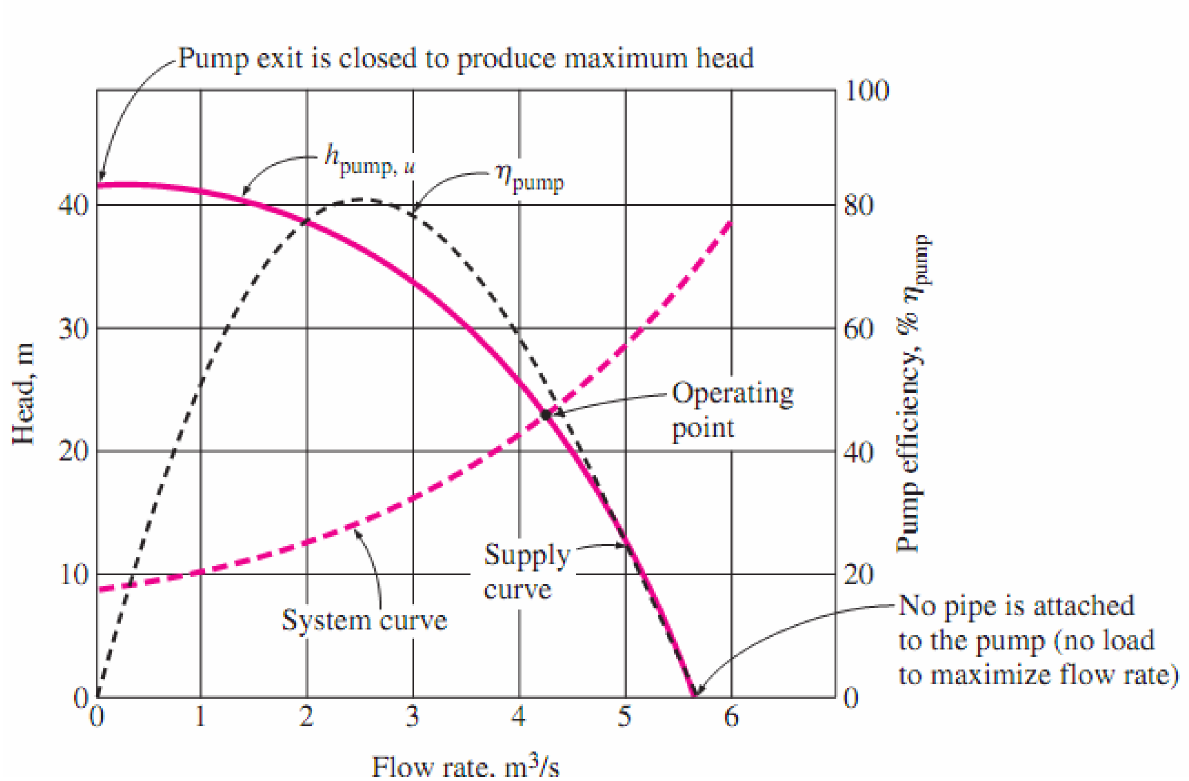
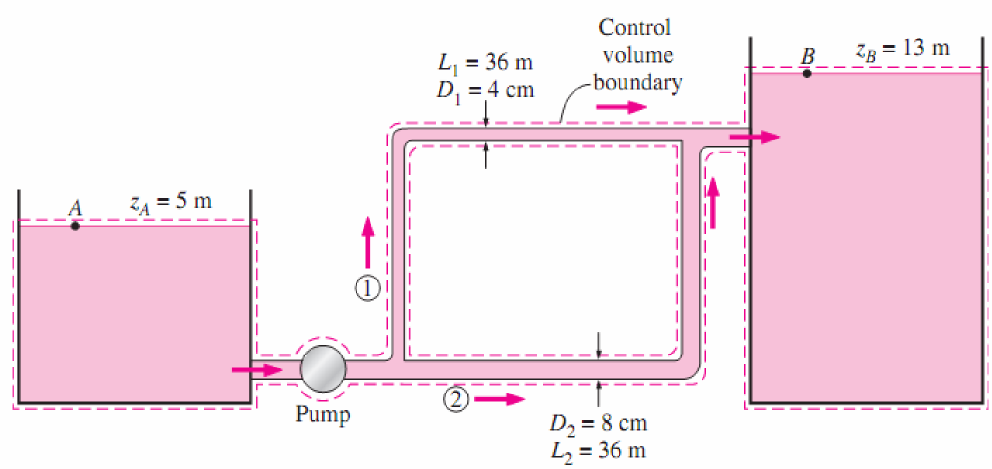


FIG.4

Characteristic pump curves for centrifugal pumps, the system curve for a piping system, and the operating point.

Ex.1 Water at 20°C is to be pumped from a reservoir (z_A = 5 m) to another reservoir at a higher elevation (z_B = 13 m) through two 36-m-long pipes connected in parallel, as shown in Fig.. The pipes are made of commercial steel, and the diameters of the two pipes are 4 and 8 cm. Water is to be pumped by a 70 percent efficient motor–pump combination that draws 8kW of electric power during operation. The minor losses and the head loss in pipes that connect the parallel pipes to the two reservoirs are considered to be negligible. Determine the total flow rate between the reservoirs and the flow rate through each of the parallel pipes.



Properties The density and dynamic viscosity of water at 20°C are $\rho = 998\ \text{kg/m}^3$ and $\mu = 1.002 \times 10^{-3}$ kg/m · s. The roughness of commercial steel pipe is $\varepsilon = 0.000045$ m.

Using

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

Answer.

$$\dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump},\,u}}{\eta_{\text{pump-motor}}} \rightarrow \quad 8000\ \text{W} = \frac{(998\ \text{kg/m}^3)\dot{V}(9.81\ \text{m/s}^2)h_{\text{pump},\,u}}{0.70}$$

We choose points A and B at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus $P_A = P_B = P_{atm}$) and that the fluid velocities at both points are zero ($V_A = V_B = 0$), the energy equation for a control volume between these two points simplifies to

$$\frac{P_A}{\rho g} + \alpha_A \frac{V_A^2}{2g}^{\nearrow 0} + z_A + h_{\text{pump},\,u} = \frac{P_B}{\rho g} + \alpha_B \frac{V_B^2}{2g}^{\nearrow 0} + z_B + h_L \rightarrow h_{\text{pump},\,u} = (z_B - z_A) + h_L$$

or

$$h_{\text{pump},\,u} = (13 - 5) + h_L$$

where

$$h_L = h_{L,1} = h_{L,2}$$

We designate the 4-cm-diameter pipe by 1 and the 8-cm-diameter pipe by 2. The average velocity, the Reynolds number, the friction factor, and the head loss in each pipe are expressed as:-

$$V_1 = \frac{\dot{V}_1}{A_{c,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \quad \rightarrow \quad V_1 = \frac{\dot{V}_1}{\pi(0.04\ \text{m})^2/4}$$

$$V_2 = \frac{\dot{V}_2}{A_{c,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \quad \rightarrow \quad V_2 = \frac{\dot{V}_2}{\pi(0.08\ \text{m})^2/4}$$

$$\text{Re}_1 = \frac{\rho V_1 D_1}{\mu} \quad \rightarrow \quad \text{Re}_1 = \frac{(998\ \text{kg/m}^3)V_1(0.04\ \text{m})}{1.002 \times 10^{-3}\ \text{kg/m}\cdot\text{s}}$$

$$\text{Re}_2 = \frac{\rho V_2 D_2}{\mu} \quad \rightarrow \quad \text{Re}_2 = \frac{(998\ \text{kg/m}^3)V_2(0.08\ \text{m})}{1.002 \times 10^{-3}\ \text{kg/m}\cdot\text{s}}$$

$$\frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f_1}} = -2.0\log\left(\frac{0.000045}{3.7 \times 0.04} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f_2}} = -2.0\log\left(\frac{0.000045}{3.7 \times 0.08} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$h_{L,1} = f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} \quad \rightarrow \quad h_{L,1} = f_1\frac{36\text{ m}}{0.04\text{ m}}\frac{V_1^2}{2(9.81\text{ m/s}^2)}$$

$$h_{L,2} = f_2\frac{L_2}{D_2}\frac{V_2^2}{2g} \quad \rightarrow \quad h_{L,2} = f_2\frac{36\text{ m}}{0.08\text{ m}}\frac{V_2^2}{2(9.81\text{ m/s}^2)}$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2$$

