

Phys101 Lectures 23-27

Fluids

Key points:

- **Pressure and Pascal's Principle**
- **Buoyancy and Archimedes' Principle**
- **Bernoulli's Equation**
- **Poiseuille's Law**

Ref: 13-1,2,3,4,5,6,7,8,9,10,11,12.

13-1 Phases of Matter

The three common phases of matter are **solid, liquid, and gas.**

A solid has a definite shape and size.

A liquid has a fixed volume but can be any shape.

A gas can be any shape and also can be easily compressed.

Liquids and gases both flow, and are called fluids.

13-2 Density and Specific Gravity

The density ρ of a substance is its mass per unit volume:

$$\rho = \frac{m}{V}.$$

The SI unit for density is kg/m^3 . Density is also sometimes given in g/cm^3 ; to convert g/cm^3 to kg/m^3 , multiply by 1000. Why?

$$1 \text{ g} / \text{cm}^3 = 1 \frac{\text{g}}{\text{cm}^3} \frac{1 \text{ kg}}{1000 \text{ g}} \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^3 = \frac{10^6}{10^3} \text{ kg} / \text{m}^3 = 1000 \text{ kg} / \text{m}^3$$

Water at 4°C has a density of $1 \text{ g}/\text{cm}^3 = 1000 \text{ kg}/\text{m}^3$.

The specific gravity of a substance is the ratio of its density to that of water.

13-3 Pressure in Fluids

Pressure is defined as the force per unit area.

$$\text{pressure} = P = \frac{F}{A}.$$

Pressure is a scalar; the units of pressure in the SI system are pascals:

$$1 \text{ Pa} = 1 \text{ N/m}^2.$$

**i-clicker question 23-1:
unit conversion from cm^2 to m^2 .**

500 cm^2 is equal to

- A) 50.0 m^2 .**
- B) 5.00 m^2 .**
- C) 0.500 m^2 .**
- D) 0.0500 m^2 .**
- E) 0.00500 m^2 .**

$$500\text{cm}^2 = 500\text{cm}^2 \times \left(\frac{1\text{m}}{100\text{cm}}\right)^2 = \frac{500}{10000}\text{m}^2 = 0.05\text{m}^2$$

Example 13-2: Calculating pressure.

The two feet of a 60-kg person cover an area of 500 cm².

(a) Determine the pressure exerted by the two feet on the ground.

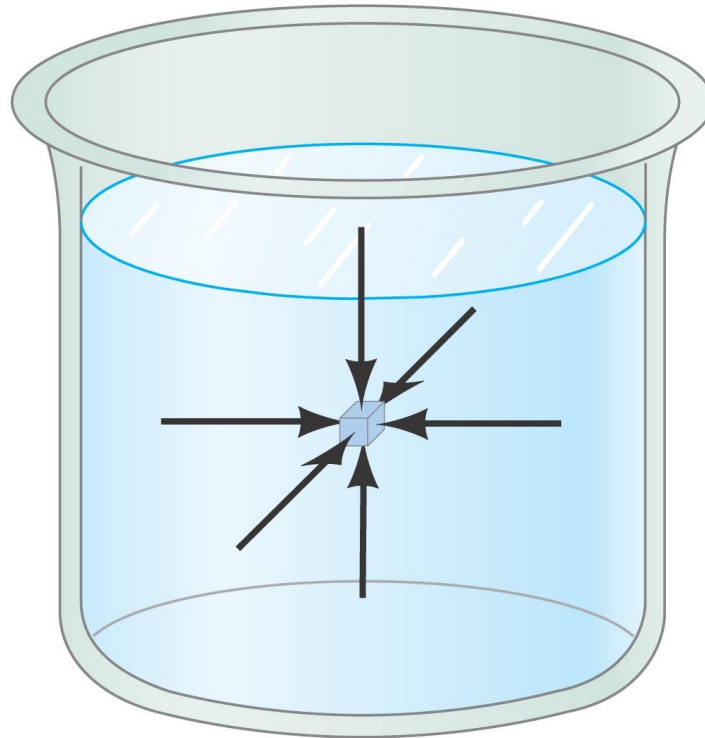
(b) If the person stands on one foot, what will the pressure be under that foot?

$$(a) \quad P = \frac{F}{A} = \frac{mg}{A} = \frac{60 \times 9.8}{0.050} = 1.2 \times 10^4 \text{ N} / \text{m}^2$$

$$(b) \quad P = \frac{F}{A} = \frac{mg}{A} = \frac{60 \times 9.8}{0.025} = 2.4 \times 10^4 \text{ N} / \text{m}^2$$

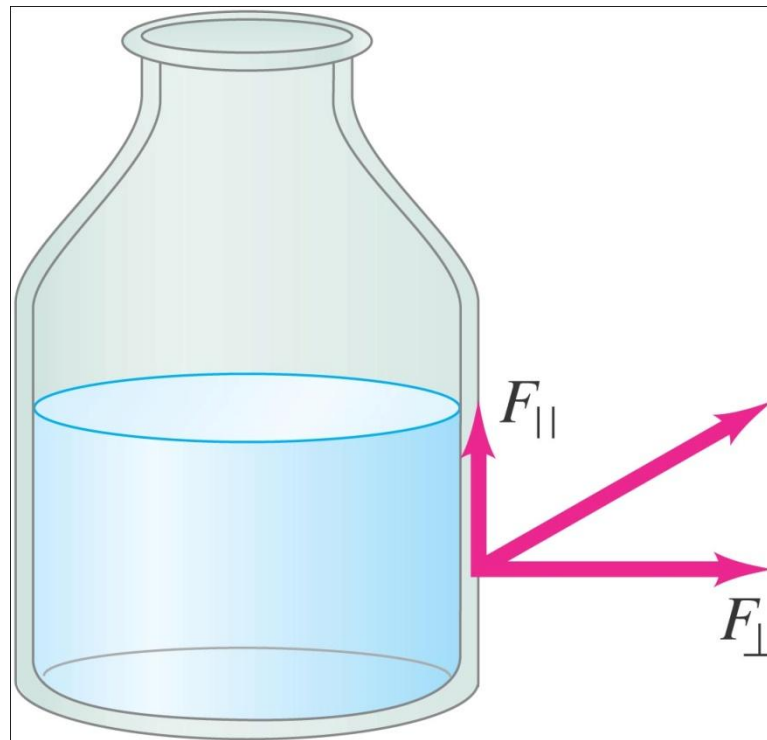
13-3 Pressure in Fluids

Pressure is the same in every direction in a static fluid at a given depth; if it were not, the fluid would flow.



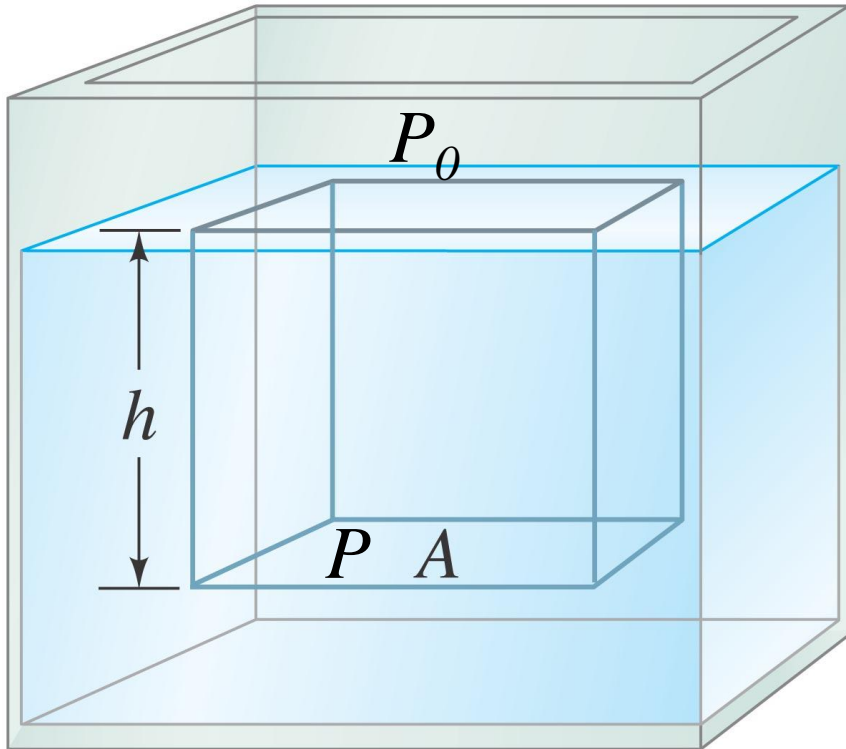
13-3 Pressure in Fluids

For a fluid at rest, there is also no component of force **parallel** to any solid surface—once again, if there were, the fluid would flow.



13-3 Pressure in Fluids

The pressure at a depth h below the surface of the liquid is due to the weight of the liquid above it. We can quickly calculate:



$$P = \frac{F}{A} = \frac{\rho Ahg}{A}$$

$$P = \rho gh.$$

This is the pressure due to the liquid. It equals the pressure in on A if what above the liquid surface is vacuum.

However, if there is an external pressure on the surface of the liquid such as the atmospheric pressure P_0 , then the actual pressure on A should be $P = \rho gh + P_0$.

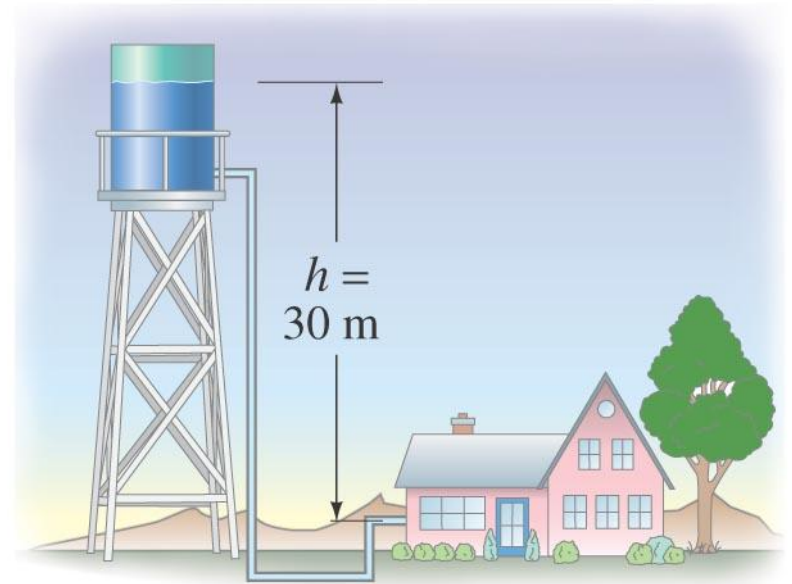
In general,

$$P - P_0 = \Delta P = \rho gh$$

Example 13-3: Pressure at a faucet.

The surface of the water in a storage tank is 30 m above a water faucet in the kitchen of a house. Calculate the difference in water pressure between the faucet and the surface of the water in the tank.

$$\begin{aligned}\Delta P &= \rho gh \\ &= (1.0 \times 10^3 \text{ kg} / \text{m}^3)(9.8 \text{ m} / \text{s}^2)(30 \text{ m}) \\ &= 2.9 \times 10^5 \text{ N} / \text{m}^2 \\ &= 2.9 \times 10^5 \text{ Pa}\end{aligned}$$



13-4 Atmospheric Pressure and Gauge Pressure

At sea level the atmospheric pressure is about 1.013×10^5 N/m²; this is called 1 atmosphere (atm).

