James O. Wilkes

Fluid Mechanics for Chemical Engineers

Second Edition, with Microfluidics and CFD

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Dedicated to the memory of

Terence Robert Corelli Fox

Shell Professor of Chemical Engineering
University of Cambridge, 1946–1959

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PREFACE

T HIS text has evolved from a need for a single volume that embraces a wide range of topics in fluid mechanics. The material consists of two parts—four chapters on *macroscopic* or relatively large-scale phenomena, followed by ten chapters on *microscopic* or relatively small-scale phenomena. Throughout, I have tried to keep in mind topics of industrial importance to the chemical engineer. The scheme is summarized in the following list of chapters.

Part I—Macroscopic Fluid Mechanics

1. Introduction to Fluid Mechanics
2. Mass, Energy, and Momentum Balances
3. Fluid Friction in Pipes
4. Flow in Chemical Engineering Equipment

Part II—Microscopic Fluid Mechanics

5. Differential Equations of Fluid Mechanics
6. Solution of Viscous-Flow Problems
7. Laplace's Equation, Irrotational and Porous-Media Flows
8. Boundary-Layer and Other Nearly Unidirectional Flows
9. Turbulent Flow
10. Bubble Motion, Two-Phase Flow, and Fluidization
11. Non-Newtonian Fluids
12. Microfluidics and Electrokinetic Flow Effects
13. An Introduction to Computational Fluid Dynamics and FlowLab
14. COMSOL (FEMLAB) Multi-physics for Solving Fluid Mechanics Problems

In our experience, an undergraduate fluid mechanics course can be based on Part I plus selected parts of Part II, and a graduate course can be based on much of Part II, supplemented perhaps by additional material on topics such as approximate methods and stability.

Second edition. I have attempted to bring the book up to date by the major addition of Chapters 12, 13, and 14—one on microfluidics and two on CFD (computational fluid dynamics). The choice of software for the CFD presented a difficulty; for various reasons, I selected FlowLab and COMSOL Multiphysics, but there was no intention of "promoting" these in favor of other excellent CFD programs.[1] The use of CFD examples in the classroom really makes the subject

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[1] The software name "FEMLAB" was changed to "COMSOL Multiphysics" in September 2005, the first release under the new name being COMSOL 3.2.

come "alive," because the previous restrictive necessities of "nice" geometries and constant physical properties, etc., can now be lifted. Chapter 9, on turbulence, has also been extensively rewritten; here again, CFD allows us to venture beyond the usual flow in a pipe or between parallel plates and to investigate further practical situations such as turbulent mixing and recirculating flows.

Example problems.   There is an average of about six completely worked examples in each chapter, including several involving COMSOL (dispersed throughout Part II) and FlowLab (all in Chapter 13). The end of each example is marked by a small, hollow square: □. All the COMSOL examples have been run on a Macintosh G4 computer using FEMLAB 3.1, but have also been checked on a PC; those using a PC or other releases of COMSOL/FEMLAB may encounter slightly different windows than those reproduced here. The format for each COMSOL example is: (a) problem statement, (b) details of COMSOL implementation, and (c) results and discussion (however, item (b) can easily be skipped for those interested only in the results).

The numerous end-of-chapter problems have been classified roughly as easy (E), moderate (M), or difficult/lengthy (D). The University of Cambridge has given permission—kindly endorsed by Professor J.F. Davidson, F.R.S.—for several of their chemical engineering examination problems to be reproduced in original or modified form, and these have been given the additional designation of "(C)".

Acknowledgments.   I gratefully acknowledge the written contributions of my former Michigan colleague Stacy Birmingham (non-Newtonian fluids), Brian Kirby of Cornell University (microfluidics), and Chi-Yang Cheng of Fluent, Inc. (FlowLab). Although I wrote most of the COMSOL examples, I have had great help and cooperation from COMSOL Inc. and the following personnel in particular—Philip Byrne, Bjorn Sjodin, Ed Fontes, Peter Georen, Olof Hernell, Johan Linde, and Rémi Magnard. At Fluent, Inc., Shane Moeykens was instrumental in identifying Chi-Yang Cheng as the person best suited to write the FlowLab chapter. Courtney Esposito and Jordan Schmidt of The MathWorks kindly helped me with MATLAB, needed for the earlier 2.3 version of FEMLAB.

I appreciate the assistance of several other friends and colleagues, including Nitin Anturkar, Stuart Churchill, John Ellis, Kevin Ellwood, Scott Fogler, Leenaporn Jongpaiboonkit, Lisa Keyser, Kartic Khilar, Ronald Larson, Susan Montgomery, Donald Nicklin, the late Margaret Sansom, Michael Solomon, Sandra Swisher, Rasin Tek, Robert Ziff, and my wife Mary Ann Gibson Wilkes. Also very helpful were Bernard Goodwin, Elizabeth Ryan, and Michelle Housley at Prentice Hall PTR, and my many students and friends at the University of Michigan and Chulalongkorn University in Bangkok. Others are acknowledged in specific literature citations.

Further information.   The website `http://www.engin.umich.edu/~fmche` is maintained as a "bulletin board" for giving additional information about the

book—hints for problem solutions, *errata*, how to contact the authors, etc.—as proves desirable. My own Internet address is `wilkes@umich.edu`. The text was composed on a Power Macintosh G4 computer using the TEXtures "typesetting" program. Eleven-point type was used for the majority of the text. Most of the figures were constructed using MacDraw Pro, Excel, and KaleidaGraph.

Professor Terence Fox, to whom this book is dedicated, was a Cambridge engineering graduate who worked from 1933 to 1937 at Imperial Chemical Industries Ltd., Billingham, Yorkshire. Returning to Cambridge, he taught engineering from 1937 to 1946 before being selected to lead the Department of Chemical Engineering at the University of Cambridge during its formative years after the end of World War II. As a scholar and a gentleman, Fox was a shy but exceptionally brilliant person who had great insight into what was important and who quickly brought the department to a preeminent position. He succeeded in combining an industrial perspective with intellectual rigor. Fox relinquished the leadership of the department in 1959, after he had secured a permanent new building for it (carefully designed in part by himself).

T.R.C. Fox

Fox was instrumental in bringing Peter Danckwerts, Kenneth Denbigh, John Davidson, and others into the department. He also accepted me in 1956 as a junior faculty member, and I spent four good years in the Cambridge University Department of Chemical Engineering. Danckwerts subsequently wrote an appreciation[2] of Fox's talents, saying, with almost complete accuracy: "Fox instigated no research and published nothing." How times have changed—today, unless he were known personally, his résumé would probably be cast aside and he would stand little chance of being hired, let alone of receiving tenure! However, his lectures, meticulously written handouts, enthusiasm, genius, and friendship were a great inspiration to me, and I have much pleasure in acknowledging his positive impact on my career.

James O. Wilkes
August 18, 2005

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[2] P.V. Danckwerts, "Chemical engineering comes to Cambridge," *The Cambridge Review*, pp. 53–55, February 28, 1983.

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Some Greek Letters

α	alpha	ν	nu
β	beta	ξ, Ξ	xi
γ, Γ	gamma	o	omicron
δ, Δ	delta	π, ϖ, Π	pi
ϵ, ε	epsilon	ρ, ϱ	rho
ζ	zeta	$\sigma, \varsigma, \Sigma$	sigma
η	eta	τ	tau
$\theta, \vartheta, \Theta$	theta	υ, Υ	upsilon
ι	iota	ϕ, φ, Φ	phi
κ	kappa	χ	chi
λ, Λ	lambda	ψ, Ψ	psi
μ	mu	ω, Ω	omega

Chapter 1

INTRODUCTION TO FLUID MECHANICS

1.1 Fluid Mechanics in Chemical Engineering

A knowledge of fluid mechanics is essential for the chemical engineer because the majority of chemical-processing operations are conducted either partly or totally in the fluid phase. Examples of such operations abound in the biochemical, chemical, energy, fermentation, materials, mining, petroleum, pharmaceuticals, polymer, and waste-processing industries.

There are two principal reasons for placing such an emphasis on fluids. First, at typical operating conditions, an enormous number of materials normally exist as gases or liquids, or can be transformed into such phases. Second, it is usually more efficient and cost-effective to work with fluids in contrast to solids. Even some operations with solids can be conducted in a quasi-fluidlike manner; examples are the fluidized-bed catalytic refining of hydrocarbons, and the long-distance pipelining of coal particles using water as the agitating and transporting medium.

Although there is inevitably a significant amount of theoretical development, almost *all* the material in this book has some application to chemical processing and other important practical situations. Throughout, we shall endeavor to present an understanding of the *physical* behavior involved; only then is it really possible to comprehend the accompanying theory and equations.

1.2 General Concepts of a Fluid

We must begin by responding to the question, "What *is* a fluid?" Broadly speaking, a fluid is a substance that will deform *continuously* when it is subjected to a tangential or *shear* force, much as a similar type of force is exerted when a water-skier skims over the surface of a lake or butter is spread on a slice of bread. The rate at which the fluid deforms continuously depends not only on the magnitude of the applied force but also on a property of the fluid called its *viscosity* or resistance to deformation and flow. Solids will also deform when sheared, but a position of equilibrium is soon reached in which elastic forces induced by the deformation of the solid exactly counterbalance the applied shear force, and further deformation ceases.

A simple apparatus for shearing a fluid is shown in Fig. 1.1. The fluid is contained between two concentric cylinders; the outer cylinder is stationary, and the inner one (of radius R) is rotated steadily with an angular velocity ω. This shearing motion of a fluid can continue indefinitely, provided that a source of energy—supplied by means of a torque here—is available for rotating the inner cylinder. The diagram also shows the resulting *velocity profile*; note that the velocity in the direction of rotation varies from the peripheral velocity $R\omega$ of the inner cylinder down to zero at the outer stationary cylinder, these representing typical *no-slip* conditions at both locations. However, if the intervening space is filled with a solid—even one with obvious elasticity, such as rubber—only a limited rotation will be possible before a position of equilibrium is reached, unless, of course, the torque is so high that *slip* occurs between the rubber and the cylinder.

Fixed cylinder

Rotating cylinder

Rotating cylinder

R

Fluid

Fixed cylinder

Velocity profile

ω

Fluid

$R\,\omega$

(a) Side elevation   (b) Plan of section across A-A (not to scale)
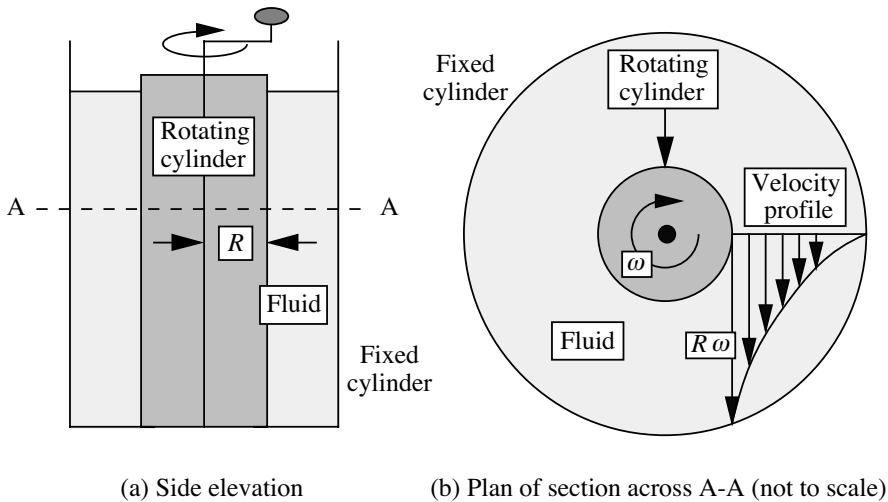
Fig. 1.1   *Shearing of a fluid.*

There are various classes of fluids. Those that behave according to nice and obvious simple laws, such as water, oil, and air, are generally called *Newtonian* fluids. These fluids exhibit constant viscosity but, under typical processing conditions, virtually no elasticity. Fortunately, a very large number of fluids of interest to the chemical engineer exhibit Newtonian behavior, which will be assumed throughout the book, except in Chapter 11, which is devoted to the study of non-Newtonian fluids.

A fluid whose viscosity is not constant (but depends, for example, on the intensity to which it is being sheared), or which exhibits significant elasticity, is termed *non-Newtonian.* For example, several polymeric materials subject to deformation can "remember" their recent molecular configurations, and in attempting to recover their recent states, they will exhibit *elasticity* in addition to viscosity. Other fluids, such as drilling mud and toothpaste, behave essentially as solids and

will *not* flow when subject to *small* shear forces, but *will* flow readily under the influence of *high* shear forces.

Fluids can also be broadly classified into two main categories—liquids and gases. *Liquids* are characterized by relatively high densities and viscosities, with molecules close together; their volumes tend to remain constant, roughly independent of pressure, temperature, or the size of the vessels containing them. *Gases*, on the other hand, have relatively low densities and viscosities, with molecules far apart; generally, they will rapidly tend to fill the container in which they are placed. However, these two states—liquid and gaseous—represent but the two extreme ends of a continuous *spectrum* of possibilities.
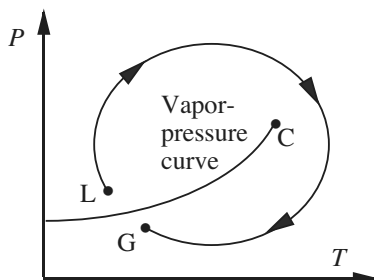
Fig. 1.2   When does a liquid become a gas?

The situation is readily illustrated by considering a fluid that is initially a gas at point G on the pressure/temperature diagram shown in Fig. 1.2. By increasing the pressure, and perhaps lowering the temperature, the vapor-pressure curve is soon reached and crossed, and the fluid condenses and apparently becomes a liquid at point L. By continuously adjusting the pressure and temperature so that the clockwise path is followed, and circumnavigating the critical point C in the process, the fluid is returned to G, where it is presumably once more a gas. But where does the transition from liquid at L to gas at G occur? The answer is at no single point, but rather that the change is a continuous and gradual one, through a whole spectrum of intermediate states.

1.3   Stresses, Pressure, Velocity, and the Basic Laws

Stresses.   The concept of a *force* should be readily apparent. In fluid mechanics, a force per unit area, called a *stress*, is usually found to be a more convenient and versatile quantity than the force itself. Further, when considering a specific surface, there are two types of stresses that are particularly important.

1. The first type of stress, shown in Fig. 1.3(a), acts *perpendicularly* to the surface and is therefore called a *normal* stress; it will be tensile or compressive, depending on whether it tends to stretch or to compress the fluid on which it acts. The normal stress equals F/A, where F is the normal force and A is the area of the surface on which it acts. The dotted outlines show the volume changes caused

by deformation. In fluid mechanics, *pressure* is usually the most important type of *compressive* stress, and will shortly be discussed in more detail.

2. The second type of stress, shown in Fig. 1.3(b), acts *tangentially* to the surface; it is called a *shear* stress τ, and equals F/A, where F is the tangential force and A is the area on which it acts. Shear stress is transmitted through a fluid by interaction of the molecules with one another. A knowledge of the shear stress is very important when studying the flow of viscous Newtonian fluids. For a given rate of deformation, measured by the time derivative $d\gamma/dt$ of the small angle of deformation γ, the shear stress τ is directly proportional to the *viscosity* of the fluid (see Fig. 1.3(b)).
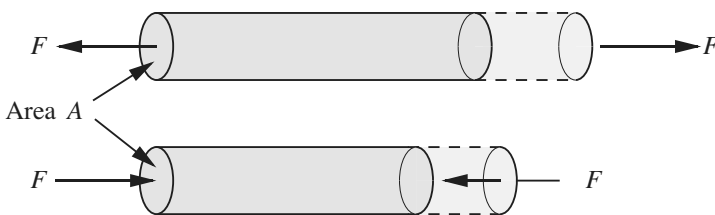
Fig. 1.3(a)   Tensile and compressive normal stresses F/A, acting on a cylinder, causing elongation and shrinkage, respectively.
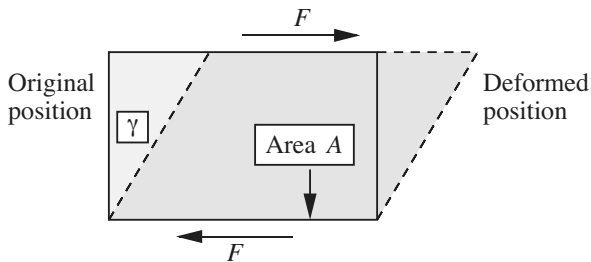
Fig. 1.3(b)   Shear stress $\tau = F/A$, acting on a rectangular parallelepiped, shown in cross section, causing a deformation measured by the angle γ (whose magnitude is exaggerated here).

