I.C.E.™

Inspiring Creativity & Endeavour

IES | GATE | PSU

FLUID MECHANICS

For

MECHANICAL ENGINEERING

CIVIL ENGINEERING

FLUID MECHANICS

SYLLABUS

Fluid properties; fluid statics, manometry, buoyancy, forces on submerged bodies, stability of floating bodies; control-volume analysis of mass, momentum and energy; fluid acceleration; differential equations of continuity and momentum; Bernoulli's equation; dimensional analysis; viscous flow of incompressible fluids, boundary layer, elementary turbulent flow, flow through pipes, head losses in pipes, bends and fittings.
Turbomachinery: Impulse and reaction principles, velocity diagrams, Pelton-wheel, Francis and Kaplan turbines.

ANALYSIS OF GATE PAPERS

MECHANICAL			
Exam Year	1 Mark Ques.	2 Mark Ques.	Total
2003	1	6	13
2004	2	9	20
2005	1	3	7
2006	3	7	17
2007	3	7	17
2008	1	5	11
2009	-	7	14
2010	4	3	10
2011	1	3	7
2012	3	2	7
2013	2	2	6
2014 Set-1	1	5	11
2014 Set-2	2	3	8
2014 Set-3	3	3	9
2014 Set-4	2	2	6
2015 Set-1	1	4	9
2015 Set-2	2	2	6
2015 Set-3	2	2	6
2016 Set-1	3	3	9
2016 Set-2	2	3	8
2016 Set-3	3	3	9
2017 Set-1	4	3	10
2017 Set-2	4	3	10
2018 Set-1	2	3	8
2018 Set-2	3	4	11

CIVIL			
Exam Year	1 Mark Ques.	2 Mark Ques.	Total
2003	3	8	19
2004	6	10	26
2005	3	7	17
2006	3	7	17
2007	2	7	16
2008	1	7	15
2009	1	3	7
2010	3	2	7
2011	3	2	7
2012	3		3
2013	3	2	7
2014 Set-1	2	7	16
2014 Set-2	2	4	10
2015 Set-1	3	4	11
2015 Set-2	3	3	9
2016 Set-1	1	3	7
2016 Set-2	1	3	7
2017 Set-1	2	3	8
2017 Set-2	1	2	5
2018 Set-1	2	3	8
2018 Set-2	1	2	5

CONTENTS

Topics **Page No**

3 PRESSURE & FLUID STATICS

3.1 PRESSURE

Pressure is defined as the normal force exerted by fluid per unit area. We speak of pressure only when we deal with a gas or a liquid. The counterpart of pressure in solids is normal stress.

$$P = \frac{F}{A}$$

Since pressure is defined as force per unit area, it has the unit of Newtons per square meter (N/m^2), which is called Pascal (Pa)

1bar = 10^5 Pa = 0.1 MPa = 100 kPa

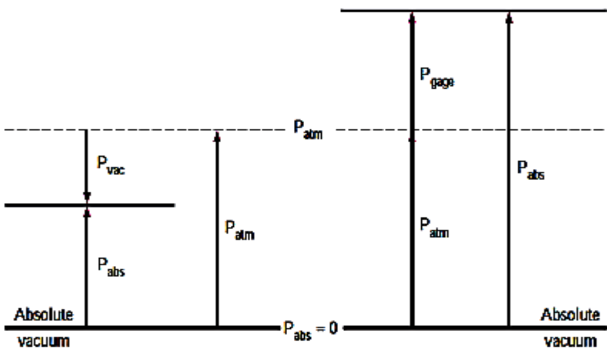
3.1.1 ABSOLUTE, GAUGE, ATMOSPHERIC & VACUUM PRESSURE

1) The pressure values must be stated with respect to a reference level. If the reference level is vacuum (i.e. absolute zero pressure), pressures are termed **absolute pressure**.
2) Most pressure gauges indicate a pressure difference—the difference between the measured pressure and the ambient level (usually atmospheric pressure). Pressure levels measured with respect to atmospheric pressure are termed **gauge pressures**
3) Pressures below atmospheric pressure are called **vacuum pressures**.

   Absolute, gauge, and vacuum pressures are all positive quantities and are related to each other by

$$P_{absolute} = P_{atmospheric} + P_{gauge}$$

$$P_{vacuum} = P_{atmospheric} + P_{absolute}$$



3.1.2 PRESSURE AT A POINT

Pressure is the compressive force per unit area, and it gives the impression of being a vector. However, in fluids under static conditions, pressure is found to be independent of the orientation of the area. This concept is explained by **Pascal's law** which **states that the pressure at a point in a fluid at rest is equal in magnitude in all directions**. Pressure has magnitude but not a specific direction, and thus it is a scalar quantity.

$$P_x = P_y = P_z$$

3.1.3 PRESSURE VARIATION IN A STATIC FLUID (HYDROSTATIC LAW):

For fluids at rest or moving on a straight path at constant velocity, all components of acceleration are zero. In fluids at rest, the pressure remains constant in any horizontal direction (P is independent of x and y) and varies only in the vertical direction.

As a result of gravity, these relations are applicable for both compressible and incompressible fluids.

$$\frac{dp}{dz} = -\rho(g), \quad \frac{dp}{dx} = 0, \quad \frac{dp}{dy} = 0$$

The negative sign is taken because dz is taken positive in upward direction and pressure decrease in upward direction.

For incompressible fluid ρ is constant.

$$\int_{p_o}^{p} dp = -\rho g \int_{z_o}^{z} dz$$

$$P - P_o = -\rho g(z - z_o)$$

$$P - P_o = \rho g h$$

For compressible fluid, ρ varies with pressure, i.e. $\rho = f(P)$

For gases, variation of density with pressure can be expressed by ideal gas equation

$$\rho = \frac{P}{RT}$$

Where,
P is pressure
T is temperature

3.1.4 FLUIDS IN RIGID-BODY MOTION

When fluid is in stationary container, the pressure remains constant along horizontal direction. The pressure varies only along vertical direction. When a fluid is placed in an accelerated container, initially fluid splashes and there is a relative motion between fluid & container boundary. After some time, the liquid comes to rest and attains fixed shape relative to container. The pressure varies in the direction of acceleration.

1) **When container accelerates in vertical direction**

   Case1: Downward acceleration of a_z

   $$\frac{dp}{dz} = -\rho(g - a_z), \quad \frac{dp}{dx} = 0, \quad \frac{dp}{dy} = 0$$

   Case 2: upward acceleration of a_z

   $$\frac{dp}{dz} = -\rho(g + a_z), \quad \frac{dp}{dx} = 0, \quad \frac{dp}{dy} = 0$$

2) **When container accelerates in horizontal direction**

   Case1: Acceleration in positive x direction

   $$\frac{dp}{dz} = -\rho(g), \quad \frac{dp}{dx} = -\rho a_x, \quad \frac{dp}{dy} = 0$$

   Case 2: Acceleration in negative x direction

   $$\frac{dp}{dz} = -\rho(g), \quad \frac{dp}{dx} = -\rho(-a_x), \quad \frac{dp}{dy} = 0$$

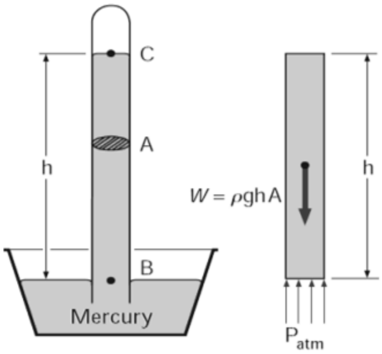
3.2 THE BAROMETER & ATMOSPHERIC PRESSURE

Atmospheric pressure is measured by a device called **barometer;** thus, the atmospheric pressure is often referred to as the barometric pressure. The pressure at point B is equal to the atmospheric pressure, and the pressure at C can be taken to be zero since there is only mercury vapour above point C and the pressure is very low relative to P_{atm} and can be neglected for an excellent approximation. Writing a force balance in the vertical direction gives

$$P_{atm} = \rho g h$$

Where,
ρ is the density of mercury,
g is the local gravitational acceleration,
h is the height of the mercury column above the free surface.

Note that the length and the cross-sectional area of the tube have no effect on the height of the fluid column of a barometer. A frequently used pressure unit is the *standard atmosphere*, which is defined as the pressure produced by a column of mercury 760 mm in height at 0°C $(\rho_{Hg}$ is $13,595\,kg/m^3)$. If water instead of mercury were used to measure the standard atmospheric pressure, a water column of about 10.3m would be needed.



3.3 PRINCIPLES OF FLUID STATICS

1) When fluid is at rest, in a continuous fluid, fluid at the same elevation has the same pressure.
2) The pressure at the bottom of a column of fluid is equal to the pressure at the top, plus density multiplied by gravity multiplied by the height of the column of fluid.

   A consequence of the second principle is that when different columns of fluid stack on top of one another, the pressures due to each column simply add up.

3.3.1 HYDRAULIC LIFT

A consequence of the pressure in a fluid remaining constant in the horizontal direction is that the pressure applied to a confined fluid increases the pressure throughout by the same amount. This is called **Pascal's law.** The application of Pascal's law in hydraulic lift is shown in fig

$P_1 = P_2$

$F_1 / A_1 = F_2 / A_2$

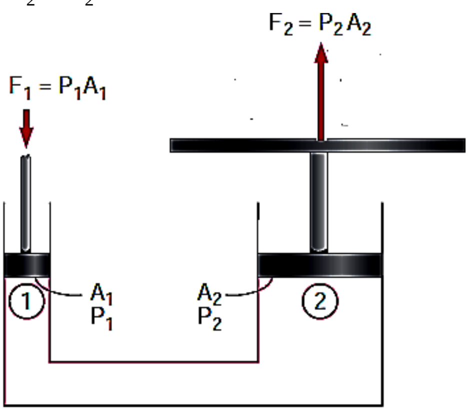


Fig.: Schematic of a hydraulic lift

3.4 PRESSURE MEASUREMENT

Hydrostatic law indicates the pressure difference b/w two points in a static fluid. A device based on this principle is called **manometer**, and it is commonly used to measure small and moderate pressure differences. A manometer mainly consists of a glass or plastic U tube containing one or more fluids such as mercury, water, alcohol, or oil**.** The pressure of a fluid is measured by the following.

1) Manometers
   a. Simple
   - Piezometer
   - U-tube manometer
   - Single column manometer

   b. Differential
   - U-tube differential manometer
   - Inverted U-tube manometer
2) Mechanical gauge

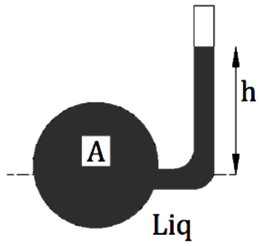
3.4.1 SIMPLE MANOMETER

1) **Piezometer:** It is the simplest kind of manometer. It does not have any high density liquid. The tube is connected to the point where pressure is to be measured. The liquid rises in tube to balance the pressure at 'A'.
   The gauge pressure P_A is given by

   $P_A = \rho g h$

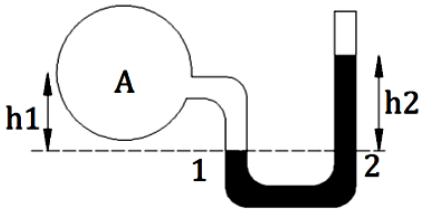
   Where

   ρ is the density of liquid inside vessel/pipe

   Piezometer is used to measure low pressures. The height of column increases if pressure is high. E.g. height of water column is 10.3 m if gauge pressure at 'A' is 1atm ($10^5 N/m^2$).

2) **U-tube manometer:** It has a glass U-tube with liquid having density higher than the density of fluid in the container.

   The atmospheric pressure exists and should be taken into account for evaluation of absolute pressure, but while evaluating gauge pressure it is not accounted in equations.
   The gauge pressure at 'A' is given by

   $P_1 = P_A + \rho_c g h_1$

   $P_2 = \rho_m g h_2$

   Where,

ρ_C is the density of fluid in container, it can be water or oil

ρ_m is the density of manometric fluid. Usually mercury is chosen as manometric fluid

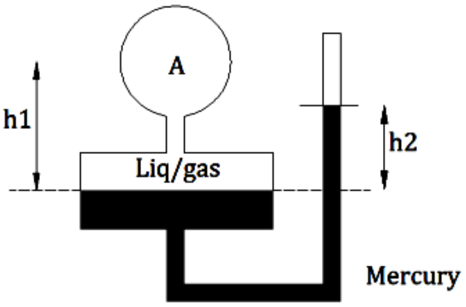
h_2 is difference in mercury level

P_A is pressure in vessel/pipe in gauge

From principle of fluid statics, when fluid is at rest, fluid at the same elevation has the same pressure.

