LSTT Strengthening Secondary Education in Practice

## Mathematics

For Secondary School
Form 1


## About this book

This book is one of a series of three created by the project Strengthening Secondary Education in Practice: Language Supportive Teaching and Textbooks in Tanzania (LSTT). The books are intended as an example of the design of language supportive learning materials specifically for use in Tanzanian secondary schools. We hope that the ideas in this book will be taken up, adapted and developed further by educators, authors and publishers.

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# ISTI Strengthening Secondary Education in Practice Language Supportive Teaching and Textbooks 

## Books in the series

Biology for Secondary School - Form 1:
A language supportive textbook, Specimen Chapters
English for Secondary School - Form 1:
A language supportive textbook, Specimen Chapters
Mathematics for Secondary School - Form 1:
A language supportive textbook, Specimen Chapters

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## How to use this book

## Learning Mathematics and English together

This textbook supports Tanzanian students in Form I. When Form I students start learning subjects in English, they often cannot use the language well enough to learn Mathematics effectively. Good teaching builds on students' previous learning. For Form 1 students, their previous learning was in Kiswahili.

Form I students may find it hard to read in English, to talk in English, to listen to the teacher talking in English or to write in English. They also do not have the general and mathematical vocabulary needed to understand and express knowledge about Mathematics. For this reason, materials for learning Mathematics in English need to be:

- Language accessible. This means it is written in a simple way, with content communicated through diagrams and activities.
- Language supportive. This means that the textbook helps students to develop the English that they need to learn Mathematics. It also means that the book helps Form I students to recall their mathematical knowledge from primary school and translate this into English.
- Tanzanian. Mathematics is used in Tanzania on a daily basis. Mathematics was developed by men and women from different parts of the world in response to human needs, to solve problems and for fun. This book represents the multicultural background of Mathematics and its use in Tanzania.

This textbook was written by Mathematics Education specialists working together with Language specialists. It has been trialled by teachers and students in community schools in Dodoma, Lindi and Morogoro regions. Their feedback informed the final version of this book.

We think the result is a great book that will be easy for teachers and students to use. Above all, it will show students that learning Mathematics can be exciting, fun and useful. We hope you enjoy using the book as much as we enjoyed writing it.

## Using Kiswahili for Learning

Students who are still developing their ability to learn in English will learn both English and Mathematics quicker, if they are sometimes allowed to express their ideas in Kiswahili. Talking in Kiswahili helps students to remember and build on what they learnt in primary school. However, Kiswahili should be used strategically to support learning of and learning in English. Here are some examples:

- When you introduce a new topic to students, revise a topic from primary school or set a problem, you should allow students to discuss it briefly in Kiswahili in small groups or pairs.
- Make sure students know the meaning of mathematical vocabulary, for example, by referring them to the 'useful words' lists in the book or writing a vocabulary list on one side of the board.
- Give students support to express mathematical ideas in English. It can help them if they first discuss in pairs in Kiswahili how to express their idea in English - two heads are better than one. This may help them to write sentences that are grammatically accurate and use mathematical vocabulary correctly.
- Talking in Kiswahili helps students to talk in English. For example, when they have to talk about a new mathematical concept in English, they can talk about it first in a group in Kiswahili. This helps them understand the concept better and they are then better able to talk about the new concept in English.

This textbooks shows teachers and students how to use to use Kiswahili systematically and strategically to improve learning of Mathematics and learning of English.

## How the textbook helps students to learn

The textbook helps students to learn in many different ways. It has:


| Challenge Activities | In every class, some students can solve mathematical problems <br> quicker than others can. Some students enjoy Mathematics more <br> than others. These activities are designed to challenge and stretch <br> those students. They may not be suitable for everyone. |
| :--- | :--- |
| Worked examples | Each chapter has plenty of worked examples, with the steps <br> sometimes explained in both English and Kiswahili. Particular <br> attention is given to extracting mathematical information from <br> word problems. |
| Did you know? | These boxes give contextual information. Some give examples of <br> how Mathematics is used in Tanzania every day. Some explain <br> the origin of mathematical ideas, showing that Mathematics is <br> truly international. |
| Revision exercise | Each chapter ends with a 'revision exercise' with questions that <br> test learning of the entire chapter content. |
| What have I learned? | Each chapter ends with a checklist that students can use to <br> quickly review their learning and identify areas, which they may <br> need to practice further. |
| To remember | In Mathematics, students need to remember some information. <br> They will use this later as they continue with the curriculum. <br> This information is summarised in one place at the end of each <br> chapter. |

## Some Useful Teaching Strategies

Here are some useful strategies that will help you when you teach using this textbook.

## Teaching from the Front:

- Build on students' existing knowledge. This knowledge will be in Kiswahili or their mother tongue. Use Kiswahili to elicit this knowledge or to 'brainstorm'. Then introduce the English vocabulary for expressing their ideas in English. Support them to construct short statements in English. Writing and talking in English activities in this book are designed to give this support.
- Use diagrams, pictures and activities to help students to build concepts. Each chapter begins with pictures or activities for this purpose.
- Write key concepts on the board. You may ask students, working in pairs, to say them out to each other so that they practice talking about Mathematics in English. Make sure they know the meaning of key words by translating them into Kiswahili.
- When you explain an idea in Kiswahili, also show the students how to express the idea in English. Make sure they know the meaning of key English words. If they can only understand Kiswahili explanations, try to move them gradually from Kiswahili to English by teaching them new English vocabulary in context, showing them how to construct English sentences and allowing them time to practice constructing statements in English, including time collaborating in small groups or pairs.
- Check regularly whether students understand you. Ask questions to check this. Short answers (e.g. yes/no questions) are easy to answer. If you ask questions that require a longer answer and the learners cannot answer in English, accept their answers in Kiswahili. You can then translate them or give structured support to enable students to translate themselves.
- Remember that Form I students have to concentrate very hard to listen to English. If you talk for a long time in English, it will be difficult for them to keep focused on what you are saying.


## When students talk:

- Make sure students know what you expect them to do. Make sure they know the meaning of 'instruction' verbs used in the book, e.g. describe, discuss, explain, compare etc.
- Very few Form 1 students can express their mathematical reasoning in English. If you ask a student to demonstrate a solution on the board, accept explanations in Kiswahili.
- When students talk in English, try not to correct their English while they are speaking. Correct after they have finished, but without discouraging them.
- Never humiliate a student because he or she cannot talk English and do not allow students to humiliate or laugh at another student's English. Mutual respect should be part of the classroom culture. This will give the students confidence to try out English.
- If students cannot talk in pairs or groups in English about a concept, ask them to talk first in Kiswahili. As they finish, tell them that you are going to ask one or two pairs or groups to report in English. Give them a few minutes to decide what they will say in English. Help them with the useful vocabulary.
- When students work in pairs or groups, go round and listen. Help them where necessary.


## When Students read the textbook

- Ask students to work briefly in pairs or small groups and say what they know about the topic. Put a question on the board for them to answer. It doesn't matter if what they say is incorrect. A 3 -minute discussion will be enough. Then ask them to read the text.
- At first ask students to look at the glossary before reading the text. As they get better at reading, students can refer to the glossary as they read.
- If there is a picture or diagram, you can ask students look at this and talk about it in English or Kiswahili.
- Fill-the-blank activities make students think about what they are reading and helps them to understand the meaning. We found when piloting the textbook that students did not read any explanation in English. We have used fill-the-blank exercises to encourage them to read short texts. Students may complete these on their own, in pairs or in small groups.
- Get a few students to report to the whole class about what they understood. If a learner has understood the text but cannot explain it in English, accept an answer in Kiswahili, and translate for the class.


## When students write:

- Demonstrate to students how an activity should be done, and then ask the students to do it.
- It is useful for students to sometimes work in pairs when they write in English. They can discuss how to construct sentences, which words to use, how to spell etc. It is good if they discuss this in English, but it is just as good if they discuss in Kiswahili
- When students write, go round and read. Help them where necessary.
- When they have finished writing, it is sometimes useful to get one or two students to read their sentences out loud to the whole class, or even to dictate a sentence to you to put on the board. However, this kind of activity can take time, so keep it short.


## KWA MWANAFUNZI

Kitabu cha kiada kinawasaidia wanafunzi kujifunza kwa njia mbalimbali. Kina:

| makala rahisi | Kitabu kimeandikwa katika sentensi rahisi kwa Kiingereza <br> ili kiweze kueleweka kwa urahisi. |
| :--- | :--- |
| vielelezo na michoro | Hivi husaidia kufafanua dhana za kimahesabu. Vinawasaidia <br> wanafunzi kuelewa na kukumbuka dhana za kimahesabu na <br> kuwajengea stadi za kutambua changamoto za kimahesabu. |
| Mtajifunza kuhusu <br> yafuatayo | Kila sura inaanza na orodha ya malengo ya kujifunza <br> yaliyoelezwa katika Kiingereza. |
| maneno muhimu | Kila sura inaanza na tafsiri inayoonesha tafsiri ya maneno <br> muhimu yanayoonekana kwenye sura. Hii inawasaidia <br> wanafunzi kuhusianisha vitu walivyojifunza shule ya msingi. <br> Kwa nyongeza, tafsiri zimetumika katika kila sura pale msamiati <br> mpya ulipotumiwa. |
| mazoezi | Kitabu hiki kimezingatia kujifunza Hisabati kwa njia ya mazoezi. <br> Wanafunzi wanaweza kujifunza dhana mpya au wakaboresha <br> uelewa wao kwa kufanya mazoezi haya yaliyotungwa kwa <br> ustadi. |
| mazoezi ya kusoma, | Kujifunza kiingereza kwa ajili ya somo la Hisabati, wanafunzi <br> wanahitaji kusoma, kuzungumza na kuandika vitu mbalimbali <br> kuhusu Hisabati kwa kingereza. Kuna mazoezi ya kusaidia <br> maendeleo yao katika Kiingereza, ukijumuisha <br> zoezi kwa wanafunzi wawiliwawili na zoezi katika kikundi. |
| kuzungumza na kuandika | Kwa kila darasa, baadhi ya wanafunzi wanaweza kukokotoa <br> maswali haraka kuliko wanafunzi wengine. Baadhi wanafurahia <br> Hisabati kuliko wengine. Haya mazoezi yametungwa ili kuleta <br> changamoto na kuwajenga hao wanafunzi. |
| mazoezi changamoto | Kila sura ina mifano toshelevu iliyokokotolewa, katika hatua <br> ambazo wakati mwingine zimeelezwa kwa lugha zote yaani <br> Kiingereza na Kiswahili. Mkazo hasa umewekwa katika kuibua <br> mawazo ya kimahesabu kutoka mazoezi ya Hisabati. |
| Je, uliwahi kujua? | Visanduku hivi vinatoa habari ya kimuktadha. Vingine vinatoa <br> mifano juu ya jinsi Hisabati inavyotumika kila siku Tanzania. <br> Vingine vinaeleza chimbuko la mawazo ya kihisabati, <br> yakionesha kwamba Hisabati ni somo la kimataifa. |
| mifano iliyokokotolewa |  |


| zoezi la marudio | Kila sura inahitimishwa na 'zoezi la marudio' lenye maswali <br> yanayopima ujifunzaji wa maudhui yote kwenye sura. |
| :--- | :--- |
| Nimejifunza nini? | Kila sura inahitimishwa na orodha ya kupima kama wanafunzi <br> wanaweza kutafakari haraka ujifunzaji wao na kutambua <br> maeneo, ambayo wanaweza kuhitaji kufanyia mazoezi zaidi. |
| kukumbuka | Katika Hisabati, wanafunzi wanatakiwa kukumbuka baadhi ya <br> taarifa. Watatumia taarifa hizi baadaye kadri wanavyoendelea na <br> mtaala. Taarifa hii imeandikwa kwa ufupi katika sehemu moja <br> mwisho wa kila sura. |

## VIDOKEZO VYA NAMNA YA KUJIFUNZA

Bila shaka unajua umuhimu wa kumsilikiliza mwalimu akizungumza Kiingereza. Pia ni muhimu wewe uzoee kuzungumza, kusoma na kuandika Kiingereza wakati ukijifunza masomo mbalimbali. Matumizi ya Kiswahili yanaweza wakati mwingine kukusaidia kufanya hivyo. Usijisikie vibaya kutumia Kiswahili wakati ukijifunza masomo kwa Kiingereza. Utaona kuwa matumizi ya Kiswahili yanakusaidia kuelewa dhana mpya unazojifunza kwa Kiingereza. Matumizi ya Kiswahili yatakusaidia kujifunza Kiingereza kwa urahisi na kwa ufanisi zaidi ili ukitumie kujifunzia masomo mengine. Vifuatavyo ni vidokezo vitakavyokusaidia kujifunza.

- Unaposoma makala kwa Kiingereza, zungumza na mwenzako kwa Kiswahili kuhusu kile mlichosoma. Mkifanya hivyo, mtaelewa vizuri zaidi dhana mpya zilizo kwenye makala hiyo.
- Mara nyingi, utaona vigumu kuzungumza kuhusu dhana kwa Kiingereza. Zungumza kuhusu dhana hizo kwa Kiswahili kwanza. Kisha mwalimu atakapowataka muwasilishe kazi yenu kwa darasa zima kwa Kiingereza, tumieni muda mfupi kuamua katika kikundi chenu jinsi hasa mtakavyofanya hivyo. Mtaona kwamba, baada ya kujadili dhana mpya kwa Kiswahili, itakuwa rahisi kwenu kuzungumzia dhana hizo kwa Kiingereza.
- Unapokuwa na kitabu hiki nyumbani, tumia muda mfupi kuzungumzia mada mojawapo na mzazi au jamaa yako. Unaweza kutumia Kiingereza kama jamaa yako huyo anakifahamu. Lakini itakuwa vizuri hata mkitumia Kiswahili kwani kwa namna hiyo, mtazungumza vizuri na kuielewa kwa kina zaidi mada hiyo. Utakapokuwa shuleni na kuzungumza au kuandika kuhusu mada hiyo itakuwia rahisi kufanya hivyo kwa Kiingereza. Uelewa mzuri kwa Kiswahili utasaidia uelewa kwa Kiingereza.
- Iwapo huna nakala yako ya kitabu hiki, unaweza kujitengenezea orodha yako ya msamiati. Chukua daftari uandike maneno mapya ya Kiingereza na maana zake kwa Kiswahili. Maneno hayo na maana zake utavipata kwenye kitabu hiki, kwenye kamusi, au hata kwa kumuuliza mwalimu au rafiki yako. Unaweza pia kuandika tafsiri za dhana muhimu kwa Kiingereza na Kiswahili.
- Zungumza na kaka yako, dada au mzazi kuhusu mambo mnayofanya shuleni. Ni muhimu kuzungumza kuhusu picha au michoro iliyoko katika kitabu hiki.
- Uliza ndugu zako hao maswali kuhusu mada ulizojifunza. Unaweza kuuliza maswali kwa Kiswahili au kwa Kiingereza kama wapo ndugu wanaokifahamu.


## Chapter 1:

Numbers

## In this chapter you will learn about numbers.

## You will learn about:

- the idea of numbers;
- base ten number system;
- place value of digits in the base ten number system;
- natural and whole numbers;
- operations (+,-, x, $\div$ ) with whole numbers (up to 10 digits);
- factors and multiples of whole numbers; and
- integers.


## Some useful words:

## base ten <br> kizio cha kumi

number system
mfumo wa namba
place value
nafasi ya namba
digits
tarakimu
natural numbers
namba za kuhesabia
whole numbers
namba nzima
operation
tendo
factor
kigawe
multiple
kigawo
integer
namba kamili

### 1.1 The idea of numbers

## ${ }^{K}$

## What is a number?

Think on your own. Discuss in pairs. Now share your ideas with the class.
Figure 1.1 The idea of number


Some useful words:
represent
wakilisha
representation
uwakilishaji
symbol alama


Zero, one, two, three are all numbers.
We represent them by the numerals $\mathbf{0}, \mathbf{1}, \mathbf{2}, 3$.

## Writing numbers

Count the number of the students in the classroom.

Write the number as a word

- Write the number as a numeral.

These are two ways to represent a number. We can represent a number as a word or as a numeral. There are many number systems for representing numbers. Look at the figure 1.2.

Figure 1．2 Representation of＇eight＇in different number systems

## 8 八人く ゥ VIII

All the symbols in figure 1.2 represent the number＇ $\mathbf{8}$＇．They are numerals in different number systems．

Figure 1．3 A numeral has one or more digits

```
163
digit digit digit
```


## Je，uliwahi kujua？

Kielelezo 1.2 kinaonesha namna nane inavyoandikwa katika mifumo ya namba ifuatayo： mfumo wa Kihindi－Kiarabu； mfumo wa Kichina；mfumo wa Kiarabu；mfumo wa Kigujarati； mfumo wa Kithailendi na mfumo wa Kirumi cha Kale．

Kati ya mifumo hii ya namba， unafikiri ni mifumo ya namba ipi inayo tumika Tanzania kwa sasa？

Figure 1．4 We can use a numeral，words or things to represent a number

|  | $\begin{array}{ll} \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta & \Delta \\ \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta & \Delta \\ \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta & \Delta \\ \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta & \\ \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \\ \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \end{array}$ <br> things |
| :---: | :---: |
| Fill each blank with one of these words： <br> digit numeral one or more represent ten words <br> We can use a numeral， $\qquad$ or things to $\qquad$ （2）a number．A $\qquad$ is a symbol or group of symbols that stands for a number．A $\qquad$ is a single symbol in a numeral．A numeral has $\qquad$ 5）digits．We use the Hindu－Arabic number system．It has $\qquad$ ${ }_{(6)}$ digits． These are $0,1,2,3,4,5,6,7,8$ and 9 ． | Je，uliwahi kujua？ <br> Mfumo wa namba wa Kihindi－ Kiarabu ni mfumo unaotumika sana katika ulimwengu wa sasa． Ulivumbuliwa na wanahisabati wa Kihindi kati ya karne ya 1 na karne ya 4 BK．Katika karne ya 9， mfumo huo ulikuwa ukitumiwa na wanahisabati wa Kiajemi na Kiarabu huko Mashariki ya Kati． <br> Huenda ulianza kutumika mapema sehemu za Pwani za Tanzania lakini ushahidi wa kwanza tulionao ni maandishi yaliyopo kwenye Kizimkazi Zanzibar ya mwaka 1107 BK． |

## Je, uliwahi kujua?

Mfumo Kale wa namba wa Misri ulitumika Misri kwa takribani miaka 4000, kuanzia yapata miaka 3000 KK.


