Physics 101: Lecture 02

Forces: Equilibrium Examples

- Today's lecture will cover Textbook Sections 2.1-2.7

Phys 101 URL:

http://courses.physics.illinois.edu/phys101/

Read the course web page!

Overview

- Last Lecture
  - ➔ **Newton's Laws of Motion**
    - » FIRST LAW: Inertia
    - » SECOND LAW: $F_{net} = ma$
    - » THIRD LAW: Action/reaction pairs
  - ➔ Gravity $$W = G\frac{M_{Earth}m}{r_{Earth}^2} = m\left(G\frac{M_{Earth}}{r_{Earth}^2}\right)$$
    $$\simeq mg \quad \text{(near Earth's surface!)}$$
- Today
  - ➔ **Forces as Vectors**
  - ➔ **Free Body Diagrams to Determine F_{net}**
    - » Draw coordinate axes, each direction is independent.
    - » Identify/draw all force vectors
  - ➔ Friction: kinetic $f = \mu_k N$; static $f \leq \mu_s N$
  - ➔ Contact Forces – Springs and Tension

Forces as Vectors

- Last lecture we calculated the force of gravity on a book (i.e. its WEIGHT):

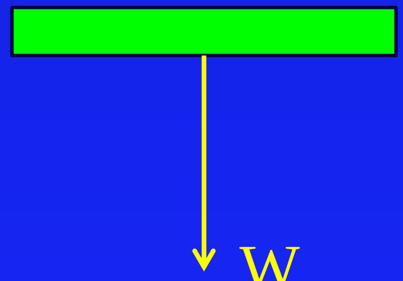
  Calculate the gravitational force on a 3 kg book held 1 meter above the surface of the earth.

$$W = G\, M_{Earth}\, m / r_{Earth}^{2}$$

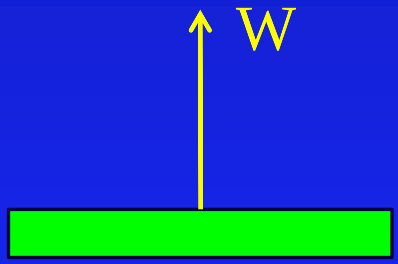
$$= (6.7\text{x}10^{-11}\ \text{m}^3 / (\text{kg s}^2))\ (6\text{x}10^{24}\ \text{kg})\ (3\ \text{kg}) / (6.4\text{x}10^{6} + 1)^2\ \text{m}^2$$

$$= \ 29.4\ \text{kg m/s}^2 = 29.4\ \text{N}$$

- We missed something: The direction!



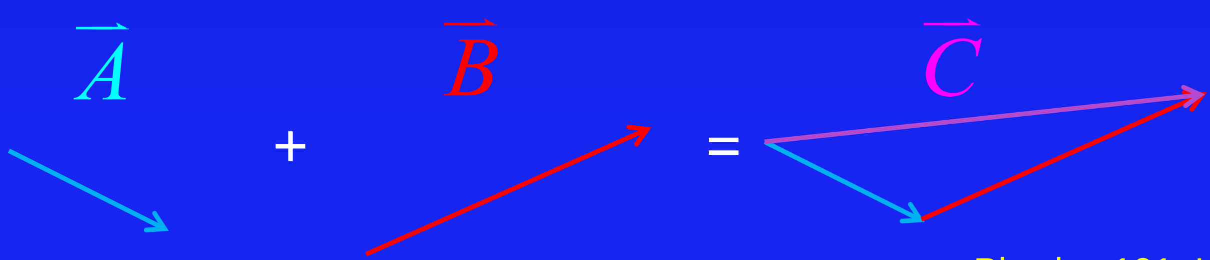
is different than



Forces as Vectors

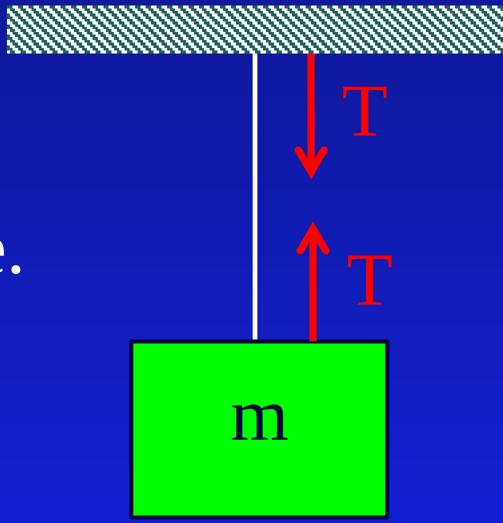
- A quantity which has both magnitude and direction is called a VECTOR; FORCES are VECTORS
- Usually drawn as an arrow pointing in the proper direction, where the length indicates the magnitude