This is a system of 13 unknowns, and their simultaneous solution by an equation solver gives

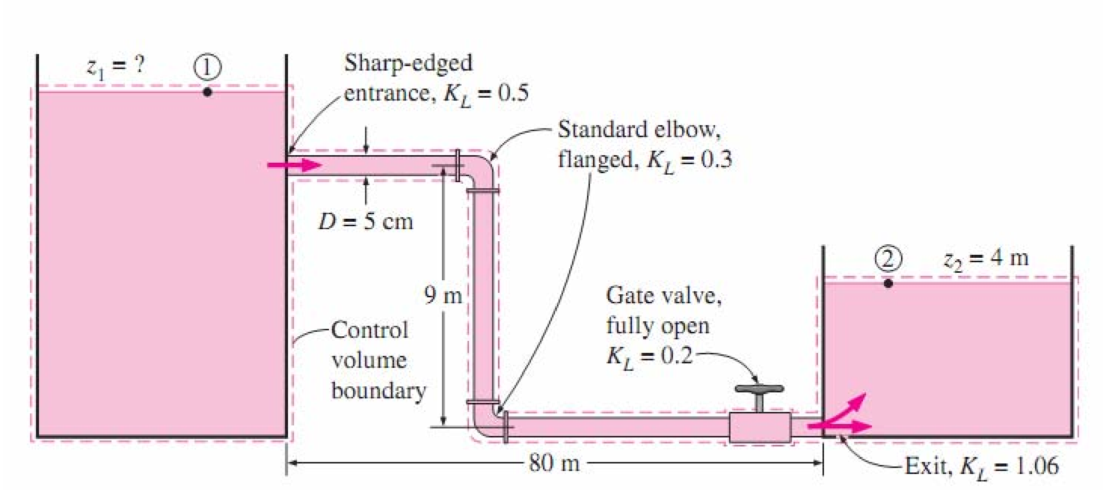
$\dot{V} = \mathbf{0.0300\ m^3/s}$, $\dot{V}_1 = \mathbf{0.00415\ m^3/s}$, $\dot{V}_2 = \mathbf{0.0259\ m^3/s}$

$V_1 = 3.30$ m/s, $V_2 = 5.15$ m/s, $h_L = h_{L,1} = h_{L,2} = 11.1$ m, $h_{pump} = 19.1$ m

$\text{Re}_1 = 131{,}600$, $\text{Re}_2 = 410{,}000$, $f_1 = 0.0221$, $f_2 = 0.0182$

Note that Re > 4000 for both pipes, and thus the assumption of turbulent flow is verified.

Ex.2 Water at 10°C flows from a large reservoir to a smaller one through a 5-cm diameter cast iron piping system, as shown in Fig.. Determine the elevation z1 for a flow rate of 6 L/s.



Properties The density and dynamic viscosity of water at 10°C are $\rho = 999.7\ \text{kg/m}^3$ and $\mu = 1.307 \times 10^{-3}$ kg/m · s. The roughness of cast iron pipe is $\varepsilon = 0.00026$ m.

Using
$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

Answer

The piping system involves 89 m of piping, a sharp-edged entrance (K_L = 0.5), two standard flanged elbows (K_L = 0.3 each), a fully open gate valve (K_L = 0.2), and a submerged exit (K_L =1.06). We choose points 1 and 2 at the free surfaces of the two reservoirs. Noting that the fluid at both points is open to the atmosphere (and thus P1 = P2 = Patm) and that the fluid velocities at both points are zero (V1 = V2 = 0), the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g}\nearrow^0 + z_1 = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g}\nearrow^0 + z_2 + h_L \quad \rightarrow \quad z_1 = z_2 + h_L$$

where

$$h_L = h_{L,\text{ total}} = h_{L,\text{ major}} + h_{L,\text{ minor}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are:-

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.006\ \text{m}^3/\text{s}}{\pi(0.05\ \text{m})^2/4} = 3.06\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(999.7\ \text{kg/m}^3)(3.06\ \text{m/s})(0.05\ \text{m})}{1.307 \times 10^{-3}\ \text{kg/m}\cdot\text{s}} = 117{,}000$$

The flow is turbulent since Re > 4000. Noting that $\varepsilon/D = 0.00026/0.05 = 0.0052$, the friction factor can be determined from the Colebrook equation (or the Moody chart),

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{0.0052}{3.7} + \frac{2.51}{117{,}000\sqrt{f}}\right)$$

It gives $f = 0.0315$. The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{ entrance}} + 2K_{L,\text{ elbow}} + K_{L,\text{ valve}} + K_{L,\text{ exit}}$$
$$= 0.5 + 2 \times 0.3 + 0.2 + 1.06 = 2.36$$

Then the total head loss and the elevation of the source become

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} = \left(0.0315\,\frac{89\ \text{m}}{0.05\ \text{m}} + 2.36\right)\frac{(3.06\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 27.9\ \text{m}$$

$$z_1 = z_2 + h_L = 4 + 27.9 = \mathbf{31.9\ m}$$

PUMPS IN PIPE SYSTEMS

Considered systems that have not involved a pump. If a pump is included in the pipe system and the flow rate is specified, the solution is straightforward using the methods we have already developed .On the other hand, if the discharge is not known, which is commonly the case, a trial-and-error solution is required. The reason for this is that the pump head HP depends upon the discharge, as shown by the pump characteristic curve, the solid curve in Fig.1. Pump manufacturers provide the characteristic curves. Figure 2 shows a complete set of curves for a manufactured centrifugal pump; included are sets of head versus discharge curves for various impeller sizes , as well as efficiency and power curves. The power requirement for a pump is given by the expression

$$\dot{W}_p = \frac{\gamma \dot{\forall} H_P}{\eta}$$

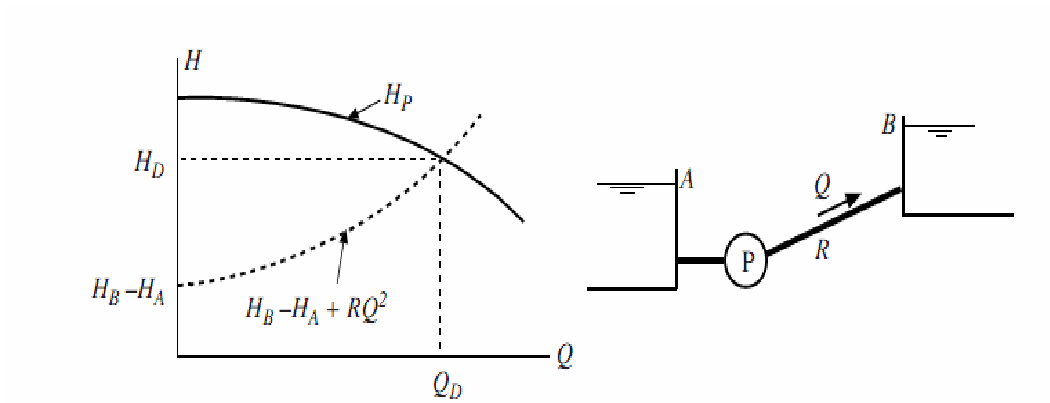


Figure 1 Pump and system demand curves

Determining the discharge in a pumped line requires an additional relation, namely the demand curve ,which is generated by writing the energy balance across the system for varying discharges. Referring to the pump–pipe system in Fig.1, the energy equation for the pipe including the pump is a quadratic in $\dot{\forall}$

$$H_P = (H_B - H_A) + RQ^2$$

Where R the friction and minor loss terms be represented by a resistant ,or loss, coefficient