Figure 1.5 Ancient Egyptian number system.

| 1 | 10 | 100 | 1000 |  |
| :---: | :---: | :---: | :---: | :---: |
|  | 1 |  |  |  |
|  |  |  |  |  |
| a stroke |  |  |  |  |
|  | heel bone | coil of rope |  |  |
|  |  |  |  |  |

Figure 1.6 Ancient Egyptian numbers


In groups, look at figures 1.5 and 1.6.

1. Write down the number of students in the class using the Ancient Egyptian number system.

2 Write the following numbers using the Ancient Egyptian number system:
7, 14, 69, 101, 165, 472, 530, 806, 1050 and 1876.
3. Create your own number system.

### 1.2 The base ten number system

There are different number systems. We use a base ten number system. In base ten:

18 means eighteen. The ' $\mathbf{1}$ ' represents one group of ten.
$\mathbf{2 5}$ means twenty five. The ' $\mathbf{2}$ ' represents two groups of ten.
Figure 1.7 Counting beans in base ten

Figure 1.8

Look at figure 1.8. Fill the blanks with the words:

In English, we say there are $\qquad$ sticks.
The $\qquad$ (2) for this number is 12 . The ' 1 ' in ' 12 'represents a complete group of $\qquad$ (3). The $\qquad$ (4) ${ }^{\text {in ' }} 12$ ' represents the two remaining ones. We group in tens because we count in (5).


## '2' base ten numeral ten twelve



Look at figure 1.9. We can see that for the numeral 148,

| Place value: | hundreds | tens | ones |
| :---: | :---: | :---: | :---: |
|  | 1 | 4 | 8 |
|  | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta$ |
|  | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta$ |
|  | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta$ |
|  | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta$ |
|  | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta$ |  |
|  | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta$ |  |
|  | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta$ |  |
|  | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta$ |  |
|  | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ | $\Delta \Delta \Delta \Delta \Delta$ |  |
|  | $\Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta \Delta$ |  | $\Delta$ |
|  |  |  |  |

The place value of the digit $\mathbf{8}$ is one. The ' 8 ' in 148 represents eight ones.
The place value of the digit $\mathbf{4}$ is ten. The ' 4 ' in 148 represents four tens.
The place value of the digit $\mathbf{1}$ is one hundred. The ' 1 ' in 148 represents one hundred.

## E

## Consider the numeral 56 and complete the sentences below.

The place value of the digit $\mathbf{6}$ is $\qquad$
The ' 6 ' in 56 represents six
The place value of the digit $\mathbf{5}$ is $\qquad$ . .

The ' 5 ' in 56 represents five

## Consider the numeral 470 and complete the sentences below.

The place value of the digit $\mathbf{0}$ is $\qquad$
The ' 0 ' in 470 represents zero $\qquad$
The place value of the digit 7 is $\qquad$
The ' 7 ' in 470 represents seven $\qquad$
The place value of the digit $\mathbf{4}$ is $\qquad$
The ' 4 ' in 470 represents four
Consider the numeral 5039 and complete the sentences below.
The place value of the digit $\mathbf{9}$ is $\qquad$ .
The ' 9 ' in 5039 represents
The place value of the digit $\mathbf{3}$ is $\qquad$ . .

The ' 3 ' in 5039 represents $\qquad$
The place value of the digit $\mathbf{0}$ is $\qquad$ . .
The ' 0 ' in 5039 represents $\qquad$
The place value of the digit $\mathbf{5}$ is $\qquad$ . .

The ' 5 ' in 5039 represents $\qquad$

Look again at figure 1.9.
What happens to the top row as you move from right to left? If we added another column on the left, what would be the word at the top? And if we added another column, what would be the word at the top?

Because we count in base ten, the value of each column is ten times the value of the column on its right.

For example, consider the numeral 3461:
Table 1.1 Place value table

| Place value | thousands | hundreds | tens | ones |
| :--- | :--- | :--- | :--- | :--- |
|  | $10 \times 10 \times 10$ | $10 \times 10$ | 10 | 1 |
| digits | 3 | 4 | 6 | 1 |
| value of digit | 3000 | 400 | 60 | 1 |

' $\mathbf{1}$ ' in 3461 represents $1 \times 1$. Its place value is 1 .
' $\mathbf{6}$ ' in 3461 represents $6 \times 10=60$. Its place value is 10 .
' $\mathbf{4}$ ' in 3461 represents $4 \times 10 \times 10=4 \times 100=400$. Its place value is 100 .
' $\mathbf{z}^{\prime}$ in 3461 represents $3 \times 10 \times 10 \times 10=3 \times 1000=3000$.
Its place value is 1000 .
Write down the place value and value of each digit in the numeral 3,156,904.

The place value of 4 is 1 . Its value is 4 .
The place value of 0 is 10 . Its value is 0 .
The place value of 9 is 100 . Its value is 900 .
The place value of 6 is 1000 . Its value is 6000 .
The place value of 5 is 10000 . Its value is 50000 .
The place value of 1 is 100000 . Its value is 100000 .
The place value of 3 is 1000000 . Its value is 3000000 .

## Exercise 1.1

Work in pairs and take it in turns to answer the questions.
Check your partner's answers are correct.

1. What is place value of ' $\mathbf{3}$ ' in the following numbers? Say your answers out loud.
(a) 43
(b) 436
(c) 3978
(d) 230984
2. Construct a place value table like that in figure 1.10 for each of the numerals below.
(a) 356
(b) 7602
(c) 509417
(d) 7103562

Worked example

## Ex 1.1

Fanya kazi mkiwa wawili na kujibu maswali.
Angalia kama majibu ya mwanafunzi mwenzio yako sahihi.

## Worked example

## Je, uliwahi kujua?

Mfumo wa vizio wa Kimataifa
Katika kitabu hiki, tunafuata mfumo wa vizio wa kimataifa (unafupishwa kama S.I) kwa ajili ya kuandika tarakimu. Mfumo huu umeafikiwa na wanasayansi duniani kote.

Katika namba kubwa, tarakimu zimetenganishwa katika makundi ya namba tatu. Utaratibu huu haufuatwi kwa namba zenye tarakimu nne peke yake. Utaratibu huu hutumika wakati unapohitaji tarakimu kuwa katika msitari katika jedwali (kwa mfano, tazama jedwali 1.3).

Siyo kila mtu ni mwanasayansi. Wakati mwingine, unaweza kuona tarakimu katika makundi matatu matatu ambayo yametenganishwa kwa kutumia mikato (,). 76,483,522 na 76483522 na 76483522 zote zikimaanisha kitu kilekile. Unaposoma Hisabati, unatakiwa kuandika 76483522.

5,023 na 5023 na 5023 zote zinamaanisha kitu kilekile isipokwa unatakiwa kuandika 5023.
3. Look at the numerals below. What is the place value of each digit? What is the value of each digit? Say your answers out loud.
(a) 12
(d) 50
(g) 560
(j) 7833
(m) 721044
(b) 36
(e) 458
(h) 805
(k) 60755
(n) 5882441
(c) 7
(f) 949
(i) 2321
(I) 95052

### 1.3 Representing numbers with words

English and Kiswahili both count in a base ten number system. So, place value can help us write a number as a word.

$$
\begin{aligned}
& \text { For example, write } 53,217 \text { in words. } \\
& 53 \begin{array}{rlrl}
534 & =5 \times 10000+3 \times 1000+2 \times 100+4 \times 10+7 \times 1 \\
& =50000+3000+200 & +40 & +7 \\
& =53000 & +200 & +47
\end{array}
\end{aligned}
$$

Answer: fifty three thousand two hundred and forty-seven


Look at the second line of the worked example above.
$50000+3000+200+40+7$
We say 53247 is written in expanded form.
Figure 1.10 Place values for 52,347


Sometimes we write numbers with a space or comma (,) between every three digits. This helps you say large numbers as a word.

For example, consider $\mathbf{6 0 2 8 4 3} 501$.

Figure 1.11 Writing large numbers as a word

# 602843501 

six hundred and two million
eight hundred and forty-three thousand
five hundred and one

We write this as:
six hundred and two million eight hundred and forty three thousand five hundred and one


## What do you notice about the English and Kiswahili number systems?

Do you know any other number systems?
Do they use a base ten?


Ex 1.2

1. Write down the numbers below in words.
(a) 11
(c) 321
(e) 2063
(g) 81945
(i) 6700389
(b) 18
(d) 708
(f) 8890
(h) 560308
(j) 104863502
2. Find a partner. Read your answers out loud to your partner.
3. Write the following numbers in numerals.
a) two hundred and thirty five
b) three hundred and fifteen
c) six thousand three hundred and fifty eight
d) eight thousand eight hundred and eighty eight
e) sixteen thousand one hundred and seven
f) two million five hundred and six thousand
g) five million three hundred thousand six hundred and ten
h) seventeen million
i) twenty-eight million six hundred and twenty
j) one billion four hundred and six thousand and two

### 1.4 Natural and Whole numbers

In groups, count out loud the number of students in your class. Then count the number of windows in your classroom.

1. When you counted, which number did you start with?
2. In each case, what was the last number you said?
3. Are the last numbers in the two cases the same?
4. If the answer in 3 above is no, why are the two different?
5. When can the two numbers be the same?

Counting begins with $\mathbf{1}$ and goes on with $\mathbf{2 , 3}, \mathbf{4} \ldots$
The three dots (...) mean that the numbers continue without end.
The numbers we use for counting are called natural numbers or counting numbers.

Natural numbers can be shown on the number line as in the diagram below.
The arrow, like $\cdots \cdot$ means the numbers continue without end
Figure 1.12 Natural numbers shown on a number line


Figure 1.13 Place ten pens on your desk


## Place ten pens on your desk.

- Say out loud, in English, the number of pens on the table.
- Remove one pen. How many pens are on the table?
- Remove another pen. How many pens are on the table now?
- Repeat until only one pen remains on the table.
- Now remove the last pen. How many pens are on the table now? Write down the number as a word and as a numeral.

If we add zero ' $\mathbf{0}$ ' to the natural numbers, we call the new series of
numbers whole numbers. So, the whole numbers are $\mathbf{0}, \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots$

Whole numbers are denoted by $\mathbf{W}$.
Figure 1.14 Whole numbers shown on a number line.


- Natural numbers that are divisible by $\mathbf{2}$ are called even numbers.
- Natural numbers that cannot be divided by $\mathbf{2}$ are called odd numbers.
- A natural number that can only be divided by $\mathbf{1}$ and itself is called a prime number. (1 is not a prime number.)
four, five, and so on are called $\qquad$ . They can also be represented as $1,2,3,4,5, \cdots$. Natural numbers can be represented on a line. This line together with the numbers is called a $\qquad$
The arrow to the right of the number line means that the natural numbers: have no $\qquad$


### 1.5 Even, odd and prime numbers

In pairs, discuss the following questions.

- Which natural numbers can be divided by 2 ? What do we call this group of numbers?
- Which natural numbers cannot be divided by 2 ? What do we call this group of numbers?
- What do you notice about the numbers 3, 5, 7, 11 and 13? What do we call these numbers?
Fill the blanks with the words:
number line zero end natural numbers
If there are no cows in your classroom, then we say that there are ......................(1) cows in that classroom. The numbers one, two, three, ${ }^{(4)}$.


## Je, uliwahi kujua?

Mfumo kale wa namba wa Misri ulikuwa ni miongoni mwa mifumo ya kwanza kuwa na alama ya sifuri, moyo pamoja na koo. Ilikuwa pia ni alama ya urembo. Kwa mara ya kwanza matumizi yake yalifahamika mwaka 1740 KK.


Find the prime numbers between 1 and 100

1. Copy figure 1.15.
2. Cross out 1 because it is not a prime number.
3. Cross out all the even numbers except 2.2 is a prime number.
4. Cross out all numbers that can be divided by 3 except 3 . 3 is a prime number.
5. Cross out all numbers that can be divided by 5 except 5 .
6. Cross out all numbers that can be divided by 7 except 7 .
7. The next prime number is 11 . You have already crossed out all numbers that can be divided by 11 except 11 .
8. Write down the numbers that have not been crossed out. These are all the prime numbers between 1 and 100 .

Figure 1.15 The numbers 1-100

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

## E

## Complete the sentences:

Even numbers are numbers that
For example,
are even numbers.

Odd numbers are numbers that
For example are odd numbers.

A prime number is a number that $\qquad$
For example. $\qquad$ are prime numbers.

1. a) Draw a number line like that in figure 1.15 . Circle all the natural numbers.
b) Draw a number line up to 10 . Circle the even numbers.
c) Draw a number line up to 10 . Circle the prime numbers.
2. Circle the following numbers on separate number lines:
a) Even numbers less than 20 .
b) Odd numbers less than 20 .
c) Prime numbers less than 20 .
3. Write down all the prime numbers between 0 and 40 .
4. Write down all natural numbers below 20 .
5. Write down all prime numbers between 10 and 50 .
6. Write down the first three prime numbers after 60 .
7. Write down a number, which is both even and prime.
8. Which of the numbers, $2,3,5,7,9,13,15$ and 19 , are not prime numbers?
9. Write a number that is a whole number but not a natural number.
10. a) Think of two even numbers. Add them together. Is the answer an even number or an odd number?
b) Now add two different even numbers.

Is the answer an even number or an odd number?
11. a) Think of two odd numbers. Add them together.

Is the answer an even number or an odd number?
b) Now add two different odd numbers.

Is the answer an even number or an odd number?
12. a) Take an even number and an odd number. Add them together.

Is the answer an even number or an odd number?
b) Now add a different even and odd number.

Is the answer an even number or an odd number?
13. a) Think of two prime numbers. Add them together.

Is the answer a prime number?
2) Now add two different prime numbers.

Is the answer a prime number?

Some useful words:
add
jumlisha, ongeza
addition
ongezeko
subtract
toa, ondoa
subtraction
punguzo
operations
matendo (mfano. $\div,-,+$ )
multiplication
kuzidisha
division
kugawanya


### 1.6 Addition and subtraction using number lines

Whole numbers can be operated on by addition (+), subtraction (-), multiplication $(\times)$ and division $(\div)$.

These are called operations.

### 1.6.1 Addition (+) using number lines

When you add two or more numbers you find its sum.
For example, we say:
The sum of 2 and 5 is 7 .
The sum of 8 and 11 is 19 .
Whole numbers can be added using a number line.
Example 1: Show $3+5$ on a number line
Figure 1.15


Answer: $3+5=8$
Example 2: Show $4+7$ on a number line
Figure 1.16


Answer: $4+7=11$

### 1.6.2 Subtraction (-) using number lines

Subtraction is the opposite of addition. To subtract is to take away a number from another.

The subtraction $\operatorname{sign}(-)$ is sometimes called minus.
Worked example

For example, consider: 12-5 = $\mathbf{7}$
We can say:
Twelve subtract five equals seven.
or: Twelve take away five equals seven.
or: Twelve minus five equals seven.
or, we can say: The difference between twelve and five is seven.

## Look at the questions below:

What is 15 subtract 7 ?
What is 15 take away 7 ?
What is 15 minus 7 ?
What do you get if you subtract 7 from 15 ?
What do you get if you take 7 away from 15 ?
What is the difference between 15 and 7 ?
What is the difference between 7 and 15 ?
They all mean the same thing: $\mathbf{1 5 - 7 =}$ ?

## How many ways can you say in words 20 - 8 = ?

We can also use a number line to help us do a subtraction.
Example 1: Show $12-8$ on a number line.
Figure 1.17


Answer: 12-8=5
Example 2: Use a number line to subtract 4 from 9.
Figure 1.18


Answer: $9-4=5$

## Exercise 1.4

1. Complete the following sentences:
a) 11 add 9 is $\qquad$
b) 4 $\qquad$ 3 is 7
c) The sum of 6 and 8 is $\qquad$
d) The $\qquad$ of 5 and 10 is 15
e) The sum of 4 and $\qquad$ is 7

## Some useful words:

take away
ondoa, toa
minus
kutoa
difference tofauti

Worked example

Ex 1.4

## Some useful words:

Horizontal
a mlalo
Vertical
a wima
2. a) Use a number line to find the sum of 11 and 9
b) Use a number line to find the sum of 15 and 4
3. Show the following additions on a number line:
a) $5+3=$
b) $8+7=$
c) $11+6=$
d) $13+7=$
4. Complete the following sentences:
a) 11 subtract 5 is $\qquad$
b) 23 $\qquad$ 8 is 15.
c) 35 take away 9 is $\qquad$
d) 46 $\qquad$ 12 is 34.
e) 30 minus 6 is $\qquad$ ..
f) 53 $\qquad$ 8 is 45 .
g) The difference between 25 and 18 is $\qquad$
h) The $\qquad$ . $\qquad$ 58 and 42 is 16.
5. a) Use a number line to find 19 take away 4
b) Use a number line to subtract 8 from 17
c) Use a number line to find the difference between 13 and 6
6. Show the following subtractions on a number line:
a) $5-3=$
b) $18-7=$
c) $12-6=$
d) $13-7=$

### 1.7 Horizontal and vertical addition and subtraction

Number lines can only help with small numbers. With larger numbers we can do horizontal and vertical additions and subtractions.