Another unit of pressure is the bar:

$$1 \text{ bar} = 1.00 \times 10^5 \text{ N/m}^2.$$

Standard atmospheric pressure is just over 1 bar.

This is 10 times as large as the pressure we apply on our feet ! However, it does not **crush us, as our body maintains an internal pressure that balances it.**

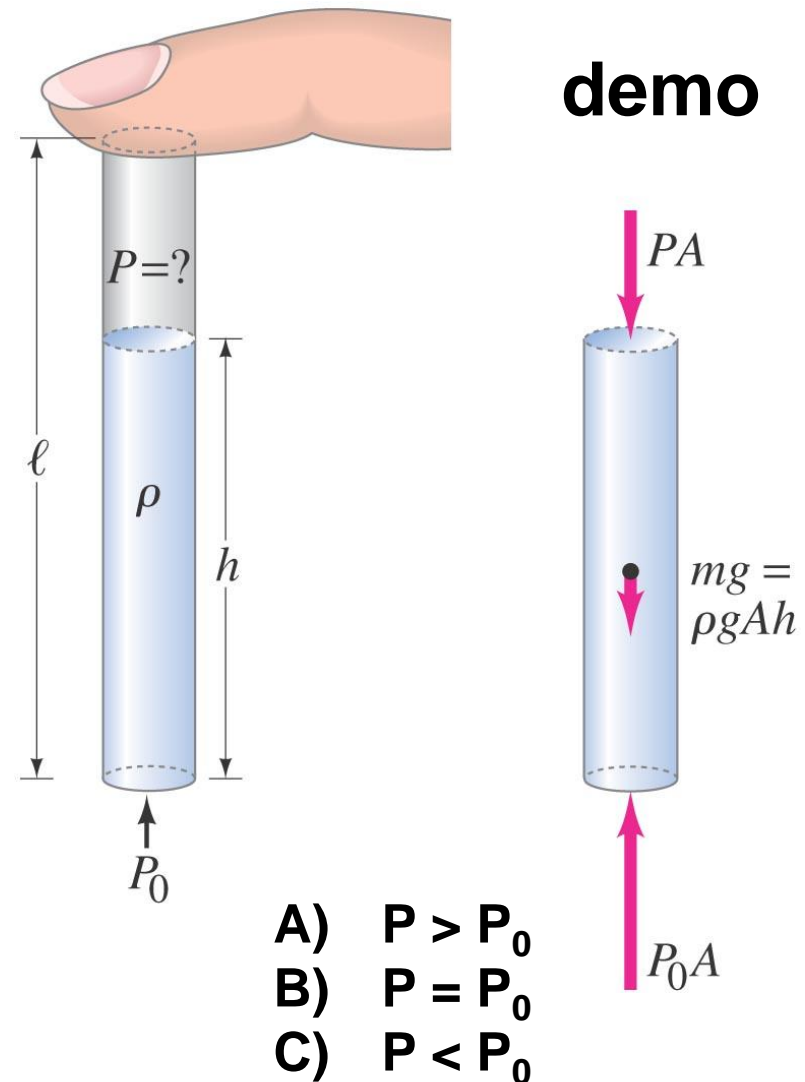
The atmospheric pressure is lower on tall mountains. It drops by 50% at an elevation of about 5000m.

Why?

I-clicker question 23-2

Conceptual Example 13-6: Finger holds water in a straw.

You insert a straw of length l into a tall glass of water. You place your finger over the top of the straw, capturing some air above the water but preventing any additional air from getting in or out, and then you lift the straw from the water. You find that the straw retains most of the water. Does the air in the space between your finger and the top of the water have a pressure P that is greater than, equal to, or less than the atmospheric pressure P_0 outside the straw?



If $P=0$, how much water can be held by the atmospheric pressure?

$$P_0 A = mg = \rho g A h, \quad h = \frac{P_0}{\rho g} = \frac{1.013 \times 10^5 \text{ N/m}^2}{(1.0 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = 10 \text{ m}$$

demo

13-4 Atmospheric Pressure and Gauge Pressure

Most pressure gauges measure the pressure above the atmospheric pressure—this is called the gauge pressure.

The absolute pressure is the sum of the atmospheric pressure and the gauge pressure.

$$P = P_0 + P_G.$$

i-clicker question 23-2

“The pressure in a flat tire is zero”. Here “the pressure” refers to:

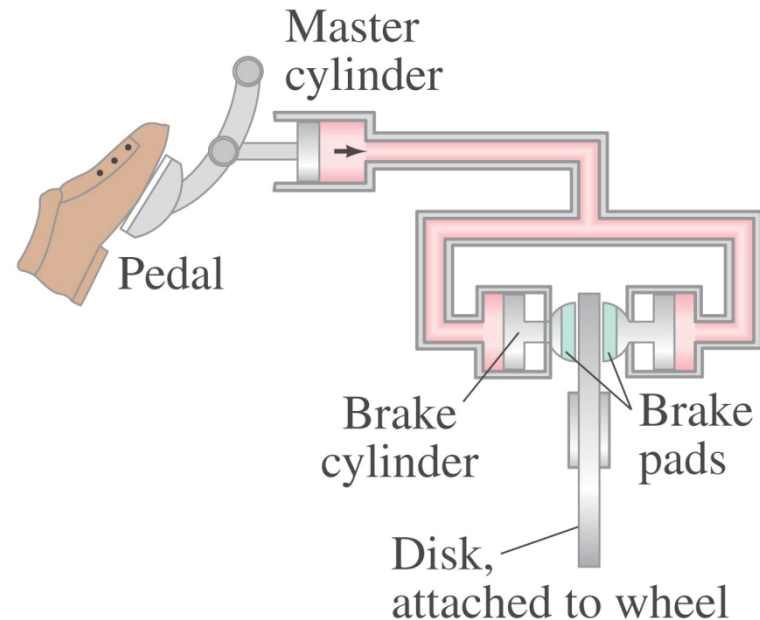
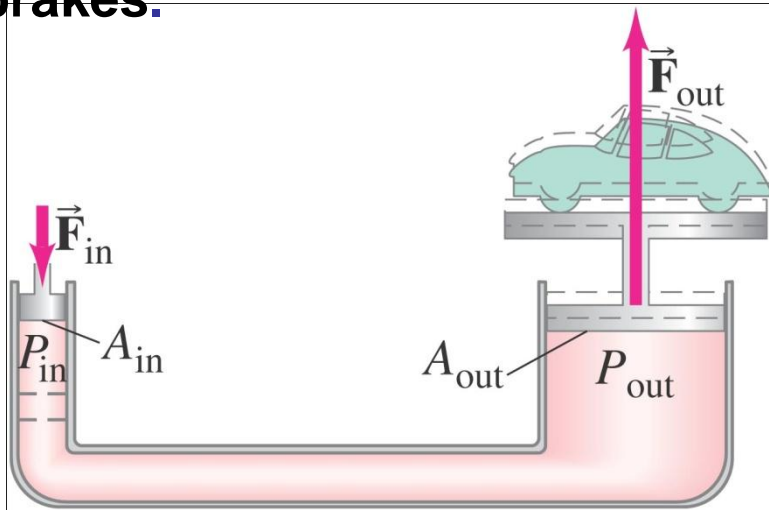
- A. The absolute pressure.
- B. The gauge pressure.

Note: The normal pressure in a tire is typically 30 psi (pounds per square inches). 1 psi = 6895 pa. The atmospheric pressure is about 15 psi.

13-5 Pascal's Principle

If an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount.

This principle is used, for example, in hydraulic lifts and hydraulic brakes.

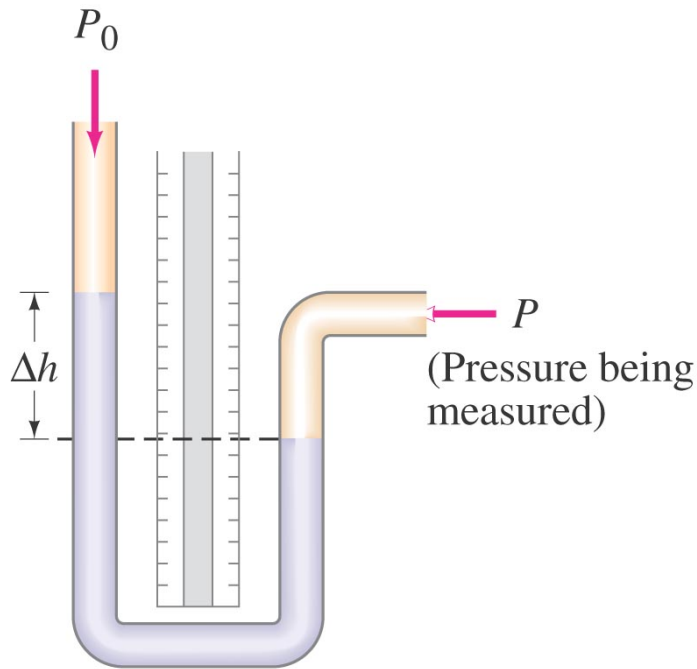


$$P_{in} = P_{out}$$

$$\frac{F_{in}}{A_{in}} = \frac{F_{out}}{A_{out}}, \quad F_{out} = \frac{A_{out}}{A_{in}} F_{in}$$

$$\therefore \text{if } A_{out} \gg A_{in}, \quad F_{out} \gg F_{in}$$

13-6 Measurement of Pressure; Gauges and the Barometer



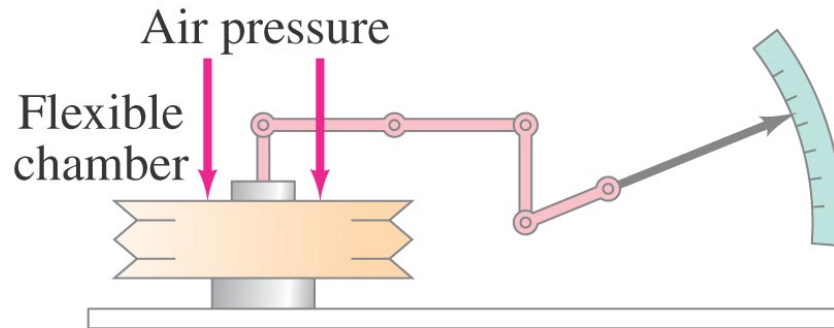
Open-tube manometer

There are a number of different types of pressure gauges. This one is an open-tube manometer. The pressure in the open end is **atmospheric pressure**; the **pressure being measured** will cause the fluid to **rise** until the pressures on both sides at the same height are **equal**.

