Formulas to Determine Stone Size for Highway Embankment Protection

ROBERT M. CARMANY, Assistant Design Engineer, California Division of Highways

•A SHOALING water formula to calculate stone size is derived by first, introducing a new concept of the effect of face slope, second, relating forces to potential breaker height, and third, relating breaker height to depth of water. Wave forecasting thus becomes unnecessary under most conditions.

The shoaling water formula is then modified for application to stream-bank and deep-water shore protection.

SHOAL WATER WAVES

Let line CP (Fig. 1) be drawn between the center of gravity of an outer stone and its point of contact with the stone below. If all stones were perfect spheres and perfectly arranged, the line CP would be parallel to the face slope α. But with irregularly shaped stones the direction of CP will vary and CP for the most precariously situated stone will make the greatest angle with the face slope α.

Experiments were made with small stones arranged as riprap in which all but the outer stones were held rigidly in plaster of Paris. The face angle was tilted upward until the first uncemented stone fell out (Fig. 2). Repetitions of this experiment indicated that if the stones are fairly well placed, the face slope will reach an angle of 65 or 70 degrees before the line CP of the least stable stone reaches the vertical and the stone falls out (Fig. 3).

Let ρ represent this maximum angle of α. Then all angles of α less than ρ will be in a more stable condition as regards wave action, although the stability at angles greater than the angle of repose will be insufficient to resist the forces of gravity. The angle ρ may be compared to the angle of shear within a granular material such as sand. The angle of repose does not represent the angle of shear within the material prior to resting at the angle of repose. The interior angle of shear will be found to be closer to twice the angle

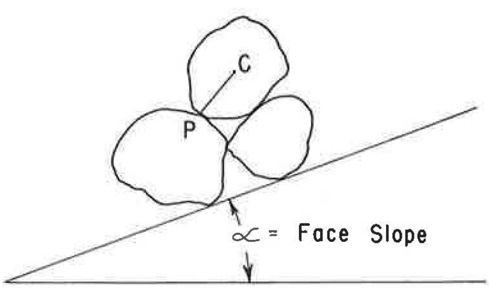

Figure 1. Typical outer stone.

Figure 2. Outer stones stable at $\alpha = 70° +$.

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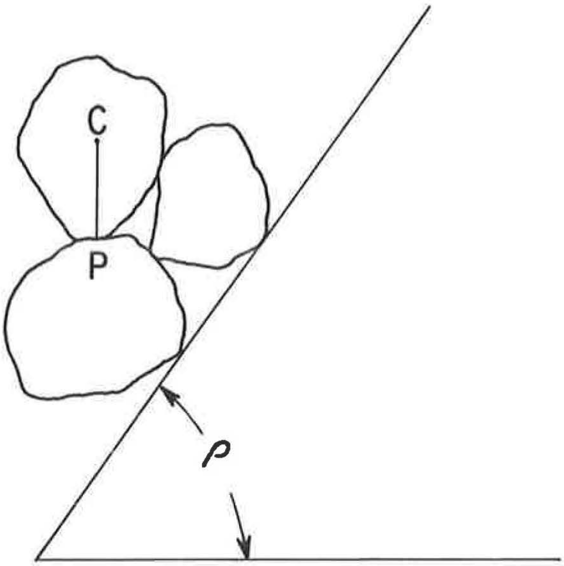

Figure 3. Typical outer stone at equilibrium.

of repose, and riprap, although not sufficiently stable to stand unaided steeper than the angle of repose, does increase in resistance to wave action starting from an angle much steeper.

In the case of the perfectly arranged uniform spheres, resistance to instability would increase starting at $\rho = 90^{0}$. For irregularly shaped stones, ρ will be in the neighborhood of 65 or 70 degrees. A lesser value has not been used to introduce a factor of safety. The actual experimental value has been used which in turn should reflect the proper relation between the face angle of the riprap and the size of stone. A factor or factors of safety may be introduced later into the final formula.

The minimum force that will dislodge the stone will be that required to rotate the stone about its point of contact P (Fig. 1). The vector diagram for this condition is shown in Figure 4.

Because the force to dislodge the stone will be supplied by the water, W_S will be the submerged weight of the stone and may be expressed:

$$W_S = k_1 d^3 (den_r - den_w) \quad (1)$$

in which

d = a linear dimension of the stone;
den_r = density of the rock;
den_w = density of the water; and
K_1 = a constant such that $k_1 d^3$ equals the volume of the stone.

