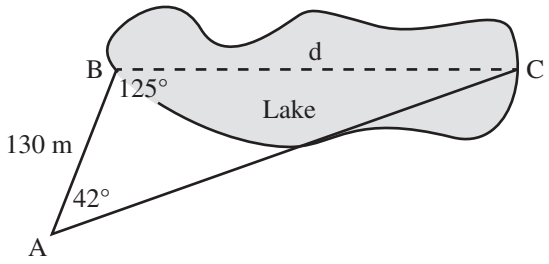
3.4 Applied Problems

The Law of Sines and the Law of Cosines are particularly useful for solving applied problems. Please remember when using the Law of Cosines in an SSS situation to find the **largest** angle first and in SAS, after finding the missing side, find the **smallest** angle first.

Example 1 (Surveying)

To measure the length of a lake, a baseline AB is established and measured to be 130 m. Angles A and B are measured to be 42° and 125° respectively. How long is the lake?

▶ *Solution:*



Find $\angle C$, then use the Law of Sines.

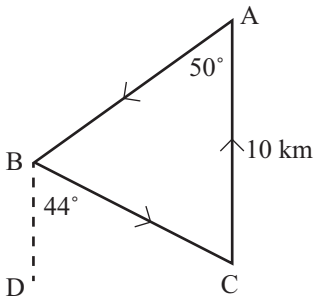
$$\angle C = 180° - 125° - 42°$$
$$= 13°$$
$$\frac{\sin 13°}{130} = \frac{\sin 42°}{d}$$
$$d = \frac{130 \sin 42°}{\sin 13°}$$
$$= 387 \text{ metres}$$

The lake is 387 metres long.

Example 2 (Navigation)

The course for a boat race starts at point A, and heads in a direction S50°W to point B, then in a direction S44°E to point C, and finally back north to point A. The distance from A to C is 10 km. Find the total distance of the boat race.

▶ *Solution:*



Since BD and AC are parallel, C = 44°.

$$\angle B = 180° - 50° - 44°$$
$$= 86°$$

Use Law of Sines.

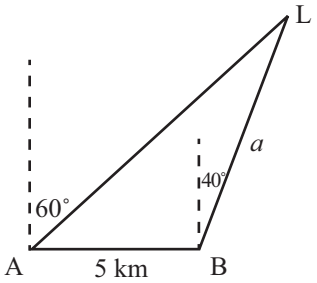
$$\frac{\sin 86°}{10} = \frac{\sin 50°}{a} \rightarrow a = \frac{10 \sin 50°}{\sin 86°} = 7.68 \text{ km}$$
$$\frac{\sin 86°}{10} = \frac{\sin 44°}{c} \rightarrow c = \frac{10 \sin 44°}{\sin 86°} = 6.96 \text{ km}$$

Boat race length is 10 + 7.68 + 6.96 = 24.6 km.

Example 3 (Distance)

A ship is heading due east and passes rock A. At this time, the bearing to a lighthouse L is N60°E. After travelling 5 km, the bearing is N40°E. How far is the ship from the lighthouse?

▶ *Solution:*



$\angle A = 30°, \angle B = 130°$ and $\angle L = 20°$.

Use Law of Sines.

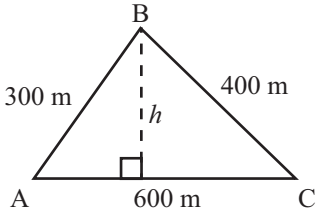
$$\frac{\sin 30°}{a} = \frac{\sin 20°}{5} \rightarrow a = \frac{5\sin 30°}{\sin 20°} = 7.3 \text{ km}$$

The ship is 7.3 km from the lighthouse.

Example 4 (Area)

The length of the sides of a triangular parcel of land are approximately 300 m, 400 m and 600 m. Approximate the area of the parcel of land.