$$R = \frac{1}{2gA^2}\left(f\frac{L}{D} + \sum K\right)$$

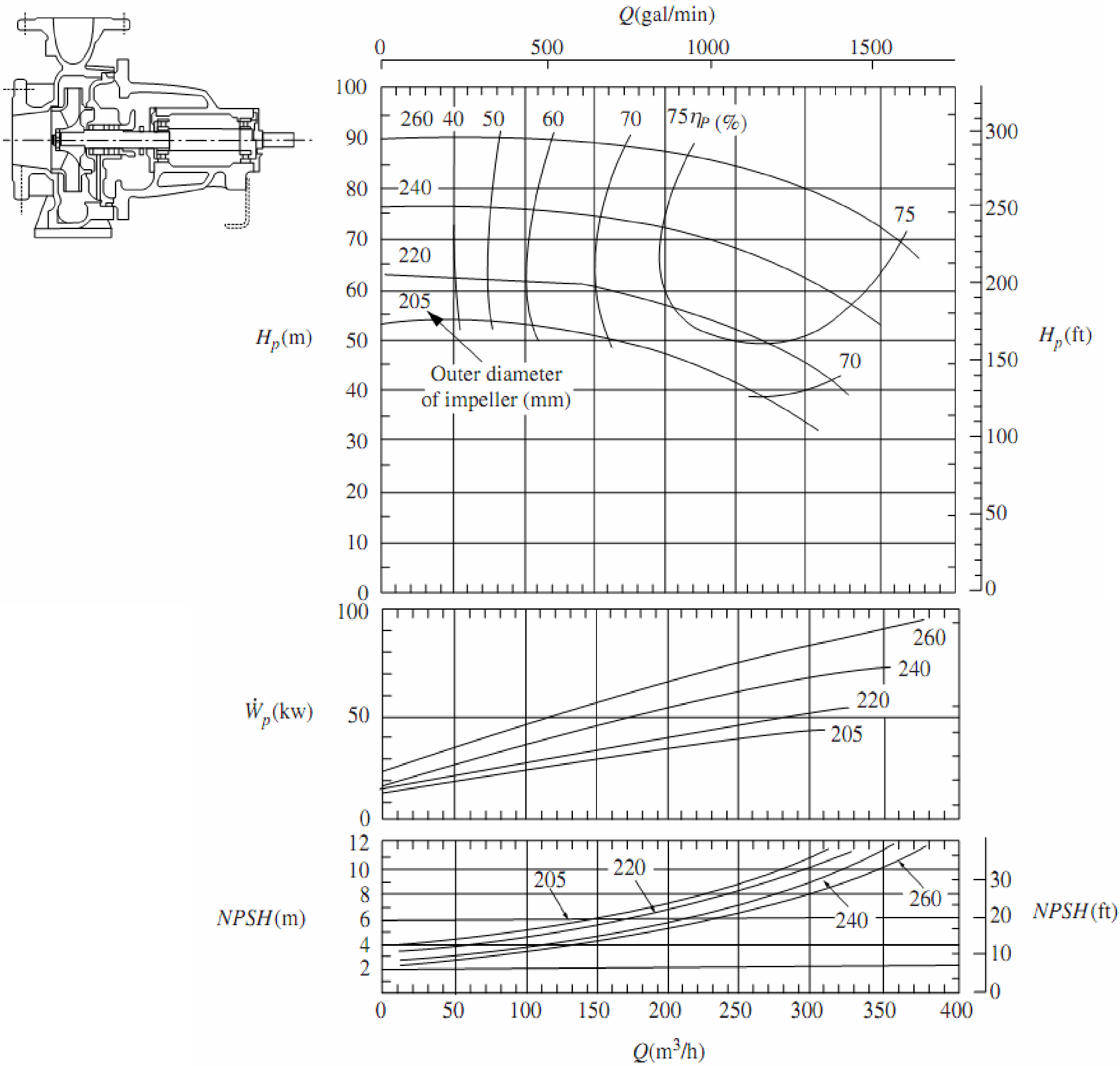
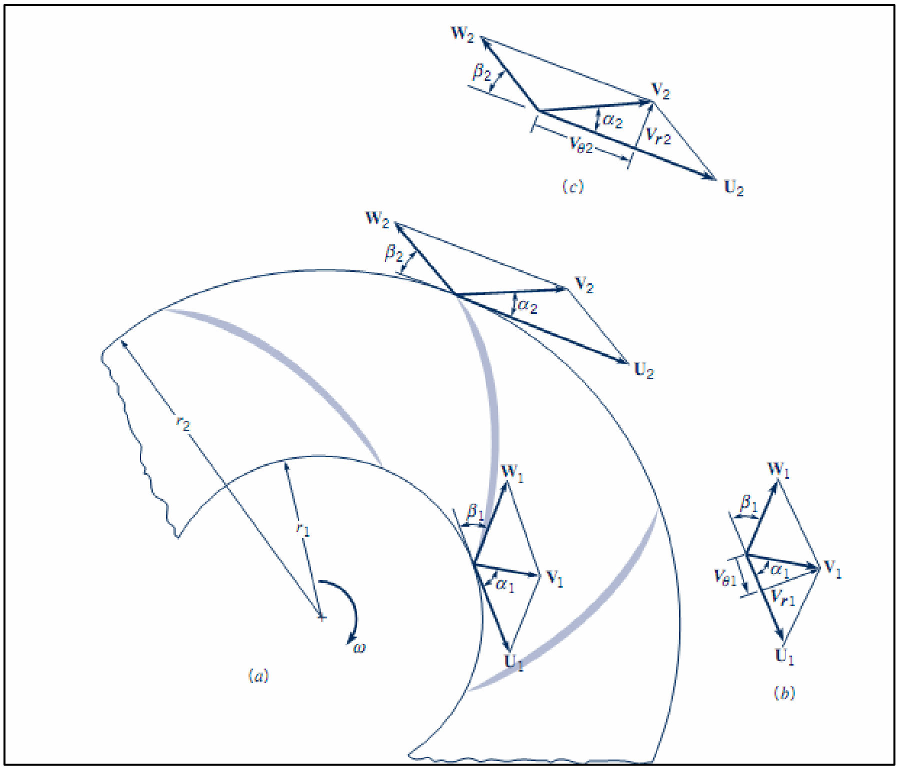


Figure 2 Centrifugal pump and performance curves for four different impellers. Water at 20°C is the pumped liquid.

Theoretical Considerations

Although flow through a pump is very complex (unsteady and three-dimensional), the basic theory of operation of a centrifugal pump can be developed by considering the average one-dimensional flow of the fluid as it passes between the inlet and the outlet sections of the impeller as the blades rotate. As shown in Fig.3, for a typical blade passage, the absolute velocity,V_1 of the fluid entering the pas- sage is the vector sum of the velocity of the blade, U_1 rotating in a circular path with angular velocity and the relative velocity, W_1 within the blade passage so that $V_1 = W_1 + U_1$ Similarly, at the exit $V_2 = W_2 + U_2$. Note that

$U_1 = r_1\omega$ and $U_2 = r_2\omega$. Fluid velocities are taken to be average velocities over the inlet and exit sections of the blade passage. The relationship between the various velocities is shown graphically in Fig.3.



The moment-of-momentum equation indicates that the shaft torque, required to rotate the pump impeller is given by applied to a pump with That is,

$$T_{\text{shaft}} = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1})$$

$$T_{\text{shaft}} = \rho Q(r_2 V_{\theta 2} - r_1 V_{\theta 1})$$

where $V_{\theta 1}$ and $V_{\theta 2}$ are the tangential components of the absolute velocities, $\mathbf{V}_1$ and $\mathbf{V}_2$

For a rotating shaft, the power transferred, $\dot{W}_{\text{shaft}}$, is given by

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\omega$$

$$\dot{W}_{\text{shaft}} = \rho Q\omega(r_2 V_{\theta 2} - r_1 V_{\theta 1})$$

Since $U_1 = r_1\omega$ and $U_2 = r_2\omega$ we obtain

$$\dot{W}_{\text{shaft}} = \rho Q(U_2 V_{\theta 2} - U_1 V_{\theta 1})$$

The shaft power per unit mass of flowing fluid is

$$w_{\text{shaft}} = \frac{\dot{W}_{\text{shaft}}}{\rho Q} = U_2 V_{\theta 2} - U_1 V_{\theta 1}$$

For incompressible pump flow

$$w_{\text{shaft}} = \left(\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}}\right) - \left(\frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}}\right) + loss$$

$$U_2 V_{\theta 2} - U_1 V_{\theta 1} = \left(\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}}\right) - \left(\frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}}\right) + loss$$

Dividing both sides of this equation by the acceleration of gravity, g, we obtain

$$\frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g} = H_{\text{out}} - H_{\text{in}} + h_L$$

where H is total head defined by

$$H = \frac{p}{\rho g} + \frac{V^2}{2g} + z$$

and h_L = loss/g is head loss.