$P_A = \rho_m g h_2 - \rho_w g h_1$

3) **Single Column Manometers:** In this manometer a large cross-sectional area reservoir is placed in one of the limbs. When pressure is applied, the fluid lowers slightly in the reservoir as compared to the fluid rise in the other limb.



Gauge Pressure at point A is given as

$$P_A = \frac{ah_2}{A}.(\rho_m g - \rho_c g) + (\rho_m g h_2 - \rho_c g h_1)$$

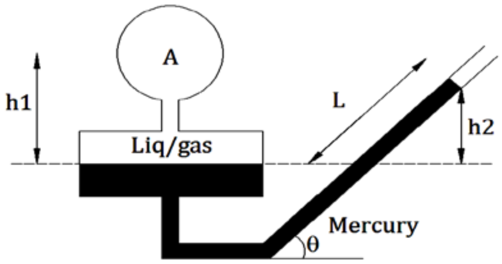
Since A>>α

$$P_A = \rho_m g h_2 - \rho_c g h_1$$

Salient features of Single column manometer:

i) modified form of U-tube manometer
ii) large cross-sectional area (100 times)
iii) due to large area of cross section and small change of pressure, the change in level of reservoir will be very small and can be neglected.

4) **Inclined Single Column Manometer:** This manometer is more sensitive than straight column. The liquid rises more in the column due to inclination.



$$h_2 = L\sin\theta$$

$$P_A = \frac{ah_2}{A}.(\rho_2 g - \rho_1 g) + (\rho_2 g h_2 - \rho_1 g h_1)$$

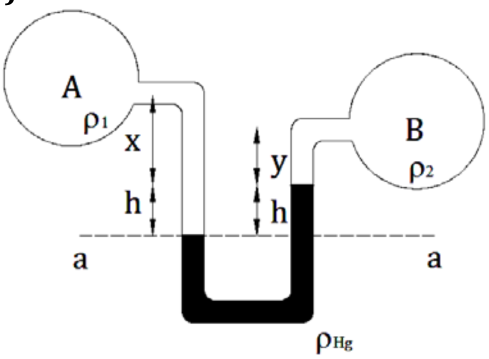
$$P_A = \rho_2 g_2 h_2 - \rho_1 g_1 h_1$$

$$P_A = \rho_2 g_2 L\sin\theta - \rho_1 g h_1$$

3.4.2 DIFFERENTIAL MANOMETERS

Differential Manometers are devices used for measuring the difference of pressure between two points in a pipe or two different pipes. It contains of a U-tube with manometric liquid. The manometric liquid can be of higher density or lower density than pipe liquid.

1) **U-tube differential manometer:**



Pressure above a-a

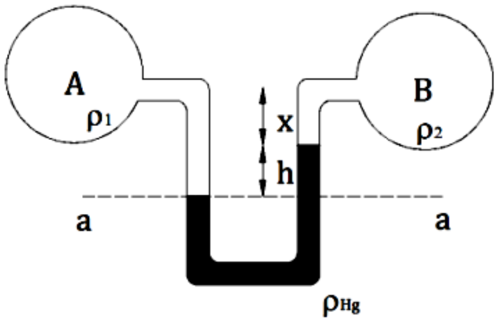
- In the left limb $= \rho_1 g(h + x) + P_A$
- In the right limb $= P_B + \rho_2 gy + \rho_{Hg} gh$

By equating

$$P_A + \rho_1 g(h + x) = P_B + \rho_2 gy + \rho_{Hg} g.H$$

$$P_A - P_B = \rho_2 gy + \rho_{Hg}.g.H - \rho_1 g(h + x)$$

<u>A & B are at same level :</u>

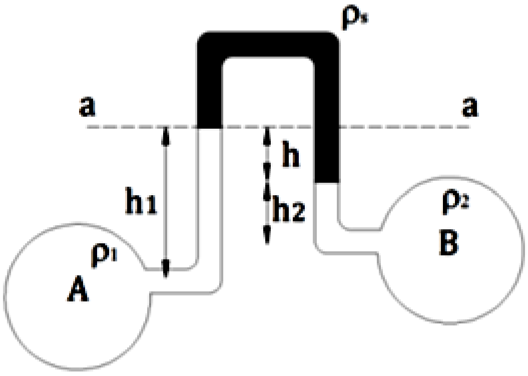


$$P_A - P_B = \rho_2 gx + \rho_{Hg} gh - \rho_1 g(h + x)$$

If liquid is same, then $\rho_1 gx = \rho_2 gx$

$$\therefore P_A - P_B = gh(\rho_{Hg} - \rho_1)$$

2) Inverted U-tube differential Manometer:



$$P_A - \rho_1 gh_1 = P_B - \rho_2 gh_2 - \rho_s gh$$

$$P_A - P_B = \rho_1 gh_1 - \rho_2 gh_2 - \rho_s gh$$

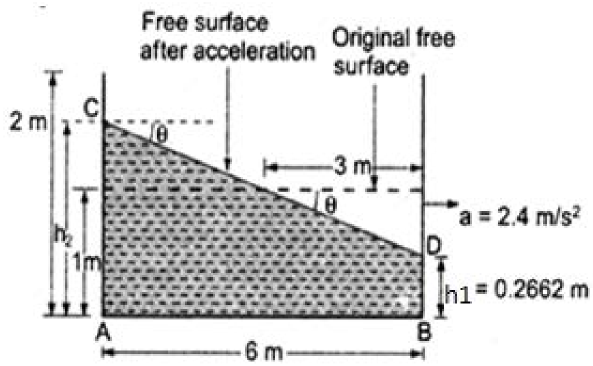
It is used for measuring difference of low pressures

SOLVED EXAMPLES

Example:

A rectangular tank is moving horizontally in the direction of its length with a constant acceleration of $2.4 m/s^2$. The length, width and depth of the tank are 6m, 2.5m and 2m respectively. If the depth of water in the tank is 1m and tank is open at the top then, Calculate:

I. angle of the water surface with the horizontal
II. the maximum and minimum pressure intensities at the bottom



Solution:

Given:

Constant acceleration $a = 2.4 m/s^2$

Length = 6m; Width = 2.5m and depth = 2m,

Depth of water in tank, h = 1m

i) The angle of the water surface to the horizontal

Let θ = the angle of water surface to the horizontal

Using equation, we get

$$\tan\theta = -\frac{a}{g} = -\frac{2.4}{9.81} = -0.2446$$

(the –ve sign shows that the free surface of water is sloping downward as shown in Fig)

$\therefore \quad \tan\theta = -0.2446$ (slope downward)

$\therefore \quad \theta = \tan^{-1} 0.2446 = 13.7446^\circ$ or $13^\circ 44.6'$

ii) The maximum and minimum pressure intensities at the bottom of the tank

From the figure, depth of water at the front end,

$$h_1 = 1 - 3\tan\theta = 1 - 3 \times 0.2446 = 0.2662m$$

Depth of water at the rear end :

$$h_2 = 1 + 3\tan\theta = 1 + 3 \times 0.2446 = 1.7338m$$

The pressure intensity will be maximum at the bottom, where depth of water is maximum.

Now, the maximum pressure intensity at the bottom will be at point A and it is given by,

$$P_{max} = \rho \times g \times h_2$$

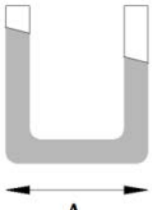
$$= 1000 \times 9.81 \times 1.7338 N/m^2 = 17008.5 N/m^2$$

The minimum pressure intensity at the bottom will be at point B and it is given by

$p_{min} = \rho \times g \times h_1$

$= 1000 \times 9.81 \times 0.2662 = 2611.4\, N/m^2$

Example:
A U-tube as shown in figure, filled with water to mid level is used to measure the acceleration when fixed on moving equipment. Determine the acceleration a_x as a function of the angle θ and the distance A between legs.



Solution:
This is similar to the formation of free surface with angle θ

$\tan\theta = -a_x / (g + a_y)$

As $a_y = 0$, $\tan\theta = -a_x / g$

The acute angle θ will be given by,

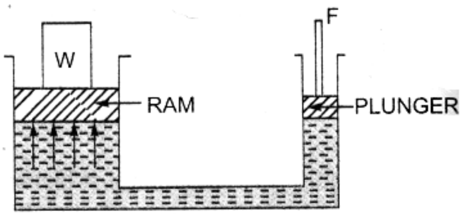
$\theta = \tan^{-1}(a_x / g)$

$a_x = g \times \tan\theta$

As $\tan\theta = 2h / A$

$h = A\, a_x / 2g$

Example:
A hydraulic press has a ram of 30cm diameter and a plunger of 4.5cm diameter. Find the weight lifted by the hydraulic press when the force applied at the plunger is 500N.



Solution:
Given:
Dia. of ram $D = 30cm = 0.3m$
Dia. Of plunger, $d = 4.5cm = 0.045m$
Force on plunger, $F = 500N$
Let the weight lifted $= W$

Area of ram, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.3)^2 = 0.07068 m^2$

Area of plunger,

$a = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.045)^2 = .00159 m^2$

Pressure intensity due to plunger

$= \frac{\text{Force on plunger}}{\text{Area of plunger}} = \frac{F}{a} = \frac{500}{.00159} N/m^2$

Due to Pascal's law, the intensity of pressure will be equally transmitted in all directions. Hence the pressure intensity at the ram

$= \frac{500}{.00159} = 314465.4 N/m^2$

But pressure intensity at ram

$= \frac{\text{Weight}}{\text{Area of ram}} = \frac{W}{A} = \frac{W}{.07068} N/m^2$

$\frac{W}{.07068} = 314465.4$

∴ Weigth

$= 314465.4 \times 0.7068 = 22222N = 22.222kN$

Example:
The diameters of a small piston and a large piston of hydraulic jack are 3 cm and 10cm respectively. A force of 80 N is applied on the small piston. Find the load lifted by the large piston when:

a) The pistons are at the same level.
b) Small piston is 40cm above the large piston.

The density of the liquid in the jack is given as $1000 kg/m^3$

Solution:
Given:
Dia. of small piston, $d = 3cm$
∴ Area of small piston,

$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times 3^2 = 7.068 cm^2$

Dia. of large piston, $D = 10cm$

∴ Area of larger piston,

$A = \frac{\pi}{4} \times 10^2 = 78.54 cm^2$

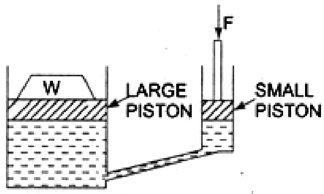
Force on small piston, $F = 80N$

Let the load lifted $= W$

a) When the pistons are at the same level

Pressure intensity on small piston

$\frac{F}{a} = \frac{80}{7.068} N/m^2$



This is transmitted equally to the large piston.

∴ Pressure intensity on the large piston

$= \frac{80}{7.068}$

∴ Force on the large piston

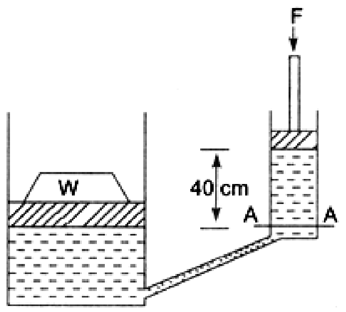
= pressure x area

$= \frac{80}{7.068} \times 78.54N = 888.96N$

b) When the small piston is 40cm above the large piston

Pressure intensity on the small piston

$\frac{F}{a} = \frac{80}{7.068} N/m^2$



∴ Pressure intensity at section A-A

$= \frac{F}{a} +$ Pressure intensity due to height of 40cm of liquid.

Pressure intensity due to 40cm of liquid

$= \rho \times g \times h = 1000 \times 9.81 \times 0.4 N/m^2$

$= \frac{1000 \times 9.81 \times .40}{10^4} N/cm^2 = 0.3924 N/cm^2$

∴ Pressure intensity at section A-A

$= \frac{80}{7.068} + 0.3924$

$11.32 + 0.3924 = 11.71 \ N/cm^2$

∴ Pressure intensity transmitted to the large piston $= 11.71 N/cm^2$

∴ Force on the large piston = Pressure × Area of the large piston

$= 11.71 \times A = 11.71 \times 78.54 = 919.7 N$

Example:

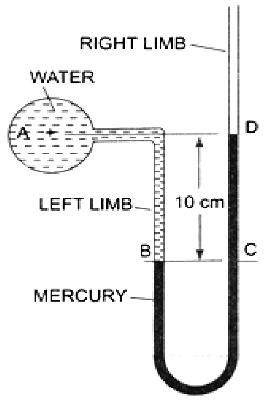
A U-tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The right limb of the manometer contains mercury and is open to water in the main line, if the difference in level of mercury is in the left limb. Determine the pressure of water in the main line, if the difference in level of mercury in the limbs of U-tube is 10 cm and the free surface of mercury is in level with the centre of the pipe. If the pressure of water in pipe line is reduced to $9810 N/m^2$, calculate the new difference in the level of mercury. Sketch the arrangements in both cases.