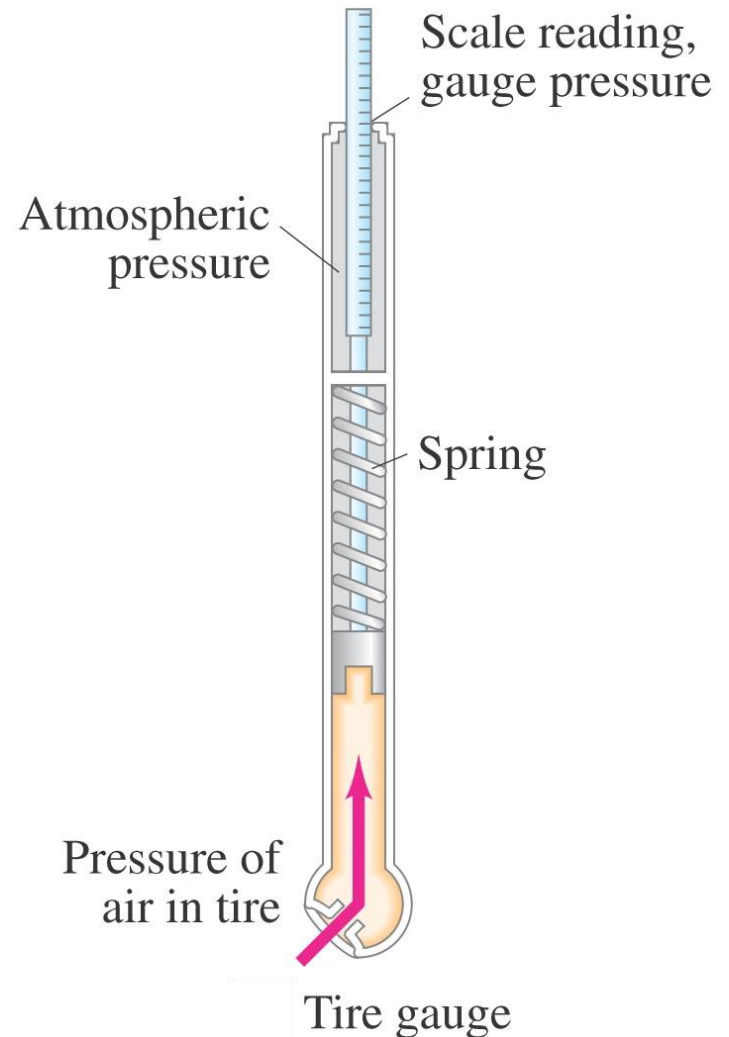
$$P = \rho g \Delta h + P_0$$

13-6 Measurement of Pressure; Gauges and the Barometer

Here are two more devices for measuring pressure: **the aneroid gauge and the tire pressure gauge.**



Aneroid gauge (used mainly for air pressure and then called an aneroid barometer)



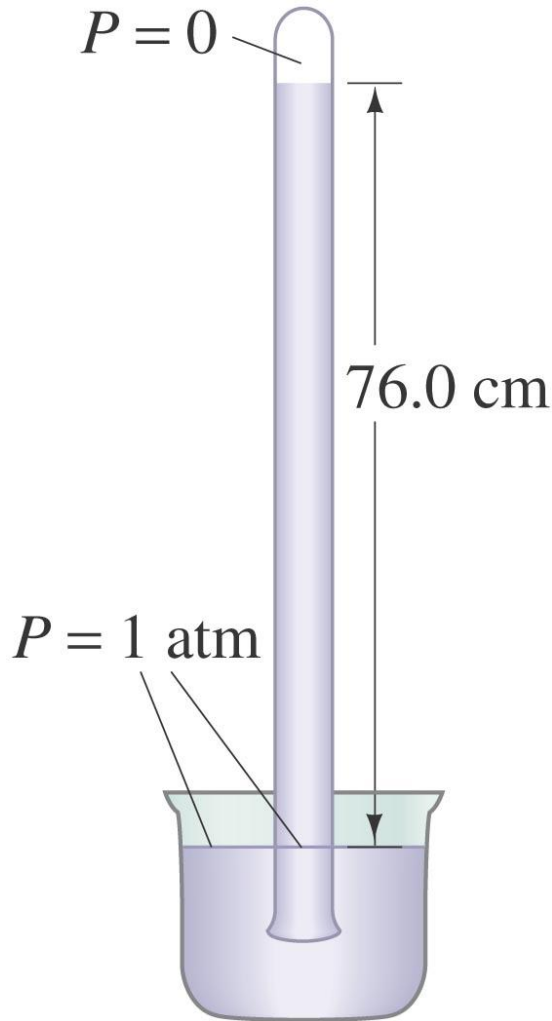
13-6 Measurement of Pressure; Gauges and the Barometer

Pressure is measured in a variety of different units. This table gives the conversion factors.

TABLE 13–2 Conversion Factors Between Different Units of Pressure

In Terms of $1 \text{ Pa} = 1 \text{ N/m}^2$	1 atm in Different Units
$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$	$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$
$= 1.013 \times 10^5 \text{ Pa} = 101.3 \text{ kPa}$	
$1 \text{ bar} = 1.000 \times 10^5 \text{ N/m}^2$	$1 \text{ atm} = 1.013 \text{ bar}$
$1 \text{ dyne/cm}^2 = 0.1 \text{ N/m}^2$	$1 \text{ atm} = 1.013 \times 10^6 \text{ dyne/cm}^2$
$1 \text{ lb/in.}^2 = 6.90 \times 10^3 \text{ N/m}^2$	$1 \text{ atm} = 14.7 \text{ lb/in.}^2$
$1 \text{ lb/ft}^2 = 47.9 \text{ N/m}^2$	$1 \text{ atm} = 2.12 \times 10^3 \text{ lb/ft}^2$
$1 \text{ cm-Hg} = 1.33 \times 10^3 \text{ N/m}^2$	$1 \text{ atm} = 76.0 \text{ cm-Hg}$
$1 \text{ mm-Hg} = 133 \text{ N/m}^2$	$1 \text{ atm} = 760 \text{ mm-Hg}$
$1 \text{ torr} = 133 \text{ N/m}^2$	$1 \text{ atm} = 760 \text{ torr}$
$1 \text{ mm-H}_2\text{O} (4^\circ\text{C}) = 9.80 \text{ N/m}^2$	$1 \text{ atm} = 1.03 \times 10^4 \text{ mm-H}_2\text{O} (4^\circ\text{C})$

13-6 Measurement of Pressure; Gauges and the Barometer



This is a **mercury barometer**, developed by Torricelli to measure **atmospheric pressure**. The **height of the column of mercury** is such that the pressure in the tube at the surface level is **1 atm**.

Therefore, pressure is often quoted in **millimeters (or inches) of mercury**.

13-6 Measurement of Pressure; Gauges and the Barometer



Any liquid can serve in a Torricelli-style barometer, but the most dense ones are the most convenient. This barometer uses water.

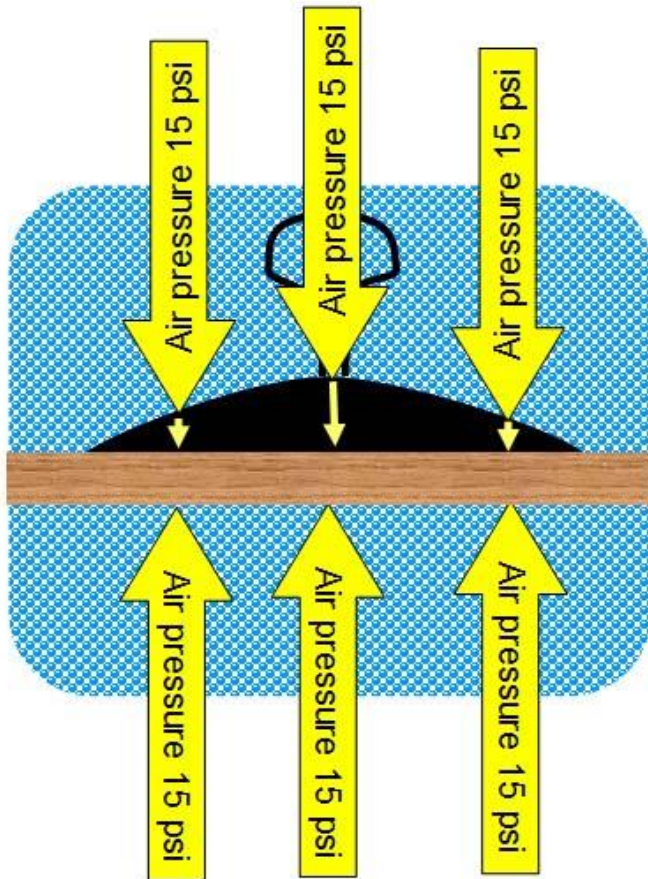
$$P_0 = \rho g h,$$

$$h = \frac{P_0}{\rho g}$$

$$\begin{aligned} &= \frac{1.013 \times 10^5 \text{ N} / \text{m}^2}{(1.0 \times 10^3 \text{ kg} / \text{m}^3)(9.8 \text{ m} / \text{s}^2)} \\ &= 10 \text{ m} \end{aligned}$$

Conceptual Example 13-7: Suction.

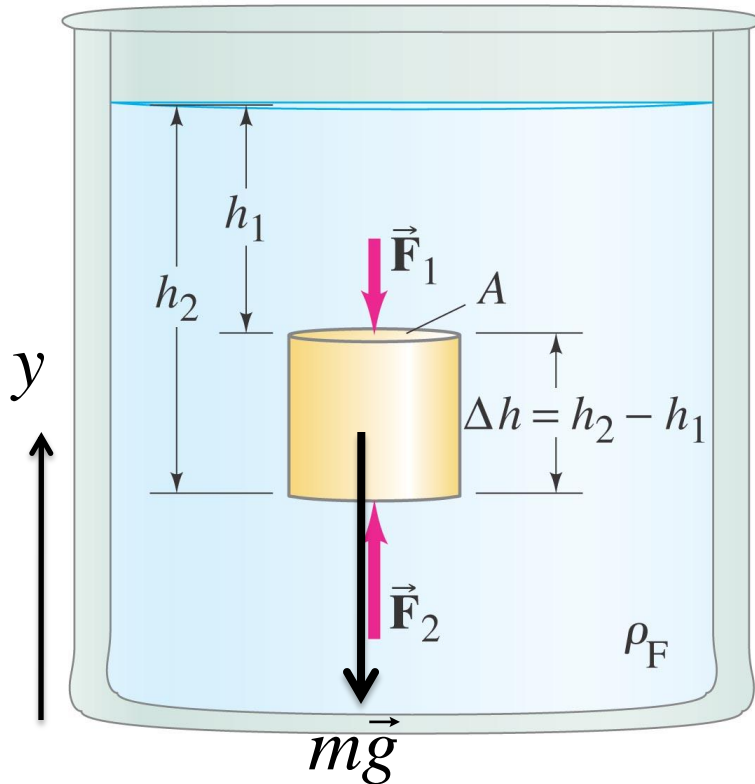
A student suggests suction-cup shoes for Space Shuttle astronauts working on the exterior of a spacecraft. Having just studied this Chapter, you gently remind him of the fallacy of this plan. What is it?



There is no atmosphere in the outer space.

13-7 Buoyancy and Archimedes' Principle

This is an object submerged in a fluid. There is an upward force on the object due to fluid pressure because the pressures at the top and bottom of it are different.



This upward force is called the buoyant force

$$\begin{aligned} F_B &= F_2 - F_1 = \rho_F g A (h_2 - h_1) \\ &= \rho_F g A \Delta h \\ &= \rho_F V g \\ &= m_F g. \end{aligned}$$

Which is equal to the weight of the fluid that takes up the same volume as the object.

