

### Formulas from Trigonometry:

$$\sin^2 A + \cos^2 A = 1$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$$

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\sin A \sin B = \frac{1}{2} \{ \cos(A - B) - \cos(A + B) \}$$

$$\sin A \cos B = \frac{1}{2} \{ \sin(A - B) + \sin(A + B) \}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$$

$$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$$

$$\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

$$\cos A - \cos B = 2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(B - A)$$

$$\cos A \cos B = \frac{1}{2} \{ \cos(A - B) + \cos(A + B) \}$$

$$\cos(\theta) = \sin(\theta + \pi/2)$$

### Differentiation Formulas:

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + \frac{du}{dx} v$$

$$\text{Chain rule: } \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

$$\frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}, \quad \left(-\frac{\pi}{2} < \sin^{-1} u < \frac{\pi}{2}\right)$$

$$\frac{d}{dx} \tan^{-1} u = \frac{1}{1+u^2} \frac{du}{dx}, \quad \left(-\frac{\pi}{2} < \tan^{-1} u < \frac{\pi}{2}\right)$$

$$\frac{d}{dx} \ln u = \frac{1}{u} \frac{du}{dx}$$

$$\frac{d}{dx} \left(\frac{u}{v}\right) = \frac{v(du/dx) - u(dv/dx)}{v^2}$$

$$\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$$

$$\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx} \cos^{-1} u = \frac{-1}{\sqrt{1-u^2}} \frac{du}{dx}, \quad (0 < \cos^{-1} u < \pi)$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$$\frac{d}{dx} \log_a u = \frac{\log_a e}{u} \frac{du}{dx}, \quad a \neq 0, 1$$

### Integration Formulas:

$$\text{Integration by parts: } \int u dv = uv - \int v du$$

$$\int \frac{du}{u} = \ln |u|$$

$$\int a^u du = \frac{a^u}{\ln a}, \quad a > 0, a \neq 1$$

$$\int \cos u du = \sin u$$

$$\int \sin^2 u du = \frac{u}{2} - \frac{\sin 2u}{4} = \frac{1}{2}(u - \sin u \cos u)$$

$$\int \cos^2 u du = \frac{u}{2} + \frac{\sin 2u}{4} = \frac{1}{2}(u + \sin u \cos u)$$

$$\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left( \frac{u-a}{u+a} \right)$$

$$\int \frac{du}{\sqrt{u^2 + a^2}} = \ln(u + \sqrt{u^2 + a^2})$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2}$$

$$\int x \sin ax dx = \frac{\sin ax}{a^2} - \frac{x \cos ax}{a}$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a}$$

$$\int x^2 \cos ax dx = \frac{2x}{a^2} \cos ax + \left( \frac{x^2}{a} - \frac{2}{a^3} \right) \sin ax$$

$$\int \tan^2 ax dx = \frac{\tan ax}{a} - x$$

$$\int \ln x dx = x \ln x - x$$

$$\int e^u du = e^u$$

$$\int \sin u du = -\cos u$$

$$\int \tan u du = -\ln |\cos u|$$

$$\int \tan^2 u du = \tan u - u$$

$$\int \frac{du}{u^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a}$$

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a}$$

$$\int \frac{du}{\sqrt{u^2 - a^2}} = \ln(u + \sqrt{u^2 - a^2})$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2}$$

$$\int x^2 \sin ax dx = \frac{2x}{a^2} \sin ax + \left( \frac{2}{a^3} - \frac{x^2}{a} \right) \cos ax$$

$$\int x \cos ax dx = \frac{\cos ax}{a^2} + \frac{x \sin ax}{a}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a}$$

$$\int x e^{ax} dx = \frac{e^{ax}}{a} \left( x - \frac{1}{a} \right)$$

$$\int x \ln x dx = \frac{x^2}{2} \left( \ln x - \frac{1}{2} \right)$$

**Summation Formulas:**

$$\sum_{k=N_1}^{N_2} \alpha^k = \frac{\alpha^{N_1} - \alpha^{N_2+1}}{1 - \alpha}, \quad \alpha \neq 1$$