Pressure.   In virtually all *hydrostatic* situations—those involving fluids at rest—the fluid molecules are in a state of *compression*. For example, for the swimming pool whose cross section is depicted in Fig. 1.4, this compression at a typical point P is caused by the downwards gravitational weight of the water *above* point P. The degree of compression is measured by a scalar, p—the *pressure*.

A small inflated spherical balloon pulled down from the surface and tethered at the bottom by a weight will still retain its spherical shape (apart from a small distortion at the point of the tether), but will be diminished in size, as in Fig. 1.4(a). It is apparent that there must be forces acting *normally inward* on the

surface of the balloon, and that these must essentially be uniform for the shape to remain spherical, as in Fig. 1.4(b).

(a) (b)
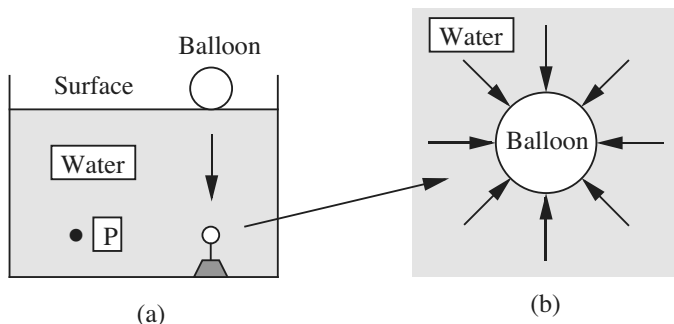
Fig. 1.4 (a) Balloon submerged in a swimming pool; (b) enlarged view of the compressed balloon, with pressure forces acting on it.

Although the pressure p is a scalar, it typically appears in tandem with an area A (assumed small enough so that the pressure is uniform over it). By definition of pressure, the surface experiences a normal compressive force $F = pA$. Thus, pressure has units of a force per unit area—the same as a stress.

The value of the pressure at a point is *independent* of the orientation of any area associated with it, as can be deduced with reference to a differentially small wedge-shaped element of the fluid, shown in Fig. 1.5.
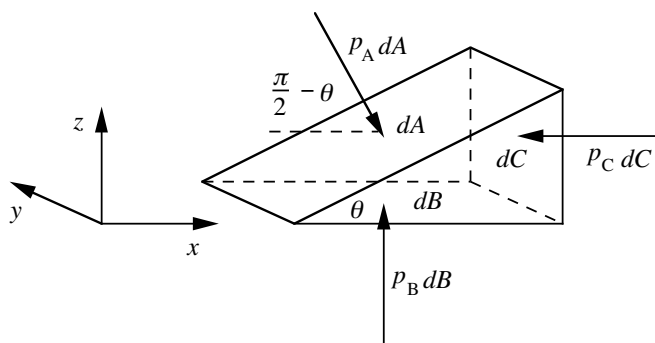
Fig. 1.5 Equilibrium of a wedge of fluid.

Due to the pressure there are three forces, $p_A dA$, $p_B dB$, and $p_C dC$, that act on the three rectangular faces of areas dA, dB, and dC. Since the wedge is not moving, equate the two forces acting on it in the horizontal or x direction, noting that $p_A dA$ must be resolved through an angle $(\pi/2 - \theta)$ by multiplying it by $\cos(\pi/2 - \theta) = \sin\theta$:

$$p_A dA \sin\theta = p_C dC. \tag{1.1}$$

The vertical force $p_B dB$ acting on the bottom surface is omitted from Eqn. (1.1) because it has no component in the x direction. The horizontal pressure forces

acting in the y direction on the two triangular faces of the wedge are also omitted, since again these forces have no effect in the x direction. From geometrical considerations, areas dA and dC are related by:

$$dC = dA \sin \theta. \tag{1.2}$$

These last two equations yield:

$$p_A = p_C, \tag{1.3}$$

verifying that the pressure is independent of the orientation of the surface being considered. A force balance in the z direction leads to a similar result, $p_A = p_B$.[1]

For *moving* fluids, the normal stresses include both a pressure *and* extra stresses caused by the motion of the fluid, as discussed in detail in Section 5.6.

The amount by which a certain pressure exceeds that of the atmosphere is termed the *gauge* pressure, the reason being that many common pressure gauges are really *differential* instruments, reading the difference between a required pressure and that of the surrounding atmosphere. *Absolute* pressure equals the gauge pressure plus the atmospheric pressure.

Velocity.   Many problems in fluid mechanics deal with the *velocity* of the fluid at a point, equal to the rate of change of the position of a fluid particle with time, thus having both a magnitude and a direction. In some situations, particularly those treated from the *macroscopic* viewpoint, as in Chapters 2, 3, and 4, it sometimes suffices to ignore variations of the velocity with position. In other cases—particularly those treated from the *microscopic* viewpoint, as in Chapter 6 and later—it is invariably essential to consider variations of velocity with position.

(a)   (b)
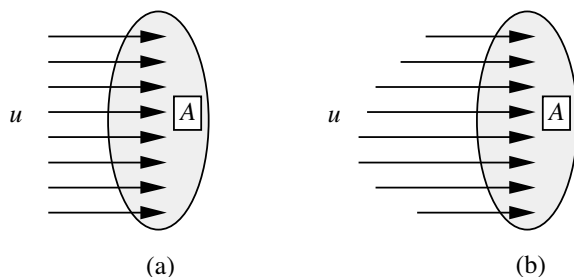
Fig. 1.6   Fluid passing through an area A:
(a) Uniform velocity, (b) varying velocity.

Velocity is not only important in its own right, but leads immediately to three *fluxes* or flow rates. Specifically, if u denotes a *uniform* velocity (not varying with position):

---

[1] Actually, a force balance in the z direction demands that the gravitational weight of the wedge be considered, which is proportional to the *volume* of the wedge. However, the pressure forces are proportional to the *areas* of the faces. It can readily be shown that the volume-to-area effect becomes vanishingly small as the wedge becomes infinitesimally small, so that the gravitational weight is inconsequential.

1. If the fluid passes through a plane of area A normal to the direction of the velocity, as shown in Fig. 1.6, the corresponding *volumetric* flow rate of fluid through the plane is $Q = uA$.
2. The corresponding *mass* flow rate is $m = \rho Q = \rho u A$, where ρ is the (constant) fluid density. The alternative notation with an overdot, $\dot{m}$, is also used.
3. When velocity is multiplied by mass it gives *momentum*, a quantity of prime importance in fluid mechanics. The corresponding *momentum* flow rate passing through the area A is $\dot{\mathcal{M}} = mu = \rho u^2 A$.

If u and/or ρ should vary with position, as in Fig. 1.6(b), the corresponding expressions will be seen later to involve *integrals* over the area A: $Q = \int_A u\, dA$, $m = \int_A \rho u\, dA$, $\dot{\mathcal{M}} = \int_A \rho u^2\, dA$.

Basic laws.   In principle, the *laws* of fluid mechanics can be stated simply, and—in the absence of relativistic effects—amount to conservation of mass, energy, and momentum. When applying these laws, the procedure is first to identify a system, its boundary, and its surroundings; and second, to identify how the system interacts with its surroundings. Refer to Fig. 1.7 and let the quantity X represent either mass, energy, or momentum. Also recognize that X may be *added* from the surroundings and transported into the system by an amount X_{in} across the boundary, and may likewise be *removed* or transported out of the system to the surroundings by an amount X_{out}.
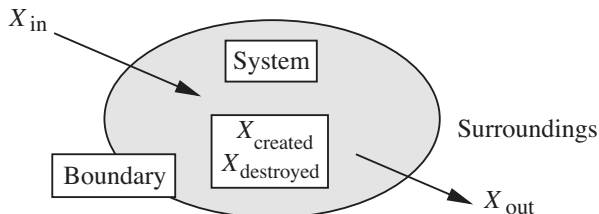
Fig. 1.7   *A system and transports to and from it.*

The general conservation law gives the increase ΔX_{system} in the X-content of the system as:

$$X_{\text{in}} - X_{\text{out}} = \Delta X_{\text{system}}. \tag{1.4a}$$

Although this basic law may appear intuitively obvious, it applies *only* to a very restricted selection of properties X. For example, it is not *generally* true if X is another extensive property such as volume, and is quite meaningless if X is an intensive property such as pressure or temperature.

In certain cases, where X^i is the mass of a definite chemical species i, we may also have an amount of creation X^i_{created} or destruction $X^i_{\text{destroyed}}$ due to chemical reaction, in which case the general law becomes:

$$X^i_{\text{in}} - X^i_{\text{out}} + X^i_{\text{created}} - X^i_{\text{destroyed}} = \Delta X^i_{\text{system}}. \tag{1.4b}$$

The conservation law will be discussed further in Section 2.1, and is of such fundamental importance that in various guises it will find numerous applications throughout all of this text.

To solve a physical problem, the following information concerning the fluid is also usually needed:

1. The physical properties of the fluid involved, as discussed in Section 1.4.
2. For situations involving fluid *flow*, a *constitutive equation* for the fluid, which relates the various stresses to the flow pattern.

1.4 Physical Properties—Density, Viscosity, and Surface Tension

There are three physical properties of fluids that are particularly important: density, viscosity, and surface tension. Each of these will be defined and viewed briefly in terms of molecular concepts, and their dimensions will be examined in terms of mass, length, and time (M, L, and T). The physical properties depend primarily on the particular fluid. For liquids, viscosity also depends strongly on the temperature; for gases, viscosity is *approximately* proportional to the square root of the absolute temperature. The density of gases depends almost directly on the absolute pressure; for most other cases, the effect of pressure on physical properties can be disregarded.

Typical processes often run almost isothermally, and in these cases the effect of temperature can be ignored. Except in certain special cases, such as the flow of a compressible gas (in which the density is not constant) or a liquid under a very high shear rate (in which viscous dissipation can cause significant internal heating), or situations involving exothermic or endothermic reactions, we shall ignore any variation of physical properties with pressure and temperature.

Densities of liquids. Density depends on the mass of an individual molecule and the number of such molecules that occupy a unit of volume. For liquids, density depends primarily on the particular liquid and, to a much smaller extent, on its temperature. Representative densities of liquids are given in Table 1.1.[2] (See Eqns. (1.9)–(1.11) for an explanation of the specific gravity and coefficient of thermal expansion columns.) The accuracy of the values given in Tables 1.1–1.6 is adequate for the calculations needed in this text. However, if highly accurate values are needed, particularly at extreme conditions, then specialized information should be sought elsewhere.

Density.   The *density* ρ of a fluid is defined as its mass per unit volume, and indicates its inertia or resistance to an accelerating force. Thus:

$$\rho = \frac{\text{mass}}{\text{volume}} \ [=] \ \frac{\text{M}}{\text{L}^3}, \tag{1.5}$$

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[2] The values given in Tables 1.1, 1.3, 1.4, 1.5, and 1.6 are based on information given in J.H. Perry, ed., *Chemical Engineers' Handbook,* 3rd ed., McGraw-Hill, New York, 1950.

in which the notation "[=]" is consistently used to indicate the *dimensions* of a quantity.[3] It is usually understood in Eqn. (1.5) that the volume is chosen so that it is neither so small that it has no chance of containing a representative selection of molecules nor so large that (in the case of gases) changes of pressure cause significant changes of density throughout the volume. A medium characterized by a density is called a *continuum*, and follows the classical laws of mechanics—including Newton's law of motion, as described in this book.

Table 1.1 Specific Gravities, Densities, and
Thermal Expansion Coefficients of Liquids at 20 °C

Liquid	Sp. Gr.	Density, ρ		α
	s	kg/m^3	lb$_m$/ft^3	°C^{-1}
Acetone	0.792	792	49.4	0.00149
Benzene	0.879	879	54.9	0.00124
Crude oil, 35°API	0.851	851	53.1	0.00074
Ethanol	0.789	789	49.3	0.00112
Glycerol	1.26 (50 °C)	1,260	78.7	—
Kerosene	0.819	819	51.1	0.00093
Mercury	13.55	13,550	845.9	0.000182
Methanol	0.792	792	49.4	0.00120
n-Octane	0.703	703	43.9	—
n-Pentane	0.630	630	39.3	0.00161
Water	0.998	998	62.3	0.000207

Degrees A.P.I. (American Petroleum Institute) are related to specific gravity s by the formula:

$$°\text{A.P.I.} = \frac{141.5}{s} - 131.5. \tag{1.6}$$

Note that for water, °A.P.I. = 10, with correspondingly *higher* values for liquids that are *less* dense. Thus, for the crude oil listed in Table 1.1, Eqn. (1.6) indeed gives $141.5/0.851 - 131.5 \doteq 35$ °A.P.I.

Densities of gases.   For *ideal* gases, $pV = nRT$, where p is the *absolute* pressure, V is the volume of the gas, n is the number of moles (abbreviated as "mol" when used as a unit), R is the gas constant, and T is the *absolute* temperature. If M_w is the molecular weight of the gas, it follows that:

$$\rho = \frac{nM_w}{V} = \frac{M_w p}{RT}. \tag{1.7}$$

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[3] An early appearance of the notation "[=]" is in R.B. Bird, W.E. Stewart, and E.N. Lightfoot, *Transport Phenomena*, Wiley, New York, 1960.

Thus, the density of an ideal gas depends on the molecular weight, absolute pressure, and absolute temperature. Values of the gas constant R are given in Table 1.2 for various systems of units. Note that degrees Kelvin, formerly represented by "°K," is now more simply denoted as "K."

Table 1.2   Values of the Gas Constant, R

Value	Units
8.314	J/g-mol K
0.08314	liter bar/g-mol K
0.08206	liter atm/g-mol K
1.987	cal/g-mol K
10.73	psia ft^3/lb-mol °R
0.7302	ft^3 atm/lb-mol °R
1,545	ft lb$_\text{f}$/lb-mol °R

For a *nonideal* gas, the compressibility factor Z (a function of p and T) is introduced into the denominator of Eqn. (1.7), giving:

$$\rho = \frac{nM_\text{w}}{V} = \frac{M_\text{w}p}{ZRT}. \tag{1.8}$$

Thus, the extent to which Z deviates from unity gives a measure of the nonideality of the gas.

The *isothermal compressibility* of a gas is defined as:

$$\beta = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T,$$

and equals—at constant temperature—the fractional *decrease* in volume caused by a unit *increase* in the pressure. For an ideal gas, $\beta = 1/p$, the reciprocal of the absolute pressure.

The *coefficient of thermal expansion* α of a material is its isobaric (constant pressure) fractional increase in volume per unit rise in temperature:

$$\alpha = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p. \tag{1.9}$$

Since, for a given mass, density is inversely proportional to volume, it follows that for moderate temperature ranges (over which α is essentially constant) the density of most liquids is approximately a linear function of temperature:

$$\rho \doteq \rho_0[1 - \alpha(T - T_0)], \tag{1.10}$$

where ρ_0 is the density at a reference temperature T_0. For an ideal gas, $\alpha = 1/T$, the reciprocal of the absolute temperature.

The *specific gravity s* of a fluid is the ratio of the density ρ to the density ρ_{SC} of a reference fluid at some *standard condition*:

$$s = \frac{\rho}{\rho_{SC}}. \tag{1.11}$$

For *liquids*, ρ_{SC} is usually the density of water at $4\,^{\circ}C$, which equals 1.000 g/ml or 1,000 kg/m^3. For *gases*, ρ_{SC} is sometimes taken as the density of air at $60\,^{\circ}F$ and 14.7 psia, which is approximately 0.0759 lb$_m$/ft^3, and sometimes at $0\,^{\circ}C$ and one atmosphere absolute; since there is no *single* standard for gases, care must obviously be taken when interpreting published values. For *natural gas*, consisting primarily of methane and other hydrocarbons, the *gas gravity* is defined as the ratio of the molecular weight of the gas to that of air (28.8 lb$_m$/lb-mol).

Values of the molecular weight M_w are listed in Table 1.3 for several commonly occurring gases, together with their densities at standard conditions of atmospheric pressure and $0\,^{\circ}C$.

Table 1.3   *Gas Molecular Weights and Densities*
(the Latter at Atmospheric Pressure and $0\,^{\circ}C$)

Gas	M_w	Standard Density kg/m^3	lb$_m$/ft^3
Air	28.8	1.29	0.0802
Carbon dioxide	44.0	1.96	0.1225
Ethylene	28.0	1.25	0.0780
Hydrogen	2.0	0.089	0.0056
Methane	16.0	0.714	0.0446
Nitrogen	28.0	1.25	0.0780
Oxygen	32.0	1.43	0.0891

Viscosity.   The viscosity of a fluid measures its resistance to flow under an applied shear stress, as shown in Fig. 1.8(a). There, the fluid is ideally supposed to be confined in a relatively small gap of thickness h between one plate that is stationary and another plate that is moving steadily at a velocity V relative to the first plate.