The minimum force necessary to dislodge the stone will be

$$F_1 = k_1 d^3 (den_r - den_w) \sin(\rho - \alpha) \quad (2)$$

The farther the face slope of the riprap is laid back, the more force and energy will be required to dislodge the stone. The required energy could be supplied by the water without the necessary force, but it is hardly likely that the necessary force to move the stone would be of such short duration that the energy was insufficient to displace it. Therefore, assume that for stability the stone must be of sufficient weight to resist the maximum force exerted by the water. The resistance of a body to high-velocity fluid flow is approximately proportional to the square of a lineal dimension of the body, the square of the velocity of the

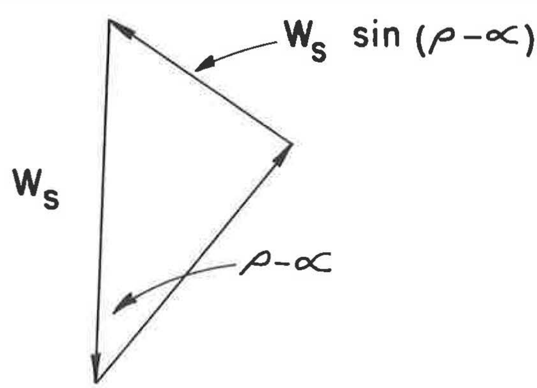

Figure 4. Vector diagram for an outer stone at equilibrium.

fluid past the body, and the density. Therefore, the force exerted by the water on a stone may be expressed as

$$F_2 = k_2 d^2 v^2 den_w \tag{3}$$

Along streams the velocity will not be the average stream velocity, but will vary with location. Along tangents it will be the bank velocity, but at bends it will be the impinging velocity which may approach the thread velocity.

Along larger rivers, lakes and oceans, it will be the maximum velocity resulting from wave action.

Energy contained in deep-water swells is approximately proportional to the length of the wave and the square of the height (1). In general, the more energy involved, the greater will be the damage potential. But this is not the whole story for one can easily visualize that a low wave of extremely long length could have a large amount of energy but exert very little force in deep water. In shoaling water energy concentrates; it depends on the ratio of wave height to wave length, wave period, conditions of shoaling and other things as to how the energy will be concentrated and how much of this energy will be finally dissipated against the riprap. A rather simple way of summarizing the resultant of all these factors on the damage potential at the riprap is to measure the maximum height to which this available energy and any additional that is added by backwash will lift a breaking wave at the riprap. The height of the breaking wave will enable calculation of the maximum velocity of the water impinging on the riprap by the simple relation:

$$v^2 = k_3 H_b \tag{4}$$

in which

v = velocity of the water at the trough, and
H_b = breaker height crest to trough.

Eq. 4 neglects the forward velocity of the breaking wave which should be small compared to the downward velocity at the trough. At higher elevations in the riprap the forward velocity would become more important but still the resultant velocity would be less.

For critical equilibrium F_1 must equal F_2, hence

$$K_1 d^3 (den_r - den_w) \sin (\rho - \alpha) = k_2 d^2 v^2 den_w \tag{5}$$

The weight of the stone in air may be expressed:

$$W = k_4 d^3 den_r$$

or (6)

$$d^3 = \frac{W}{k_4 den_r}$$

Substituting Eqs. 4 and 6 into 5 and combining all constants results in the following for the required weight of stone:

$$W = \frac{k_5 H_b^3 den_w^3 den_r}{(den_r - den_w)^3 \sin^3(\rho - \alpha)} \tag{7}$$

Most of California's highway embankments along the shore, especially the ocean shore, are not in deep water and some are not even wet except at high tide, so usually waves generated in deep water will be shoaling as they approach the embankment. Under these circumstances there will be a maximum size wave that will reach it still in possession of most of its deep-water energy. This maximum size wave which will expend its energy upon the embankment would ordinarily break at this depth of water. Larger deep-water waves will break in deeper water and will have spent a large portion of this energy before reaching the embankment. Waves that would ordinarily break in shallower water will, of course, reach the embankment but will contain less energy. Thus, the wave that would ordinarily break at the depth of water near the embankment will expend a maximum amount of energy upon the protection.

The height of this breaking wave, which is the criterion of damage potential adopted in Eq. 7, bears a relation to the depth of water.

For various beach slopes and wave shapes, Figure 5 (2) shows the relation between the height of breaker and the depth of water at which the wave will break when shoaling.

