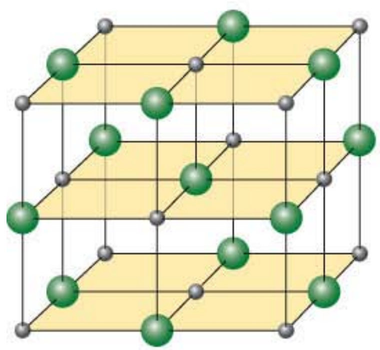
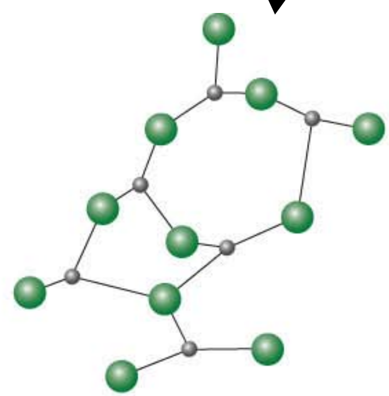
Fluids

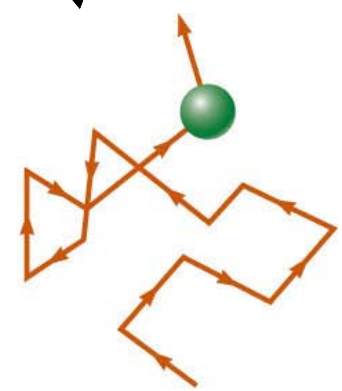
States of matter

Solids: crystalline, amorphous

Fluids: liquids, gasses







	crystalline		liquids		gasses
Inter-atomic forces	strong	>	strong	>>	very weak
Density	high	>	high	>>	low (pressure dependent)

Density is an important **material parameter**.

For a *uniform* material the **density** is defined as its **total mass divided** by its **total volume**.

$$\text{Density: } \rho = \frac{m}{V} \quad \left(\frac{kg}{m^3}\right)$$

(a scalar, Greek symbol: ρ, rho)

For a *non-uniform* material we can still define the local density as that of the mass Δm per very small volume element ΔV

$$\rho(x) = \frac{\Delta m}{\Delta V}$$

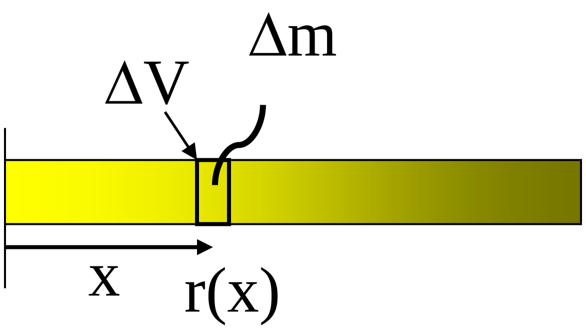


Table 9.1 Densities of Common Substances (at 0°C and 1 atm unless otherwise indicated)

Gases	Density (kg/m^3)	Liquids	Density (kg/m^3)	Solids	Density (kg/m^3)
Hydrogen	0.090	Gasoline	680	Polystyrene	100
Helium	0.18	Ethanol	790	Cork	240
Steam (100°C)	0.60	Oil	800–900	Wood (pine)	350–550
Nitrogen	1.25	Water (0°C)	999.87	Wood (oak)	600–900
Air (20°C)	1.20	Water (3.98°C)	1000.00	Ice	917
Air (0°C)	1.29	Water (20°C)	1001.80	Wood (ebony)	1000–1300
Oxygen	1.43	Seawater	1025	Bone	1500–2000
Carbon dioxide	1.98	Blood (37°C)	1060	Concrete	2000
		Mercury	13 600	Quartz, granite	2700
				Aluminum	2702
				Iron, steel	7860
				Copper	8920
				Lead	11 300
				Gold	19 300
				Platinum	21 500

How much **volume is there around atoms** (or molecules) in solids, liquids and gasses (typically)?

Consider a mole (Avogadro's number: 6.02×10^{23}) from each class:

↓	atomic or molec. weight	Density	$\frac{\text{volume}}{\text{mole}}$
iron (solid)	$55.8\frac{\text{g}}{\text{mole}}$	$7.86\frac{\text{g}}{\text{cm}^3}$	$\frac{55.8\frac{\text{g}}{\text{mole}}}{7.86\frac{\text{g}}{\text{cm}^3}} = 7.1\frac{\text{cm}^3}{\text{mole}}$
mercury (liquid)	$200.6\frac{\text{g}}{\text{mole}}$	$13.6\frac{\text{g}}{\text{cm}^3}$	$\frac{200.6\frac{\text{g}}{\text{mole}}}{13.6\frac{\text{g}}{\text{cm}^3}} = 14.8\frac{\text{cm}^3}{\text{mole}}$
air (N_2) at STP	$28.0\frac{\text{g}}{\text{mole}}$	$0.00125\frac{\text{g}}{\text{cm}^3}$	$22.4\frac{\text{liter}}{\text{mole}} = 22,400\frac{\text{cm}^3}{\text{mole}}$

↶ work backwards

Since each of these volumes contain the **same number** (Avogadro's #) of objects (atoms or molecules) the **ratio** of these volumes gives the **relative volumes** around the objects in each.

	$\frac{\text{volume}}{\text{mole}}$
iron (solid)	$7.10 \frac{cm^3}{mole}$
mercury (liquid)	$14.8 \frac{cm^3}{mole}$
air (N_2) at STP	$22,400 \frac{cm^3}{mole}$

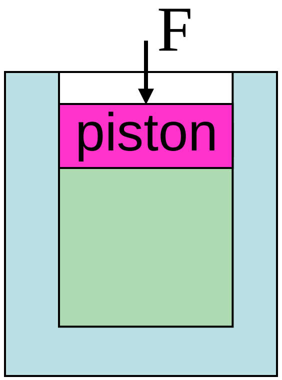
So an atom of liquid mercury has 14.8/7.1 ~ **2 times** the volume around it that an atom of Fe has in solid iron

But a molecule of N_2 in air (at STP) has 22,400/7.1 ~ **3150 times** the volume around it that an atom of Fe has in solid iron

This makes **gasses** far more **compressible** than solids or liquids.

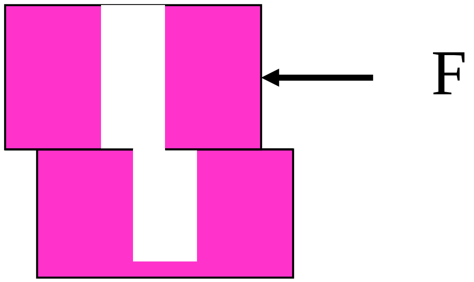
If we try to compress a gas in a container with a piston the volume will change with the applied force. A **gas** is said to be **compressible**.

For a **liquid**, because its molecules are already effectively in "contact", it's volume will change little, so to reasonable **approximation** (and certainly compared to gasses) liquids are often treated as being **incompressible**.



