5

Foundation Analysis and Design

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This chapter illustrates application of the 2009 Edition of the *NEHRP Recommended Provisions* to the design of foundation elements. Example 5.1 completes the analysis and design of shallow foundations for two of the alternative framing arrangements considered for the building featured in Example 6.2. Example 5.2 illustrates the analysis and design of deep foundations for a building similar to the one highlighted in Chapter 7 of this volume of design examples. In both cases, only those portions of the designs necessary to illustrate specific points are included.

The force-displacement response of soil to loading is highly nonlinear and strongly time dependent. Control of settlement is generally the most important aspect of soil response to gravity loads. However, the strength of the soil may control foundation design where large amplitude transient loads, such as those occurring during an earthquake, are anticipated.

Foundation elements are most commonly constructed of reinforced concrete. As compared to design of concrete elements that form the superstructure of a building, additional consideration must be given to concrete foundation elements due to permanent exposure to potentially deleterious materials, less precise construction tolerances and even the possibility of unintentional mixing with soil.

Although the application of advanced analysis techniques to foundation design is becoming increasingly common (and is illustrated in this chapter), analysis should not be the primary focus of foundation design. Good foundation design for seismic resistance requires familiarity with basic soil behavior and common geotechnical parameters, the ability to proportion concrete elements correctly, an understanding of how such elements should be detailed to produce ductile response and careful attention to practical considerations of construction.

In addition to the *Standard* and the *Provisions* and *Commentary*, the following documents are either referenced directly or provide useful information for the analysis and design of foundations for seismic resistance:

ACI 318	American Concrete Institute. 2008. *Building Code Requirements and Commentary for Structural Concrete*.
Bowles	Bowles, J. E. 1988. *Foundation Analysis and Design*. McGraw-Hill.
CRSI	Concrete Reinforcing Steel Institute. 2008. *CRSI Design Handbook*. Concrete Reinforcing Steel Institute.
ASCE 41	ASCE. 2006. *Seismic Rehabilitation of Existing Buildings*.
Kramer	Kramer, S. L. 1996. *Geotechnical Earthquake Engineering*. Prentice Hall.
LPILE	Reese, L. C. and S. T. Wang. 2009. *Technical Manual for LPILE Plus 5.0 for Windows*. Ensoft.
Rollins et al. (a)	Rollins, K. M., Olsen, R. J., Egbert, J. J., Jensen, D. H., Olsen, K. G.and Garrett, B. H. (2006). "Pile Spacing Effects on Lateral Pile Group Behavior: Load Tests." *Journal of Geotechnical and Geoenvironmental Engineering*, ASCE, Vol. 132, No. 10, p. 1262-1271.
Rollins et al. (b)	Rollins, K. M., Olsen, K. G., Jensen, D. H, Garrett, B. H., Olsen, R. J.and Egbert, J. J. (2006). "Pile Spacing Effects on Lateral Pile Group Behavior: Analysis."

Journal of Geotechnical and Geoenvironmental Engineering, ASCE, Vol. 132, No. 10, p. 1272-1283.

Wang & Salmon — Wang, C.-K. and C. G. Salmon. 1992. *Reinforced Concrete Design* . HarperCollins.

Several commercially available programs were used to perform the calculations described in this chapter. SAP2000 is used to determine the shears and moments in a concrete mat foundation; LPILE, in the analysis of laterally loaded single piles; and spColumn, to determine concrete pile section capacities.

5.1 SHALLOW FOUNDATIONS FOR A SEVEN-STORY OFFICE BUILDING, LOS ANGELES, CALIFORNIA

This example features the analysis and design of shallow foundations for two of the three framing arrangements for the seven-story steel office building described in Section 6.2 of this volume of design examples. Refer to that example for more detailed building information and for the design of the superstructure.

5.1.1 Basic Information

5.1.1.1 Description. The framing plan in Figure 5.1-1 shows the gravity load-resisting system for a representative level of the building. The site soils, consisting of medium dense sands, are suitable for shallow foundations. Table 5.1-1 shows the design parameters provided by a geotechnical consultant. Note the distinction made between *bearing pressure* and *bearing capacity*. If the long-term, service-level loads applied to foundations do not exceed the noted bearing pressure, differential and total settlements are expected to be within acceptable limits. Settlements are more pronounced where large areas are loaded, so the bearing pressure limits are a function of the size of the loaded area. The values identified as bearing capacity are related to gross failure of the soil mass in the vicinity of loading. Where loads are applied over smaller areas, punching into the soil is more likely.

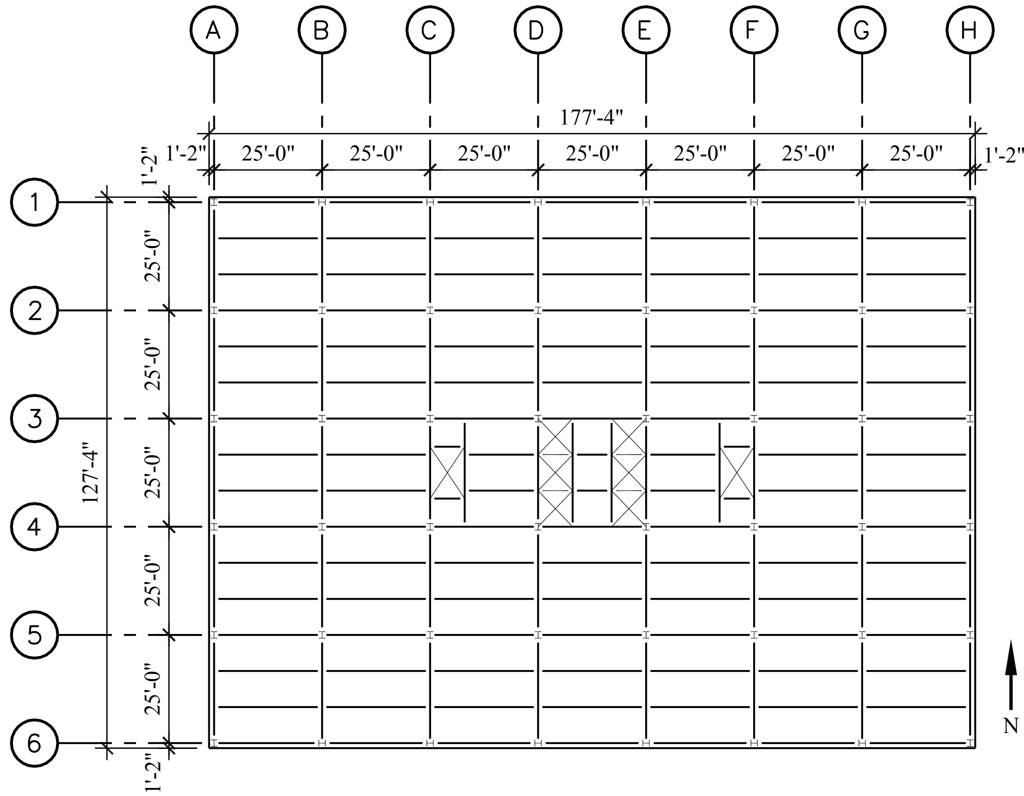


Figure 5.1-1 Typical framing plan

Because bearing capacities are generally expressed as a function of the minimum dimension of the loaded area and are applied as limits on the maximum pressure, foundations with significantly non-square loaded areas (tending toward strip footings) and those with significant differences between average pressure and maximum pressure (as for eccentrically loaded footings) have higher calculated bearing capacities. The recommended values are consistent with these expectations.

Table 5.1-1 Geotechnical Parameters

Parameter	Value
Basic soil properties	Medium dense sand (SPT) $N = 20$ $\gamma = 125$ pcf Angle of internal friction = 33 degrees

Table 5.1-1 Geotechnical Parameters

Parameter	Value
Net bearing pressure (to control settlement due to sustained loads)	$\leq$ 4,000 psf for $B \leq 20$ feet $\leq$ 2,000 psf for $B \geq 40$ feet (may interpolate for intermediate dimensions)
Bearing capacity (for plastic equilibrium strength checks with factored loads)	2,000B psf for concentrically loaded square footings 3,000B' psf for eccentrically loaded footings where B and B' are in feet, B is the footing width and B' is an average width for the compressed area. Resistance factor, $\phi = 0.7$ [This ϕ factor for cohesionless soil is specified in *Provisions* Part 3 Resource Paper 4; the value is set at 0.7 for vertical, lateral and rocking resistance.]
Lateral properties	Earth pressure coefficients: ▪ Active, $K_A = 0.3$ ▪ At-rest, $K_0 = 0.46$ ▪ Passive, $K_P = 3.3$ "Ultimate" friction coefficient at base of footing = 0.65 Resistance factor, $\phi = 0.7$

The structural material properties assumed for this example are as follows:

- f'_c = 4,000 psi
- f_y = 60,000 psi

5.1.1.2 Seismic Parameters. The complete set of parameters used in applying the *Provisions* to design of the superstructure is described in Section 6.2.2.1 of this volume of design examples. The following parameters, which are used during foundation design, are duplicated here.

- Site Class = D
- S_{DS} = 1.0
- Seismic Design Category = D

5.1.1.3 Design Approach.

5.1.1.3.1 Selecting Footing Size and Reinforcement. Most foundation failures are related to excessive movement rather than loss of load-carrying capacity. In recognition of this fact, settlement control should be the first issue addressed. Once service loads have been calculated, foundation plan dimensions should be selected to limit bearing pressures to those that are expected to provide adequate settlement performance. Maintaining a reasonably consistent level of service load-bearing pressures for all of the individual footings is encouraged since it will tend to reduce differential settlements, which are usually of more concern than are total settlements.

Once a preliminary footing size that satisfies serviceability criteria has been selected, bearing capacity can be checked. It would be rare for bearing capacity to govern the size of footings subjected to sustained loads. However, where large transient loads are anticipated, consideration of bearing capacity may become important.

The thickness of footings is selected for ease of construction and to provide adequate shear capacity for the concrete section. The common design approach is to increase footing thickness as necessary to avoid the need for shear reinforcement, which is uncommon in shallow foundations.

Design requirements for concrete footings are found in Chapters 15 and 21 of ACI 318. Chapter 15 provides direction for the calculation of demands and includes detailing requirements. Section capacities are calculated in accordance with Chapters 10 (for flexure) and 11 (for shear). Figure 5.1-2 illustrates the critical sections (dashed lines) and areas (hatched) over which loads are tributary to the critical sections. For elements that are very thick with respect to the plan dimensions (as at pile caps), these critical section definitions become less meaningful and other approaches (such as strut-and-tie modeling) should be employed. Chapter 21 provides the minimum requirements for concrete foundations in Seismic Design Categories D, E and F, which are similar to those provided in prior editions of the *Provisions*.

For shallow foundations, reinforcement is designed to satisfy flexural demands. ACI 318 Section 15.4 defines how flexural reinforcement is to be distributed for footings of various shapes.

Section 10.5 of ACI 318 prescribes the minimum reinforcement for flexural members where tensile reinforcement is required by analysis. Provision of the minimum reinforcement assures that the strength of the cracked section is not less than that of the corresponding unreinforced concrete section, thus preventing sudden, brittle failures. Less reinforcement may be used as long as “the area of tensile reinforcement provided is at least one-third greater than that required by analysis.” Section 10.5.4 relaxes the minimum reinforcement requirement for footings of uniform thickness. Such elements need only satisfy the shrinkage reinforcement requirements of Section 7.12. Section 10.5.4 also imposes limits on the maximum spacing of bars.

5.1.1.3.2 Additional Considerations for Eccentric Loads. The design of eccentrically loaded footings follows the approach outlined above with one significant addition: consideration of overturning stability. Stability calculations are sensitive to the characterization of soil behavior. For sustained eccentric loads, a linear distribution of elastic soil stresses is generally assumed and uplift is usually avoided. If the structure is expected to remain elastic when subjected to short-term eccentric loads (as for wind loading), uplift over a portion of the footing is acceptable to most designers. Where foundations will be subjected to short-term loads and inelastic response is acceptable (as for earthquake loading), plastic soil stresses may be considered. It is most common to consider stability effects on the basis of statically applied loads even where the loading is actually dynamic; that approach simplifies the calculations at the expense of increased conservatism. Figure 5.1-3 illustrates the distribution of soil stresses for the various assumptions. Most textbooks on foundation design provide simple equations to describe the conditions shown in Parts b, c and d of the figure; finite element models of those conditions are easy to develop. Simple hand calculations can be performed for the case shown in Part f. Practical consideration of the case shown in Part e would require modeling with inelastic elements, but that offers no advantage over direct consideration of the plastic limit. (All of the discussion in this section focuses on the common case in which foundation elements may be assumed to be rigid with respect to the supporting soil. For the interested reader, Chapter 4 of ASCE 41 provides a useful discussion of foundation compliance, rocking and other advanced considerations.)

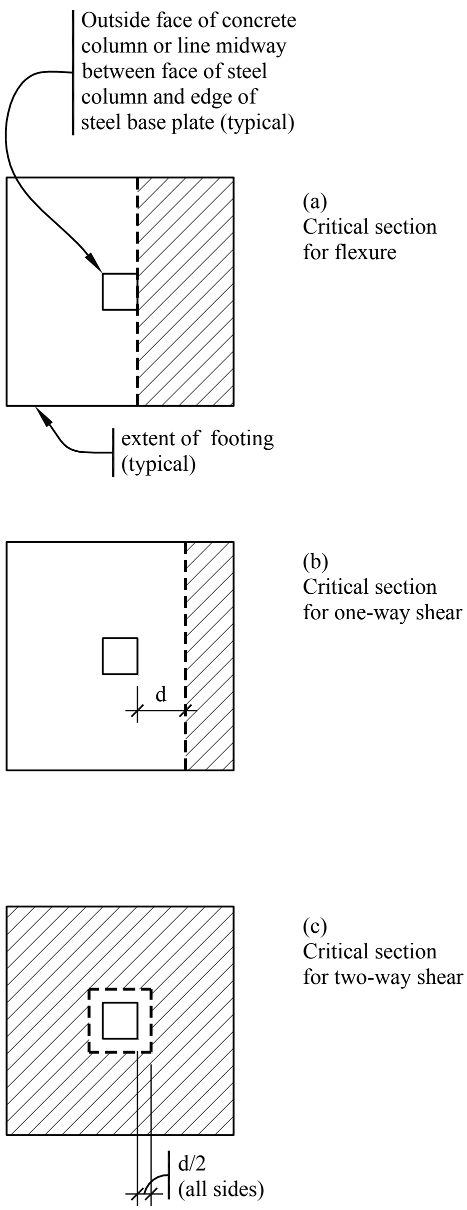


Figure 5.1-2 Critical sections for isolated footings

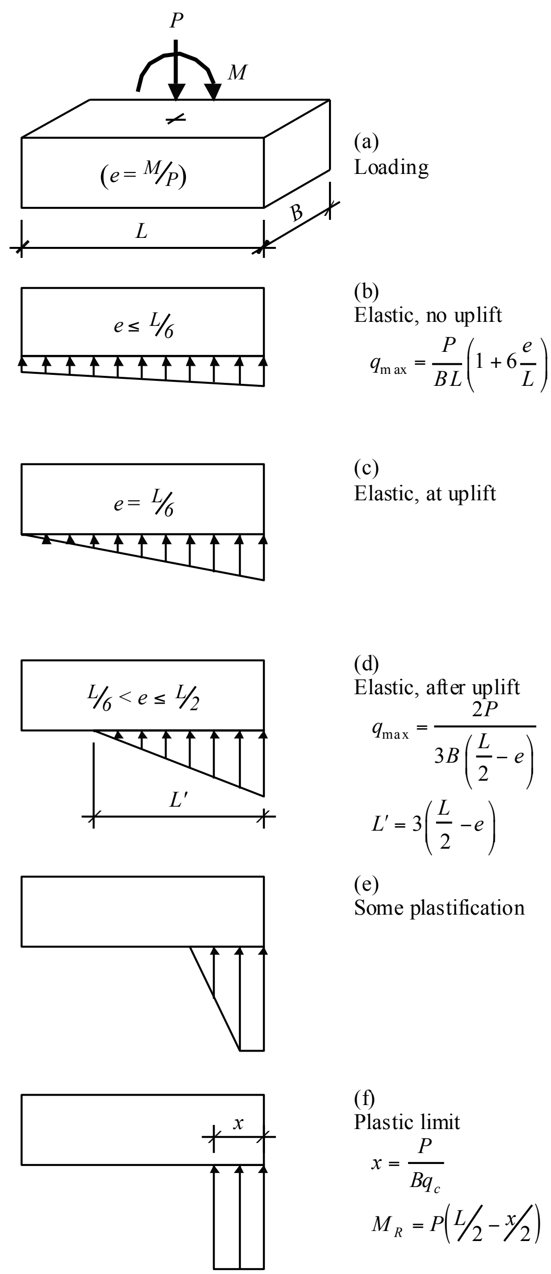


Figure 5.1-3 Soil pressure distributions

5.1.2 Design for Gravity Loads

Although most of the examples in this volume do not provide detailed design for gravity loads, it is provided in this section for two reasons. First, most of the calculation procedures used in designing shallow foundations for seismic loads are identical to those used for gravity design. Second, a complete gravity design is needed to make the cost comparisons shown in Section 5.1.5 below meaningful.

Detailed calculations are shown for a typical interior footing. The results for all three footing types are summarized in Section 5.1.2.5.

5.1.2.1 Demands. Dead and live load reactions are determined as part of the three-dimensional analysis described in Section 6.2 of this volume of design examples. Although there are slight variations in the calculated reactions, the foundations are lumped into three groups (interior, perimeter and corner) for gravity load design and the maximum computed reactions are applied to all members of the group, as follows:

- Interior: D = 387 kips
  L = 98 kips
- Perimeter: D = 206 kips
  L = 45 kips
- Corner: D = 104 kips
  L = 23 kips

The service load combination for consideration of settlement is $D + L$. Considering the load combinations for strength design defined in Section 2.3.2 of the *Standard*, the controlling gravity load combination is $1.2D + 1.6L$.

5.1.2.2 Footing Size. The preliminary size of the footing is determined considering settlement. The service load on a typical interior footing is calculated as:

$$P = D + L = 387 \text{ kips} + 98 \text{ kips} = 485 \text{ kips}$$

Since the footing dimensions will be less than 20 feet, the allowable bearing pressure (see Table 5.1-1) is 4,000 psf. Therefore, the required footing area is 487,000 lb/4,000 psf = 121.25 ft^2.