The ideal head rise possible, hi ,is $h_i = \frac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g}$

The actual head rise, $H_{out} - H_{in} = h_a$, , is always less than the ideal head rise, hi , by an amount equal to the head loss, hL, in the pump.

$$h_i = \frac{1}{2g}\left[(V_2^2 - V_1^2) + (U_2^2 - U_1^2) + (W_1^2 - W_2^2)\right]$$

Or

$$h_i = \frac{U_2 V_{\theta 2}}{g}$$

From Fig.3 c obtained $\cot \beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$

Or can be expressed as

$$h_i = \frac{U_2^2}{g} - \frac{U_2 V_{r2} \cot \beta_2}{g}$$

The flowrate, $\dot{\forall}$, is related to the radial component of the absolute velocity through the equation

$$\dot{\forall} = 2\pi r_2 b_2 V_{r2}$$

where is $\boldsymbol{b_2}$ the impeller blade height at the radius $\boldsymbol{r_2}$.

$$h_i = \frac{U_2^2}{g} - \frac{U_2 \cot \beta_2}{2\pi r_2 b_2 g} \dot{\forall}$$

This equation is graphed in the margin and shows that the ideal or maximum head rise for a centrifugal pump varies linearly with $\dot{\forall}$ for a given blade geometry and angular velocity.

Ex.1 Water is pumped at the rate of 1400 gpm through a centrifugal pump operating at a speed of 1750 rpm. The impeller has a uniform blade height, b, of 2 in. with r_1 =1.9 in. and r_2 = 7in., and the exit blade angle $\beta_2 = 23^o$ is as show in fig.3 . Assume ideal flow conditions and that the tangential velocity component $V_{\theta 1}$, of the water entering the blade is zero ($\alpha_1 = 90^o$)

Determine (a) the tangential velocity component, at the exit, (b) the ideal head rise, and (c) the power, transferred to the fluid.

Ans.

$$U_2 = r_2\omega = (7/12 \text{ ft})(2\pi \text{ rad/rev}) \frac{(1750 \text{ rpm})}{(60 \text{ s/min})}$$

Since the flowrate is given, it follows that

$$Q = 2\pi r_2 b_2 V_{r2}$$

or

$$V_{r2} = \frac{Q}{2\pi r_2 b_2} = \frac{1400 \text{ gpm}}{(7.48 \text{ gal/ft}^3)(60 \text{ s/min})(2\pi)(7/12 \text{ ft})(2/12 \text{ ft})} = 5.11 \text{ ft/s}$$

$$\cot \beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

so that

$$V_{\theta 2} = U_2 - V_{r2} \cot \beta_2 = (107 - 5.11 \cot 23°) \text{ ft/s} = 95.0 \text{ ft/s}$$ **(Ans)**

(b) the ideal head rise is given by

$$h_i = \frac{U_2 V_{\theta 2}}{g} = \frac{(107 \text{ ft/s})(95.0 \text{ ft/s})}{32.2 \text{ ft/s}^2} = 316 \text{ ft}$$ **(Ans)**

(c) with $V_{\theta 1} = 0$, the power transferred to the fluid is given by the equation

$$\dot{W}_{\text{shaft}} = \rho Q U_2 V_{\theta 2} = \frac{(1.94 \text{ slugs/ft}^3)(1400 \text{ gpm})(107 \text{ ft/s})(95.0 \text{ ft/s})}{[1(\text{slug} \cdot \text{ft/s}^2)/\text{lb}](7.48 \text{ gal/ft}^3)(60 \text{ s/min})}$$

$$= (61{,}500 \text{ ft} \cdot \text{lb/s})(1 \text{ hp}/550 \text{ ft} \cdot \text{lb/s}) = 112 \text{ hp}$$ **(Ans)**

Note that the ideal head rise and the power transferred to the fluid are related through the relationship

$$\dot{W}_{\text{shaft}} = \rho g Q h_i$$

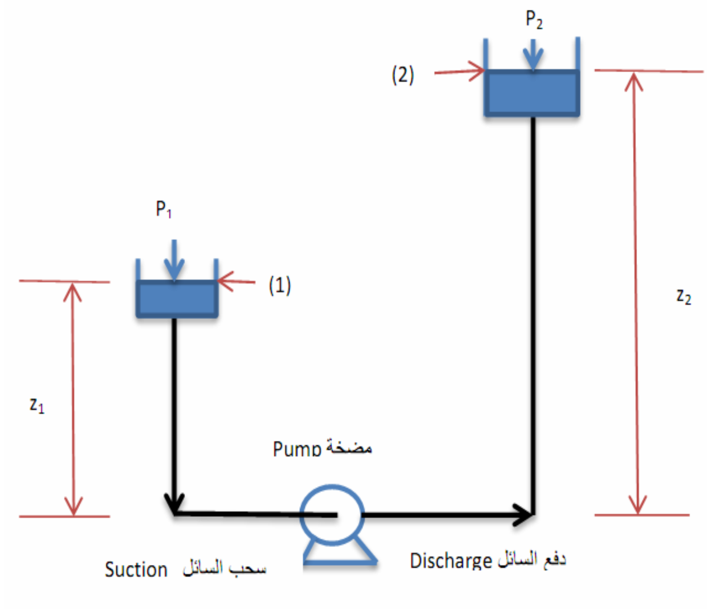
Net positive suction head (NPSH)

The total suction head hs can be defined by applying the energy equation to point (1) and the suction in Fig 1 as follows:

$$\frac{\bar{u}_2^2}{2\,\alpha} + \frac{P_s}{\rho\, g} + h_{fs} = z_1 + \frac{P_1}{\rho\, g}$$

The total suction head hs is the sum of the pressure head and kinetic head at the pump inlet, hence the equation becomes:

$$h_s = z_1 + \frac{P_1}{\rho\, g} - h_{fs}$$



$\bar{u}_s$ denotes the average velocity at the pump inlet, Ps is the pressure at the inlet and hfs represents the friction head in the suction line. In the case of pumping a liquid from a low location, z1 will be negative. The net positive suction head (NSPH) is defined as the pressure head that must be added to the liquid vapor pressure head hv to effect pumping. In other words, the pump will not draw and pump the liquid unless the total suction head equals or exceeds the sum of the net positive suction head and the vapour pressure head. If it is less, the liquid will partially vaporise forming bubbles in the suction line causing cavitation; this will disrupt pumping of the liquid, thus:

$$h_s = NPSH + \frac{P_v}{\rho\, g}$$

Pv is the vapor pressure of the liquid.

and

$$z_1 + \frac{P_1}{\rho\, g} - h_{fs} = NPSH + \frac{P_v}{\rho\, g}$$

$$\therefore NPSH = z_1 + \frac{P_1 - P_v}{\rho\, g} - h_{fs}$$

Centrifugal pumps

Centrifugal pumps are extensively used in industry to pump water, oil, corrosive liquids and slurries, such as concrete mix. The liquid is drawn axially to enter the rotating part of the pump, known as the impeller. It is then pushed to the volute, by centrifugal force, to exit from the discharge port. The impeller consists of a number of curved vanes; these are fixed between two circular plates forming passages for the liquid. The impeller is driven by a motor through a shaft attached to it. The shaft passes into the pump casing through a stuffing box containing sealing rings, see Fig 2.

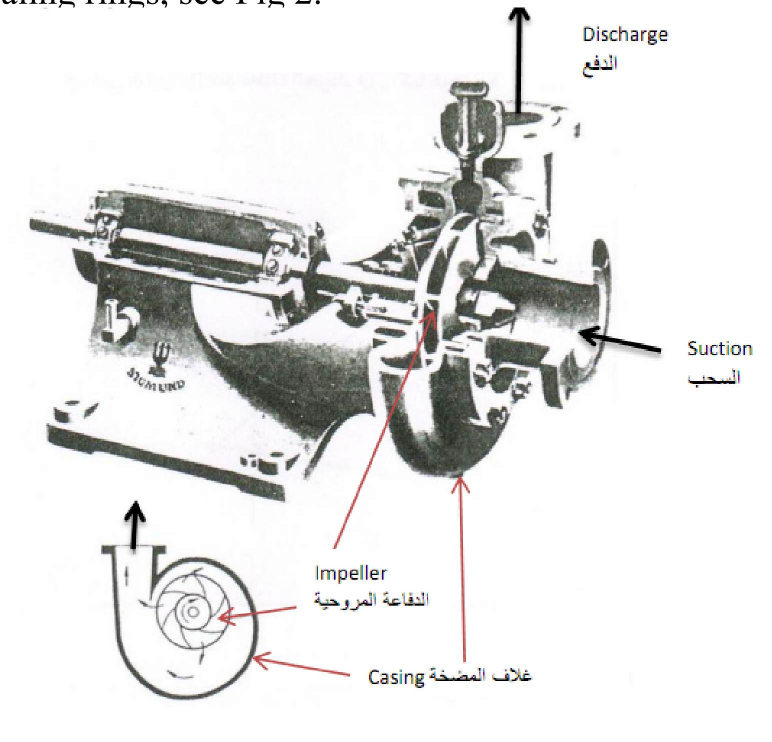


Fig 2 Centrifugal pump

Performance and characteristic curves

The performance of a centrifugal pump is illustrated by plotting characteristic curves for each rotation speed of the impeller. There are three different types of curves; these are the total head Δh vs. the volumetric flow rate Q, the efficiency η vs flow rate Q and the power PE vs. flow rate Q, as shown in Fig 3.