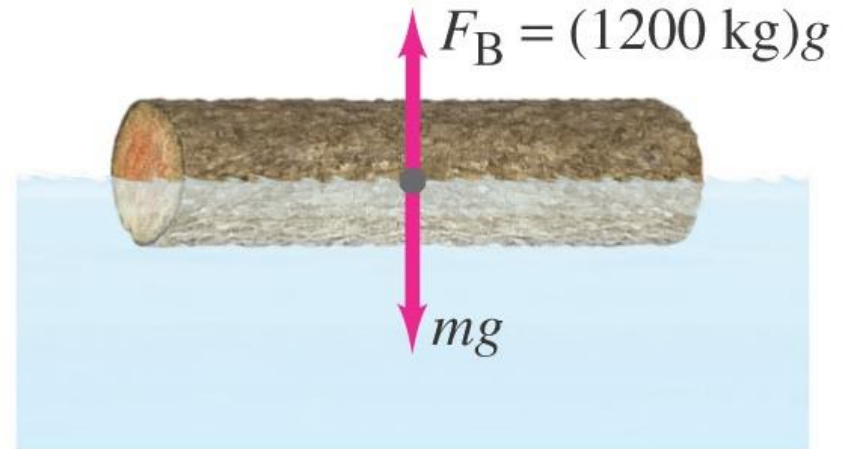
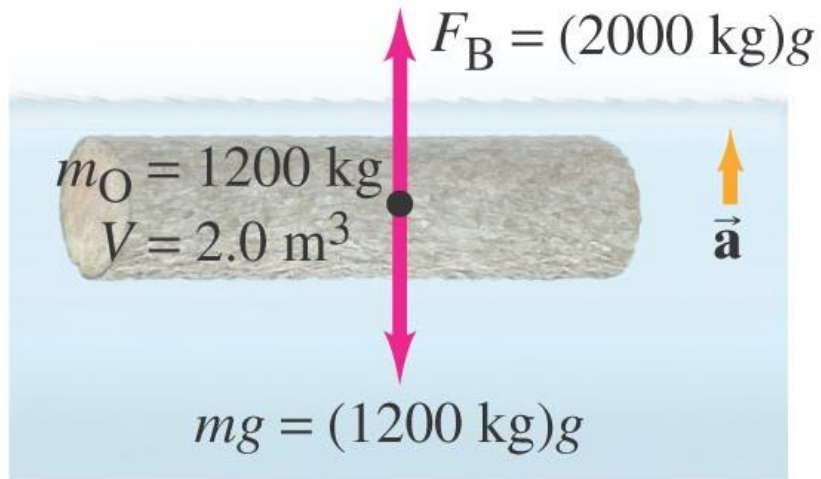
$$y\text{-comp of net force: } F_2 - F_1 - mg = m_F g - mg = Vg(\rho_F - \rho)$$

Therefore, float if $(\rho < \rho_F)$; but sink if $(\rho > \rho_F)$.

demo

13-7 Buoyancy and Archimedes' Principle

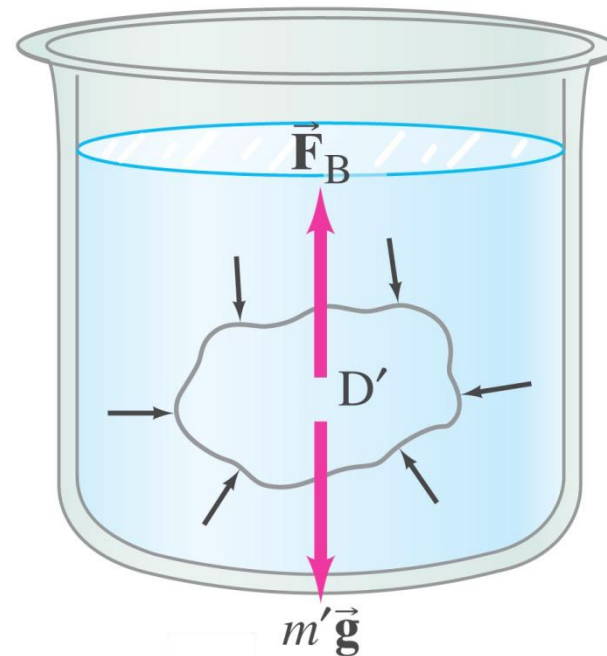
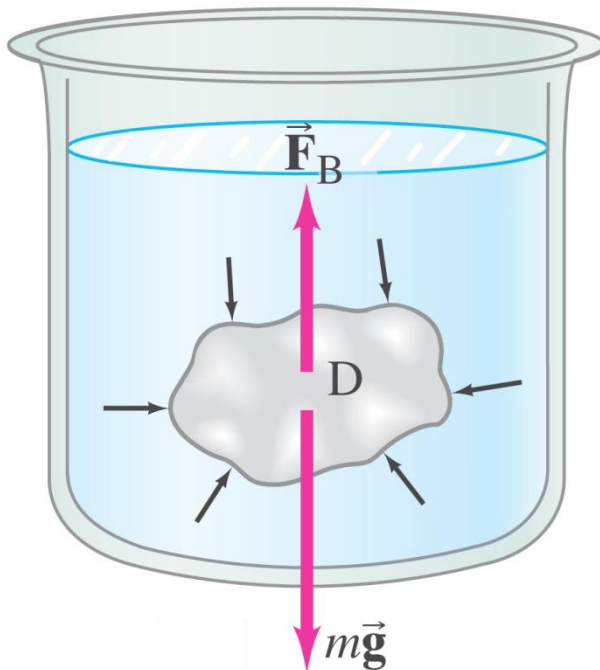
If an object's **density** is less than that of water, there will be an **upward net force** on it, and it will rise until it is **partially out of the water**.



13-7 Buoyancy and Archimedes' Principle

Archimedes' principle:

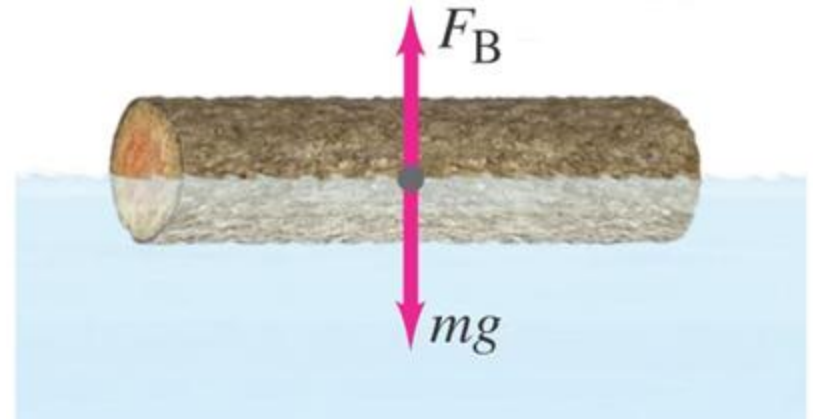
The buoyant force on an object immersed in a fluid is equal to the weight of the fluid displaced by that object.



I-clicker question 23-3: Conceptual Example 13-8: Two pails of water.

Consider two identical pails of water filled to the brim. One pail contains only water, the other has a piece of wood floating in it. Which pail has the greater weight?

- A. The pail with wood.
- B. The pail without wood.
- C. The two pails have the same weight.



The buoyant force on the wood object equals the weight of the wood object (static equilibrium).

Also, the buoyant force is equal to the weight of the spilled (i.e., displaced) water (Archimedes' principle).

In other words, the weight of wood is the same as the weight of the spilled water.

Therefore, putting the wood doesn't change the weight of the pail.

Example 13-9: Recovering a submerged statue.

A 70-kg ancient statue lies at the bottom of the sea. Its volume is $3.0 \times 10^4 \text{ cm}^3$. How much force is needed to lift it?

If the statue were in the air, the force needed to lift it would be mg . i.e., you need to overcome the gravity.

Now the statue is submerged in water. The buoyant force due to water is helping you. The force you need now is

$$F = mg - F_B$$

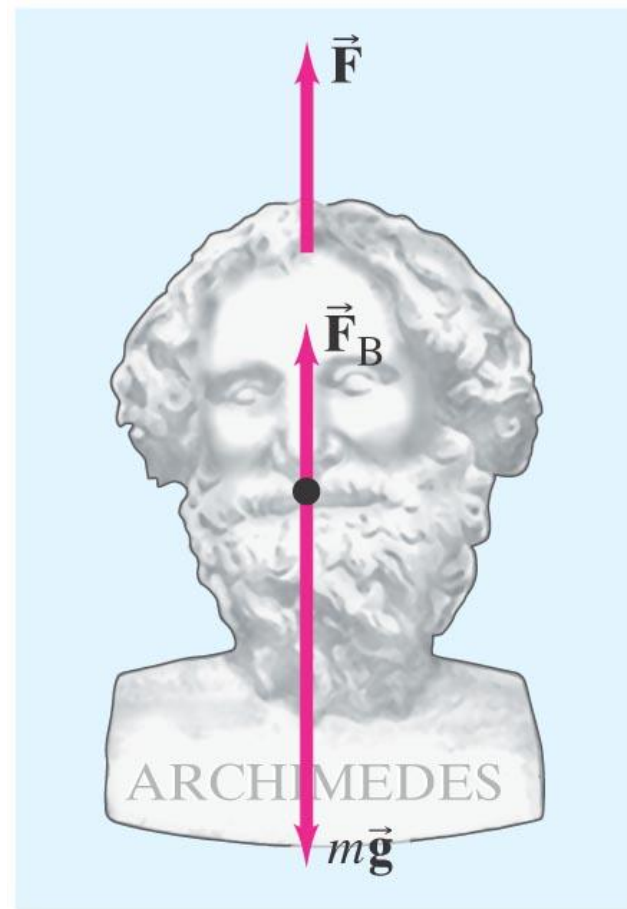
$$\left(\because \vec{F} + m\vec{g} + \vec{F}_B = 0 \quad \Rightarrow \quad F - mg + F_B = 0 \right)$$

$$F = mg - \rho_{\text{water}} Vg$$

$$= 70 \times 9.8 - 1000 \times 0.03 \times 9.8 = 390 \text{ N}$$

The apparent weight of the statue is 390N.

demo



Example 13-10: Archimedes: Is the crown gold?

When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold?

Idea: density of Au: $19.3 \times 10^3 \text{ kg/m}^3$

From the FBDs:

$$F_T - mg = 0$$

$$F'_T + F_B - mg = 0$$

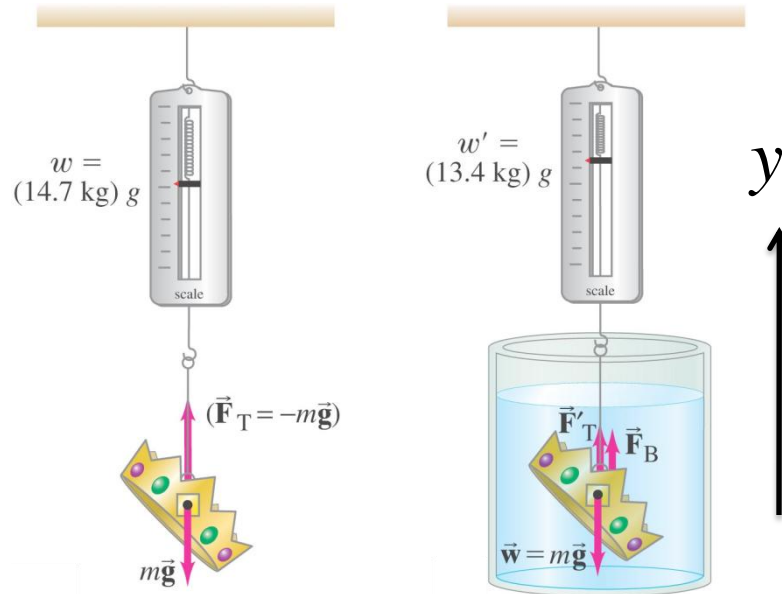
$$W - mg = 0$$

$$W' + F_B - mg = 0$$

$$\frac{W}{W - W'} = \frac{\rho}{\rho_F}$$

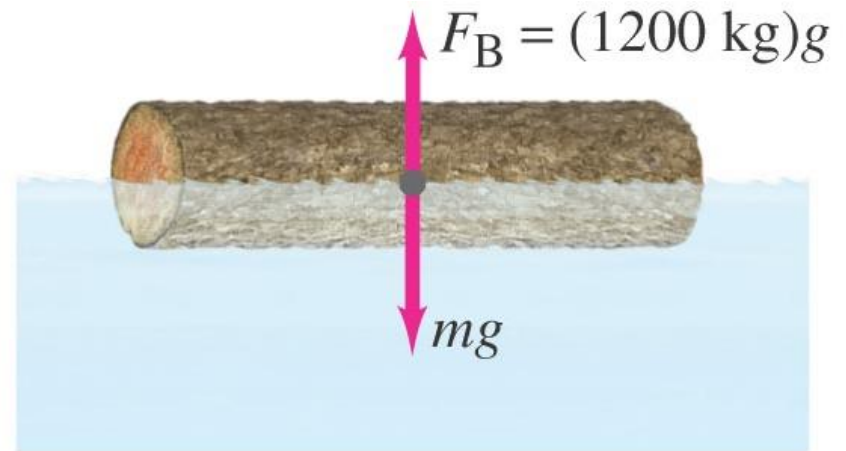
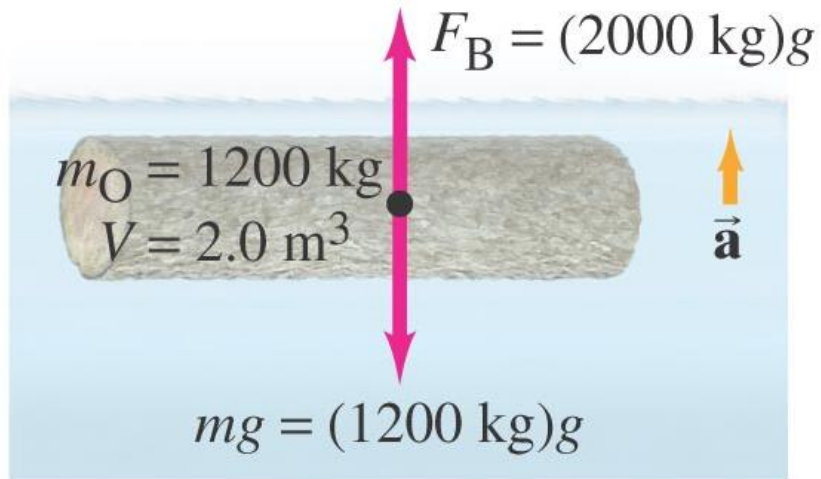
$$\frac{\rho_C}{\rho_{\text{water}}} = \frac{W}{W - W'} = \frac{14.7}{14.7 - 13.4} = 11.3$$

The density of the crown is $11.3 \times 10^3 \text{ kg/m}^3$. It's made of lead (or some alloy) !



13-7 Buoyancy and Archimedes' Principle

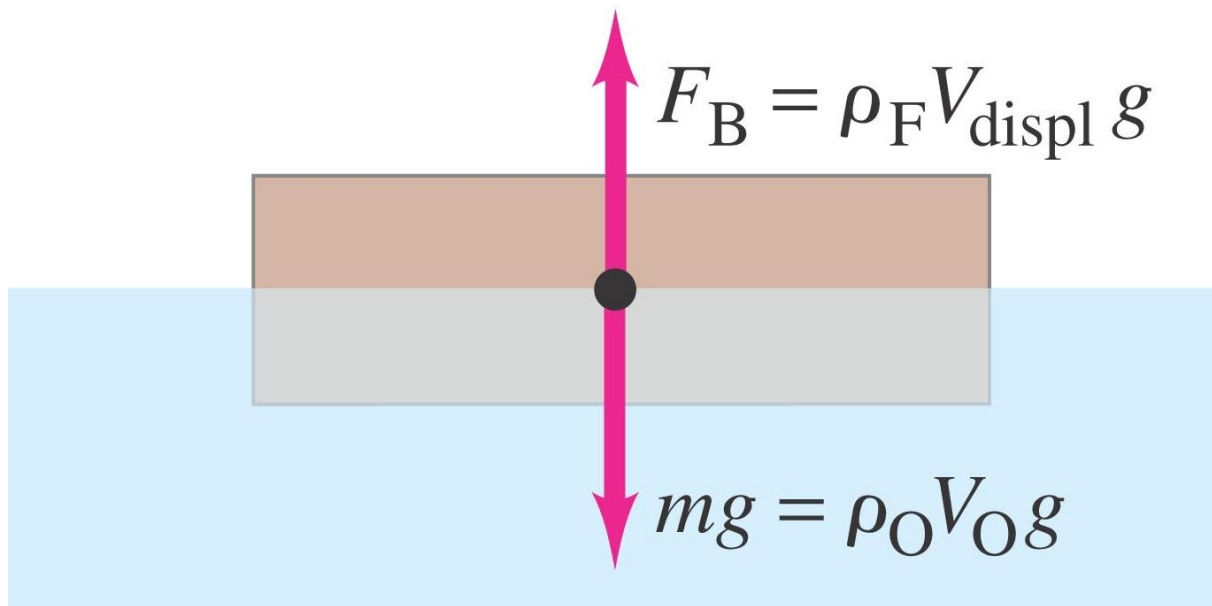
If an object's **density** is less than that of water, there will be an **upward net force** on it, and it will rise until it is **partially out of the water**.



13-7 Buoyancy and Archimedes' Principle

For a floating object, the fraction that is submerged is given by the ratio of the object's density to that of the fluid.

$$\frac{V_{\text{displ}}}{V_O} = \frac{\rho_O}{\rho_F}$$



$$F_B = mg$$

$$\rho_F V_{\text{displ}} g = \rho_O V_O g$$

$$\frac{V_{\text{displ}}}{V_O} = \frac{\rho_O}{\rho_F}$$

Example 13-11: Hydrometer calibration.

A hydrometer is a simple instrument used to measure the specific gravity of a liquid by indicating how deeply the instrument sinks in the liquid. This hydrometer consists of a glass tube, weighted at the bottom, which is 25.0 cm long and 2.00 cm² in cross-sectional area, and has a mass of 45.0 g. How far from the end should the 1.000 mark be placed?

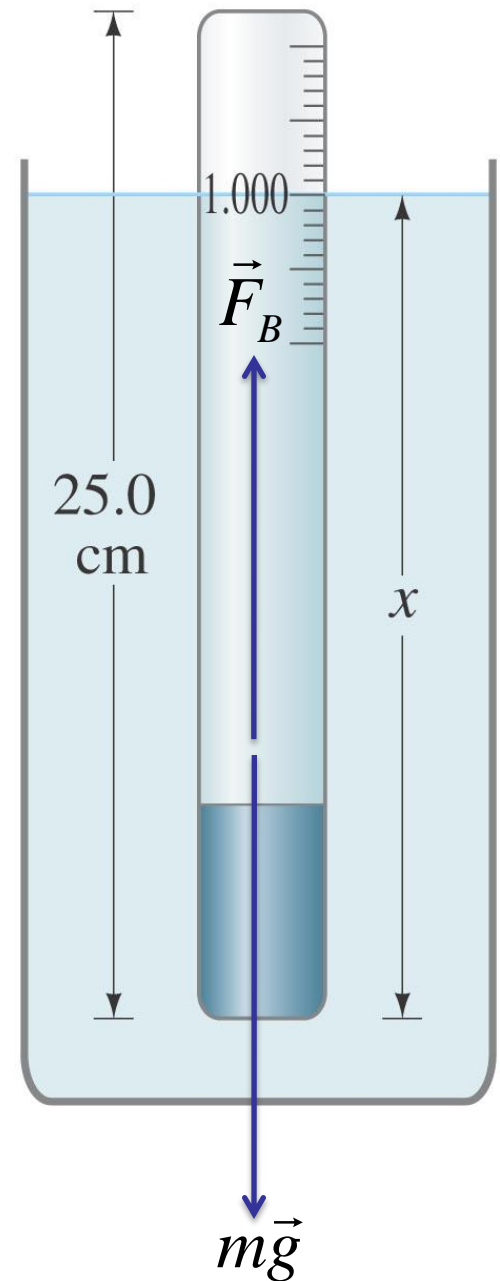
$$mg = F_B = \rho_F Vg = \rho_F Ahg$$

$$h = \frac{mg}{\rho_F Ag} = \frac{m}{\rho_F A}$$

When the specific gravity is 1.000,

$$\rho_F = \rho_{\text{Water}} = 1.000 \times 10^3 \text{ kg} / \text{m}^3$$

$$h = \frac{45.0 \times 10^{-3}}{1.000 \times 10^3 \times 2.00 \times 10^{-4}} = 0.225 \text{ m}$$



Example 13-12: Helium balloon.

What volume V of helium is needed if a balloon is to lift a load of 180 kg (including the weight of the empty balloon)?

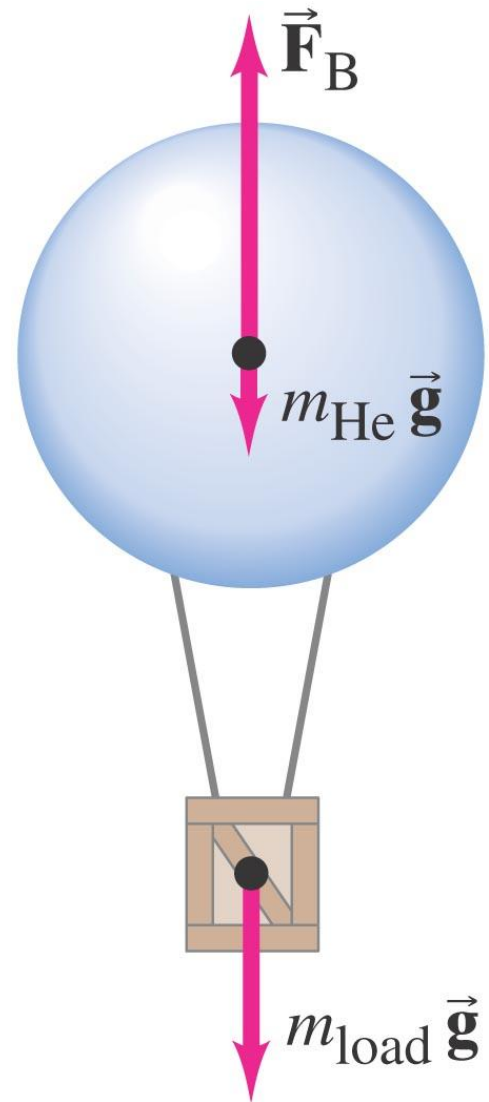
$$F_B = m_{load}g + m_{He}g$$

$$\rho_{air}Vg = m_{load}g + \rho_{He}Vg$$

Here we ignore the volume of the load.

$$\rho_{air}V = m_{load} + \rho_{He}V$$

$$V = \frac{m_{load}}{(\rho_{air} - \rho_{He})} = \frac{180}{1.29 - 0.179} = 160m^3$$



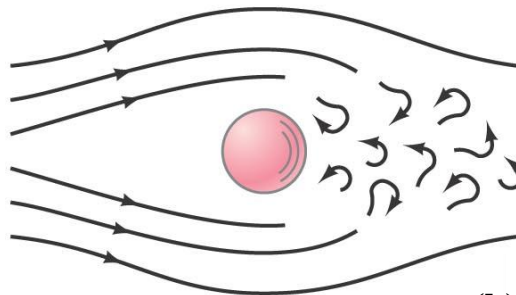
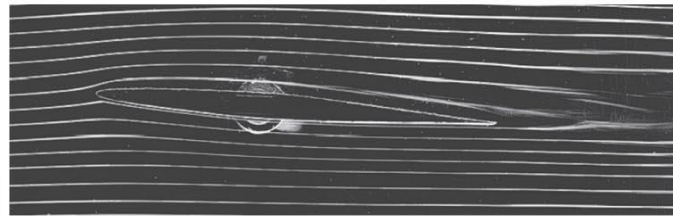
13-8 Fluids in Motion; Flow Rate and the Equation of Continuity

If the flow of a fluid is smooth, it is called **streamline or laminar flow (a)**.