Check a footing that is 11'-0" by 11'-0":

$$P_{allow} = 11 \text{ ft}(11 \text{ ft})(4{,}000 \text{ psf}) = 484{,}000 \text{ lb} = 484 \text{ kips} \approx 485 \text{ kips (demand)} \qquad \text{OK}$$

The strength demand is:

$$P_u = 1.2(387 \text{ kips}) + 1.6(98 \text{ kips}) = 621 \text{ kips}$$

As indicated in Table 5.1-1, the bearing capacity (q_c) is 2,000B = 2,000 × 11 = 22,000 psf = 22 ksf.

The design capacity for the foundation is:

$$\phi P_n = \phi q_c B^2 = 0.7(22 \text{ ksf})(11 \text{ ft})^2 = 1{,}863 \text{ kips} > 621 \text{ kips} \qquad \text{OK}$$

For use in subsequent calculations, the factored bearing pressure q_u = 621 kips/(11 ft)^2 = 5.13 ksf.

5.1.2.3 Footing Thickness. Once the plan dimensions of the footing are selected, the thickness is determined such that the section satisfies the one-way and two-way shear demands without the addition of shear reinforcement. Demands are calculated at critical sections, shown in Figure 5.1-2, which depend on the footing thickness.

Check a footing that is 26 inches thick:

For the W14 columns used in this building, the side dimensions of the loaded area (taken halfway between the face of the column and the edge of the base plate) are approximately 16 inches. Accounting for cover and expected bar sizes, d = 26 - (3 + 1.5(1)) = 21.5 in.

One-way shear:

$$V_u = 11\left(\frac{11-\frac{16}{12}}{2} - \frac{21.5}{12}\right)(5.13) = 172 \text{ kips}$$

$$\phi V_n = \phi V_c = (0.75)2\sqrt{4{,}000}\,(11\times 12)(21.5)\left(\tfrac{1}{1{,}000}\right) = 269 \text{ kips} > 172 \text{ kips} \qquad \text{OK}$$

Two-way shear:

$$V_u = 621 - \left(\tfrac{16+21.5}{12}\right)^2(5.13) = 571 \text{ kips}$$

$$\phi V_n = \phi V_c = (0.75)4\sqrt{4{,}000}\,\left[4\times(16+21.5)\right](21.5)\left(\tfrac{1}{1{,}000}\right) = 612 \text{ kips} > 571 \text{ kips} \qquad \text{OK}$$

5.1.2.4 Footing Reinforcement. Footing reinforcement is selected considering both flexural demands and minimum reinforcement requirements. The following calculations treat flexure first because it usually controls:

$$M_u = \frac{1}{2}(11)\left(\frac{11-\frac{16}{12}}{2}\right)^2(5.13) = 659 \text{ ft-kips}$$

Try nine #8 bars each way. The distance from the extreme compression fiber to the center of the top layer of reinforcement, $d = t$ - cover - $1.5d_b$ = 26 - 3 - 1.5(1) = 21.5 in.

$T = A_s f_y$ = 9(0.79)(60) = 427 kips

Noting that $C = T$ and solving the expression $C = 0.85 f'_c\, b\, a$ for a produces a = 0.951 in.

$$\phi M_n = \phi T\left(d - \tfrac{a}{2}\right) = 0.90(427)\left(21.5 - \tfrac{0.951}{2}\right)\left(\tfrac{1}{12}\right) = 673 \text{ ft-kips} > 659 \text{ ft-kips} \qquad \text{OK}$$

The ratio of reinforcement provided is ρ = 9(0.79)/[(11)(12)(26)] = 0.00207. The distance between bars spaced uniformly across the width of the footing is s = [(11)(12)-2(3+0.5)]/(9-1) = 15.6 in.

According to ACI 318 Section 7.12, the minimum reinforcement ratio = 0.0018 < 0.00207 OK and the maximum spacing is the lesser of 5 × 26 in. and 18 = 18 in. > 15.6 in. OK

5.1.2.5 Design Results. The calculations performed in Sections 5.1.2.2 through 5.1.2.4 are repeated for typical perimeter and corner footings. The footing design for gravity loads is summarized in Table 5.1-2; Figure 5.1-4 depicts the resulting foundation plan.

Table 5.1-2 Footing Design for Gravity Loads

Location	Loads	Footing Size and Reinforcement; Soil Capacity	Critical Section Demands and Design Strengths
Interior	D = 387 kip L = 98 kip P = 485 kip P_u = 621 kip	11'-0" × 11'-0" × 2'-2" deep 9-#8 bars each way P_{allow} = 484 kip ϕP_n = 1863 kip	One-way shear: V_u = 172 kip ϕV_n = 269 kip Two-way shear: V_u = 571 kip ϕV_n = 612 kip Flexure: M_u = 659 ft-kip ϕM_n = 673 ft-kip
Perimeter	D = 206 kip L = 45 kip P = 251 kip P_u = 319 kip	8'-0" × 8'-0" × 1'-6" deep 9-#6 bars each way P_{allow} = 256 kip ϕP_n = 716 kip	One-way shear: V_u = 88.1 kip ϕV_n = 123 kip Two-way shear: V_u = 289 kip ϕV_n = 302 kip Flexure: M_u = 222 ft-kip ϕM_n = 234 ft-kip
Corner	D = 104 kip L = 23 kip P = 127 kip P_u = 162 kip	6'-0" × 6'-0" × 1'-2" deep 6-#5 bars each way P_{allow} = 144 kip ϕP_n = 302 kip	One-way shear: V_u = 41.5 kip ϕV_n = 64.9 kip Two-way shear: V_u = 141 kip ϕV_n = 184 kip Flexure: M_u = 73.3 ft-kip ϕM_n = 75.2 ft-kip

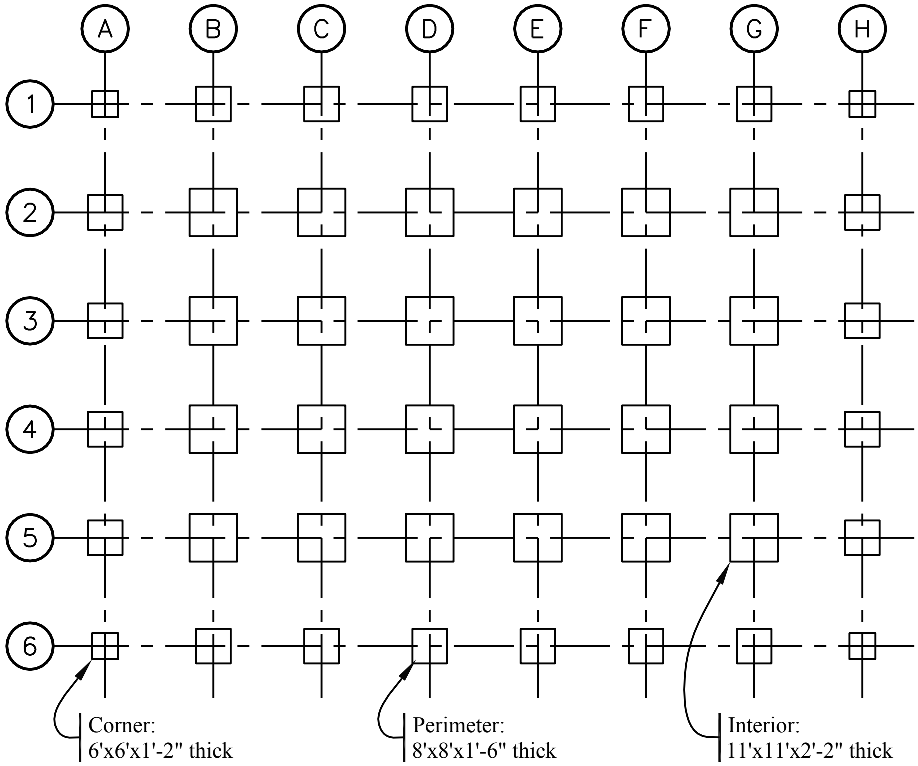


Figure 5.1-4 Foundation plan

5.1.3 Design for Moment-Resisting Frame System

Framing Alternate A in Section 6.2 of this volume of design examples includes a perimeter moment-resisting frame as the seismic force-resisting system. A framing plan for the system is shown in Figure 5.1-5. Detailed calculations are provided in this section for a combined footing at the corner and focus on overturning and sliding checks for the eccentrically loaded footing; settlement checks and design of concrete sections would be similar to the calculations shown in Section 5.1.2. The results for all footing types are summarized in Section 5.1.3.4.

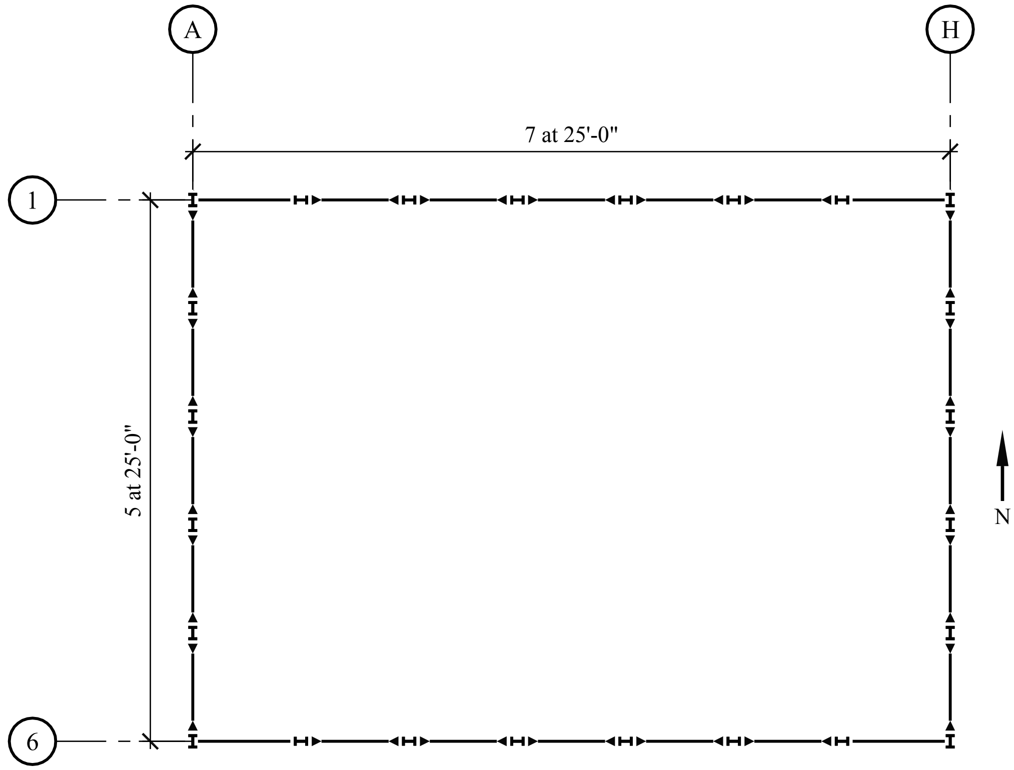


Figure 5.1-5 Framing plan for moment-resisting frame system

5.1.3.1 Demands. A three-dimensional analysis of the superstructure, in accordance with the requirements for the equivalent lateral force (ELF) procedure, is performed using the ETABS program. Foundation reactions at selected grids are reported in Table 5.1-3.

Table 5.1-3 Demands from Moment-Resisting Frame System

Location	Load	*Fx*	*Fy*	*Fz*	*Mxx*	*Myy*
A-5	*D*			-203.8		
	L			-43.8		
	Ex	-13.8	4.6	3.8	53.6	-243.1
	Ey	0.5	-85.1	-21.3	-1011.5	8.1
A-6	*D*			-103.5		
	L			-22.3		
	Ex	-14.1	3.7	51.8	47.7	-246.9
	Ey	0.8	-68.2	281.0	-891.0	13.4

Note: Units are kips and feet. Load *Ex* is for loads applied toward the east, including appropriately amplified counter-clockwise accidental torsion. Load *Ey* is for loads applied toward the north, including appropriately amplified clockwise accidental torsion.

Section 6.2.3.5 of this volume of design examples outlines the design load combinations, which include the redundancy factor as appropriate. A large number of load cases result from considering two senses of accidental torsion for loading in each direction and including orthogonal effects . The detailed calculations presented here are limited to two primary conditions, both for a combined foundation for columns at Grids A-5 and A-6: the downward case ($1.4D + 0.5L + 0.3Ex + 1.0Ey$) and the upward case ($0.7D + 0.3Ex + 1.0Ey$).

Before loads can be computed, attention must be given to *Standard* Section 12.13.4. That Section states that "overturning effects at the soil-foundation interface are permitted to be reduced by 25 percent" where the ELF procedure is used and by 10 percent where modal response spectrum analysis is used. Because the overturning effect in question relates to the global overturning moment for the *system*, judgment must be used in determining which design actions may be reduced. If the seismic force-resisting system consists of isolated shear walls, the shear wall overturning moment at the base best fits that description. For a perimeter moment-resisting frame, most of the global overturning resistance is related to axial loads in columns. Therefore, in this example column axial loads (Fz) from load cases Ex and Ey are multiplied by 0.75 and all other load effects remain unreduced.

5.1.3.2 Downward Case (1.4*D* + 0.5*L* + 0.3*Ex* + 1.0*Ey*). In order to perform the overturning checks, a footing size must be assumed. Preliminary checks (not shown here) confirmed that isolated footings under single columns were untenable. Check overturning for a footing that is 9 feet wide by 40 feet long by 5 feet thick. Furthermore, assume that the top of the footing is 2 feet below grade (the overlying soil contributes to the resisting moment). (In these calculations the $0.2S_{DS}D$ modifier for vertical accelerations is used for the dead loads *applied to* the foundation but not for the weight of the foundation and soil. This is the author's interpretation of the *Standard*. The footing and soil overburden are not subject to the same potential for dynamic amplification as the dead load of the superstructure and it is not common practice to include the vertical acceleration on the weight of the footing and the overburden. Furthermore, for footings that resist significant overturning, this issue makes a significant difference in design.)
Combining the loads from columns at Grids A-5 and A-6 and including the weight of the foundation and overlying soil produces the following loads at the foundation-soil interface:

P = applied loads + weight of foundation and soil
$= 1.4(-203.8 - 103.5) + 0.5(-43.8 - 22.3) + 0.75[0.3(3.8 + 51.8) + 1.0(-21.3 + 281)]$
$- 1.2[9(40)(5)(0.15) + 9(40)(2)(0.125)]$
$= -688$ kips.

M_{xx} = direct moments + moment due to eccentricity of applied axial loads
$= 0.3(53.6 + 47.7) + 1.0(-1011.5 - 891.0)$
$+ [1.4(-203.8) + 0.5(-43.8) + 0.75(0.3)(3.8) + 0.75(1.0)(-21.3)](12.5)$
$+ [1.4(-103.5) + 0.5(-22.3) + 0.75(0.3)(51.8) + 0.75(1.0)(281)](-12.5)$
$= -6{,}717$ ft-kips.

$M_{yy} = 0.3(-243.1 - 246.9) + 1.0(8.1 + 13.4)$
$= -126$ ft-kips. (The resulting eccentricity is small enough to neglect here, which simplifies the problem considerably.)

$V_x = 0.3(-13.8 - 14.1) + 1.0(0.5 + 0.8)$
$= -7.11$ kips.

$V_y = 0.3(4.6 + 3.7) + 1.0(-85.1 - 68.2)$
$= -149.2$ kips.

Note that the above load combination does not yield the maximum downward load. Reversing the direction of the seismic load results in P = -1,103 kips and M_{xx} = 2,964 ft-kips. This larger axial load does not control the design because the moment is so much less that the resultant is within the kern and no uplift occurs.

The following soil calculations use a different sign convention than that in the analysis results noted above; compression is positive for the soil calculations. The eccentricity is as follows:

$e = |M/P| = 6{,}717/688 = 9.76$ ft

Figure 5.1-3 shows the elastic and plastic design conditions and their corresponding equations. Where e is less than $L/2$, a solution to the overturning problem exists; however, as e approaches $L/2$, the bearing pressures increase without bound. Since e is greater than $L/6 = 40/6 = 6.67$ feet, uplift occurs and the maximum bearing pressure is:

$$q_{\max} = \frac{2P}{3B\left(\frac{L}{2} - e\right)} = \frac{2(688)}{3(9)\left(\frac{40}{2} - 9.76\right)} = 4.98 \text{ ksf}$$

and the length of the footing in contact with the soil is:

$$L' = 3\left(\frac{L}{2} - e\right) = 3\left(\frac{40}{2} - 9.76\right) = 30.7 \text{ ft}$$

The bearing capacity $q_c = 3{,}000B' = 3{,}000 \times \min(B, L'/2) = 3{,}000 \times \min(9, 30.7/2) = 27{,}000$ psf = 27 ksf. ($L'/2$ is used as an adjustment to account for the gradient in the bearing pressure in that dimension.)

The design bearing capacity $\phi q_c = 0.7(27 \text{ ksf}) = 18.9 \text{ ksf} > 4.98$ ksf OK

The foundation satisfies overturning and bearing capacity checks. The upward case, which follows, will control the sliding check.

5.1.3.3 Upward Case (0.7*D* + 0.3*Ex* + 1.0*Ey*). For the upward case the loads are:

P = -332 kips

M_{xx} = -5,712 ft-kips

M_{yy} = -126 ft-kips (negligible)

V_x = -7.1 kips

V_y = -149 kips

The eccentricity is:

$e = |M/P| = 5{,}712/332 = 17.2$ feet

Again, e is greater than L/6, so uplift occurs and the maximum bearing pressure is:

$$q_{\max} = \frac{2(332)}{3(10)\left(\frac{40}{2} - 17.2\right)} = 8.82 \text{ ksf}$$

and the length of the footing in contact with the soil is:

$$L' = 3\left(\frac{40}{2} - 17.2\right) = 8.4 \text{ ft}$$

The bearing capacity q_c = 3,000 × min(9, 8.4/2) = 12,500 psf = 12.5 ksf.

The design bearing capacity ϕq_c = 0.7(12.5 ksf) = 8.78 ksf < 8.82 ksf. NG

Using an elastic distribution of soil pressures, the foundation fails the bearing capacity check (although stability is satisfied). Try the plastic distribution. Using this approach, the bearing pressure over the entire contact area is assumed to be equal to the design bearing capacity. In order to satisfy vertical equilibrium, the contact area times the design bearing capacity must equal the applied vertical load P. Because the bearing capacity used in this example is a function of the contact area and the value of P changes with the size, the most convenient calculation is iterative.