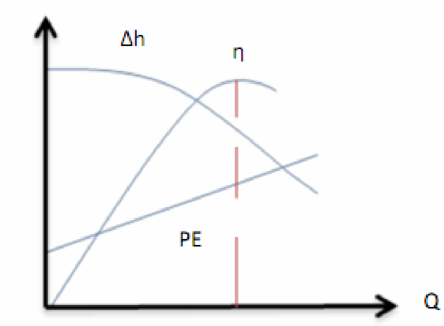


Fig 3 Characteristic curves for a centrifugal pump

The optimum operating conditions for running the pump is determined by identifying the duty point. The duty point is the intersection of the vertical from the maximum efficiency with the total head curve. It is noted in Fig 4 that the total head decreases as the flow rate increases. If we plot the total system head Δhs vs. the flow rate, we will that it increases with the increase of the flow; thus the reverse of the characteristic curve of the total head known as the pump curve.

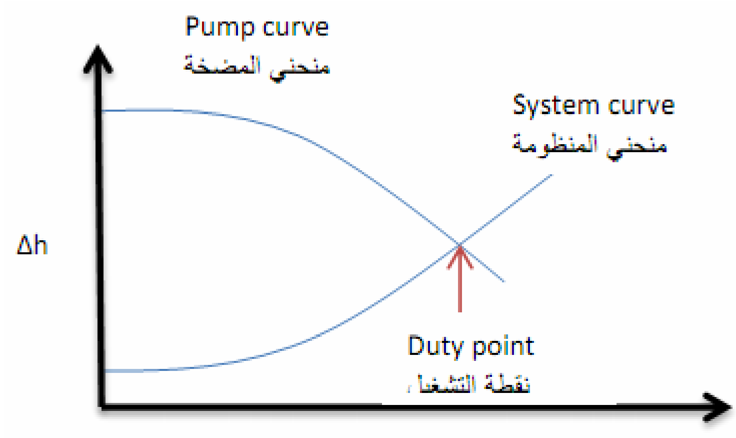


Fig 4 Duty point

Relationships for Centrifugal pumps

The power required to operate a centrifugal pump PE is a function of the liquid density ρ, the impeller diameter D and the rotation speed N, thus:

$$PE = f\,(\rho, D, N)$$

The specific speed is considered as a characteristic number to specify the type of pump; it is calculated at the peak specific efficiency of operation. When the non-dimensional specific speed of two pumps is constant, the pumps will be geometrically similar. Further, we can apply the following relationships to such similar pumps:

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3$$

$$\frac{\Delta h_1}{\Delta h_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2$$

$$\frac{PE_1}{PE_2} = \left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^5$$

The following relationships can be used to when the impeller size is changed for the same pump:

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)$$

$$\frac{\Delta h_1}{\Delta h_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2$$

$$\frac{PE_1}{PE_2} = \left(\frac{N_1}{N_2}\right)^3\left(\frac{D_1}{D_2}\right)^3$$

specific speed

$$\frac{N^2\,Q}{(g\,\Delta h)^{3/2}} = N_s$$

Ex.1 A test run was conducted for a centrifugal pump with an impeller diameter of 0.3 m. The peak efficiency was achieved at a speed of 900 rpm and flow rate 1.8 m^3/min. The pump drives the liquid into a system to overcome a total head of 15.2 m. If a geometrically similar pump is used to deliver double the quantity against a head of 18.3 m, compute the impeller diameter and the speed of the second pump.

Solution

Given data for the first pump:

$$\Delta h_1 = 15.2\text{ m},\ Q_1 = 1.8/60\text{ m}^3/\text{s},\ D_1 = 0.3\text{ m}$$

Data for the similar pump:

$$\Delta h_2 = 18.3\text{ m},\ Q_2 = 2 \times 1.8/60\text{ m}^3/\text{s}$$

We have:

$$\frac{Q_1}{Q_2} = \left(\frac{N_1}{N_2}\right)\left(\frac{D_1}{D_2}\right)^3$$

$$\frac{\Delta h_1}{\Delta h_2} = \left(\frac{N_1}{N_2}\right)^2\left(\frac{D_1}{D_2}\right)^2$$

Substituting in the above equations:

$$\frac{1.8/60}{2 \times 1.8/60} = (900/N_2)\,(0.3/D_2)^3$$

$$1/2 = (900/N_2)\,(0.3/D_2)^3 \qquad \text{A}$$

Using equation

$$\frac{\Delta h_1}{\Delta h_2} = \left(\frac{N_1}{N_2}\right)^2 \left(\frac{D_1}{D_2}\right)^2$$

$$15.2/18.3 = (900/N_2)\,(0.3/D_2)^2 \qquad \text{B}$$

Solving equation A and B, we arrive at;

$$D_2 = 0.405 \text{ m}$$

$$N_2 = 730 \text{ rpm}$$

The rotation speed can also be determined using the non-dimensional specific speed:

$$\frac{N^2\,Q}{(g\,\Delta h)^{3/2}} = N_s$$

Since the two pumps are geometrically similar, the following relationship is applied:

$$\frac{N_1^2 Q_1}{(g\,\Delta h_1)^{3/2}} = \frac{N_2^2 Q_2}{(g\,\Delta h_2)^{3/2}}$$

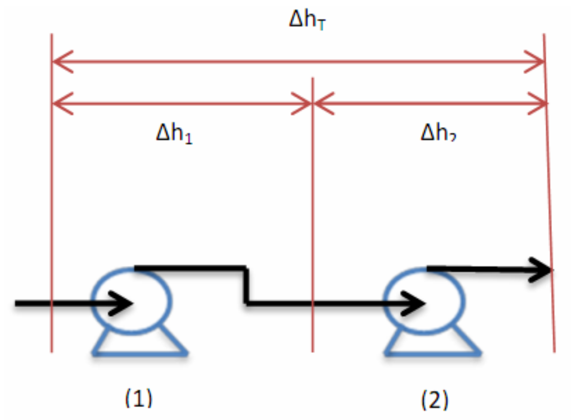
$$\frac{900^2 \times \frac{1.8}{60}}{(9.81 \times 15.2)^{3/2}} = \frac{N_2^2\,(\frac{1.8}{60} \times 2)}{(9.81 \times 18.3)^{3/2}}$$

N_2 = <u>730 rpm</u>

TWO OR MORE PUMPS IN FLOW SYSTEM

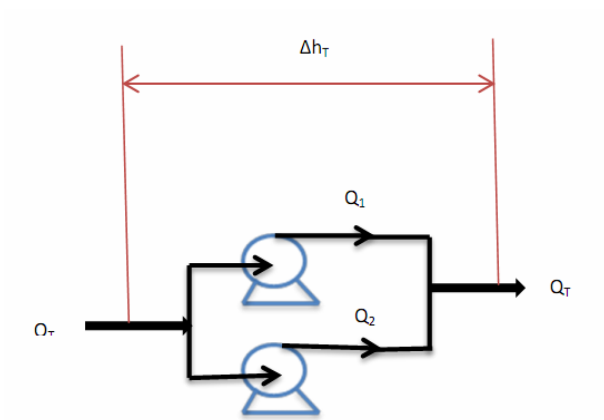
In some cases, the pumping of liquids requires more than one pump. The way the pumps are arranged is either in series or in parallel. For a series connection, the discharge line of the first pump is attached to the suction line of the second. The total head ΔhT for the two pumps will be equal to the sum of the two heads $\Delta h1$ and $\Delta h2$. The discharge remains unchanged, that is ($QT = Q1 = Q2$), see fig 5.