Solution:

Given:

Difference in mercury level =10 cm =0.1m

The arrangement is shown in fig (a)



1st Part

Let $P_A =$ (pressure of water in pipe line (i.e., at point A)

The points B and C lie on the same horizontal line. Hence pressure at B should be equal to pressure at C. But pressure at B = Pressure at A + Pressure due to 10cm (or 0.1 m) of water

$= p_A + \rho \times g \times h$

Where $\rho = 1000 kg/m^3$ and h = 0.1m

$= p_A + 1000 \times 9.81 \times 0.1$

$= p_A + 981 N/m^2$ ....(i)

Pressure at C = pressure at D+ Pressure due to 10 cm of mercury

$= 0 + \rho_0 \times g \times h_0$

Where ρ_0 for mercury $= 13.6 \times 1000 kg/m^3$

And $h_0 = 10 cm = 0.1 m$

∴ Pressure at C

$= 0 + (13.6 \times 1000) \times 9.81 \times .01$

$= 13341.6 N$ ....(ii)

But pressure at B is equal to pressure at C. Hence equating the equations (i) and (ii), we get,

$p_A + 981 = 13341.6$

∴ $p_A = 13341.6 - 981$

$= 12360.6 \frac{N}{m^2}$

2nd Part

Given, $p_A = 9810 N/m^2$

Find new difference of mercury level. The arrangement is shown in following figure. In this case, pressure at A is $9810 N/m^2$ which is less than the $12360.6 N/m^2$. Hence mercury in left limb will rise. The rise of mercury in left limb will be equal to the fall of mercury in right limb as the total volume of mercury remains same.

Let x =Rise of mercury in left limb in cm.

Then fall of mercury in right limb = x cm

The points B, C and D shows the initial conditions whereas points B*, C* and D* show the final conditions.

pressure at B* = Pressure at C*

Or

Pressure at A + Pressure due to (10-x)cm of water

= Pressure at D* + Pressure due to (10-2x) cm of mercury

Or $p_A + \rho_1 \times g \times h_1 = p_D^* + \rho_2 \times g \times h_2$

Or $1910 + 1000 \times 9.81 \times \left(\frac{10 - x}{100}\right)$

$= 0 + (13.6 \times 1000) \times 9.81 \times \left(\frac{10 - 2x}{100}\right)$

Dividing by 9.81, we get

1000 + 100 - 10x = 1360 - 272x

Or

272x - 10x = 1360 - 1100

Or

262x = 260

∴ $x = \frac{260}{262} = 0.992 cm$

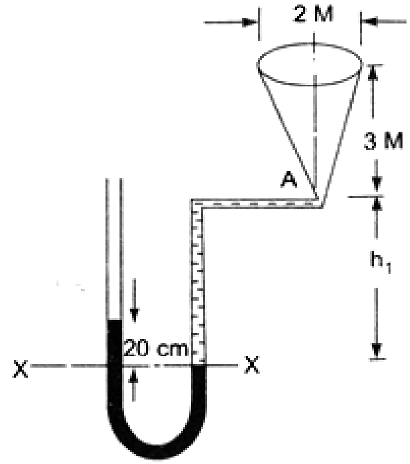
∴ New difference of mercury

$= 10 - 2x cm = 10 - 2 \times 0.992$

$= 8.016 cm$

Example:

Fig. shows a conical vessel having its outlet at A to which a U-tube monometer is connected. The reading of the manometer given in the figure shows when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.



Solution:

Vessel is empty:

Given:

Difference of mercury level

$h_2 = 20 cm$

Let h_1 = Height of water above X-X

S.G. of mercury, $S_2 = 13.6$

S.G. of water, $S_1 = 1.0$

Density of mercury,

$\rho_2 = 13.6 \times 1000$

Density of water,

$\rho_1 = 1000$

Equating the pressure above datum line X-X, we have,

$\rho_2 \times g \times h_2 = \rho_1 \times g \times h_1$

Or $13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$

$h_1 = 2.72\text{m}$ of water.

Vessel is full of water:

When vessel is full of water, the pressure in the right limb will increase and mercury level in the right limb will go down. Let the distance through which mercury goes down in the right limb be, y cm as shown in following figure. The mercury will rise in the left by a distance of y cm. Now the datum line is Z-Z. Equating the pressure above the datum line Z-Z,

Pressure in left limb =Pressure in right limb

$$13.6 \times 1000 \times 9.81 \times \left(0.2 + \frac{2y}{100}\right)$$

$$= 1000 \times 9.81 \times \left(3 + h_1 + y/100\right)$$

$$13.6 \times \left(0.2 + 2y/100\right) = \left(3 + 2.72 + y/100\right)$$

Or

$$2.72 + 27.2y/100 = 3 + 2.72 + y/100$$

Or

$$\left(27.2y\text{-}y\right)/100 = 3.0$$

Or

$$26.2y = 3 \times 100 = 300$$

$$\therefore \quad y = \frac{300}{26.2} = 11.45\text{cm}$$

The difference of mercury level in two limbs

$= \left(20 + 2y\right)$cm of mercury

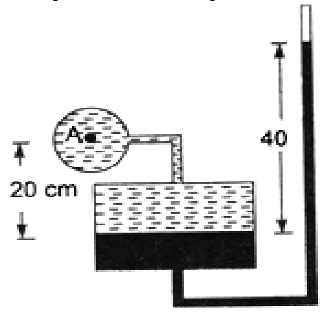
$= 20 + 2 \times 11.45 = 20 + 22.90$

$= 42.90$cm of mecury

$\therefore$ Reading of monometer = 42.90 cm

Example:

A single column manometer is connected to a pipe containing a liquid of S.G. 0.9 as shown in Fig. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube for the manometer reading shown in Fig. The specific gravity of mercury is 13.6



Solution:

Given:

Sp. gr. of liquid in pipe, $S_1 = 0.9$

$\therefore$ Density $\rho_1 = 900\text{kg/m}^3$

Sp. gr. of heavy liquid, $S_2 = 13.6$

$\therefore$ Density, $\rho_2 = 13.6 \times 1000$

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of liquid, $h_1 = 20\text{cm} = 0.2\text{m}$

Rise of mercury in right limb,

$h_2 = 40\text{cm} = 0.4\text{m}$

Let p_A = Pressure in pipe

Using equation,

$$p_A = \frac{a}{A} h_2 \left[\rho_2 g - \rho_1 g\right] + h_2 \rho_2 g - h_1 \rho_1 g,$$

we get,

$$= \frac{1}{100} \times 0.4\left[13.6 \times 1000 \times 9.81 \text{-} 900 \times 9.81\right] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100}\left[133416\text{-}8829\right] + 53366.4\text{-}1765.8$$

$$= 533.664 + 53366.4 - 1765.8\,\text{N/m}^2$$

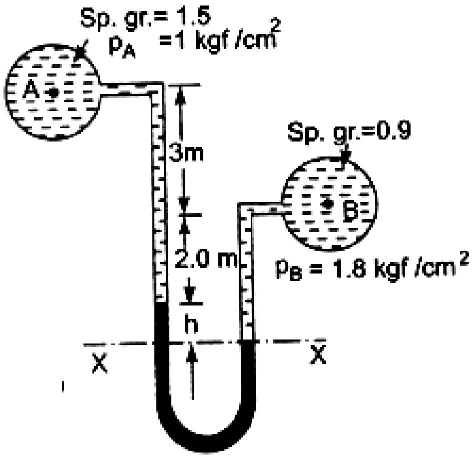
$$= 52134\,\text{N/m}^2$$

$$= 5.21\,\text{N/cm}^2$$

Example:

A differential manometer is connected at the two points A and B of two pipes as shown in fig. The pipe A contains a liquid of

S.G. = 1.5 while pipe B contains a liquid of S.G. = 0.9. The pressures at A and B are $1 kgf/cm^2$ and $1.80 kgf/cm^2$ respectively. Find the difference in mercury level in the differential manometer.



Solutions:
Given:
S.G. of liquid at A, $S_1 = 1.5 \therefore \rho_1 = 1500$
S.G. of liquid at B, $S_2 = 1.5 \therefore \rho_2 = 1500$
Pressure at A,

$$P_A = 1 kgf/cm^2 = 1\times10^4 kgf/m^2$$
$$= 10^4 \times 9.81 N/m^2 (\because 1kgf = 9.81N)$$

Pressure at B,

$$P_B = 1.8 kgf/cm^2$$
$$= 1.8\times10^4 kgf/m^2$$
$$= 1.8\times10^4\times9.81 N/m^2 (\because 1kgf = 9.81N)$$

Density of mercury $= 13.6\times1000 kg/m^3$
Taking X-X as datum line
Pressure above X-X in the left limb

$$= 13.6\times1000\times9.81\times h + 1500\times9.81\times(2+3) + p_A$$
$$= 13.6\times1000\times9.81\times h + 7500\times9.81 + 9.81\times10^4$$

Pressure above X-X in the right limb

$$= 900\times9.81\times(h+2) + p_B$$
$$= 900\times9.81\times(h+2) + 1.8\times10^4\times9.81$$

Equating the two pressures, we get

$$13.6\times1000\times9.81h + 7500\times9.81 + 9.81\times10^4$$
$$= 900\times9.81\times(h+2) + 1.8\times10^4\times9.81$$

Dividing by 1000×9.81, we get

$$13.6h + 7.5 + 10 = (h+2.0)\times.9 + 18$$

Or

$$13.6h + 17.5 = 0.9h + 1.8 + 18 = 0.9h + 19.8$$

Or

$$(13.6 - 0.9)h = 19.8 - 17.5 \text{ or } 12.5h = 2.3$$
$$\therefore h = \frac{2.3}{12.7} = 0.181 m = 18.1 cm$$

3.5 HYDROSTATIC FORCES ON SURFACES

In fluid statics, there is no relative motion between adjacent fluid layers, and thus there are no shear (tangential) stresses in the fluid trying to deform it. The only stress we deal with in fluid statics is the normal stress, which is the pressure, and the variation of pressure is only due to the weight of the fluid. The force exerted on a surface by a fluid at rest is normal to the surface at the point of contact since there is no relative motion between the fluid and the solid surface, and thus no shear forces can act parallel to the surface.
Fluid statics is used to determine the forces acting on floating or submerged bodies and the forces developed by devices like hydraulic presses and car jacks. The design of many engineering systems such as water dams and liquid storage tanks requires the determination of the forces acting on the surfaces using fluid statics.

3.5.1 TOTAL PRESSURE

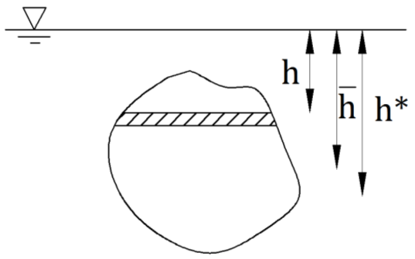
Force is exerted by a static fluid on a surface, either plane or curved when fluid comes in contact with the surfaces. This force always acts normal to the surface.

3.5.2 CENTRE OF PRESSURE

It is defined as the point of application of the total pressure on the surface. The submerged surfaces may be
1) Vertical plane submerged
2) Horizontal plane surface
3) Inclined plane
4) Curved surface

3.5.3 VERTICAL PLANE SURFACE SUBMERGED IN LIQUID

Consider a plane vertical surface of arbitrary shape immersed in a liquid as shown
A = Total area of surface
$\bar{h}$ = Distance of C. G. of the area from free surface of liquid
G = Centre of Gravity of plane surface
P = Centre of Pressure
h^* = Distance of centre of pressure from free surface of liquid.



a) Total Pressure

Pressure Intensity at strip $= \rho gh$
Area of strip $dA = b.dh$
Force on strip $dF = \rho.g.h.b.dh$
Total pressure force on the whole surface is

$$\int_s dF = \int \rho ghbdh$$

$$\int_s dF = \rho g \int h.dA$$

$$F = \rho.g.\bar{h}.A$$

$\int h.dA$ is moment of surface area about free surface of liquid is equal moment of C.G. about free surface.

$$\int h.dA = A.\bar{h}$$

b) Centre of Pressure: (h^*)

Principle of Moments: Moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis.