- This is an example of VECTOR ADDITION: to add vectors, you place them head to tail, and draw the RESULTANT from the start of the first to the end of the last



Another Example of a Force: Tension

- Tension in an Ideal String, T:
  - ➔ Direction is parallel to string (only pulls)
  - ➔ Magnitude of tension is equal everywhere.



- Now we are ready to do some physics!

QUESTION:

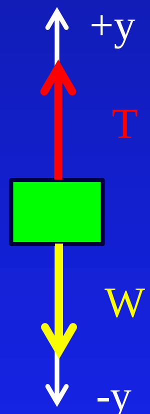
We suspend a mass m = 5 kg from the ceiling using a string. What is the tension in the string?

Newton's 2^{nd} Law and Equilibrium Systems

We suspend a mass m = 5 kg from the ceiling using a string. What is the tension in the string?

- Every single one of these problems is done the same way!

➔ Step 1: Draw a simple picture (called a Free Body Diagram), and label your axes!



➔ Step 2: Identify and draw all force vectors Weight, W Tension, T

➔ Step 3: Use your drawing to write down Newton's 2^{nd} law

$F_{Net} = ma$ In equilibrium, everything is balanced! $a = 0$

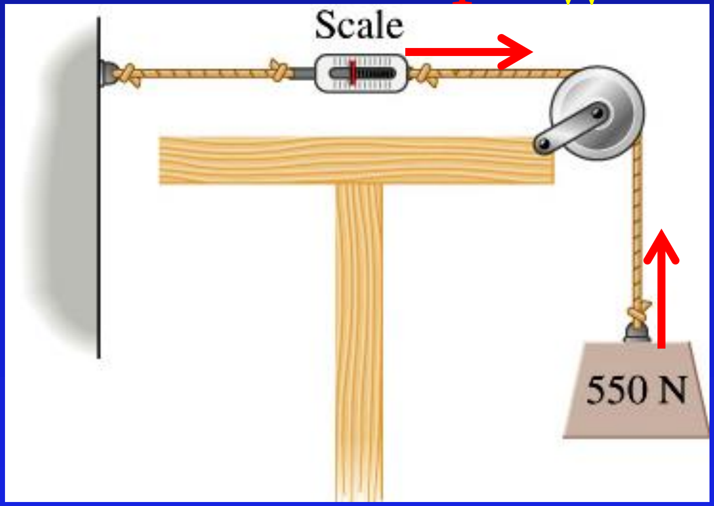
$T - W = 0$

$T = W = mg = (5\ \text{kg})*(9.8\ \text{m/s}^2) = 49\ \text{N}$

Checkpoint!

- What does scale 1 read?

- A) 225 N B) 550 N C) 1100 N



The magnitude of tension in a ideal string is equal everywhere.

Tension ACT

- Two boxes are connected by a string over a frictionless pulley. **In equilibrium**, box 2 is lower than box 1. Compare the weight of the two boxes.

A) Box 1 is heavier

B) Box 2 is heavier

C) They have the same weight

Step 1 – Draw!

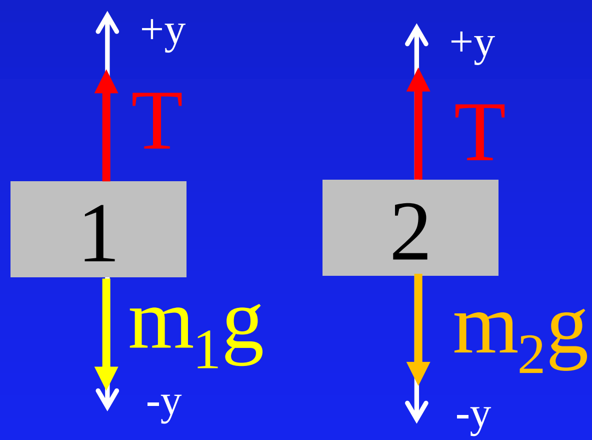
Step 2 – Forces!