By iteration, the length of contact area is L' = 4.19 feet.

The bearing capacity q_c = 3,000 × min(10, 4.19) = 12,570 psf = 12.57 ksf. (No adjustment to L' is needed as the pressure is uniform.)

The design bearing capacity ϕq_c = 0.7(12.6 ksf) = 8.80 ksf.

(8.80)(4.19)(9) = 332 kips = 332 kips, so equilibrium is satisfied.

The resisting moment, $M_R = P\,(L/2\text{-}L'/2)$ = 33 (40/2 - 4.19/2) = 5,944 ft-kip > 5,712 ft-kip. OK

Therefore, using a plastic distribution of soil pressures, the foundation satisfies overturning and bearing capacity checks.

The calculation of demands on concrete sections for strength checks should use the same soil stress distribution as the overturning check. Using a plastic distribution of soil stresses defines the upper limit of static loads for which the foundation remains stable, but the extreme concentration of soil bearing tends to drive up shear and flexural demands on the concrete section. It should be noted that the foundation may remain stable for larger loads if they are applied dynamically; even in that case, the strength demands on the concrete section will not exceed those computed on the basis of the plastic distribution.

For the sliding check, initially consider base traction only. The sliding demand is:

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(-7.11)^2 + (-149.2)^2} = 149.4 \text{ kips}$$

As calculated previously, the total compression force at the bottom of the foundation is 332 kips. The design sliding resistance is:

$\phi V_c = \phi \times$ friction coefficient $\times P = 0.7(0.65)(332 \text{ kips}) = 151 \text{ kips} > 149.4 \text{ kips}$ OK

If base traction alone had been insufficient, resistance due to passive pressure on the leading face could be included. Section 5.2.2.2 below illustrates passive pressure calculations for a pile cap.

5.1.3.4 Design Results. The calculations performed in Sections 5.1.3.2 and 5.1.3.3 are repeated for combined footings at middle and side locations. Figure 5.1-6 shows the results.

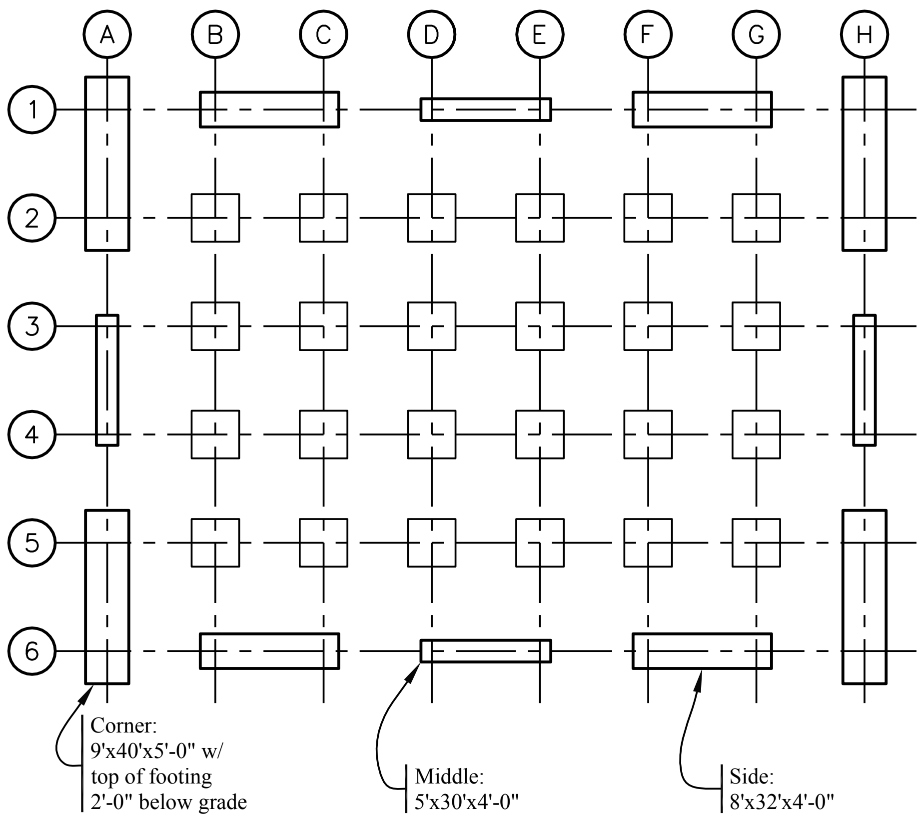


Figure 5.1-6 Foundation plan for moment-resisting frame system

One last check of interest is to compare the flexural stiffness of the footing with that of the steel column, which is needed because the steel frame design was based upon flexural restraint at the base of the columns. Using an effective moment of inertia of 50 percent of the gross moment of inertia and also using the distance between columns as the effective span, the ratio of *EI/L* for the smallest of the combined footings is more than five times the *EI/h* for the steel column. This is satisfactory for the design assumption.

5.1.4 Design for Concentrically Braced Frame System

Framing Alternate B in Section 6.2 of this volume of design examples employs a concentrically braced frame system at a central core to provide resistance to seismic loads. A framing plan for the system is shown in Figure 5.1-7.

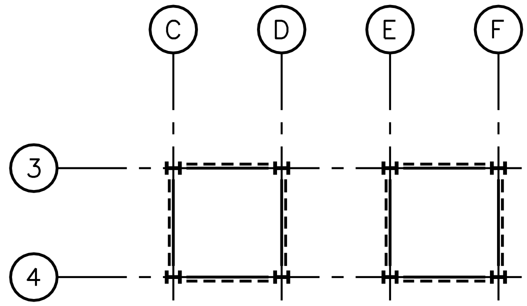


Figure 5.1-7 Framing plan for concentrically braced frame system

5.1.4.1 Check Mat Size for Overturning. Uplift demands at individual columns are so large that the only practical shallow foundation is one that ties together the entire core. The controlling load combination for overturning has minimum vertical loads (which help to resist overturning), primary overturning effects (M_{xx}) due to loads applied parallel to the short side of the core and smaller moments about a perpendicular axis (M_{yy}) due to orthogonal effects. Assume mat dimensions of 45 feet by 95 feet by 7 feet thick, with the top of the mat 3'-6" below grade. Combining the factored loads applied to the mat by all eight columns and including the weight of the foundation and overlying soil produces the following loads at the foundation-soil interface:

- P = -7,849 kips
- M_{xx} = -148,439 ft-kips
- M_{yy} = -42,544 ft-kips
- V_x = -765 kips
- V_y = -2,670 kips

Figure 5.1-8 shows the soil pressures that result from application in this controlling case, depending on the soil distribution assumed. In both cases the computed uplift is significant. In Part a of the figure, the contact area is shaded. The elastic solution shown in Part b was computed by modeling the mat in SAP2000 with compression only soil springs (with the stiffness of edge springs doubled as recommended by Bowles). For the elastic solution, the average width of the contact area is 11.1 feet and the maximum soil pressure is 16.9 ksf.

The bearing capacity q_c = 3,000 × min(95, 11.1/2) = 16,650 psf = 16.7 ksf.

The design bearing capacity ϕq_c = 0.7(16.7 ksf) = 11.7 ksf < 16.9 ksf. NG

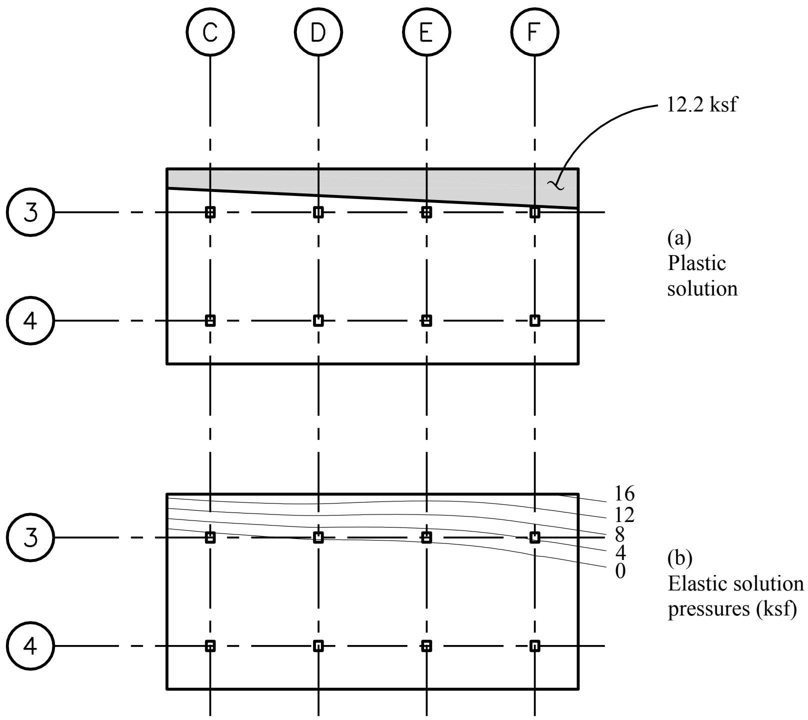


Figure 5.1-8 Soil pressures for controlling bidirectional case

As was done in Section 5.1.3.3 above, try the plastic distribution. The present solution has an additional complication as the off-axis moment is not negligible. The bearing pressure over the entire contact area is assumed to be equal to the design bearing capacity. In order to satisfy vertical equilibrium, the contact area times the design bearing capacity must equal the applied vertical load *P*. The shape of the contact area is determined by satisfying equilibrium for the off-axis moment. Again the calculations are iterative.

Given the above constraints, the contact area shown in Figure 5.1-8 is determined. The length of the contact area is 4.13 feet at the left side and 8.43 feet at the right side. The average contact length, for use in determining the bearing capacity, is (4.13 + 8.43)/2 = 6.27 feet. The distances from the center of the mat to the centroid of the contact area are as follows:

$$\bar{x} = 5.42 \text{ ft}$$

$$\bar{y} = 19.24 \text{ ft}$$

The bearing capacity is q_c = 3,000 × min(95, 6.27) = 18,810 psf = 18.81 ksf.

The design bearing capacity is ϕq_c = 0.7(18.8 ksf) = 13.2 ksf.

(13.2)(6.27)(95) = 7,863 kips ≈ 7,849 kips, confirming equilibrium for vertical loads.

(7,849)(5.42) = 42,542 ft-kips ≈ 42,544 ft-kips, confirming equilibrium for off-axis moment.

The resisting moment, $M_{R,xx} = P\bar{y} = 7{,}849(19.24) = 151{,}015 \text{ ft-kips} > 148{,}439 \text{ ft-kips}$. OK

So, the checks of stability and bearing capacity are satisfied. The mat dimensions are shown in Figure 5.1-9.

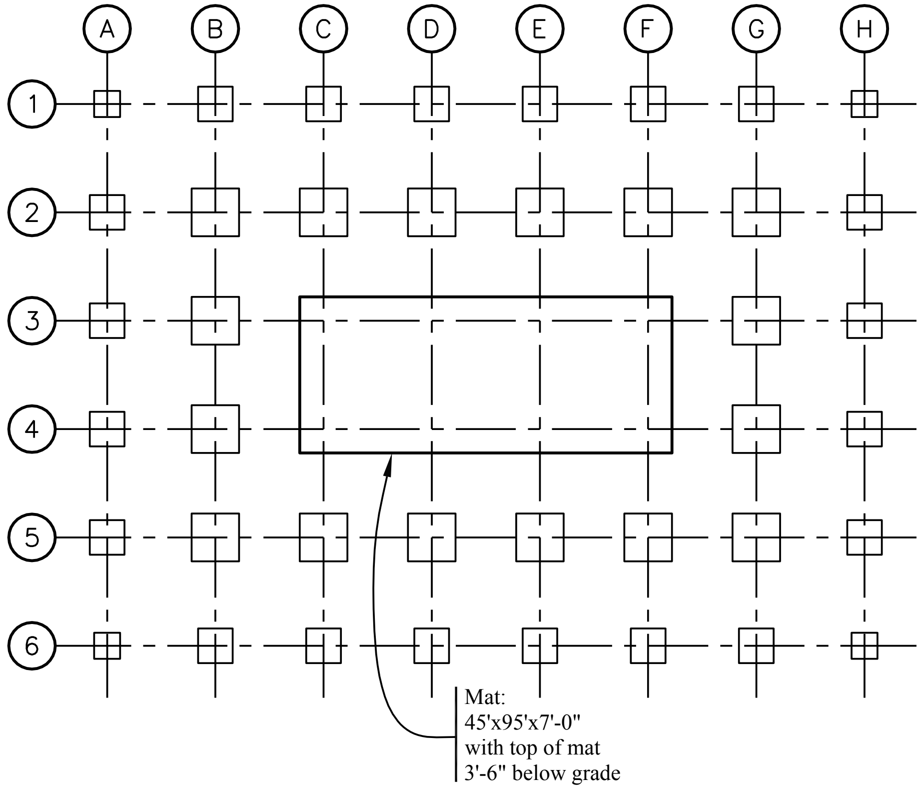


Figure 5.1-9 Foundation plan for concentrically braced frame system

5.1.4.2 Design Mat for Strength Demands. As was previously discussed, the computation of strength demands for the concrete section should use the same soil pressure distribution as was used to satisfy stability and bearing capacity. Because dozens of load combinations were considered and hand calculations were used for the plastic distribution checks, the effort required would be considerable. The same analysis used to determine elastic bearing pressures yields the corresponding section demands directly. One approach to this dilemma would be to compute an additional factor that must be applied to selected elastic cases to produce section demands that are consistent with the plastic solution. Rather than provide such calculations here, design of the concrete section will proceed using the results of the elastic analysis. This is conservative for the demand on the concrete for the same reason that it was unsatisfactory for the soil: the edge soil pressures are high (that is, we are designing the concrete for a peak soil pressure of 16.9 ksf, even though the plastic solution gives 13.2 ksf).

Standard Section 12.13.3 requires consideration of parametric variation for soil properties where foundations are modeled explicitly. This example does not illustrate such calculations.

Concrete mats often have multiple layers of reinforcement in each direction at the top and bottom of their thickness. Use of a uniform spacing for the reinforcement provided in a given direction greatly increases the ease of construction. The minimum reinforcement requirements defined in Section 10.5 of ACI 318 were discussed in Section 5.1.1.3 above. Although all of the reinforcement provided to satisfy

Section 7.12 of ACI 318 may be provided near one face, for thick mats it is best to compute and provide the amount of required reinforcement separately for the top and bottom halves of the section. Using a bar spacing of 10 inches for this 7-foot-thick mat and assuming one or two layers of bars, the section capacities indicated in Table 5.1-4 (presented in order of decreasing strength) may be precomputed for use in design. The amount of reinforcement provided for Marks B, C and D are less than the basic minimum for flexural members, so the demands should not exceed three-quarters of the design strength where those reinforcement patterns are used. The amount of steel provided for Mark D is the minimum that satisfies ACI 318 Section 7.12.

Table 5.1-4 Mat Foundation Section Capacities

Mark	Reinforcement	A_s (in.^{2} per ft)	ϕM_n (ft-kip/ft)	$3/4\phi M_n$ (ft-kip/ft)
A	2 layers of #10 bars at 10 in. o.c.	3.05	1,012	Not used
B	2 layers of #9 bars at 10 in. o.c.	2.40	Not used	601
C	2 layers of #8 bars at 10 in. o.c.	1.90	Not used	477
D	#8 bars at 10 in. o.c.	0.95	Not used	254

Note: Where the area of steel provided is less than the minimum reinforcement for flexural members as indicated in ACI 318 Sec. 10.5.1, demands are compared to 3/4 of ϕM_n as permitted in Sec. 10.5.3.

To facilitate rapid design, the analysis results are processed in two additional ways. First, the flexural and shear demands computed for the various load combinations are enveloped. Then the enveloped results are presented (see Figure 5.1-10) using contours that correspond to the capacities shown for the reinforcement patterns noted in Table 5.1-4.

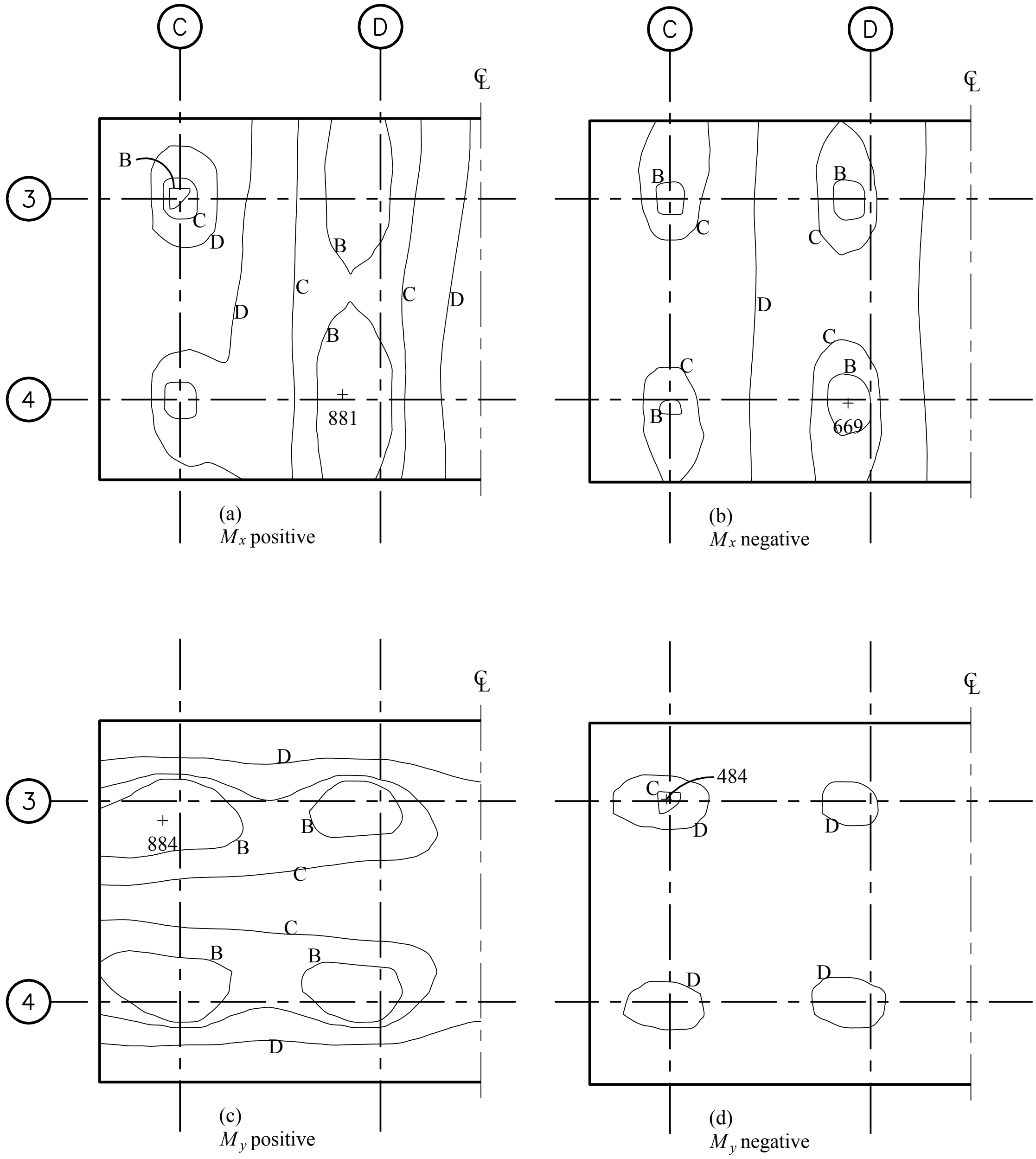


Figure 5.1-10 Envelope of mat foundation flexural demands

Using the noted contours permits direct selection of reinforcement. The reinforcement provided within a contour for a given mark must be that indicated for the next higher mark. For instance, all areas within Contour B must have two layers of #10 bars. Note that the reinforcement provided will be symmetric about the centerline of the mat in both directions. Where the results of finite element analysis are used in the design of reinforced concrete elements, averaging of demands over short areas is appropriate. In Figure 5.1-11, the selected reinforcement is superimposed on the demand contours. Figure 5.1-12 shows a section of the mat along Gridline C.