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1 - a}, \quad |a| < 1$$

$$\sum_{k=0}^{\infty} ka^k = \frac{a}{(1 - a)^2}, \quad |a| < 1$$

$$\sum_{k=0}^n a^k = \frac{1 - a^{n+1}}{1 - a}, \quad a \neq 1$$

$$\sum_{k=0}^n ka^k = \frac{a\{1 - (n + 1)a^n + na^{n+1}\}}{(1 - a)^2}$$

**Signals:**

$$\delta(t) = \frac{d}{dt}u(t) \qquad \delta[n] = u[n] - u[n - 1]$$

$$\langle x_1(t), x_2(t) \rangle = \int_{-\infty}^{\infty} x_1(t)x_2^*(t) dt \qquad \langle x_1[n], x_2[n] \rangle = \sum_{n=-\infty}^{\infty} x_1[n]x_2^*[n]$$

$$\mathcal{E}v\{x(t)\} = \frac{1}{2}\{x(t) + x(-t)\} \qquad \mathcal{O}d\{x(t)\} = \frac{1}{2}\{x(t) - x(-t)\}$$

**Complex Exponential Signals:**

$e^{j\omega_0 t}$	$e^{j\omega_0 n}$
Distinct signals for distinct $\omega_0$	Identical signals for values of $\omega_0$ separated by multiples of $2\pi$
Periodic for any choice of $\omega_0$	Periodic only if $\omega_0/(2\pi) = m/N \in \mathbb{Q}$
Fundamental frequency $\omega_0$	Fundamental frequency $\omega_0/m$
Fundamental period: $\omega_0 = 0$ : undefined $\omega_0 \neq 0$ : $2\pi/\omega_0$	Fundamental period: $\omega_0 = 0$ : one $\omega_0 \neq 0$ : $2\pi m/\omega_0$

**Systems:**

System  $H$  is linear if  $H\{ax_1(t) + bx_2(t)\} = aH\{x_1(t)\} + bH\{x_2(t)\}$ .

System  $H$  is time invariant if  $H\{x(t - t_0)\} = y(t - t_0)$ .

System  $H$  is memoryless if the current output does not depend on future or past inputs.

System  $H$  is invertible if distinct input signals produce distinct output signals.

System  $H$  is invertible if an inverse system  $G$  exists which “undoes” the action of  $H$ .

System  $H$  is causal if the current output does not depend on future inputs.

LTI system  $H$  is causal iff  $h(t) = 0 \forall t < 0$ .

Bounded:  $x(t)$  is bounded if  $\exists B \in \mathbb{R}, B > 0$ , such that  $|x(t)| \leq B \forall t \in \mathbb{R}$ .

System  $H$  is BIBO stable if every bounded input signal produces a bounded output signal.

LTI system  $H$  is BIBO stable iff  $\int_{-\infty}^{\infty} |h(t)| dt < \infty$ .

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t - \tau) d\tau = \int_{-\infty}^{\infty} x(t - \tau)h(\tau) d\tau$$

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n - k] = \sum_{k=-\infty}^{\infty} x[n - k]h[k]$$

$$s(t) = \int_{-\infty}^t h(\tau) d\tau \qquad h(t) = \frac{d}{dt}s(t)$$

$$s[n] = \sum_{k=-\infty}^n h[k] \qquad h[n] = s[n] - s[n - 1]$$

**Fourier Series Representation for a Periodic  $x(t)$  with Period  $T$ :**

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_T t}$$

$$a_k = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-jk\Omega_T t} dt$$

$$X(j\Omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\Omega_T)$$

$$\Omega_T = \frac{2\pi}{T}$$

**Sampling:**

sampling interval:  $T$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t + nT)$$