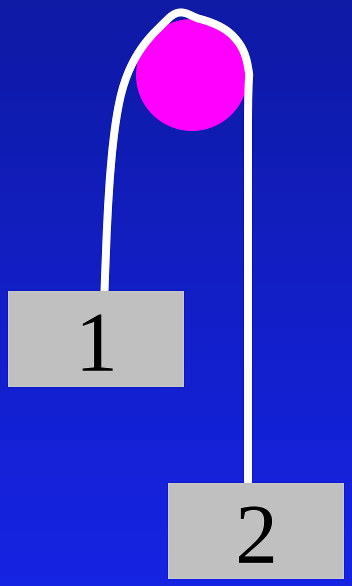
Step 3 – Newton's 2nd!

$$F_{Net} = m\,a$$

1) $T - m_1\,g = 0$

2) $T - m_2\,g = 0$



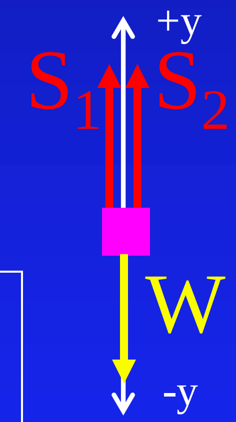


Another Force Example: Springs

- Force exerted by a spring is directly proportional to its displacement x (stretched or compressed).

$$F_{spring} = -k\,x$$

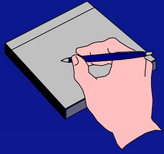
- Example: When a 5 kg mass is suspended from a spring, the spring stretches x_1 = 8 cm. If it is hung by two identical springs, they will stretch x_2 =



A) 4 cm B) 8 cm C) 16 cm

1 Spring	2 Springs
$S_1 - W = 0$	$S_1 + S_2 - W = 0$
$S_1 = W$	$kx_2 + kx_2 = 2kx_2 = W = mg$
$kx_1 = mg$	$x_2 = mg/(2k) = (5kg)*(9.8m/s^2)/(2*612.5N)$
$k = mg/x_1 = 612.5$ N/m	So: x_2 = 4 cm.

2 Dimensional Equilibrium!



Calculate force of hand to keep a book sliding at ***constant speed (i.e. a = 0)***, if the mass of the book is 1 Kg, μ_s=.84 and μ_k=.75

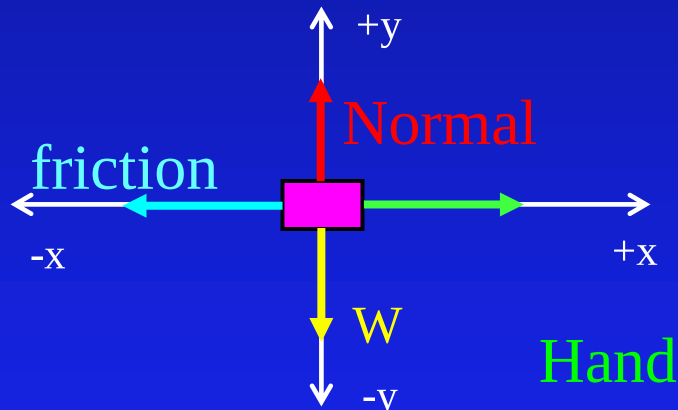
We do exactly the same thing as before, except in both x and y directions!

Step 1 – Draw!

Step 2 – Forces!

Step 3 – Newton's 2nd ($F_{Net} = ma$)!

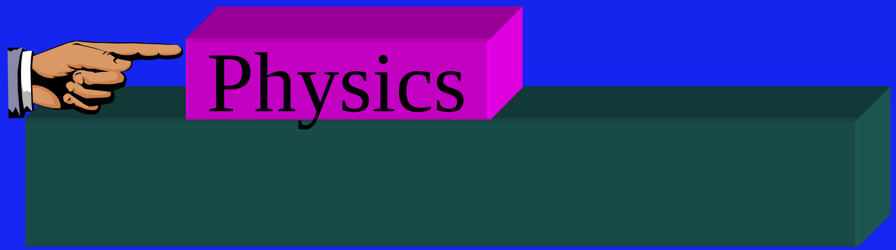
Treat x and y independently!



$$F_{Net,\,y} = N - W = ma_y = 0$$

$$F_{Net,\,x} = H - f = ma_x = 0$$

This is what we want!