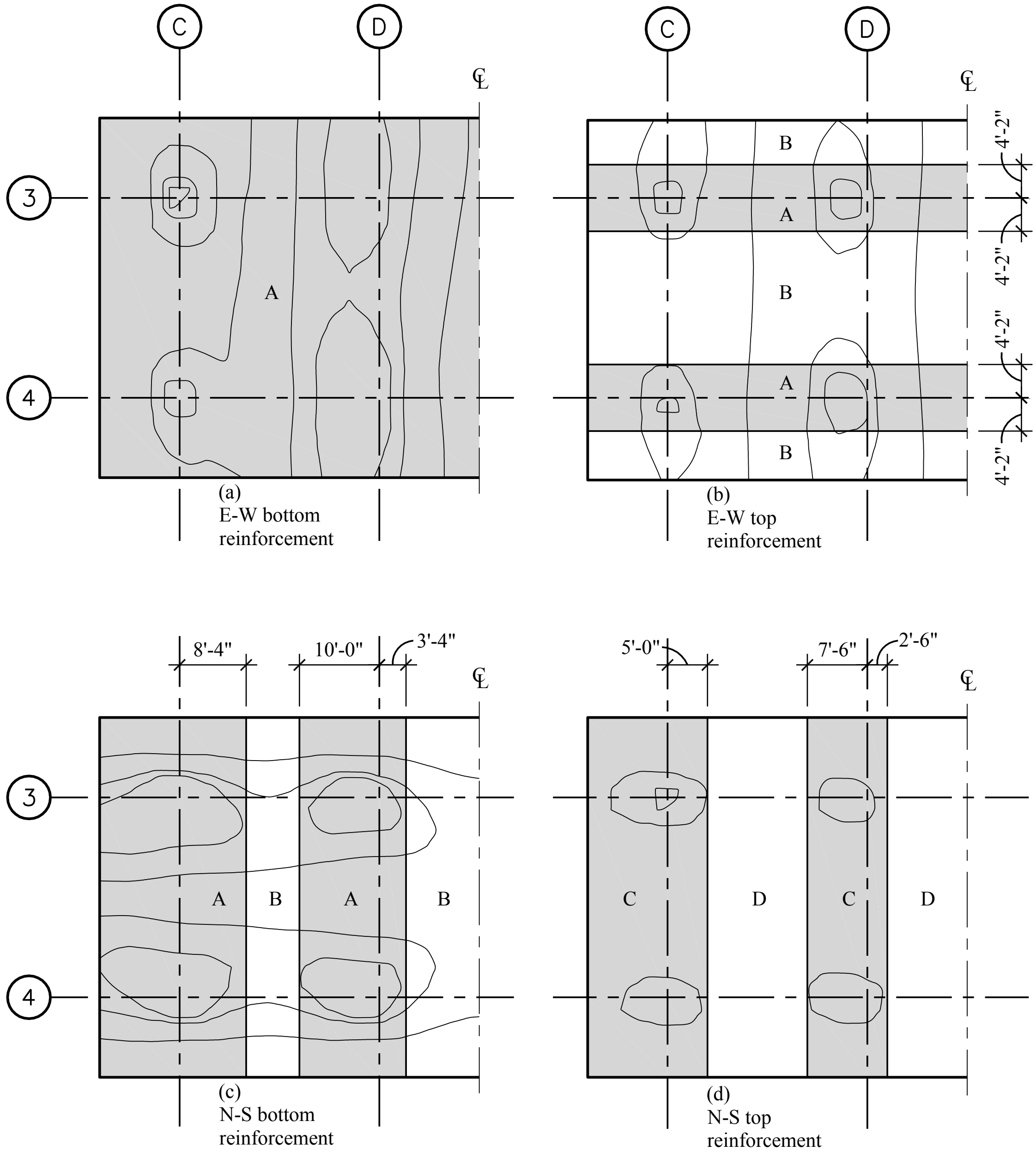


Figure 5.1-11 Mat foundation flexural reinforcement

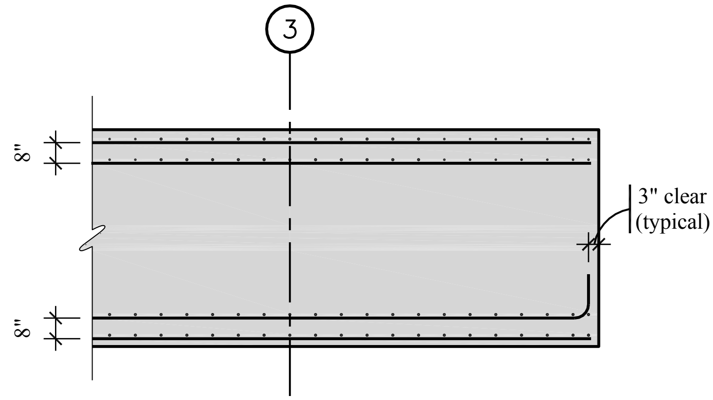


Figure 5.1-12 Section of mat foundation

Figure 5.1-13 presents the envelope of shear demands. The contours used correspond to the design strengths computed assuming $V_s = 0$ for one-way and two-way shear. In the hatched areas the shear stress exceeds $\phi 4\sqrt{f_c'}$ and in the shaded areas it exceeds $\phi 2\sqrt{f_c'}$. The critical sections for two-way shear (as discussed in Section 5.1.1.3) also are shown. The only areas that need more careful attention (to determine whether they require shear reinforcement) are those where the hatched or shaded areas are outside the critical sections. At the columns on Gridline D, the hatched area falls outside the critical section, so closer inspection is needed. Because the perimeter of the hatched area is substantially smaller than the perimeter of the critical section for punching shear, the design requirements of ACI 318 are satisfied.

One-way shears at the edges of the mat exceed the $\phi 2\sqrt{f_c'}$ criterion. Note that the high shear stresses are not produced by loads that create high bearing pressures at the edge. Rather, they are produced by loads that create large bending stresses parallel to the edge. The distribution of bending moments and shears is not uniform across the width (or breadth) of the mat, primarily due to the torsion in the seismic loads and the orthogonal combination. It is also influenced by the doubled spring stiffnesses used to model the soil condition. However, when the shears are averaged over a width equal to the effective depth (d), the demands are less than the design strength.

In this design, reinforcement for punching or beam shear is not required. If shear reinforcement cannot be avoided, standee bars may be used both to chair the upper decks of reinforcement and to provide resistance to shear in which case they may be bent thus: .

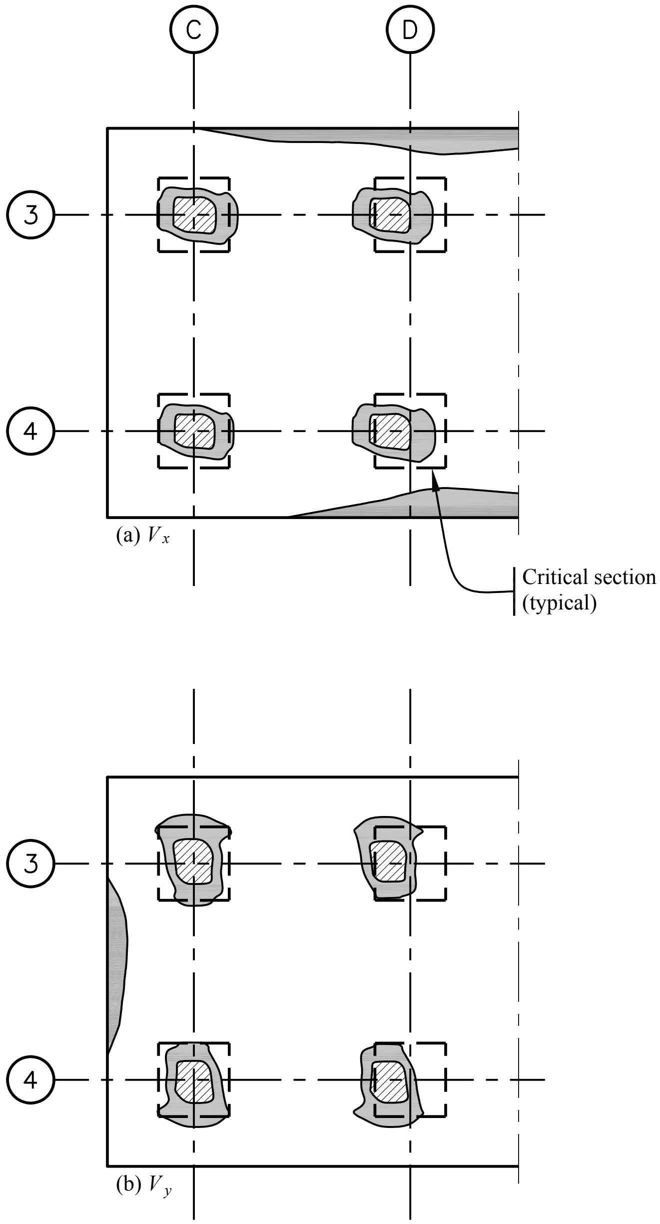


Figure 5.1-13 Critical sections for shear and envelope of mat foundation shear demands

5.1.5 Cost Comparison

Table 5.1-5 provides a summary of the material quantities used for all of the foundations required for the various conditions considered. Corresponding preliminary costs are assigned. The gravity-only condition does not represent a realistic case because design for wind loads would require changes to the foundations; it is provided here for discussion. It is obvious that design for lateral loads adds cost as compared to a design that neglects such loads. However, it is also worth noting that braced frame systems usually have substantially more expensive foundation systems than do moment frame systems. This condition occurs for two reasons. First, braced frame systems are stiffer, which produces shorter periods and higher design forces. Second, braced frame systems tend to concentrate spatially the demands on the foundations. In this case the added cost amounts to approximately $0.80/ft^2, which is an increase of perhaps 4 or 5 percent to the cost of the structural system.

Table 5.1-5 Summary of Material Quantities and Cost Comparison

Design Condition	Concrete at Gravity Foundations	Concrete at Lateral Foundations	Total Excavation	Total Cost
Gravity only (see Figure 5.1-4)	310 cy at $350/cy = $108,600		310 cy at $30/cy = $9,300	$117,900
Moment frame (see Figure 5.1-6)	233 cy at $350/cy = $81,600	507 cy at $400/cy = $202,900	770 cy at $30/cy = $23,100	$307,600
Braced frame (see Figure 5.1-9)	233 cy at $350/cy = $81,600	1,108 cy at $400/cy = $443,300	1895 cy at $30/cy = $56,800	$581,700

5.2 DEEP FOUNDATIONS FOR A 12-STORY BUILDING, SEISMIC DESIGN CATEGORY D

This example features the analysis and design of deep foundations for a 12-story reinforced concrete moment-resisting frame building similar to that described in Chapter 7 of this volume of design examples.

5.2.1 Basic Information

5.2.1.1 Description. Figure 5.2-1 shows the basic design condition considered in this example. A 2×2 pile group is designed for four conditions: for loads delivered by a corner and a side column of a moment-resisting frame system for Site Classes C and E. Geotechnical parameters for the two sites are given in Table 5.2-1.

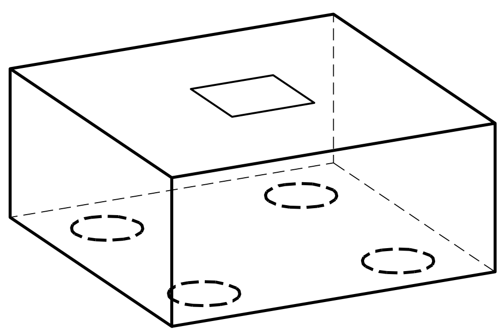


Figure 5.2-1 Design condition: Column of concrete moment-resisting frame supported by pile cap and cast-in-place piles

Table 5.2-1 Geotechnical Parameters

<table>
<tr><th>Depth</th><th>Class E Site</th><th>Class C Site</th></tr>
<tr><td>0 to 3 feet</td><td>Loose sand/fill

γ = 110 pcf
Angle of internal friction = 28 degrees
Soil modulus parameter, k = 25 pci

Neglect skin friction
Neglect end bearing</td><td>Loose sand/fill

γ = 110 pcf
Angle of internal friction = 30 degrees
Soil modulus parameter, k = 50 pci

Neglect skin friction
Neglect end bearing</td></tr>
<tr><td>3 to 30 feet</td><td>Soft clay

γ = 110 pcf
Undrained shear strength = 430 psf
Soil modulus parameter, k = 25 pci
Strain at 50 percent of maximum stress, ε_{50} = 0.01

Skin friction (ksf) = 0.3
Neglect end bearing</td><td rowspan="2">Dense sand (one layer: 3- to 100-foot depth)

γ = 130 pcf
Angle of internal friction = 42 degrees
Soil modulus parameter, k = 125 pci

Skin friction (ksf)* = 0.3 + 0.03/ft $\leq$ 2
End bearing (ksf)* = 65 + 0.6/ft $\leq$ 150</td></tr>
<tr><td>30 to 100 feet</td><td>Medium dense sand

γ = 120 pcf
Angle of internal friction = 36 degrees
Soil modulus parameter, k = 50 pci

Skin friction (ksf)* = 0.9 + 0.025/ft $\leq$ 2
End bearing (ksf)* = 40 + 0.5/ft $\leq$ 100</td></tr>
<tr><td>Pile cap resistance</td><td>300 pcf, ultimate passive pressure</td><td>575 pcf, ultimate passive pressure</td></tr>
<tr><td>Resistance factor, ϕ</td><td>0.8 for vertical, lateral and rocking resistance of cohesive soil</td><td>0.7 for vertical, lateral and rocking resistance of cohesionless soil</td></tr>
<tr><td>Safety factor for settlement</td><td>2.5</td><td>2.5</td></tr>
</table>

*Skin friction and end bearing values increase (up to the maximum value noted) for each additional foot of depth below the top of the layer. (The values noted assume a minimum pile length of 20 ft.)

The structural material properties assumed for this example are as follows:

- f'_c = 3,000 psi
- f_y = 60,000 psi

5.2.1.2 Seismic Parameters.

- Site Class = C and E (both conditions considered in this example)
- S_{DS} = 1.1
- Seismic Design Category = D (for both conditions)

5.2.1.3 Demands. The unfactored demands from the moment frame system are shown in Table 5.2-2.

Table 5.2-2 Gravity and Seismic Demands

Location	Load	*Vx*	*Vy*	*P*	*Mxx*	*Myy*
Corner	*D*			-460.0		
	L			-77.0		
	Vx	55.5	0.6	193.2	4.3	624.8
	Vy	0.4	16.5	307.5	189.8	3.5
	ATx	1.4	3.1	26.7	34.1	15.7
	ATy	4.2	9.4	77.0	103.5	47.8
Side	*D*			-702.0		
	L			-72.0		
	Vx	72.2	0.0	0.0	0.0	723.8
	Vy	0.0	13.9	181.6	161.2	1.2
	ATx	0.4	1.8	2.9	18.1	4.2
	ATy	1.2	5.3	8.3	54.9	12.6

Note: Units are kips and feet. Load *Vy* is for loads applied toward the east. *ATx* is the corresponding accidental torsion case. Load *Vx* is for loads applied toward the north. *ATy* is the corresponding accidental torsion case.

Using Load Combinations 5 and 7 from Section 12.4.2.3 of the *Standard* (with $0.2S_{DS}D = 0.22D$ and taking $\rho = 1.0$), considering orthogonal effects as required for Seismic Design Category D and including accidental torsion, the following 32 load conditions must be considered.

$$1.42D + 0.5L \pm 1.0Vx \pm 0.3Vy \pm \max(1.0ATx, 0.3ATy)$$

$$1.42D + 0.5L \pm 0.3Vx \pm 1.0Vy \pm \max(0.3ATx, 1.0ATy)$$

$$0.68D \pm 1.0Vx \pm 0.3Vy \pm \max(1.0ATx, 0.3ATy)$$

$$0.68D \pm 0.3Vx \pm 1.0Vy \pm \max(0.3ATx, 1.0ATy)$$

5.2.1.4 Design Approach. For typical deep foundation systems, resistance to lateral loads is provided by both the piles and the pile cap. Figure 5.2-2 shows a simple idealization of this condition. The relative contributions of these piles and pile cap depend on the particular design conditions, but often both effects are significant. Resistance to vertical loads is assumed to be provided by the piles alone regardless of whether their axial capacity is primarily due to end bearing, skin friction, or both. Although the behavior of foundation and superstructure are closely related, they typically are modeled independently. Earthquake loads are applied to a model of the superstructure, which is assumed to have fixed supports. Then the support reactions are seen as demands on the foundation system. A similar substructure technique is usually applied to the foundation system itself, whereby the behavior of pile cap and piles are considered separately. This section describes that typical approach.