$F_t.h^* = \sum$moments about free surface of liquid. ...(1)

$$\sum \text{moments} = \int dA.h.\rho gh$$

$$= \rho g \int bh^2 dh$$

$$= \rho gb \int h^2 dh$$

Where,

$\int dA.h^2 = I_o$ is the moment of Inertia of surface about free surface of liquid.

$$\sum \text{moments} = \rho g I_0 \qquad ...(2)$$

$$\therefore \quad F_t.h^* = \rho g I_o$$

$$h^* = \frac{\rho g I_o}{\rho g\bar{h}.A}$$

$$h^* = \frac{I_0}{\bar{h}A}$$

Where,
$\bar{h}$ is the distance of C.G. from free surface
A is the area.
From II axis theorem

$$I_o = I_{C.G.} + A\bar{h}^2$$

$$h^* = \frac{I_{C.G.} + A\bar{h}^2}{\bar{h}.A}$$

1) h^* lies below the C.G. of the surface
2) It is independent of the density of liquid & depends only on surface area.

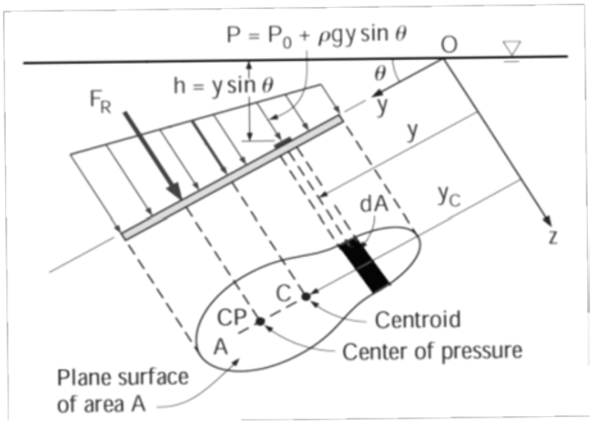
3.5.4 HORIZONTAL PLANE SURFACE SUBMERGED IN LIQUID

As every point of the surface is at the same depth from free surface of the liquid, the pressure intensity will be equal on the entire surface and equal to $P = \rho gh$ where h is depth of surface

$$F_1 = \rho g\, h \times \text{Area}$$

$$\bar{h} = h = h^*$$

3.5.5 INCLINED PLANE SUBMERGED IN LIQUID



Let

A = Total area of surface

$\bar{h}$ = Distance of C. G. of the area from free surface of liquid

G = Centre of Gravity of plane surface

P = Centre of Pressure

h* = Distance of centre of pressure from free surface of liquid.

a) Total Pressure

Pressure intensity on the strip $P = \rho g h$

Pressure force dF on the strip $dF = P \times dA = \rho g h dA$

Total pressure force on the whole area,

$$F = \int dF = \int \rho g h dA$$

From fig. $\sin\theta = \frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*}$

$\therefore \quad h = y\sin\theta$

$\therefore \quad F = \int \rho g \, y \sin\theta dA$

But $\int y \, dA = A\bar{y}$ is the moment of surface at distance 'y'

$\therefore \quad F = \rho g \sin\theta A\bar{y}$

$\therefore \quad F = \rho g A \bar{h} \qquad (\because \bar{h} = \bar{y}\sin\theta)$

Note: the above expression of force is for fluid with no pressure acting on the surface. If pressure acts on the surface

$$F = P_0 A + \rho g A\bar{h}$$

b) Centre of Pressure

Pressure force on the strip, $dF = \rho g h \, dA = \rho g y \sin\theta dA$

Moment of the force, dF, about axis O-O $= dF \times y$

$= \rho g y \sin\theta dA.y$

Sum of moments of all such forces about $O - O = \int \rho g \sin\theta y^2 dA$

$$= \rho g \sin\theta \int y^2 dA$$

Where,

$\int y^2 dA =$ Moment of Inertia of the surface about $O - O = I_0$

$\therefore$ Sum of moments of all force $= \rho g \sin\theta I_o$

$$F \times y^* = \rho g \sin\theta I_o$$

$$y^* = \frac{\rho g \sin\theta I_o}{\rho g A\bar{h}}$$

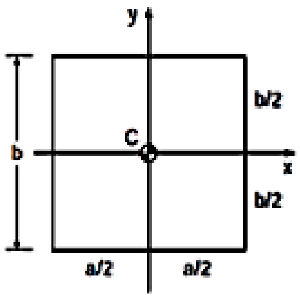
$$y^* = \frac{I_o \sin\theta}{A\bar{h}}$$

$$h^* = \frac{I_0 \sin^2\theta}{A\bar{h}}$$

$$h^* = \frac{\sin^2\theta}{A\bar{h}}(I_G + A\overline{y^2})$$

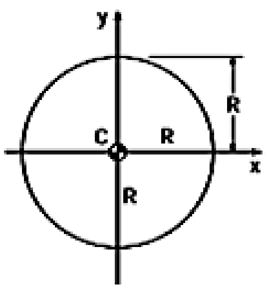
$$h^* = \frac{\sin^2\theta}{A\bar{h}}\left(I_G + \frac{A\overline{h^2}}{\sin^2\theta}\right)$$

a) Rectangle



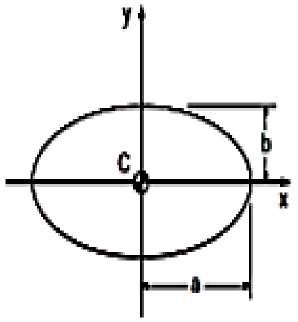
$$A = ab, \; I_{xx,C} = \frac{ab^3}{12}$$

b) Circle



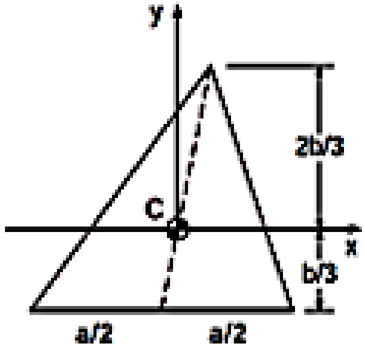
$$A = \pi R^2, I_{xx,C} = \frac{\pi R^4}{4}$$

c) Triangle



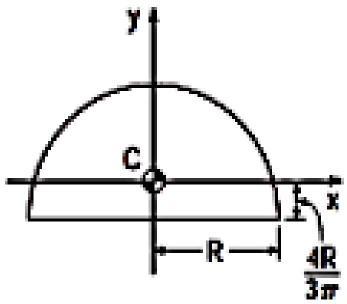
$$A = \pi ab, I_{xx,C} = \frac{\pi ab^3}{4}$$

d) Triangle



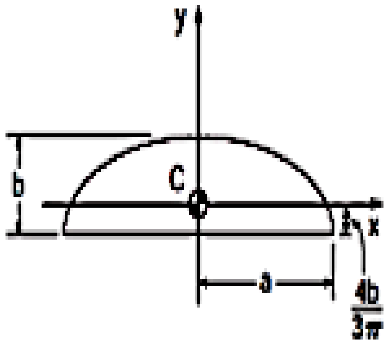
$$A = \frac{ab}{2}, I_{xx,C} = \frac{ab^3}{36}$$

e) Semicircle



$$A = \frac{\pi R^2}{2}, I_{xx,C} = 0.109757R^4$$

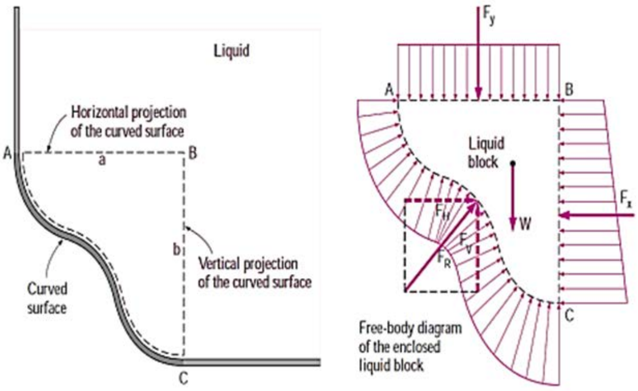
f) Semi ellipse



$$A = \frac{\pi ab^2}{2}, I_{xx,C} = 0.109757ab^3$$

3.5.6 CURVED SURFACE SUB-MERGED IN LIQUID

For a submerged curved surface, the determination of the resultant hydrostatic force is more involved since it typically requires the integration of the pressure forces that change direction along the curved surface. The way to determine the resultant hydrostatic force F_R acting on a two-dimensional curved surface is to determine the horizontal and vertical components F_x and F_y separately. This is done by considering the free-body diagram of the liquid block enclosed by the curved surface and the two plane surfaces (one horizontal and one vertical) passing through the two ends of the curved surface. Note that the vertical surface of the liquid block considered is simply the projection of the curved surface on a vertical plane, and the horizontal surface is the projection of the curved surface on a horizontal plane.



The resultant force acting on the curved surface is given by

$$F_R = \sqrt{F_x^2 + F_y^2}$$

Inclination of resultant with horizontal is given by

$$\tan\theta = \frac{F_y}{F_x}$$

1) The horizontal component of the hydrostatic force acting on a curved surface is equal (in both magnitude and the line of action) to the hydrostatic

force acting on the vertical projection of the curved surface.

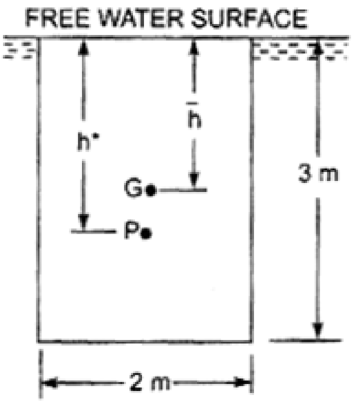
2) The vertical component of the hydrostatic force acting on a curved surface is equal to the weight of liquid supported by the curved surface.

Example:
A rectangular plane surface is 2m wide and 3m deep. It lies in vertical plane in water. Determine the total pressure and position of centre of pressure on the plane surface when it's upper edge is horizontal and (a) coincides with water surface, (b) 2.5m below the free water surface.

Solution:
Given:
Width of plane surface, b=2m
Depth of plane surface, d=3m

a) Upper edge coincides with water surface

Total pressure is given by equation as $F = \rho g A\bar{h}$



Where,
$\rho = 1000 kg/m^3,\ g = 9.81 m/s^2$

$A = 3\times2 = 6m^2,\ \bar{h} = \frac{1}{2}\times3 = 1.5m$

$\therefore F = 1000\times9.81\times6\times1.5$

$= 88290N$

Depth of centre of pressure is given by equation as

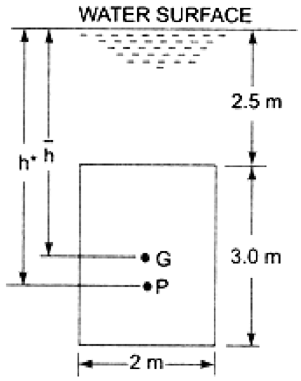
$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

Where,
$I_G =$ M.O.I. about C.G. of the area of surface

$$= \frac{bd^3}{12} = \frac{2\times3^3}{12} = 4.5m^4$$

$$\therefore h^* = \frac{4.5}{6\times1.5} + 1.5 = 0.5 + 1.5 = 2.0m$$

b) Upper edge is 2.5, below water surface



Total pressure (F) is given by
$F = \rho g A\bar{h}$
Where,
$\bar{h}$ =Distance of C.G. from free surface of water

$= 2.5 + \frac{3}{2} = 4.0m$

$\therefore\ F = 1000\times9.81\times6\times4.0$
$= 235440N$

Center of pressure is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

Where
$I_G = 4.5, A = 6.0, \bar{h} = 4.0$

$$h^* = \frac{4.5}{6.0\times4.0} + 4.0$$

$= 0.1875 + 4.0 = 4.1875 = 4.1875m.$

Example:
A circular opening, 3m diameter, in a vertical side of a tank is closed by disc of 3m diameter which can rotate a horizontal diameter.
Calculate:

i) The force on the disc, and
ii) The torque required to maintain the disc in equilibrium in the vertical position when the head of water above the horizontal diameter is 4m.