These data are the results of wave tank experiments. They show that for a steep beach slope (1:10), waves with a height to length ratio of approximately $H_0/L_0 = 0.02$) ($H_0/T^2 = 0.1$) will produce the highest breakers for any given depth. Their height can be as much as 1.25 times this depth of water d_b (flatter slopes give lower breaker heights). Thus, considering all factors most unfavorable except depth of water which is known, d_b may be substituted for H_b in Eq. 7 and absorb the value of 1.25 in the constant.

$$W = \frac{K_6 \, d_b^3 \, den_w^3 \, den_r}{(den_r - den_w)^3 \sin^3(\rho - \alpha)} \tag{8}$$

All experimental work relating stone size to breaker height has been done on a small scale with artificial waves less than 10 in. high. Considerable trouble can result from extrapolating into the unknown from an empirical or partially empirical formula derived from small-scale experiments. However, once the form is determined from small-scale tests, it may be fitted to full-scale experience by means of the constant. Fortunately for construction, but unfortunately from the standpoint of furnishing data to develop an equation, very few failures resulted that could be directly attributed to wave action. On the Ventura County coast, 3-ton stones resting on a 1.5 to 1 slope were displaced during a severe storm. The depth of water at this time was approximately 7 ft. Damage was minor (Fig. 6). Waves in this area at other times have been estimated to be 7 ft high but did no damage.

On this same coast in an area of greater exposure, 12-ton stones successfully resisted waves estimated to be 12 ft. With no experience with designs failing as a result of insufficient stone size, the constant K_6 has been set at 0.003 to agree with minimum stone sizes which proved adequate. As data build up, it is quite possible that this constant may be reduced.

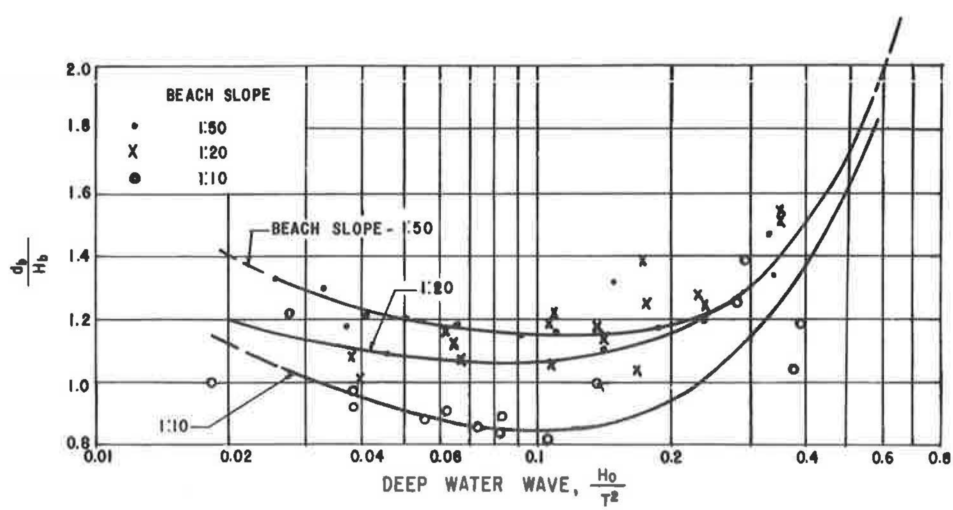

Figure 5. Breaker depth index (2).

Figure 6. Displaced stones after severe storms.

STREAM FLOW

Whether it be by wave action or stream flow, it is the velocity of water against the riprap that displaces the stones. As stated previously, it seems reasonable that the breaking wave concentrates and transforms the energy into high velocity flow, and neglecting forward velocity, the relation between the height of breaker and velocity of water against the riprap is

$$H_b = \frac{v^2}{2g} = \frac{v^2}{64.4} \qquad (9)$$

From Iversen's data (Fig. 5), the maximum height of breaker in relation to depth would be $H_b = 1.25\ d_b$. Therefore, $d_b = v^2/80$.

Substituting $v^2/80$ for d_b in the shallow water formula gives:

$$W = \frac{0.003 \frac{v^6}{80^3} den_w^{\,3}\, den_r}{(den_r - den_w)^3 \sin^3(\rho-\alpha)} = \frac{1.17 \times 10^{-5} v^6\, den_w^{\,3}\, den_r}{(den_r - den_w)^3 \sin^3(\rho-\alpha)} \qquad (10)$$

The constant of 1.17 has been increased to 2. The values obtained then agree with a table that had been in general use in California and which was based on experience of the Division of Highways Joint Bank Protection Committee.