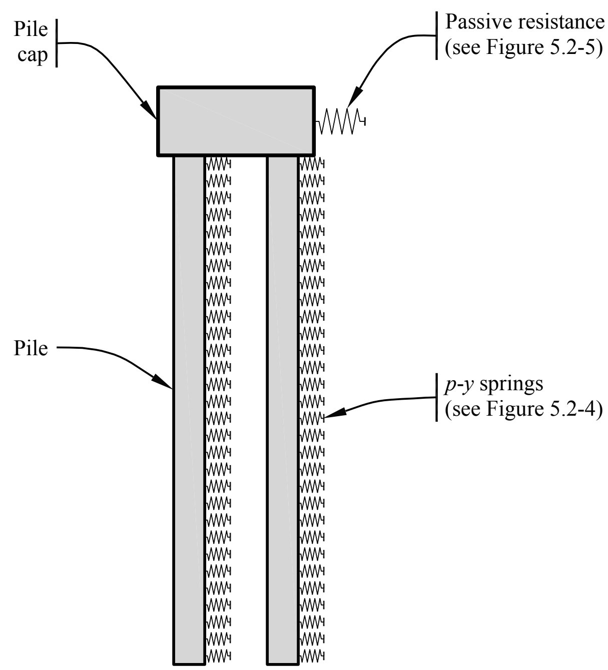


Figure 5.2-2 Schematic model of deep foundation system

5.2.1.4.1 Pile Group Mechanics. With reference to the free body diagram (of a 2×2 pile group) shown in Figure 5.2-3, demands on individual piles as a result of loads applied to the group may be determined as follows:

$V = \frac{V_{group} - V_{passive}}{4}$ and $M = V \times \ell$, where ℓ is a characteristic length determined from analysis of a laterally loaded single pile.

$P_{ot} = \frac{V_{group}h + M_{group} + 4M - h_p V_{passive}}{2s}$, where s is the pile spacing, h is the height of the pile cap and h_p is the height of $V_{passive}$ above Point O.

$P_p = \frac{P_{group}}{4}$ and $P = P_{ot} + P_p$

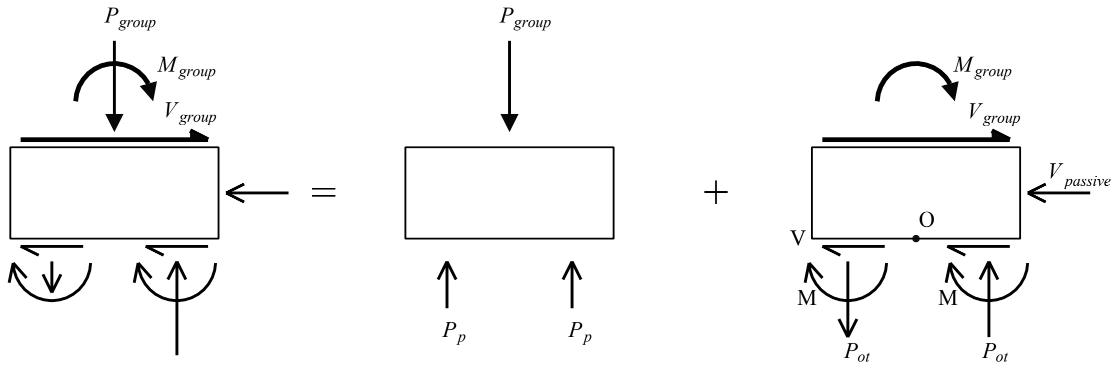


Figure 5.2-3 Pile cap free body diagram

5.2.1.4.2 Contribution of Piles. The response of individual piles to lateral loads is highly nonlinear. In recent years it has become increasingly common to consider that nonlinearity directly. Based on extensive testing of full-scale specimens and small-scale models for a wide variety of soil conditions, researchers have developed empirical relationships for the nonlinear *p-y* response of piles that are suitable for use in design. Representative *p-y* curves (computed for a 22-inch-diameter pile) are shown in Figure 5.2-4. The stiffness of the soil changes by an order of magnitude for the expected range of displacements (the vertical axis uses a logarithmic scale). The *p-y* response is sensitive to pile size (an effect not apparent in the figure, which is based on a single pile size); soil type and properties; and, in the case of sands, vertical stress, which increases with depth. Pile response to lateral loads, like the *p-y* curves on which the calculations are based, is usually computed using computer programs like LPILE.

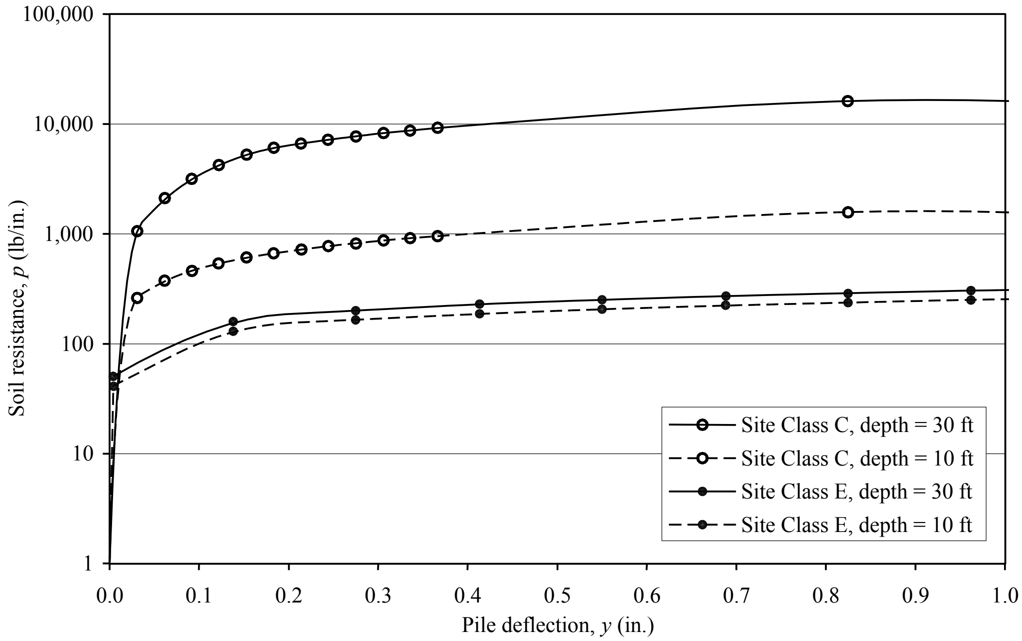


Figure 5.2-4 Representative *p*-*y* curves
(note that a logarithmic scale is used on the vertical axis)

5.2.1.4.3 Contribution of Pile Cap. Pile caps contribute to the lateral resistance of a pile group in two important ways: directly as a result of passive pressure on the face of the cap that is being pushed into the soil mass and indirectly by producing a fixed head condition for the piles, which can significantly reduce displacements for a given applied lateral load. Like the *p*-*y* response of piles, the passive pressure resistance of the cap is nonlinear. Figure 5.2-5 shows how the passive pressure resistance (expressed as a fraction of the ultimate passive pressure) is related to the imposed displacement (expressed as a fraction of the minimum dimension of the face being pushed into the soil mass).

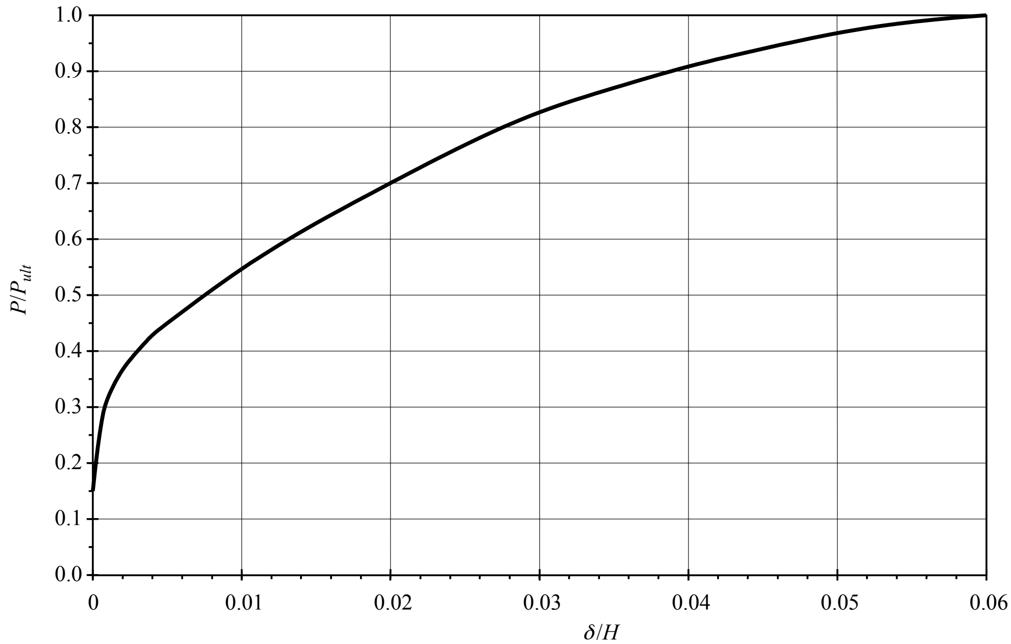


Figure 5.2-5 Passive pressure mobilization curve (after ASCE 41)

5.2.1.4.4 Group Effect Factors. The response of a group of piles to lateral loading will differ from that of a single pile due to pile-soil-pile interaction. (Group effect factors for axial loading of very closely spaced piles may also be developed but are beyond the scope of the present discussion.)

Full-size and model tests show that the lateral capacity of a pile in a pile group versus that of a single pile (termed "efficiency") is reduced as the pile spacing is reduced. The observed group effects are associated with shadowing effects. Various researchers have found that leading piles are loaded more heavily than trailing piles when all piles are loaded to the same deflection. The lateral resistance is primarily a function of row location within the group, rather than pile location within a row. Researchers recommend that these effects may be approximated by adjusting the resistance value on the single pile *p-y* curves (that is, by applying a *p*-multiplier).

Based on full-scale testing and subsequent analysis, Rollins et al. recommend the following *p*-multipliers (f_m), where D is the pile diameter or width and s is the center-to-center spacing between rows of piles in the direction of loading.

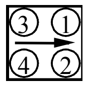
First (leading) row piles: $f_m = 0.26\ln\left(s/D\right) + 0.5 \le 1.0$

Second row piles: $f_m = 0.52\ln\left(s/D\right) \le 1.0$

Third or higher row piles: $f_m = 0.60\ln\left(s/D\right) - 0.25 \le 1.0$

Because the direction of loading varies during an earthquake and the overall efficiency of the group is the primary point of interest, the average efficiency factor is commonly used for all members of a group in the analysis of any given member. In that case, the average *p*-reduction factor is as follows:

$$\bar{f}_m = \frac{1}{n}\sum_{i=1}^{n} f_{mi}$$

For a 2×2 pile group thus 

with $s = 3D$, the group effect factor is calculated as follows:

For piles 1 and 2, in the leading row, $f_m = 0.26\ln(3) + 0.5 = 0.79$.

For piles 3 and 4, in the second row, $f_m = 0.52\ln(3) = 0.57$.

So, the group effect factor (average *p*-multiplier) is $\bar{f}_m = \dfrac{0.79 + 0.79 + 0.57 + 0.57}{4} = 0.68$.

Figure 5.2-6 shows the group effect factors that are calculated for pile groups of various sizes with piles at several different spacings.

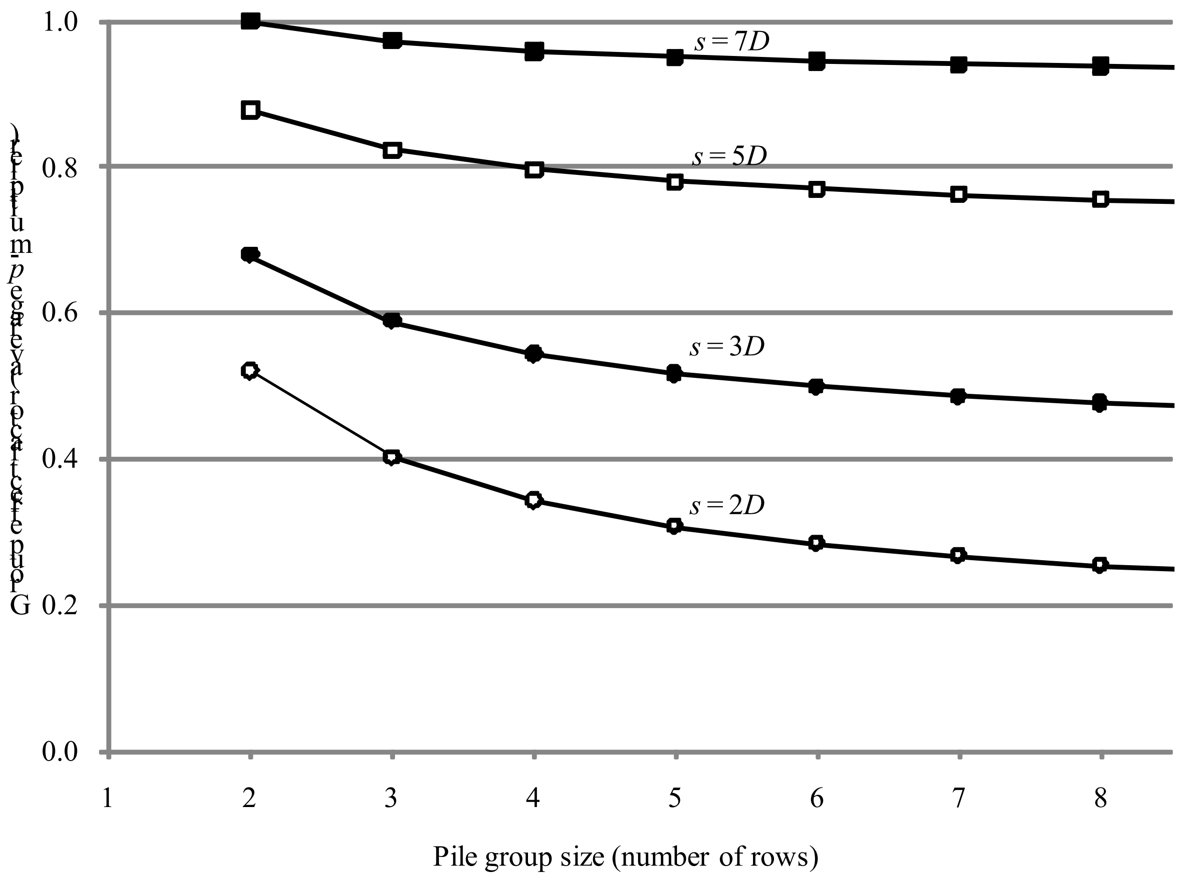


Figure 5.2-6 Calculated group effect factors

5.2.2 Pile Analysis, Design and Detailing

5.2.2.1 Pile Analysis. For this design example, it is assumed that all piles will be fixed-head, 22-inch-diameter, cast-in-place piles arranged in 2×2 pile groups with piles spaced at 66 inches center-to-center. The computer program LPILE Plus 5.0 is used to analyze single piles for both soil conditions shown in Table 5.2-1 assuming a length of 50 feet. Pile flexural stiffness is modeled using one-half of the gross moment of inertia because of expected flexural cracking. The response to lateral loads is affected to some degree by the coincident axial load. The full range of expected axial loads was considered in developing this example, but in this case the lateral displacements, moments and shears were not strongly affected; the plots in this section are for zero axial load. A *p*-multiplier of 0.68 for group effects (as computed at the end of Section 5.2.1.4) is used in all cases. Figures 5.2-7, 5.2-8 and 5.2-9 show the variation of shear, moment and displacement with depth (within the top 30 feet) for an applied lateral load of 15 kips on a single pile with the group reduction factor. It is apparent that the extension of piles to depths beyond 30 feet for the Class E site (or approximately 25 feet for the Class C site) does not provide additional resistance to lateral loading; piles shorter than those lengths would have reduced lateral resistance. The trends in the figures are those that should be expected. The shear and displacement are maxima at the pile head. Because a fixed-head condition is assumed, moments are also largest at the top of the pile. Moments and displacements are larger for the soft soil condition than for the firm soil condition.