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT) \delta(t - nT)$$

sampling frequency:  $\Omega_T = \frac{2\pi}{T}$

$$P(j\Omega) = \frac{2\pi}{T} \sum_{n=-\infty}^{\infty} \delta(\Omega - n\Omega_T)$$

$$X_p(j\Omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(\Omega - k\Omega_T)$$

$$\Omega_T > 2\Omega_{\max}$$

**Discrete-Time Processing of Continuous-Time Signals:**

continuous-time frequency:  $\Omega$

discrete-time frequency:  $\omega$

sampling interval:  $T$

$$H_a(j\Omega) = \begin{cases} H_d(e^{j\Omega T}), & |\Omega| < \frac{\Omega_T}{2} \\ 0, & |\Omega| > \frac{\Omega_T}{2} \end{cases}$$

$$H_d(e^{j\omega}) = H_a\left(\frac{\omega}{T}\right)$$

(fundamental period)

$$H_d(e^{j\omega}) = \sum_{k=-\infty}^{\infty} H_a\left(\frac{\omega - 2\pi k}{T}\right)$$

$$\Omega = \frac{\omega}{T}$$

$$\omega = \Omega T$$

sampling frequency:  $\Omega_T = \frac{2\pi}{T}$

**Properties of the Fourier Transform:**

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)e^{-j\Omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega)e^{j\Omega t} d\Omega$$

Property	Aperiodic signal	Fourier transform
Linearity	$ax(t) + by(t)$	$aX(j\Omega) + bY(j\Omega)$
Time Shifting	$x(t - t_0)$	$e^{-j\Omega t_0} X(j\Omega)$
Frequency Shifting	$e^{j\Omega_0 t} x(t)$	$X(j(\Omega - \Omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\Omega)$
Time Reversal	$x(-t)$	$X(-j\Omega)$
Time and Frequency Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{j\Omega}{a}\right)$
Convolution	$x(t) * y(t)$	$X(j\Omega)Y(j\Omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} X(j\Omega) * Y(j\Omega)$
Differentiation in Time	$\frac{d}{dt}x(t)$	$j\Omega X(j\Omega)$
Integration	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{j\Omega} X(j\Omega) + \pi X(0)\delta(\Omega)$
Differentiation in Frequency	$tx(t)$	$j \frac{d}{d\Omega} X(j\Omega)$
Duality	$X(t)$	$2\pi x(-\Omega)$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\Omega) = X^*(-j\Omega) \\ \text{Re}\{X(j\Omega)\} = \text{Re}\{X(-j\Omega)\} \\ \text{Im}\{X(j\Omega)\} = -\text{Im}\{X(-j\Omega)\} \\  X(j\Omega)  =  X(-j\Omega)  \\ \angle X(j\Omega) = -\angle X(-j\Omega) \end{cases}$
Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\Omega)$ real and even
Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\Omega)$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ [ $x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ [ $x(t)$ real]	$\text{Re}\{X(j\Omega)\}$ $j\text{Im}\{X(j\Omega)\}$

Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\Omega)|^2 d\Omega$$

## Basic Fourier Transform Pairs:

Signal	Fourier transform
$\sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\Omega - \Omega_0)$
$e^{j\Omega_0 t}$	$2\pi \delta(\Omega - \Omega_0)$
$\cos \Omega_0 t$	$\pi[\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)]$
$\sin \Omega_0 t$	$\frac{\pi}{j}[\delta(\Omega - \Omega_0) - \delta(\Omega + \Omega_0)]$
$x(t) = 1$	$2\pi \delta(\Omega)$
<p>Periodic square wave</p> $x(t) = \begin{cases} 1, &  t  < T_1 \\ 0, & T_1 <  t  \leq \frac{T}{2} \end{cases}$ <p>and</p> $x(t + T) = x(t)$	$\sum_{k=-\infty}^{\infty} \frac{2 \sin k\Omega_0 T_1}{k} \delta(\Omega - k\Omega_0)$
$\sum_{n=-\infty}^{\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\Omega - \frac{2\pi k}{T}\right)$
$x(t) = \begin{cases} 1, &  t  < T_1 \\ 0, &  t  > T_1 \end{cases}$	$\frac{2 \sin \Omega T_1}{\Omega}$
$\frac{\sin Wt}{\pi t}$	$X(j\Omega) = \begin{cases} 1, &  \Omega  < W \\ 0, &  \Omega  > W \end{cases}$
$\delta(t)$	1
$u(t)$	$\frac{1}{j\Omega} + \pi \delta(\Omega)$
$\delta(t - t_0)$	$e^{-j\Omega t_0}$
$e^{-at}u(t), \quad \text{Re}\{a\} > 0$	$\frac{1}{a + j\Omega}$
$te^{-at}u(t), \quad \text{Re}\{a\} > 0$	$\frac{1}{(a + j\Omega)^2}$
$\frac{t^{n-1}}{(n-1)!} e^{-at}u(t), \quad \text{Re}\{a\} > 0$	$\frac{1}{(a + j\Omega)^n}$