Solution:

Given:

Dia. of opening $d = 3m$

Area, $A = \frac{\pi}{4} \times 3^2 = 7.0685 m^2$

Depth of C.G. $\bar{h} = 4m$

i) Force on the disc is given by equation as

$F = \rho g A\bar{h} = 1000 \times 9.81 \times 7.0685 \times 4.0$

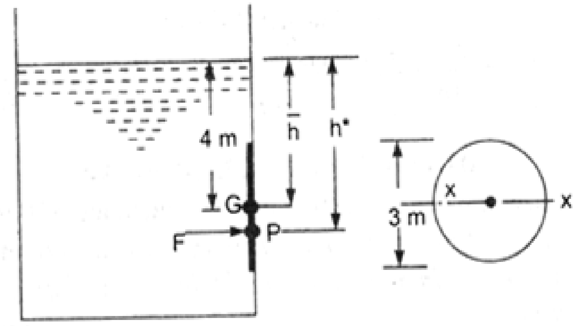
$= 277368 N = 277.368 kN$

ii) To find the torque required to maintain the disc in equilibrium, first calculate the point application of force acting on the disc, i.e., center of pressure of the force F. The depth of centre of pressure (h*) is given by equation as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times 4.0} + 4.0$$

$$\left\{ \because I_G = \frac{\pi}{64} d^4 \right\}$$

$$= \frac{d^2}{16 \times 4.0} + 4.0 = \frac{3^2}{16 \times 4.0} + 4.0 = 0.14 + 4.0 = 4.14m$$



The force F is acting at a distance of 4.14 m from free surface. Moment of this force about horizontal diameter X-X

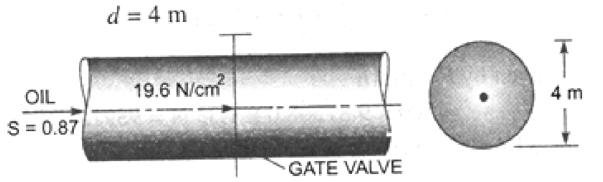
$= F(h^* - \bar{h}) = 277368(4.14 - 4.0) = 38831 Nm$

Hence a torque of 38831 Nm must be applied on the disc in the clockwise direction.

Example:

A pipe line which is 4m in diameter contains a gate valve. The pressure at the centre of the pipe is $19.6 N/cm^2$. If the pipe is filled with oil of S.G. 0.87; find the force exerted by the oil upon the gate and position of centre of pressure.

Solution:



Given:

Dia. of pipe, $d = 4m$

∴ Area,

$A = \frac{\pi}{4} \times 4^2 = 4pm^2$

∴Density of oil $\rho_0 = 0.87 \times 1000 = 870 kg/m^3$

∴ Weight density of oil,

$w_0 = \rho_0 \times g = 870 \times 9.81 N/m^3$

Pressure at the centre of pipe,

$p = 19.6 N/cm^2 = 19.6 \times 10^4 N/m^3$

∴ Pressure head at the centre

$$= \frac{p}{w_0} = \frac{19.6 \times 10^4}{870 \times 9.81} = 22.988 m$$

∴ The height of equivalent free oil surface from the centre of pipe =22.988m

The depth of C.G. of the gate valve from free oil surface $\bar{h} = 22.988 m$

$F = \rho g A\bar{h}$

Where $\rho =$ density of oil $= 870 kg/m^3$

$F = 870 \times 9.81 \times 4\pi \times 22.988 = 2465500 N = 2.465 MN$

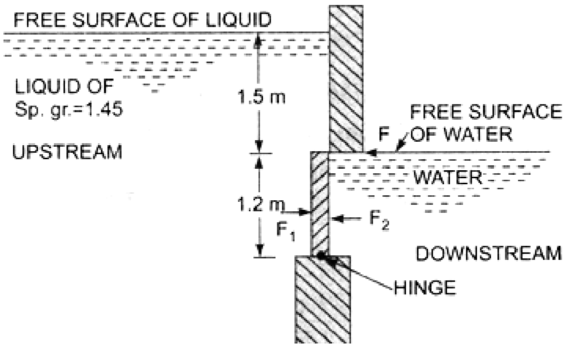
(ii)Position of centre of pressure (h*) is given as

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} = \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2 \times \bar{h}} + \bar{h} = \frac{d^2}{16h} + \bar{h} = \frac{4^2}{16 \times 22.988} + 22.988$$

$= 0.043 + 22.988 = 23.031 m$

Or, centre of pressure is below the centre of the pipe by a distance of 0.043m

Example:
A vertical sluice gate is used to cover an opening in a dam. The opening is 2m wide and 1.2 m high. On the upstream of the gate, the liquid of S.G. 1.45 lies upto a height of 1.5m above the top of the gate. Find the resultant force acting on the gate and position of centre of pressure. Find also the force acting horizontally at the top of the gate which is capable of opening it. Assume that the gate is hinged at the bottom.



Solution:
Given:
Width of gate, $b = 2m$
Depth of gate, $d = 1.2m$
∴ Area,
$A = b \times d = 2 \times 1.2 = 2.4m^2$
Sp. gr. of liquid $= 1.45$
∴ Density of liquid,
$\rho_1 = 1.45 \times 1000 = 1450 kg/m^3$
Let $F_1 =$ Force exerted by the fluid of sp. gr 1.45 on gate
$F_2 =$ Force exerted by water on the gate.
The force
$F_1 =$ is given by $F_1 = \rho_1 g \times A \times \bar{h}_1$
Where
$\rho_1 = 1.45 \times 1000 = 1450 kg/m^2$
$\bar{h}_1 =$ Depth of C.G. of gate from free surface of liquid

$$= 1.5 + \frac{1.2}{2} = 2.1m$$

$\therefore F_1 = 1450 \times 9.81 \times 2.4 \times 2.1 = 71691N$
Similarly, $F_2 = \rho_2 g.A\bar{h}_2$
Where $\rho_2 = 1000 kg/m^3$
$\bar{h}_2 =$ Depth of C.G. of gate from free surface of water

$$= \frac{1}{2} \times 1.2 = 0.6m$$

$\therefore F_2 = 1000 \times 9.81 \times 2.4 \times 0.6 = 14126N$

(i)Resultant force on the gate
$= F_1 - F_2 = 71691 - 14126 = 57565N$

(ii) Position of centre of pressure of resultant force.
The force F_1 will be acting at a depth of h_1^* from free surface of liquid, given by the relation

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}_1$$

where

$$I_G = \frac{bd^3}{12} = \frac{2 \times 1.2^3}{12} = 0.288 m^4$$

∴

$$h_1^* = \frac{.288}{2.4 \times 2.1} + 2.1 = 0.0571 + 2.1 = 2.1571m$$

∴ Distance of F_1 from hinge
$= (1.5 + 1.2) - h_1^* = 2.7 - 2.1571 = 0.5429m$
The force F_2 will be acting at a depth of h_2^* from free surface of water and is given by

$$h_2^* = \frac{I_G}{A\bar{h}_2} + \bar{h}_2$$

Where
$I_G = 0.288 m^4, \bar{h}_2 = 0.6m, A = 2.4m^2$

$$h_2^* = \frac{.288}{2.4 \times 0.6} + 0.6 = 0.2 + 0.6 = 0.8m$$

Distance of F_2 from hinge
$= 1.2 - 0.8 = 0.4m$
The resultant force 57565N will be acting at a distance given by

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{57565}$$

$$= \frac{38921 - 5650.4}{57565} \text{ m above hinge}$$

$= 0.578 m$ above the hinge

(iii)Force at the top of gate which is capable of opening the gate.

Let F is the force required on the top of the gate to open it as shown in fig.

Taking the moments of F, F_1 and F_2 about the hinge, we get

$F \times 1.2 + F_2 \times 0.4 = F_1 \times .5429$

Or

$$F = \frac{F_1 \times .5429 - F_2 \times 0.4}{1.2}$$

$$= \frac{71691 \times .5429 - 14126 \times 0.4}{1.2} = \frac{38921 - 5650.4}{1.2}$$

$= 27725.5N.$

Example:

A tank contains water up to a height of 0.5m above the base. An immiscible liquid of sp. gr. 0.8 is filled on the top of water up to 1m height. Calculate:

i) total pressure on one side of the tank,

ii) the position of centre of pressure from one side of the tank, which is 2m wide

Solution:

Given:

Depth of water $= 0.5m$

Depth of liquid $= 1m$

Sp. gr of liquid $= 0.8$

Density of liquid,

$\rho_1 = 0.8 \times 1000 = 800 kg/m^3$

Density of water,

$\rho_2 = 1000 kg/m^3$

Width of tank $= 2m$

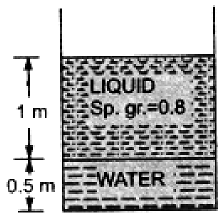
(i) Total pressure on one side is calculated by drawing pressure diagram, which is shown in fig .

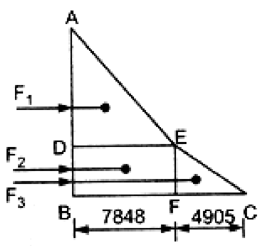
Intensity of pressure on top, $p_A = 0$

Intensity of pressure on D (or DE),

$p_D = \rho_1 g.h_1$

$= 800 \times 9.81 \times 1.0 = 7848 N/m^2$





Intensity of pressure on base (or BC),

$p_B = \rho_1 g h_1 + \rho_2 g \times 0.5$

$$= 7848 + 1000 \times 9.81 \times 0.5 = 7848 + 4905 = \frac{12753N}{m^2}$$

Now Force,

$F_1 =$ Area of $\Delta ADE \times$ Width of tank

$$= \frac{1}{2} \times AD \times DE \times 2.0 = \frac{1}{2} \times 1 \times 7848 \times 2.0 = 7848N$$

Force,

$F_2 =$ Areaof rectangle DBFE $\times$ Width of tank

$= 0.5 \times 7848 \times 2 = 7848N$

$F_3 =$ Area of $\Delta EFC \times$ Width of tank

$$= \frac{1}{2} \times EF \times FC \times 2.0 = \frac{1}{2} \times 0.5 \times 4905 \times 2.0 = 2452.5N$$

$\therefore$ Total force $F = F_1 + F_2 + F_3$

$= 7848 + 7848 + 2452.5 = 18148.5N$

(ii) Centre of pressure (h*). Taking the moments of all forces about A, we get

$$F \times h^* = F_1 \times \frac{2}{3} AD + F_2 \left(AD + \frac{1}{2} BD \right) + F_3 \left[AD + \frac{2}{3} BD \right]$$

$$18148.5 \times h^* = 7848 \times \frac{2}{3} \times 1 + 7848 \left(1.0 + \frac{0.5}{2} \right) + 2452.5 \left(1.0 + \frac{2}{3} \times .5 \right)$$

$= 5232 + 9810 + 3270 = 18312$

$$\therefore\ h^* = \frac{18312}{18148.5} = 1.009 m \text{ from top}$$

Example:

A circular plate 3.0m diameter is immersed in water in such a way that their greatest and least depths below the free surface are 4m and 1.5m respectively. Determine the total pressure on one face of the plate and position of the centre of pressure.

Solution:

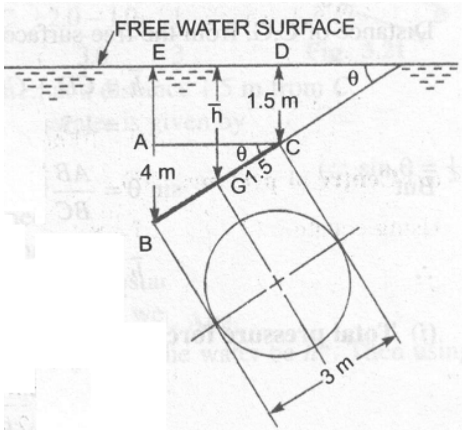
Given

Dia. of plate, $d = 3.0m$

$\therefore$ Area,

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (3.0)^2 = 7.0685 m^2$$

Distance $DC = 1.5m, BE = 4m$



Distance of C.G. from free surface

$= \bar{h} = CD + GC\sin\theta = 1.5 + 1.5\sin\theta$

But

$$\sin\theta = \frac{AB}{BC} = \frac{BE - AE}{BC} = \frac{4.0 - DC}{3.0} = \frac{4.0 - 1.5}{3.0}$$

$$= \frac{2.5}{3.0} = 0.8333$$

$\therefore\ \bar{h} = 1.5 + 1.5 \times .8333 = 1.5 + 1.249 = 2.749\,m$

i) Total Pressure (F)

$F = \rho g A\bar{h}$

$= 1000 \times 9.81 \times 7.0685 \times 2.749 = 190621\,N$

ii) Centre of pressure (h*)

Using equation, we have

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

Where

$$I_G = \frac{\pi}{64}(d)^4 = \frac{\pi}{64}(3)^4 = 3.976\,m^4$$

$$h^* = \frac{3.976 \times (.8333) \times .8333}{7.0685 \times 2.749} + 2.749 = 0.1420 + 2.749$$

$= 2.891\,m.$

Example:

If in the above problem, the given circular plate is having a concentric circular hole of diameter 1.5m, then calculate the total pressure and position of the centre of pressure on one face of the plate.