Examples of horizontal addition are,
(a) $432+216=648$
(b) $1895+104=1999$
(c) $25+197=222$

Examples of horizontal subtraction are,
(a) $45-12=33$
(b) $165-32=133$
(c) $254-121=133$

## Try the following additions:

(a) $14+5=$
(b) $340+28=$
(c) $78+17=$
(d) $78+35=$

Which questions were the easiest?
Which questions were the hardest?
How did you find the answers? Discuss with a friend.
Now try these subtractions:
(e) $29-6=$
(f) $345-21=$
(g) $56-8=$
(h) $345-168=$

Which questions were the easiest?
Which questions were the hardest?
How did you find the answers? Discuss with a friend.

### 1.7.1 Horizontal addition

In horizontal addition, we start by adding the digits with the same place value.

For example: $341+228=$ ?

| hundreds | ens | ones | + | hundreds | tens | ones | $=$ | hundreds | tens | ones |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 4 | 1 |  | 2 | 2 | 8 |  | $3+2$ | $4+2$ | $1+8$ |
|  |  |  |  |  |  |  | $=$ | 5 | 6 | 9 |

Answer: $341+228=569$

Now consider: $248+234=$ ?

| hundreds | tens | ones |  | ndreds | tens | ones | $=$ | hundreds tens |  | ones |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 4 | 8 | $+$ | 2 | 3 | 4 |  | $2+2$ | $4+3$ | $8+4$ |
| We cannot have more than 9 in a place value. |  |  |  |  |  |  | $=$ | 4 | 7 | 12 |
| So, we 'carry 1 ' to the next place value. |  |  |  |  |  |  | $=$ | 4 | 7 | $10+2$ |
|  |  |  |  |  |  |  | $=$ | 4 | $7+1$ | 2 |
|  |  |  |  |  |  |  | $=$ | 4 | 8 | 2 |

Answer: $248+254=482$

## Some useful words:

Carry
beba, chukua

## Worked example

### 1.7.2 Horizontal subtraction

In horizontal subtraction, we start by subtracting numbers in the same place value.

For example: 678-243

| hundreds | ens | ones | - | hundreds | tens | ones | $=$ | hundreds | tens | ones |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 7 | 8 |  | 2 | 4 | 3 |  | 6-2 | 7-4 | 8-3 |
|  |  |  |  |  |  |  | $=$ | 4 | 3 | 5 |

Answer: $678-243=435$

Now consider: 341-228


Answer: $341-228=113$

### 1.7.3 Vertical addition

When there are numbers to carry it is easier to use vertical addition.

## For example:



Answer $=648$
Now consider: $26+197=$ ?


Answer $=223$

| For example: | $100 \mathrm{~s}_{10 \mathrm{~s}}^{1 s}$ |
| :---: | :---: |
|  | 897 |
|  | 458 |
|  | 302 |
| carry $\begin{gathered}\text { Cat } \\ \text { thousands } \\ \text { to } 1000 \mathrm{~s}\end{gathered}$ | +515 |
| to 1000s column | $\underline{+2+1+2}$ |
|  | 2172 |
| Answer $=2172$ |  |

### 1.7.4 Vertical subtraction

What is $678-243$ ?


Answer $=435$

Worked example

Some useful words:
borrow azima, kopa

Now consider 856 - 219.


$$
\begin{array}{ll}
\|\| \\
\hline 637
\end{array}
$$

Answer = 637.

### 1.8 Word problems with addition and subtraction

| Words or phrases mostly used | Sign |
| :--- | :---: |
| Addition, sum of, increased by, greater than, plus, <br> taller than, more than, larger than, older than, total | + |
| Subtract, difference, decreased by, less by, minus, <br> shorter than, reduced by, younger than. | - |

## Look at the worked example for 856 - 219 above.

``` What do you think 'borrow' means?
Look at the worked example for 856-219 above.
What do you think 'borrow' means?
```

Worked example

## Worked example

First read the question carefully. Pick out the mathematical information. What are you asked to find? Second, write this down using numerals.
Kwanza, soma swali kwa umakini ili kubaini taarifa za kihisabati na ujue unachotakiwa kukifanya. Kisha, ziandike kwa kutumia tarakimu.

Pick out the mathematical information.

## Bainisha taarifa za kihisabati.

Pick out the mathematical information.

## Bainisha taarifa za

kihisabati.

## Worked Example 1

A teacher bought pens from school shop. She kept fifteen pens and gave the rest to the students. She gave ten pens to boys and seven pens to girls. What is the total of the pens that the teacher bought from school shop?

Teacher kept fifteen pens and gave ten pens to boys and gave seven pens to girls, total = ?
15
10
7

$$
15+10+7=32
$$

Answer: 32 pens

## Worked Example 2

Salma went to the market. She spent a thousand shillings on rice, five hundred shillings on spinach and one thousand three hundred shillings on meat. How much did she spend in total?
Salma went to the market. She spent a thousand shillings on rice, five hundred shillings on spinach and one thousand three hundred shillings on meat. How much did she spend in total?

| Rice | $\mathbf{1 0 0 0}$ |
| :--- | ---: |
| Spinach | $\mathbf{5 0 0}$ |
| Meat | $\mathbf{1 3 0 0}$ |
| Total | $\mathbf{2 8 0 0}$ |

Salma spent Sh 2800.

## Worked Example 3

George bought a bag of rice containing fifty kilogrammes. He cooked nine kilogrammes for a wedding ceremony. He gave fifteen kilogrammes to his aunt. How many kilogrammes of rice remained?
George bought a bag of rice containing fifty kilogrammes. He cooked nine kilogrammes for a wedding ceremony. He gave fifteen kilogrammes to his aunt. How many kilogrammes of rice remained?

## $50-3 \quad 9 \quad-\quad 15 \quad=26$

Bag of $\mathbf{5 0} \mathbf{k g}$, cooked $\mathbf{9 k g}$, gave away $\mathbf{1 5} \mathbf{k g}$, remained?
Answer: 26kg remained

## Worked Example 4

Mr. Kazi had 675 goats. After a few days, 227 goats died from eating poisonous food. How many goats did Mr. Kazi remain with?

Mr. Kazi had $\mathbf{6 7 5}$ goats. After a few days, 227 goats died from eating poisonous food. How many goats did Mr. Kazi remain with?

Mr. Kazi had 675 goats, 227 died, remained with?

$$
675-227=448
$$

Answer: Mr. Kazi remained with 448 goats

Worked example

Pick out the mathematical information.

## Bainisha taarifa za kihisabati.

## Exercise 1.5

## Ex 1.5

1. Work out the following additions:
a) $82516+1563=$
h) 12785
b) $6734+347=$
c) $7629+134=$
d) $992728+739=$
е) $743+109=$
f) 7618
+1385
j) 9375
$\qquad$
i) 802079 $+147883$
g) 75288
$+12816$
2. Work out the following subtractions:
a) $35328-21619=$
h) 284

b) $12657-1253=$ | -29 |
| :--- |

c) $84216-3248=$
d) $3213-1362=$
e) $47810-3109=$
i) 984723
f) 6485 $-24629$
$\qquad$
$\qquad$
g) 76896
j) 3568245923 $-143213641$
3. 2000 students were selected to join Form One. 1013 students were girls. Only 684 girls reported and 892 boys reported.
a) Number of boys who reported to school and the number of girls who reported to school is $\qquad$ . .
b) The difference between students who reported to school and those who did not report to school is $\qquad$ ..
c) The number of girls who did not report to school is
$\qquad$ .
4. Write nine sentences using words from each column in the table 1.2. For example: Twelve plus five equals seventeen.

How many mathematically correct sentences can you find?

## Table 1.2

| Twelve | plus |  | equals | seventeen. |
| :---: | :---: | :---: | :---: | :---: |
| Twenty |  | three |  | twenty-five. |
| Forty |  | five |  | forty. |
| Sixty-two |  | fifteen |  | fifty-seven. |
| One hundred and six | minus | twenty-two forty-three |  | eighty-four. one hundred |
| One hundred and fifteen |  | seventy-five |  | and thirtyseven. |

5. Juma has 25000 shillings and spends 2600 shillings. How much does he still have?
6. 2680 dogs were tested for rabies. 15 of them were found to be infected. How many dogs were not infected?
7. James bought mangoes at Kariakoo Market. He ate five mangoes and gave the rest to his friends. He gave Jane three mangoes, Ali two mangoes and Asha four mangoes. How many mangoes did James buy from Kariakoo Market?

### 1.9 Multiplication of whole numbers

Multiplication is the repeated addition of the same number.
For example, three beans plus three beans plus three beans plus three beans equals twelve beans.

Figure 1.16


Figure 1.17


So, $\mathbf{3}$ multiplied by $\mathbf{4}$ equals $\mathbf{1 2}$ or, $\mathbf{3 \times 4 = 1 2}$

## Example 1

Show $5 \times 2$ on the number line
Figure 1.18


Answer: $5 \times 2=10$

1. Show $2 \times 6$ on the number line.
2. Show $6 \times 2$ on the same number line in question 1 .

3. Compare the answers for question 1 and 2.
4. What can you conclude?

## Some useful words:

Swap
badilisha, weka kitu mbadala

Namba mbili au zaidi zinapozidishwa, jawabu linalopatikana, huitwa product.
Namba inayotumika kuzidisha namba nyingine, huitwa multiplier.

Namba inayozidishwa na namba nyingine, huitwa multiplicand.

## Worked example



You can see that $2 \times 6=6 \times 2=12$
Therefore, if we swap around the two numbers being multiplied the product is the same.

When two or more numbers are multiplied the result is called the product.

The number used to multiply another number is called a multiplier.

The number that is multiplied by another number is called a multiplicand.

So, when we swap the multiplicand and the multiplier the product is the same.


## Example 2

Find $12 \times 2$.
Solution using number line


Answer: $12 \times 2=24$

We can also use vertical multiplication so solve $12 \times 2$
Example 1: Find $12 \times 2$.
$12 \longleftarrow$ 1st number (multiplicand)
$\mathbf{x ~} \mathbf{2} \longleftarrow 2$ 2nd number (multiplier) below
Example 2: Solve $213 \times 32$

| $\mathbf{2 1 3}$ | $\longleftarrow$ multiplicand |  |
| ---: | :--- | :--- |
| $\mathbf{x ~ 3 2}$ | $\longleftarrow$ multiplier |  |
| 6390 | $\longleftarrow 30 \times 213$ | (i) |
| +426 | $\longleftarrow 2 \times 213$ | (ii) |
| $\mathbf{6 8 1 6}$ | $\boxed{ } 5816$ | (i) + (ii) |

Answer: $213 \times 32=6816$

Example 3: Solve $421 \times 364$


Answer: $421 \times 364=153244$

### 1.10 Division of whole numbers

## Division is like repeated subtraction.

For example, how many times can you subtract 3 from 12 before you reach zero?

$$
12-3-3-3-3=0
$$

We can subtract 3 from 12 four times, or $12 \div 3=4$.
Figure 1.20


Division is also like sharing. So, if we share $\mathbf{1 5}$ sweets equally between $\mathbf{3}$ children, each child will get $\mathbf{5}$ sweets because $\mathbf{1 5} \div \mathbf{3 = 5}$.

The number that divides another number is called a divisor.
The number, which is to be divided, is called a dividend.
The number obtained when another number divides a number is called a quotient.


Sometimes when we divide one number by another there is a remainder.
For example, $11 \div 2=5$ remainder 1 :


Namba inayogawanya namba nyingine, huitwa divisor.
Namba inayogawanywa, huitwa dividend.

Namba inayopatikana wakati namba moja inagawanya nyingine, huitwa quotient.

## Worked example

1. Namba inagawanyika kwa 2 kama inaishia na $0,2,4,6$ and 8 .
2. Namba inagawanyika kwa 3 kama jumla ya tarakimu zake inagawanyika kwa 3.
3. Namba inagawanyika kwa 4 kama tarakimu mbili za mwisho zinaifanya igawanyike kwa 4.
4. Namba inagawanyika kwa 5 kama inaishia na 0 na 5.
5. Namba inagawanyika kwa 6 kama inagawanyika kwa 2 na 3.
6. Namba inagawanyika kwa 7 kama tarakimu ya mwisho inayotolewa mara mbili kutoka kwenye namba iliyopatikana baada ya kuondoa tarakimu ya mwisho inagawanyika kwa 7.
7. Namba inagawanyika kwa 8 kama namba inayoundwa na tarakimu tatu za mwisho inagawanyika kwa 8.
8. Namba inagawanyika kwa 9 kama jumla ya tarakimu zake inagawanyika kwa 9.

Example 1: Find $495 \div 6$
82 remainder 3
$6 \mathbf{4 9 5} 4$ is less than 6
$-\frac{48}{15} \quad \begin{array}{r}49 \div 6=8 \text { remainder } 1 \\ 15 \div 6=2 \text { remainder } 3\end{array}$
$-12$
3
Answer: $495 \div 6=82$ remainder 3
Example 2: Solve $8657 \div 24$
' $\mathbf{r}$ ' is short for 'remainder'


Answer: $8657 \div 24=360$ r 17

### 1.10.1 Divisibility

A number is divisible by another number if it can be divided without a remainder. For example, 15 is divisible by 5 because $15 \div 5=3$ (no remainder), but 15 is not divisible by 7 because $15 \div 7=2$ remainder 1 .

Divisibility test: This is how you can quickly work out if a number is divisible by a divisor:

1. A number is divisible by $\mathbf{2}$ if it ends with $\mathbf{0 , 2 , 4 , 6}$ and $\mathbf{8}$.
2. A number is divisible by $\mathbf{3}$ if the sum of its digits is divisible by $\mathbf{3}$.
3. A number is divisible by $\mathbf{4}$ if the last two digits make a number divisible by 4 .
4. A number is divisible by $\mathbf{5}$ if it ends with $\mathbf{0}$ and $\mathbf{5}$.
5. A number is divisible by $\mathbf{6}$ if it is divisible by $\mathbf{2}$ and $\mathbf{3}$.
6. A number is divisible by $\mathbf{7}$ if twice the last digit subtracted from the number formed after removing the last digit is divisible by $\mathbf{7}$.
7. A number is divisible by $\mathbf{8}$ if the number formed by the last three digits is divisible by 8 .
8. A number is divisible by $\mathbf{9}$ if the sum of its digits is divisible by $\mathbf{9}$.

Use the rules above to complete the following table
Table 1.3

| divisible by: | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{3 6}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\times$ | $\checkmark$ | $\times$ | $x$ | $\checkmark$ |
| $\mathbf{1 4 0}$ | $\checkmark$ | $\times$ | $\checkmark$ | $\checkmark$ |  | $\times$ | $\checkmark$ | $x$ |
| $\mathbf{2 2 5}$ |  |  |  |  |  |  |  |  |
| $\mathbf{2 5 6}$ |  |  |  |  |  |  |  |  |
| $\mathbf{3 7 8}$ |  |  |  |  |  |  |  |  |
| $\mathbf{5 4 0}$ |  |  |  |  |  |  |  |  |
| $\mathbf{8 2 6}$ |  |  |  |  |  |  |  |  |
| $\mathbf{1 2 9 6}$ |  |  |  |  |  |  |  |  |
| $\mathbf{2 1 6 9}$ |  |  |  |  |  |  |  |  |
| $\mathbf{3 1 8 5}$ |  |  |  |  |  |  |  |  |
| $\mathbf{3 9 5 6}$ |  |  |  |  |  |  |  |  |
| $\mathbf{2 6 6 3 7}$ |  |  |  |  |  |  |  |  |
| $\mathbf{2 3 9 4 9 0}$ |  |  |  |  |  |  |  |  |

## Exercise 1.6

Ex 1.5

1. Show the following on the number line:
a) $3 \times 4$
b) $9 \times 2$
c) $13 \times 2$
d) $3 \times 3$
2. Solve:
a) $5 \times 6$
b) $8 \times 9$
c) $10 \times 23$
d) $12 \times 15$
e) $140 \times 20$
f) $13 \times 7$
g) $18 \times 9$
h) $53 \times 14$
i) $48 \times 18$
j) $90 \times 100$
k) $6819 \times 12$
I) $21318 \times 98$
m) $148 \times 278$
n) $684 \times 834$
o) $7259 \times 978$
3. Write ten sentences using words from each column in table 1.4. For example: Twelve multiplied by eight equals to ninety-six.
How many mathematically correct sentences can you find?
Table 1.4

|  |  |  |  | zero. |
| :--- | :--- | :--- | :--- | :--- |
| Four |  | zero |  | four. <br> Six |
| Nine | multiplied | by |  | nine. <br> five <br> sixty-six. |
| Twelve |  | eight | equals | seventy-two. <br> Twinety-six. |
| Twenty |  | eleven |  | one hundred. <br> one hundred and |
| twenty. |  |  |  |  |

4. Solve:
(a) $24 \div 8$
(f) $372 \div 4$
(k) $1411 \div 17$
(b) $49 \div 7$
(g) $106484 \div 6$
(I) $15750 \div 21$
(c) $72 \div 12$
(h) $636 \div 12$
(m) $4116 \div 42$
(d) $900 \div 10$
(i) $3469 \div 13$
(n) $95481 \div 309$
(e) $132 \div 11$
(j) $36063025 \div 25$
(o) $956484 \div 728$
5. Are the following numbers are divisible by $2,3,4,5,6,7,8$, or 9 ?
(a) 540
(c) 2169
(e) 26637
(b) 826
(d) 3956
(f) 239490
6. Write sentences using words from each column in table 1.23. For example: Ninety-six divided by eight equals twelve. How many mathematically correct sentences can you make?