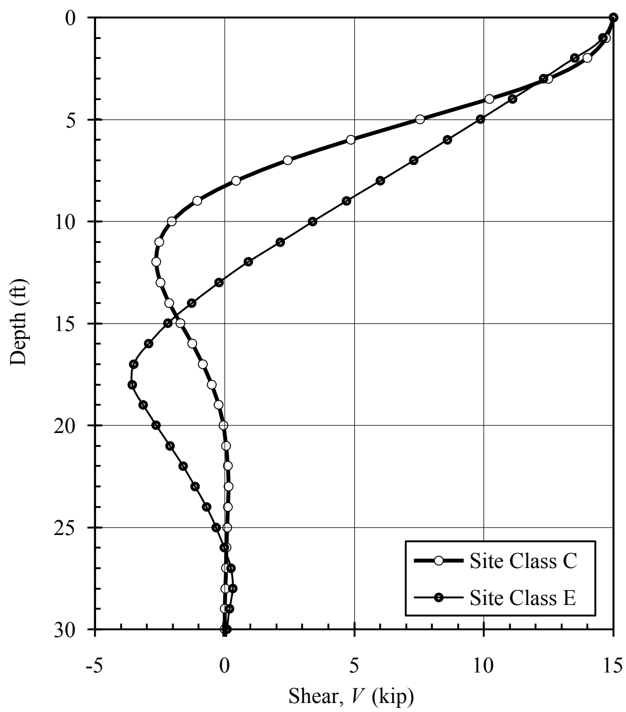


Figure 5.2-7 Results of pile analysis-shear versus depth (applied lateral load is 15 kips)

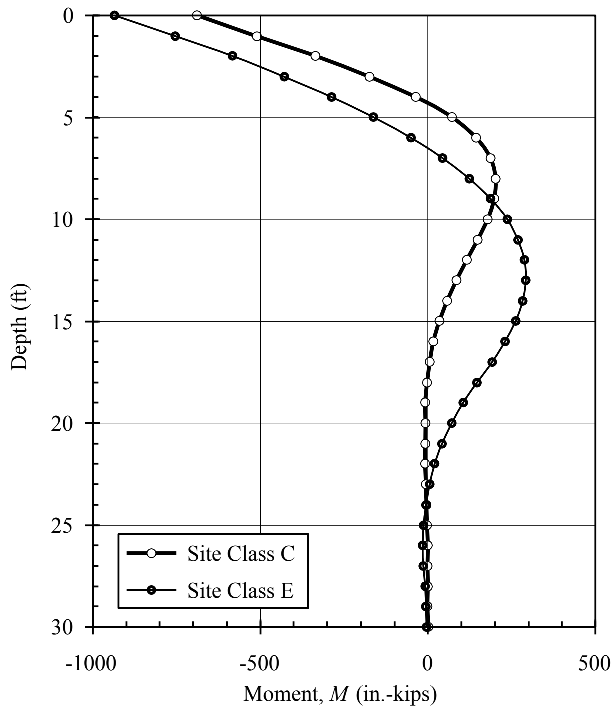


Figure 5.2-8 Results of pile analysis-moment versus depth (applied lateral load is 15 kips)

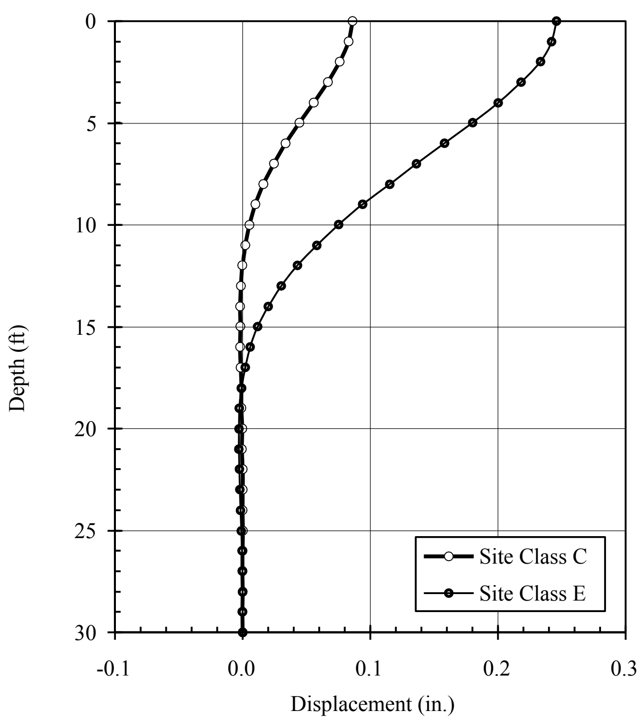


Figure 5.2-9 Results of pile analysis-displacement versus depth (applied lateral load is 15 kips)

The analyses performed to develop Figures 5.2-7 through 5.2-9 are repeated for different levels of applied lateral load. Figures 5.2-10 and 5.2-11 show how the moment and displacement at the head of the pile are related to the applied lateral load. It may be seen from Figure 5.2-10 that the head moment is related to the applied lateral load in a nearly linear manner; this is a key observation. Based on the results shown, the slope of the line may be taken as a characteristic length that relates head moment to applied load. Doing so produces the following:

- ℓ = 46 in. for the Class C site
- ℓ = 70 in. for the Class E site

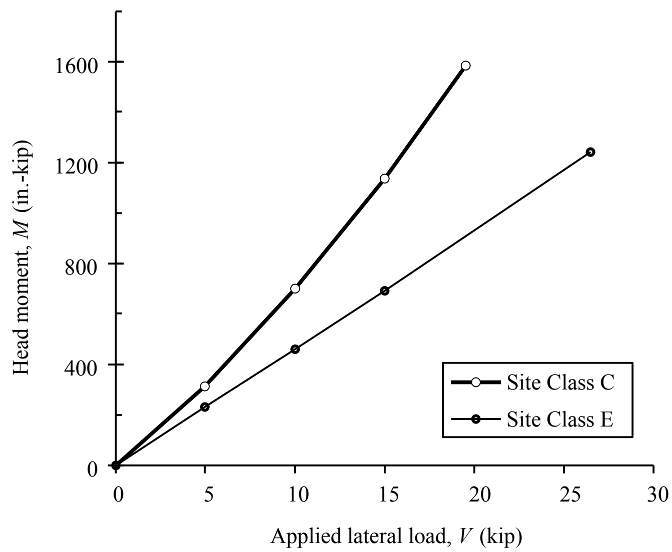


Figure 5.2-10 Results of pile analysis – applied lateral load versus head moment

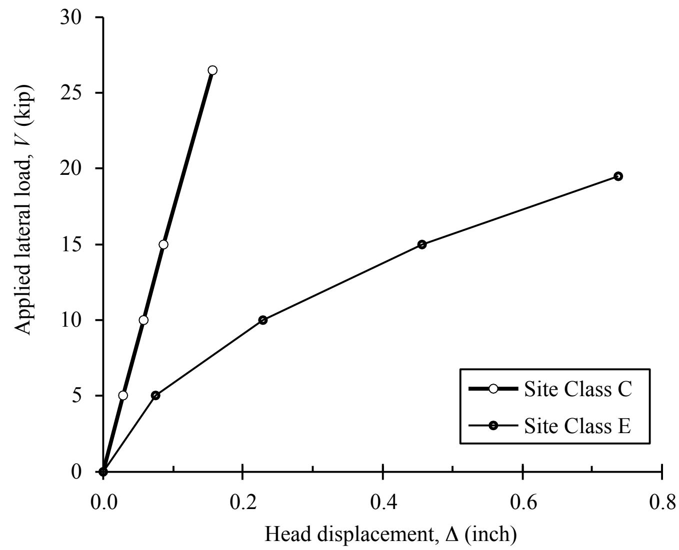


Figure 5.2-11 Results of pile analysis – head displacement versus applied lateral load

A similar examination of Figure 5.2-11 leads to another meaningful insight. The load-displacement response of the pile in Site Class C soil is essentially linear. The response of the pile in Site Class E soil is somewhat nonlinear, but for most of the range of response a linear approximation is reasonable (and useful). Thus, the effective stiffness of each individual pile is:

- k = 175 kip/in. for the Class C site
- k = 40 kip/in. for the Class E site

5.2.2.2 Pile Group Analysis. The combined response of the piles and pile cap and the resulting strength demands for piles are computed using the procedure outlined in Section 5.2.1.4 for each of the 32 load combinations discussed in Section 5.2.1.3. Assume that each 2×2 pile group has a 9'-2" × 9'-2" × 4'-0" thick pile cap that is placed 1'-6" below grade.

Check the Maximum Compression Case under a Side Column in Site Class C

Using the sign convention shown in Figure 5.2-3, the demands on the group are as follows:

- P = 1,224 kip
- M_{yy} = 222 ft-kips
- V_x = 20 kips
- M_{yy} = 732 ft-kips
- V_y = 73 kips

From preliminary checks, assume that the displacements in the x and y directions are sufficient to mobilize 30 percent and 35 percent, respectively, of the ultimate passive pressure:

$$V_{passive,x} = 0.30(575)\left(\frac{18}{12} + \frac{48}{2(12)}\right)\left(\frac{48}{12}\right)\left(\frac{110}{12}\right)\left(\frac{1}{1000}\right) = 22.1 \text{ kips}$$

and

$$V_{passive,y} = 0.35(575)\left(\frac{18}{12} + \frac{48}{2(12)}\right)\left(\frac{48}{12}\right)\left(\frac{110}{12}\right)\left(\frac{1}{1000}\right) = 25.8 \text{ kips}$$

and conservatively take $h_p = h/3$ = 16 inches.

Since $V_{passive,x} > V_x$, passive resistance alone is sufficient for this case in the x direction. However, in order to illustrate the full complexity of the calculations, reduce $V_{passive,x}$ to 4 kips and assign a shear of 4.0 kips to each pile in the x direction. In the y direction, the shear in each pile is as follows:

$$V = \frac{73 - 25.8}{4} = 11.8 \text{kips}$$

The corresponding pile moments are:

M = 4.0(46) = 186 in.-kips for x-direction loading

and

M = 11.8(46) = 543 in.-kips for y-direction loading

The maximum axial load due to overturning for x-direction loading is:

$$P_{ot} = \frac{20(48) + 222(12) + 4(184) - 16(4)}{2(66)} = 32.5 \text{kips}$$

and for y-direction loading (determined similarly), P_{ot} = 106.4 kips.

The axial load due to direct loading is P_p = 1224/4 = 306 kips.

Therefore, the maximum load effects on the most heavily loaded pile are the following:

P_u = 32.5 + 106.4 + 306 = 445 kips

$$M_u = \sqrt{(184)^2 + (543)^2} = 573 \text{in.-kips}$$

The expected displacement in the y direction is computed as follows:

$\delta = V/k$ = 11.8/175 = 0.067 in., which is 0.14 percent of the pile cap height (h)

Reading Figure 5.2-5 with δ/H = 0.0014, $P/P_{ult} \approx 0.34$, so the assumption that 35 percent of P_{ult} would be mobilized was reasonable.

5.2.2.3 Design of Pile Section. The calculations shown in Section 5.2.2.2 are repeated for each of the 32 load combinations under each of the four design conditions. The results are shown in Figures 5.2-12 and 5.2-13. In these figures, circles indicate demands on piles under side columns and squares indicate demands on piles under corner columns. Also plotted are the ϕP-ϕM design strengths for the 22-inch-diameter pile sections with various amounts of reinforcement (as noted in the legends). The appropriate reinforcement pattern for each design condition may be selected by noting the innermost capacity curve that envelops the corresponding demand points. The required reinforcement is summarized in Table 5.2-4, following calculation of the required pile length.

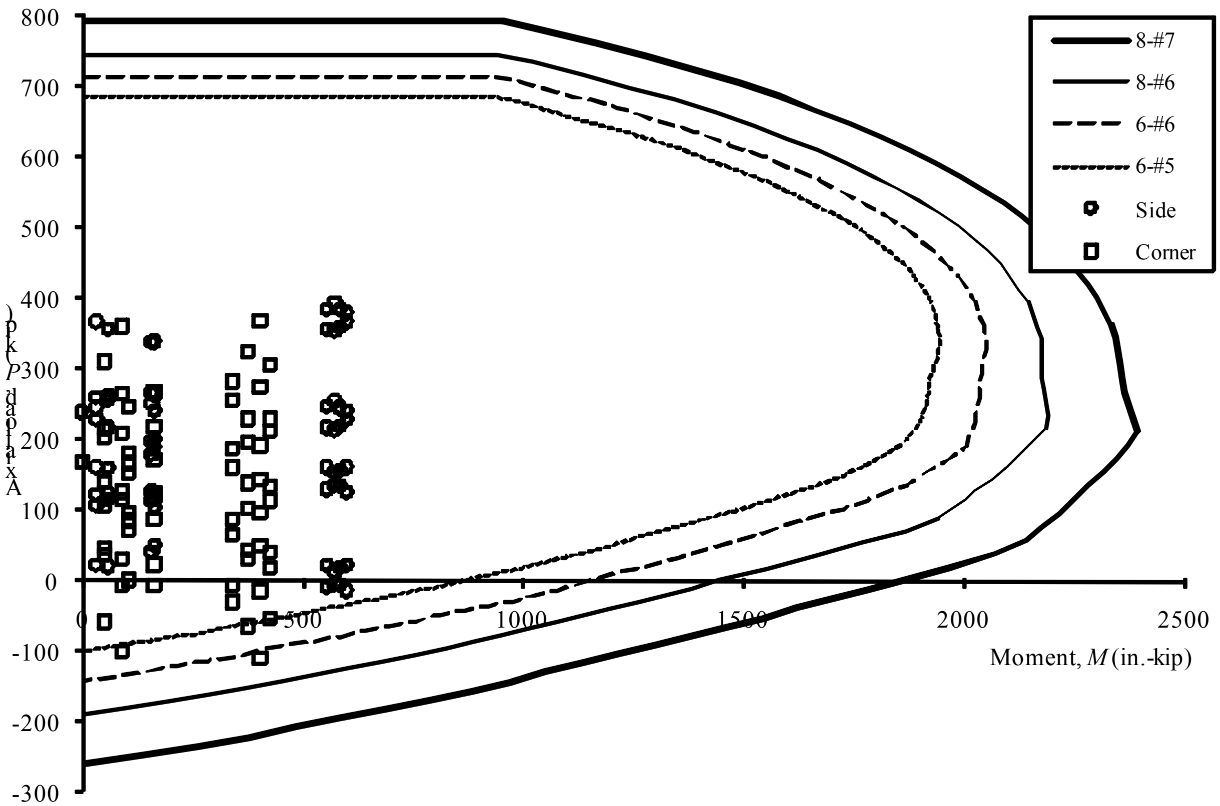


Figure 5.2-12 P-M interaction diagram for Site Class C

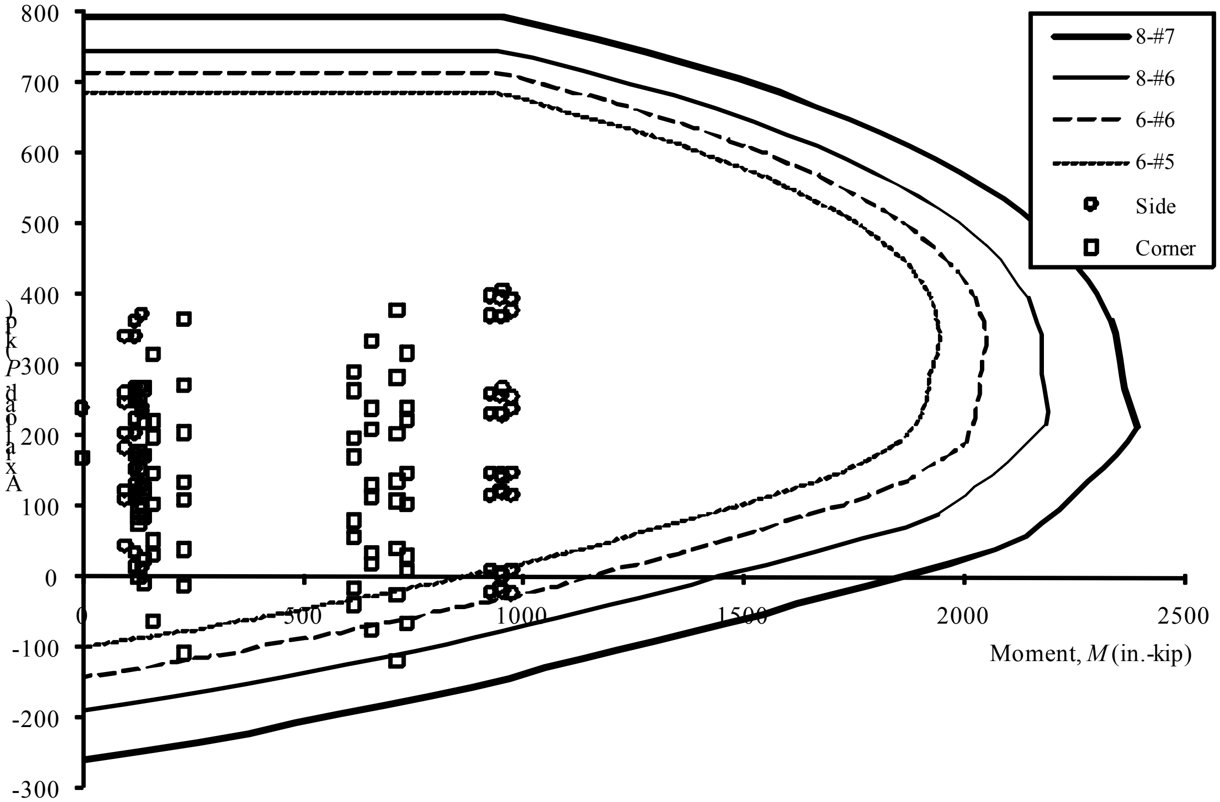


Figure 5.2-13 P-M interaction diagram for Site Class E

5.2.2.4 Pile Length for Axial Loads. For the calculations that follow, recall that skin friction and end bearing are neglected for the top 3 feet in this example. The design is based on having 1'-6" of soil over a 4'-0" deep pile cap.

5.2.2.4.1 Length for Settlement. Service loads per pile are calculated as $P = (P_D + P_L)/4$.

Check the pile group under the side column in Site Class C, assuming L = 52.5 feet – 5.5 feet = 47 feet:

$P = (752 + 114)/4 = 217$ kips.

P_{skin} = average friction capacity × pile perimeter × pile length for friction
$= 0.5[0.3 + 2.5(0.03) + 0.3 + 49.5(0.03)]\pi(22/12)(44) = 292$ kips

P_{end} = end bearing capacity at depth × end bearing area
$= [65 + 49.5(0.6)](\pi/4)(22/12)^2 = 250$ kips

$P_{allow} = (P_{skin} + P_{end})/\text{S.F.} = (292 + 250)/2.5 = 217$ kips = 217 kips (demand) OK

Check the pile group under the corner column in Site Class E, assuming L = 49 feet:

$P = (460 + 77)/4 = 134$ kips

P_{skin} = [friction capacity in first layer + average friction capacity in second layer] × pile perimeter
= $[24.5(0.3) + (24.5/2)(0.9 + 0.9 + 24.5[0.025])]\pi(22/12)$ = 212 kips

$P_{end} = [40 + 24.5(0.5)](\pi/4)(22/12)^2$ = 138 kips

$P_{allow} = (212 + 138)/2.5$ = 140 kips > 134 kips OK

5.2.2.4.2 Length for Compression Capacity. All of the strength-level load combinations (discussed in Section 5.2.1.3) must be considered.

Check the pile group under the side column in Site Class C, assuming L = 49 feet:

As seen in Figure 5.1-12, the maximum compression demand for this condition is P_u = 394 kips.

$P_{skin} = 0.5[0.3 + 0.3 + 47(0.03)]\pi(22/12)(47)$ = 272 kips

$P_{end} = [65 + 47(0.6)](\pi/4)(22/12)^2$ = 246 kips

$\phi P_n = \phi(P_{skin} + P_{end}) = 0.75(272 + 246)$ = 389 kips ≈ 390 kips OK

Check the pile group under the corner column in Site Class E, assuming L = 64 feet:

As seen in Figure 5.2-13, the maximum compression demand for this condition is P_u = 340 kips.