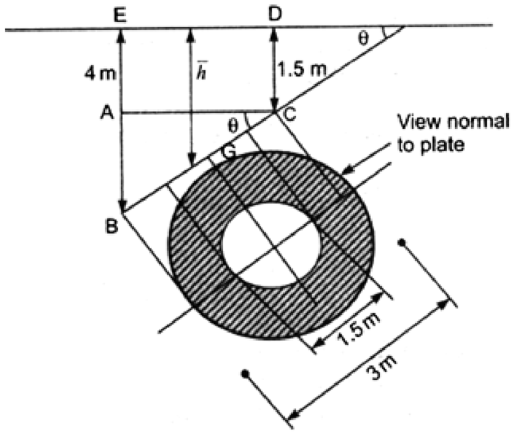
Solution:

Given: [referring to given figure]

Dia. of plate, $d = 3.0\,m$

$\therefore$ Area of solid plate

$$= \frac{\pi}{4}d^2 = \frac{\pi}{4}(3)^2 = 7.0685\,m^2$$



Dia. of hole in the plate, $d_0 = 1.5\,m$

$\therefore$ Area of hole $= \frac{\pi}{4}d_0^2 = \frac{\pi}{4}(1.5)^2 = 1.7671\,m^2$

$\therefore$ Area of the given plate A

= Area of solid plate – Area of hole

$= 7.0685 - 1.7671 = 5.3014\,m^2$

Distance of $CD = 1.5, BE = 4\,m$

Distance of C.G. from the free surface,

$\bar{h} = CD + GC\sin\theta$

$= 1.5 + 1.5\sin\theta$

But

$$\sin\theta = \frac{AB}{BC} = \frac{BE - AE}{BC} = \frac{4 - 1.5}{3} = \frac{2.5}{3}$$

$$\therefore\ \bar{h} = 1.5 + 1.5 \times \frac{2.5}{3} = 1.5 + 1.25 = 2.75\,m$$

i) Total pressure force (F)

$F = \rho g A\bar{h}$

$= 1000 \times 9.81 \times 5.3014 \times 2.75$

$= 143018\,N = 143.018\,kN$

ii) Position of centre of pressure (h*)

Using equation, we have

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

where

$$I_G = \frac{\pi}{64}\left[d^4 - d_0^4\right] = \frac{\pi}{64}\left[3^4 - 1.5^4\right]m^4$$

$$A = \frac{\pi}{4}\left[d^2 - d_0^2\right] = \frac{\pi}{4}\left[3^2 - 1.5^2\right]m^2$$

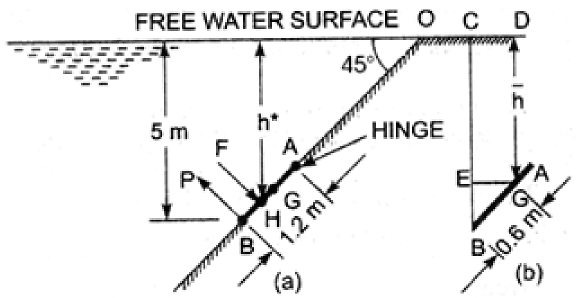
$$\sin\theta = \frac{2.5}{3} \text{ and } \bar{h} = 2.75$$

$$\therefore \quad h^* = \frac{\frac{\pi}{64}\left[3^4 - 1.5^4\right]\times\left(\frac{2.5}{3}\right)^2}{\frac{\pi}{4}\left[3^2 - 1.5^2\right]\times 2.75} + 2.75$$

$= 0.177 + 2.75 = 2.927\,\text{m}$

Example:

An inclined rectangular sluice gate AB, 1.2m x 5m size as shown in fig is installed to control the discharge of water. The end A is hinged. Determine the force normal to the gate applied at B to open it.



Solution:

Given:

$A = \text{Area of gate} = 1.2 \times 5.0 = 6.0\,\text{m}^2$

Depth of C.G. of the gate from free surface of the water $= \bar{h}$

$= DG = BC - BE$

$= 5.0 - BG \sin 45°$

$$5.0 - 0.6 \times \frac{1}{\sqrt{2}} = 4.576\text{m}$$

The total pressure force (F) acting on the gate,

$F = \rho g A \bar{h}$

$= 1000 \times 9.81 \times 6.0 \times 4.576$

$= 269343\text{N}$

This force is acting at H, where the depth of h from free surface is given by

$$h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

Where, $I_G = \text{M.O.I. of gate}$

$$= \frac{bd^3}{12} = \frac{5.0 \times 1.2^3}{12} = 0.72\,\text{m}$$

$\therefore$ Depth of centre of pressure

$$h^* = \frac{0.72 \times \sin^2 45°}{6 \times 4.576} + 4.576 = .013 + 4.576 = 4.589\,\text{m}$$

But from fig, $\dfrac{h^*}{OH} = \sin 45°$

$\therefore$

$$OH = \frac{h^*}{\sin 45°} = \frac{4.589}{\frac{1}{\sqrt{2}}} = 4.589 \times \sqrt{2} = 6.489\,\text{m}$$

Distance, $BO = \dfrac{5}{\sin 45°} = 5 \times \sqrt{2} = 7.071\,\text{m}$

Distance, $BH = BO - OH = 0.071 - 6.489 = 0.582\text{m}$

$\therefore$ Distance,

$AH = AB - BH = 1.2 - 0.582 = 0.618\,\text{m}$

Taking the moments about the hinge A

$P \times AB = F \times (AH)$

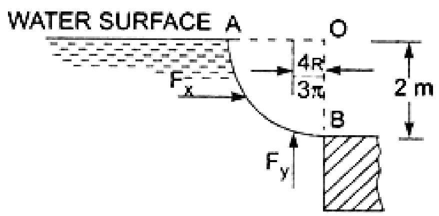
Where P is the force normal to the gate applied at B

$\therefore\ P \times 1.2 = 269343 \times 0.618$

$$\therefore \quad P = \frac{269343 \times 0.618}{1.2} = 138708\text{N}$$

Example:

Fig shows a quadrant shaped gate of radius 2m. Find the resultant force due to water per meter length of the gate. Find also the angle at which the total force will act.



Solution:

Given:

Radius of gate =2m

Width of gate =1m

Horizontal Force,

F_x = Force on the projected area of the curved surface on vertical plane

= Force on $BO = \rho g A \bar{h}$

Where,

$$A = \text{Area of BO} = 2 \times 1 = 2\,\text{m}^2, \bar{h} = \frac{1}{2} \times 2 = 1\,\text{m}$$

$F_x = 1000 \times 9.81 \times 2 \times 1 = 19620\text{N}$

This will act at depth of $\frac{2}{3}\times 2=\frac{4}{3}$m from free surface of liquid,

Vertical Force,

F_y = Weight of water (imagined) supported by AB

$=\rho g\times$ Areaof AOB$\times 1.0$

$=1000\times 9.81\times\frac{\pi}{4}(2)^4\times 1.0=30819\text{N}$

This will act a distance of

$\frac{4R}{3\pi}=\frac{4\times 0.2}{3\pi}=0.848\,\text{m from OB.}$

$\therefore$ Resultant force, F is given by

$F=\sqrt{F_x^2+F_y^2}$

$=\sqrt{19620^2+30819^2}$

$=36534.4\text{N}.$

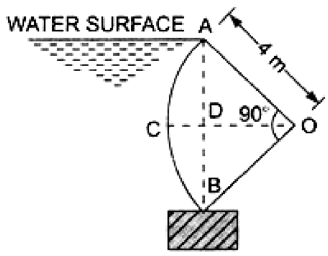
The angle made by the resultant with horizontal is given by

$\tan\theta=\frac{F_y}{F_x}=\frac{30819}{19620}=1.5708$

$\therefore\ \theta=\tan^{-1}1.5708=57°31'$

Example:

Find the horizontal and vertical component of water pressure acting on the face of a sector gate of 90° with radius 4m as shown in fig. Take width of gate as unity.



Solution:

Given:

Radius of gate, R=4m

Horizontal component of force acting on the gate is

F_x = Force on area of gate projected on vertical plane

= Force on area ADB

$=\rho g A\bar{h}$

Where $A=AB\times$ Width of gate

$=2\times AD\times 1\qquad (\because AB=2AD)$

$=2\times 4\times\sin 45°=8\times .707=5.656\text{m}^2$

$\{\because AD=4\sin 45°\}$

$\bar{h}=\frac{AB}{2}=\frac{5.656}{2}=2.828\text{m}$

$\therefore\ F_x=1000\times 9.81\times 5.656\times 2.828\text{N}=156911\text{N}$

Vertical component

F_y = Weight of water supported or enclosed by the curved surface

= Weight of water in portion ACBDA

$=\rho g\times$ Area of ACBDA$\times$ Width of gate

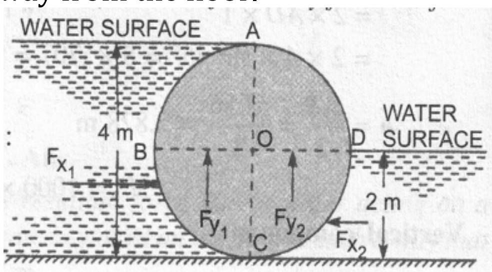
$=1000\times 9.81\times[\text{Area of sector ACBOA}-\text{Area of }\Delta ABO]\times 1$

$=9810\times\left[\frac{\pi}{4}R^2-\frac{AO\times BO}{2}\right]\qquad[\because\ \Delta AOB\text{ is a right angled}]$

$=9810\times\left[\frac{\pi}{4}4^2-\frac{4\times 4}{2}\right]=44796\text{N}$

Example:

A cylindrical gate of 4m diameter & 2m long has water on its both sides as shown in Fig. Determine the magnitude, location and direction of the resultant force exerted by the water on the gate. Find also the least weight of the cylinder so that it may not be lifted away from the floor.



Solution:

Given:

Dia. of gate =4m

Radius =2m

(i) The force acting on the left sides of the cylinder are

The horizontal component, F_{x_1}

Where F_{x_1} = Force of water on area projected on vertical plane

= Force on area AOC

$=\rho g A\bar{h}$

Where A = AC × width = 4×2= 8cm²

$$\bar{h} = \frac{1}{2} \times 4 = 2m$$

$F_{x_1} = 1000 \times 9.81 \times 8 \times 2$

$= 156960N$

F_{y_1} = Weight of water enclosed by ABCOA

$$= 1000 \times 9.81 \times \left[\frac{\pi}{2} R^2\right] \times 2.0 = 9810 \times \frac{\pi}{2} \times 2^2 \times 2.0 = 123276N$$

Right Side of the Cylinder

$F_{x_2} = \rho g A_2 \bar{h}_2 =$ Force on vertical area CO

$$= 1000 \times 9.81 \times (2 \times 2) \times \frac{2}{2} = 39240N$$

F_{y_2} = Weight of water enclosed by DOCD

$$= \rho g \times \left[\frac{\pi}{4} R^2\right] \times \text{Width of gate}$$

$$= 1000 \times 9.81 \times \frac{\pi}{4} \times 2^2 \times 2 = 61638N$$

∴ Resultant force in the direction of x,

$F_x = F_{x_1} - F_{x_2} = 156960 - 39240 = 117720N$

Resultant force in the direction of y,

$F_y = F_{y_1} + F_{y_2} = 123276 + 61638 = 184914N$

i) Resultant force, F is given as

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(117720)^2 + (184914)^2} = 219206N$$

ii) Direction of resultant force is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{184914}{117720} = 1.5707$$

$\therefore \quad \theta = 57°31'$

iii) Location of the resultant force

Force, F_{x_1} acts at a distance of $\frac{2 \times 4}{3} = 2.67m$ from the top surface of water on left side, while F_{x_2} acts at a distance of $\frac{2}{3} \times 2 = 1.33m$ from free surface on the right side of the cylinder .The resultant force F_x in the direction of x will act at a distance of y from the bottom as

$$F_x \times y = F_{x_1}[4 - 2.67] - F_{x_2}[2 - 1.33]$$

Or

$117720 \times y = 156960 \times 1.33 - 39240 \times .67$

$= 208756.8 - 26290.8 = 182466$

$\therefore \quad y = \frac{182466}{117720} = 1.55m$ from the bottom

Force F_{y_1} acts at a distance $\frac{4R}{3\pi}$ from AOC or at a distance $\frac{4 \times 2.0}{3\pi} = 0.8488m$ from AOC towards left of AOC.