It may be noted that this increase in the constant is the equivalent of an increase in stream velocity of less than 10 percent.

DEEP-WATER WAVES

The California Division of Highways experience with deep-water protection is very limited. By substituting significant wave height $H_{1/3}$ for H_b and using a constant of 231×10^{-5}, Eq. 7 then agrees with Iribarren's formula for a slope of $1\frac{3}{4}$-1 and with the Army Engineers' method (1) for all slopes between $1\frac{1}{4}$-1 and 3-1 providing the Army K is always taken for the most severe wave shape. Its relation to the formulas of other investigators is shown in Figure 7.

The formulas in slightly altered form used by the California Division of Highways (3) are as follows:

Shoal water:

$$W = \frac{0.003 d_b^3\, sg_r \csc^3(\rho-\alpha)}{\left(\frac{sg_r}{sg_w} - 1\right)^3} \qquad (11)$$

Stream flow:

$$W = \frac{0.00002\, v^2\, sg_r \csc^3(\rho-\alpha)}{(sg_r - 1)^3} \qquad (12)$$

Deep water:

$$W = \frac{0.00231\, H_{1/3}^{\,3}\, sg_r \csc^3(\rho-\alpha)}{\left(\frac{sg_r}{sg_w} - 1\right)^3} \qquad (13)$$

$\rho = 70^0$ for broken rock.

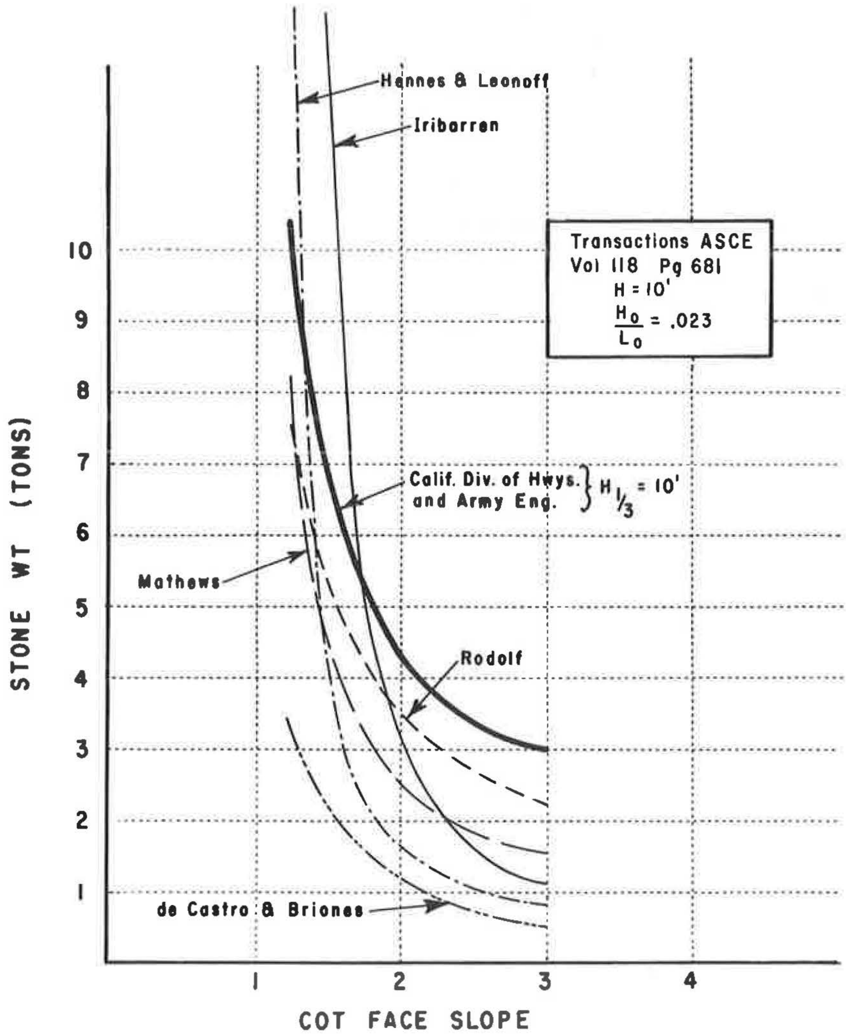

Figure 7. Comparison of formulas (4).

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