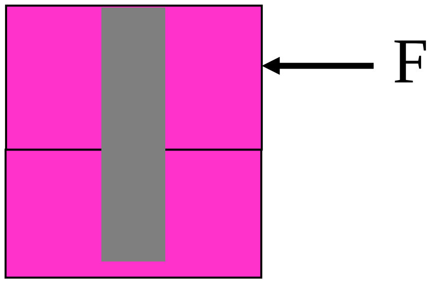
A defining characteristic of a fluid (liquid or gas) is a material that **cannot support a shear stress**.

Consider an unusual split container with a smooth interface between the top and bottom halves.

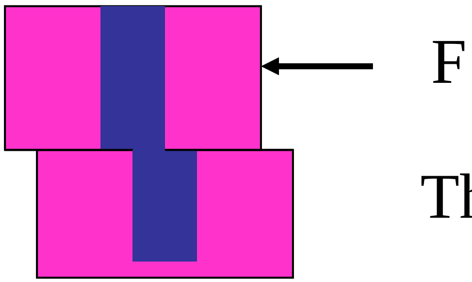


A force F applied to its top will cause its acceleration (in accord with Newton's 2^{nd} law).

If the space of in the container is filled with a solid like e.g. iron the force will act to try and shear the material at the split but unless the force is very large (and the container material very strong) the iron will resist being sheared.



If however the volume is instead filled by a fluid the resistance to the shearing will be largely the inertial resistance to the additional mass.



The fluid does not resist being **sheared**.

Because of this **fluids** will **adjust** their **shape** to **conform** to the forces acting on them. Under gravity they will fill the bottom of any container shape in which they are placed.

The inability of fluids to support a shear stress makes it difficult to talk about the force on a fluid. If your hand pushes on a fluid it yields, flowing around you.

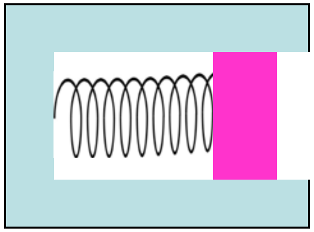
This ability to flow and change shape makes it difficult to talk of fluids as discrete objects. If two bodies of fluid meet they merge and mix making them no longer distinguishable.

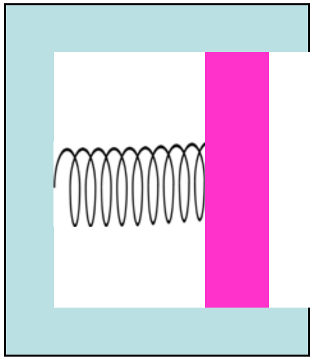
Parameters that are more useful in (quantitatively) describing fluids are the **density** and **pressure**.

To explore **forces** in a **fluid** we resort to a type of gauge described in the text consisting of piston cylinder arrangement **evacuated** behind the piston, but containing a **calibrated spring**.

$$F = -k\Delta x, \quad \text{(k known)}$$

By reading off Δx, we can determine force acting on the piston.





For experimentation we also make these with different piston areas.

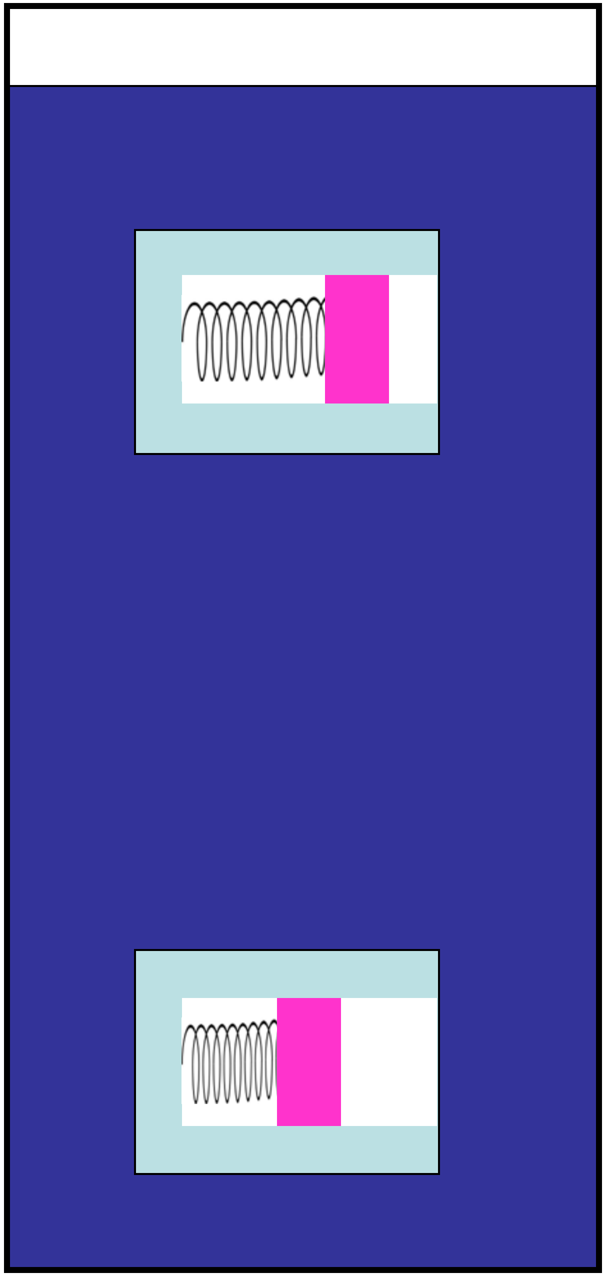
With these at our disposal we can perform a series of **experiments** in a tall tank of water.

We would find that:

1) The **force** (as read off by the spring compression), is **greater** the **deeper** we go into the tank. Depending linearly on the depth.

2) The **force** at a given level is **independent** of the **orientation** of the pressure gauge.

3) The **lateral dimensions** of the **tank don't matter** (double the tank width and all force readings are un-changed).