Table 1.5

| Zero |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Thirty-two |  | one |  | zero. |
| Seventy-two |  | two |  | four. |
| Ninety | divided by | five |  | six. |
| Ninety-six |  | eight | equals | nine. |
| One hundred |  | ten |  | twelve. |
| One hundred <br> and thirty-two |  | eleven |  | ten. |
| twenty. |  |  |  |  |

### 1.11 Word problems involving multiplication and division

Different phrases can mean multiply or divide. Look out for these:

| Words or phrases mostly used | Signs |
| :--- | :---: |
| Multiply, times, product of | $\mathbf{x}$ |
| Divide, share, each, per, shared between, <br> shared among, shared equally | $\div$ |

## Example 1

If a cow gives three litres of milk in one day, what is the total number of litres of milk the same cow gives over thirty days? Assume the cow gives the same amount every day.
If a cow gives three litres of milk in one day, what is the total number of litres of milk the same cow gives over thirty days?

3 litres in 1 day $\longleftarrow$ what you know
? litres in 30 days $\longleftarrow$ what you are asked to find
$\mathbf{3 x 3 0}$ litres in 30 days
90 litres in 30 days
Answer: The cow gives 90 litres over 30 days

## Example 2

Heleni bought 8 exercise books from Seif Bookshop. If one exercise book costs 300 shillings. How much money did she spend?
Heleni bought $\mathbf{8}$ exercise books from Seif Bookshop. If one exercise book costs 300 shillings, how much money did she spend?

$$
\begin{array}{ll}
\mathbf{8} \text { exercise book costs } & \text { Sh } 300 \\
\mathbf{8} \text { exercise books cost? }
\end{array}
$$

Answer: She spent $\mathbf{2 4 0 0}$ shillings

Worked example

First read the question carefully.
Pick out the important information. What are you asked to find? Second, write this down using numerals.
Kwanza, soma swali kwa umakini ili kubaini taarifa muhimu na ujue unachotakiwa kukifanya. Kisha, ziandike kwa kutumia tarakimu.

Pick out the mathematical information.

Bainisha taarifa za kihisabati.

Pick out the mathematical information.

## Bainisha taarifa za kihisabati.

First read the question carefully. Pick out the important information. What are you asked to find? Second, write this down using numerals.

Kwanza, soma swali kwa umakini ili kubaini taarifa muhimu na ujue unachotakiwa kukifanya. Kisha, ziandike kwa kutumia tarakimu.

Pick out the mathematical information.

Bainisha taarifa za kihisabati.

## Example 3:

A fast running car covers 113 kilometres in one hour. How many kilometres will the same car cover in 11 hours?

113 km in 1 hour what you know
? km in 11 hours what you are asked to find

| $\mathbf{1 1 3}$ |
| ---: |
| $\mathbf{X} \quad \mathbf{1} \mathbf{1}$ |
| 113 |
| +1130 |
| 1243 |

Answer: The car covers 1243 km in 11 hours.

## Example 4:

Neema has picked 12 passion fruit. She wants to divide them equally between herself and her three friends. How many passion fruit will each friend get?
Neema has a passion fruit vine in her garden. One day, she picked 12 passion fruit from her garden. She wants to divide them equally between herself and her three neighbours. How many passion fruit: will each friend get?

12 fruit, divide between $(1+3=4)$ people
Answer: Each person gets $12 \div 4=3$ passion fruit

## Example 5:

A Tanzania national football team was given 45 million shillings. This money was shared equally among 15 players. How much did each player receive?

A Tanzania national football team was given $\mathbf{4 5}$ million shillings. This money was shared equally among 15 players. How much did each player receive?

15 players have Sh 45000000
1 player has Sh $45000000 \div 15$

$$
\text { = Sh } 3000000
$$

Answer: Each player received 3 million shillings

## Example 6

Mrs Komba pays Mr. Baruti 649600 shillings for 116 metres of fabric. How much does the fabric cost per metre?

Mrs Komba pays Mr. Baruti $\mathbf{6 4 9} \mathbf{6 0 0}$ shillings for $\mathbf{1 1 6}$ metres of fabric. How much does the fabric cost per metre?

116 m costs Sh 649 600, 1 m costs Sh ?
$\mathbf{1} \mathbf{~ m}$ costs Sh $649600 \div 116$
Answer: The fabric costs 5600 shillings per metre.

## Exercise 1.7

Pick out the mathematical information.

## Bainisha taarifa za kihisabati.

1. 10 cows eat 9 bags of maize in one hour. How many bags will the same cows eat in 24 hours?
2. One box of chalk contains 12 pieces of chalk. How many pieces of chalk will 10 boxes have?
3. One mini bus carries 30 passengers at once. How many passengers will one hundred mini buses carry?
4. One student can eat 6 pancakes from a supermarket. How many pancakes can 202 students eat?
5. Mr. Juma gave his children 353500 shillings to share equally between them. If he has seven children, how much money did each child get?
6. The product of three numbers is 2730 . Two of the numbers are 13 and 30 . Find the other number.
7. 200 sweets are divided equally among 10 students. How many sweets will each student receive?
8. Mr. Majuto bought 988 bags of cashew nuts for his 76 students. He asked them to share those bags equally. How many bags did each student receive?
9. 600 matches will be divided equally among 30 students. How many matches will each student receive?
10. Mrs James collects 400 eggs every day. Each egg is sold for 300 shillings. How much money does Mrs James earn per day if all eggs are sold?
11. The United Nations Organization will distribute equally 1600 books among 8 schools in Dar es Salaam District. How many books will each school receive?
12. It is reported that 502 tourists visit mountain Kilimanjaro every month. How many tourists will visit mountain Kilimanjaro in a year?
13. Write down the place value of digit 3 in each of the following numbers.
a) 5328
b) 143692
c) 1038
d) 6238920
e) 713
14. Write down each of the following numbers in expanded form. (Hint: see page 17)
a) 7261
(b) 630795
c) 10123
d) 1969200011
e) 493
15. Write each of the following numbers in numerals:
a) Eighty five thousand seven hundred and ten
b) One hundred twenty three thousand.
c) Nine million six hundred and seventy thousand and thirty three.
d) Six million two hundred and one thousand and twelve.
e) Seven billion two hundred and three thousand one hundred and one.
16. Write down each of the following numbers in words.
a) 250063072
b) 516918300
c) 197810
d) 300175450
e) 19528148
17. Use the following digits to write the largest five digit numbers.
a) $0,1,5,7,9$
b) $3,7,8,1,0$
18. Write all three digit numbers that can be formed using the digits 3,8 and 5 without repetition.
19. Write down all:
a) Whole numbers between 5 and 11
b) Even numbers between 16 and 30 .
c) Prime numbers between 6 and 40 .
d) Odd numbers between 10 and 25 .
20. Work out:
a) $8975+2517$
b) $3217+6892$
c) $7819-2861$
d) $6812-2497$
e) $2418 \times 126$
f) $1634 \times 120$
g) $21952 \div 28$
h) $36063025 \div 35$
21. Halima bought 20 oranges at 200 shillings each and 10 mangoes at 500 shillings each. How much did she spend in total?
22. Mr Mchome has 600 cows. He sold 15 cows each at 500000 shillings.
a) How much money did he earn?
b) How many cows was he left with?
23. James' school is 5 km from his home. If he goes to and from school daily, how many kilometres does he travel in 197 days?
24. Mrs Matumla receives a bank statement showing that she has Shillings 6005345 in her account. However she knows that after the bank statement was sent she spent another Shillings 140000 on developing her farm. How much money did she really have?
25. A worker earns Shillings 72000 in a week. If he works 40 hours in a week, how much does he earn per hour?

## Challenge activity

Write the numbers 1 to 6 in the circles so that each side of the triangle adds up to the same number.


1. In how many different ways can you solve this problem?

Show the different solutions.
2. Design and write another problem similar to this one.

## What have I learned?

Work with your partner. Check what have you have learned and what you need help with.

|  | Learned | Need <br> help |
| :--- | :--- | :--- | :--- |
| 1. Finding the place value of digits in a number | $\square$ | $\square$ |
| 2. Writing a number in expanded form | $\square$ | $\square$ |
| 3. Identifying even, odd and prime numbers | $\square$ | $\square$ |
| 4. Horizontal addition and subtraction of whole numbers | $\square$ | $\square$ |
| 5. Vertical addition and subtraction of whole numbers | $\square$ | $\square$ |
| 6. Solving addition and subtraction word problems | $\square$ | $\square$ |
| 7. Multiplication of whole numbers | $\square$ |  |
| 8. Division of whole numbers | $\square$ | $\square$ |
| 9. Divisibility test | $\square$ |  |
| 10. Solving multiplication and division word problems | $\square$ | $\square$ |

## To remember

1. Natural or counting numbers are $1,2,3,4,5, \ldots$
2. Whole numbers are $0,1,2,3,4, \ldots$
3. Even numbers can be divided by 2 .
4. They are $2,4,6,8,10, \ldots$
5. Odd numbers cannot be divided by 2 .
6. They are $1,3,5,7,9, \ldots$
7. Prime numbers can be divided by 1 and by themselves only.
8. They are $2,3,5,7,11, \ldots$
9. Mutiplicand $\times$ multiplier $=$ product
10. Dividend $\div$ divider $=$ quotient + remainder

## Chapter 2:

## Introducing Algebra

In this chapter you will learn how to solve problems using algebra. In algebra we use letters to write rules and formulae. You will learn:

- how to use letters in algebra;
- how to simplify an algebraic expression;
- how to form and solve equations with one unknown; and
- how to translate a word problem into an equation in one unknown.


## Some useful words:

algebra
aljebra
algebraic expression
mitajo wa kialjebra
constant
namba isiyobadilika
coefficient
namba inayozidishwa na
'variable'
formula
kanuni
multiplier
kizidisho
simplify
rahisisha
term
kijisehemu cha mtajo
variable
kitu kinacho-badilika
the value of $p$
thamani yap
equation
mlinganyo
solve for the value of $p$
tafuta thamani ya p
letters
herufi
simplify this expression rahisisha mitajo hii
like terms
mitajo inayofanana
patterns
mpangilio

## Je, uliwahi kujua?

Aljebra ilianza Afrika, wamisri walianza kutumia aljebra miaka 3500 iliyopita.
Chimbuko la neno 'aljebra' ni jina la kitabu cha hisabati kilichoandikwa kwa kiarabu mwaka 825 BK.Kitabu hiki kiliitwa 'Aljebar w'al almugabalah'.

### 2.1 Introducing algebra



## Stick Patterns

Try this activity in small groups. You will need about 20 matchsticks.

- Arrange the sticks to make a square.

How many sticks have you used?

- Add three sticks to make two squares.

How many sticks have you used?

- Add another three sticks to make three squares.

How many sticks have you used now?
Figure 2.1 Matchstick squares


- Carry on adding sticks. Complete the table below.

Table 2.1 Squares and sticks

| Number of <br> squares | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> sticks | 4 | 7 | 10 |  |  |  |  |  |

- On your own, complete these sentences using two different numbers.
To make $\qquad$ squares we need $\qquad$ match sticks.
- In groups, talk about how you found the answers. Did you all get the answer in the same way?

Fill in the blanks using the following words:
variable number seven changing two changes
The number of squares changes. It can take the value one, $\qquad$ three and so on. The $\qquad$ of squares is a variable. A variable is a
to vary/to change
kubadalika
variable
kitu kinachobadilika
to change
kubadilika
Kutengeneza miraba tunahitaji njiti za kibiriti


## Some useful words

mething that $\qquad$ or "varies". The number of value measuring something that match sticks keeps on $\qquad$ depending on how many squares we make. The number of matchsticks can be four, $\qquad$ ten and so on. Therefore the number of matchsticks is also a $\qquad$ . .

## Everyday variables

In groups think of at least five different quantities you know that change or vary. What are they called in English?

Write your answers in table 2.2. Compare your answers with other groups.

Table 2.2: Quantities that vary

| Quantities that vary | Example |
| :--- | :--- |
| Temperature | Body temperature <br> Room temperature, etc. |
|  |  |
|  |  |
|  |  |
|  |  |

## Use of Letters in Algebra

In algebra, we use a letter to stand for a variable.
For example, in the stick pattern below, we will call the number of squares ' $\mathbf{n}$ '.

Figure 2.2 Stick patterns again


## Some useful words

to vary/to change
kubadalika
variable
kitu kinachobadilika
temperature
joto
body temperature
joto la mwili
room temperature
joto la chumba

In algebra, we use a letter to stand for a variable.

## Some useful words

formula (s.), formulae (pl.)

## kanuni


$n$ ni kitu kinachobadilika. Inaweza kuwa na thamani yoyote. Tunasema, 'ni thamani ya n isiyobadilika'.


Figure 2.3: Engineers use formulae to calculate the safe height for a building.

To make 1 square we need 4 match sticks.
$3 \times 1+1=4$
To make 2 squares we need 7 match sticks.
$3 \times 2+1=7$
To make 3 squares we need 10 match sticks.
$3 \times 3+1=10$
To make 10 squares we need 31 match sticks.
$3 \times 10+1=31$
Now, let's call the number of squares $\boldsymbol{n}$.
To make $\mathbf{n}$ squares we need $\mathbf{3 x n + 1}$ match sticks.
$\mathbf{3 x} \boldsymbol{n}+\mathbf{1}$ is a formula for the number of match sticks.
Usually, we do not write the multiply $(x)$ sign, therefore:

## Number of matchsticks $=3 n+1$

How many matchsticks do you need to make 11 squares?
11 is the variable ( $n$ ), therefore,
Number of matchsticks $=3 \times 11+1$
Answer: 34 matchsticks.
How many matchsticks do you need to make 42 squares?
Use the formula $3 n+1$.
How many matchsticks do you need to make 100 squares?

## Je, uliwahi kujua?

Watu wengi hutumia kanuni za kihisabati. Mfamasi hutumia kanuni za kihisabati kukokotoa viwango vya madawa. Mhasibu hutumia fomyula kukokotoa mauzo na gharama. Wahandisi hutumia fomyula kukokotoa uimara wa madaraja au kimo salama kwa ajili ya jengo. Kompyuta zinaweza kufanya ukokotoaji mwingi haraka sana kwa kutumia kanuni za kihisabati.

### 2.2 Algebraic expressions

## Forming an algebraic expression

In algebra, letters are treated like numbers. They can be added, subtracted, multiplied or divided. For example:

$$
\begin{aligned}
& a+a+a=3 a \\
& y+y+y+y+y+y+y=7 y
\end{aligned}
$$

Different letters, numbers and signs can be combined to make an algebraic expression. For example:

$$
\begin{aligned}
& 8 x+2 y+5 \\
& 2 n+3 x+2 y-5 r
\end{aligned}
$$

Figure 2.4 Helen's Monday sales


Some useful words
to sell
kuuza
sale
uzo
total sales
jumla ya mauzo

## Worked example

Helen owns a shop that sells credit recharge vouchers with the value $500,1000,5000$ and 10000 shillings. Look at figure 2.4.

## What were Helen's total sales on Monday?

Helen can work out an algebraic formula for her sales.
First, she needs to replace each variable with a letter. The variables are the number of each kind of voucher that could be sold in a day.

She could name the variables as follows.
Number of 500 shilling vouchers sold per day -a,
Number of 1,000 shilling vouchers sold per day - b,
Number of 2,000 shilling vouchers sold per day - c,
Number of 5,000 shilling vouchers sold per day - d, and
Number of 10,000 shilling vouchers sold per day - e,
Her total sales per day in shillings can be expressed by the algebraic formula.

Total sales $=a \times 500+b \times 1000+c \times 2000+d \times 5000+e \times 10000$
$=500 a+1000 b+2000 c+5000 d+10000 e$
On Monday: $a=1, b=3, \mathrm{c}=2, d=0, e=0$
Total sales $=1 \times 500+3 \times 1000+2 \times 2000+0 \times 5000+0 \times 10000$

$$
\begin{aligned}
& =500+3000+4000+0+0 \\
& =7500
\end{aligned}
$$

Helen's Monday sales were 7,500 shillings.

## Exercise 2.1.1

Work in groups to answer the following questions:

1. What is algebra?
2. Write the algebraic expression that would summarise the following:
a) A teacher marks $x$ students' exercise books every half an hour. If the teacher continues to mark at the same speed for six hours, what is the total number of exercise books that the teacher will have marked at the end of the six hours?
b) Kaundime bought $b$ bananas from market, and $c$ from a shop. On her way back home she gave three bananas to her neighbour's children. How many bananas did she have when she reached home?
c) Temba bought $p$ oranges from Kisutu market, and then twice as many from Kariakoo market. On her way back home she gave $q$ bananas to her neighbour's children. How many bananas did she have when she reached home?
d) Joni had 20 goats that he reared at his farm on the outskirts of Dar es Salaam. He bought $x$ goats at an auction in Vinguguti. How many goats in all does he have now?
3. Think of word problems that would be translated into the following algebraic expressions:
a) $3 a+5 a$
b) $5 x+y$
c) $3 v+2 v-8$
d) $60-6 p$
e) $50+9 q$

## Simplifying algebraic expressions

In pairs, consider the algebraic expression $3 x+2 y-x$
Each part of the expression is called a term.
So, $3 x, 2 y$ and $-x$ are terms,
$3 x$ and $-x$ are like terms and
$3 x$ and $2 y$ are unlike terms.

We can simplify expressions by adding and subtracting like terms.
We cannot add and subtract unlike terms.