**Properties of the Discrete-Time Fourier Transform:**

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \qquad x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega})e^{j\omega n} d\omega$$

Property	Aperiodic signal	Fourier transform
Linearity	$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
Time Shifting	$x[n - n_0]$	$e^{-j\omega n_0} X(e^{j\omega})$
Frequency Shifting	$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Time Reversal	$x[-n]$	$X(e^{-j\omega})$
Time Expansion	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi} \int_{2\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)}) d\theta$
Differencing in Time	$x[n] - x[n - 1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - e^{-j\omega}} X(e^{j\omega})$
Differentiation in Frequency	$nx[n]$	$j \frac{d}{d\omega} X(e^{j\omega})$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \text{Re}\{X(e^{j\omega})\} = \text{Re}\{X(e^{-j\omega})\} \\ \text{Im}\{X(e^{j\omega})\} = -\text{Im}\{X(e^{-j\omega})\} \\  X(e^{j\omega})  =  X(e^{-j\omega})  \\ \angle X(e^{j\omega}) = -\angle X(e^{-j\omega}) \end{cases}$
Symmetry for Real and Even Signals	$x[n]$ real and even	$X(e^{j\omega})$ real and even
Symmetry for Real and Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ [x[n] real] $x_o[n] = \mathcal{O}d\{x[n]\}$ [x[n] real]	$\text{Re}\{X(e^{j\omega})\}$ $j\text{Im}\{X(e^{j\omega})\}$

Parseval's Relation for Aperiodic Signals

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |X(e^{j\omega})|^2 d\omega$$

### Basic Discrete-Time Fourier Transform Pairs:

Signal	Fourier transform
$\sum_{k=(N)} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l)$
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - 2\pi l)$
Periodic square wave $x[n] = \begin{cases} 1, &  n  < N_1 \\ 0, & N_1 <  n  \leq N/2 \end{cases}$ and $x[n + N] = x[n]$	$2\pi \sum_{k=-\infty}^{\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$
$\sum_{k=-\infty}^{\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$
$a^n u[n], \quad  a  < 1$	$\frac{1}{1 - ae^{-j\omega}}$
$x[n] = \begin{cases} 1, &  n  \leq N_1 \\ 0, &  n  > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$
$\frac{\sin Wn}{\pi n}, \quad 0 < W < \pi$	$X(e^{j\omega}) = \begin{cases} 1, & 0 \leq  \omega  \leq W \\ 0, & W <  \omega  \leq \pi \end{cases}$ $X(e^{j\omega})$ is $2\pi$ -periodic
$\delta[n]$	1
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi \delta(\omega - 2\pi k)$
$\delta[n - n_0]$	$e^{-j\omega n_0}$
$(n + 1)a^n u[n], \quad  a  < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n], \quad  a  < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$

**Properties of the Laplace Transform:**

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st} dt$$

$$x(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st} ds$$

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	$R$
	$x_1(t)$	$X_1(s)$	$R_1$
	$x_2(t)$	$X_2(s)$	$R_2$
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time Shifting	$x(t - t_0)$	$e^{-st_0} X(s)$	$R$
$s$ -Domain Shifting	$e^{s_0 t} x(t)$	$X(s - s_0)$	$R + s_0$
Time Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{s}{a}\right)$	$aR$
Conjugation	$x^*(t)$	$X^*(s^*)$	$R$
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Time Differentiation	$\frac{d}{dt} x(t)$	$sX(s)$	At least $R$
$s$ -Domain Differentiation	$-tx(t)$	$\frac{d}{ds} X(s)$	$R$
Time Integration	$\int_{-\infty}^t x(\tau) d\tau$	$\frac{1}{s} X(s)$	At least $R \cap \{\text{Re}\{s\} > 0\}$