▶ *Solution:*



Find A by Law of Cosines.

$$400^2 = 300^2 + 600^2 - 2(300)(600)\cos A$$
$$\cos A = 0.805$$
$$A = 36.34°$$
$$\sin 36.34° = \frac{h}{300}$$
$$h = 300\sin 36.34°$$
$$= 177.76 \text{ m}$$

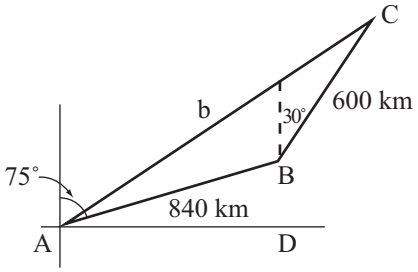
$$Area = \frac{1}{2} base \times height$$
$$= \frac{1}{2}(600)(177.76)$$
$$= 53\,327 \text{ m}^2$$

The parcel of land is approximately 53 327 m^2.

Example 5 (Navigation)

A plane flies 840 km from A to B at a bearing of N75°E. Then it flies 600 km from B to C with a bearing of N30°E. Find the distance from C to A.

▶ *Solution:*



$\angle BAD = 15°$, thus $\angle B = 135°$

Use Law of Cosines.

$$b^2 = 840^2 + 600^2 - 2(840)(600)\cos 135°$$
$$b = 1334$$

The distance from C to A is 1334 km.

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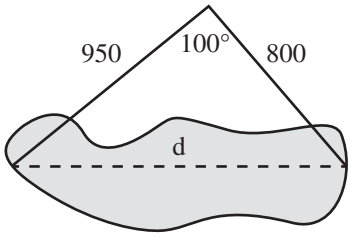
Example 6 (Surveying)

To approximate the length of a lake, a surveyor triangulates the distance to one side to be 950 m and to the other 800 m. If the angle between the two measures is 100°, how long is the lake?

▶ *Solution:*



Use Law of Cosines.

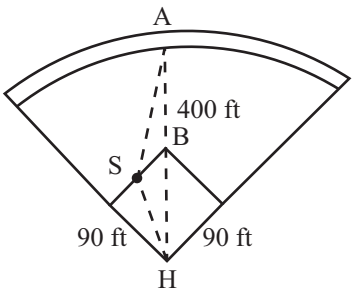
$$d^2 = 950^2 + 800^2 - 2(950)(800)\cos 100°$$

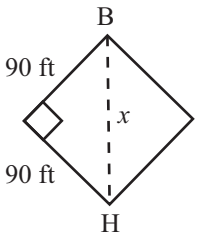
$$d = 1344$$

The lake is 1344 m long.

Example 7 (Angle)

The distance from home plate to centre field at Yankee Stadium is 400 ft. What is the angle A between short stop (half way between 2nd and 3rd base) and home plate?





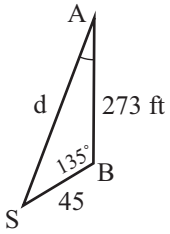
$$x^2 = 90^2 + 90^2$$

$$x = 127 \text{ feet}$$

$$AB = 400 - 127$$

$$= 273 \text{ feet}$$

▶ *Solution:*



By Law of Cosine.

$$d^2 = 45^2 + 273^2 - 2(45)(273)\cos 135°$$

$$d = 306.5 \text{ ft}$$

By Law of Sine

$$\frac{\text{Sin } A}{45} = \frac{\text{Sin } 135°}{306.5}$$

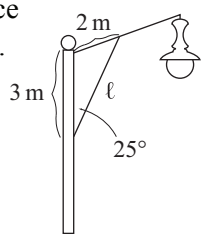
$$\text{Sin } A = 0.10382$$

$$A = 6.0°$$

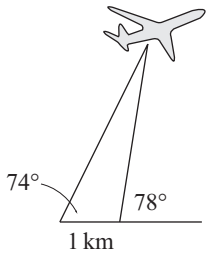
The angle between short stop and centre field is 6.0°.

3.4 Exercise Set

1. Find the length, l, of the brace required to support the lamp.



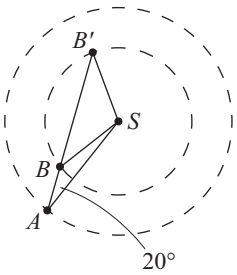
2. A plane is sighted by two observers 1 km apart at angles 74° and 78°. How high is the plane?