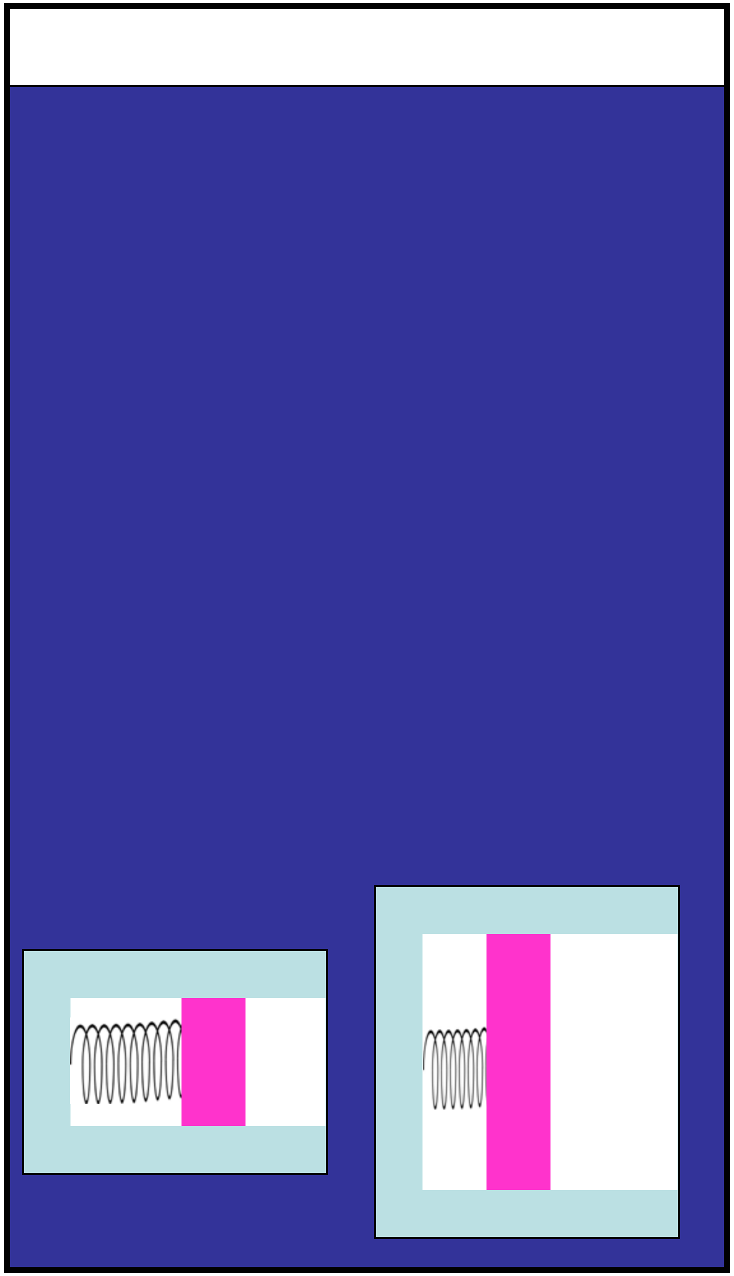
4) For each depth, if we use a device that has **double** the **piston area**, the **force** measured by the spring is **doubled**.

The last of these tells us that if we define the **pressure** as the **force per area** of the piston

$$p = \frac{F}{A}$$

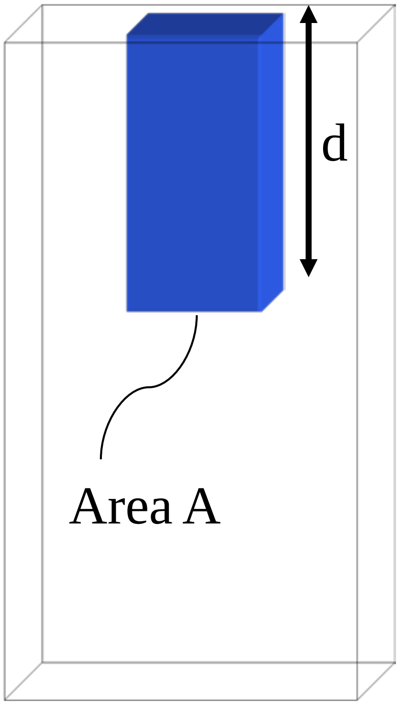
Then two **different sized gauges** read the **same** values of **pressure** at the same depth.

The bigger piston is pushed in twice as far because the fluid exerts its force on a per area basis.



With this realization we can understand the **lack of dependence** on the tanks **lateral dimensions** and the **linear dependence on depth**.

Consider the column of water having depth d and area at the bottom A. The pressure at the bottom of this column is the force of the weight of the water above it pressing down on the bottom face, divided by its area.



$$p = \frac{F}{A} = \frac{mg}{A}$$

But since $\rho = \frac{m}{V} \longrightarrow m = \rho V = \rho A d,$

Then,

$$p = \frac{F}{A} = \frac{mg}{A} = \frac{(\rho \cancel{A} d)g}{\cancel{A}} = \rho g d$$

Which depends **linearly** on the **depth**, d, but **not** the **area**, A, of the column.

$\longrightarrow$ Independent of tank width.

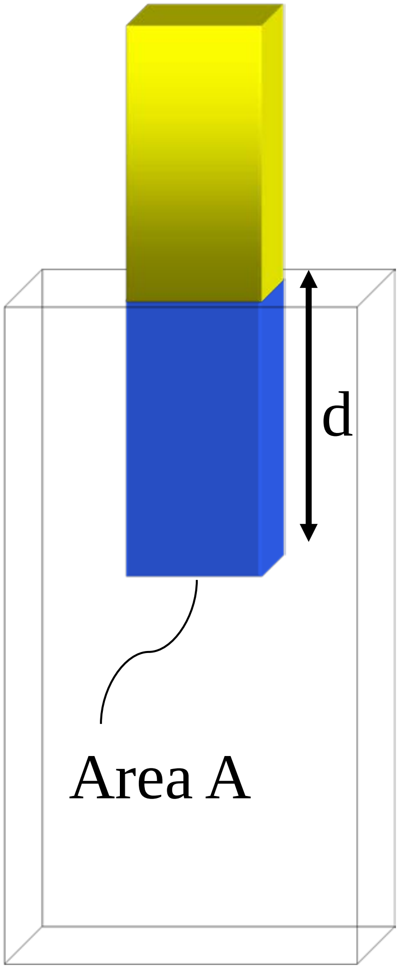
The **density** does **depend slightly** on the **depth** however we ignore this for liquids, assuming them to be **incompressible** (careful not a good assumption for gasses).

The **orientation independence** of the gauge reading tells us that **pressure is a scalar** quantity.

Finally, if we watched carefully we would have observed when our pressure **gauge** was **evacuated** (not under water) that the **piston sank in a little bit**. Considering this we realize that we are living at the bottom of an enormous tank of fluid: the atmosphere.

What we observed was the **pressure due to the atmosphere** (initially equal on both sides of the piston) pushing the piston in as the atmosphere inside was removed.

This pressure of the atmosphere is also due to the weight of the column of the atmosphere sitting above our column of water.



If the pressure of the atmosphere at the water surface is p_o then the ***absolute*** pressure at the bottom of our water column, at depth d is

$$p = p_o + \rho g d$$

The pressure ρgd, referenced to atmospheric pressure taken as zero is called the ***gauge*** pressure.

The SI unit of pressure is the Pascal: $1\,\text{Pa} = 1\frac{\text{N}}{\text{m}^2}$

There are also a number of other pressure units in common use that have the following equivalence,

$$1\,\text{atm} = 1.01\text{x}10^5\ \text{Pa} = 760\ \text{Torr} = 14.7\ \text{lb}/\text{in}^2$$

And there are a number of different types of pressure gauges in use.