Calculate force of hand to keep the book sliding at a ***constant speed (i.e. a = 0)***, if the mass of the book is 1 Kg, μ_s = .84 and μ_k=.75.

$F_{Net,\,y} = N - W = 0$ $\qquad N = W$

$F_{Net,\,x} = H - f = 0$ $\qquad H = f$

- Magnitude of frictional force is proportional to the normal force and always opposes motion!
  - ➔ $f_{kinetic} = \mu_k N$ $\qquad \mu_k$ coefficient of Kinetic (sliding) friction
  - ➔ $f_{static} \leq \mu_s N$ $\qquad \mu_s$ coefficient of Static (stationary) friction

$$H = f = \mu_k N = \mu_k W = \mu_k mg$$

$$= (0.75)*(1 \text{ kg})*(9.8 \text{ m/s}^2)$$

$$H = 7.35 \text{ N}$$

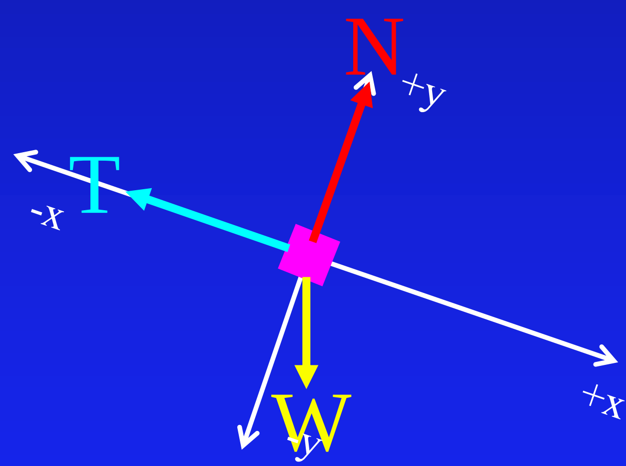
Forces in 2 Dimensions: Ramp

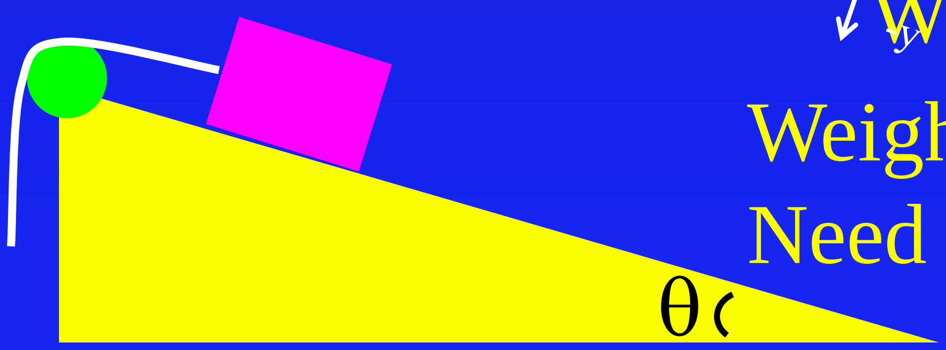
Calculate tension in the rope necessary to keep the 5 kg block from sliding down a **frictionless** incline of 20 degrees.

Step 1 - Draw!

You should draw axes parallel and perpendicular to motion!

Step 2 - Forces!