In practice, the situation would essentially be realized by a fluid occupying the space between two concentric cylinders of large radii rotating relative to each other, as in Fig. 1.1. A steady force F to the right is applied to the upper plate (and, to preserve equilibrium, to the left on the lower plate) in order to maintain a

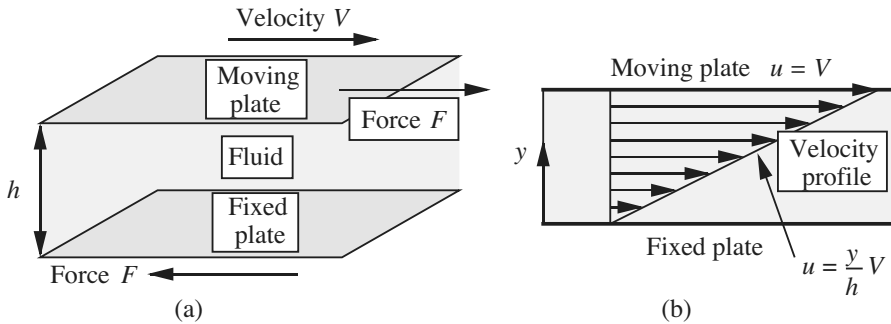constant motion and to overcome the viscous friction caused by layers of molecules sliding over one another.

Fig. 1.8   (a) Fluid in shear between parallel plates; (b) the ensuing linear velocity profile.

Under these circumstances, the velocity u of the fluid to the right is found experimentally to vary linearly from zero at the lower plate ($y = 0$) to V itself at the upper plate, as in Fig. 1.8(b), corresponding to no-slip conditions at each plate. At any intermediate distance y from the lower plate, the velocity is simply:

$$u = \frac{y}{h}V. \tag{1.12}$$

Recall that the *shear stress* τ is the tangential applied force F per unit area:

$$\tau = \frac{F}{A}, \tag{1.13}$$

in which A is the area of each plate. Experimentally, for a large class of materials, called *Newtonian* fluids, the shear stress is directly proportional to the velocity *gradient*:

$$\tau = \mu\frac{du}{dy} = \mu\frac{V}{h}. \tag{1.14}$$

The proportionality constant μ is called the *viscosity* of the fluid; its dimensions can be found by substituting those for F (ML/T^2), A (L^2), and du/dy (T^{-1}), giving:

$$\mu \; [=] \; \frac{M}{LT}. \tag{1.15}$$

Representative units for viscosity are g/cm s (also known as *poise*, designated by P), kg/m s, and lb$_m$/ft hr. The *centipoise* (cP), one hundredth of a poise, is also a convenient unit, since the viscosity of water at room temperature is approximately 0.01 P or 1.0 cP. Table 1.11 gives viscosity conversion factors.

The viscosity of a fluid may be determined by observing the pressure drop when it flows at a known rate in a tube, as analyzed in Section 3.2. More sophisticated

methods for determining the *rheological* or flow properties of fluids—including viscosity—are also discussed in Chapter 11; such methods often involve containing the fluid in a small gap between two surfaces, moving one of the surfaces, and measuring the force needed to maintain the other surface stationary.

Table 1.4   Viscosity Parameters for Liquids

Liquid	a	b	a	b
	(T in K)		(T in °R)	
Acetone	14.64	−2.77	16.29	−2.77
Benzene	21.99	−3.95	24.34	−3.95
Crude oil, 35° API	53.73	−9.01	59.09	−9.01
Ethanol	31.63	−5.53	34.93	−5.53
Glycerol	106.76	−17.60	117.22	−17.60
Kerosene	33.41	−5.72	36.82	−5.72
Methanol	22.18	−3.99	24.56	−3.99
Octane	17.86	−3.25	19.80	−3.25
Pentane	13.46	−2.62	15.02	−2.62
Water	29.76	−5.24	32.88	−5.24

The *kinematic viscosity* ν is the ratio of the viscosity to the density:

$$\nu = \frac{\mu}{\rho}, \tag{1.16}$$

and is important in cases in which significant viscous *and* gravitational forces coexist. The reader can check that the dimensions of ν are L^2/T, which are identical to those for the diffusion coefficient $\mathcal{D}$ in mass transfer and for the thermal diffusivity $\alpha = k/\rho c_{\mathrm{p}}$ in heat transfer. There is a definite analogy among the three quantities—indeed, as seen later, the value of the kinematic viscosity governs the rate of "diffusion" of momentum in the laminar and turbulent flow of fluids.

Viscosities of liquids. The viscosities μ of liquids generally vary approximately with absolute temperature T according to:

$$\ln \mu \doteq a + b \ln T \qquad \text{or} \qquad \mu \doteq e^{a+b \ln T}, \tag{1.17}$$

and—to a good approximation—are independent of pressure. Assuming that μ is measured in centipoise and that T is either in degrees Kelvin or Rankine, appropriate parameters a and b are given in Table 1.4 for several representative liquids. The resulting values for viscosity are approximate, suitable for a first design only.

Viscosities of gases. The viscosity μ of many gases is approximated by the formula:

$$\mu \doteq \mu_0 \left(\frac{T}{T_0}\right)^n,\tag{1.18}$$

in which T is the absolute temperature (Kelvin or Rankine), μ_0 is the viscosity at an absolute reference temperature T_0, and n is an empirical exponent that best fits the experimental data. The values of the parameters μ_0 and n for atmospheric pressure are given in Table 1.5; recall that to a *first* approximation, the viscosity of a gas is independent of pressure. The values μ_0 are given in centipoise and correspond to a reference temperature of $T_0 \doteq 273$ K $\doteq 492\,°$R.

Table 1.5   *Viscosity Parameters for Gases*

Gas	μ_0, cP	n
Air	0.0171	0.768
Carbon dioxide	0.0137	0.935
Ethylene	0.0096	0.812
Hydrogen	0.0084	0.695
Methane	0.0120	0.873
Nitrogen	0.0166	0.756
Oxygen	0.0187	0.814

Surface tension.[4] Surface tension is the tendency of the surface of a liquid to behave like a stretched elastic membrane. There is a natural tendency for liquids to minimize their surface area. The obvious case is that of a liquid droplet on a horizontal surface that is not wetted by the liquid—mercury on glass, or water on a surface that also has a thin oil film on it. For small droplets, such as those on the left of Fig. 1.9, the droplet adopts a shape that is almost perfectly spherical, because in this configuration there is the least surface area for a given volume.

Fig. 1.9   *The larger droplets are flatter because grav-ity is becoming more important than surface tension.*

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[4] We recommend that this subsection be omitted at a first reading, because the concept of surface tension is somewhat involved and is relevant only to a small part of this book.

For larger droplets, the shape becomes somewhat flatter because of the increasingly important gravitational effect, which is roughly proportional to a^3, where a is the approximate droplet radius, whereas the surface area is proportional only to a^2. Thus, the ratio of gravitational to surface tension effects depends roughly on the value of $a^3/a^2 = a$, and is therefore increasingly important for the larger droplets, as shown to the right in Fig. 1.9. Overall, the situation is very similar to that of a water-filled balloon, in which the water accounts for the gravitational effect and the balloon acts like the surface tension.

A fundamental property is the *surface energy*, which is defined with reference to Fig. 1.10(a). A molecule I, situated in the *interior* of the liquid, is attracted equally in all directions by its neighbors. However, a molecule S, situated in the *surface*, experiences a net attractive force into the bulk of the liquid. (The vapor above the surface, being comparatively rarefied, exerts a negligible force on molecule S.) Therefore, work has to be done against such a force in bringing an interior molecule to the surface. Hence, an energy σ, called the *surface energy*, can be attributed to a unit area of the surface.
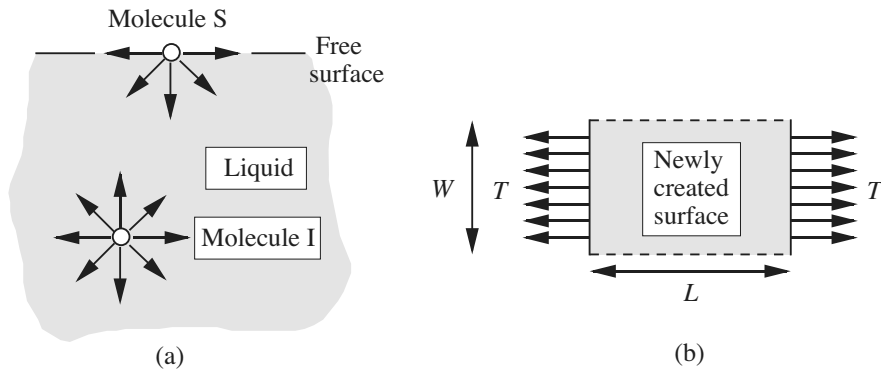
Fig. 1.10   (a) Molecules in the interior and surface of a liquid; (b) newly created surface caused by moving the tension T through a distance L.

An equivalent viewpoint is to consider the surface tension T existing per unit distance of a line drawn in the surface, as shown in Fig. 1.10(b). Suppose that such a tension has moved a distance L, thereby creating an area WL of fresh surface. The work done is the product of the force, TW, and the distance L through which it moves, namely TWL, and this must equal the newly acquired surface energy σWL. Therefore, $T = \sigma$; both quantities have units of force per unit distance, such as N/m, which is equivalent to energy per unit area, such as J/m^2.

We next find the amount $p_1 - p_2$ by which the pressure p_1 inside a liquid droplet of radius r, shown in Fig. 1.11(a), exceeds the pressure p_2 of the surrounding vapor. Fig. 1.11(b) illustrates the equilibrium of the upper hemisphere of the droplet, which is also surrounded by an imaginary cylindrical "control surface" ABCD, on which forces in the vertical direction will soon be equated. Observe that the

internal pressure p_1 is trying to blow apart the two hemispheres (the lower one is not shown), whereas the surface tension σ is trying to pull them together.
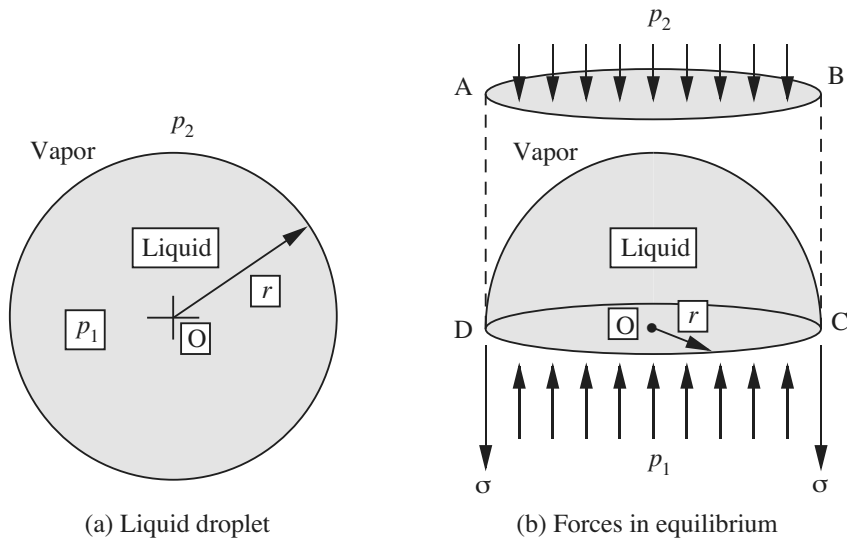

(a) Liquid droplet   (b) Forces in equilibrium

Fig. 1.11   Pressure change across a curved surface.

In more detail, there are two different types of forces to be considered:

1. That due to the *pressure difference* between the pressure inside the droplet and the vapor outside, each acting on an area πr^2 (that of the circles CD and AB):

$$(p_1 - p_2)\pi r^2. \tag{1.19}$$

2. That due to surface tension, which acts on the circumference of length $2\pi r$:

$$2\pi r\sigma. \tag{1.20}$$

At equilibrium, these two forces are equated, giving:

$$\Delta p = p_1 - p_2 = \frac{2\sigma}{r}. \tag{1.21}$$

That is, there is a *higher* pressure on the concave or droplet side of the interface. What would the pressure change be for a *bubble* instead of a droplet? Why?

More generally, if an interface has principal radii of curvature r_1 and r_2, the increase in pressure can be shown to be:

$$p_1 - p_2 = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right). \tag{1.22}$$

For a sphere of radius r, as in Fig. 1.11, both radii are equal, so that $r_1 = r_2 = r$, and $p_1 - p_2 = 2\sigma/r$. Problem 1.31 involves a situation in which $r_1 \neq r_2$. The radii r_1 and r_2 will have the same sign if the corresponding centers of curvature are on the same side of the interface; if not, they will be of opposite sign. Appendix A contains further information about the curvature of a surface.
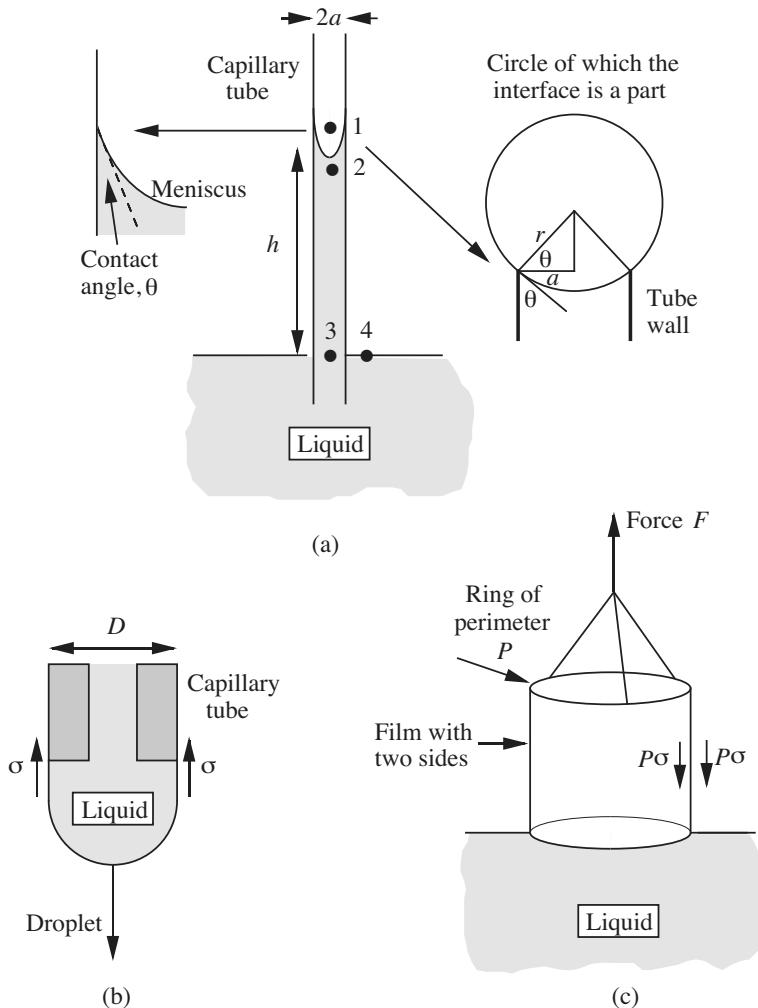
Fig. 1.12 Methods for measuring surface tension.

A brief description of simple experiments for measuring the surface tension σ of a liquid, shown in Fig. 1.12, now follows:

(a) In the *capillary-rise* method, a narrow tube of internal radius a is dipped vertically into a pool of liquid, which then rises to a height h inside the tube; if the *contact angle* (the angle between the free surface and the wall) is θ, the meniscus will be approximated by part of the surface of a sphere; from the geometry shown in the enlargement on the right-hand side of Fig. 1.12(a) the radius of the sphere is seen to be $r = a/\cos\theta$. Since the surface is now concave on the air side, the reverse of Eqn. (1.21) occurs, and $p_2 = p_1 - 2\sigma/r$, so that p_2 is *below* atmospheric pressure p_1. Now follow the path 1–2–3–4, and observe that $p_4 = p_3$ because points

3 and 4 are at the same elevation in the same liquid. Thus, the pressure at point 4 is:

$$p_4 = p_1 - \frac{2\sigma}{r} + \rho g h.$$

However, $p_4 = p_1$ since both of these are at atmospheric pressure. Hence, the surface tension is given by the relation:

$$\sigma = \frac{1}{2}\rho g h r = \frac{\rho g h a}{2\cos\theta}. \tag{1.23}$$

In many cases—for complete wetting of the surface—θ is essentially zero and $\cos\theta = 1$. However, for liquids such as mercury in glass, there may be a complete *non*-wetting of the surface, in which case $\theta = \pi$, so that $\cos\theta = -1$; the result is that the liquid level in the capillary is then *depressed* below that in the surrounding pool.