HITT

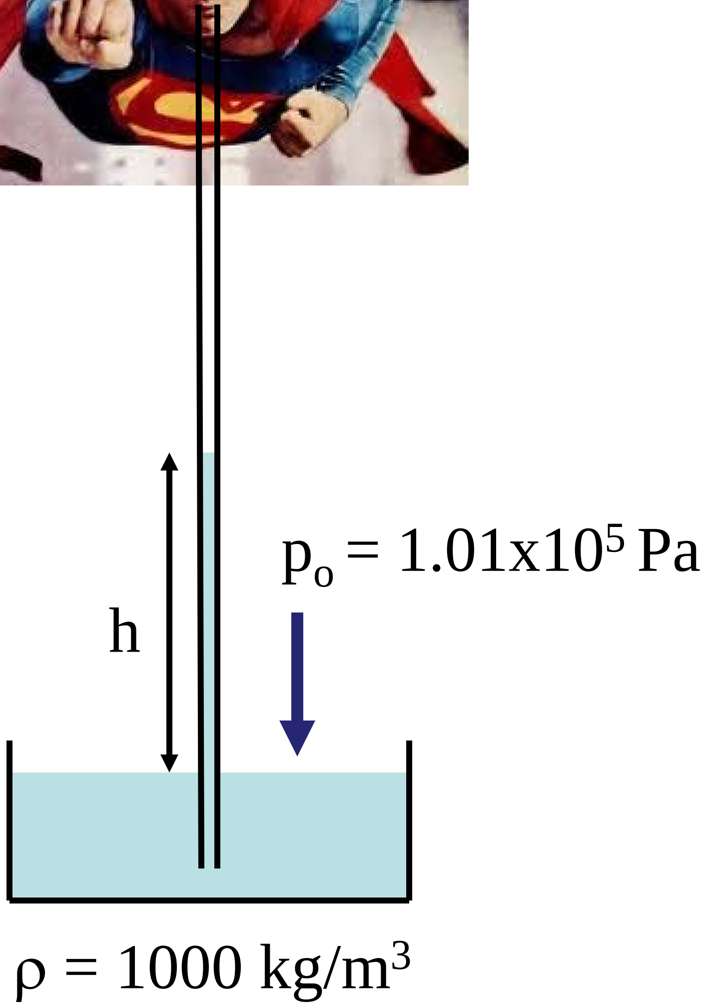
Superman (of course) has super suction, capable of pulling an infinite vacuum.

Given an appropriate straw how high could he suck the fresh water from a lake?

A) as high as he wants

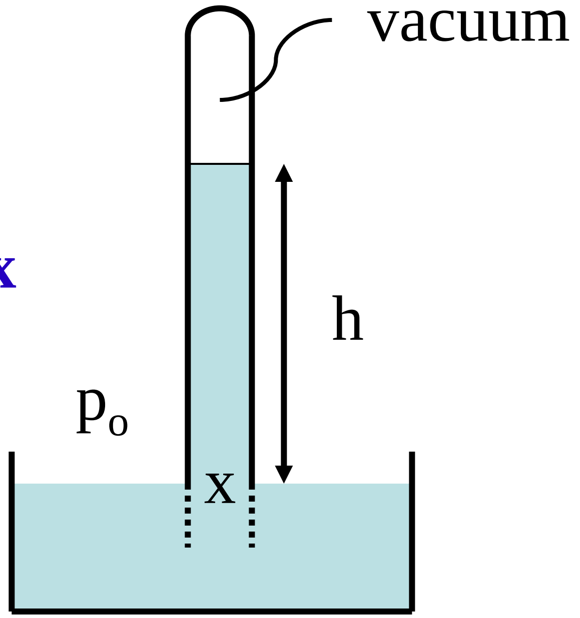
B) 32.0 m

C) 10.3 m



A device for measuring **atmospheric pressure** is the **barometer**. This is a close bottomed tube, filled to overflowing with a fluid and then **turned over** in an **open bath** of the same fluid.

Done **on the moon** which has essentially **no atmosphere**, the **fluid** would just **run out** until h = 0.



On earth where the pressure on the exterior fluid surface is around 1 atm the fluid **at point x** inside the tube has pressure equal to p_o, from the atmospheric pressure p_o outside the tube.

That means a **force up** given by solving

$$p_o = \frac{F}{A}$$

with A the cross-section of the tube.

Since the fluid is stationary, there must be an equal but opposite force down **at point x** having the same magnitude, but where we recognize the force as being due to the weight of the fluid column,

$$p_o = \frac{F}{A} = \frac{mg}{A}$$

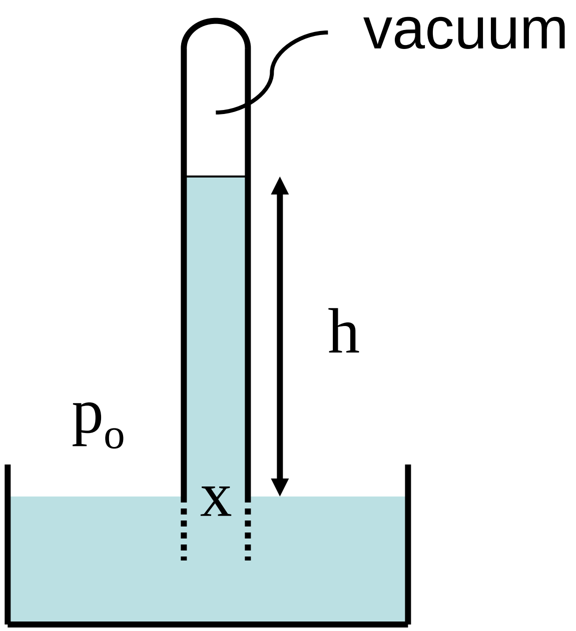
But since the density (rho) $\rho = \frac{m}{V}$ this can be rearranged to give,

$$m = \rho V = \rho A h,$$

which substituted above gives

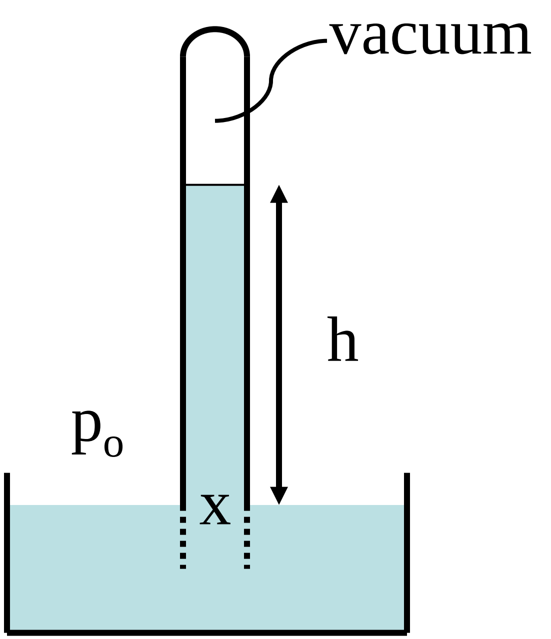
$$p_o = \frac{F}{A} = \frac{mg}{A} = \frac{\rho g A h}{A}$$

$$p_o = \rho g h$$



$p_o = \rho g h$

Taking h = 0 as zero pressure (absolute) the height of the fluid column is proportional to the pressure.



Mercury is often used. The height of a column of mercury at sea level is on average **760 mm**, so you will often hear atmospheric pressures quoted in mm of mercury (also called Torr) around this value.

The density of mercury depends on the temperature, and since the pressure reading depends on density, for accurate readings of atmospheric pressure this must be corrected for the local temperature.