Simplify: $\quad 3 x+2 y-x$
Group together like terms.
Add or subtract like terms.

$$
\begin{aligned}
3 x+2 y-x & =3 x-x+2 y \\
& =2 x+2 y
\end{aligned}
$$

## Simplify: $\quad 5 p q-3 p+2 p q$

Group together like terms.
Add or subtract like terms.

$$
\begin{aligned}
5 p q-3 p+2 p q & =5 p q+2 p q-3 p \\
& =7 p q-3 p
\end{aligned}
$$

Figure 2.5 Like and unlike terms


In pairs, look at this expression:
$3 a b+6 a-2 b+5 a b-3 a+5 b-a b$
$3 a b, 5 a b$ and $-a b$ are like terms. Find two pairs of like terms.
$6 a$ and $-3 a$ are unlike terms. Find two more pairs of unlike terms.

## Now simplify:

$3 a b+6 a-2 b+5 a b-3 a+5 b-a b$
Compare your answer to another pair.

## Example 1:

Simplify: $9 x+3 y-2 x-y$

$$
\begin{aligned}
9 x+3 y-2 x-y & =9 x-2 x+3 y-y \\
& =7 x+2 y
\end{aligned}
$$

## Example 2:

Simplify: $3 a b+6 x-2 y+5 a b-3 x+5 y-a b$

$$
\begin{aligned}
3 a b+6 x-2 y+5 a b-3 x+5 y-a b= & 3 a b+5 a b-a b+6 x-3 x-2 y+5 y \\
& =7 a b+3 x+3 y
\end{aligned}
$$

## Exercise 2.2

1. Simplify the following algebraic expressions.

Example: $3 x+2 x+x=6 x$
a) $7 y+2 y$
b) $9 r+2 r-r$
c) $8 n-5 n+2 n$
d) $6 m+15 m+3 m-2 m$
e) $5 a+35 a-2 a+4 a$
f) $8 m+4 m-2 m+3 m$
g) $12+3 c+7+9 c-1-4 c$
h) $7 n-3+20 n+5+3 n$
i) $r+9 r-2 r+6$
j) $2 x-5+7 x+2 y$
2. Given that $a=-5, c=1, m=7, n=4, r=5, x=2$, and $y=-3$, find the value of each expression in question '1.' above.

## Describing algebraic expressions

The parts of an algebraic expression are called terms.
For example, this algebraic expression has four terms.


## Some useful words

term
mtajo
constant
namba isiyobadilika
coefficient
namba inayozidishwa na herufi kwenye mtajo

The first term is $\mathbf{3 x}$.
The second term is $\mathbf{2 y}$.
The third term is $\mathbf{- 5 n}$.
The fourth term is $\mathbf{3}$. It is called 'the constant' because it does not vary.
$x, y$ and $n$ represent different variables. A number which multiplies the variable is called coefficient of the variable.


Ex 2.3


## Exercise 2.3

Working in pairs, fill in blanks in the sentences below. When you have finished, find another pair of students, and compare your answers.

For the algebraic expression, $6 y-3 x+7 z+9$ :

1. There are $\qquad$ terms.
2. $\qquad$ is the coefficient of $6 y$.
3. -3 is the $\qquad$ of $\qquad$ . .
4. 7 is the $\qquad$ of $\qquad$ . .
5. $\qquad$ is the second term.
6. $6 y$ is the $\qquad$ term.
7. $7 z$ is the $\qquad$ term.
8. 9 is the $\qquad$

## Multiplication and division of algebraic expressions

Letters in algebra are multiplied in the same way as numbers. For example,

1. $4 \times 2=8$
$2 \times 4=8$
Remember multiplication is commutative. $\therefore 4 \times 2=2 \times 4=8$
2. $a \times b=a b$
$a \times b=a b$
$a b$ and $b a$ are like terms.
$\therefore a \times b=b \times a=a b=b a$
3. $2 \times 7 n=7 n \times 2=14 n$
4. $\quad 9 m \times 3 n=9 \times 3 \times m \times n=27 m n$

Algebraic expressions can be multiplied by a number or another expression.

Each term in the expression must be multiplied by the multiplier.

## Example 1

Multiply $7 x+2 y+3 z-6 m$ by 3 .
$\mathbf{3}$ is the multiplier.
Multiply each term by 3

$$
\begin{aligned}
(7 x+2 y+3 z-6 m) \times 3 & =3(7 x+2 y+3 z-6 m) \\
& =(3 \times 7 x)+(3 \times 2 y)+(3 \times 3 z)-(3 \times 6 m) \\
& =21 x+6 y+9 z-18 m
\end{aligned}
$$

## Example 2

Multiply $5 a+3 b-2 c$ by $3 y$.
$3 \boldsymbol{y}$ is the multiplier.
Multiply each term by $\mathbf{3 y}$.

$$
\begin{aligned}
(5 a+3 b-2 c) \times 3 y & =3 y(5 a+3 b-2 c) \\
& =(3 y \times 5 a)+(3 y \times 3 b)-(3 y \times 2 c) \\
& =(3 \times 5 \times y \times a)+(3 \times 3 \times y \times b)-(3 \times 2 \times y \times c) \\
& =15 y a+9 y b-6 y c
\end{aligned}
$$

## Example 3

Complete the following statement: $8 m+6 n-12 p=2(\quad)$.
Divide each term by 2.
$8 m+6 n-12 p=2(4 m+3 n-6 p)$

## Example 4

Divide $(9 b y+6 b x+21 b z-3 b m) \div 3 b$
Divide each term by $\mathbf{3 b}$.

$$
\begin{aligned}
(9 b y+6 b x+21 b z-3 b m) \div 3 b= & \frac{9 b y}{3 b}+\frac{6 b x}{3 b}+\frac{21 b z}{3 b}-\frac{3 b m}{3 b} \\
& =3 y+2 x+7 z-m
\end{aligned}
$$

Worked example

3 ni kizidisho.
Zidish kila mtajo na 3.
$3 y$ ni kizidisho.
Zidisha kila mtajo na $3 y$.

## Gawanya kila mtajo kwa 2.

Gawanya kila mtajo kwa 3 b.

1. Multiply each of the following
a) $3 a+7 b+3 c$ by 3
b) $5 x-2 y+8 z$ by $7 m$
c) $-6 m+7 n-12 p$ by -4
d) $9 d+5 e-2 f$ by $-2 b$
e) $2 a-7 b+9 c$ by $x y$
2. Open the brackets
a) $8(3 a-2 b+c)$
b) $6 m(7 x+3 y-2 z)$
c) $7 x y(3 a+2 b-c)$
d) $-3 u v(5 x+7 y-8)$
e) $\frac{1}{3} x(9 a+21 b-c)$
3. Fill in blanks with the correct expressions
a) $25 x+10 y-15 z=5($ $\qquad$ ..)
b) $8 a b-16 a x+4 a z=-4 a($ $\qquad$ ..)
c) $18 m p-6 m r+27 m q+12 m x=3 m($. $\qquad$ ..)
d) $12 a r y+6 a r x-18 a r z+30 a q r=6 a r($ $\qquad$ ..)
e) $-42 m x y+14 n x y-63 p x y+21 x y z=-7 x y($. $\qquad$ ..)
4. Divide the following expressions
a) $3 b r x+2 b x y-b x z$ by $x$
b) $20 a x-15 a y+65 a z$ by $5 a$
c) $6 a m+12 u v-4 x y$ by 2
d) $30 x y-16 x z+10 n x$ by $-2 x$
e) $21 m n+18 x y-6 r z$ by -3

### 2.3.5 Forming algebraic expressions

You formed an algebraic expression in Activity 2.4 for Helen's total sale of Tigo vouchers in a day. You were given a word problem and formed an expression that solves that problem.

To form algebraic expressions, follow these three steps:
Step 1: Identify the variables. Select letter(s) to stand for variable(s).
Step 2: Identify the operation signs and form an expression.
Step 3: Simplify the expression if possible.
Activity 2.6 will help you to identify the operation signs (step 2).

## Talking about mathematical operations

Table 2.3 shows some words and phrases used in English to talk about mathematical operations. In groups, read the words and phrases out loud and translate the words and phrases into Kiswahili.

Table 2.3 Words and phrases describing mathematical operations

| Words or phrases | Signs |
| :--- | :---: |
| addition, sum of, increased by, greater than, plus, <br> taller than, more than, larger than, older than. | $\mathbf{+}$ |
| subtract, difference, decreased by, less than, minus, <br> shorter than, reduced by, younger than. | $\mathbf{-}$ |
| multiply, times, product. | $\times$ |
| divided, division. | $\div$ |
| equals, is, gives, results, totals | $=$ |

Use these words to fill in the blanks in the word problems below. You may use a word more than once.
litres older expression algebraic than less km

1. Ann is four years $\qquad$ Ali. Suppose Ali is $x$ years old. Write an algebraic $\qquad$ for Ann's age.
2. A car can travel 60 km on one litre of petrol. Write an $\qquad$ for the number of $\qquad$ it can travel with $x$ of petrol.
3. Mr Ashad sells 1 kg of fertiliser for $x$ Shillings. Ms. Komba sells 1 kg of fertiliser for 300 Shillings more $\qquad$ Mr. Ashad. Write an $\qquad$
$\qquad$ for 5 kg of Ms. Komba's fertiliser.
4. Now make up your own word problem in English.
$\qquad$
$\qquad$ for $x$ S
gs more
n..... for  d. .

Hatua ya1: Baini vitu
vinavyobadilika. Chagua
herufi zinazowakilisha vitu vinavyobadilika.
Hatua ya 2: Baini alama za matendo na unda mitajo.
Hatua ya 3: Rahisisha mitajo kama inawezekana.

## Worked example

## Bei la chungwa inabadilika.

## Bei la nanasi inabadilika

## Example 1

Ali bought seven oranges and three pineapples. Write down the expression for the total cost of the fruits.
Step 1:
The cost of an orange is a variable.
The cost of a pineapple is a variable.
Let the cost of one orange be $\boldsymbol{x}$ shillings.
Let the cost of one pineapple be $\boldsymbol{y}$ shillings.
Then, the cost of 7 oranges $\quad=7 \times x=7 \times$ shillings and the cost of buying 3 apples $=3 \times y=3 y$ shillings
Step 2:
Replace 'and' by ' + '. (See table 2.3)
The total cost of all fruits will be .... shillings
$\therefore$ The total cost of all fruits will be $=(7 x+3 y)$ shillings

## Example 2

Rebecca is five years younger than her brother Alex. Find an algebraic expression for the sum of their ages after two years
a) using Rebecca's present age as the variable.
b) using Alex's present age as the variable.
a) Let Rebecca's present age be $x$ years
$\therefore$ Alex's $^{\prime}$ present age is $x+5$ years
After two years, Rebecca's age will be $x+2$ years, and Alex's age will be $x+5+2=x+7$ years old.
$\therefore$ After two years, the sum of their ages $=x+2+x+7=2 x+9$ years.


## In pairs, fill in the blanks to answer part (b)

b) Let Alex's present age be $y$ years
$\therefore$ Rebecca's present age is $\qquad$ years.

After two years, Alex's age will be $\qquad$ years and Rebecca's will be $\qquad$ years old.
$\therefore$ After two years, the sum of their ages will be $\qquad$ years.

Working in pairs, now use table 3a to answer part (a) and table 3b to answer part (b).

Table 3a

| Rebecca's <br> present age | Alex's <br> present age | Rebecca's <br> age after <br> two years | Alex's age <br> after two <br> years | Sum of their <br> ages after <br> two years |
| :---: | :---: | :--- | :---: | :---: |
| $x$ | $x+5$ | $x+2$ | $x+7$ | $2 x+9$ |
| 1 | 6 |  |  | 11 |
| 2 | 7 |  |  | 13 |
| 3 | 8 |  |  | - |
| 4 | 9 |  |  | - |
| 5 | - |  |  | - |
| 6 | - |  |  | - |

## Table $3 b$

| Alex's <br> present age | Rebecca's <br> present age | Alex's age <br> after two <br> years | Rebecca's <br> age after <br> two years | Sum of their <br> ages after <br> two years |
| :---: | :---: | :---: | :---: | :---: |
| $y$ | $y-5$ | $y+2$ | $y-3$ | $2 y-1$ |
| 6 | 1 |  |  | 11 |
| 7 | 2 |  |  | 13 |
| 8 | 3 |  |  | - |
| 9 | 4 |  |  | - |
| 10 | - |  |  | - |
| 11 | - |  |  | - |

What have you learned from this table? Discuss with your partner.
Sometimes, in word problems, you choose which variable you will represent with a letter. If you chose a different variable from your friend, you will get a different answer for the algebraic expression. But you could both be right!

## Exercise 2.5

Represent the following statements algebraically:

1. The total number of legs in a herd of cattle. Let the number of cattle be $c$.
2. Total number of students' eyes in a classroom. Let the number of students in the classroom be $s$.
3. Total cost of all exercise books and pencils, if one exercise book costs 1,500 and one pencil costs 50 shillings. Let the number of exercise books be $x$ and the number of pencils be $y$.
4. Juma's age if Juma is three years younger than his sister Asha. You select the variable.

## Ex 2.5

## Some useful words

Equation
mlinganyo
Solution
jawabu au jibu
Value
thamani
Solve
fumbua
True
kweli
The unknown
Kinachotafutwa au
kitafutwacho/ thamani isiyofahamika
Linear equation mlinganyo wenye kinachotafutwa kimoja

Hatua ya 1: Tambua swali
linauliza kitu gani, yaani, tambua kitu kisichojulikana.
Hatua ya 2: Fafanua
herufi inayowakilisha kitu kisichofahamika.

Hatua ya 3: Eleza taarifa tajwa kama mlinganyo kwa kutumia herufi.
Hatua ya 4: Fumbua mlinganyo ili kupata jibu.
5. The sum of three consecutive even numbers. Let the first even number be $x$.
6. Think of a number, add seven to it and multiply the result by two. What answer do you get? Write this as an algebraic expression.
7. The price of one litre of fresh milk is twice as much as that of curdled milk. Let the price of curdled milk be $x$. Find the cost of buying three litres of fresh milk and two litres of curdled milk.
8. Mr Masanja can sell a cow for $x$ Shillings and a goat for $y$ Shillings. If he sells seven cows and twelve goats, what is his total sales in Shillings?
9. Mrs Jangala had 50 eggs in her basket. If she sold a certain number $a$ and $b$ broke, how many eggs were left in her basket?
10. A tailor has two pieces of fabric for making two different wedding dresses. One piece is two times as long as the other. Find the total length of the two pieces of fabric. You select the variable.

### 2.4 Forming and solving equations with one unknown

Look at table 2.4
a) After 2 years the sum of Rebecca and Alex's ages will be 13 years. How old is Rebecca now?
b) Suppose, after 2 years the sum of Rebecca and Alex's ages will be 17 years, how old is Rebecca now?

Making a table takes a long time.
We can write these problems as an equation.
Let Rebecca's age be $x$
Then the sum of Rebecca and Alex's ages after two years is $2 x+1$
So we can write problem (a) as: $\quad 2 x+1=13$
And we can write problem (b) as: $\quad 2 x+1=17$
These relations are called equations. They have only one variable or unknown, therefore they are called linear equations.

Steps to follow when forming an equation from a mathematical statement:
Step 1: Identify what the question is asking for, i.e. identify the unknown variable.

Step 2: Define a letter to stand for the unknown.

Step 3: Express the given information as an equation using the letter.

Step 4: Solve the equation to get the solution.

## Formulate equations from mathematical statements

## Example 1

Three times a certain number gives 72 . Find the number.

## Solution

Three times a certain number gives 72


## Example 2

A number is doubled and eight added. The result is the same as multiplying the same number by five and subtracting seven.

## Solution

Let the number be $y$
Step 1: $y$ is doubled and eight is added
Double $y$ : $\quad y \times 2=2 y$
Add 8: $\quad 2 y+8$
Step 2: Multiply $y$ by five and subtract seven.
Multiply the number by five: $\quad y \times 5=5 y$
Subtract seven:
5y-7
The result is the same
$\therefore 2 y+8=5 y-7$

## Exercise 2.6

1. The sum of a number $x$ and 11 is 35 . Express this as an equation and solve the equation to find $x$.
2. The sum of three consecutive odd numbers is 57 .

If the first number is $x$, write an equationand solve it to find $x$.
3. When $x$ is taken away from 32 and the result divided by 3 the answer is -15 . Write an equation and solve it to find the value of $x$.
4. The product of 7 and another number is the same as three times the sum of 8 and the number. Form an equation to find the number.
5. Leila is twice as old as her sister Ramla, and their brother Juma is 3 years younger than Ramla. The sum of their ages is 53 . Form an equation and solve it to find their ages.
6. When a certain number is increased by 12 the result is 7 times the number. Write the equation for finding the number.
7. The sum of three consecutive counting numbers is 156 . Write an equation and find the middle number.
8. The width of a rectangle is 4 m shorter than its length. If its perimeter is 72 m , write an equation and find the length of the rectangle.
9. George, Rajabu and William shared 100000 shillings among themselves. George got 15000 shillings less than William and Rajabu got 20000 shillings less than George. Write and solve an equation to find how much each one of them got.
10. In a wedding ceremony, the number of women who attended is the same as the number of men who attended and three times the number of children. Write and solve an equation to find the number of children that attended if a total of 217 people attended.

## Solving equations with one unknown

Equations can be solved by the balancing method. This method uses the same principle of a beam balance used to weigh things.

Suppose the beam balance is balanced and you want to keep it balanced.

- If you add weight on the left arm, you must add the same weight to the right arm.
- If you remove weight from the left arm, you must remove the same weight from the right arm.
Whatever you do to one side you have to do to the other. The same is true for equations.