(b) In the *drop-weight* method, a liquid droplet is allowed to form very slowly at the tip of a capillary tube of outer diameter D. The droplet will eventually grow to a size where its weight just overcomes the surface-tension force $\pi D \sigma$ holding it up. At this stage, it will detach from the tube, and its weight $w = Mg$ can be determined by catching it in a small pan and weighing it. By equating the two forces, the surface tension is then calculated from:

$$\sigma = \frac{w}{\pi D}. \tag{1.24}$$

(c) In the *ring tensiometer,* a thin wire ring, suspended from the arm of a sensitive balance, is dipped into the liquid and gently raised, so that it brings a thin liquid film up with it. The force F needed to support the film is measured by the balance. The downward force exerted on a unit length of the ring by one side of the film is the surface tension; since there are two sides to the film, the total force is $2P\sigma$, where P is the circumference of the ring. The surface tension is therefore determined as:

$$\sigma = \frac{F}{2P}. \tag{1.25}$$

In common with most experimental techniques, all three methods described above require slight modifications to the results expressed in Eqns. (1.23)–(1.25) because of imperfections in the simple theories.

Surface tension generally appears only in situations involving either free surfaces (liquid/gas or liquid/solid boundaries) or interfaces (liquid/liquid boundaries); in the latter case, it is usually called the *interfacial* tension.

Representative values for the surface tensions of liquids at $20\,°$C, in contact either with air or their vapor (there is usually little difference between the two), are given in Table 1.6.[5]

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[5] The values for surface tension have been obtained from the *CRC Handbook of Chemistry and Physics,* 48th ed., The Chemical Rubber Co., Cleveland, OH, 1967.

Table 1.6 Surface Tensions

Liquid	σ dynes/cm
Acetone	23.70
Benzene	28.85
Ethanol	22.75
Glycerol	63.40
Mercury	435.5
Methanol	22.61
n-Octane	21.80
Water	72.75

1.5 Units and Systems of Units

Mass, weight, and force.   The *mass M* of an object is a measure of the amount of matter it contains and will be constant, since it depends on the number of constituent molecules and their masses. On the other hand, the *weight w* of the object is the gravitational force on it, and is equal to Mg, where g is the local gravitational acceleration. Mostly, we shall be discussing phenomena occurring at the surface of the earth, where g is approximately 32.174 ft/s^2 = 9.807 m/s^2 = 980.7 cm/s^2. For much of this book, these values are simply taken as 32.2, 9.81, and 981, respectively.

Table 1.7 Representative Units of Force

System	Units of Force	Customary Name
SI	kg m/s^2	newton
CGS	g cm/s^2	dyne
FPS	lb$_\text{m}$ ft/s^2	poundal

Newton's second law of motion states that a force F applied to a mass M will give it an acceleration a:

$$F = Ma, \tag{1.26}$$

from which is apparent that force has *dimensions* ML/T^2. Table 1.7 gives the corresponding *units* of force in the SI (meter/kilogram/second), CGS (centimeter/gram/second), and FPS (foot/pound/second) systems.

The poundal is now an archaic unit, hardly ever used. Instead, the *pound force*, lb_f, is much more common in the English system; it is defined as the gravitational force on 1 lb_m, which, if left to fall freely, will do so with an acceleration of 32.2 ft/s^2. Hence:

$$1 \ lb_f = 32.2 \ lb_m \ \frac{ft}{s^2} = 32.2 \ poundals. \tag{1.27}$$

Table 1.8   SI Units

Physical Quantity	Name of Unit	Symbol for Unit	Definition of Unit
Basic Units			
Length	meter	m	–
Mass	kilogram	kg	–
Time	second	s	–
Temperature	degree Kelvin	K	–
Supplementary Unit			
Plane angle	radian	rad	—
Derived Units			
Acceleration			m/s^2
Angular velocity			rad/s
Density			kg/m^3
Energy	joule	J	$kg \ m^2/s^2$
Force	newton	N	$kg \ m/s^2$
Kinematic viscosity			m^2/s
Power	watt	W	$kg \ m^2/s^3$ (J/s)
Pressure	pascal	Pa	$kg/m \ s^2$ (N/m^2)
Velocity			m/s
Viscosity			$kg/m \ s$

When using lb_f in the ft, lb_m, s (FPS) system, the following conversion factor, commonly called "g_c," will almost invariably be needed:

$$g_c = 32.2 \ \frac{lb_m \ ft/s^2}{lb_f} = 32.2 \ \frac{lb_m \ ft}{lb_f \ s^2}. \tag{1.28}$$

Some writers incorporate g_c into their equations, but this approach may be confusing since it virtually implies that one particular set of units is being used, and hence tends to rob the equations of their generality. Why not, for example, also incorporate the conversion factor of 144 in^2/ft^2 into equations where pressure is expressed in lb$_f$/in^2? We prefer to omit all conversion factors in equations, and introduce them only as needed in evaluating expressions numerically. If the reader is in any doubt, units should *always* be checked when performing calculations.

SI Units. The most systematically developed and universally accepted set of units occurs in the *SI* units or *Système International d'Unités*[6]; the subset we mainly need is shown in Table 1.8.

The basic units are again the meter, kilogram, and second (m, kg, and s); from these, certain *derived* units can also be obtained. Force (kg m/s^2) has already been discussed; energy is the product of force and length; power amounts to energy per unit time; surface tension is energy per unit area or force per unit length, and so on. Some of the units have names, and these, together with their abbreviations, are also given in Table 1.8.

Table 1.9   Auxiliary Units Allowed in Conjunction with SI Units

Physical Quantity	Name of Unit	Symbol for Unit	Definition of Unit
Area	hectare	ha	10^4 m^2
Kinematic viscosity	stokes	St	10^{-4} m^2/s
Length	micron	μm	10^{-6} m
Mass	tonne	t	10^3 kg = Mg
	gram	g	10^{-3} kg = g
Pressure	bar	bar	10^5 N/m^2
Viscosity	poise	P	10^{-1} kg/m s
Volume	liter	l	10^{-3} m^3

Tradition dies hard, and certain other "metric" units are so well established that they may be used as *auxiliary* units; these are shown in Table 1.9. The *gram* is the classic example. Note that the basic SI unit of mass (kg) is even represented in terms of the gram, and has not yet been given a name of its own!

Table 1.10 shows some of the acceptable prefixes that can be used for accommodating both small and large quantities. For example, to avoid an excessive number of decimal places, 0.000001 s is normally better expressed as 1 μs (one microsecond). Note also, for example, that 1 μkg should be written as 1 mg—one prefix being better than two.

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[6] For an excellent discussion, on which Tables 1.8 and 1.9 are based, see *Metrication in Scientific Journals*, published by The Royal Society, London, 1968.

Table 1.10   Prefixes for Fractions and Multiples

Factor	Name	Symbol	Factor	Name	Symbol
10^{-12}	pico	p	10^3	kilo	k
10^{-9}	nano	n	10^6	mega	M
10^{-6}	micro	μ	10^9	giga	G
10^{-3}	milli	m	10^{12}	tera	T

Some of the more frequently used conversion factors are given in Table 1.11.

Example 1.1—Units Conversion

Part 1. Express 65 mph in (a) ft/s, and (b) m/s.

Solution

The solution is obtained by employing conversion factors taken from Table 1.11:

$$\text{(a)}\quad 65\ \frac{\text{mile}}{\text{hr}} \quad\times\quad \frac{1}{3{,}600}\ \frac{\text{hr}}{\text{s}} \quad\times\quad 5{,}280\ \frac{\text{ft}}{\text{mile}} \quad=\quad 95.33\ \frac{\text{ft}}{\text{s}}\ .$$

$$\text{(b)}\quad 95.33\ \frac{\text{ft}}{\text{s}} \quad\times\quad 0.3048\ \frac{\text{m}}{\text{ft}} \quad=\quad 29.06\ \frac{\text{m}}{\text{s}}\ .$$

Part 2. The density of $35\,^{\circ}$API crude oil is 53.1 $\text{lb}_\text{m}/\text{ft}^3$ at $68\,^{\circ}$F and its viscosity is 32.8 $\text{lb}_\text{m}/\text{ft hr}$. What are its density, viscosity, and kinematic viscosity in SI units?

Solution

$$\rho = \ 53.1\ \frac{\text{lb}_\text{m}}{\text{ft}^3} \quad\times\quad 0.4536\ \frac{\text{kg}}{\text{lb}_\text{m}} \quad\times\quad \frac{1}{0.3048^3}\ \frac{\text{ft}^3}{\text{m}^3} \quad=\quad 851\ \frac{\text{kg}}{\text{m}^3}\ .$$

$$\mu = \ 32.8\ \frac{\text{lb}_\text{m}}{\text{ft hr}} \quad\times\quad \frac{1}{2.419}\ \frac{\text{centipoise}}{\text{lb}_\text{m}/\text{ft hr}} \quad\times\quad 0.01\ \frac{\text{poise}}{\text{centipoise}} \quad=\quad 0.136\ \text{poise.}$$

Or, converting to SI units, noting that P is the symbol for poise, and evaluating ν:

$$\mu = 0.136\ \text{P}\ \times\ 0.1\ \frac{\text{kg/m s}}{\text{P}} \quad=\quad 0.0136\ \frac{\text{kg}}{\text{m s}}\ .$$

$$\nu = \frac{\mu}{\rho} = \frac{0.0136\ \text{kg/m s}}{851\ \text{kg/m}^3} \quad=\quad 1.60\times 10^{-5}\ \frac{\text{m}^2}{\text{s}} \quad (= 0.160\ \text{St}). \qquad \square$$

Example 1.2—Mass of Air in a Room   **25**

Table 1.11   Commonly Used Conversion Factors

Area	1 mile^2	=	640 acres
	1 acre	=	0.4047 ha
Energy	1 BTU	=	1,055 J
	1 cal	=	4.184 J
	1 J	=	0.7376 ft lb$_f$
	1 erg	=	1 dyne cm
Force	1 lb$_f$	=	4.448 N
	1 N	=	0.2248 lb$_f$
Length	1 ft	=	0.3048 m
	1 m	=	3.281 ft
	1 mile	=	5,280 ft
Mass	1 lb$_m$	=	0.4536 kg
	1 kg	=	2.205 lb$_m$
Power	1 HP	=	550 ft lb$_f$/s
	1 kW	=	737.6 ft lb$_f$/s
Pressure	1 atm	=	14.696 lb$_f$/in^2
	1 atm	=	1.0133 bar
	1 atm	=	1.0133×10^5 Pa
Time	1 day	=	24 hr
	1 hr	=	60 min
	1 min	=	60 s
Viscosity	1 cP	=	2.419 lb$_m$/ft hr
	1 cP	=	0.001 kg/m s
	1 cP	=	0.000672 lb$_m$/ft s
	1 lb$_f$ s/ft^2	=	4.788×10^4 cP
Volume	1 ft^3	=	7.481 U.S. gal
	1 U.S. gal	=	3.785 l
	1 m^3	=	264.2 U.S. gal

Example 1.2—Mass of Air in a Room

Estimate the mass of air in your classroom, which is 80 ft wide, 40 ft deep, and 12 ft high. The gas constant is R = 10.73 psia ft^3/lb-mol °R.

Solution

The volume of the classroom, shown in Fig. E1.2, is:

$$V = 80 \times 40 \times 12 = 3.84 \times 10^4 \text{ ft}^3.$$
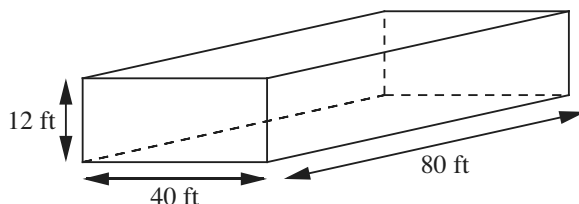
Fig. E1.2   *Assumed dimensions of classroom.*

If the air is approximately 20% oxygen and 80% nitrogen, its mean molecular weight is $M_w = 0.8 \times 28 + 0.2 \times 32 = 28.8$ lb$_m$/lb-mol. From the gas law, assuming an absolute pressure of $p = 14.7$ psia and a temperature of $70\,°F = 530\,°R$, the density is:

$$\rho = \frac{M_w p}{RT} = \frac{28.8 \text{ (lb}_m/\text{lb mol)} \times 14.7 \text{ (psia)}}{10.73 \text{ (psia ft}^3/\text{lb mol }°R) \times 530 \text{ (}°R)} = 0.0744 \text{ lb}_m/\text{ft}^3.$$

Hence, the mass of air is:

$$M = \rho V = 0.0744 \text{ (lb}_m/\text{ft}^3) \times 3.84 \times 10^4 \text{ (ft}^3) = 2{,}860 \text{ lb}_m. \qquad \square$$

For the rest of the book, manipulation of units will often be less detailed; the reader should always check if there is any doubt.

1.6   Hydrostatics

Variation of pressure with elevation.   Here, we investigate how the pressure in a stationary fluid varies with elevation z. The result is useful because it can answer questions such as "What is the pressure at the summit of Mt. Annapurna?" or "What forces are exerted on the walls of an oil storage tank?" Consider a hypothetical differential cylindrical element of fluid of cross-sectional area A, height dz, and volume $A\,dz$, which is also surrounded by the same fluid, as shown in Fig. 1.13. Its weight, being the downwards gravitational force on its mass, is $dW = \rho A\,dz\,g$. Two completely equivalent approaches will be presented:

Method 1.   Let p denote the pressure at the base of the cylinder; since p changes at a rate dp/dz with elevation, the pressure is found either from Taylor's expansion or the definition of a derivative to be $p + (dp/dz)dz$ at the top of the cylinder.[7] (Note that we do not anticipate a reduction of pressure with elevation here; hence, the plus sign is used. If, indeed—as proves to be the case—pressure falls with increasing elevation, then the subsequent development will tell us that

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[7] Further details of this fundamental statement can be found in Appendix A and *must* be fully understood, because similar assertions appear repeatedly throughout the book.

dp/dz is negative.) Hence, the fluid exerts an *upward* force of pA on the base of the cylinder, and a *downward* force of $[p + (dp/dz)dz]A$ on the top of the cylinder.

Next, apply Newton's second law of motion by equating the net upward force to the mass times the acceleration—which is zero, since the cylinder is stationary:

$$\underbrace{pA - \left(p + \frac{dp}{dz}dz\right)A}_{\text{Net pressure force}} - \underbrace{\rho A\, dz\, g}_{\text{Weight}} = \underbrace{(\rho A\, dz)}_{\text{Mass}} \times 0 = 0. \tag{1.29}$$

Cancellation of pA and division by $A\, dz$ leads to the following *differential equation,* which governs the rate of change of pressure with elevation:

$$\frac{dp}{dz} = -\rho g. \tag{1.30}$$
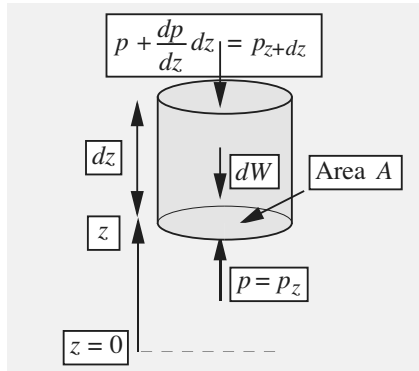
Fig. 1.13   *Forces acting on a cylinder of fluid.*

Method 2.   Let p_z and p_{z+dz} denote the pressures at the base and top of the cylinder, where the elevations are z and $z+dz$, respectively. Hence, the fluid exerts an *upward* force of $p_z A$ on the base of the cylinder, and a *downward* force of $p_{z+dz}A$ on the top of the cylinder. Application of Newton's second law of motion gives:

$$\underbrace{p_z A - p_{z+dz}A}_{\text{Net pressure force}} - \underbrace{\rho A\, dz\, g}_{\text{Weight}} = \underbrace{(\rho A\, dz)}_{\text{Mass}} \times 0 = 0. \tag{1.31}$$

Isolation of the two pressure terms on the left-hand side and division by $A\, dz$ gives:

$$\frac{p_{z+dz} - p_z}{dz} = -\rho g. \tag{1.32}$$

As dz tends to zero, the left-hand side of Eqn. (1.32) becomes the derivative dp/dz, leading to the same result as previously:

$$\frac{dp}{dz} = -\rho g. \tag{1.30}$$

The same conclusion can also be obtained by considering a cylinder of *finite* height Δz and then letting Δz approach zero.