HITT

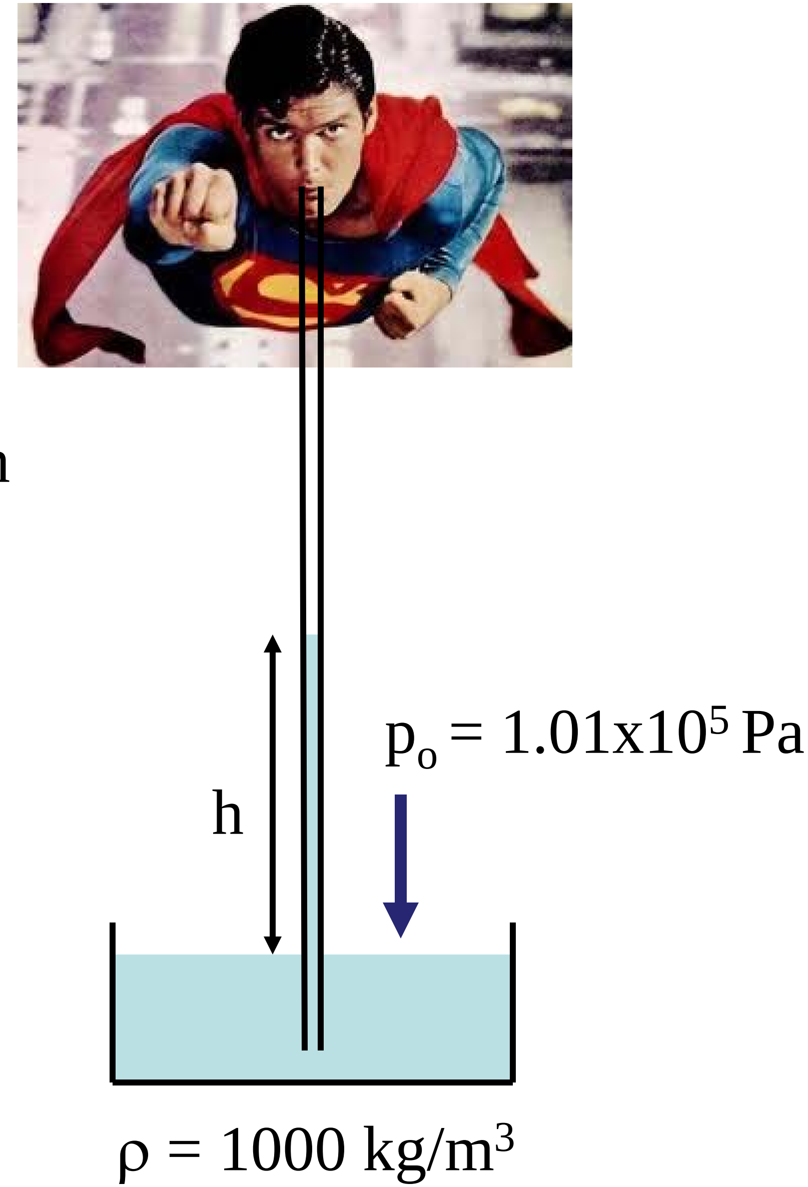
Superman (of course) has super suction, capable of pulling an infinite vacuum.

Given an appropriate straw how high could he suck the fresh water from a lake?

A) as high as he wants

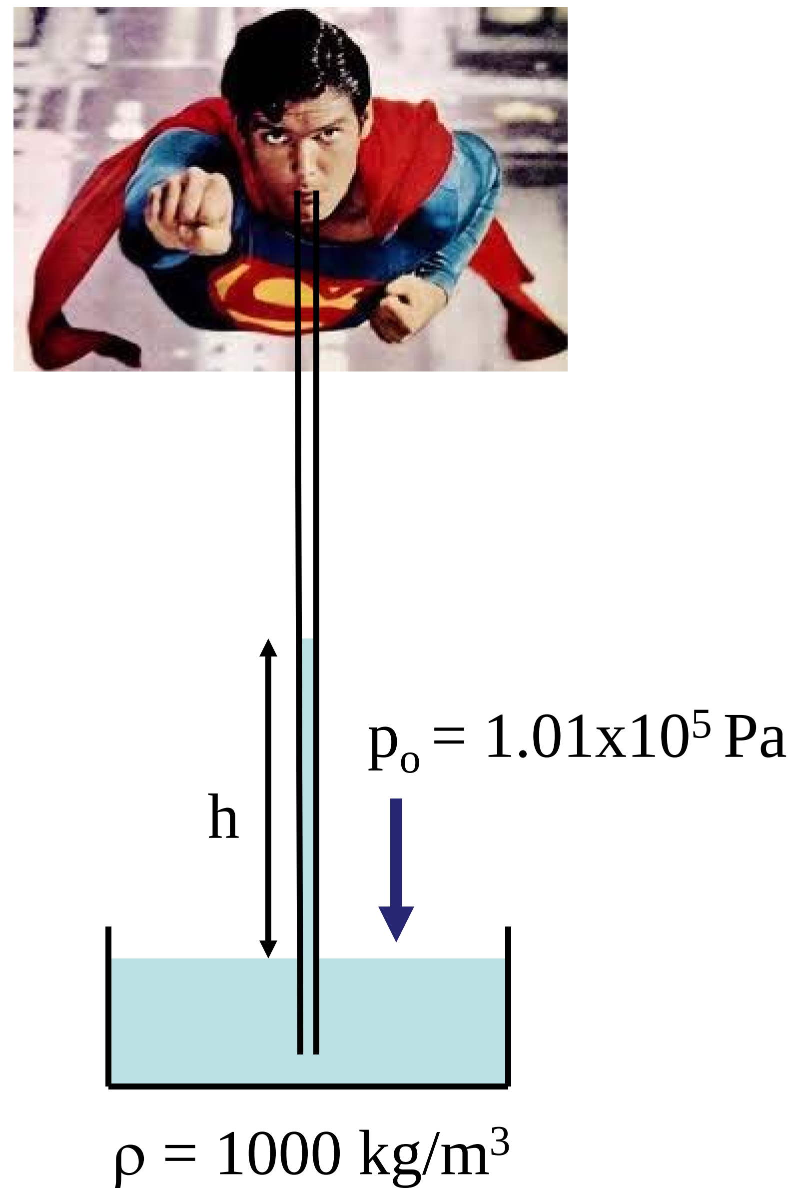
B) 32.0 m

C) 10.3 m



HITT

The barometer also has a vacuum in the upper part of the tube. Making that vacuum more perfect changes the height of the water negligibly.