Fig.5 Series arrangement



If the two pumps are connected in parallel, the total head will remain unchanged, that is: $\Delta hT = \Delta h1 = \Delta h2$, whilst the discharge will be equal to the sum of the tow pump outputs ($QT = Q1 + Q2$), see Fig.6.

Fig.6 Parallel arrangement



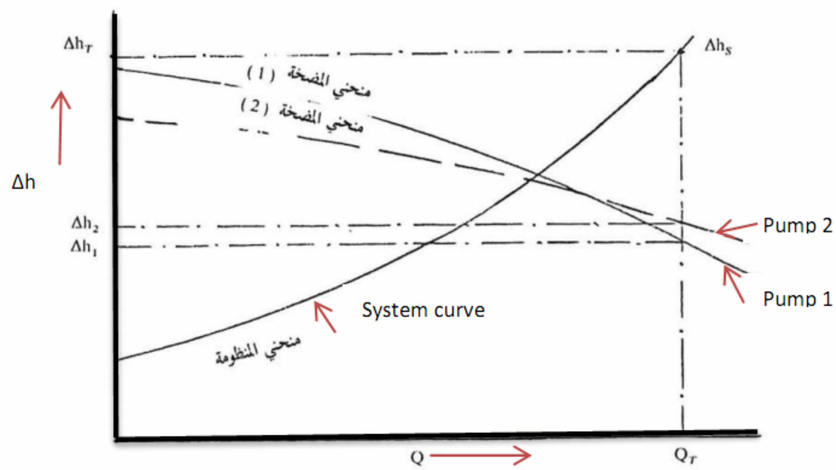
The duty point for two pumps connected in series can be estimated as follows:

(a) We plot the characteristic curve for each pump separately (Δh vs. Q); also, the total head vs discharge (ΔhT vs. Q), as shown in Fig.7

(b) We draw a vertical line to the horizontal axis, intersecting the three curves at $\Delta h1$, $\Delta h2$ and Δhs.

(c) We calculate the total head for the two pumps: $\Delta h_T = \Delta h_1 + \Delta h_2$

Fig.7. Duty point for two pumps in series

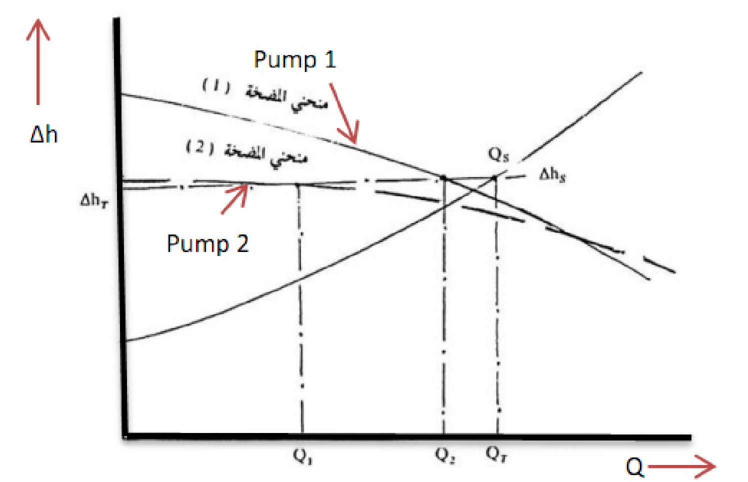


(d) We compare ΔhT to Δhs, and if the two values are far apart, we repeat the steps (b), (c) and (d) until ΔhT= Δhs. At that condition, we obtain the duty point defined by the last value of the total head ΔhT and the corresponding total discharge QT.

Similarly, we can find the duty point for parallel connection:
(a) We plot the two characteristics for the two pumps and the total head curve for the system, as shown in Fig 8
(b) We draw a line parallel to the axis, intersecting the three curves at Q1, Q2, and Q3.
(c) We calculate the total discharge QT as follows: $Q_T = Q_1 + Q_2$

(d) We compare Qt to Qs; if the values differ significantly, we repeat the steps (b), (c) and (d) until the values get closer; at the last calculated values of QT and ΔhT the duty point is specified.

Fig 8 Duty point for two pumps in parallel



ADVANTAGES AND DISADVANTAGES OF CENTRIFUGAL PUMPS

The advantages of a centrifugal pump are:

(a) The pump assembly is simple and therefore it can be manufactured from various materials such as steel and plastics etc

(b) The rotation speed is high enough to allow coupling to an electric motor

(c) The pump delivers a pulsation-free discharge

(d) It has a relatively low maintenance cost

(e) The pump will not get damaged upon sudden blockage, provided it is not left operating for a long time

(f) For a given discharge, it has a smaller size compared to other types. Thus, it can be manufactured as a submersible pump. A good example is the pumping of refrigerated liquefied petroleum gas (LPG) from a storage tank

(g) The pump can be used to pump slurries and it is widely used to pump sewage waste

The common disadvantages are:

(a) Low livery pressure and for higher pressures, costly multistage pumps are manufactured

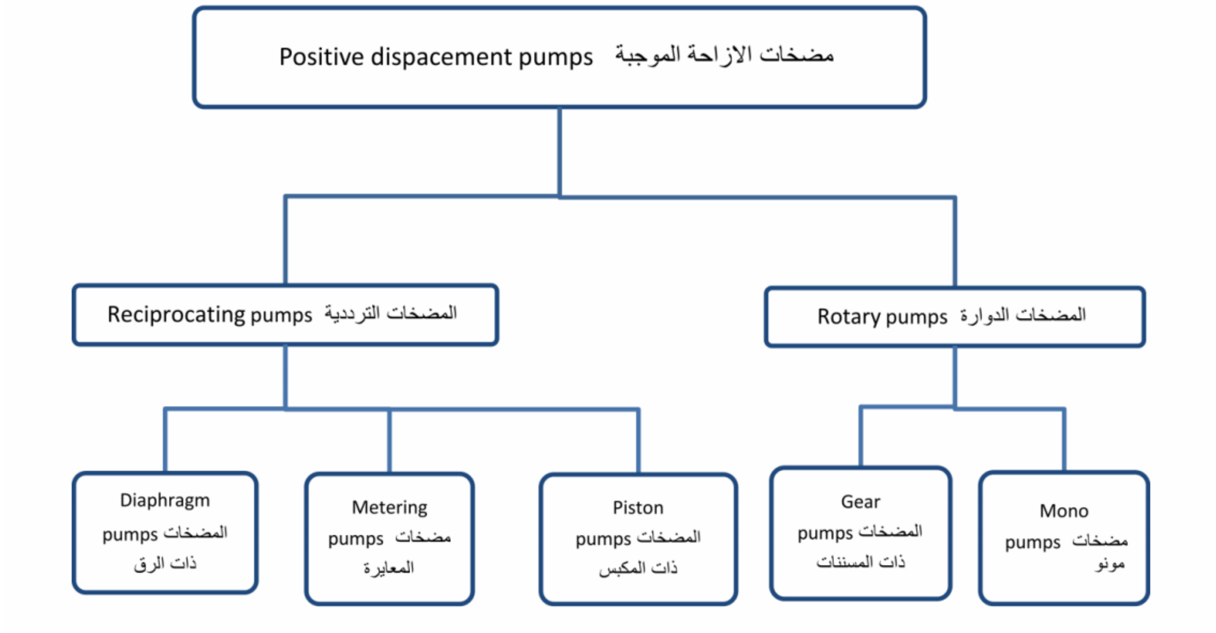
(b) It is required to fill the casing and the suction line with liquid before running the pump, this is normally known as priming

(c) A check valve must be included in the discharge line to prevent the backflow of the liquid after operation

(d) The pump is not used for viscous liquids

POSITIVE DISPLACEMENT PUMPS

The pumps are classified as follows



PISTON PUMPS-DOSING PUMPS

This pump consists of a piston reciprocating inside a cylinder having four valves as shown in Fig 9.

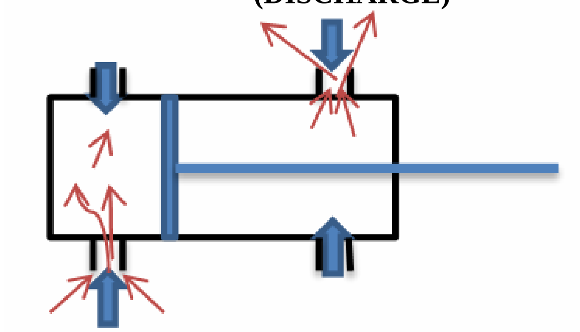


Fig.9 Piston-cylinder arrangement

NOTE how the valves open on the two sides of the piston causing the suction of the liquid on one side and a high-pressure discharge on the other. Changing the direction of the piston movement will reverse the process. A motor, steam engine or internal combustion engine can be used to drive the pump.