Note that Eqn. (1.30) predicts a pressure *decrease* in the vertically upward direction at a rate that is proportional to the local density. Such pressure variations can readily be detected by the ear when traveling quickly in an elevator in a tall building, or when taking off in an airplane. The reader must thoroughly understand *both* the above approaches. For most of this book, we shall use Method 1, because it eliminates the steps of taking the limit of $dz \to 0$ and invoking the definition of the derivative.

Pressure in a liquid with a free surface.   In Fig. 1.14, the pressure is p_s at the free surface, and we wish to find the pressure p at a depth H below the free surface—of water in a swimming pool, for example.
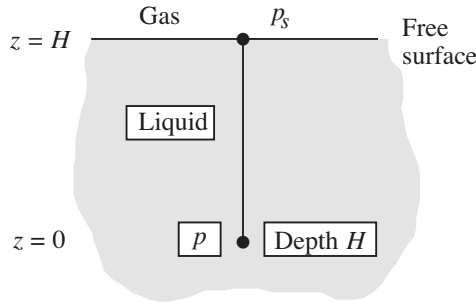
Fig. 1.14   Pressure at a depth H.

Separation of variables in Eqn. (1.30) and integration between the free surface ($z = H$) and a depth H ($z = 0$) gives:

$$\int_{p_s}^{p} dp = -\int_{H}^{0} \rho g \, dz. \tag{1.33}$$

Assuming—quite reasonably—that ρ and g are constants in the liquid, these quantities may be taken outside the integral, yielding:

$$p = p_s + \rho g H, \tag{1.34}$$

which predicts a *linear* increase of pressure with distance downward from the free surface. For large depths, such as those encountered by deep-sea divers, very substantial pressures will result.

Example 1.3—Pressure in an Oil Storage Tank   **29**

Example 1.3—Pressure in an Oil Storage Tank

What is the absolute pressure at the bottom of the cylindrical tank of Fig. E1.3, filled to a depth of H with crude oil, with its free surface exposed to the atmosphere? The specific gravity of the crude oil is 0.846. Give the answers for (a) $H = 15.0$ ft (pressure in lb_f/in^2), and (b) $H = 5.0$ m (pressure in Pa and bar). What is the purpose of the surrounding dike?
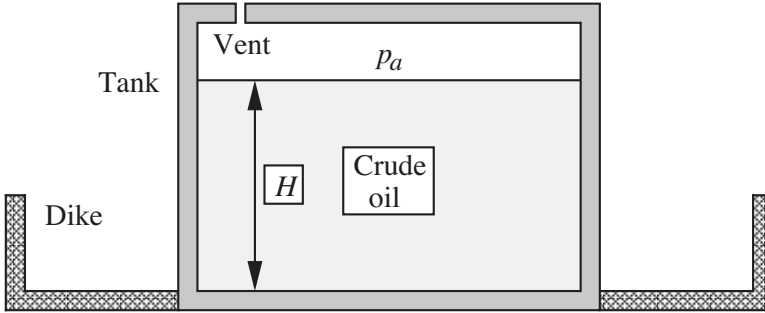
Fig. E1.3   *Crude oil storage tank.*

Solution

(a) The pressure is that of the atmosphere, p_a, plus the increase due to a column of depth $H = 15.0$ ft. Thus, setting $p_s = p_a$, Eqn. (1.34) gives:

$$p = p_a + \rho g H$$

$$= 14.7 + \frac{0.846 \times 62.3 \times 32.2 \times 15.0}{144 \times 32.2}$$

$$= 14.7 + 5.49 = 20.2 \text{ psia.}$$

The reader should check the units, noting that the 32.2 in the numerator is g [=] ft/s², and that the 32.2 in the denominator is g_c [=] lb_m ft/lb_f s².

(b) For SI units, no conversion factors are needed. Noting that the density of water is 1,000 kg/m³, and that $p_a \doteq 1.01 \times 10^5$ Pa absolute:

$$p = 1.01 \times 10^5 + 0.846 \times 1{,}000 \times 9.81 \times 5.0 = 1.42 \times 10^5 \text{ Pa} = 1.42 \text{ bar.}$$

In the event of a tank rupture, the dike contains the leaking oil and facilitates prevention of spreading fire and contamination of the environment.

Epilogue

When he arrived at work in an oil refinery one morning, the author saw first-hand the consequences of an inadequately vented oil-storage tank. Rain during the night had caused partial condensation of vapor inside the tank, whose pressure had become sufficiently lowered so that the external atmospheric pressure had crumpled the steel tank just as if it were a flimsy tin can. The refinery manager was not pleased.   □

Example 1.4—Multiple Fluid Hydrostatics

The U-tube shown in Fig. E1.4 contains oil and water columns, between which there is a long trapped air bubble. For the indicated heights of the columns, find the specific gravity of the oil.
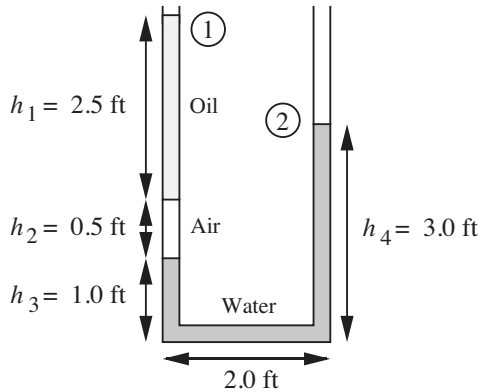
Fig. E1.4   Oil/air/water system.

Solution

The pressure p_2 at point 2 may be deduced by starting with the pressure p_1 at point 1 and adding or subtracting, as appropriate, the hydrostatic pressure changes due to the various columns of fluid. Note that the width of the U-tube (2.0 ft) is irrelevant, since there is no change in pressure in the horizontal leg. We obtain:

$$p_2 = p_1 + \rho_o g h_1 + \rho_a g h_2 + \rho_w g h_3 - \rho_w g h_4, \qquad (E1.4.1)$$

in which ρ_o, ρ_a, and ρ_w denote the densities of oil, air, and water, respectively. Since the density of the air is very small compared to that of oil or water, the term containing ρ_a can be neglected. Also, $p_1 = p_2$, because both are equal to atmospheric pressure. Equation (E1.4.1) can then be solved for the specific gravity s_o of the oil:

$$s_o = \frac{\rho_o}{\rho_w} = \frac{h_4 - h_3}{h_1} = \frac{3.0 - 1.0}{2.5} = 0.80. \qquad \square$$

Pressure variations in a gas.   For a gas, the density is no longer constant, but is a function of pressure (and of temperature—although temperature variations are usually less significant than those of pressure), and there are two approaches:

1. For *small* changes in elevation, the assumption of constant density can still be made, and equations similar to Eqn. (1.34) are still approximately valid.
2. For moderate or large changes in elevation, the density in Eqn. (1.30) is given by Eqn. (1.7) or (1.8), $\rho = M_w p/RT$ or $\rho = M_w p/ZRT$, depending on whether

Example 1.5—Pressure Variations in a Gas   **31**

the gas is ideal or nonideal. It is understood that *absolute* pressure and temperature must *always* be used whenever the gas law is involved. A separation of variables can still be made, followed by integration, but the result will now be more complicated because the term dp/p occurs, leading—at the simplest (for an isothermal situation)—to a decreasing *exponential* variation of pressure with elevation.

Example 1.5—Pressure Variations in a Gas

For a gas of molecular weight M_w (such as the earth's atmosphere), investigate how the pressure p varies with elevation z if $p = p_0$ at $z = 0$. Assume that the temperature T is constant. What approximation may be made for *small* elevation increases? Explain how you would proceed for the nonisothermal case, in which $T = T(z)$ is a known function of elevation.

Solution

Assuming ideal gas behavior, Eqns. (1.30) and (1.7) give:

$$\frac{dp}{dz} = -\rho g = -\frac{M_w p}{RT} g. \tag{E1.5.1}$$

Separation of variables and integration between appropriate limits yields:

$$\int_{p_0}^{p} \frac{dp}{p} = \ln \frac{p}{p_0} = -\int_{0}^{z} \frac{M_w g}{RT}\, dz = -\frac{M_w g}{RT} \int_{0}^{z} dz = -\frac{M_w g z}{RT}, \tag{E1.5.2}$$

since $M_w g / RT$ is constant. Hence, there is an exponential decrease of pressure with elevation, as shown in Fig. E1.5:

$$p = p_0 \exp\left(-\frac{M_w g}{RT} z \right). \tag{E1.5.3}$$

Since a Taylor's expansion gives $e^{-x} = 1 - x + x^2/2 - \ldots$, the pressure is *approximated* by:

$$p \doteq p_0 \left[1 - \frac{M_w g}{RT} z + \left(\frac{M_w g}{RT} \right)^2 \frac{z^2}{2} \right]. \tag{E1.5.4}$$

For *small* values of $M_w g z / RT$, the last term is an insignificant second-order effect (compressibility effects are unimportant), and we obtain:

$$p \doteq p_0 - \frac{M_w p_0}{RT} g z = p_0 - \rho_0 g z, \tag{E1.5.5}$$

in which ρ_0 is the density at elevation $z = 0$; this approximation—essentially one of constant density—is shown as the dashed line in Fig. E1.5 and is clearly

applicable only for a small change of elevation. Problem 1.19 investigates the upper limit on z for which this linear approximation is realistic. If there are significant elevation changes—as in Problems 1.16 and 1.30—the approximation of Eqn. (E1.5.5) *cannot* be used with any accuracy. Observe with caution that the Taylor's expansion is only a vehicle for demonstrating what happens for small values of $M_w gz/RT$. Actual calculations for larger values of $M_w gz/RT$ should be made using Eqn. (E1.5.3), *not* Eqn. (E1.5.4).

$$p = p_0 - \rho_0 gz$$

Exact variation of pressure
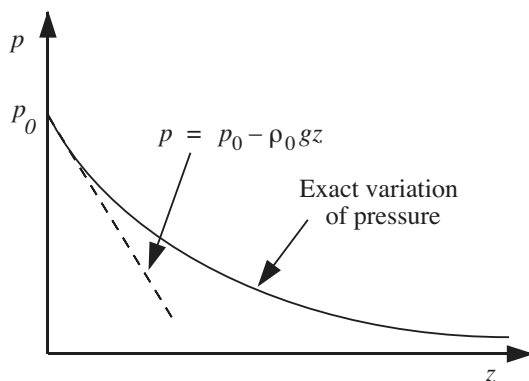
Fig. E1.5   Variation of gas pressure with elevation.

For the case in which the temperature is not constant, but is a known function $T(z)$ of elevation (as might be deduced from observations made by a meteorological balloon), it must be included *inside* the integral:

$$\int_{p_1}^{p_2} \frac{dp}{p} = -\frac{M_w g}{R} \int_0^z \frac{dz}{T(z)}. \tag{E1.5.6}$$

Since $T(z)$ is unlikely to be a simple function of z, a numerical method—such as Simpson's rule in Appendix A—will probably have to be used to approximate the second integral of Eqn. (E1.5.6). $\square$

Total force on a dam or lock gate.   Fig. 1.15 shows the side and end elevations of a dam or lock gate of depth D and width W. An expression is needed for the total horizontal force F exerted by the liquid on the dam, so that the latter can be made of appropriate strength. Similar results would apply for liquids in storage tanks. Gauge pressures are used for simplicity, with $p = 0$ at the free surface and in the air outside the dam. Absolute pressures could also be employed, but would merely add a constant atmospheric pressure everywhere, and would eventually be canceled out. If the coordinate z is measured from the bottom of the liquid upward, the corresponding *depth* of a point below the free surface is $D - z$. Hence, from Eqn. (1.34), the differential horizontal force dF on an infinitesimally small rectangular strip of area $dA = W\,dz$ is:

$$dF = pW\,dz = \rho g(D - z)W\,dz. \tag{1.35}$$
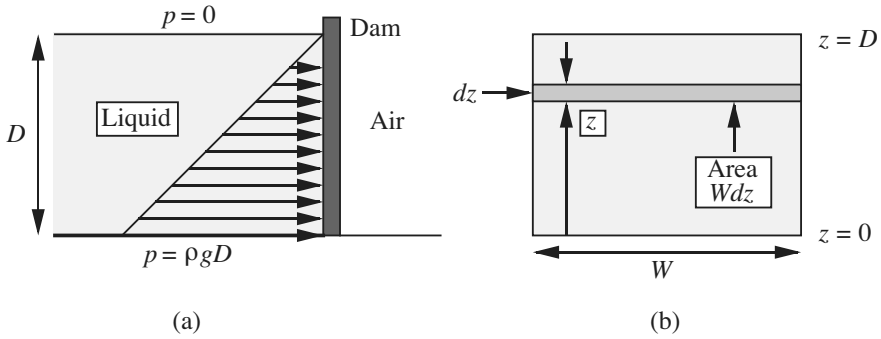
Fig. 1.15   Horizontal thrust on a dam:
(a) side elevation, (b) end elevation.

Integration from the bottom ($z = 0$) to the top ($z = D$) of the dam gives the total horizontal force:

$$F = \int_0^F dF = \int_0^D \rho g W (D - z)\, dz = \frac{1}{2} \rho g W D^2. \tag{1.36}$$

Horizontal pressure force on an arbitrary plane vertical surface. The preceding analysis was for a regular shape. A more general case is illustrated in Fig. 1.16, which shows a *plane* vertical surface of *arbitrary* shape. Note that it is now slightly easier to work in terms of a *downward* coordinate h.
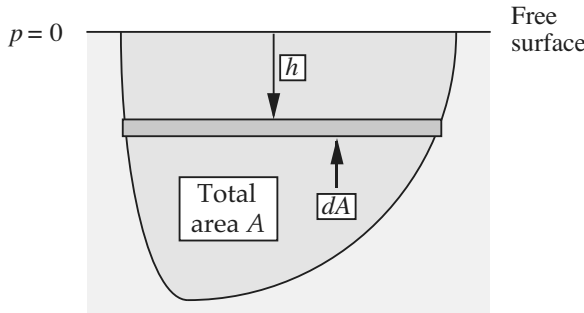
Fig. 1.16   Side view of a pool of liquid
with a submerged vertical surface.

Again taking gauge pressures for simplicity (the gas law is not involved), with $p = 0$ at the free surface, the total horizontal force is:

$$F = \int_A p\, dA = \int_A \rho g h\, dA = \rho g A \frac{\int_A h\, dA}{A}. \tag{1.37}$$

But the depth h_c of the centroid of the surface is defined as:

$$h_c \equiv \frac{\int_A h\, dA}{A}. \tag{1.38}$$

Thus, from Eqns. (1.37) and (1.38), the total force is:

$$F = \rho g h_c A = p_c A, \qquad (1.39)$$

in which p_c is the pressure at the centroid.

The advantage of this approach is that the location of the centroid is already known for several geometries. For example, for a rectangle of depth D and width W:

$$h_c = \frac{1}{2}D \quad \text{and} \quad F = \frac{1}{2}\rho g W D^2, \qquad (1.40)$$

in agreement with the earlier result of Eqn. (1.36). Similarly, for a vertical circle that is just submerged, the depth of the centroid equals its radius. And, for a vertical triangle with one edge coincident with the surface of the liquid, the depth of the centroid equals one-third of its altitude.
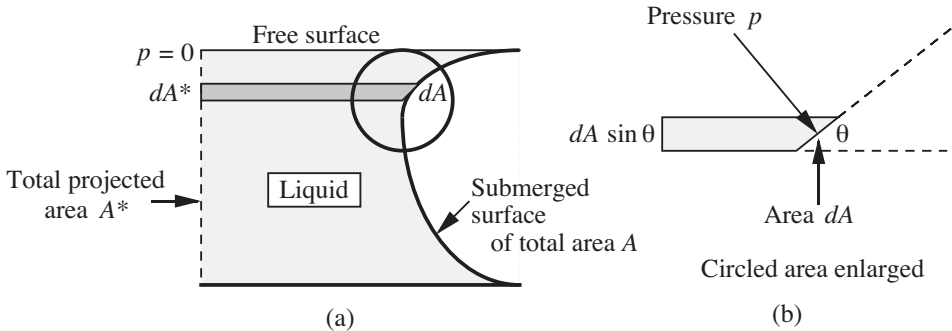
Fig. 1.17 *Thrust on surface of uniform cross-sectional shape.*

Horizontal pressure force on a curved surface.   Fig. 1.17(a) shows the cross section of a submerged surface that is no longer plane. However, the shape *is* uniform normal to the plane of the diagram.

In general, as shown in Fig. 1.17(b), the local pressure force $p\,dA$ on an element of surface area dA does not act horizontally; therefore, its horizontal component must be obtained by projection through an angle of $(\pi/2 - \theta)$, by multiplying by $\cos(\pi/2 - \theta) = \sin\theta$. The total horizontal force F is then:

$$F = \int_A p \sin\theta \, dA = \int_{A^*} p \, dA^*, \qquad (1.41)$$

in which $dA^* = dA\sin\theta$ is an element of the projection of A onto the hypothetical vertical plane A*. The integral of Eqn. (1.41) can be obtained readily, as illustrated in the following example.