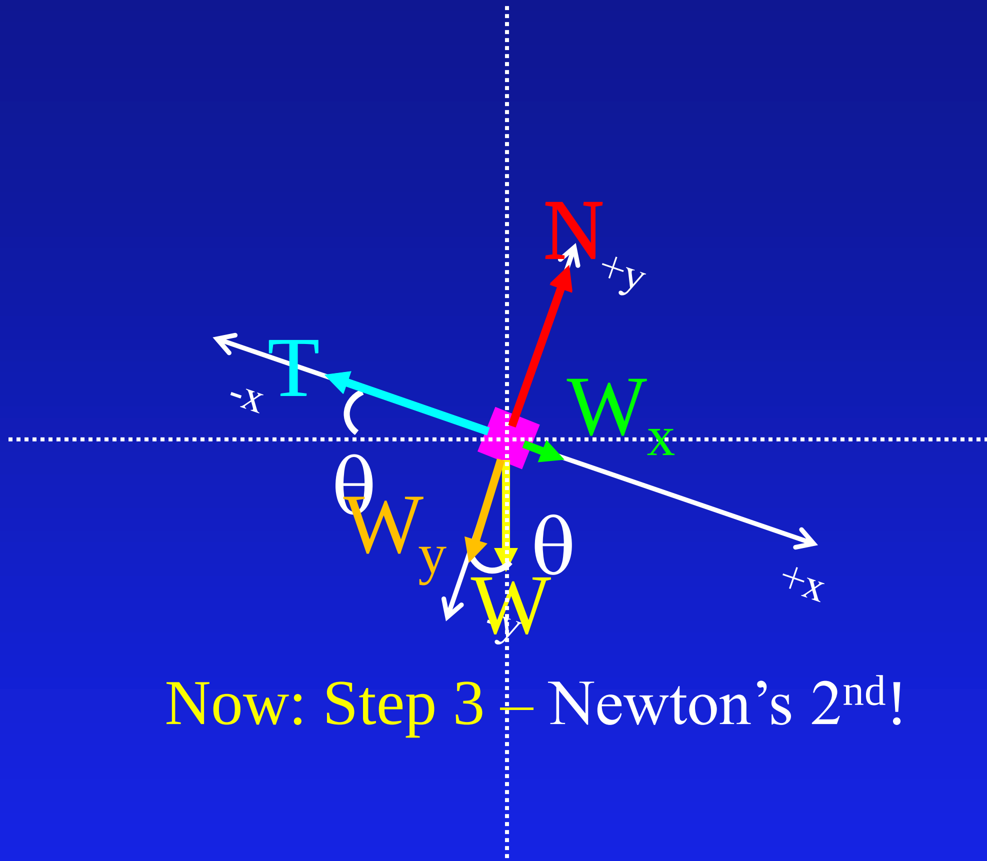




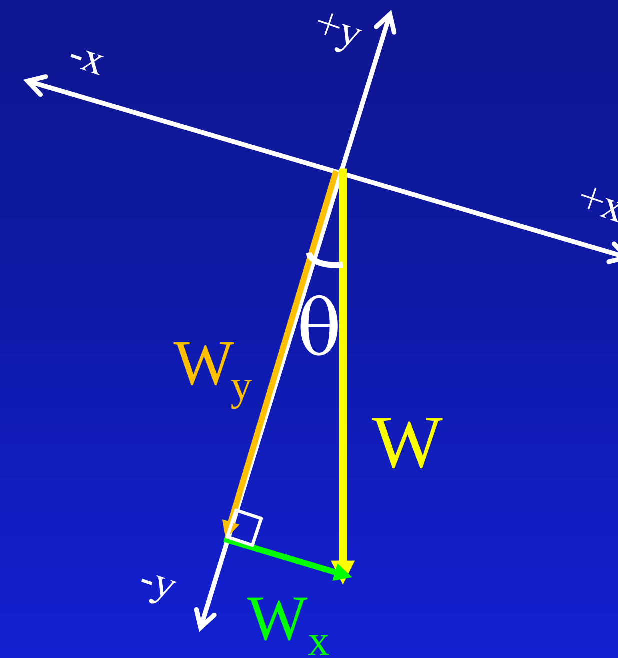
Weight is not in x or y direction!

Need to DECOMPOSE it!

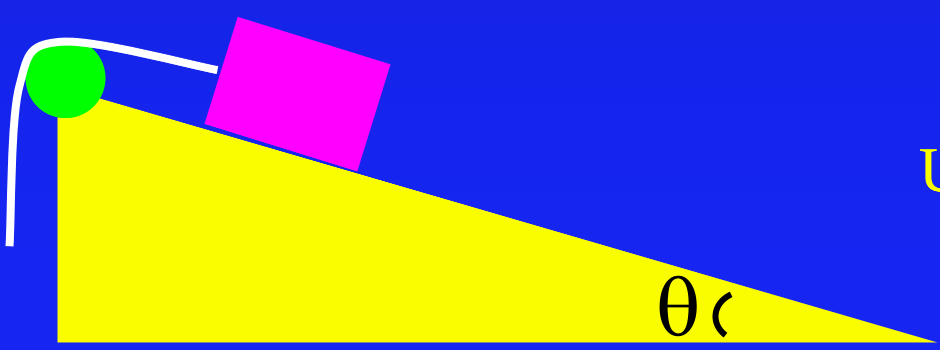
Vector Decomposition



Now: Step 3 – Newton's 2nd!