The piston pump delivers high pressures with a pulsating discharge due to the reciprocating motion of the piston.

DIAPHRAGM PUMPS

This pump is made of a rubbery material diaphragm separating two compartments. A piston-cylinder arrangement is fixed on one side of the diaphragm; this is to induce the reciprocating motion directly or indirectly though liquid. The motion is transferred to the other compartment where the liquid is withdrawn. This pump is used to pump corrosive liquids. See Fig.10.

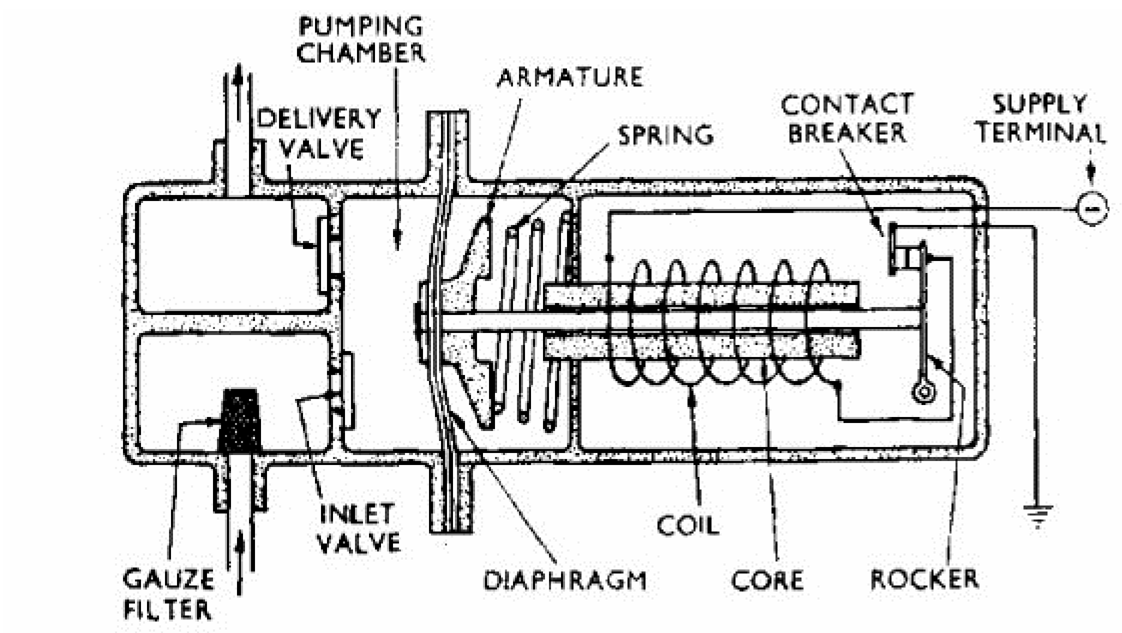
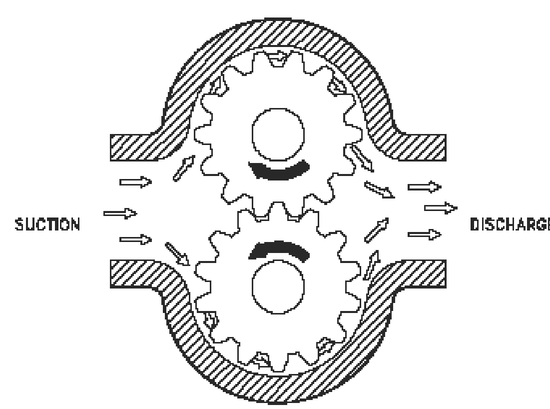


Fig 8.10 Diaphragm pump

GEAR PUMPS

The liquid is pumped as a result of the gears rotating inside the casing as shown in Fig11. High pressures can be generated by gear pumps and therefore can be used to pump viscous liquids. The pump delivers steady and smooth discharges and can be attached directly to an electric motor. The pump does not require priming prior to operation. These pumps are used extensively in the petroleum industry to pump viscous liquids. They are also used to handle food stuff and sticky materials.

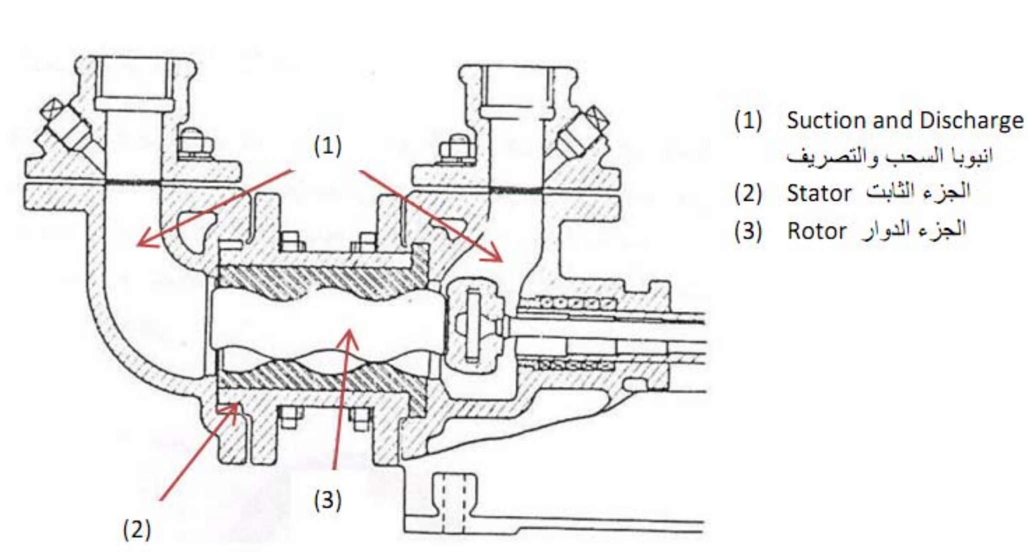
Fig 11 Gear pump



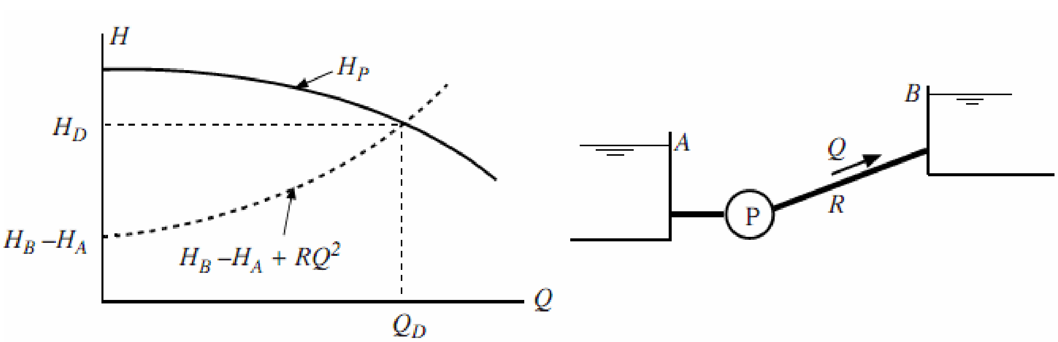
MONO PUMPS

This pump consists of a helical rotor sealed tightly against a helical hole in a plastic stator. The liquid is pushed through the cavities formed by the rotation of the rotor inside the stator. This pump is used to pump slurries to filtration equipment; the pump must not be operated in dry conditions, see Fig.12

Fig12 Mono pump



EX 1. Estimate the discharge in the pipe system shown in Fig. 10.2, and in addition find the required pump power. For the pipe, L = 700 m, D = 300 mm, f = 0.02, and H_B - H_A = 30 m. Use the 240-mm curve in Fig. for the pump head–discharge relation.