Figure 2.6: The principle of a beam balance



## Example 1

Solve the equation: $x+7=15$
Solution

$x+7=15$
$x+7-7=15-7$
Subtract seven from both sides.
Answer: $x=8$
Check: $x+7=8+7=15$


## Example 2

Solve the equation: $\boldsymbol{x}-\mathbf{5}=\mathbf{1 3}$
Solution
$x-5=13$

$x-5+5=13+5$
Add five to both sides.
Answer: $x=18$
Check: $x-5=18-5=13$


## Example 3

Solve the equation: $3 x=27$
Solution
$3 x=27$

$\frac{3 x}{3}=\frac{27}{3}$
$x=9$
Check: $3 x=3 \times 9=27$


## Example 4

Solve the equation $\frac{x}{7}=8$
Solution
$\frac{x}{7}=8$
$\frac{x}{7} \times 7=8 \times 7$
Multiply by 7 on both sides.
$x=56$
Check $\frac{x}{7}=\frac{56}{7}=8$

## Example 5

Solve the equation $\frac{x}{3}+\frac{x}{5}=4$
Solution
$\frac{x}{3}+\frac{x}{5}=4$
$15\left(\frac{x}{3}\right)+15\left(\frac{x}{5}\right)=15 \times 4$
Multiply each term by LCM of 3 and 5
$5 x+3 x=60$
$8 x=60$
$\frac{8 x}{8}=\frac{60}{8}$
$x=7 \frac{4}{8}=7 \frac{1}{2}$

## Example 6

Solve the equation $\frac{2}{x+1}=\frac{5}{x-3}$, for $x \neq-1, x \neq 3$
Solution
$\frac{2}{x+1}=\frac{5}{x-3}$
$\frac{2}{x+1}(x+1)(x-3)=\frac{5}{x-3}(x+1)(x-3)$
Multiply each term by $(x+1)(x-3)$
$2(x-3)=5(x+1)$
$2 x-6=5 x+5$
$2 x-6+6=5 x+5+6$
$2 x=5 x+11$
$2 x-5 x=5 x-5 x+11$
$-3 x=11$
$\frac{-3 x}{-3}=\frac{11}{-3}$
$x=-3 \frac{2}{3}$

Discuss how you would solve $\frac{2}{3} x-4=8$
Discuss why in $\frac{60}{x}=6, x$ should not be 0 .

Ex 2.7

## Exercise 2.7

Solve the following equations

1. $x-9=15$
2. $x+7=3$
3. $5 x=30$
4. $\frac{3 x}{7}=6$
5. $\frac{x}{3}=7$
6. $\frac{x+5}{x-3}=2$
7. $\frac{9}{x}=6$
8. $\frac{x}{3}-5=20$
9. $\frac{2}{x+3}=8$
10. $3(x+2)=x-10$

## Exercise 2.8

1. The difference between 36 and another number is 13 . Find the number.
2. Find two consecutive odd numbers such that the sum of 3 times the smaller number and 7 times the larger is 124 .
3. Helen has 600 shillings, how many oranges can she buy if each orange costs 50 shillings.
4. Three fifths of a number is added to two thirds of the number. If the result is 38 . Find the number
5. The length of a rectangle is 5 m longer than its width. If its perimeter is 70 m , find the width of the rectangle.
6. The sum of the number of girls and boys in class is 45 . The number of girls exceeds that of boys by 9 ; find the number of boys and the number of girls in the class.
7. James is 3 cm taller than Harold and Lina is 19 cm shorter than Harold. The sum of their heights is 350 cm . Find the height of each.
8. A woman earns half of what her husband's earns. One month they spent $\frac{2}{3}$ of their total earnings and saved what was left. They saved 200000 shillings. How much does the woman earn in a month?
9. A member of a football club is charged two third of the normal fee for the entry into a football match. Find the normal entry fee if members paid 400 shillings less than the normal charge.
10. The profit $2,400,000$ shillings of a certain company owned by three partners, Ali, Joseph and Halima is to be shared among them. Ali gets twice as much as Joseph and Halima gets 100000 shillings less than Ali. Find each person's share.

## Revision Exercise 2

1. Simplify the following algebraic expressions:
a) $5 p+8 q-2 p+r$
b) $-9 z+12$
c) $16 a+2 b-3-6 a+7 b$
d) $3 x+7 y-3 z+4 y-x+2 z-7$
e) $7 e-3 k+10-5 f-10 e+11 k$
2. Multiply each of the following:
a) $2 x+7 y-3 c$ by 4
b) $5 x-7 y+2 z$ by $3 m$
c) $6 m-3 n+5 p$ by -7
d) $9 d+5 e-2 f$ by $-2 b$
e) $2 a-7 b+9 c$ by $x y$
3. Open the brackets:
a) $7(3 a-2 b+c+2 d-e)$
b) $-3 m(5 x-y+7 z)$
4. Fill in blanks with the correct expressions:
a) $35 x+14 y-56 z=7$ ( $\qquad$ .)
b) $12 a b-28 a x+4 a z=4 a($ $\qquad$ .)
c) $30 m p-5 m r+45 m q+15 m x=5 m($................ )
d) 12 yar $+6 x a r-18 z a r+30$ qar $=$ $\qquad$ ( ................ )
e) $-56 x y m+21 x y n-63 x y p+35 x y q=$ $\qquad$
$\qquad$
5. Divide the following expressions
a) 7 arx $+24 x y-a x z$ by $x$
b) $25 a x-55 a y+15 a z$ by $5 a$
C) $6 m a+12 u v-4 x y$ by 2
d) $32 x y-18 x z+16 x n$ by $-2 x$
e) $21 m n+18 x y-6 z r$ by -3
6. To convert miles into kilometres you multiply the number of miles by 1.6. Write an algebraic expression to represent this.
7. Thermometers usually give the temperature in both Celsius $\left({ }^{\circ} \mathrm{C}\right)$ and Farenheit ( ${ }^{\circ}$ F). To convert Celsius into Farenheit multiply by 9, divide by 5 and then add 32. Write this as an algebraic expression.
8. Solve the following equations
a) $2 x+9=7-(3 x-4)$
b) $\frac{x+7}{5}=3+\frac{x}{3}$
c) $\frac{7}{x+1}=\frac{1}{x-5}$
d) $\frac{x}{2}-\frac{x}{7}=\frac{1}{3}$
e) $\frac{5}{3}(3 x-2)=15$
9. James's age is five times the age of Charles. If the sum of their age is 48 years, find Charles' age.
10. When a certain number is subtracted from 70 and the result divided by 2 , the result is 3 times the original number. Find the number.
11. The cost of a handbag is 10000 shillings less than that of pair of shoes. The cost of a skirt is half the cost of a pair of shoes. If the total cost of all three items is 70000 shillings, find the cost of a pair of shoes.
12. One third of Rehema's money plus two fifth of Zaina's money is 28000 shillings. If Zaina's money exceed that of Rehema's by 4000 shillings find the amount of money each has.
13. When a number is added to both numerator and denominator of $\frac{5}{9}$, the result is $\frac{2}{3}$, what is the number?

## Challenge activity

## More stick patterns

Using the same steps, find an algebraic expression for the number of match sticks needed to make $n$ rhombuses using this pattern.

Figure 2.8


Try making up your own patterns and find the formula to go with them.

## What Have I learned?

Work with your partner. Try to check what you have learned. Review what you need help with.

1. Forming an algebraic expression.
2. Simplifying algebraic expressions.
3. Forming equation with one unknown.
4. Solving equation with one unknown.
5. Solve a word problem leading to equation in one unknown.

## To remember

Algebra is related to arithmetic and is applicable and useful in our daily life activities.

1. Something whose value can change or keeps on changing is called a variable. We use letters to represent variables.
2. The expression which contains letters or group of letters separated by + or - is called an algebraic expression. For example, $2 x, 5 a+3 b$, $5 x y-2 a x+3 x b$ are algebraic expressions.
3. The number which multiplies a variable is called the coefficient of the variable.
4. The variable together with its coefficient and the constant number in any algebraic expression forms a term. For example, the expression $2 x+5 y+7$ has three terms, $2 x$ the first term, $5 y$ the second term and 7 the third term.
5. The constant number appearing in an algebraic expression is a constant term.
6. An algebraic expression can be added, subtracted, multiplied and divided.
7. Like terms are collected when simplifying an algebraic expression.
8. A statement which connects two expressions with an equal sign ( $=$ ) is called an equation. For example, $2 y-3=5$
9. The unknown variable in an equation is represented by a letter. For example $x$ is the unknown variable in the equation $3 x+5=10$
10. If the value is substituted for an unknown the equation may be true or false. For example $3 x=12$ is true when $x=4$ and false when $x=6$
11. To solve an equation means to find the value of unknown variable that makes the equation to be true.
12. The steps for solving a word problem of algebra with one unknown are:

Step 1: Identify the unknown that you need to find.
Step 2: Define the letter that will stand for the unknown.
Step 3: Write the given information in algebraic form. In other words, write an equation to express the conditions given in the question using the chosen letter.
Step 4: Solve the equation to find the unknown.

## Chapter 3:

## Inequalities

In this chapter you will learn how to solve problems using algebra of inequalities. We use letters and symbols to write inequalities.

## You will learn:

- how to use inequality symbols;
- how to form and solve inequalities; and
- how to solve a word problem leading to an inequality.


## Some useful words:

true
kweli
false
siyo kweli
inequality
kutokuwa sawa
is not equal to
si sawa na
less than
ndogo kuliko
greater than
kubwa kuliko

### 3.1 Representing inequalities



In chapter seven, we studied equations. In this chapter, we are going to study inequality and comparison.

## Talking about inequalities

1. The whole class should line up in age order, with youngest on the left and oldest on the right. Turn to the person on your left. Say, "My age is more than your age." Turn to the person on your right. Say, "My age is less than your age."
2. Now line up in height order and repeat the exercise for height.

Figure 3.1

3. Go and find a short stick or a pencil. In groups of five arrange your sticks in order of length. Repeat the activity above but now you say, "My stick is longer than your stick", or "My stick is shorter than your stick".

Figure 3.2


In pairs, read each of the following statements aloud and say whether they are true or false.

Mwanaisha is heavier than Faraji. $\qquad$ false
$\qquad$
Faraji is lighter than Georgina.
Georgina is heavier than Mwanaisha. $\qquad$
Mwainaisha is not heavier than Georgina. $\qquad$
Georgina is heavier than Faraji. $\qquad$
Mwanaisha is heavier than Faraji. $\qquad$


Talk with a friend about figure 3.4. Compare the heights of the basketball players. Try to write three sentences in English. Your sentences should include the phrases:
taller than shorter than equal to

Figure 3.4: American basketball players


Talk about picture 3.5 with your friend. Compare the speed of the athletes. Try to write three sentences in English. Your sentences should include the phrases:
faster than slower than

Figure 3.5: Women's 100m race at the 2007 World Athletics Championships in Japan.


## Inequality symbols

In mathematics we use special symbols to write about inequality.
Look at the number 7 on the number line. Numbers less than 7 are on its left. Numbers greater the 7 are on its right.


3 is less than 7 is written as $\mathbf{3}<\mathbf{7}$.
On the number line, 3 is to the left of 7 .
9 is greater than 7 is written as $\mathbf{9 > 7}$.
On the number line, 9 is to the right of 7 .

Work with a partner. Read the following inequalities aloud in English, using "less than" or "greater than". Fill the blanks to write the inequalities in words.
$3<7 \quad$ Three is ...less........ ...than
$8<9 \quad$ Eight is .................. .................. nine.
$12>4 \quad$ Twelve is .................. .................. four.
$5>0 \quad$ Five is $\qquad$ zero.
$0<5$
Zero is $\qquad$ five.
$13 \cdot 8>13 \cdot 2$ Thirteen point eight is $\qquad$
$\qquad$ thirteen point two. $\frac{1}{3}<\frac{1}{2} \quad$ One third is $\qquad$ one half.

Look at the inequality: $x>4$. We can show it on the number line.
Note that $x$ cannot be four, and this fact is shown using unfilled O at point four (4).

$x$ can be any number greater than 4 e.g. $4 \cdot 0001,4 \cdot 2,5$ and so on.

Now look at the expression: $x<5$, shown on a number line.


Discuss with a partner. What values can $x$ take?

## Ex 3.1

(1) Mstatili ni pembenne.
(2) Pembetatu ni pembenne.
(3) Ama mstatili au pembetatu ni pembenne.

## Exercise 3.1

Represent the following inequalities on a number line:

$$
\begin{aligned}
& y>6 \\
& p>12 \\
& x<-5 \\
& q>-6 \\
& t<3.5 \\
& r>-6.5
\end{aligned}
$$

## Less than, greater than or equal to statements

## Consider the following sentences:

(1) A rectangle is a quadrilateral.
(2) A triangle is a quadrilateral.
(3) Either a rectangle or a triangle is a quadrilateral.

Statement (1) is true. Statement (2) is false. What about statement (3)?
Statement (3) is true. If one part of an 'either ... or' statement is true then the whole statement is true.

Look at the statements below. Are they true or false?

1. A triangle has either 3 or 4 sides.
2. A square has either 4 or 3 sides.
3. Either a square has 4 sides or a triangle has 3 sides.
4. A square has either 5 or 6 sides.

Now look at the following statements. Are they true or false?
$8>7$
$8=7$

Either $8>7$ or $8=7$.
We can write statement (3) above as $8 \geq 7$
Figure 3.3


Are the statements below true or false? Discuss in groups of two or three. You can draw the number lines to support your answer.

| 8 $\leq 12$ | true |
| :--- | :--- |
| $5=7$ | $\ldots \ldots . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . ~$ |

Now look at the inequation $x \geq 4$. We can show this on the number line. The circle at 4 is shaded to show that 4 is included.


An inequality is a mathematical sentence that uses phrases involving one of the following symbols $<, \leq,>, \geq$ and $\neq$.

## How to read inequalities in English

$50 \neq 65$ is read as " 50 is not equal to 65 "
$50<65$ is read as " 50 is less than 65 "
$50 \leq 65$ is read as " 50 is less or equal to 65 "
$65>50$ is read as " 65 is greater than 50 "
$65 \geq 50$ is read as " 65 is greater or equal to 50 "

In pairs, read the following inequalities aloud to each other in English and write them down in words.

| $12 \neq 14$ | $16<20$ | $-7 \leq 0$ | $2>-3$ |
| :--- | :--- | :--- | :--- |
| $101 \geq 100$ | $22>19$ | $5 \leq 10$ | $-4 \neq 4$ |
| $5 \leq 5$ | $-12<-11$ | $75 \neq 80$ | $30 \geq 30$ |

## Representing inequalities on the number line

Represent the following inequalities on the number line if:

1. $x$ is any number
2. $x$ is an integer
3. $x$ is any number,
a) $x>3$

b) $x \leq 1$

c) $1 \leq x<5$

4. $x$ is an integer
a) $x>3$


Worked example

## Ex 3.2

## Exercise 3.2

Represent the following inequalities on a number line if:

1. $x$ is any number
2. $x$ is an integer
a) $x>5$
b) $x<-3$
c) $x \geq-2$
d) $x \leq 1$
e) $x<4$
f) $x>-1$
g) $-2<x<5$
h) $3 \leq x \leq 7$
i) $-6<x \leq-1$
j) $2 \leq x<8$
3. Complete the inequalities by inserting >or < in the space provided.
a) -9 ........ 2
b) 7 ........ 4
c) 8 ........ 7
d) $-5 \ldots \ldots . .5$

### 3.2 Forming Linear Inequations

In chapter two, you wrote word problems as equations. An equation is an algebraic expression with " $=$ ". For example: $x-8=12,3 x-5=16$

You can write some word problems as inequations. An inequation is an algebraic expression with $\neq,>,<, \geq$ or $\leq$ instead of $=$.

For example, $x+4>8,2 p-6 \leq 5$.
Table 3.1

| Words or phrases | Symbols | Kiswahili meaning |
| :--- | :--- | :--- |
| not equal to | $\neq$ | si sawa na |
| older than | $>$ | ana umri mkubwa kuliko |
| greater than | $>$ | kubwa kuliko |
| taller than | $>$ | mrefu kuliko |
| heavier than | $>$ | -zito kuliko |
| more than | nyingi kuliko |  |
| less than | $<$ | kidogo kuliko |
| younger than | ana umri mdogo kuliko |  |
| smaller than | $<$ | -dogo kuliko |
| shorter than | -fupi kuliko |  |
| lighter than | $<$ | nyepesi kuliko, ... |
| greater or equal to, $\ldots$ | $\geq$ | kubwa kuliko au sawa na, $\ldots$ |
| less or equal to, $\ldots$ | $\leq$ | ndogo kuliko au sawa na, ... |

## Write the following inequalities as mathematical expressions:

(1) Mariamu is taller than 1.5 m . Let Mariamu's height be $x \mathrm{~m}$.
(2) Sauda is shorter than 1.3 m . Let Sauda's height be $y \mathrm{~m}$.
(3) Juliana is over 18 years old. Let Juliana's age be a years.
(4) Lulu is less than 20 years old. Let Lulu's age be $b$ years.

1) Let Mariamu's height be $x \mathrm{~m}$

Mariamu is taller than 1.5 m

In pairs, write the inequalities for sentences (2), (3) and (4). Refer to table 3.1.

Hatua ya 2: Fafanua herufi inayowakilisha kitu kisichofahamika.

Hatua ya 2: Bainisha taarifa za kihisabati


## Worked example

## Hatua ya 1: Fafanua

herufi inayowakilisha kitu kisichofahamika.

Hatua ya 2: Bainisha taarifa za kihisabati.

Hatua ya 1: Fafanua herufi inayowakilisha kitu kisichofahamika.

Hatua ya 2: Bainisha taarifa za kihisabati.

## Hatua ya 1: Fafanua

herufi inayowakilisha kitu kisichofahamika.

Hatua ya 2: Bainisha taarifa za kihisabati.