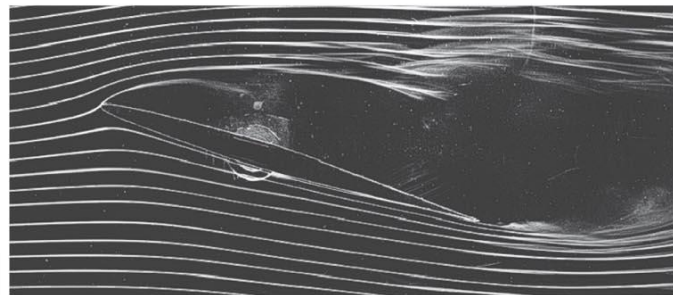
Above a certain speed, the flow becomes **turbulent (b)**. Turbulent flow has **eddies**; the **viscosity** of the fluid is much greater when eddies are present.



(a)



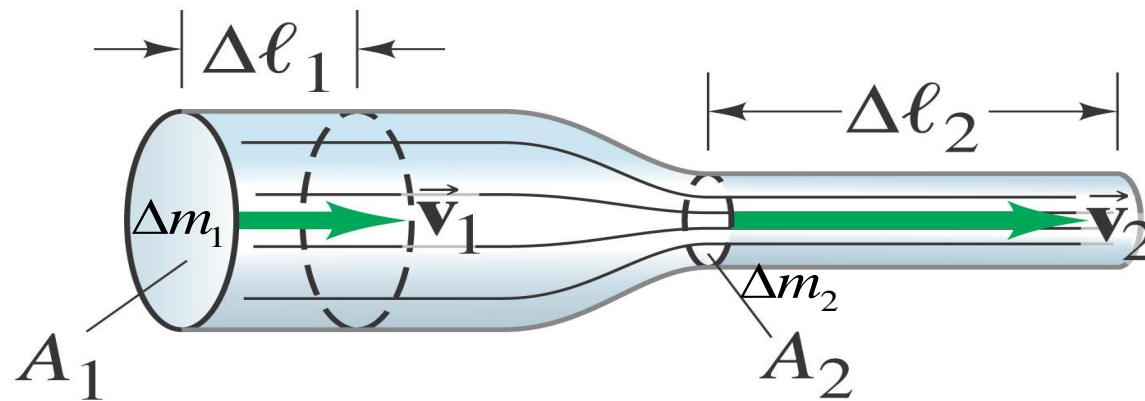
(b)



Flow Rate and the Equation of Continuity

We will deal with laminar flow.

The mass flow rate is the mass that passes a given point per unit time. The flow rates at any two points must be equal, as long as no fluid is being added or taken away.



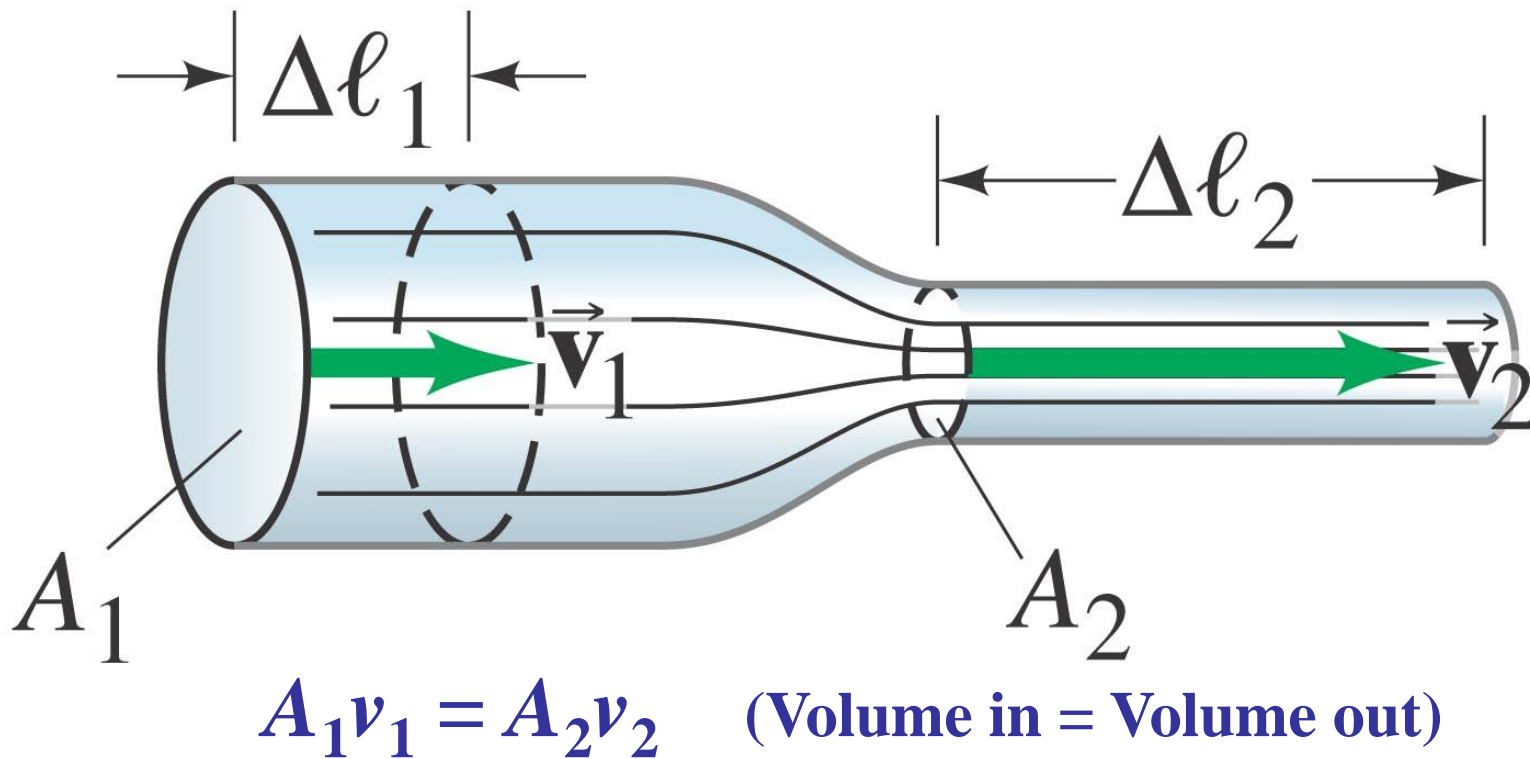
This gives us the equation of continuity:

Since
$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}, \quad (\text{in} = \text{out}, \Delta m_1 = \Delta m_2)$$

then
$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2.$$

13-8 Fluids in Motion; Flow Rate and the Equation of Continuity

If the density doesn't change—typical for liquids—this simplifies to $A_1v_1 = A_2v_2$. Where the pipe is **wider**, the **flow is slower**.



Example 13-13: Blood flow.

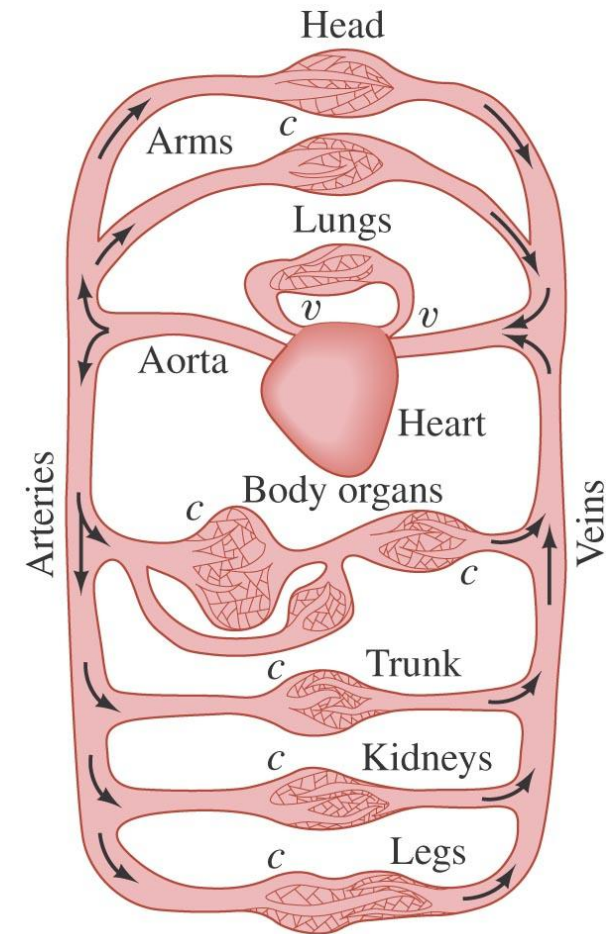
In humans, blood flows from the heart into the aorta, from which it passes into the major arteries. These branch into the small arteries (arterioles), which in turn branch into myriads of tiny capillaries. The blood returns to the heart via the veins. The radius of the aorta is about 1.2 cm, and the blood passing through it has a speed of about 40 cm/s. A typical capillary has a radius of about 4×10^{-4} cm, and blood flows through it at a speed of about 5×10^{-4} m/s. Estimate the number of capillaries that are in the body.

$$A_1 v_1 = N A_2 v_2$$

$N A_2$ is the total cross sectional area of all the capillaries

$$N = \frac{A_1 v_1}{A_2 v_2} = \frac{\pi R^2 v_1}{\pi r^2 v_2} = \frac{R^2 v_1}{r^2 v_2}$$

$$N = \frac{(1.2 \times 10^{-2})^2 \times 0.4}{(4 \times 10^{-6})^2 \times 5 \times 10^{-4}} = 7 \times 10^9$$



v = valves
 c = capillaries

Bernoulli's Equation

In time interval Δt , m_1 moves in and m_2 moves out. Continuity requires

$$m_1 = m_2 = m = \rho A_1 v_1 \Delta t = \rho A_2 v_2 \Delta t$$

Work done by external pressure:

$$W_p = P_1 A_1 v_1 \Delta t - P_2 A_2 v_2 \Delta t = \frac{m}{\rho} (P_1 - P_2)$$