Split W into COMPONENTS parallel to axes



Note that

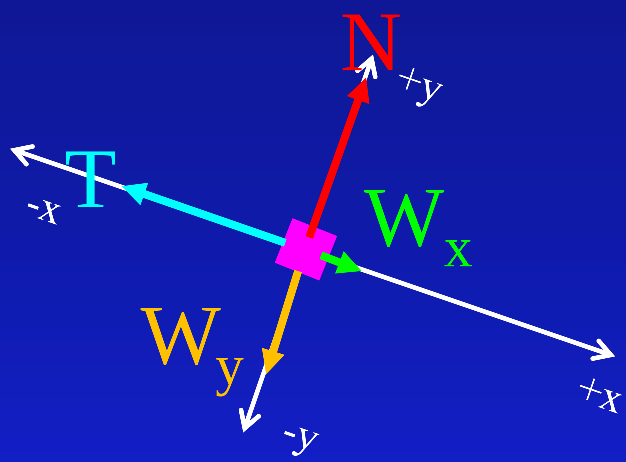
$$\vec{W} = \vec{W_y} + \vec{W_x}$$

Using Trig:

$$W_x = W \sin\theta$$

$$W_y = W \cos\theta$$

Calculate force necessary to keep the 5 kg block from sliding down a frictionless incline of 20 degrees.



Now: Step 3 – Newton's 2nd!

x direction:

$$F_{net,\,x} = ma_x$$

System is in equilibrium (a = 0)!

$$F_{net,\,x} = 0$$

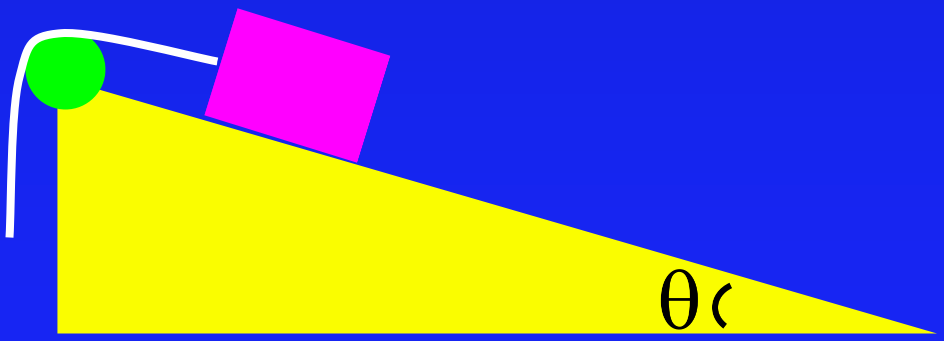
$$W_x - T = 0$$

$$T = W_x = W \sin\theta$$
$$= mg \sin\theta$$
$$= (5kg)(9.8m/s^2)\sin(20^o)$$

$$T = 16.8\ N$$

$$W_x = W \sin\theta$$

$$W_y = W \cos\theta$$



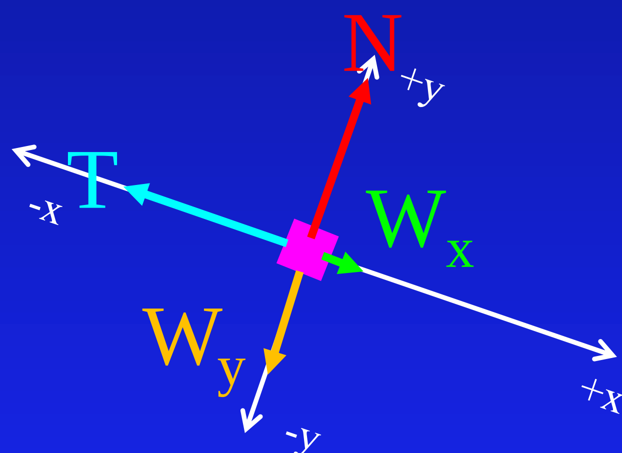
Normal Force ACT

What is the normal force of the ramp on the block?