1. Lucy has 2000 shillings in her Vodacom mobile phone account and a call to her aunt in South Africa costs $\underline{500}$ shillings to connect and then 400 shillings per minute. How many minutes can she talk for if Vodacom charge by the minute?
Let number of minutes be $x$.
The call costs 500 shillings and then 400 shillings per minute.
Total cost Shillings must be less than 2000.
Cost in Shillings is $500+400 x$.
$\therefore 500+400 x<2000$
2. Rajabu wishes to buy plough oxen. He has 50,000 shillings to spend. Each ox costs 12000 shillings. He also has to pay 20,000 shillings registration fee for veterinary services. How many oxen can Rajabu buy?
Let the number of animals Rajabu can buy be $x$.
The cost in shillings is $\underline{12000}$ per ox plus $\underline{20000}$ registration fee.
Total cost in shillings must be less or equal to 50000 .
$\therefore x$ oxen cost $12000 x+2000$

$$
\therefore 12000 x+20000<50000
$$

3. Mama Komba wants to buy iron sheets for roofing her house. The iron sheets are sold in batches of twelve. The most expensive iron sheets cost 300000 shillings for one batch. The cheapest cost 150000 shillings for a batch. Write the number of batches Mama Komba can buy as an inequation if she spends 2000000 shillings on her roof. Let the number of batches of iron sheets she is able to buy be $\operatorname{Sh} x$ most expensive iron sheets cost Sh 300000 for one batch total cost must be less than Sh 2000000

$$
\begin{gathered}
\therefore 300000 x<2000000 \\
3 x<20
\end{gathered}
$$

cheapest cost Sh 150000 for a batch

$$
\therefore 150000 x>2000000
$$

$15 x>200$
$3 x>40$
Answer: $20<3 x<40$
(We can use $<$ rather than $\leq$ because $x$ must be a whole number).

## In groups, discuss and translate the following into Kiswahili and formulate its inequality.

Fatuma is travelling by Mohamed Trans Bus to Mwanza. She has 30,000 shillings to pay for luggage. The freight cost of a 30 kilogram suitcase is 5000 shillings and each extra kilogram of luggage costs 3000 shillings. What is the amount of luggage she can carry?

## Exercise 3.3

## Form linear inequalities from the following word problems.

1. When the variable $x$ is added to 12 , the result is greater or equal to 20.
2. The height $h$ of a tree in metres is more than 40 metres.
3. $y$ is a whole number. If three-quarters of $y$ is subtracted from 1 , the result is always greater than 0 .
4. Jorum's height in metres $y$, is not equal to that of Suzanna.
5. More millimetres of rain fell today than yesterday. 15 millimetres of rain fell yesterday.
6. Mashaka harvested 8 times more maize from his shamba than his neighbour Marwa. The total harvest for the two neighbours is less than 100 bags. Write an inequality to represent the number of bags harvested by Marwa.
7. When 50 is added to a certain number, the sum is less than 75 . Write an inequality representing this sentence.
8. The sum of the three consecutive integers is less than 1 . Write an inequality representing this statement.
9. A man is 26 years older than his daughter. If the sum of their ages is greater than 50 but less than 100, write an inequality for the possible ages of the man.
10. Anifa is 6 years older than Maria. The sum of their ages is less than or equal to 40 but greater than or equal to 20. Write an inequality for Maria's age.

### 3.3 Solving inequations with one unknown

## Solving inequations with one unknown without changing sign

## Consider the inequality:

$$
8>4 \quad \text { True }
$$

This is a true statement. Now add 3 to both sides:

$$
\begin{array}{ll}
8+3>4+3 \\
11>7 & \text { True }
\end{array}
$$

Now try subtracting 5 from both sides:

$$
\begin{array}{ll}
8-5>4-5 \\
3>-1 & \text { True }
\end{array}
$$

Try multiplying both sides by 2 :

$$
\begin{array}{ll}
8 \times 2>4 \times 2 & \\
16>8 & \text { True }
\end{array}
$$

1. In your groups discuss in Kiswahili the steps you will follow to solve : the following equations:
a) $6 x+8=26$
b) $4 y-20=64$
c) $\frac{5 p}{4}+12=2$
2. In (1), replace $=$ with $>$. Follow the same steps to solve the inequations:
a) $6 x+8>26$
b) $4 y-20>64$
c) $\frac{5 p}{4}+12>2$
3. Now in (1), replace $=$ with $<$. Follow the same steps to solve the inequations:
a) $6 x+8<26$
b) $4 y-20<64$
c) $\frac{5 p}{4}+12<2$

Conclusion: Inequations can be solved in the same way as equations.

## Solve for $x$ in the following

1. $x+3<7$

$$
\begin{aligned}
& x+3<7 \\
& x+3-3<7-3 \quad-3 \text { both sides } \\
& \underline{x<4}
\end{aligned}
$$

2. $x-5<3$

$$
\begin{aligned}
& x-5<3 \\
& x-5+5<3+5 \quad+5 \text { both sides } \\
& \underline{x<8}
\end{aligned}
$$

3. $x+2>5$

$$
\begin{aligned}
& x+2>5 \\
& x+2-2>5-2 \quad-2 \text { both sides } \\
& x>3
\end{aligned}
$$

4. $x-7>2$

$$
\begin{aligned}
& x-7>2 \\
& x-7+7>2+7 \quad+7 \text { both sides } \\
& \underline{x>9}
\end{aligned}
$$

5. $3 x+2 \leq 11$

$$
\begin{array}{ll}
3 x+2 \leq 11 & \\
3 x+2-2 \leq 11-2 & -2 \text { both sides } \\
3 x \leq 9 & \\
\frac{3 x}{3} \leq \frac{9}{3} & \div 3 \text { both sides } \\
x \leq 3
\end{array}
$$

6. $4 x-3 \geq 7$

$$
\begin{array}{ll}
4 x-3 \geq 7 & \\
4 x-3+3 \leq 7+3 & +3 \text { both sides } \\
4 x \leq 10 & \\
\frac{4 x}{4} \leq \frac{10}{4} & \div 4 \text { both sides } \\
x \leq 2 \frac{1}{2} & \\
\hline
\end{array}
$$

## Exercise 3.4

Solve the following linear inequalities

1. $x-7 \leq-3$
2. $x+5>2$
3. $x-2 \geq 7$
4. $x-1<3$
5. $5 x-3>-1$
6. $3 x+5 \leq-2$
7. $\frac{1}{3} x+2<7$
8. $\frac{1}{2} x-\frac{2}{3} \geq 5$
9. $2 x-\frac{2}{5} \leq-13$
10. $5 x-1 \geq 4$

## Solving inequations with a sign change

## Consider again:

$$
8>4 \quad \text { True }
$$

Now multiply both sides by -2 :

$$
\begin{aligned}
& 8 \times(-2)>4 \times(-2) \\
& -16>-8 \quad \text { False! }
\end{aligned}
$$

When we multiply an inequation by a negative number, we have to reverse the inequality.

$$
-16<-8 \quad \text { True }
$$

Now divide both sides by -2 :

$$
\begin{aligned}
& 8 \div(-2)>4 \div(-2) \\
& -4>-2 \quad \text { False! }
\end{aligned}
$$

When we divide an inequation by a negative number, we also have to reverse the inequality.

$$
-4<-2 \quad \text { True }
$$

Conclusion: We can solve inequalities in the same way as equations but when multiplying by a negative number remember to change the direction of the inequality.

1. $13-\frac{y}{3}>9$
$13-\frac{y}{3}>9$
$-\frac{y}{3}>-4 \quad-13$ both sides
$y<12 \quad x(-3)$ both sides
2. $7-12 c \leq 43$

$$
\begin{array}{ll}
7-12 c \leq 43 & \\
-12 c \leq 36 & -7 \text { both sides } \\
c \geq 3 & \div(-12) \text { both sides }
\end{array}
$$

## Multiplicative inverse

There is another way to think about these problems. To find the unknown multiply by the multiplicative inverse of the coefficient.
[Definition] When you multiply a number by its multiplicative inverse the answer is always equal to 1 .

$$
\text { a number } \times \text { its multiplicative inverse }=\mathbf{1}
$$

For example:

$$
2 \times \frac{1}{2}=1
$$

therefore, the multiplicative inverse of 2 is $\frac{1}{2}$.
Take another example:

$$
5 \times \frac{1}{5}=1
$$

therefore, the multiplicative inverse of 5 is $\frac{1}{5}$.
But it is also true that

$$
\frac{1}{5} \times 5=1
$$

therefore, the multiplicative inverse of $\frac{1}{5}$ is 5 .
The multiplicative inverse of a negative number is also a negative number.
For example, find the multiplicative of -4 .

$$
-4 \times\left(-\frac{1}{4}\right)=1
$$

Therefore, the multiplicative inverse of -4 is $-\frac{1}{4}$.
and the multiplicative inverse of $-\frac{1}{4}$ is -4 .

- What is the multiplicative inverse of $\mathbf{1 0}$ ?
- What is the multiplicative inverse of $-\frac{\mathbf{1}}{\mathbf{2}}$ ?
- What is the multiplicative inverse of $\mathbf{1 ?}$
- What is the multiplicative inverse of $\mathbf{- 1}$ ?


## The multiplicative inverse is sometimes called the reciprocal.

As a general rule:
The multiplicative inverse of $x$ is $\frac{1}{x}($ where $x \neq 0)$.

## Worked example

Solve for $x$ in the following linear inequalities

1) $3-x<5$

$$
3-x<5
$$

$3-x-3<5-3 \quad-3$ both sides
$-x<2$
$(-1) \times-x>2 \times(-1) \times(-1)$ [inverse of -1 ], change direction of inequality
$x>-2$
2) $4-7 x \geq-11$
$4-7 x \geq-11$
$4-7 x-4 \geq-11-4+4$ both sides
$-7 x \geq-15$
$-7 x \times-\frac{1}{7} \leq-15 x-\frac{1}{7} \quad \times\left(-\frac{1}{7}\right)$ [inverse of -7$]$, change direction of inequality
$x \leq \frac{15}{7}$
$x \leq 2 \frac{1}{7}$
3) $-3 x-7<1$
$-3 x-7<1$
$-3 x-7+7<1+7+7$ both sides
$-3 x<8$
$-3 x \times-\frac{1}{3}>8 \times-\frac{1}{3} \quad \times\left(-\frac{1}{3}\right)$ [inverse of -3 ], change direction of inequality
$x>-\frac{8}{3}$
$x>-2 \frac{2}{3}$
4) $3-x<2 x-6 \leq 7$

Taking the first two expressions: Taking the last two expressions:

$$
\begin{array}{ccc}
3-x<2 x-6 & 2 x-6 \leq 7 \\
3-x-3<2 x-6-3 & & 2 x-6+6 \leq 7+6 \\
-x<2 x-9 & 2 x \leq 13 \\
-x-2 x<2 x-9-2 x & & \frac{2 x}{2} \leq \frac{13}{2} \\
-3 x<-9 & x \leq \frac{13}{2} \\
-3 x \times \frac{1}{-3}>-9 \times \frac{1}{-3} & x \leq 6 \frac{1}{2} \\
x>3 & \therefore 3<x \leq 6 \frac{1}{2} &
\end{array}
$$

Look at each inequality on a number line


Therefore, combining the two inequalities as the value of $x$ should satisfy the two inequalities we obtain

$$
3<x \leq 6 \frac{1}{2}
$$



$$
\therefore 3<x \leq 6 \frac{1}{2}
$$

## Exercise 3.5

## Ex 3.5

Solve the following linear inequalities involving changes in direction of the inequality:

1) $7-3 x<1$
2) $-6 x+5 \geq 3$
3) $-3 x+7 \leq-1$
4) $-\frac{2}{5} x-3>9$
5) $-x+\frac{1}{2}<7$
6) $5-2 x \geq 7$
7) $1-5 x<3$
8) $-\frac{3}{5} x-5 \geq-2$
9) $x+1 \leq 4-2 x<x+7$
10) $11-3 x \leq 5 \leq x+3$

Ex 3.6

## Solving word problems leading to inequalities

## Exercise 3.6

## Solve the following word problem leading to linear inequalities.

1. If 7 is added to the variable $x$, the result is greater than 17 . Find the range of possible values of the number $x$.
2. Lucy's age is more than twice that of her brother. If her brother is 6 years old, how old is Lucy?
3. The sum of three consecutive whole numbers is at least 14 . Find the range of possible values of the smallest number.
4. The sum of two numbers is at most 10 . If one of the numbers is 3 . Find the other number.
5. Twice the price of a kilogram of maize flour plus the price of a kilogram of beans is less than or equal to 5000 shillings. If the price of beans is 2000 shillings per kilogram. Find the maximum price of a kilogram of maize flour.
6. Halima is 7 years older than Jane. The sum of their ages is less than or equal to 50. Find Jane's age.
7. A coach passenger removed some items of mass 5 kg from her luggage so it was less than 30 kg . How much did she have originally?
8. The product of two numbers is at most 54 . If one of the numbers is 9 and the other is $x$, express the value of $x$ as an inequality.
9. In Tanzania, the minimum voting age is 18 years old. Alex was able to vote in an election eight years ago. How old is Alex now? Express your answer as an inequality.
10. A man is 34 years older than his daughter. If the sum of their ages is less than or equal to 60 , find the range of possible ages of his daughter.

## Revision Exercise 3

1. Solve the following inequalities and draw a number line to illustrate your answer.
a) $5 x-7<-12$
b) $15 \leq 3-4 x$
c) $4 y+5>13$
d) $\frac{3}{2} m-\frac{3}{2} \geq 3$
e) $5-3 x<20$
f) $-2 x+7 \geq-3$
g) $x-2<3 x+4 \leq 16$
h) $7(y+3)-11>9 y+2$
i) $0 \leq 9(1-y) \leq 18$
j) $\frac{-2 x+3}{2} \geq 3 x+1$
2. Solve the following inequalities and draw a number line to illustrate the answer, if $x$ is an integer.
a) $3 x+1<4$
b) $8 x<5 x-10$
c) $-3 x \geq 21$
d) $5-2 y>1$
e) $2 x+1 \leq 11<6 x-1$
f) $2 x+2 \leq \frac{1}{4} x+9$
g) $6 x+\frac{3}{4} \geq \frac{1}{2}$
h) $\frac{1}{2}(3-x)>2+x$
i) $x+1<7 \leq x+3$
j) $\frac{5 x}{2 x-1}>3$
3. Write down the inequalities illustrated on the number lines below.

4. Emmanuel is 3 years younger than Grace. The sum of their ages is at least 39. Write an inequality to represent Grace's age.
5. Mrs Sijaona has 30,000 shillings in her handbag. She needs to buy linen material for making her dress. If one metre for the material costs 8000 shillings, how many metres of linen material can she buy?
6. The sum of three consecutive even numbers is at most 14 . Find the possible values of the smallest even number.
7. Juma has 80000 shillings to spend on his shopping. He needs to buy a pair of trousers and a shirt. If all the pairs of trousers in the shop cost more than 50000 shillings, how much can he spend, in shillings, on the shirt?
8. In Tanzania, the minimum voting age is 18 years old. Lulu will be able to vote in an election after five years. How old is Lulu now? Express your answer as an inequality.
9. Massawe harvested 8 sacks more than his neighbour Mushi. However, the total harvest for the two neighbours did not exceed 50 sacks. How many sacks of maize did Mushi harvest?

## Challenge activity

1. If $-1 \leq x \leq 5$ and $-4<y<7$, where $x$ and $y$ are integers what is:
(a) the greatest value $x+y$ can take; and
(b) the least value $x+y$ can take.
2. Given $\left\{\begin{array}{l}4 x+3 y<27 \\ 3 x-y>17\end{array}\right.$

Find the range of values that $x$ "and " $y$ can take.
3. What is the secret message hidden in the inequality:
$9 x-7 i>3(3 x-7 u)$
(Hint: simplify the inequality).
4. Solve the following riddle:

If you triple me and take away ten, you will have less than half a century.

Double me and add five and you will have more than five fives.
What values can I take?
Write the answer as an inequality.

## What have I learned?

Work with your partner. Try to check what you have learned. Review what you need help with.

1. How to form linear inequalities.
2. How to represent linear inequalities on the number line.
3. How to solve linear inequalities which do not involve a change of sign.
4. How to solve linear inequalities which involve a change of sign.
5. How to solve word problems leading to linear inequalities.

## To remember

1. An inequality is a mathematical sentence that uses phrases involving one of the following symbols $<, \leq,>, \geq$ and $\neq$.
2. Inequalities can be solved in the same way as equations.

But when multiplying or dividing by a negative number, change the direction of the inequality.
3. The multiplicative inverse or reciprocal of any number when multiplied by that number gives 1 .

In other words,
The multiplicative inverse or reciprocal of $x$ is $\frac{1}{x}$

## Answers

## Chapter 1

## Ex 1.1

1. (a) 1
(b) 10
(c) 1000
(d) 10000
2. (a) 356

| Place value | Hundreds | Tens | Ones |
| :--- | :--- | :--- | :--- |
|  | $10 \times 10$ | 10 | 1 |
| Digits | 3 | 5 | 6 |
| Value of digit | 300 | 50 | 6 |
| taller than | $>$ | mrefu kuliko |  |

(b) 7602

| Place value | Thousands | Hundreds | Tens | Ones |
| :--- | :--- | :--- | :--- | :--- |
|  | $10 \times 10 \times 10$ | $10 \times 10$ | 10 | 1 |
| Digits | 7 | 6 | 0 | 2 |
| Value of digit | 7000 | 600 | 0 | 2 |

(c) 509417

| Place value | Thousands | Hundreds | Tens | Ones |
| :--- | :--- | :--- | :--- | :--- |
|  | $10 \times 10 \times 10$ | $10 \times 10$ | 10 | 1 |
| Digits | 7 | 6 | 0 | 2 |
| Value of digit | 7000 | 600 | 0 | 2 |

(d) 7103562

| Place <br> value | Millions | Hundred thousands | Ten thousands | Thousands | Hundreds | Tens | Ones |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | $10 \times 10 \times 10 \times$ <br> $10 \times 10 \times 10$ | $10 \times 10 \times 10 \times 10 \times 10$ | $10 \times 10 \times 10 \times 10$ | $10 \times 10 \times 10$ | $10 \times 10$ | 10 | 1 |
| Digits | 7 | 1 | 0 | 3 | 5 | 6 | 2 |
|  | 7000000 | 100000 | 0 | 3000 | 500 | 60 | 2 |

3. (a) The place value of 2 is 1 . Its value is 2 .