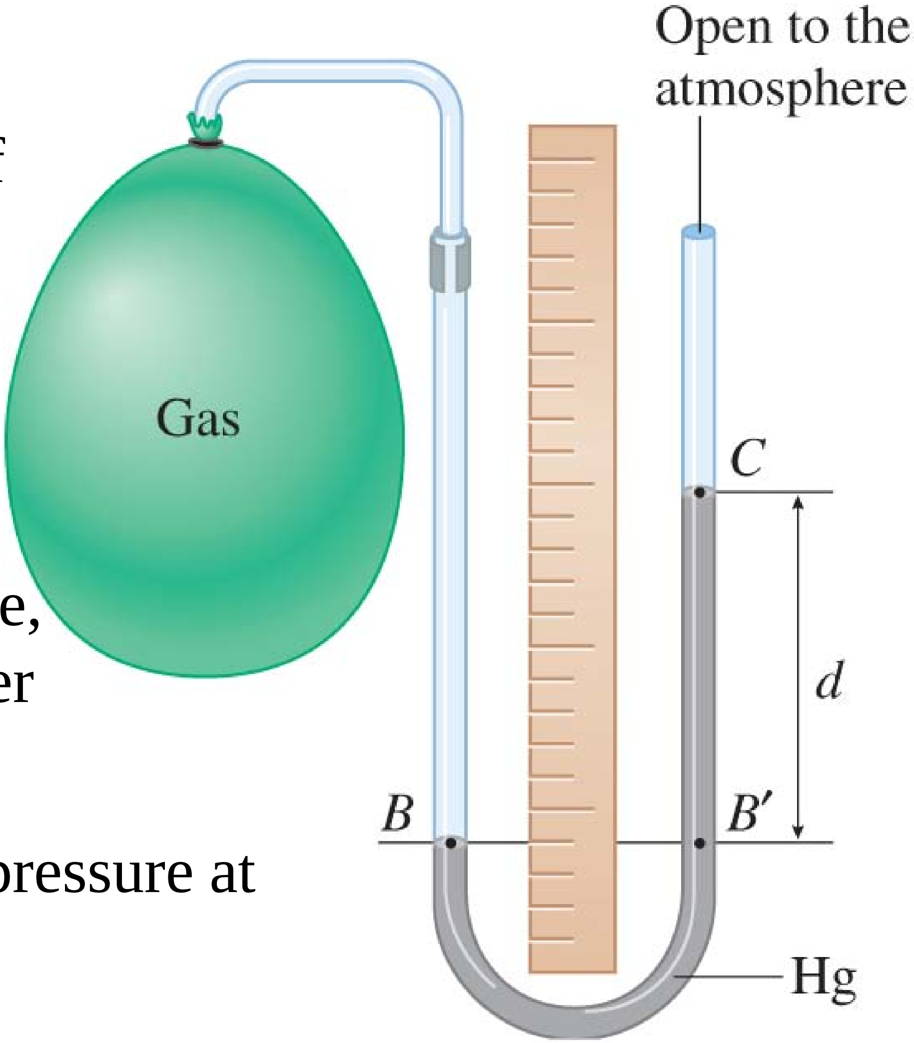
There we found $p_o = \rho g h$

so,

$$h = \frac{p_o}{\rho g} = \frac{1.01\times10^5\ \text{Pa}}{\left(1000\,\frac{\text{kg}}{\text{m}^2}\right)\left(9.8\,\frac{\text{m}}{\text{s}^2}\right)} = 10.3\,\text{m}$$

The open tube **manometer** works similarly to measure the *gauge* pressure of a gas inside an otherwise closed volume.

Before the gas volume is attached the **liquid level** on the two sides of the open U tube are **equal**.



After the gas volume is attached, if the gas pressure is > 1 atm, the liquid level on that side will be forced down by this added pressure, causing the liquid level on the other side to climb.

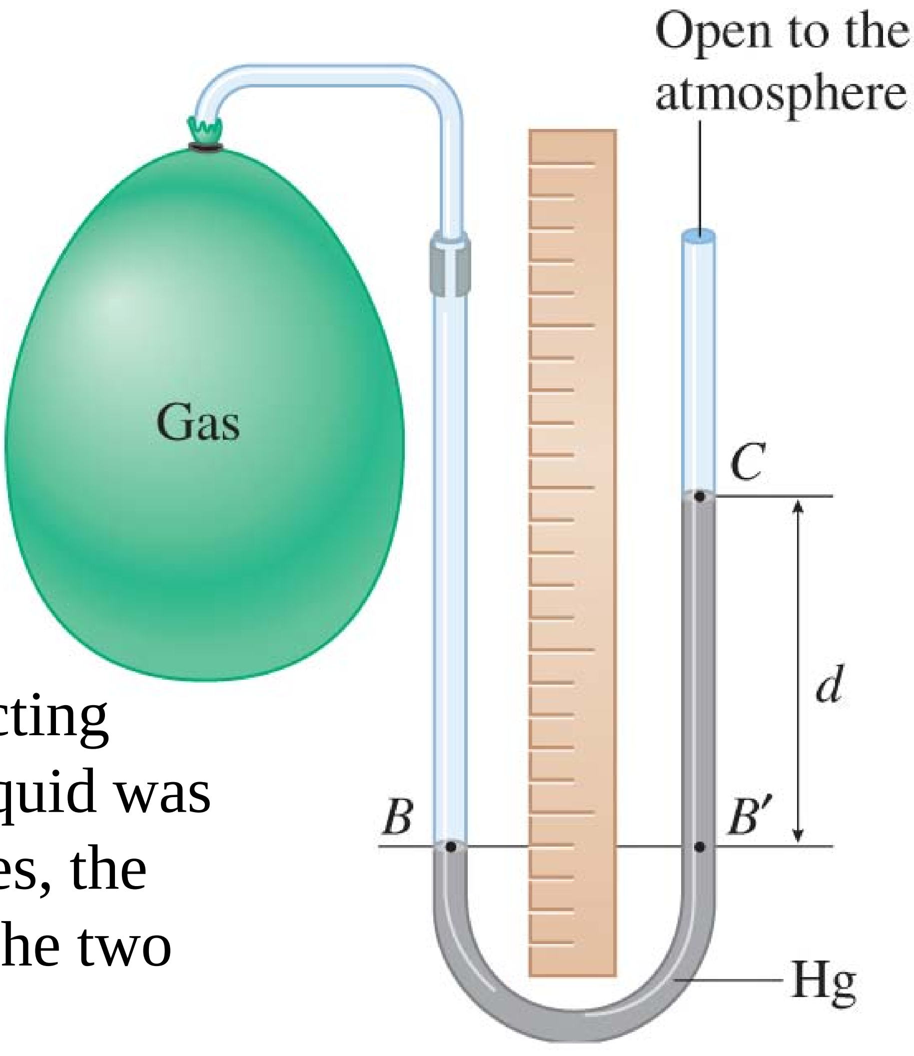
When they are in equilibrium the pressure at B and B′ must be equal.

But at B′ the pressure due to the column of height d above B′ (and B) is,

$$p_{B'} = \frac{F}{A} = \frac{mg}{A} = \frac{\rho g A d}{A}$$

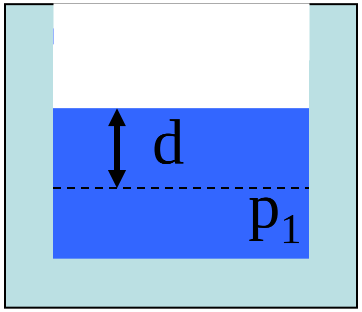
$$p_{B'} = \rho g d = p_g$$

Which is the gauge pressure (i.e. relative to the atmospheric pressure) because before connecting the pressurized gas, when the liquid was at the same level on the two sides, the atmosphere pressed equally on the two sides.



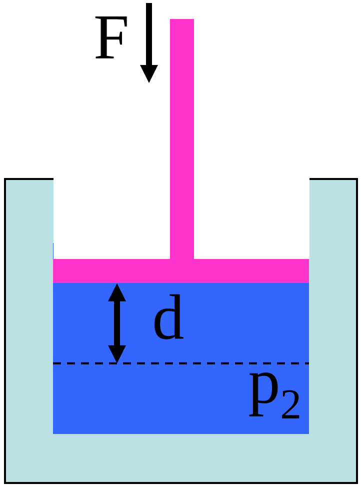
Pascal's Principle

Consider the circumstance to the right in which we have a cylinder of cross-section A filled with fluid.