## Laplace Transforms of Elementary Functions:

Signal	Transform	ROC
$\delta(t)$	1	All $s$
$u(t)$	$\frac{1}{s}$	$\text{Re}\{s\} > 0$
$-u(-t)$	$\frac{1}{s}$	$\text{Re}\{s\} < 0$
$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\text{Re}\{s\} > 0$
$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\text{Re}\{s\} < 0$
$e^{-\alpha t}u(t)$	$\frac{1}{s + \alpha}$	$\text{Re}\{s\} > -\alpha$
$-e^{-\alpha t}u(-t)$	$\frac{1}{s + \alpha}$	$\text{Re}\{s\} < -\alpha$
$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s + \alpha)^n}$	$\text{Re}\{s\} > -\alpha$
$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s + \alpha)^n}$	$\text{Re}\{s\} < -\alpha$
$\delta(t - T)$	$e^{-sT}$	All $s$
$[\cos \omega_0 t]u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\text{Re}\{s\} > 0$
$[e^{-\alpha t} \cos \omega_0 t]u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega_0^2}$	$\text{Re}\{s\} > -\alpha$
$[e^{-\alpha t} \sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s + \alpha)^2 + \omega_0^2}$	$\text{Re}\{s\} > -\alpha$
$u_n(t) = \frac{d^n}{dt^n} \delta(t)$	$s^n$	All $s$
$u_{-n}(t) = \underbrace{u(t) * \dots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\text{Re}\{s\} > 0$

**Properties of the Unilateral Laplace Transform:**

$$\mathcal{X}(s) = \int_{0^-}^{\infty} x(t)e^{-st} dt$$

$$x(t) = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} \mathcal{X}(s)e^{st} ds$$

Property	Signal	Unilateral Laplace Transform
	$x(t)$	$\mathcal{X}(s)$
	$x_1(t)$	$\mathcal{X}_1(s)$
	$x_2(t)$	$\mathcal{X}_2(s)$
Linearity	$ax_1(t) + bx_2(t)$	$a\mathcal{X}_1(s) + b\mathcal{X}_2(s)$
$s$ -Domain Shifting	$e^{s_0 t} x(t)$	$\mathcal{X}(s - s_0)$
Time Scaling	$x(at), \quad a > 0$	$\frac{1}{a} \mathcal{X}\left(\frac{s}{a}\right)$
Conjugation	$x^*(t)$	$\mathcal{X}^*(s^*)$
Convolution (assumes $x_1(t)$ and $x_2(t)$ are zero $\forall t < 0$ ).	$x_1(t) * x_2(t)$	$\mathcal{X}_1(s)\mathcal{X}_2(s)$
Time Differentiation	$\frac{d}{dt} x(t)$	$s\mathcal{X}(s) - x(0^-)$
	$\frac{d^n}{dt^n} x(t)$	$s^n \mathcal{X}(s) - s^{n-1} x(0^-) - \dots - x^{(n-1)}(0^-)$
$s$ -Domain Differentiation	$-tx(t)$	$\frac{d}{ds} \mathcal{X}(s)$
Time Integration	$\int_{0^-}^t x(\tau) d\tau$	$\frac{1}{s} \mathcal{X}(s)$

**Properties of the  $z$ -Transform:**

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$x[n] = \frac{1}{2\pi j} \oint_C X(z)z^{n-1} dz$$