Example 1.6—Hydrostatic Force on a Curved Surface   **35**

Example 1.6—Hydrostatic Force on a Curved Surface

A submarine, whose hull has a circular cross section of diameter D, is just submerged in water of density ρ, as shown in Fig. E1.6. Derive an equation that gives the total horizontal force F_x on the left half of the hull, for a distance W normal to the plane of the diagram. If $D = 8$ m, the circular cross section continues essentially for the total length $W = 50$ m of the submarine, and the density of sea water is $\rho = 1,026$ kg/m^3, determine the total horizontal force on the left-hand half of the hull.

Solution

The force is obtained by evaluating the integral of Eqn. (1.41), which is identical to that for the rectangle in Fig. 1.15:

$$F_x = \int_{A^*} p\, dA = \int_{z=0}^{z=D} \rho g W (D - z)\, dz = \frac{1}{2} \rho g W D^2. \qquad \text{(E1.6.1)}$$

Insertion of the numerical values gives:

$$F_x = \frac{1}{2} \times 1{,}026 \times 9.81 \times 50 \times 8.0^2 = 1.61 \times 10^7 \text{ N}. \qquad \text{(E1.6.2)}$$
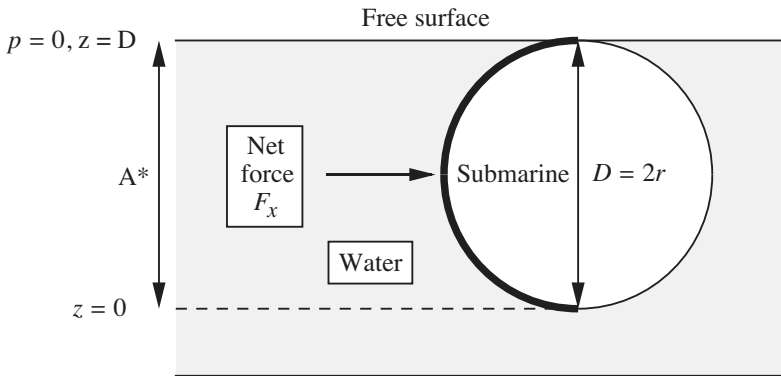
Fig. E1.6   *Submarine just submerged in seawater.*

Thus, the total force is considerable—about 3.62×10^6 lb$_\text{f}$. □
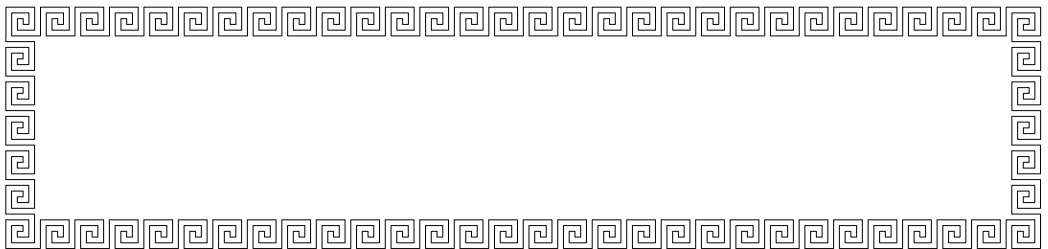
Archimedes, ca. 287–212 B.C. Archimedes was a Greek mathematician and inventor. He was born in Syracuse, Italy, where he spent much of his life, apart from a period of study in Alexandria. He was much more interested in mathematical research than any of the ingenious inventions that made him famous. One invention was a "burning mirror," which focused the sun's rays to cause intense heat. Another was the rotating Archimedean screw, for raising a continuous stream of water. Presented with a crown supposedly of pure gold, Archimedes tested the possibility that it might be "diluted" by silver by separately immersing the crown and an equal weight of pure gold into his bath, and observed the difference in the overflow. Legend has it that he was so excited by the result that he ran home without his clothes, shouting "εὕρηκα, εὕρηκα", "I have found it, I have found it." To dramatize the effect of a lever, he said, "Give me a place to stand, and I will move the earth." He considered his most important intellectual contribution to be the determination of the ratio of the volume of a sphere to the volume of the cylinder that circumscribes it. [Now that calculus has been invented, the reader might like to derive this ratio!] Sadly, Archimedes was killed during the capture of Syracuse by the Romans.

Source: *The Encyclopædia Britannica*, 11th ed., Cambridge University Press (1910–1911).

Buoyancy forces.   If an object is submerged in a fluid, it will experience a net upward or *buoyant* force exerted by the fluid. To find this force, first examine the buoyant force on a submerged circular cylinder of height H and cross-sectional area A, shown in Fig. 1.18.
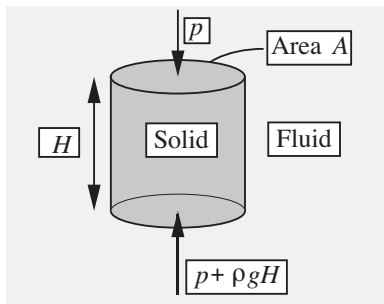
Fig. 1.18   Pressure forces on a submerged cylinder.

The forces on the curved vertical surface act horizontally and may therefore be ignored. Hence, the net *upward* force due to the difference between the opposing pressures on the bottom and top faces is:

$$F = (p + \rho gH - p)A = \rho HAg, \tag{1.42}$$

Example 1.7—Application of Archimedes' Law   **37**

which is exactly the weight of the displaced liquid, thus verifying *Archimedes' law*, (the buoyant force equals the weight of the fluid displaced) for the cylinder. The same result would clearly be obtained for a cylinder of any uniform cross section.
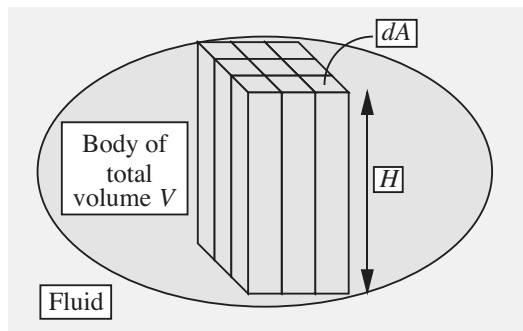
Fig. 1.19   *Buoyancy force for an arbitrary shape.*

Fig. 1.19 shows a more general situation, with a body of arbitrary shape. However, Archimedes' law still holds since the body can be decomposed into an infinitely large number of vertical rectangular parallelepipeds or "boxes" of infinitesimally small cross-sectional area dA. The effect for one box is then summed or "integrated" over all the boxes, and again gives the net upward buoyant force as the weight of the liquid displaced.

Example 1.7—Application of Archimedes' Law

Consider the situation in Fig. E1.7(a), in which a barrel rests on a raft that floats in a swimming pool. The barrel is then pushed off the raft, and may either float or sink, depending on its contents and hence its mass. The cross-hatching shows the volumes of water that are displaced. For each of the cases shown in Fig. E1.7 (b) and (c), determine whether the water level in the pool will rise, fall, or remain constant, relative to the initial level in (a).

(a) Initial   (b) Final (light barrel)   (c) Final (heavy barrel)
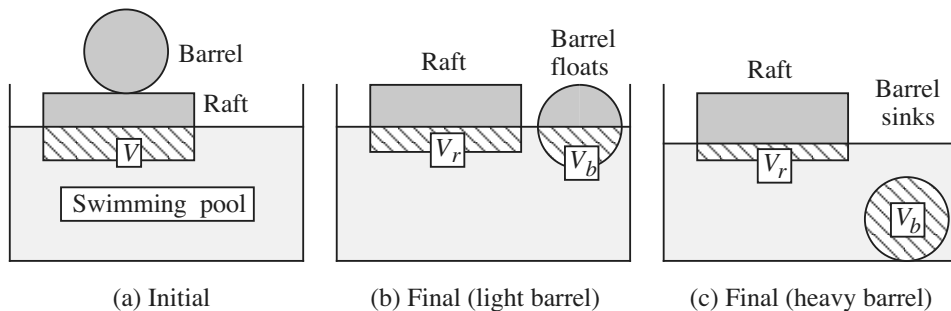
Fig. E1.7   *Raft and barrel in swimming pool: (a) initial positions, (b) light barrel rolls off and floats, (c) heavy barrel rolls off and sinks. The cross-hatching shows volumes below the surface of the water.*

Solution

Initial state. Let the masses of the raft and barrel be M_r and M_b, respectively. If the volume of displaced water is initially V in (a), Archimedes' law requires that the total weight of the raft and barrel equals the weight of the displaced water, whose density is ρ:

$$(M_r + M_b)g = V\rho g. \tag{E1.7.1}$$

Barrel floats. If the barrel floats, as in (b), with submerged volumes of V_r and V_b for the raft and barrel, respectively, Archimedes' law may be applied to the raft and barrel separately:

$$\text{Raft}: \ M_r g = V_r \rho g, \qquad \text{Barrel}: \ M_b g = V_b \rho g. \tag{E1.7.2}$$

Addition of the two equations (E1.7.2) and comparison with Eqn. (E1.7.1) shows that:

$$V_r + V_b = V. \tag{E1.7.3}$$

Therefore, since the volume of the water is constant, and the total displaced volume does not change, the level of the surface also remains *unchanged*.

Barrel sinks. Archimedes' law may still be applied to the raft, but the weight of the water displaced by the barrel no longer suffices to support the weight of the barrel, so that:

$$\text{Raft}: \ M_r g = V_r \rho g, \qquad \text{Barrel}: \ M_b g > V_b \rho g. \tag{E1.7.4}$$

Addition of the two relations in (E1.7.4) and comparison with Eqn. (E1.7.1) shows that:

$$V_r + V_b < V. \tag{E1.7.5}$$

Therefore, since the volume of the water in the pool is constant, and the total displaced volume is *reduced*, the level of the surface *falls*. This result is perhaps contrary to intuition: since the *whole* volume of the barrel is submerged in (c), it might be thought that the water level will rise above that in (b). However, because the barrel must be heavy in order to sink, the load on the raft and hence V_r are substantially reduced, so that the total displaced volume is also reduced.

This problem illustrates the need for a complete analysis rather than jumping to a possibly erroneous conclusion. ☐
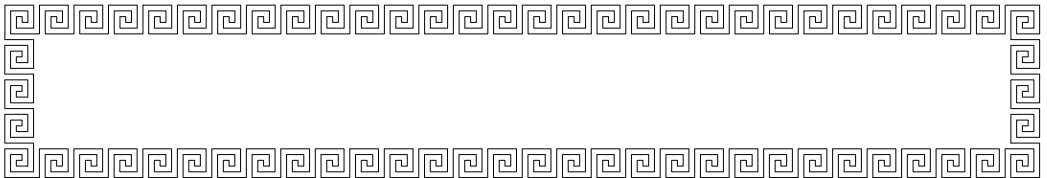
1.7   Pressure Change Caused by Rotation

Finally, consider the shape of the free surface for the situation shown in Fig. 1.20(a), in which a cylindrical container, partly filled with liquid, is rotated with an angular velocity ω—that is, at $N = \omega/2\pi$ revolutions per unit time. The analysis has applications in fuel tanks of spinning rockets, centrifugal filters, and liquid mirrors.
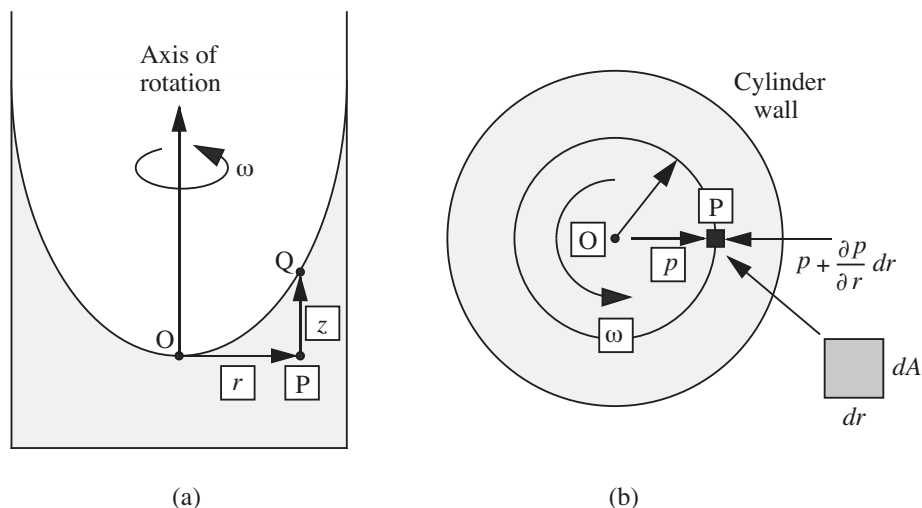
Fig. 1.20   *Pressure changes for rotating cylinder: (a) elevation, (b) plan.*

Point O denotes the origin, where $r = 0$ and $z = 0$. After a sufficiently long time, the rotation of the container will be transmitted by viscous action to the liquid, whose rotation is called a *forced vortex*. In fact, the liquid spins as if it were a *solid body*, rotating with a uniform angular velocity ω, so that the velocity in the direction of rotation at a radial location r is given by $v_\theta = r\omega$. It is therefore appropriate to treat the situation similar to the hydrostatic investigations already made.

Suppose that the liquid element P is essentially a rectangular box with cross-sectional area dA and radial extent dr. (In reality, the element has slightly tapering sides, but a more elaborate treatment taking this into account will yield identical results to those derived here.) The pressure on the inner face is p, whereas that on the outer face is $p + (\partial p/\partial r)dr$. Also, for uniform rotation in a circular path of radius r, the acceleration toward the center O of the circle is $r\omega^2$. Newton's second law of motion is then used for equating the net pressure force toward O to the mass of the element times its acceleration:

$$\underbrace{\left(p + \frac{\partial p}{\partial r}dr - p\right) dA}_{\text{Net pressure force}} = \underbrace{\rho(dA\,dr)}_{\text{Mass}}\,r\omega^2. \tag{1.43}$$

Note that the use of a *partial* derivative is essential, since the pressure now varies in both the horizontal (radial) *and* vertical directions. Simplification yields the variation of pressure in the radial direction:

$$\frac{\partial p}{\partial r} = \rho r \omega^2, \tag{1.44}$$

so that pressure *increases* in the radially outward direction.

Observe that the gauge pressure at all points on the interface is zero; in particular, $p_O = p_Q = 0$. Integrating from points O to P (at constant z):

$$\int_{p=0}^{p_P} dp = \rho \omega^2 \int_0^r r \, dr,$$

$$p_P = \frac{1}{2}\rho\omega^2 r^2. \tag{1.45}$$

However, the pressure at P can also be obtained by considering the usual hydrostatic increase in traversing the path QP:

$$p_P = \rho g z. \tag{1.46}$$

Elimination of the intermediate pressure p_P between Eqns. (1.45) and (1.46) relates the elevation of the free surface to the radial location:

$$z = \frac{\omega^2 r^2}{2g}. \tag{1.47}$$

Thus, the free surface is *parabolic* in shape; observe also that the density is not a factor, having been canceled from the equations.

There is another type of vortex—the *free* vortex—that is also important, in cyclone dust collectors and tornadoes, for example, as discussed in Chapters 4 and 7. There, the velocity in the angular direction is given by $v_\theta = c/r$, where c is a constant, so that v_θ is inversely proportional to the radial position.

Example 1.8—Overflow from a Spinning Container

A cylindrical container of height H and radius a is initially half-filled with a liquid. The cylinder is then spun steadily around its vertical axis Z-Z, as shown in Fig. E1.8. At what value of the angular velocity ω will the liquid just start to spill over the top of the container? If $H = 1$ ft and $a = 0.25$ ft, how many rpm (revolutions per minute) would be needed?
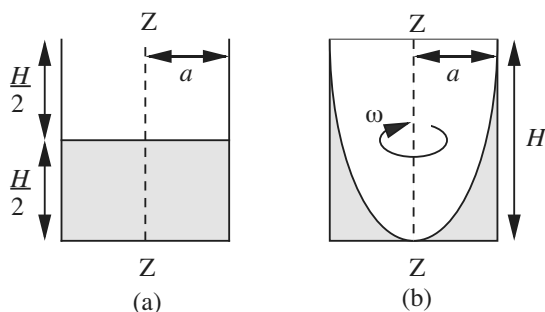
Fig. E1.8   Geometry of a spinning container:
(a) at rest, (b) on the point of overflowing.