$P_{skin} = [27(0.3) + (34/2)(0.9 + 0.9 + 34[0.025])]\pi(22/12)$ = 306 kips

$P_{end} = [40 + 34(0.5)](\pi/4)(22/12)^2$ = 150 kips

$\phi P_n = \phi(P_{skin} + P_{end}) = 0.75(306 + 150)$ = 342 kips > 340 kips OK

5.2.2.4.3 Length for Uplift Capacity. Again, all of the strength-level load combinations (discussed in Section 5.2.1.3) must be considered.

Check the pile group under side column in Site Class C, assuming L = 5 feet:

As seen in Figure 5.2-12, the maximum tension demand for this condition is P_u = -1.9 kips.

$P_{skin} = 0.5[0.3 + 0.3 + 2(0.03)]\pi(22/12)(2)$ = 3.8 kips

$\phi P_n = \phi(P_{skin}) = 0.75(3.8)$ = 2.9 kips > 1.9 kips OK

Check the pile group under the corner column in Site Class E, assuming L = 52 feet:

As seen in Figure 5.2-13, the maximum tension demand for this condition is P_u = -144 kips.

$P_{skin} = [27(0.3) + (22/2)(0.9 + 0.9 + 22[0.025])]\pi(22/12)$ = 196 kips

$\phi P_n = \phi(P_{skin}) = 0.75(196)$ = 147 kips > 144 kips OK

5.2.2.4.4 Graphical Method of Selecting Pile Length. In the calculations shown above, the adequacy of the soil-pile interface to resist applied loads is checked once a pile length is assumed. It would be possible to generate mathematical expressions of pile capacity as a function of pile length and then solve such expressions for the demand conditions. However, a more practical design approach is to pre-calculate the capacity for piles for the full range of practical lengths and then select the length needed to satisfy the demands. This method lends itself to graphical expression as shown in Figures 5.2-14 and 5.2-15.

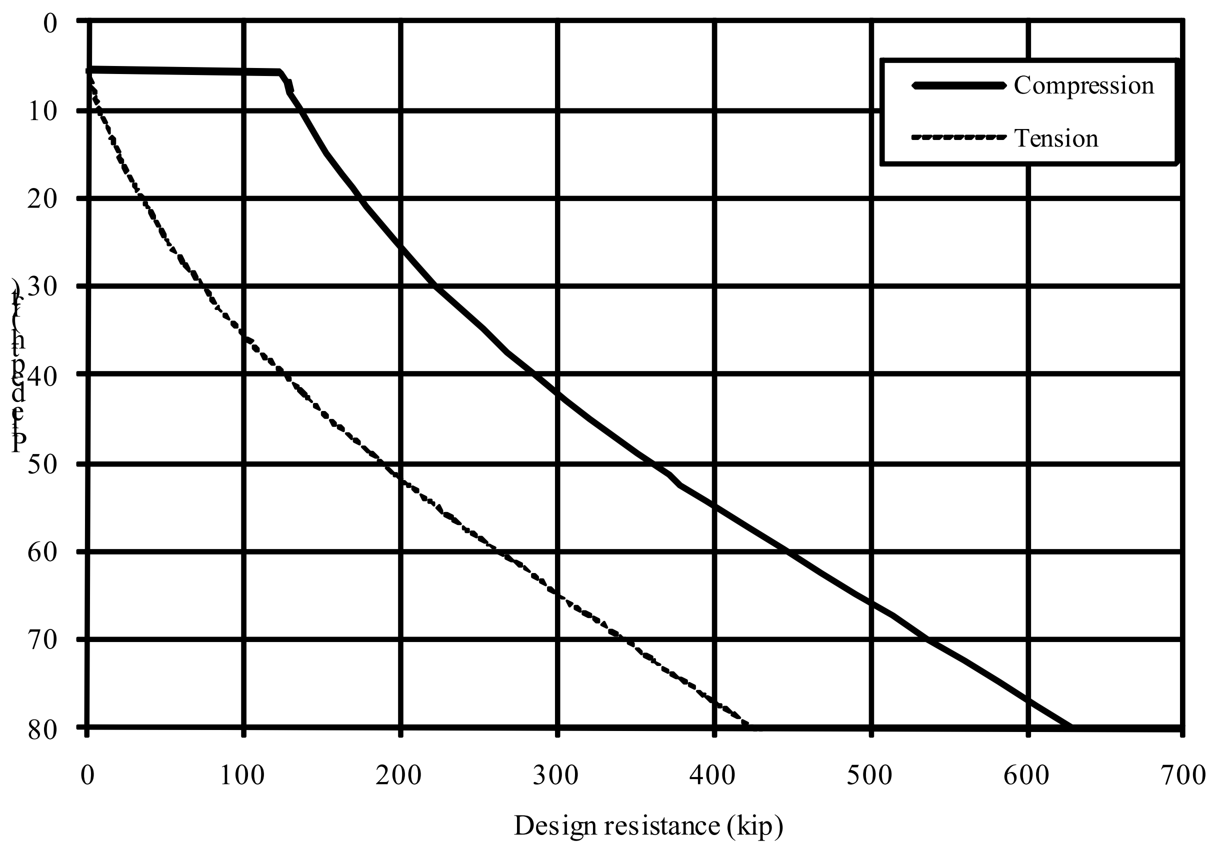


Figure 5.2-14 Pile axial capacity as a function of length for Site Class C

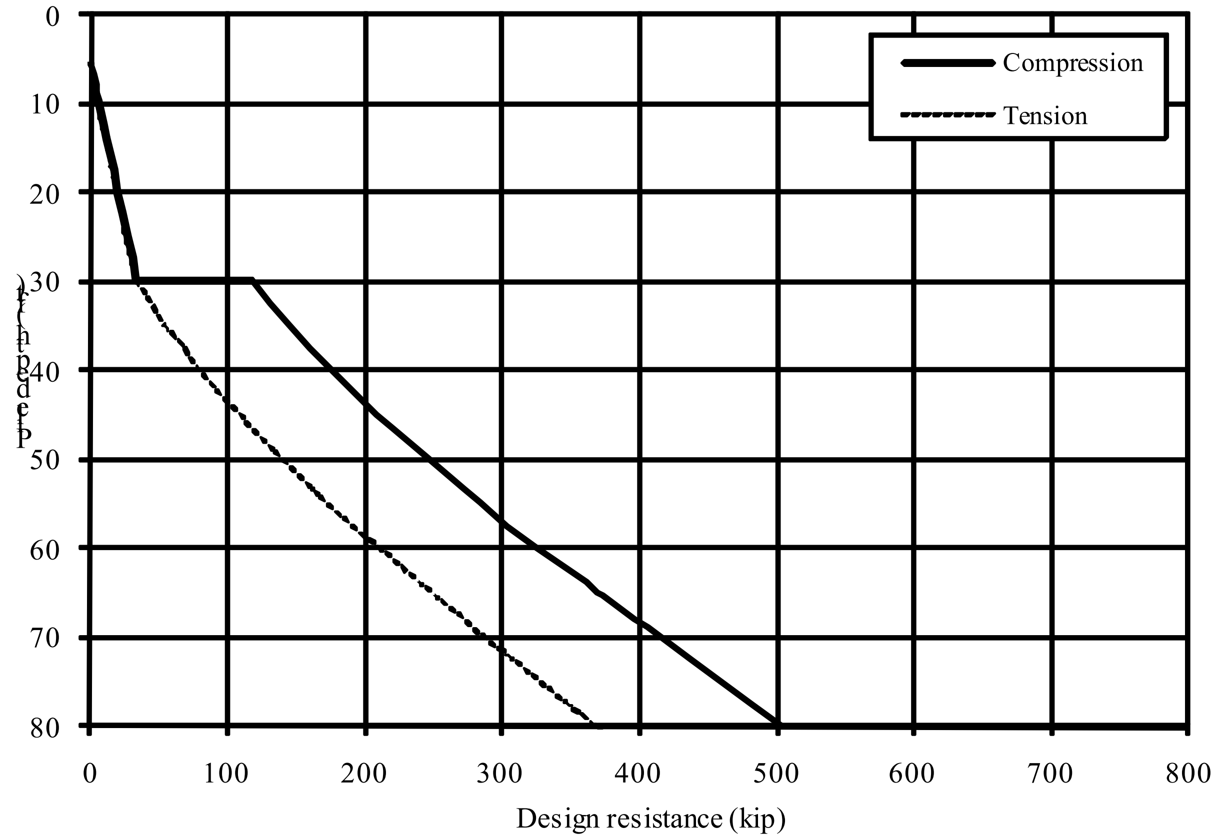


Figure 5.2-15 Pile axial capacity as a function of length for Site Class E

5.2.2.4.5 Results of Pile Length Calculations. Detailed calculations for the required pile lengths are provided above for two of the design conditions. Table 5.2-3 summarizes the lengths required to satisfy strength and serviceability requirements for all four design conditions.

Table 5.2-3 Pile Lengths Required for Axial Loads

	Piles Under Corner Column			Piles Under Side Column		
Site Class	Condition	Load	Min Length	Condition	Load	Min Length
Site Class C	**Compression**	**369 kip**	**46 ft**	**Compression**	**394 kip**	**49 ft**
	Uplift	108 kip	32 ft	Uplift	13.9 kip	8 ft
	Settlement	134 kip	27 ft	Settlement	217 kip	47 ft
Site Class E	**Compression**	**378 kip**	**61 ft**	Compression	406 kip	64 ft
	Uplift	119 kip	42 ft	Uplift	23.6 kip	17 ft
	Settlement	134 kip	48 ft	**Settlement**	**217 kip**	**67 ft**

5.2.2.5 Design Results. The design results for all four pile conditions are shown in Table 5.2-4. The amount of longitudinal reinforcement indicated in the table is that required at the pile-pile cap interface and may be reduced at depth as discussed in the following section.

Table 5.2-4 Summary of Pile Size, Length and Longitudinal Reinforcement

Site Class	Piles Under Corner Column	Piles Under Side Column
Site Class C	22 in. diameter by 46 ft long 8-#6 bars	22 in. diameter by 49 ft long 6-#5 bars
Site Class E	22 in. diameter by 61 ft long 8-#7 bars	22 in. diameter by 67 ft long 6-#6 bars

5.2.2.6 Pile Detailing. *Standard* Sections 12.13.5, 12.13.6, 14.2.3.1 and 14.2.3.2 contain special pile requirements for structures assigned to Seismic Design Category C or higher and D or higher. In this section, those general requirements and the specific requirements for uncased concrete piles that apply to this example are discussed. Although the specifics are affected by the soil properties and assigned site class, the detailing of the piles designed in this example focuses on consideration of the following fundamental items:

- All pile reinforcement must be developed in the pile cap (*Standard* Sec. 12.13.6.5).
- In areas of the pile where yielding might be expected or demands are large, longitudinal and transverse reinforcement must satisfy specific requirements related to minimum amount and maximum spacing.
- Continuous longitudinal reinforcement must be provided over the entire length resisting design tension forces (ACI 318 Sec. 21.12.4.2).

The discussion that follows refers to the detailing shown in Figures 5.2-16 and 5.2-17.

5.2.2.6.1 Development at the Pile Cap. Where neither uplift nor flexural restraint are required, the development length is the full development length for compression. Where the design relies on head fixity or where resistance to uplift forces is required (both of which are true in this example), pile reinforcement must be fully developed in tension unless the section satisfies the overstrength load condition or demands are limited by the uplift capacity of the soil-pile interface (*Standard* Sec. 12.13.6.5). For both site classes considered in this example, the pile longitudinal reinforcement is extended straight into the pile cap a distance that is sufficient to fully develop the tensile capacity of the bars. In addition to satisfying the requirements of the *Standard*, this approach offers two advantages. By avoiding lap splices to field-placed dowels where yielding is expected near the pile head (although such would be permitted by the *Standard*), more desirable inelastic performance would be expected. Straight development, while it may require a thicker pile cap, permits easier placement of the pile cap's bottom reinforcement followed by the addition of the spiral reinforcement within the pile cap. Note that embedment of the entire pile in the pile cap facilitates direct transfer of shear from pile cap to pile but is not a requirement of the *Standard*. (Section 1810.3.11 of the 2009 *International Building Code* requires that piles be embedded at least 3 inches into pile caps.)

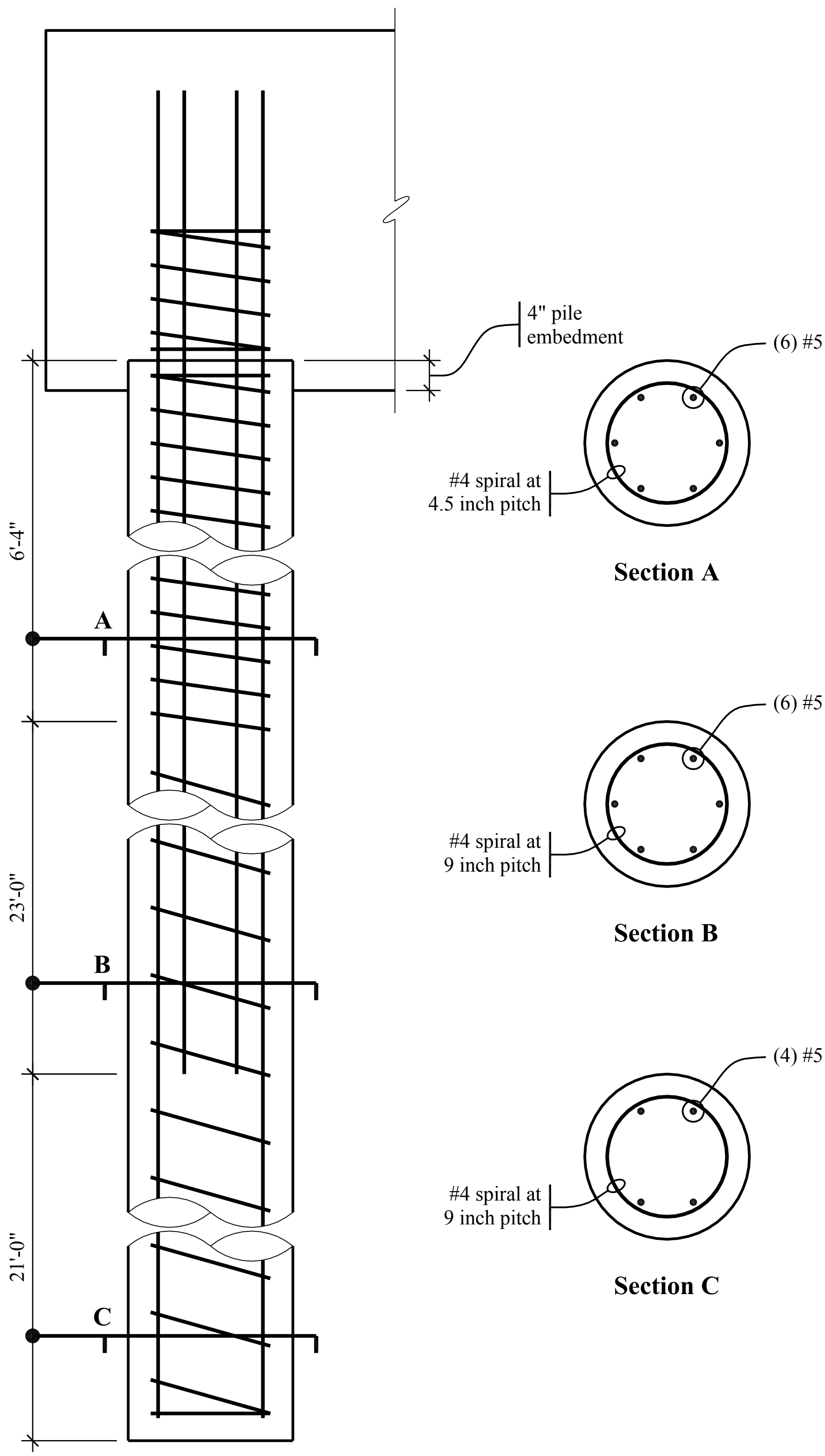


Figure 5.2-16 Pile detailing for Site Class C (under side column)

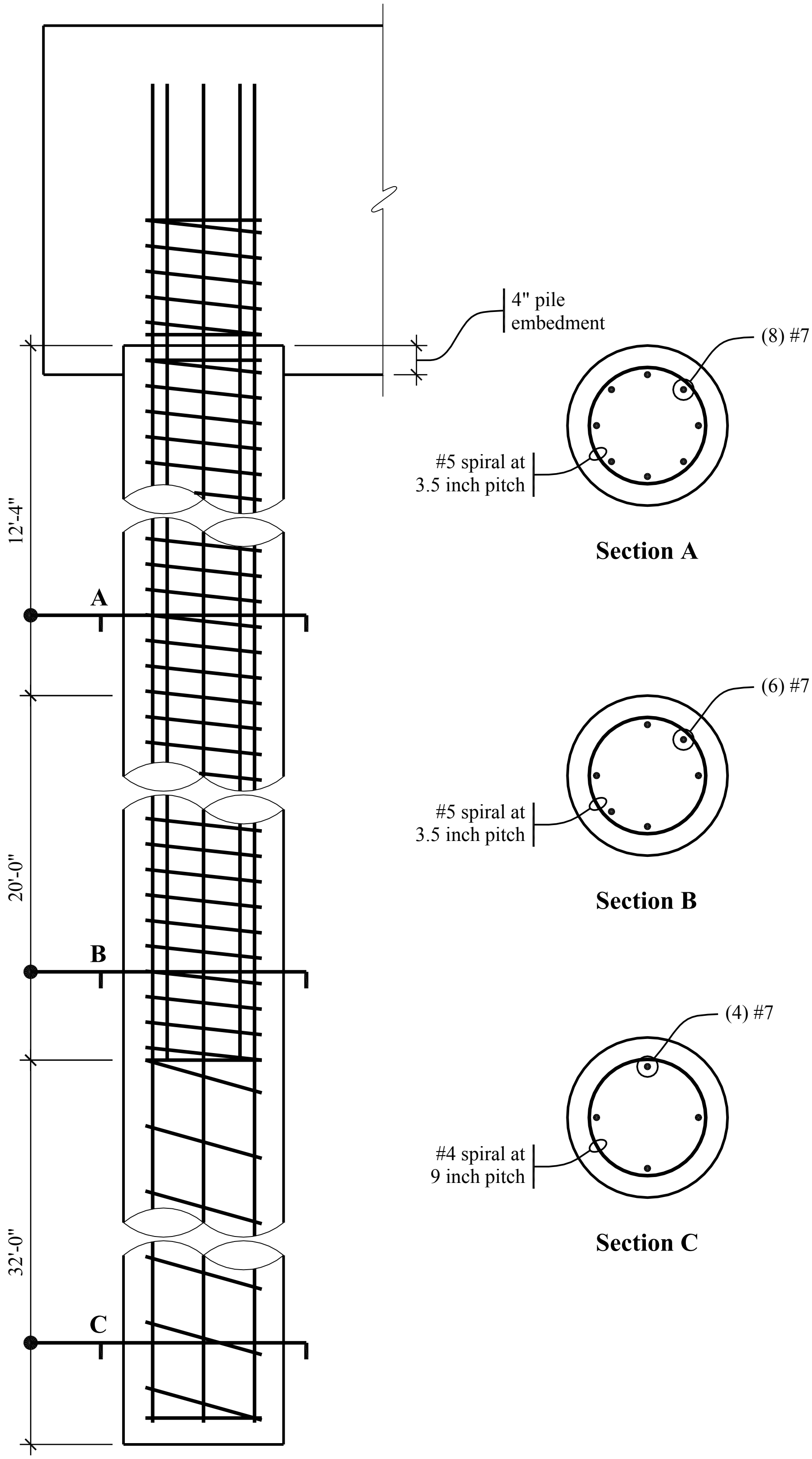


Figure 5.2-17 Pile detailing for Site Class E (under corner column)