Also F_{y_2} acts at a distance $\frac{4R}{3\pi} = 0.8488m$ from AOC towards the right of AOC. The resultant force F_y will act at a distance x from AOC which is given by

$$F_y \times x = F_{y_1} \times .8488 - F_{y_2} \times .8488$$

Or $184914 \times x = 123276 \times .8488 - 61638 \times .8488$

$= .8488[123276 - 61638] = 52318.4$

$\therefore x = \frac{52318.4}{184914} = 0.2829m$ from AOC

iv) Least weight of cylinder. The resultant force in the upward direction is

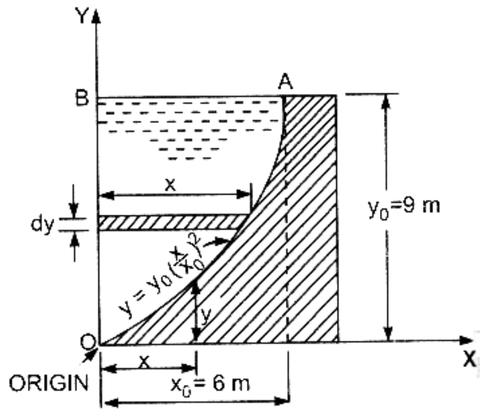
$F_y = 184914N$

Thus the weight of cylinder should not be less than the upward force F_y.

Hence, weight of cylinder should be at least 184914N

Example:

A dam has a parabolic shape $y = y_0\left(\frac{x}{x_0}\right)^2$ as shown in fig. below having $x_0 = 6m$ and $y_0 = 9m$. The fluid is water with density $= 1000 kg/m^3$. Compute the horizontal, vertical and the resultant thrust exerted by water per meter length of the dam.



Solution:

Given:

Equation of the curve OA is

$$y = y_0\left(\frac{x}{x_0}\right)^2 = 9\left(\frac{x}{6}\right)^2 = 9\times\frac{x^2}{36} = \frac{x^2}{4}$$

Or $\quad x^2 = 4y$

$\therefore \quad x = \sqrt{4y} = 2y^{1/2}$

Width of dam, $b = 1\,m$

i) Horizontal thrust exerted by water

F_x = Force exerted by water on vertical surface OB, i.e., the surface obtained by projecting the curved surface on vertical plane

$= \rho g A\bar{h}$

$$= 1000\times 9.81\times(9\times 1)\times\frac{9}{2} = 397305N$$

ii) Vertical thrust exerted by water

F_y = Weight of water supported by curved surface OA upto free surface of water

= Weight of water in the portion ABO

$= \rho g \times$ Area of OAB $\times$ Width of dam

$$= 1000\times 9.81\times\left[\int_0^9 x\times dy\right]\times 1.0$$

$$= 1000\times 9.81\times\left[\int_0^9 2y^{1/2}\times dy\right]\times 1.0$$

$(\because x = 2y^{1/2})$

$$= 19620\times\left[\frac{y^{3/2}}{(3/2)}\right]_0^9 = 19620\times\frac{2}{3}\left[9^{3/2}\right]$$

$$= 19620\times\frac{2}{3}\times 27 = 353160N$$

iii) Resultant thrust exerted by water

$$F = \sqrt{F_X{}^2 + F_Y{}^2} = \sqrt{397305^2 + 353160^2} = 531574N$$

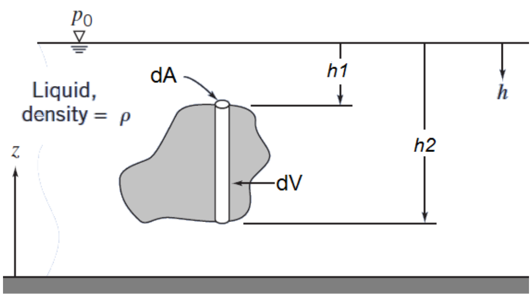
iv) Direction of resultant is given by

$$\tan\theta = \frac{F_y}{F_x} = \frac{353160}{397305} = 0.888$$

$$\theta = \tan^{-1} 0.888 = 41.63^o$$

3.6 BUOYANCY & FLOATATION

When a body is immersed in a fluid, an upward force is exerted on the body; this upward force is known as the buoyant force. This force is because of difference in pressure.



Force acting on the element because of difference in pressure on the top and bottom.

$dF = \rho g(h_2 - h_1)dA$

$F_B = \int \rho g(h_2 - h_1)dA$

$F_B = \rho g V$

Where,

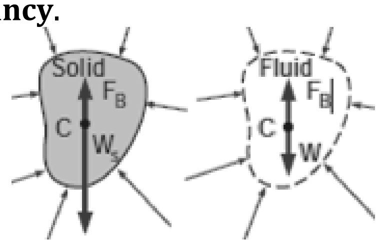
ρ is the density of fluid

V is the volume of body immersed in fluid **or** volume of fluid displaced by that body.

The relation $\rho g V$ is simply the weight of the liquid whose volume is equal to the immersed volume of the body. Thus the buoyant force acting on the body is equal to the weight of the liquid displaced by the body. Note that the buoyant force is independent of the distance of the body from the free surface. It is also independent of the density of the solid body

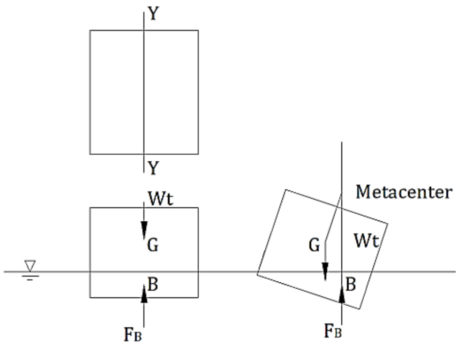
3.6.1 CENTRE OF BUOYANCY

The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume. The centroid of displaced fluid is known as **centre of buoyancy**.



3.6.2 META CENTRE

It is defined as the point about which a body starts oscillating when the body is tilled by a small angle. The Meta centre may also be defined as the intersection point of line of action of buoyant force and normal to the body when the body is tilted by an angle.



Meta centric height = GM

$$= \frac{I(\text{Moment of Inertia about yy of the plan})}{\text{Volume of fluid displaced}} - BG$$

B.G is the distance between CG & CB points.

3.6.3 OSCILLATION OF A FLOATING BODY

When body floats in the fluid and it is given a disturbance in clockwise direction or anti clock wise direction. The body oscillates about its metacenter. The time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{k^2}{GM.g}}$$

Where,

GM is Meta centric ht

K is radius of gyration

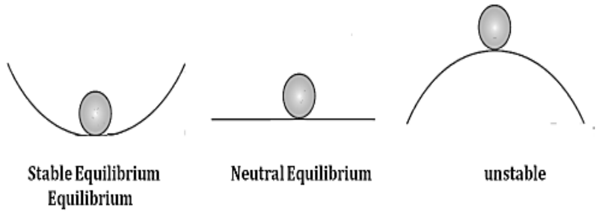
3.6.4 CONDITIONS OF EQUILIBRIUM OF SUBMERGED & FLOATING BODIES

There are 3 types of equilibrium conditions

i) Stable Equilibrium

ii) Neutral equilibrium

iii) Unstable equilibrium



i) Stable equilibrium:

Any small disturbance (someone moves the ball to the right or left) generates a restoring force (due to gravity) that returns it to its initial position.

ii) Neutral equilibrium:

If someone moves the ball to the right or left, it will stay at its new location. It has no tendency to move back to its original location, nor does it continue to move.

iii) Unstable Equilibrium:

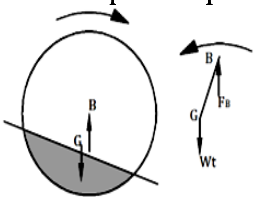
It is a situation, in which the ball may be at rest at the moment, but any disturbance, even an infinitesimal one, causes the ball to roll off the hill—it does not return to its original position; rather it moves away from it.

3.6.4.1 STABILITY IN SUBMERGED BODIES

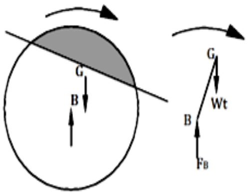
1) **Stable Equilibrium:** When $W = F_B$ and point B is above G. A small displacement in clockwise direction, gives couple due

to F_B & weight in anticlockwise direction. Thus, the body will return to its original position. Hence, equilibrium is stable.

2) **Unstable Equilibrium:** If $W = F_B$ and point B is below point 'G'. A small displacement to the body, in the clock wise direction, gives couple due to $W \& F_B$ also in the clockwise direction. Thus, body will move away from its original position. Hence, equilibrium is unstable
3) **Neutral Equilibrium**: If $F_B = W$ and B & G are at the same point, the displacement of body does not result in any couple of Wt & F_B. Body remains at its displaced position



Stable equilibrium

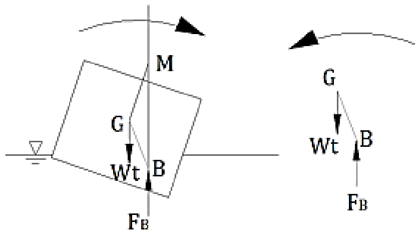


Unstable equilibrium

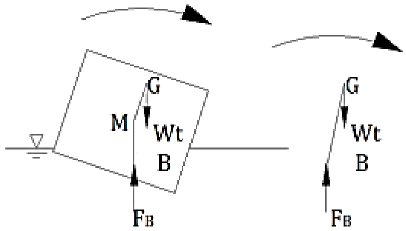
3.6.42 STABILITY IN FLOATING BODY

The stability of floating body is determined from position of metacentre(M). In case of floating body, the weight of body is equal to the buoyant force.

1) **Stable Equilibrium:** When M is above G, because of a small displacement to the body in the clock wise direction, the couple between Wt & F_B causes rotation in anti-clockwise direction.
2) **Unstable Equilibrium:** When M is below G, because of small displacement to the body in the clock wise direction, the couple between Wt & F_B causes rotation in clockwise direction.
3) **Neutral:** If M lies at the C.G. of body, the displacement of body does not result in any couple of Wt & F_B. Body remains at its displaced position.



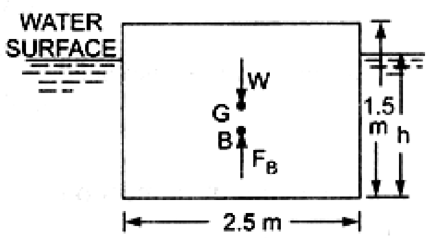
Stable equilibrium



Unstable equilibrium

Example:
Find the volume of the water displaced and position of centre of buoyancy for a wooden block of width 2.5 m and of depth 1.5m, when it floats horizontally in water. The density of wooden block is $650 kg/m^3$ and its length is 6.0m.