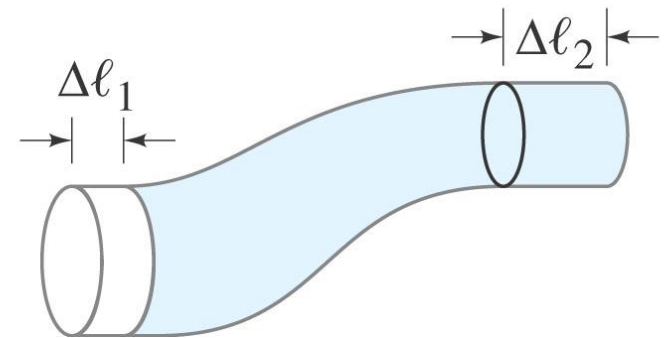
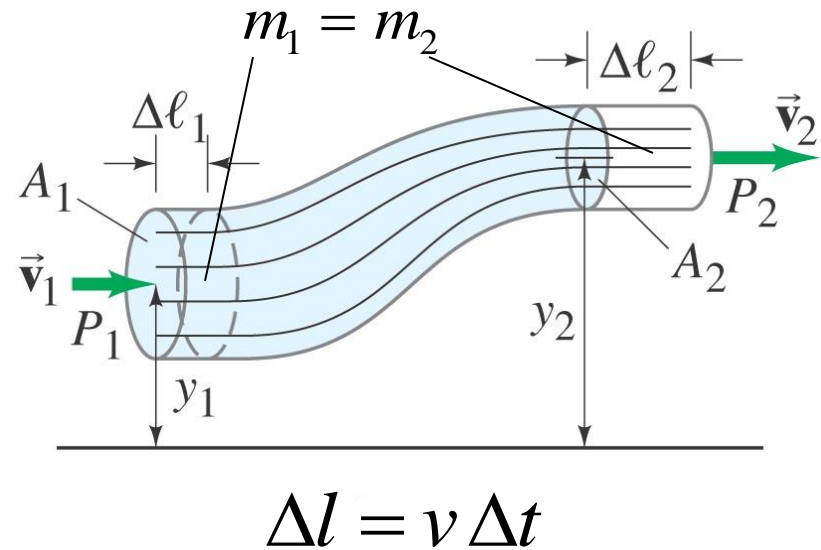
Ideally, **when there is no drag**, W_p should be equal to the gain in mechanical energy:

$$\frac{m}{\rho} (P_1 - P_2) = mgy_2 + \frac{1}{2}mv_2^2 - mgy_1 - \frac{1}{2}mv_1^2$$

$$P_1 - P_2 = \rho gy_2 + \frac{1}{2}\rho v_2^2 - \rho gy_1 - \frac{1}{2}\rho v_1^2$$

$$P_1 + \rho gy_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gy_2 + \frac{1}{2}\rho v_2^2$$

$$\text{OR: } P + \rho gy + \frac{1}{2}\rho v^2 = \text{constant}$$



This is known as Bernoulli's equation, which is a consequence of conservation of energy.

Bernoulli's principle:

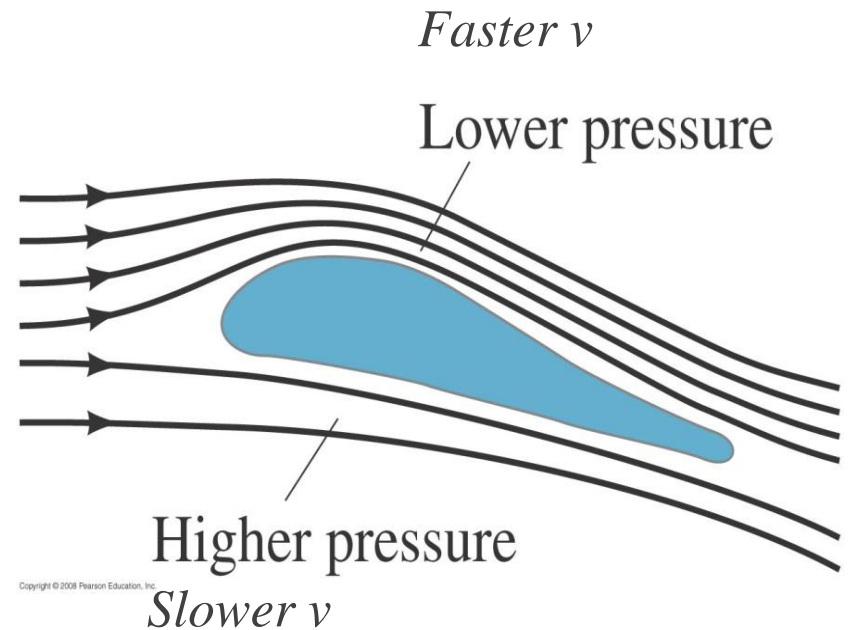
When the height y doesn't change much, Bernoulli's equation becomes

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$\text{OR: } P + \frac{1}{2} \rho v^2 = \text{constant}$$

Where the velocity of a fluid is high, the pressure is low, and where the velocity is low, the pressure is high.

Lift on an airplane wing is due to the different air speeds and pressures on the two surfaces of the wing.



Demo

Example 13-15: Flow and pressure in a hot-water heating system.

Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.5 m/s through a 4.0-cm-diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6-cm-diameter pipe on the second floor 5.0 m above? Assume the pipes do not divide into branches.

[Solution]

$$P_1 = 3.0 \text{ atm}, v_1 = 0.5 \text{ m/s}, A_1 = \pi \left(\frac{0.04}{2} \right)^2 = 0.00126 \text{ m}^2, y_1 = 0$$

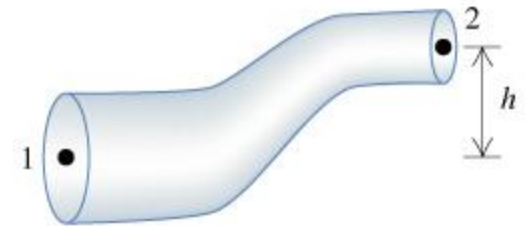
$$A_2 = \pi \left(\frac{0.026}{2} \right)^2 = 5.31 \times 10^{-6} \text{ m}^2, y_2 = h = 5.0 \text{ m}$$

$$v_1 A_1 = v_2 A_2 \Rightarrow v_2 = \frac{v_1 A_1}{A_2} = \frac{0.5 \times 0.00126}{5.31 \times 10^{-6}} = 1.19 \text{ m/s}$$

$$P_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2$$

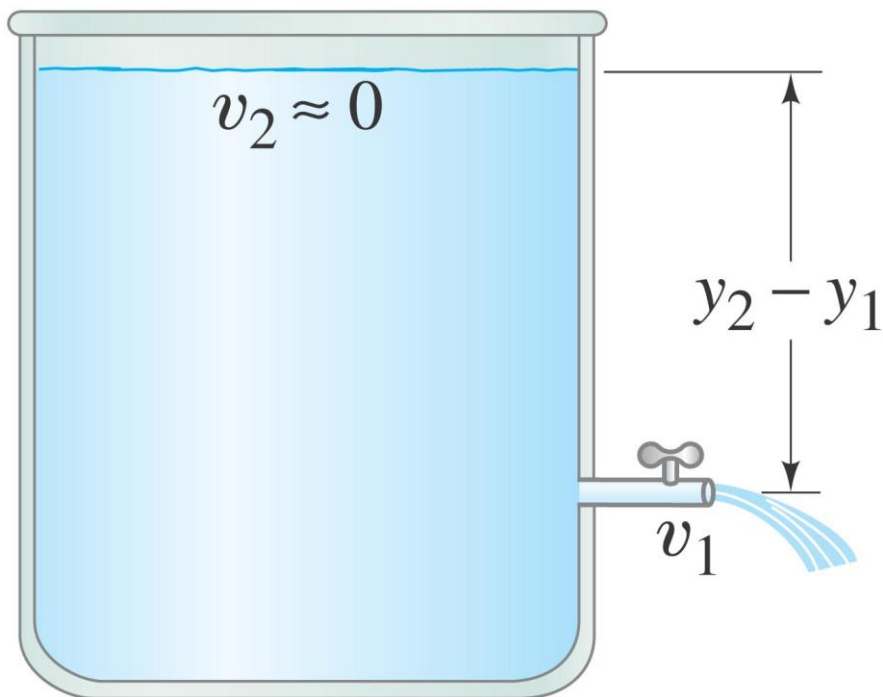
$$P_2 = P_1 + \frac{1}{2} \rho v_1^2 - \frac{1}{2} \rho v_2^2 - \rho g y_2$$

$$= 3.0 \times 10^5 + \frac{1}{2} (1000) (0.5^2 - 1.19^2) - (1000) (9.8) (5.0) = 2.5 \times 10^5 \text{ Pa}$$



13-10 Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, TIA

Using Bernoulli's principle, we find that the speed of fluid coming from a spigot on an open tank is:



$$\frac{1}{2} \rho v_1^2 + \rho g y_1 = \rho g y_2$$

or

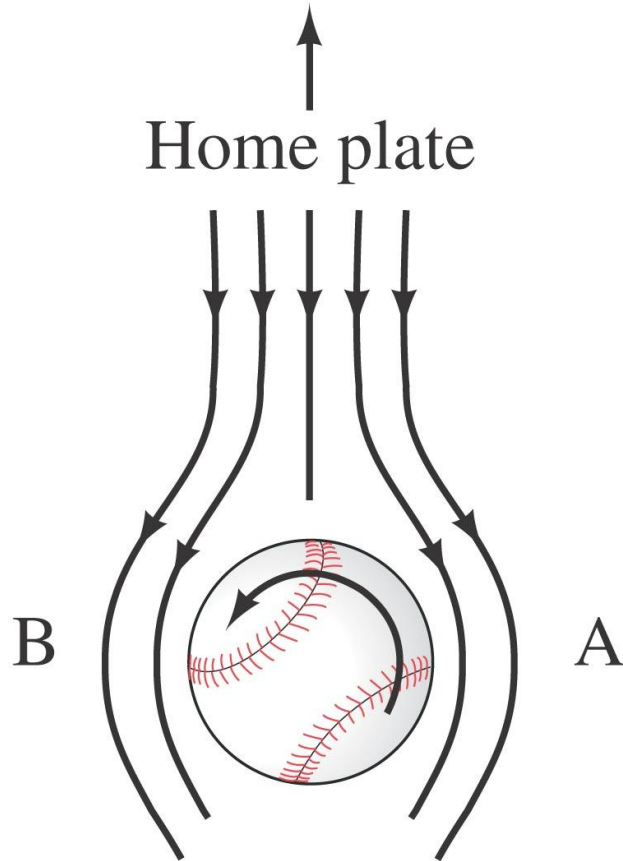
$$v_1 = \sqrt{2g(y_2 - y_1)}.$$

This is called Torricelli's theorem.

Compared to conservation of mechanical energy of a falling object:

$$mgh = \frac{1}{2} mv^2, \Rightarrow v = \sqrt{2gh}$$

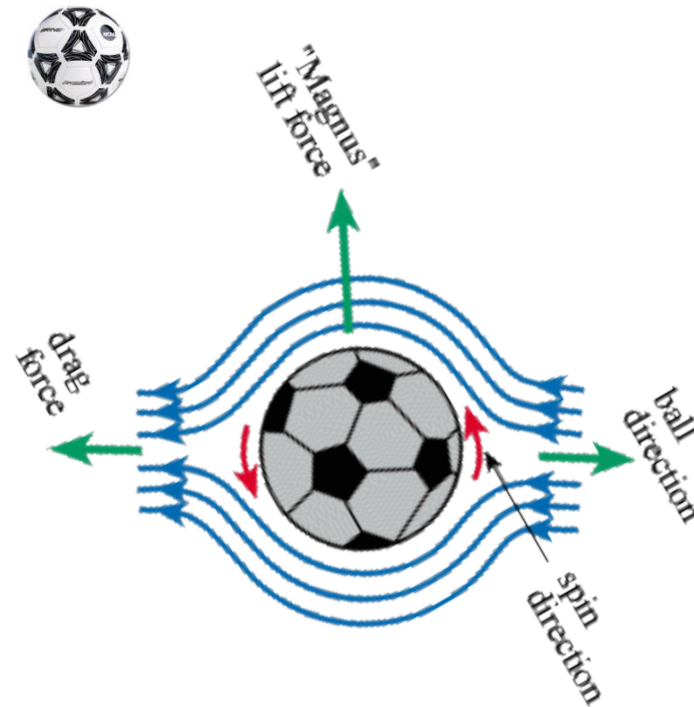
Applications of Bernoulli's Principle



The air travels faster relative to the center of the ball where the periphery of the ball is moving in the same direction as the airflow.