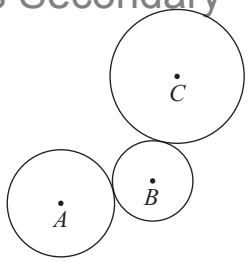
3. A hot air balloon is flying directly between two cities that are 4 km apart. The balloonist finds that the angle of depression to one city is 38° and 33° to the other city. How high above the ground is the balloon?

4. Two planes leave airport A at the same time in different directions. One plane lands at airport B, 630 km from airport A. The other plane lands at airport C some time later. If $\angle ABC = 110°$ and $\angle ACB = 40°$, how far did the second plane fly?

5. In a solar system, the distance from the Sun (S) to planets A and B are 85 and 61 million miles respectively. When $\angle A = 20°$, how far is it from planet A to planet B?



6. Three circles with radius $A = 4$ cm, $B = 3$ cm and $C = 5$ cm are shown. If $\angle CAB = 35°$, how far is it from the centre of circle A to the centre of circle C?



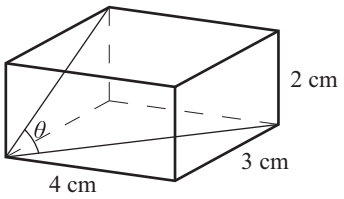
7. A plane flies 420 km from point A at a direction of 135° from due east, and then travels 240 km at a direction of 240° from due east. How far is the plane from point A?

8. Two planes leave Victoria at 9:00 a.m. One plane travels due east at 500 km/h, while the other plane travels 640 km/h N30°W. How far apart are the two planes at noon?

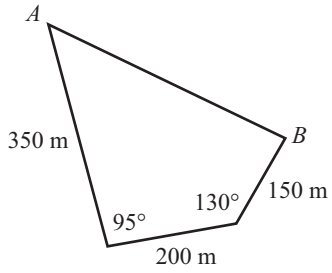
9. Two adjacent sides of a parallelogram meet at an angle of 38°, and have lengths of 4 cm and 9 cm. What is the length of the larger diagonal of the parallelogram?

10. Three circles of radius 3, 5 and 7 cm are tangent to each other. Find the largest angle formed by joining their centres.

11. The rectangular box has dimensions $4\text{ cm} \times 3\text{ cm} \times 2\text{ cm}$. Find angle θ formed by a diagonal of the base, and a diagonal of the $2\text{ cm} \times 3\text{ cm}$ side.



12. An irregular plot of land has dimension as shown. Find AB.

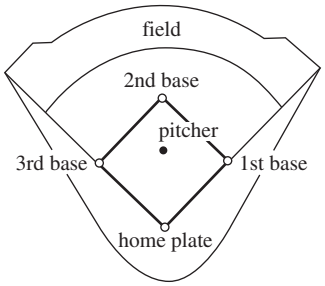


13. A fire at C is spotted from two fire lookout stations, A and B, which are 12 km apart. If station A reports $\angle BAC$ is 50°, and station B reports $\angle ABC$ is 32°, how far is the fire from station A?

14. A regular octagon is inscribed in a circle at radius 12 cm. What is the perimeter of the octagon?

15. A ship sails from port 50 km on a bearing of 20°, then 30 km further on a bearing of 80°. How far is the ship from the port?

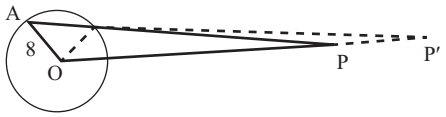
16. A baseball diamond is a square of sides 90 feet, with 60 feet the distance between the pitcher's mound and home plate. When a runner is halfway between second and third base, how far is the runner from the pitcher's mound?



Baseball Diamond

17. The flying distance between Vancouver and Calgary is 675 km. A pilot, after flying 240 km from Vancouver, finds she is 6° off course. How far is she from Calgary at this time?

18. On an engine the crankshaft is 8 cm long and the connecting rod, AP, is 25 cm long. At the time when $\angle APO$ is 15°, how far is the piston P from centre O of the crankshaft?