Solution:
Given:
Width = 2.5m
Depth = 1.5m
Length = 6.0m
Volume of the block
$= 2.5 \times 1.5 \times 6.0 = 22.50 m^3$
Density of wood, $\rho = 65 kg/m^3$



$\therefore$ Weight of block $= \rho \times g \times \text{Volume}$
$= 650 \times 9.81 \times 22.50 N = 143471 N$
For equilibrium, the weight of water displaced = Weight of wooden block
$= 143471 N$
$\therefore$ Volume of water displaced

$$=\frac{\text{Weight of water displaced}}{\text{Weight density of water}}=\frac{143471}{1000\times9.81}=14.625\text{m}^3$$

Position of center of Buoyancy:
Volume of wooden block in water
= Volume of water displaced
Or
$2.5\times h\times6.0=14.625\text{m}^3$,
Where,
h is depth of wooden block in water

$$\therefore\ h=\frac{14.625}{2.5\times6.0}=0.975\text{m}$$

$\therefore$ Centre of Buoyancy

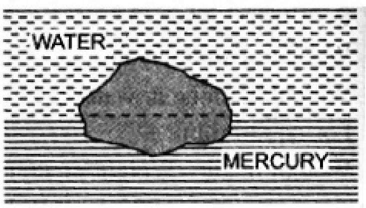
$$=\frac{0.975}{2}=0.4875\text{m from base}$$

Example:
Find the density of a metallic body which floats at the interface of mercury of S.G. 13.6 and water such that 40% of its volume is sub-merged in mercury and 60% in water.
Solution:
Let the volume of the body $=\text{Vm}^3$
Then volume of body sub-merged in mercury

$$=\frac{40}{100}V=0.4\text{Vm}^3$$

Volume of body sub-merged in water

$$=\frac{60}{100}V=0.6\text{Vm}^3$$



For the equilibrium of the body
Total buoyant force (upward force) =Weight of the body
But total buoyant force= Force of buoyancy due to water + Force of buoyancy due to mercury
Force of buoyancy due to water = Weight of water displaced by body
= Density of water × Volume of mercury displaced
$=1000\times g\times\text{Volume of body in water}$
$= 1000\times g\times0.6\times VN$
And, force of buoyancy due to mercury
= Weight of mercury displaced by body
= g × Density of water× Volume of mercury displaced
$=g\times13.6\times1000\times$ volume of body in mercury
$= g\times13.6\times1000\times0.4VN$
Weight of the body
=Density x g x Volume of body
$\therefore$ For equilibrium, we have
Total buoyant force =Weight of the body

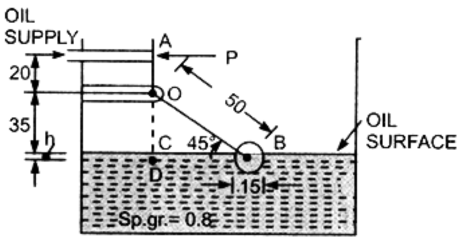
$$1000\times g\times0.6\times V+13.6\times1000\times g\times.4V=\rho\times g\times V$$

Or

$$\rho=600+13600\times.4=600+54400=6040.00\text{kg/m}^3$$

$\therefore$ Density of the body $=6040.00\text{kg/m}^3$

Example:
A float valve regulates the flow of oil of S.G. 0.8 into a cistern. The spherical float is 15 cm in diameter. AOB is a weightless link carrying the float at one end, and a valve at the other end which closes the pipe through which oil flows into the cistern. The link is mounted on a frictionless hinge at O and the angle AOB is 135°. The length of OA is 20cm, and the distance between the centre of the float and the hinge 50 cm. When of the flow is stopped AO will be vertical. The valve is to be pressed on to the seat with a force of 9.81 N to completely stop the flow of oil into the cistern. It was observed that the flow of oil is stopped when the free surface of oil in the cistern is 35 cm below the hinge. Determine the weight of the float.



Solution:
Given:
Sp. gr. of oil =0.8
$\therefore$ Density of oil

$=\rho_0 = 0.8\times 1000 = 800\,kg/m^3$

Dia. of float, D=15cm

$\angle AOB = 135°$

$OA = 20cm$

Force, P=9.81N

OB=50 cm

Let the weight be W.

When the flow of oil is stopped, the centre of float is shown in Fig. The level of oil is also shown. The centre of float is below the level of oil, by a depth 'h'

From ΔBOD,

$$\sin 45° = \frac{OD}{OB} = \frac{OC+CD}{OB} = \frac{35+h}{50}$$

$50\times\sin 45° = 35+h$ Or

$$h = 50\times\frac{1}{\sqrt{2}} - 35 = 35.355 - 35 = 0.355cm = .00355m$$

The weight of float is acting through B, but the upward buoyant force is acting through the centre of weight of oil displaced

Volume of oil displaced :

$$= \frac{2}{3}\pi r^3 + h\times\pi r^2 \left\{ r = \frac{D}{2} = \frac{15}{2} = 7.5cm \right\}$$

$$= \frac{2}{3}\times\pi\times(0.75)^3 + .00355\times\pi\times(0.75)^2 = 0.000945m^3$$

=Weight of oil displaced

$= \rho_0 \times g \times \text{Volume of oil}$

$= 800\times 9.81\times 0.00945 = 7.416N$

The buoyant force and weight of the float passes through the same vertical line, passing through B.

Let the weight of float is W. Then net vertical force on float

=Buoyant force –Weight of float = (7.416-W)

Taking moments about the hinges O, we get

$$P\times 20 = (7.416 - W)\times BD = (7.416 - W)\times 50\times\cos 45°$$

Or $9.81\times 20 = (7.416 - W)\times 35.355$

$$\therefore W = 7.416 - \frac{20\times 9.81}{35.355} = 7.416 - 5.55 = 1.866N$$

Example:

A rectangular pontoon is 5m long, 3m wide and 1.20m high. The depth of immersion of the pontoon is 0.80m in sea water. If the centre of gravity is 0.6m above the bottom of the pontoon, determine the Meta – centric height. The density for sea water $= 1025\,kg/m^3$

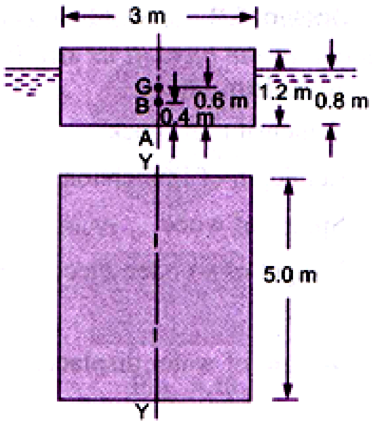
Solution:

Given:

Dimension of pontoon $= 5m\times 3m\times 1.20m$

Depth of immersion =0.8m

Distance AG=0.6m



PLAN AT WATER SURFACE

$$\text{Distance } AB = \frac{1}{2}\times\text{Depth of immersion}$$

$$= \frac{1}{2}\times .8 = 0.4m$$

Density for sea water $= 1025\,kg/m^3$

Meta-centre height GM, given by equation as

$$GM = \frac{I}{\forall} - BG$$

Where I = Moment of Inertia of the plan of the pontoon about Y-Y axis

$$= \frac{1}{12}\times 5\times 3^3\,m^4 = \frac{45}{4}m^4$$

$\forall$ = Volume of the body sub- merged in water

$= 3\times 0.8\times 5.0 = 12.0m^3$

$BG = AG = AB = 0.6 - 0.4 = 0.2m$

$$GM = \frac{45}{4}\times\frac{1}{12.0} - 0.2 = \frac{45}{48} - 0.2 = 0.9375 - 0.2 = 0.7375m$$

Example:

A solid cylinder of diameter 4.0 m has a height of 4.0m. Find the meta-centric height of the cylinder if the specific gravity of the material of cylinder = 0.6 and it is floating in water with its axis vertical. State whether the equilibrium is stable or unstable.

Solution:

Given: D=4m

Height, h=4m

S.G. = 0.6

Depth of cylinder in water = S.G x h

$= 0.6 \times 4.0 = 2.4\,m$

$\therefore$ Distance of centre of buoyancy (B) from A

$$AB = \frac{2.4}{2} = 1.2\,m$$

Distance of centre of gravity (G) from A

$$AG = \frac{h}{2} = \frac{4.0}{2} = 2.0\,m$$

$\therefore BG = AG - AB = 2.0 - 1.2 = 0.8\,m$

Now the meta-centric height GM is given by

$$GM = \frac{I}{\forall} - BG$$

Where

I=M. O. I. of the plan of the body about Y-Y axis

$$= \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (4.0)^4$$

$\forall =$ Volume of cylinder in water

$$= \frac{\pi}{4.0} \times D^2 \times \text{Depth of cylinder in water} = \frac{\pi}{4} \times 4^2 \times 2.4\,m^3$$

$$\therefore \frac{1}{\forall} = \frac{\frac{\pi}{64} \times 4^4}{\frac{\pi}{4} \times 4^2 \times 2.4} = \frac{1}{16} \times \frac{4^2}{2.4} = \frac{1}{2.4} = 0.4167\,m$$

$$GM = \frac{1}{\forall} - BG = 0.4167 - 0.8 = -0.3833\,m$$

-ve sign means that the meta–centre (M) is below the centre of gravity (G). Thus the cylinder is in unstable equilibrium.

Example:

A wooden cylinder of S.G. 0.6 and circular cross–section is required to float in oil (S.G. 0.90). Find the L/D ratio for the cylinder to float with its longitudinal axis vertical in oil, where L is the height of cylinder and D is its diameter.

Solution:

Given:

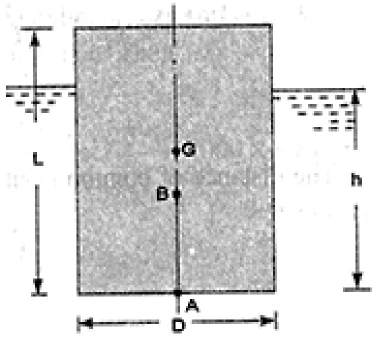
Dia of cylinder =D

Height of cylinder =L

Sp. Gr. Of cylinder $S_1 = 0.6$

Sp. Gr of oil $S_2 = 0.9$

Let the depth of cylinder immersed in oil=h



For the principle of buoyancy

Weight of cylinder = wt. of oil displaced

$$\frac{\pi}{4} \times D^2 \times L \times 0.6 \times 1000 \times 9.81 = \frac{\pi}{4} \times D^2 \times h \times 0.9 \times 1000 \times 9.81$$

Or $\quad L \times 0.6 = h \times 0.9$

$$\therefore \quad h = \frac{0.6 \times L}{0.9} = \frac{2}{3} L$$

The distance of centre of gravity G from A,

$$AG = \frac{L}{2}$$

The distance of centre of buoyancy B from A

$$AB = \frac{h}{2} = \frac{1}{2}\left[\frac{2}{3}L\right] = \frac{L}{3}$$

$$\therefore BG = AG - AB = \frac{L}{2} - \frac{L}{3} = \frac{3L - 2L}{6} = \frac{L}{6}$$

The meta–centric height GM is given by

$$GM = \frac{I}{\forall} - BG$$

Where $I=\frac{\pi}{64}D^4$ and $\forall$ = Volume of cylinder in oil $=\frac{\pi}{4}D^2\times h$

$$\therefore \frac{I}{\forall}=\left(\frac{\pi}{64}D^4/\frac{\pi}{4}D^2h\right)=\frac{1}{16}\frac{D^2}{h}=\frac{D^2}{16\times\frac{2}{3}L}=\frac{3D^2}{32L}$$

$$\left\{\because\ h=\frac{2}{3}L\right\}$$

$$\therefore\ GM=\frac{3D^2}{32L}-\frac{L}{6}$$

For stable equilibrium, GM should be +ve or,

$$GM>0 \text{ or } \frac{3D^2}{32L}-\frac{L}{6}>0$$

Or

$$\frac{3D^2}{32L}>\frac{L}{6} \text{ or } \frac{3\times6}{32}>\frac{L^2}{D^2}$$

Or

$$\frac{L^2}{D^2}<\frac{18}{32} \text{ or } \frac{9}{16}$$

$$\therefore \quad \frac{L}{D}<\sqrt{\frac{9}{16}}=\frac{3}{4}$$

$$\therefore \quad \frac{L}{D}<3/4$$

Example:
The time period of rolling of a ship of weight 29430kN in sea water is 10seconds. The centre of buoyancy of the ship is 1.5 m below the centre of gravity. Find the radius of gyration of the ship if the moment of inertia of the ship at the water line about fore and aft axis is$1000m^4$. The specific weight of sea water as $10100N/m^3$

Solution:
Given:
Time period T=10sec
Distance between centre of buoyancy and centre of gravity, BG=1.5m
Moment of Inertia, $I=10000m^4$
Weight $W=29430kN=29430\times1000N$
Let the radius of gyration =K

First calculating the meta–centric height, which is given as

$$GM=BM-BG=\frac{I}{\forall}-BG$$

Where I= Moment of Inertia
And $\forall=$ Volume of water displaced

$$=\frac{\text{Weight of ship}}{\text{Sp.weight of sea water}}=\frac{29430\times1000}{10104}=2912.6m^3$$

$$\therefore GM=\frac{10000}{2912.6}-1.5=3.433-1.5=1.933m$$

Using equation, $T=2\pi\sqrt{\frac{K^2}{GM\times g}}$

We get

$$10=2\pi\sqrt{\frac{K^2}{1.933\times9.81}}=\frac{2\pi K}{\sqrt{1.933\times9.81}}$$

Or

$$K=\frac{10\times\sqrt{1.933\times9.81}}{2\pi}=6.93m$$