A ball's path will curve due to its spin, which results in the air speeds on the two sides of the ball not being equal; therefore there is a pressure difference.

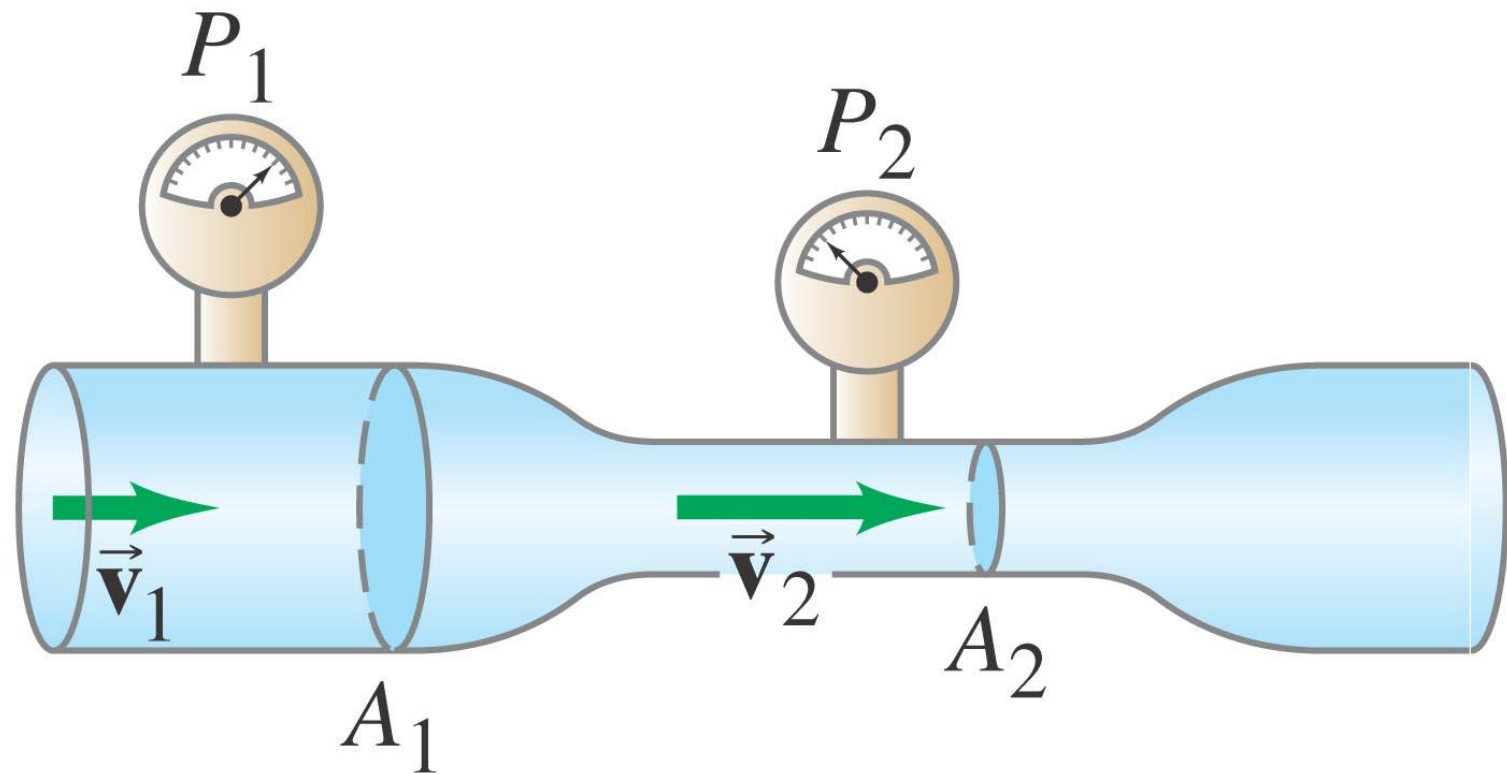
Free kick – a curving soccer ball.



Video

13-10 Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, TIA

A venturi meter can be used to measure fluid flow by measuring pressure differences.



13-11 Viscosity

Real fluids have some internal friction, called viscosity.

The viscosity can be measured; it is found from the relation

$$F = \eta A \frac{v}{\ell}$$

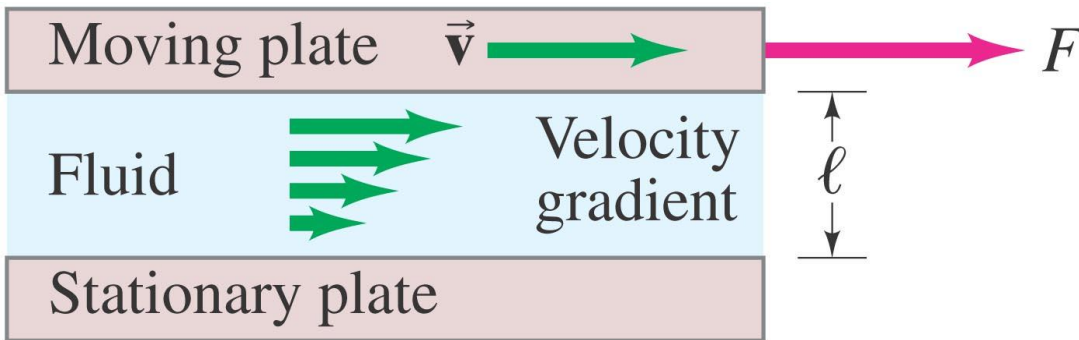


TABLE 13–3
Coefficients of Viscosity

Fluid (temperature in °C)	Coefficient of Viscosity, η (Pa·s) [†]
Water (0°)	1.8×10^{-3}
(20°)	1.0×10^{-3}
(100°)	0.3×10^{-3}
Whole blood (37°)	$\approx 4 \times 10^{-3}$
Blood plasma (37°)	$\approx 1.5 \times 10^{-3}$
Ethyl alcohol (20°)	1.2×10^{-3}
Engine oil (30°) (SAE 10)	200×10^{-3}
Glycerine (20°)	1500×10^{-3}
Air (20°)	0.018×10^{-3}
Hydrogen (0°)	0.009×10^{-3}
Water vapor (100°)	0.013×10^{-3}

[†] 1 Pa·s = 10 P = 1000 cP.

13-12 Flow in Tubes; Poiseuille's Equation, Blood Flow

The rate of flow in a fluid in a round tube depends on the viscosity of the fluid, the pressure difference, and the dimensions of the tube.

The volume flow rate is proportional to the pressure difference, inversely proportional to the length of the tube and to the pressure difference, and proportional to the fourth power of the radius of the tube.

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8\eta l}$$

This has consequences for blood flow—if the radius of the artery is half what it should be, the pressure has to increase by a factor of 16 to keep the same flow.

Example: 13-71 (MP #9).

e.g. Problem 13.71.

Blood transfusion.

Given: $l = 21 \text{ mm} = 0.021 \text{ m}$.

$$d = 0.75 \text{ mm}.$$

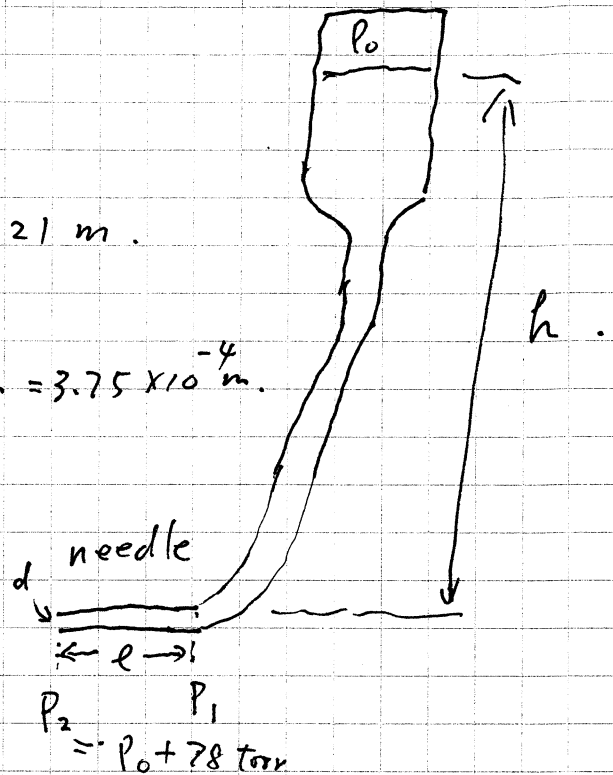
$$r = \frac{d}{2} = 0.375 \text{ mm} = 3.75 \times 10^{-4} \text{ m}.$$

$$\rho = 1.05 \times 10^3 \text{ kg/m}^3$$

$$\eta = 4 \times 10^{-3} \text{ Pa}\cdot\text{s}$$

$$P_2 = 78 \text{ Torr}$$

$$= 78 \times 133 = 1.04 \times 10^4 \text{ Pa}.$$



$$P_1 = P_0 + 78 \text{ Torr}$$

Require: flow rate: $Q = 2.5 \frac{\text{cm}^3}{\text{min}} = 2.5 \times 10^{-6} \text{ m}^3/\text{min}$
 $= 4.17 \times 10^{-8} \text{ m}^3/\text{s}.$

Since $\eta \neq 0$. (cannot be ignored), it's viscous flow.

Poiseuille's Law:

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8 \eta l}$$

$$P_1 - P_2 = \frac{8 \eta l Q}{\pi R^4}$$

$$pgh = P_1 - P_2 = \frac{8 \eta l Q}{\pi R^4}$$

$$h = \frac{1}{\rho g} \left(P_2 + \frac{8 \eta l Q}{\pi R^4} \right) = \frac{1}{\rho g} \left(1.04 \times 10^4 + \frac{8 \times 4 \times 10^{-3} \times 0.021 \times 4.17 \times 10^{-8}}{\pi (3.75 \times 10^{-4})^4} \right)$$
$$= \frac{1.09 \times 10^4}{1000 \times 9.8} = 1.11 \text{ m}.$$

Summary of Chapter 13

- Phases of matter: solid, liquid, gas
- Liquids and gases are called fluids.
- Density is mass per unit volume.
- Specific gravity is the ratio of the density of the material to that of water.
- Pressure is force per unit area.
- Pressure at a depth h is ρgh .
- External pressure applied to a confined fluid is transmitted throughout the fluid.

Summary of Chapter 13

- Atmospheric pressure is measured with a barometer.
- Gauge pressure is the total pressure minus the atmospheric pressure.
- An object submerged partly or wholly in a fluid is buoyed up by a force equal to the weight of the fluid it displaces.
- Fluid flow can be laminar or turbulent.
- The product of the cross-sectional area and the speed is constant for horizontal flow.

Summary of Chapter 13

- Where the velocity of a fluid is high, the pressure is low, and vice versa.
- Viscosity is an internal frictional force within fluids.