5.2.2.6.2 Longitudinal and Transverse Reinforcement Where Demands Are Large. Requirements for longitudinal and transverse reinforcement apply over the entire length of pile where demands are large. For uncased concrete piles in Seismic Design Category D, at least four longitudinal bars (with a minimum reinforcement ratio of 0.005) must be provided over the largest region defined as follows: the top one-half of the pile length, the top 10 feet below the ground, or the flexural length of the pile. The flexural length is taken as the length of pile from the cap to the lowest point where 0.4 times the concrete section cracking moment (see ACI 318 Section 9.5.2.3) exceeds the calculated flexural demand at that point. For the piles used in this example, one-half of the pile length governs. (Note that "providing" a given reinforcement ratio means that the reinforcement in question must be developed at that point. Bar development and cutoff are discussed in more detail in Chapter 7 of this volume of design examples.) Transverse reinforcement must be provided over the same length for which minimum longitudinal reinforcement requirements apply. Because the piles designed in this example are larger than 20 inches in diameter, the transverse reinforcement may not be smaller than 0.5 inch diameter. For the piles shown in Figures 5.2-16 and 5.2-17, the spacing of the transverse reinforcement in the top half of the pile length may not exceed the least of the following: $12d_b$ (7.5 in. for #5 longitudinal bars and 10.5 in. for #7 longitudinal bars), 22/2 = 11 in., or 12 in.

Where yielding may be expected, even more stringent detailing is required. For the Class C site, yielding can be expected within three diameters of the bottom of the pile cap ($3D = 3 \times 22 = 66$ in.). Spiral reinforcement in that region must not be less than one-half of that required in Section 21.4.4.1(a) of ACI 318 (since the site is not Class E, Class F, or liquefiable) and the requirements of Sections 21.4.4.2 and 21.4.4.3 must be satisfied. Note that Section 21.4.4.1(a) refers to Equation 10-5, which often will govern. In this case, the minimum volumetric ratio of spiral reinforcement is one-half that determined using ACI 318 Equation 10-5. In order to provide a reinforcement ratio of 0.01 for this pile section, a #4 spiral must have a pitch of no more than 4.8 inches, but the maximum spacing permitted by Section 21.4.4.2 is 22/4 = 5.5 inches or $6d_b$ = 3.75 inches, so a #4 spiral at 3.75-inch pitch is used. (Section 1810.3.2.1.2 of the 2009 *International Building Code* clarifies that ACI 318 Equation 10-5 need not be applied to piles.)

For the Class E site, the more stringent detailing must be provided "within seven diameters of the pile cap and of the interfaces between strata that are hard or stiff and strata that are liquefiable or are composed of soft to medium-stiff clay" (*Standard* Sec. 14.2.3.2.1). The author interprets "within seven diameters of ... the interface" as applying in the direction into the softer material, which is consistent with the expected location of yielding. Using that interpretation, the *Standard* does not indicate the extent of such detailing into the firmer material. Taking into account the soil layering shown in Table 5.2-1 and the pile cap depth and thickness, the tightly spaced transverse reinforcement shown in Figure 5.2-17 is provided within $7D$ of the bottom of pile cap and top of firm soil and is extended a little more than $3D$ into the firm soil. Because the site is Class E, the full amount of reinforcement indicated in ACI 318 Section 21.6.4 must be provided. In order to provide a reinforcement ratio of 0.02 for this pile section, a #5 spiral must have a pitch of no more than 3.7 inches. The maximum spacing permitted by Section 21.6.4.3 is 22/4 = 5.5 inches or $6d_b$ = 5.25 inches, so a #5 spiral at 3.5-inch pitch is used.

5.2.2.6.3 Continuous Longitudinal Reinforcement for Tension. Table 5.2-3 shows the pile lengths required for resistance to uplift demands. For the Site Class E condition under a corner column (Figure 5.2-17), longitudinal reinforcement must resist tension for at least the top 42 feet (being developed at that point). Extending four longitudinal bars for the full length and providing widely spaced spirals at such bars is practical for placement, but it is not a specific requirement of the *Standard*. For the Site Class C condition under a side column (Figure5.2-16), design tension due to uplift extends only approximately 5 feet below the bottom of the pile cap. Therefore, a design with Section C of

Figure 5.2-16 being unreinforced would satisfy the *Provisions* requirements, but the author has decided to extend very light longitudinal and nominal transverse reinforcement for the full length of the pile.

5.2.3 Other Considerations

5.2.3.1 Foundation Tie Design and Detailing. *Standard* Section 12.13.5.2 requires that individual pile caps be connected by ties. Such ties are often grade beams, but the *Standard* would permit use of a slab (thickened or not) or calculations that demonstrate that the site soils (assigned to Site Class A, B, or C) provide equivalent restraint. For this example, a tie beam between the pile caps under a corner column and a side column is designed. The resulting section is shown in Figure 5.2-18.

For pile caps with an assumed center-to-center spacing of 32 feet in each direction and given P_{group} = 1,224 kips under a side column and P_{group} = 1,142 kips under a corner column, the tie is designed as follows.

As indicated in *Standard* Section 12.13.5.2, the minimum tie force in tension or compression equals the product of the larger column load times S_{DS} divided by 10 = 1224(1.1)/10 = 135 kips.

The design strength for six #6 bars is as follows

$$\phi A_s f_y = 0.9(6)(0.44)(60) = 143 \text{ kips} > 135 \text{ kips} \qquad \text{OK}$$

According to ACI 318 Section 21.12.3.2, the smallest cross-sectional dimension of the tie beam must not be less than the clear spacing between pile caps divided by 20 = (32'-0" - 9'-2")/20 = 13.7 inches. Use a tie beam that is 14 inches wide and 16 inches deep. ACI 318 Section 21.12.3.2 further indicates that closed ties must be provided at a spacing of not more than one-half the minimum dimension, which is 14/2 = 7 inches.

Assuming that the surrounding soil provides restraint against buckling, the design strength of the tie beam concentrically loaded in compression is as follows:

$$\phi P_n = 0.8\phi[0.85f'_c(A_g - A_{st}) + f_y A_{st}]$$

$$= 0.8(0.65)[0.85(3)\{(16)(14) - 6(0.44)\} + 60(6)(0.44)] = 376 \text{ kips} > 135 \text{ kips} \qquad \text{OK}$$

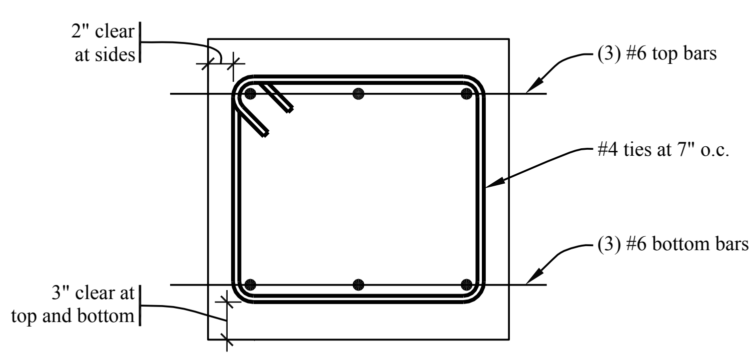


Figure 5.2-18 Foundation tie section

5.2.3.2 Liquefaction. For Seismic Design Categories C, D, E and F, *Standard* Section 11.8.2 requires that the geotechnical report address potential hazards due to liquefaction. For Seismic Design Categories D, E and F, *Standard* Section 11.8.3 further requires that the geotechnical report describe the likelihood and potential consequences of liquefaction and soil strength loss (including estimates of differential settlement, lateral movement, lateral loads on foundations, reduction in foundation soil-bearing capacity, increases in lateral pressures on retaining walls and flotation of buried structures) and discuss mitigation measures. During the design of the structure, such measures (which can include ground stabilization, selection of appropriate foundation type and depths and selection of appropriate structural systems to accommodate anticipated displacements and forces) must be considered. *Provisions* Part 3, Resource Paper 12 contains a calculation procedure that can be used to evaluate the liquefaction hazard.

5.2.3.3 Kinematic Interaction. Piles are subjected to curvature demands as a result of two different types of behavior: inertial interaction and kinematic interaction. The term *inertial interaction* is used to describe the coupled response of the soil-foundation-structure system that arises as a consequence of the mass properties of those components of the overall system. The structural engineer's consideration of inertial interaction is usually focused on how the structure *loads* the foundation and how such loads are transmitted to the soil (as shown in the pile design calculations that are the subject of most of this example) but also includes assessment of the resulting foundation movement. The term *kinematic interaction* is used to describe the manner in which the stiffness of the foundation system impedes development of free-field ground motion. Consideration of kinematic interaction by the structural engineer is usually focused on assessing the strength and ductility demands imposed directly on piles by movement of the soil. Although it is rarely done in practice, *Standard* Section 12.13.6.3 requires consideration of kinematic interaction for foundations of structures assigned to Seismic Design Category D, E, or F. Kramer discusses kinematic and inertial interaction and the methods of analysis employed in consideration of those effects and demonstrates "that the solution to the entire soil-structure interaction problem is equal to the sum of the solutions of the kinematic and inertial interaction analyses."

One approach that would satisfy the requirements of the *Standard* would be as follows:

- The geotechnical consultant performs appropriate kinematic interaction analyses considering free-field ground motions and the stiffness of the piles to be used in design.

- The resulting pile demands, which generally are greatest at the interface between stiff and soft strata, are reported to the structural engineer.

- The structural engineer designs piles for the sum of the demands imposed by the vibrating superstructure and the demands imposed by soil movement.

A more practical, but less rigorous, approach is to provide appropriate detailing in regions of the pile where curvature demands imposed directly by earthquake ground motions are expected to be significant. Where such a judgment-based approach is used, one must decide whether to provide only additional transverse reinforcement in areas of concern to improve ductility or whether additional longitudinal reinforcement should also be provided to increase strength. Section 18.10.2.4.1 of the 2009 *International Building Code* permits application of such deemed-to-comply detailing in lieu of explicit calculations and prescribes a minimum longitudinal reinforcement ratio of 0.005.

5.2.3.4 Design of Pile Cap. Design of pile caps for large pile loads is a very specialized topic for which detailed treatment is beyond the scope of this volume of design examples. CRSI notes that "most pile caps are designed in practice by various short-cut rule-of-thumb procedures using what are hoped to be conservative allowable stresses." Wang & Salmon indicates that "pile caps frequently must be designed for shear considering the member as a deep beam. In other words, when piles are located inside the critical sections *d* (for one-way action) or *d*/2 (for two-way action) from the face of column, the shear cannot be neglected." They go on to note that "there is no agreement about the proper procedure to use." Direct application of the special provisions for deep flexural members as found in ACI 318 is not possible since the design conditions are somewhat different. CRSI provides a detailed outline of a design procedure and tabulated solutions, but the procedure is developed for pile caps subjected to concentric vertical loads only (without applied overturning moments or pile head moments). Strut-and-tie models (as described in Appendix A of ACI 318) may be employed, but their application to elements with important three-dimensional characteristics (such as pile caps for groups larger than 2×1) is so involved as to preclude hand calculations.

5.2.3.5 Foundation Flexibility and Its Impact on Performance

5.2.3.5.1 Discussion. Most engineers routinely use fixed-base models. Nothing in the *Provisions* or *Standard* prohibits that common practice; the consideration of foundation flexibility and of soil-structure interaction effects (*Standard* Section 12.13.3 and Chapter 19) is "permitted" but not required. Such fixed-base models can lead to erroneous results, but engineers have long assumed that the errors are usually conservative. There are two obvious exceptions to that assumption: soft soil site-resonance conditions (e.g., as in the 1985 Mexico City earthquake) and excessive damage or even instability due to increased displacement response.

Site resonance can result in significant amplification of ground motion in the period range of interest. For sites with a fairly long predominant period, the result is spectral accelerations that increase as the structural period approaches the site period. However, the shape of the general design spectrum used in the *Standard* does not capture that effect; for periods larger than T_0, accelerations remain the same or decrease with increasing period. Therefore, increased system period (as a result of foundation flexibility) always leads to lower design forces where the general design spectrum is used. Site-specific spectra may reflect long-period site-resonance effects, but the use of such spectra is required only for Class F sites.

Clearly, an increase in displacements, caused by foundation flexibility, does change the performance of a structure and its contents—raising concerns regarding both stability and damage. Earthquake-induced instability of buildings has been exceedingly rare. The analysis and acceptance criteria in the *Standard* are not adequate to the task of predicting real stability problems; calculations based on linear, static behavior cannot be used to predict instability of an inelastic system subjected to dynamic loading. While *Provisions* Part 2 Section 12.12 indicates that structural stability was considered in arriving at the "consensus judgment" reflected in the drift limits, such considerations were qualitative. In point of fact, the values selected for the drift limits were selected considering damage to nonstructural systems (and, perhaps in some cases, control of structural ductility demands). For most buildings, application of the *Standard* is intended to satisfy performance objectives related to life safety and collapse prevention, not damage control or post-earthquake occupancy. Larger design forces and more stringent drift limits are applied to structures assigned to Occupancy Category III or IV in the hope that those measures will improve performance without requiring explicit consideration of such performance. Although foundation flexibility can affect structural performance significantly, since all consideration of performance in the context of the *Standard* is approximate and judgment-based, it is difficult to define how such changes in performance should be characterized. Explicit consideration of performance measures also tends to increase engineering effort substantially, so mandatory performance checks often are resisted by the user community.

The engineering framework established in ASCE 41 is more conducive to explicit use of performance measures. In that document (Sections 4.4.3.2.1 and 4.4.3.3.1), the use of fixed-based structural models is prohibited for "buildings being rehabilitated for the Immediate Occupancy Performance Level that are sensitive to base rotations or other types of foundation movement." In this case the focus is on damage control rather than structural stability.

5.2.3.5.2 Example Calculations. To assess the significance of foundation flexibility, one may compare the dynamic characteristics of a fixed-base model to those of a model in which foundation effects are included. The effects of foundation flexibility become more pronounced as foundation period and structural period approach the same value. For this portion of the example, use the Site Class E pile design results from Section 5.2.2.1 and consider the north-south response of the concrete moment frame building located in Berkeley (Section 7.2) as representative for this building.

5.2.3.5.2.1 Stiffness of the Structure. Calculations of the effect of foundation flexibility on the dynamic response of a structure should reflect the overall stiffness of the structure (e.g., that associated with the fundamental mode of vibration) rather than the stiffness of any particular story. Table 7-2 shows that the total weight of the structure is 43,919 kips. Table 7-3 shows that the calculated period of the fixed-base structure is 2.02 seconds and Table 7-7 indicates that 83.6 percent of the mass participates in that mode. Using the equation for the undamped period of vibration of a single-degree-of-freedom oscillator, the effective stiffness of the structure is as follows:

$$K = \frac{4\pi^2 M}{T^2} = \frac{4\pi^2\left((0.836)43{,}919/386.1\right)}{2.02^2} = 920 \text{ kip/in.}$$

5.2.3.5.2.2 Foundation Stiffness. As seen in Figure 7-1, there are 36 moment frame columns. Assume that a 2×2 pile group supports each column. As shown in Section 5.2.2.1, the stiffness of each pile is 40 kip/in. Neglecting both the stiffness contribution from passive pressure resistance and the flexibility of the beam-slab system that ties the pile caps, the stiffness of each pile group is 4 × 40 = 160 kip/in. and the stiffness of the entire foundation system is 36 × 160 = 5,760 kip/in.

5.2.3.5.2.3 Effect of Foundation Flexibility. Because the foundation stiffness is much greater than the structural stiffness, period elongation is expected to be minimal. To confirm this expectation, the period of the combined system is computed. The total stiffness for the system (springs in series) is as follows:

$$K_{combined} = \frac{1}{\dfrac{1}{K_{structure}} + \dfrac{1}{K_{fdn}}} = \frac{1}{\dfrac{1}{920} + \dfrac{1}{5760}} = 793 \text{ kip/in.}$$

Assume that the weight of the foundation system is 4,000 kips and that 100 percent of the corresponding mass participates in the new fundamental mode of vibration. The period of the combined system is as follows:

$$T = 2\pi\sqrt{\frac{M}{K}} = 2\pi\sqrt{\frac{\left[(0.836)(43{,}919) + (1.0)(4000)\right]/386.1}{793}} = 2.29 \text{ sec}$$

which is an increase of 13 percent over that predicted by the fixed-base model. For systems responding in the constant-velocity portion of the spectrum, accelerations (and thus forces) are a function of $1/T$ and relative displacements are a function of T. Therefore, with respect to the fixed-based model, the combined system would have forces that are 12 percent smaller and displacements that are 13 percent larger. In the context of earthquake engineering, those differences are not significant.