Solution

From Eqn. (1.47), the shape of the free surface is a parabola. Therefore, the air inside the rotating cylinder forms a paraboloid of revolution, whose volume is known from calculus to be exactly one-half of the volume of the "circumscribing cylinder," namely, the container.[8] Hence, the liquid at the center reaches the bottom of the cylinder *just* as the liquid at the curved wall reaches the top of the cylinder. In Eqn. (1.47), therefore, set $z = H$ and $r = a$, giving the required angular velocity:

$$\omega = \sqrt{\frac{2gH}{a^2}}.$$

For the stated values:

$$\omega = \sqrt{\frac{2 \times 32.2 \times 1}{0.25^2}} = 32.1 \ \frac{\text{rad}}{\text{s}}, \qquad N = \frac{\omega}{2\pi} = \frac{32.1 \times 60}{2\pi} = 306.5 \ \text{rpm.} \qquad \square$$

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[8] Proof can be accomplished as follows. First, note for the parabolic surface in Fig. E1.8(b), $r = a$ when $z = H$, so, from Eqn. (1.47), $\omega^2/2g = H/a^2$. Thus, Eqn. (1.47) can be rewritten as:

$$z = H\frac{r^2}{a^2}.$$

The volume of the paraboloid of air within the cylinder is therefore:

$$V = \int_{z=0}^{z=H} \pi r^2 \, dz = \int_{z=0}^{z=H} \frac{\pi a^2 z}{H} \, dz = \frac{1}{2}\pi a^2 H,$$

which is exactly one-half of the volume of the cylinder, $\pi a^2 H$. Since the container was initially just half filled, the liquid volume still accounts for the remaining half.

PROBLEMS FOR CHAPTER 1

1. *Units conversion—E*. How many cubic feet are there in an acre-foot? How many gallons? How many cubic meters? How many tonnes of water?

2. *Units conversion—E*. The viscosity μ of an oil is 10 cP and its specific gravity s is 0.8. Reexpress both of these (the latter as density ρ) in both the lb_m, ft, s system and in SI units.

3. *Units conversion—E*. Use conversion factors to express: (a) the gravitational acceleration of 32.174 ft/s^2 in SI units, and (b) a pressure of 14.7 lb$_f$/in^2 (one atmosphere) in both pascals and bars.

4. *Meteorite density—E*. The Barringer Crater in Arizona was formed 30,000 years ago by a spherical meteorite of diameter 60 m and mass 10^6 t (tonnes), traveling at 15 km/s when it hit the ground.[9] (Clearly, all figures are *estimates*.) What was the mean density of the meteorite? What was the predominant material in the meteorite? Why? If one tonne of the explosive TNT is equivalent to five billion joules, how many tonnes of TNT would have had the same impact as the meteorite?

5. *Reynolds number—E*. What is the mean velocity u_m (ft/s) and the Reynolds number Re $= \rho u_m D/\mu$ for 35 gpm (gallons per minute) of water flowing in a 1.05-in. I.D. pipe if its density is $\rho = 62.3$ lb$_m$/ft^3 and its viscosity is $\mu = 1.2$ cP? What are the units of the Reynolds number?

6. *Pressure in bubble—E*. Consider a *soap-film bubble* of diameter d. If the external air pressure is p_a, and the surface tension of the soap film is σ, derive an expression for the pressure p_b inside the bubble. *Hint*: Note that there are *two* air/liquid interfaces.
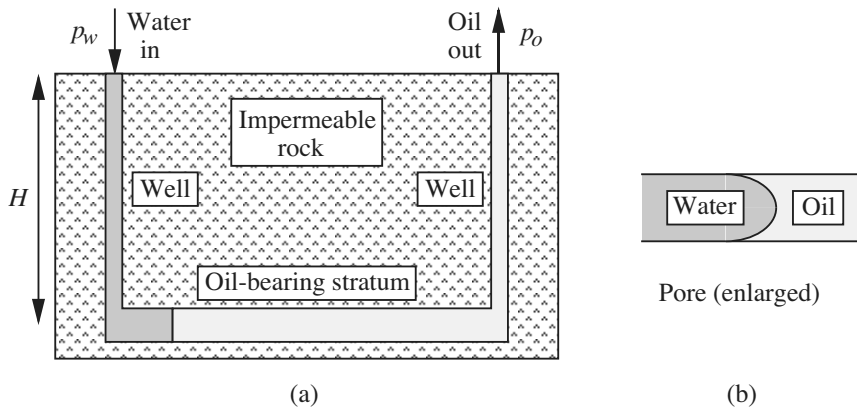
Fig. P1.7   *Waterflooding of an oil reservoir.*

[9] Richard A.F. Grieve, "Impact cratering on the earth," *Scientific American*, Vol. 262, No. 4, p. 68 (1990).

7. *Reservoir waterflooding—E*. Fig. P1.7(a) shows how water is pumped down one well, of depth H, into an oil-bearing stratum, so that the displaced oil then flows up through another well. Fig. P1.7(b) shows an enlargement of an idealized pore, of diameter d, at the water/oil interface.

If the water and oil are *just* starting to move, what water inlet pressure p_w is needed if the oil exit pressure is to be p_o? Assume that the oil completely wets the pore (*not* always the case), that the water/oil interfacial tension is σ, and that the densities of the water and oil are ρ_w and ρ_o, respectively.[10]

8. *Barometer reading—M*. In your house (elevation 950 ft above sea level) you have a barometer that registers inches of mercury. On an average day in January, you telephone the weather station (elevation 700 ft) and are told that the exact pressure there is 0.966 bar. What is the correct reading for your barometer, and to how many psia does this correspond? The specific gravity of mercury is 13.57.
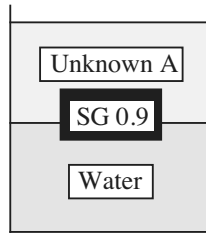
Fig. P1.9   Cylinder immersed in water and liquid A.

9. *Two-layer buoyancy—E*. As shown in Fig. P1.9, a layer of an unknown liquid A (immiscible with water) floats on top of a layer of water W in a beaker. A completely submerged cylinder of specific gravity 0.9 adjusts itself so that its axis is vertical and two-thirds of its height projects above the A/W interface and one-third remains below. What is the specific gravity of A? Solve the problem two ways—first using Archimedes' law, and then using a momentum or force balance.
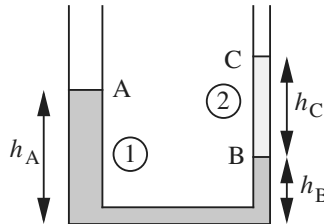
Fig. P1.10   U-tube with immiscible liquids.

10. *Differential manometer—E*. The U-tube shown in Fig. P1.10 has legs of *unequal* internal diameters d_1 and d_2, which are partly filled with immiscible liquids of densities ρ_1 and ρ_2, respectively, and are open to the atmosphere at the top.

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[10] D.L. Katz et al., *Handbook of Natural Gas Engineering*, McGraw-Hill, New York, 1959, p. 57, indicates a wide range of wettability by water, varying greatly with the particular rock formation.

If an *additional* small volume v_2 of the second liquid is added to the right-hand leg, derive an expression—in terms of ρ_1, ρ_2, v_2, d_1, and d_2—for δ, the amount by which the level at B will fall. If ρ_1 is known, but ρ_2 is unknown, could the apparatus be used for determining the density of the second liquid?

Hints: The lengths h_A, h_B, and h_C have been included just to get started; they must not appear in the final result. After adding the second liquid, consider h_C to have increased by a length Δ—a quantity that must also eventually be eliminated.
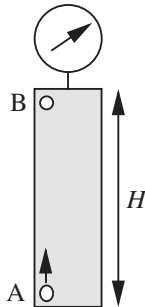
Fig. P1.11   *Bubble rising in a closed cylinder.*

11. *Ascending bubble—E.* As shown in Fig. P1.11, a hollow vertical cylinder with *rigid* walls and of height H is closed at both ends, and is filled with an *incompressible* oil of density ρ. A gauge registers the pressure at the top of the cylinder. When a small bubble of volume v_0 initially adheres to point A at the bottom of the cylinder, the gauge registers a pressure p_0. The gas in the bubble is ideal, and has a molecular weight of $M_{\rm w}$. The bubble is liberated by tapping on the cylinder and rises to point B at the top. The temperature T is constant throughout. Derive an expression in terms of any or all of the specified variables for the new pressure-gauge reading p_1 at the top of the cylinder.

12. *Ship passing through locks—M.* A ship of mass M travels uphill through a series of identical rectangular locks, each of equal superficial (bird's-eye view) area A and elevation change h. The steps involved in moving from one lock to the next (1 to 2, for example) are shown as A–B–C in Fig. P1.12. The lock at the top of the hill is supplied by a source of water. The initial depth in lock 1 is H, and the density of the water is ρ.

(a) Derive an expression for the increase in mass of water in lock 1 for the sequence shown in terms of some or all of the variables M, H, h, A, ρ, and g.
(b) If, after reaching the top of the hill, the ship descends through a similar series of locks to its original elevation, again derive an expression for the mass of water gained by a lock from the lock immediately above it.
(c) Does the mass of water to be supplied depend on the mass of the ship if: (i) it travels only uphill, (ii) it travels uphill, then downhill? Explain your answer.
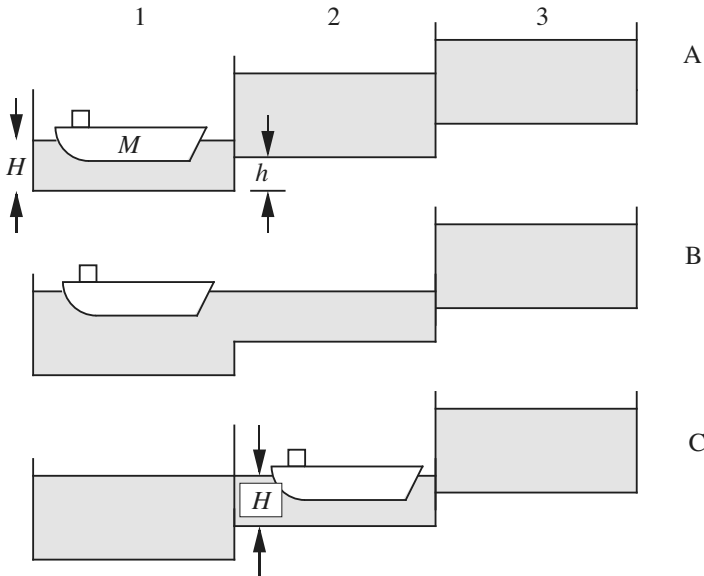
Fig. P1.12 Ship and locks.

13. *Furnace stack—E*. Air ($\rho_a = 0.08$ lb$_m$/ft^3) flows through a furnace where it is burned with fuel to produce a hot gas ($\rho_g = 0.05$ lb$_m$/ft^3) that flows up the stack, as in Fig. P1.13. The pressures in the gas and the immediately surrounding air at the top of the stack at point A are equal.
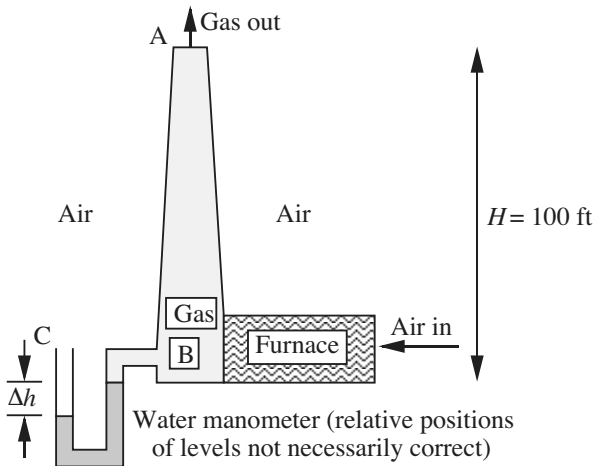
Fig. P1.13 Furnace stack.

What is the difference Δh (in.) in levels of the water in the manometer connected between the base B of the stack and the outside air at point C? Which side rises? Except for the pressure drop across the furnace (which you need not

worry about), treat the problem as one in hydrostatics. That is, ignore any frictional effects and kinetic energy changes in the stack. Also, neglect compressibility effects.
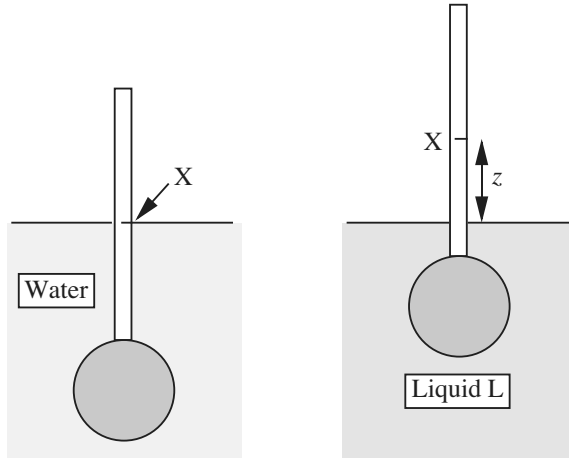
Fig. P1.14   Hydrometer in water and test liquid L.

14. *Hydrometer—E*. When a hydrometer floats in water, its cylindrical stem is submerged so that a certain point X on the stem is level with the free surface of the water, as shown in Fig. P1.14. When the hydrometer is placed in another liquid L of specific gravity s, the stem *rises* so that point X is now a height z above the free surface of L.

Derive an equation giving s in terms of z. If needed, the cross-sectional area of the stem is A, and when in water a total volume V (stem plus bulb) is submerged.
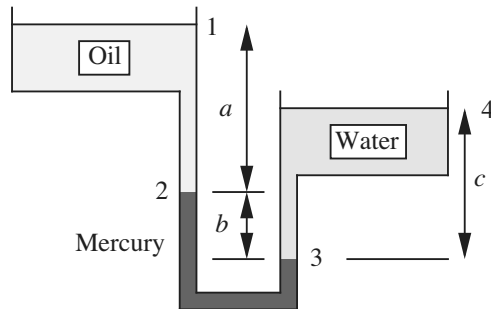
Fig. P1.15   Oil/mercury/water system.

15. *Three-liquid manometer—E*. In the hydrostatic case shown in Fig. P1.15, $a = 6$ ft and $c = 4$ ft. The specific gravities of oil, mercury, and water are $s_o = 0.8$, $s_m = 13.6$, and $s_w = 1.0$. Pressure variations in the air are negligible. What is the difference b in inches between the mercury levels, and which leg of the manometer has the higher mercury level? *Note*: In this latter respect, the diagram may or may not be correct.

16. *Pressure on Mt. Erebus—M*. On page 223 of the biography *Shackleton* (by Roland Huntford, Atheneum, New York, 1986), the Antarctic explorer's colleague, Edward Marshall, is reported as having " . . . fixed the altitude [of Mt. Erebus] by hypsometer. This was simply a small cylinder in which distilled water was boiled and the temperature measured. It was then the most accurate known method of measuring altitude. The summit of Erebus turned out to be 13,500 feet above sea level."[11]

Assuming a uniform (mean) air temperature of $-5\,°F$ (the summer *summit* temperature is $-30\,°F$), and a sea-level pressure of 13.9 psia, at what temperature did the water boil in the hypsometer? At temperatures $T = 160, 170, 180, 190,$ 200, and $210\,°F$, the respective vapor pressures of water are $p_v = 4.741, 5.992,$ 7.510, 9.339, 11.526, and 14.123 psia.

17. *Oil and gas well pressures—M*. A pressure gauge at the top of an oil well 18,000 ft deep registers 2,000 psig. The bottom 4,000-ft portion of the well is filled with oil ($s = 0.70$). The remainder of the well is filled with natural gas ($T = 60\,°F$, compressibility factor $Z = 0.80$, and $s = 0.65$, meaning that the molecular weight is 0.65 times that of air).

Calculate the pressure (psig) at (a) the oil/gas interface, and (b) the bottom of the well.

18. *Thrust on a dam—E*. Concerning the thrust on a rectangular dam, check that Eqn. (1.36) is still obtained if, instead of employing an upward coordinate z, use is made of a *downward* coordinate h (with $h = 0$ at the free surface).

19. *Pressure variations in air—M*. Refer to Example 1.5 concerning the pressure variations in a gas, and assume that you are dealing with air at $40\,°F$. Suppose further that you are using just the linear part of the expansion (up to the term in z) to calculate the absolute pressure at an elevation z above ground level. How large can z be, in miles, with the knowledge that the error amounts to no more than 1% of the exact value?

20. *Grand Coulee dam—E*. The Grand Coulee dam, which first operated in 1941, is 550 ft high and 3,000 ft wide. What is the pressure at the base of the dam, and what is the total horizontal force F lb$_f$ exerted on it by the water upstream?