The place value of 1 is 10 . Its value is 10 .
(b) The place value of 6 is 1 . Its value is 6 . The place value of 3 is 10 . Its value is 30 .
(c) The place value of 7 is 1 . Its value is 7 .
(d) The place value of 0 is 1 . Its value is 0 . The place value of 5 is 10 . Its value is 50 .
(e) The place value of 8 is 1 . Its value is 8 . The place value of 5 is 10 . Its value is 50 .
The place value of 4 is 100 . Its value is 400 .
(f) The place value of 9 is 1 . Its value is 9 .

The place value of 4 is 10 . Its value is 40 .
The place value of 9 is 100 . Its value is 900 .
$(\mathbf{g})$ The place value of 0 is 1 . Its value is 0 .
The place value of 6 is 10 . Its value is 60 .
The place value of 5 is 100 . Its value is 500 .
(h) The place value of 5 is 1 . Its value is 5 .

The place value of 0 is 10 . Its value is 0 .
The place value of 8 is 100 . Its value is 800 .
(i) The place value of 1 is 1 . Its value is 1 .

The place value of 2 is 10 . Its value is 20 .
The place value of 3 is 100 . Its value is 300 .
The place value of 2 is 1000 . Its value is 2000 .
(j) The place value of 3 is 1 . Its value is 3 .

The place value of 3 is 10 . Its value is 30 .
The place value of 8 is 100 . Its value is 800 .
The place value of 7 is 1000 . Its value is 7000 .
(k) The place value of 5 is 1 . Its value is 5 .

The place value of 5 is 10 . Its value is 50 .
The place value of 7 is 100 . Its value is 700 .
The place value of 0 is 1000 . Its value is 0 .
The place value of 6 is 10000 . Its value is 60000 .
(I) The place value of 2 is 1 . Its value is 2 .

The place value of 5 is 10 . Its value is 50 .
The place value of 0 is 100 . Its value is 0 .
The place value of 5 is 1000 . Its value is 5000 .
The place value of 9 is 10000 . Its value is 90000 .
$(\mathbf{m})$ The place value of 4 is 1 . Its value is 4 .
The place value of 4 is 10 . Its value is 40 .
The place value of 0 is 100 . Its value is 0 .
The place value of 1 is 1000 . Its value is 1000 .
The place value of 2 is 10000 . Its value is 20000 .
The place value of 7 is 100000 . Its value is 700000 .
(n) The place value of 1 is 1 . Its value is 1 .

The place value of 4 is 10 . Its value is 40 .
The place value of 4 is 100 . Its value is 400 .
The place value of 2 is 1000 . Its value is 2000 .
The place value of 8 is 10000 . Its value is 80000 .
The place value of 8 is 100000 . Its value is 800000 .
The place value of 5 is 1000000 . Its value is 5000000 .

## Ex 1.2

1. (a) Eleven
(b) Eighteen
(c) Three hundred and twenty-one
(d) Seven hundred and eight
(e) Two thousand and sixty-three
(f) Eight thousand eight hundred and ninety
(g) Eighty-one thousand nine hundred and forty five
(h) Five hundred and sixty thousand three hundred and eight
(i) Six million seven hundred thousand three hundred and eighty-nine
(j) One hundred and four million eight hundred and sixty-three thousand five hundred and two
2. (a) 235
(b) 315
(c) 6358
(d) 8888
(e) 16107
(f) 2506000
(g) 5300610
(h) 17000000
(i) 28000620
(j) 1000406002

## Ex 1.3

3. $2,3,5,7,11,13,17,19,23,29,31,37$
4. $1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19$
5. $11,13,17,19,23,29,31,37,41,43,47$
6. $61,67,71$
7. 2
8. 9,15
9. 0
10. (a) Even
(b) Even
11. (a) Even
(b) Even
12. (a) Odd
(b) Odd
13. (a) No
(b) No

Ex 1.4

1. (a) 20
(b) add
(c) 14 .
(d) sum
(e) 3
2. (a) 6
(b) subtract
(c) 26 .
(d) take away
(e) 24
(f) minus
(g) 7. (h) difference between

## Ex 1.5

1. (a) 84079
(b) 7081
(c) 7763
(d) 993467
(e) 852
(f) 9003
(g) 88104
(h) 13161
(i) 949962
(j) 9541
2. (a) 13709
(b) 11404
(c) 80968
(d) 1851
(e) 44701
(f) 5253
(g) 73330
(h) 255
(i) 960094
(j) 3425032282
3. (a) 1576
(b) 424
(c) 329
4. Twelve plus five equals seventeen.

Twenty plus five equals twenty-five.
Sixty-two plus twenty-two equals eighty-four.
Sixty-two plus seventy-five equals one hundred and thirty-seven.
One hundred and fifteen plus twenty-two equals one hundred and thirty-seven.
Twenty minus three equals seventeen.
Forty- minus fifteen equals twenty-five.
Sixty-two minus twenty-two equals forty.
One hundred and six minus twenty-two equals eighty-four.
One hundred and fifteen minus seventy-five equals forty.
6. (a) 22400 shillings
(b) 2665
(c) 14

## Ex 1.6

2. (a) 30
(b) 72
(c) 230
(d) 180
(e) 2800
(f) 91
(g) 162
(h) 742
(i) 864
(j) 9000
$\begin{array}{lll}\text { (k) } 81828 & \text { (I) } 2089164\end{array}$
(m) 41144
(n) 570456
(o) 7099302
3. Four multiplied by zero equals zero.

Four multiplied by one equals four.
Six multiplied by zero equals zero.
Six multiplied by eleven equals sixty-six.
Nine multiplied by zero equals zero.

Nine multiplied by one equals nine.
Nine multiplied by eight equals seventy-two.
Twelve multiplied by zero equals zero.
Twelve multiplied by eight equals ninety-six.
Twenty multiplied by zero equals zero.
Twenty multiplied by five equals one hundred.
4. (a) 3
(b) 7
(c) 6
(d) 90
(e) 12
(f) 93
(g) 17747 r 2
(h) 53
(i) 266 r 11
(j) 1442521
(k) 83
$\begin{array}{ll}\text { (I) } 750 & \text { (m) } 98\end{array}$
(n) 309
(o) 1313 r 620
5. (a) $2,3,4,5,6,9$
(b) 2, 7
(c) 3, 9
(d) 2, 4
(e) 3
(f) 2, 3, 5, 6, 9
6. Zero divided by one equals zero.

Zero divided by two equals zero.
Zero divided by five equals zero.
Zero divided by eight equals zero.
Zero divided by ten equals zero.
Zero divided by eleven equals zero.
Thirty-two divided by eight equals four.
Seventy-two divided by eight equals nine.
Ninety divided by ten equals nine.
Ninety-six divided eight equals twelve.
One hundred divided by five equals twenty.
One hundred divided by ten equals ten.
One hundred and thirty-two divided by eleven equals twelve.

## Ex 1.7

(1) 216
(2) 120
(3) 3000
(4) 1212
(5) 50,500 Shillings
(6) 7
(7) 20
(8) 13
(9) 20
(10) 120000 Shillings
(11) 200
(12) 6024

## Revision Exercise 1

1. (a) Hundreds or 100
(b) Thousands or 1000
(c) Tens or 10
(d) Ten thousands or 10000
(e) Ones or 1
2. (a) $7000+200+60+1$ (b) $600000+30000+700+90+5$
(c) $10000+100+20+3$
(d) $1000000000+900000000+60000000+9000000+$ $200000+10+1$
(e) $400+90+3$
3. (a) 85710
(b) 123000
(c) 9670033
(d) 6201012
(e) 7000203101
4. (a) Two hundred and fifty million sixty-three thousand and seventy-two
(b) Five hundred and sixteen million nine hundred and eighteen thousand and three hundred
(c) One hundred and ninety seven thousand eight hundred and ten
(d) Three hundred million one hundred and seventy-five thousand four hundred and fifty
(e) Nineteen million five hundred and twenty-eight thousand one hundred and forty eight
5. (a) 97510
(b) 87310
6. $853 ; 835 ; 583 ; 538 ; 385 ; 358$
7. (a) 6, 7, 8, 9, 10
(b) $18,20,22,24,26,28$
(c) $7,11,13,17,19,23,29,31,37$
(d) $11,13,15,17,19,21,23$
8. (a) 11492
(b) 10109
(c) 4958
(d) 4315
(e) 304668
(f) 1961760
(g) 784
(h) 1030372 r 5
9. 9000 Shillings
10. (a) 7500000 Shillings
(b) 585 cows
11. 1970 kilometres
12. 5865345 Shillings
13. 1800 Shillings

## Chapter 2

## Ex 2.1

2. (a) $12 x$
(b) $b+c-3$
(c) $3 p-q$
(d) $20+x$

## Ex 2.2

1. (a) $9 y$
(b) $10 r$
(c) $5 n$
(d) $22 m$
$\begin{array}{ll}\text { (e) } 42 a & \text { (f) } 13 m\end{array}$
(g) $8 c+18$
(h) $30 n+2$
(i) $8 r+6$
(j) $9 x+2 y-5$
2. (a) -27
(b) 50
(c) 20
(d) 154
(e) -210
(f) 91
(g) 26
(h) 122
(i) 46
(j) 7

Ex 2.3
(1) four
(2) 6
(3) coefficient, $-3 x$
(4) coefficient, $7 z$
(5) $-3 x$
(6) second
(7) third
(8) constant

## Ex 2.4

1. (a) $9 a+21 b+9 c$
(b) $35 m x-14 m y+56 m z$
(c) $24 m-28 n+48 p$
(d) $-18 b d-10 b e+4 b f$
(e) $2 a x y-7 b x y+9 c x y$
2. (a) $24 a-16 b+8 c$
(b) $42 m x+18 m y-12 m z$
(c) $21 a x y+14 b x y-7 c x y$
(d) $-15 u v x-21 u v y+24 u v$
(e) $3 a x+7 b x--C$
3. (a) $5(5 x+2 y-3 z)$
(b) $-4 a(-2 b+4 x-z)$
(c) $3 m(6 p-2 r+9 q+4 x)$
(d) $6 \operatorname{ar}(2 y+x-3 z+5 q)$
(e) $-7 x y(6 m-2 n+9 p-3 z)$
4. (a) $3 b r+2 b y-b z$
(b) $4 x-3 y+13 z$
(c) $3 a m+6 u v-2 x y$
(d) $-15 y+8 z-5 n$
(e) $-7 m n-6 x y+2 r z$

## Ex 2.5

(1) $4 c$
(2) $2 s$
(3) $1500 x+50 y$
(4) $x-3$
(5) $3 x+6$
(6) $2(x+7)$
(7) $8 x$
(8) $7 x+12 y$
(9) $50-a-b$
(10) $3 x$ where $x$ is length of shorter piece, or $\frac{2}{3} y$ where $y$ is length of longer piece.
For (4), (6) and (10), you may use letters other than $x$ or $y$.

## Ex 2.6

(1) $x+11=35, x=24$
(2) $3 x+6=57, x=17$
(3) $\frac{32-x}{3}=-15, x=77$
(4) $7 x=3(x+8), x=6$
(5) $2 x-3=53$, where $x$ is Leila's age; or $4 y-3=53$, where $y$ is Ramla's age; or $4 z+9=44$, where $z$ is Juma's age. Leila is 28 , Ramla is 14 and Juma is 11 .
(6) $x+13=7 x, x=2$
(7) $3 x=156, x=52$
(8) $4 x-8=72,20 \mathrm{~m}$.
(9) $3 x-5=105$, where $x$ is George's share; or $3 y+55=100$, where $y$ is Rajabu's share; or $3 z-50=100$, where $z$ is William's share. George gets Sh35,000, Rajabu gets Sh15,000 and William get Sh50,000.
(10) $7 x=217, x=31$

For (4) - (10), you may use letters other than $x, y$ and $z$.

## Ex 2.7

(1) $x=24$
(2) $x=-4$
(3) $x=6$
(4) $x=14$
(5) $x=21$
(6) $x=11$
(7) $x=\frac{3}{2}$
(8) $x=75$
(9) $x=-3 \frac{3}{4}$
(10) $x=-8$

## Ex 2.8

(1) 23
(2) 11 and 13
(3) 12
(4) 30
(5) 10 m
(6) 18 boys, 27 girls
(7) James is 125 cm , Harold is 122 cm and Lina is 103 cm .
(8) Sh200,000
(9) Sh1200
(10) Ali gets Sh1,000,000; Joseph get Sh500,000; Halima gets Sh900,000.

## Revision Exercise 2

1. (a) $3 p+8 q+r$
(b) $3(4-3 z)$
(c) $10 a+9 b-3 b$
(d) $2 x+11 y-z-7$
(e) $-3 e-5 f+8 k+10$
2. (a) $8 x+28 y-12 c$
(b) $15 m x-21 m y+6 m z$
(c) $-42 m+21 n-35 p$
(d) $-18 b d-10 b e+4 b f$
(e) $2 a x y-7 b x y+9 c x y$
3. (a) $21 a-14 b+7 c+14 d-7 e$
(b) $-15 m x+3 m y-21 m z$
4. (a) $7(5 x+2 y-8 z)$
(b) $4 a(3 b-7 x+z)$
(c) $5 m(6 p+9 q-r+3 x)$
(d) $6 \operatorname{ar}(5 q+x+2 y-3 z)$
(e) $-7 x y(8 m-3 n+9 p-5 q)$ or $7 x y(-8 m+3 n-9 p+5 q)$
5. (a) $7 a r+24 y-a z$
(b) $5 x-11 y+3 z$
(c) $3 a m+6 u v-2 x y$
(d) $-8 n-16 y+9 z$
(e) $-7 m n-6 x y+2 r z$
6. $1.6 x \quad$ 7. $\frac{9 c}{5}+32$
7. (a) $x=\frac{2}{5}$
(b) $x=-12$
(c) $x=6$
(d) $x=\frac{14}{15}$
(e) $x=3 \frac{2}{3}$
8. 8 years
9. 10
10. Sh 32,000
11. Rehema has Sh36,000; Zaina has Sh40,000.
12. 3

## Chapter 3

## Ex 3.2

3. $(\mathbf{a})<$
(b) $>$
(c) $>$
(d) $<$

## Ex 3.3

1. $x+12 \geq 8 \quad$ 2. $\mathrm{h}>40 \quad$ 3. $1-3 / 4 y>0$, where $y$ is an integer.
2. $y \neq x$
3. $x>15$
4. $9 x<100$
5. $x+50<75$
6. $3 x+3<1$, where $x$ is the lowest number or $3 x<1$, where $x$ is the middle number or $3 x-3<1$, where $x$ is the highest number
7. $50<2 x-26<100$
8. $20 \leq 2 x+6 \leq 40$

For 4. - 10. you may use a letter other than $x$ for the variable.

## Ex 3.4

1. $x \leq 4$
2. $x>-3$
3. $x \geq 9$
4. $x<4$
5. $x>-$
6. $x \leq-$
7. $x<15$
8. $x \geq 11-$
9. $x \leq-6 \cdot 7$
10. $x \geq 1$

## Ex 3.5

1. $x>2$
2. $x \leq \frac{1}{3}$
3. $x \geq \frac{8}{3}$
4. $x<-30$
5. $x>-6 \frac{1}{2}$
6. $x \leq-1$
7. $x>-\frac{2}{5}$
8. $x \leq-5$
9. $-1<x \leq 1$
10. $x \geq 2$

## Ex 3.6

1. $x>10 \quad$ 2. more than 12. 3 3. greater or equal to 4 .
2. less than or equal to 7 .
3. 1500 shillings
4. 22 or older.
5. more than 30 kg but less than 35 kg .
6. $x \leq 6$
7. $x \geq 26$ (you may use a letter other than $x$ for Alex's age in years)
8. 0 to 13 years.

## Revision Exercise 3

1. (a) $x<-1$
(b) $x \leq-3$
(c) $y>2$
(d) $m \geq 3$
(e) $x>-5$
(f) $x \leq 5$
(g) $-3<x \leq 4$
(h) $y<4$
(i) $-1 \leq y \leq 1$
(j) $x \leq \frac{1}{8}$
2. (a) $x<1$
(b) $x<-3 \frac{1}{3}$
(c) $x \leq-7$
(d) $y<2 \quad$ (e) $2<x \leq 5_{3}^{7}$
(f) $x \leq 4$
(g) $x \geq-\frac{1}{24}$
(h) $\frac{1}{3}<-\frac{1}{3}$
$\begin{array}{ll}\text { (i) } 4 \leq x<6 & \text { (j) } x<3\end{array}$
3. (a) $x \geq-8$
(b) $x<2$
(c) $-9<x \leq 5$
(d) $0<x<7$
4. $x \geq 21$
5. 3 m or less.
6. $x \leq 2$, where $x$ is an even number.
7. less than $30000 \quad$ 8. $x \geq 13$, where $x$ is Lulu's age in years.
8. less than or equal to 21 .

For 4., 6. and 8. you may use a letter other than $x$. same time as learning about Number and Algebra.