A) $N > mg$ B) $N = mg$ C) $N < mg$

$W_x = W \sin\theta$

$W_y = W \cos\theta$



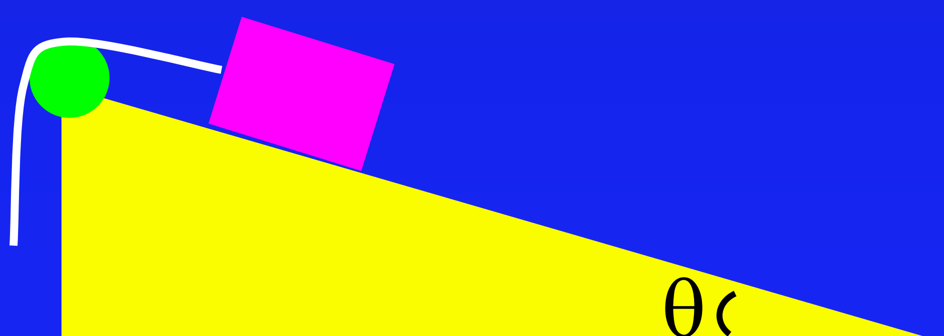
y direction:

$F_{net,\,y} = ma_y$

Equilibrium ($a = 0$)!

$F_{net,\,y} = 0$

$N - W_y = 0$



Summary

- Contact Force: Tension
  - ➔ Force parallel to string
  - ➔ Always Pulls, tension equal everywhere
- Contact Force: Spring
  - ➔ Can push or pull, force proportional to displacement
  - ➔ $F = k x$
- Contact Force: Friction
  - ➔ Static and kinetic
  - ➔ Magnitude of frictional force is proportional to N
- Two Dimensional Examples
  - ➔ Choose coordinate system; choose wisely!
  - ➔ Analyze each direction independently

Force at Angle Example

- A person is pushing a 15 kg block across a floor with $\mu_k = 0.4$ at a constant speed. If she is pushing down at an angle of 25 degrees, what is the magnitude of her force on the block?

x- direction: $F_{Net,\,x} = ma_x$

$P_x - f = P\cos(\theta) - f = 0$

$P\cos(\theta) - \mu N = 0$

$N = P\cos(\theta) / \mu$

y- direction: $F_{Net,\,y} = ma_y$

$N - W - P_y = N - W - P\sin(\theta) = 0$

$N - mg - P\sin(\theta) = 0$

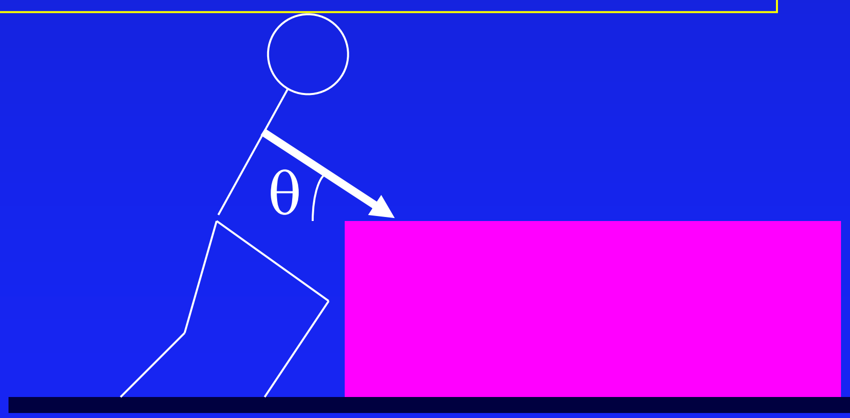
Combine:

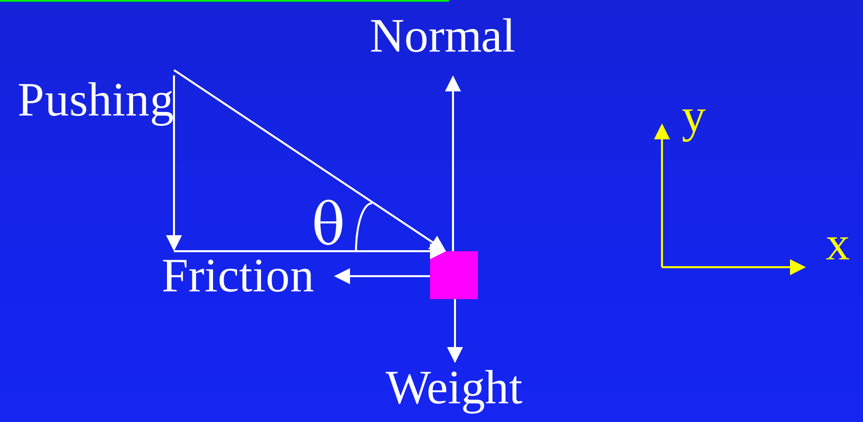
$(P\cos(\theta) / \mu) - mg - P\sin(\theta) = 0$

$P\,(\cos(\theta) / \mu \; - \sin(\theta)) = mg$

$P = m\,g / (\cos(\theta)/\mu - \sin(\theta))$

$P = 80$ N





Tension Example:

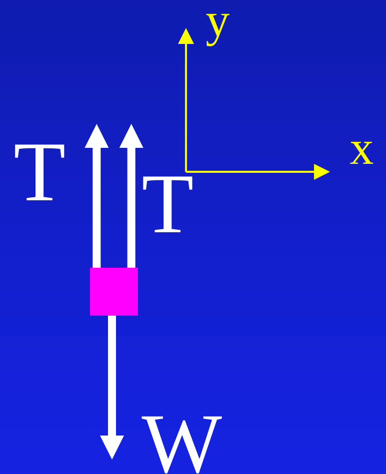
- Determine the force exerted by the hand to suspend the 45 kg mass as shown in the picture.

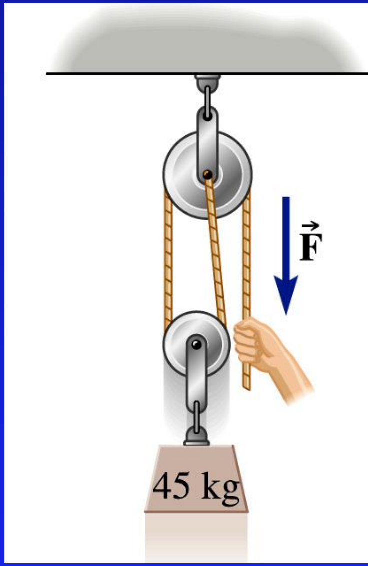
A) 220 N B) 440 N C) 660 N

D) 880 N E) 1100 N

$F_{Net} = m\,a$

$T + T - W = 0$





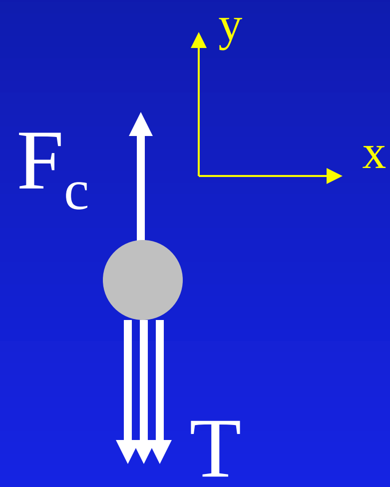
•Remember the magnitude of the tension is the same everywhere along the rope!

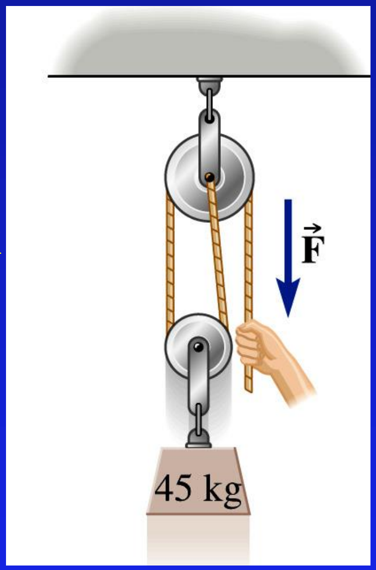
Tension ACT II

- Determine the force exerted by the ceiling to suspend pulley holding the 45 kg mass as shown in the picture.

A) 220 N B) 440 N C) 660 N

D) 880 N E) 1100 N





$\Sigma F = m\,a$

$F_c - T - T - T = 0$

•Remember the magnitude of the tension is the same everywhere along the rope!