The pressure at a depth d below the surface depends on the depth.

Let's now add a piston and apply an additional force to it. The pressure at depth d **increases** by



$$\Delta p = \frac{F}{A}$$

but that's true at every point of the fluid.

The pressure ***change*** is transmitted throughout the entire fluid.

Such a **change** in pressure would be transmitted independent of the source of the pressure change.

For example, if the temperature rises and the fluid expands, the resulting ***change*** in the pressure would occur throughout the volume of fluid.

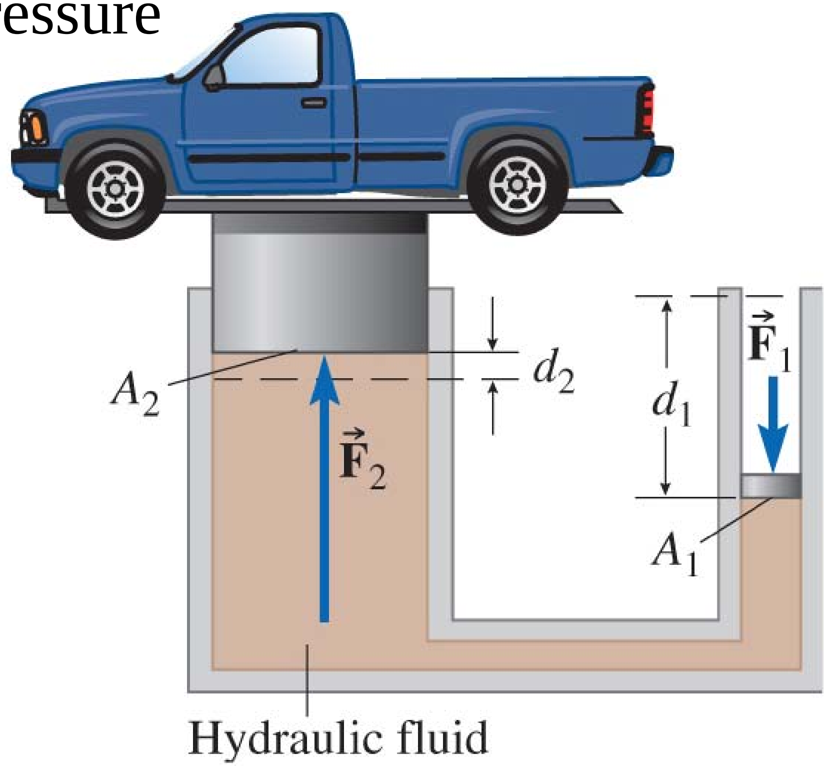
This is **Pascal's principle**: the ***change*** in pressure occurring in an incompressible fluid, in a closed container, **is transmitted undiminished to every portion of the fluid** and to the walls of the container.

This allows for a **hydraulic lift**, which consists of different area pistons/cylinders connected together as shown here.

A force $\vec{F}_1$ is applied at the **small area piston** and the force $\vec{F}_2$ occurs at the **large area piston**.

By Pascal's principle, the change in pressure is the same everywhere so

$$\Delta p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$



But then,

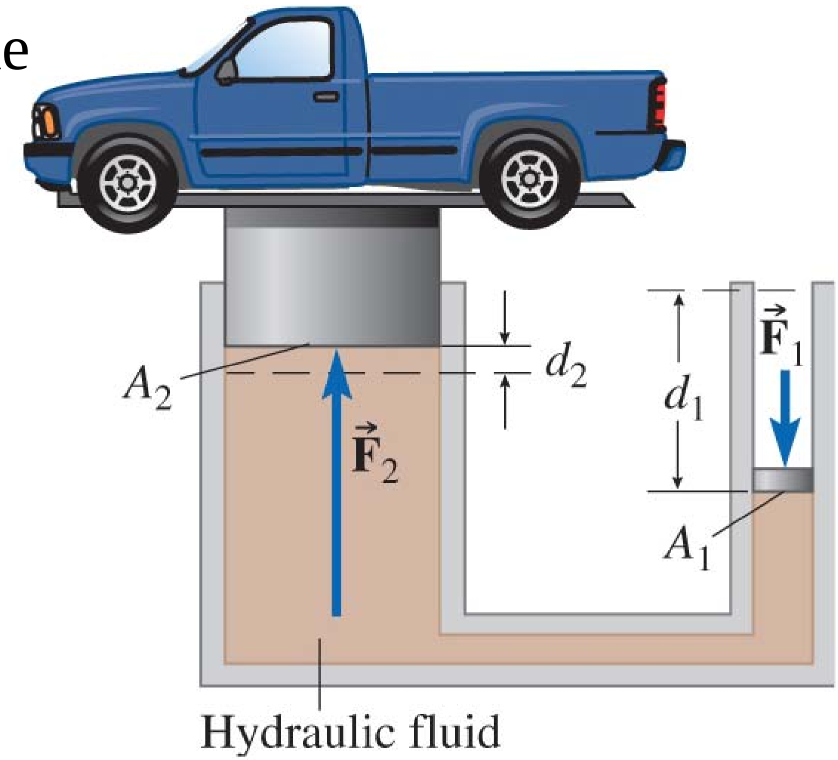
$$F_2 = \frac{A_2}{A_1} F_1$$

So force F_2 is force F_1 **multiplied** by the **ratio** of the **piston areas**.

For round cylinder/pistons:

$$F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi R_2^2}{\pi R_1^2} F_1 = \left(\frac{R_2}{R_1}\right)^2 F_1$$

So if the output cylinder has a 10 cm radius and the input cylinder a 1 cm radius the force multiplier is $(10/1)^2 = 100$.



This is how hydraulic lifts and the brakes in your car work.

Does this scheme defy conservation of energy?

(A wise thing to ask whenever we seem to be getting something seemingly extraordinary)

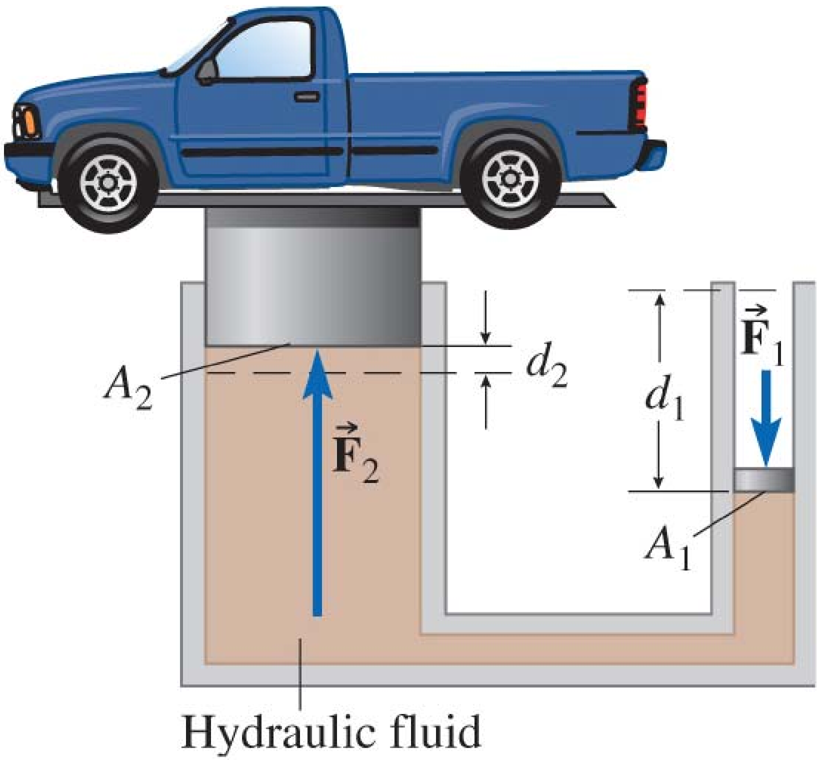
If piston 1 moves down a distance Δx_1 the **volume** of fluid it displaces is $\Delta V = \Delta x_1 A_1$. Since the fluid is incompressible this must be the **same volume displaced** by the opposing piston so,