Property	Signal	$z$ -Transform	ROC
	$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$	$R$ $R_1$ $R_2$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least $R_1 \cap R_2$
Time Shifting	$x[n - n_0]$	$z^{-n_0}X(z)$	$R$ , except possibly $z = 0$
$z$ -Domain Scaling	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	$R$
	$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	$z_0 R$
Time Reversal	$a^n x[n]$ $x[-n]$	$X\left(\frac{z}{a}\right)$ $X(z^{-1})$	$ a R$ $R^{-1}$
Time Expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases} \quad r \in \mathbb{Z}$	$X(z^k)$	$R^{\frac{1}{k}}$
Conjugation	$x^*[n]$	$X^*(z^*)$	$R$
Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least $R_1 \cap R_2$
First Difference	$x[n] - x[n - 1]$	$(1 - z^{-1})X(z)$	At least $R \cap \{ z  > 0\}$
Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - z^{-1}}X(z)$	At least $R \cap \{ z  > 1\}$
$z$ -Domain Differentiation	$nx[n]$	$-z \frac{d}{dz} X(z)$	$R$

**Common  $z$ -Transform Pairs:**

Signal	Transform	ROC
$\delta[n]$	1	All $z$
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z  < 1$
$\delta[n - m]$	$z^{-m}$	All $z$ , except 0 (if $m > 0$ ) or $\infty$ (if $m < 0$ )
$\alpha^n u[n]$	$\frac{1}{1 - \alpha z^{-1}}$	$ z  >  \alpha $
$-\alpha^n u[-n - 1]$	$\frac{1}{1 - \alpha z^{-1}}$	$ z  <  \alpha $
$n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$ z  >  \alpha $
$-n\alpha^n u[-n - 1]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$ z  <  \alpha $
$[\cos \omega_0 n]u[n]$	$\frac{1 - [\cos \omega_0]z^{-1}}{1 - [2 \cos \omega_0]z^{-1} + z^{-2}}$	$ z  > 1$
$[\sin \omega_0 n]u[n]$	$\frac{[\sin \omega_0]z^{-1}}{1 - [2 \cos \omega_0]z^{-1} + z^{-2}}$	$ z  > 1$
$[r^n \cos \omega_0 n]u[n]$	$\frac{1 - [r \cos \omega_0]z^{-1}}{1 - [2r \cos \omega_0]z^{-1} + r^2 z^{-2}}$	$ z  > r$
$[r^n \sin \omega_0 n]u[n]$	$\frac{[r \sin \omega_0]z^{-1}}{1 - [2r \cos \omega_0]z^{-1} + r^2 z^{-2}}$	$ z  > r$

**Properties of the Unilateral  $z$ -Transform:**

$$\mathcal{X}(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

$$x[n] = \frac{1}{2\pi j} \oint_C \mathcal{X}(z)z^{n-1} dz$$

Property	Signal	Unilateral $z$ -Transform
	$x[n]$ $x_1[n]$ $x_2[n]$	$\mathcal{X}(z)$ $\mathcal{X}_1(z)$ $\mathcal{X}_2(z)$
Linearity	$ax_1[n] + bx_2[n]$	$a\mathcal{X}_1(z) + b\mathcal{X}_2(z)$
Time Delay	$x[n - 1]$	$z^{-1}\mathcal{X}(z) + x[-1]$
Time Advance	$x[n + 1]$	$z\mathcal{X}(z) - zx[0]$
$z$ -Domain Scaling	$e^{j\omega_0 n}x[n]$ $z_0^n x[n]$ $a^n x[n]$	$\mathcal{X}(e^{-j\omega_0}z)$ $\mathcal{X}(z/z_0)$ $\mathcal{X}(z/a)$
Time Expansion	$x_{(k)}[n] = \begin{cases} x[m], & n = mk \\ 0, & n \neq mk \end{cases} \quad m \in \mathbb{Z}$	$\mathcal{X}(z^k)$
Conjugation	$x^*[n]$	$\mathcal{X}^*(z^*)$
Convolution (assumes $x_1[n]$ and $x_2[n]$ are zero $\forall n < 0$ ).	$x_1[n] * x_2[n]$	$\mathcal{X}_1(z)\mathcal{X}_2(z)$
First Difference	$x[n] - x[n - 1]$	$(1 - z^{-1})\mathcal{X}(z) - x[-1]$
Accumulation	$\sum_{k=0}^n x[k]$	$\frac{1}{1 - z^{-1}}\mathcal{X}(z)$
$z$ -Domain Differentiation	$nx[n]$	$-z \frac{d}{dz} \mathcal{X}(z)$