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3.5 Chapter Review

Section 3.1

1. Find each ratio to four decimal places.

a) sin 78° _____ **b)** cos 41° _____

c) tan 19° _____ **d)** sin 23.7° _____

e) cos 24.3° _____ **f)** tan 65.4° _____

2. Find the measure of angle α to one decimal place.

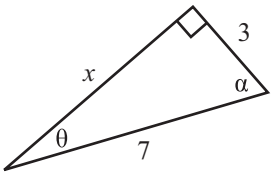
a) $\sin\alpha = 0.2138$ _____ **b)** $\cos\alpha = 0.4386$ _____

c) $\tan\alpha = 0.7458$ _____ **d)** $\sin\alpha = 0.8291$ _____

e) $\cos\alpha = 0.8352$ _____ **f)** $\tan\alpha = 3.2157$ _____

3. Solve the triangles.

a)

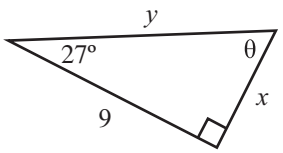


$x =$ _____

$\alpha =$ _____

$\theta =$ _____

b)



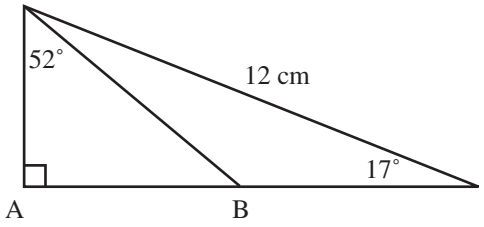
$x =$ _____

$y =$ _____

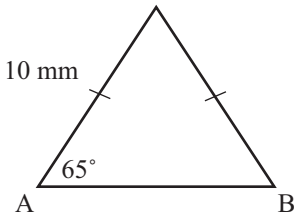
$\theta =$ _____

4. Find the length of AB, to the nearest tenth.

a)



b)

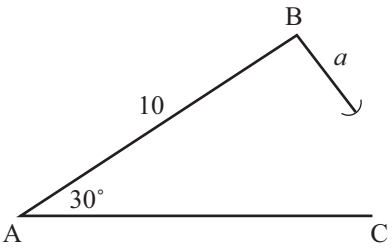


Section 3.2

5. Find the angle which gives the same value of sine as the following, $0° \le \theta \le 180°$.

a) sin 30° _____ **b)** sin 140° _____

6. Determine the value of *a* that will give 0, 1 or 2 triangles.



a) 0 solutions _____

b) 1 solution _____

c) 2 solutions _____

7. Determine if the following leads to 0, 1 or 2 triangles.

a) $\Delta ABC, \angle A = 19°, a = 25, \angle C = 30°$

b) $\Delta ABC, \angle A = 28°, a = 50, b = 20$

c) $\Delta XYZ, \angle X = 58°, x = 9.3, z = 6.8$

d) $\Delta XYZ, \angle X = 110°, x = 90, z = 100$

8. Solve ΔABC using the Law of Sines.

a) $\angle A = 65°, \angle B = 93°, c = 10$

b) $\angle B = 54°, b = 9, c = 10$

Section 3.3

9. Solve using the Law of Sines or the Law of Cosines.

a) $\angle A = 25°, b = 8, c = 14$

b) $a = 3, b = 4, c = 6$

c) $\angle C = 60°, a = 2, b = 3$

d) $\angle A = 40°, \angle B = 20°, a = 2$

e) $a = 6, b = 8, \angle A = 35°$

f) $a = 2, c = 1, \angle C = 50°$

10. Find the area of ΔABC.

a) $a = 2, b = 3, \angle C = 60°$

b) $a = 4, b = 2, c = 5$

Section 3.4

11. Find the angle from the origin O, between point A with coordinate (3, 4) and point B with coordinate (4, 3). (angle AOB)

12. A baseball diamond is 90 square feet. If the distance from home plate to straight away centre field is 400 ft, how far is it from first base to centre field?

13. A softball field is 60 square feet with the pitching rubber 46 feet from home plate. How far is it from the pitching rubber to first base?

14. Three circles of radius 2, 4 and 6 cm are tangent to one another. Find the three angles formed by the lines joining their centres.

15. Find the perimeter of a regular pentagon inscribed by a circle of radius 10 cm.

16. Find the perimeter of a regular pentagon which contains an inscribed circle of radius 10 cm.