Ans. Used this eq.

$$R = \frac{1}{2gA^2}\left(f\frac{L}{D} + \sum K\right)$$

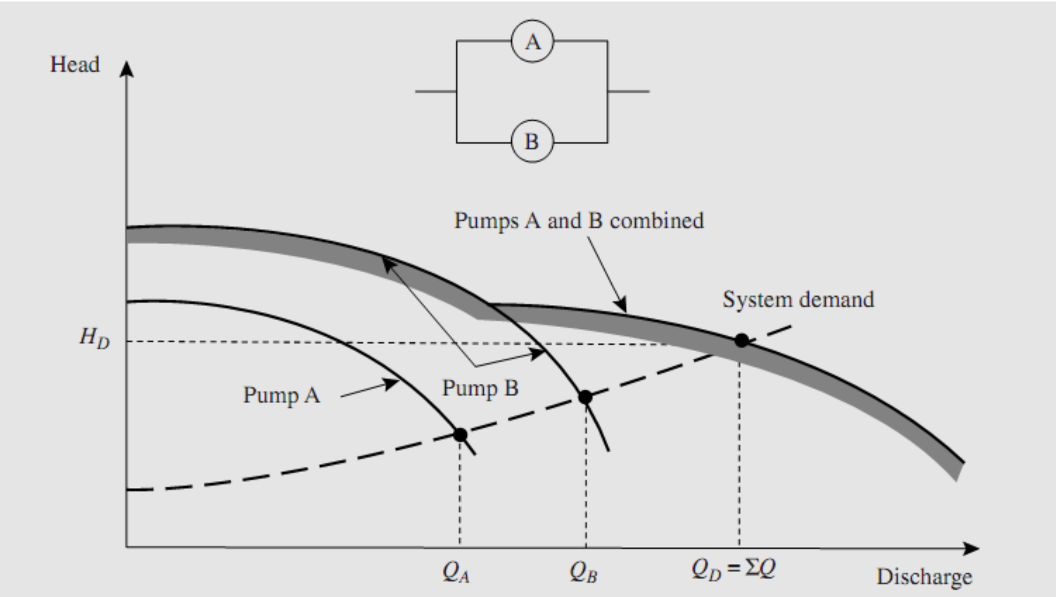
$$R = \frac{1}{2 \times 9.81(\pi \times 0.30^2/4)^2}\left(0.02 \times \frac{700}{0.30}\right) = 476\ \mathrm{s^2/m^5}$$

A trial solution is utilized to determine the pump head and discharge. The procedure is as follows: (1) guess a discharge; (2) compute HP with Eq. ($H_P = (H_B - H_A) + RQ^2$); and (3) compare that value with the one from the 240-mm pump curve. Continue estimating values of Q until the two pump heads agree. The solution is shown in the table.

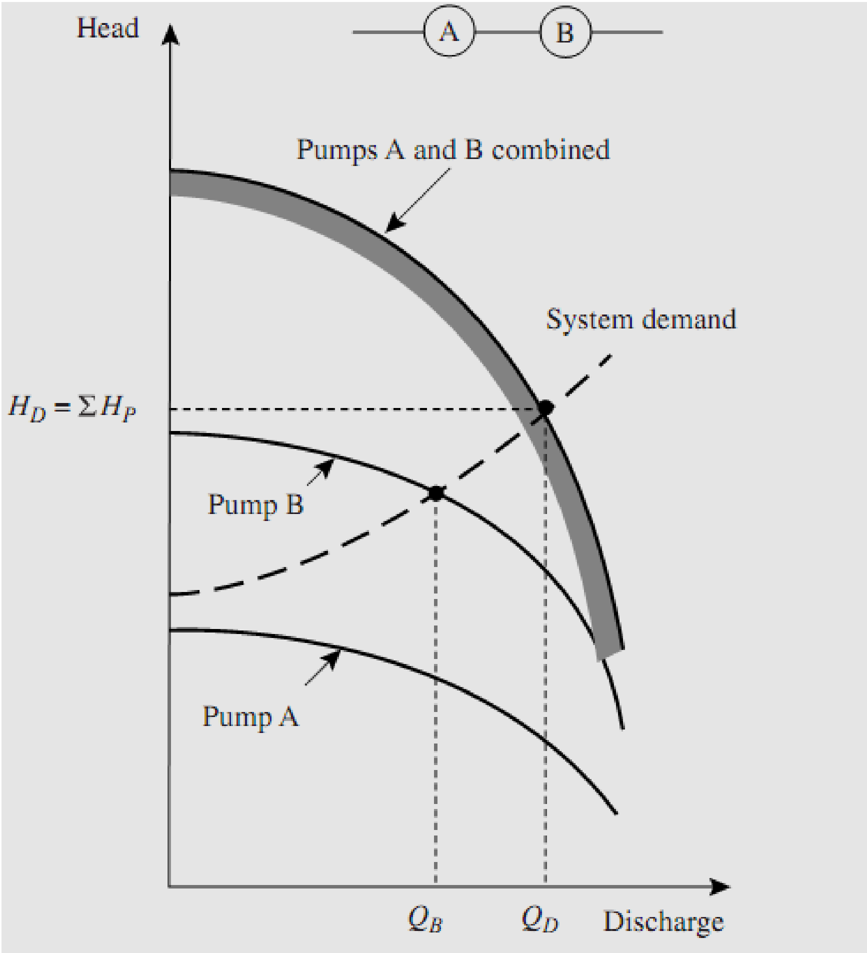
Q, m^3/h	Q, m^3/s	H_P, m **From Eq.**	H_P, m **From Fig**
150	0.042	70.8	74
250	0.069	72.3	67
200	0.056	71.5	72

Hence, the approximate solution isQ = 200 m^3/h and HP = 72 m. From Fig., the efficiency is approximately 75%, so that the required power is

$$\dot{W}_P = \frac{\gamma Q H_p}{\eta} = \frac{9800 \times 0.056 \times 72}{0.75} = 52\,700 \text{ W or } 706 \text{ hp}$$



(*a*) Parallel pumping



(*b*) Series pumping

EX.2 Water is pumped between two reservoirs in a single pipe with the value of R = 85 s^2/m^5 .For the pump characteristic curve, use HP = 22:9 + 10.7 Q - 111Q^2. Compute the discharge Q and pump head HP for:

(*a*) $H_B - H_A = 15$ m with one pump placed in operation
(*b*) $H_B - H_A = 15$ m with two identical pumps operating in parallel
(*c*) $H_B - H_A = 25$ m with two pumps operating in series

Solution: Since the pump curve is provided in quadratic form, Eqs. ($H_P = (H_B - H_A) + RQ^2$) and ($H_P(Q) = a_0 + a_1 Q + a_2 Q^2$) can be combined to eliminate HP and solve for Q.The solutions are as follows (*a*) Equate the system demand curve to the pump characteristic curve and solve the resulting quadratic equation:

$$15 + 85Q^2 = 22.9 + 10.7Q - 111Q^2$$
$$195Q^2 - 10.7Q - 7.9 = 0$$
$$Q = \frac{1}{2 \times 195}\left(10.7 + \sqrt{10.7^2 + 4 \times 195 \times 7.9}\right) = 0.23\,\text{m}^3/\text{s}$$
$$H_P = 15 + 85 \times 0.23^2 = 19.5\,\text{m}$$

(*b*) For two pumps in parallel, the characteristic curve is

$$H_P = 22.9 + 10.7\left(\frac{Q}{2}\right) - 111\left(\frac{Q}{2}\right)^2 = 22.9 + 5.35Q - 27.75Q^2$$

The system demand curve is equated to this result and solved for Q:

$$15 + 85Q^2 = 22.9 + 5.35Q - 27.75Q^2$$
$$112.8Q^2 - 5.35Q - 7.9 = 0$$
$$Q = \frac{1}{2 \times 112.8}\left(5.35 + \sqrt{5.35^2 + 4 \times 112.8 \times 7.9}\right) = 0.29\,\text{m}^3/\text{s}$$
$$H_P = 15 + 85 \times 0.29^2 = 22.2\,\text{m}$$

(*c*) With two pumps in series, the characteristic curve becomes

$$H_P = 2(22.9 + 5.35Q - 111Q^2) = 45.8 + 21.4Q - 222Q^2$$

Equate this to the system demand curve and solve for Q:

$$25 + 85Q^2 = 45.8 + 21.4Q - 222Q^2$$
$$307Q^2 - 21.4Q - 20.8 = 0$$
$$Q = \frac{1}{2 \times 307}\left(21.4 + \sqrt{21.4^2 + 4 \times 307 \times 20.8}\right) = 0.30\,\text{m}^3/\text{s}$$
$$H_P = 25 + 85 \times 0.30^2 = 32.5\,\text{m}$$