$$\Delta V = \Delta x_1 A_1 = \Delta x_2 A_2$$

Or,

$$\Delta x_2 = \frac{A_1}{A_2} \Delta x_1$$

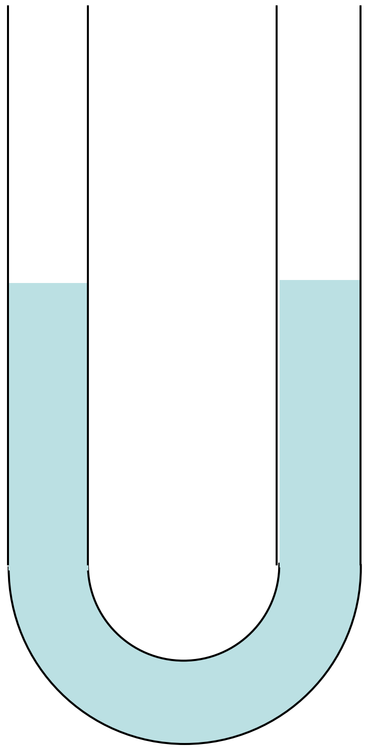
This is the **inverse ratio** of the force multiplier so the distance moved by the output piston is proportionately smaller than the distance moved by the input piston.



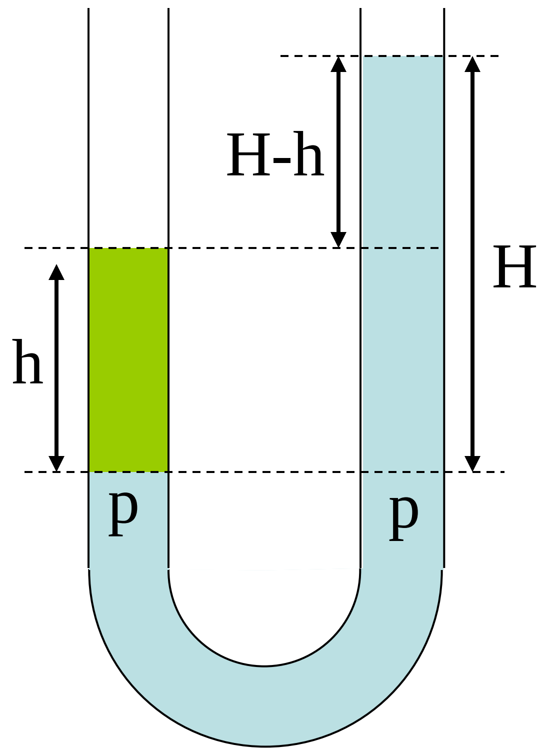
Since **work** is **force times displacement** the same work is done on both sides so that energy is conserved.

U-tubes (determining the **density** of an **immiscible** fluid)

Tube cross-sectional area A.



Initially – fluid of known density ρ_k



Add column h of fluid of unknown density ρ_u

($\rho_u > \rho_k$ case)

The pressure on the two sides at the lowest dashed line must be equal (or the fluid would move).

$$p_{left} = p_{right}$$

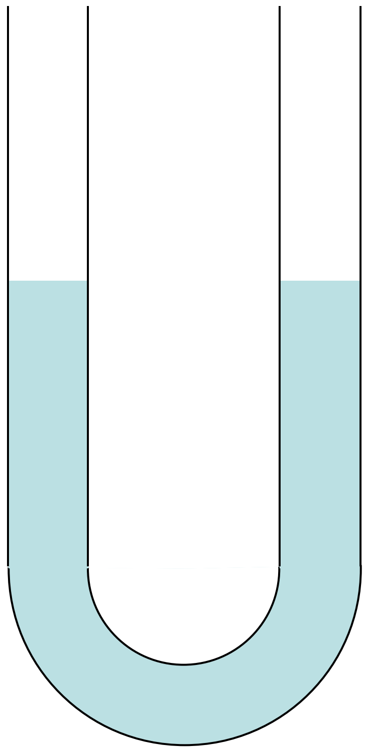
$$p_o + \frac{m_u g}{A} = p_o + \frac{m_k g}{A}$$

$$m_u = m_k$$

$$\rho_u A h = \rho_k A H$$

$$\rho_u = \frac{H}{h}\rho_k$$

If $\rho_u < \rho_k$



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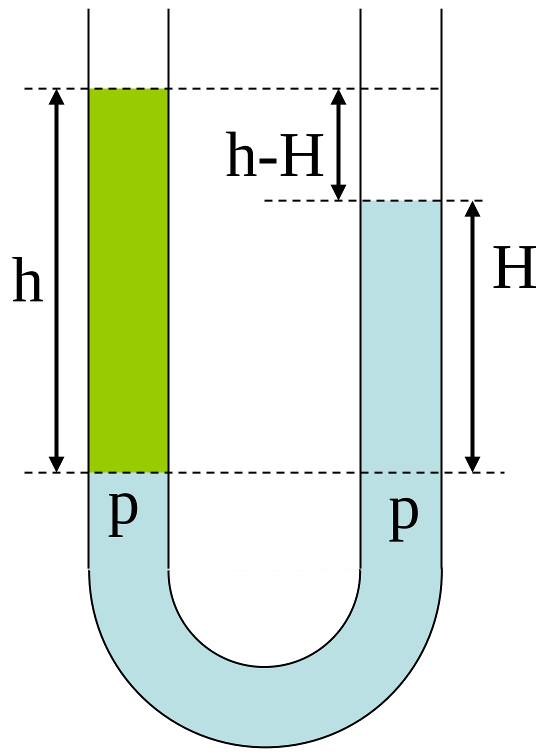

Add column h of fluid of unknown density ρ_u

The pressure on the two sides at the lowest dashed line must be equal (or the fluid would move).

$$p_{left} = p_{right}$$

$$p_o + \frac{m_u g}{A} = p_o + \frac{m_k g}{A}$$

$$m_u = m_k$$

$$\rho_u A h = \rho_k A H$$

So again,

$$\rho_u = \frac{H}{h}\rho_k$$

But now H < h.