21. *Force on V-shaped dam—M*. A vertical dam has the shape of a V that is 3 m high and 2 m wide at the top, which is just level with the surface of the water upstream of the dam. Use two different methods to determine the total force (N) exerted by the water on the dam.

22. *Rotating mercury mirror—M*. Physicist Ermanno Borra, of Laval University in Quebec, has made a 40-in. diameter telescopic mirror from a pool of

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[11] A more recent value is thought to be 12,450 feet.

mercury that rotates at one revolution every six seconds.[12] (Air bearings elimi-nate vibration, and a thin layer of oil prevents surface ripples.)

By what value Δz would the surface at the center be depressed relative to the perimeter, and what is the focal length (m) of the mirror? The mirror cost Borra $7,500. He estimated that a similar 30-meter mirror could be built for $7.5 million. If the focal length were unchanged, what would be the new value of Δz for the larger mirror? *Hint*: the equation for a parabola of focal length f is $r^2 = 4fz$.

23. *Oil and water in rotating container—E.* A cylindrical container partly filled with immiscible layers of water and oil is placed on a rotating turntable. Develop the necessary equations and prove that the shapes of the oil/air and water/oil interfaces are identical.

24. *Energy to place satellite in orbit—M.* "NASA launched a $195 million astronomy satellite at the weekend to probe the enigmatic workings of neutron stars, black holes, and the hearts of galaxies at the edge of the universe . . . The long-awaited mission began at 8:48 a.m. last Saturday when the satellite's Delta–2 rocket blasted off from the Cape Canaveral Air Station."[13]

This "X-ray Timing Explorer satellite" was reported as having a mass of 6,700 lb$_\mathrm{m}$ and being placed 78 minutes after lift-off into a 360-mile-high circular orbit (measured above the earth's surface).

How much energy (J) went directly to the satellite to place it in orbit? What was the corresponding average power (kW)? The force of attraction between a mass m and the mass M_e of the earth is GmM_e/r^2, where r is the distance of the mass from the center of the earth and G is the universal gravitational constant. The value of G is *not* needed in order to solve the problem, as long as you remember that the radius of the earth is 6.37×10^6 m, and that $g = 9.81$ m/s^2 at its surface.

25. *Central-heating loop—M.* Fig. P1.25 shows a piping "loop" that circulates hot water through the system ABCD in order to heat two floors of a house by means of baseboard fins attached to the horizontal runs of pipe (BC and DA). The horizontal and vertical portions of the pipes have lengths L and H, respectively.

The water, which has a mean density of $\overline{\rho}$ and a volume coefficient of expansion α, circulates by the action of natural convection due to a small heater, whose inlet and outlet water temperatures are T_1 and T_2, respectively. The pressure drop due to friction per unit length of piping is cu^2/D, where c is a known constant, u is the mean water velocity, and D is the internal diameter of the pipe. You may assume that the vertical legs AB and CD are insulated, and that equal amounts of heat are dissipated on each floor.

Derive an expression that gives the volumetric circulation rate of water, Q, in terms of c, D, $\overline{\rho}$, α, g, L, H, T_1, and T_2.

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[12] *Scientific American,* February 1994, pp. 76–81. There is also earlier mention of his work in *Time,* December 15, 1986.

[13] *Manchester Guardian Weekly*, January 7, 1996.
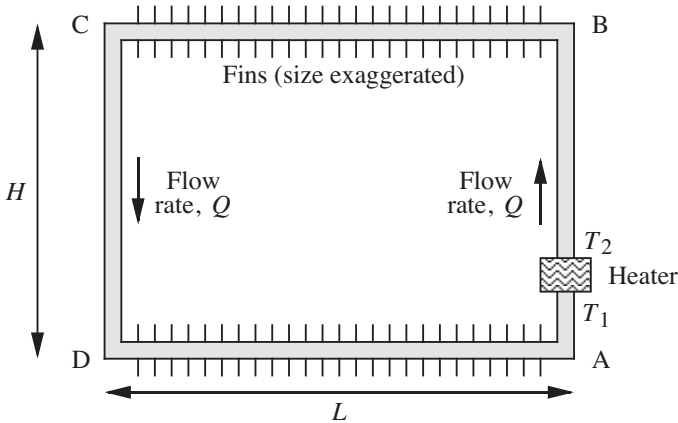
Fig. P1.25   Central-heating loop.

26. *Pressure at the center of the earth—M.* Prove that the pressure at the center of the earth is given by $p_c = 3Mg_s/8\pi R^2$, in which g_s is the gravitational acceleration at the surface, M is the mass of the earth, and R is its radius. *Hints:* Consider a small mass m inside the earth, at a radius r from the center. The force of attraction mg_r (where g_r is the *local* gravitational acceleration) between m and the mass M_r enclosed *within* the radius r is GmM_r/r^2, where G is the universal gravitational constant. Repeat for the mass at the surface, and hence show that $g_r/g_s = r/R$. Then invoke hydrostatics.

If the radius of the earth is $R = 6.37 \times 10^6$ m, and its mean density is approximately 5,500 kg/m^3, estimate p_c in Pa and psi.
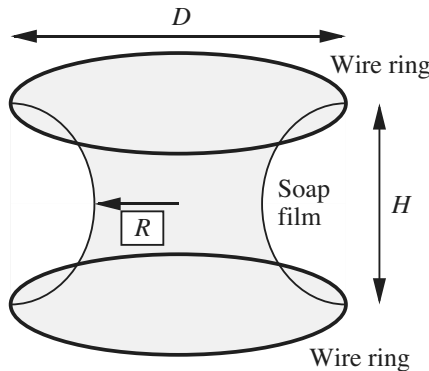
Fig. P1.27   Soap film on two rings.

27. *Soap film on wire rings—M.* As shown in Fig. P1.27, a soap film is stretched between two wire rings, each of diameter D and separated by a distance H. Prove that the radius R of the film at its narrowest point is:

$$R = \frac{1}{6}\left(2D + \sqrt{D^2 - 3H^2}\right).$$

You may assume that a section of the soap film is a circular arc, and that $D \geq \sqrt{3}\,H$. What might happen if D is less than $\sqrt{3}\,H$?

Clearly stating your assumptions, derive an expression for the radius, in terms of D and H. Is your expression exact or approximate? Explain.
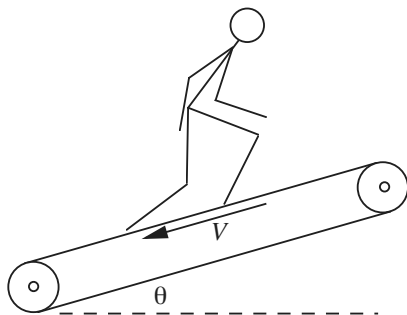
Fig. P1.28   Person on a treadmill.

28.   *Treadmill stress test—M.* What power P is needed to resist a force F at a steady velocity V? In a treadmill stress test (Fig. P1.28), you have to keep walking to keep up with a moving belt whose velocity V and angle of inclination θ are steadily increased. Initially, the belt is moving at 1.7 mph and has a grade (defined as $\tan\theta$) of 10%. The test is concluded after 13.3 min, at which stage the belt is moving at 5.0 mph and has a grade of 18%. If your mass is 163 lb_m: (a) how many HP are you exerting at the start of the test, (b) how many HP are you exerting at the end of the test, and (c) how many joules have you expended overall?

29. *Bubble rising in compressible liquid—D.* A liquid of volume V and isothermal compressibility β has its pressure increased by an amount Δp. Explain why the corresponding increase ΔV in volume is given approximately by:

$$\Delta V = -\beta V \Delta p.$$

Repeat Problem P1.11, now allowing the oil—whose density and volume are initially ρ_0 and V_0—to have a finite compressibility β. Prove that the ratio of the final bubble volume v_1 to its initial volume v_0 is:

$$\frac{v_1}{v_0} = 1 + \frac{\rho_0 g H}{p_0}.$$

If needed, assume that: (a) the bubble volume is much smaller than the oil volume, and (b) $\beta p_0 V_0 \gg v_1$. If $\rho_0 = 800$ kg/m³, $\beta = 5.5 \times 10^{-10}$ m²/N, $H = 1$ m, $p_0 = 10^5$ N/m² (initial absolute pressure at the top of the cylinder), $v_0 = 10^{-8}$ m³, and $V_0 = 0.1$ m³, evaluate v_1/v_0 and check that assumption (b) above is reasonable.
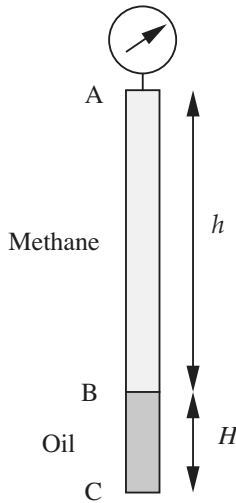
Fig. P1.30   Well containing oil and methane.

30. *Pressures in oil and gas well—M*. Fig. P1.30 shows a well that is 12,000 ft deep. The bottom $H = 2,000$-ft portion is filled with an incompressible oil of specific gravity $s = 0.75$, above which there is an $h = 10,000$-ft layer of methane (CH_4; $C = 12$, $H = 1$) at $100\,°F$, which behaves as an ideal isothermal gas whose density is *not* constant. The gas and oil are static. The density of water is 62.3 lb_m/ft^3.

(a) If the pressure gauge at the top of the well registers $p_A = 1,000$ psig, compute the *absolute* pressure p_B (psia) at the oil/methane interface. Work in terms of symbols before substituting numbers.

(b) Also compute $(p_C - p_B)$, the additional pressure (psi) in going from the interface B to the bottom of the well C.
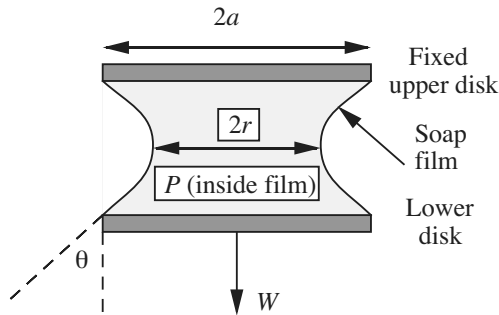
Fig. P1.31   Soap film between two disks.

31. *Soap film between disks—E (C)*. A circular disk of weight W and radius a is hung from a similar disk by a soap film with surface tension σ, as shown in Fig. P1.31. The gauge pressure inside the film is P.

First, derive an expression for the angle θ in terms of a, P, W, and σ. Then obtain an equation that relates the radius of the neck r to a, P, W, and σ. Assume that: (a) the excess pressure inside a soap film with radii of curvature r_1 and r_2 is $2\sigma(1/r_1 + 1/r_2)$, and (b) the cross section of the film forms a circular arc.

32. *Newspaper statements about the erg—E.* In the *New York Times* for January 18, 1994, the following statement appeared: "An erg is the metric unit scientists use to measure energy. One erg is the amount of energy it takes to move a mass of one gram one centimeter in one second." (This statement related to the earthquake of the previous day, measuring 6.6 on the Richter scale, in the Northridge area of the San Fernando Valley, 20 miles north of downtown Los Angeles.)

Also in the same newspaper, there was a letter of rebuttal on January 30 that stated in part: " . . . This is not correct. The energy required to move a mass through a distance does not depend on how long it takes to accomplish the movement. Thus the definition should not include a unit of time."

A later letter from another reader, on February 10, made appropriate comments about the original article and the first letter. What do you think was said in the second letter?

33. *Centroid of triangle—E.* A triangular plate held vertically in a liquid has one edge (of length B) coincident with the surface of the liquid; the altitude of the plate is H. Derive an expression for the depth of the centroid. What is the horizontal force exerted by the liquid, whose density is ρ, on one side of the plate?

34. *Blake-Kozeny equation—E.* The Blake-Kozeny equation for the pressure drop $(p_1 - p_2)$ in laminar flow of a fluid of viscosity μ through a packed bed of length L, particle diameter D_p and void fraction ε is (Section 4.4):

$$\frac{p_1 - p_2}{L} = 150 \left(\frac{\mu u_0}{D_p^2}\right) \left[\frac{(1 - \varepsilon)^2}{\varepsilon^3}\right].$$

(a) Giving your reasons, suggest appropriate units for ε.
(b) If $p_1 - p_2 = 75$ lb$_f$/in^2, $D_p = 0.1$ in., $L = 6.0$ ft, $\mu = 0.22$ P, and $u_0 = 0.1$ ft/s, compute the value of ε.

35. *Shear stresses for air and water—E.* Consider the situation in Fig. 1.8, with $h = 0.1$ cm and $V = 1.0$ cm/s. The pressure is atmospheric throughout.

(a) If the fluid is air at $20\,^\circ$C, evaluate the shear stress τ_a (dynes/cm^2). Does τ vary across the gap? Explain.
(b) Evaluate τ_w if the fluid is water at $20\,^\circ$C. What is the ratio τ_w/τ_a?
(c) If the temperature is raised to $80\,^\circ$C, does τ_a increase or decrease? What about τ_w?

36. *True/false.* Check *true* or *false*, as appropriate:[14]

(a)   When a fluid is subjected to a steady shear stress, it will reach a state of equilibrium in which no further motion occurs.   T ☐   F ☐

(b)   Pressure and shear stress are two examples of a force per unit area.   T ☐   F ☐

(c)   In fluid mechanics, the basic conservation laws are those of volume, energy, and momentum.   T ☐   F ☐

(d)   Absolute pressures and temperatures must be employed when using the ideal gas law.   T ☐   F ☐

(e)   The density of an ideal gas depends only on its absolute temperature and its molecular weight.   T ☐   F ☐

(f)   Closely, the density of water is 1,000 kg/m^3, and the gravitational acceleration is 9.81 m/s^2.   T ☐   F ☐

(g)   To convert pressure from gauge to absolute, add approximately 1.01 Pa.   T ☐   F ☐

(h)   To convert from psia to psig, add 14.7, approximately.   T ☐   F ☐

(i)   The absolute atmospheric pressure in the classroom is roughly one bar.   T ☐   F ☐

(j)   If ρ is density in g/cm^3 and μ is viscosity in g/cm s, then the kinematic viscosity $\nu = \mu/\rho$ is in stokes.   T ☐   F ☐

(k)   For a given liquid, surface tension and surface energy per unit area have identical numerical values *and* identical units.   T ☐   F ☐

(l)   A force is equivalent to a rate of transfer of momentum.   T ☐   F ☐

(m)   Work is equivalent to a rate of dissipation of power per unit time.   T ☐   F ☐

(n)   It is possible to have gauge pressures that are as low as −20.0 psig.   T ☐   F ☐

(o)   The density of air in the classroom is roughly 0.08 kg/m^3.   T ☐   F ☐

(p)   Pressure in a static fluid varies in the vertically upward direction z according to $dp/dz = -\rho g_c$.   T ☐   F ☐

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[14] Solutions to all the true/false assertions are given in Appendix B.

(q)   At any point, the rate of change of pressure with elevation is $dp/dz = -\rho g$, for both incompressible *and* compressible fluids.   T ☐   F ☐

(r)   A vertical pipe full of water, 34 ft high and open at the top, will generate a pressure of about one atmosphere (gauge) at its base.   T ☐   F ☐

(s)   The horizontal force on one side of a vertical circular disc of radius R immersed in a liquid of density ρ, with its center a distance R below the free surface, is $\pi R^3 \rho g$.   T ☐   F ☐

(t)   For a vertical rectangle or dam of width W and depth D, with its top edge submerged in a liquid of density ρ, as in Fig. 1.15, the total horizontal thrust of the liquid can also be expressed as $\int_0^D \rho g h W \, dh$, where h is the coordinate measured *downwards* from the free surface.   T ☐   F ☐

(u)   The horizontal pressure force on a rectangular dam with its top edge in the free surface is F_x. If the dam were made twice as deep, but still with the same width, the total force would be $2F_x$.   T ☐   F ☐

(v)   A solid object completely immersed in oil will experience the same upward buoyant force as when it is immersed in water.   T ☐   F ☐

(w)   Archimedes' law will not be true if the object immersed is hollow (such as an empty box with a tight lid, for example).   T ☐   F ☐

(x)   The rate of pressure change due to centrifugal action is given by $\partial p/\partial r = \rho r^2 \omega$, in which ω is the angular velocity of rotation.   T ☐   F ☐

(y)   To convert radians per second into rpm, divide by 120π.   T ☐   F ☐

(z)   The shape of the free surface of a liquid in a rotating container is a hyperbola.   T ☐   F ☐

(A)   The hydrostatic force exerted on one face of a square plate of side L that is held vertically in a liquid with one edge in the free surface is F. If the plate is lowered vertically by a distance L, the force on one face will be $3F$.   T ☐   F ☐

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