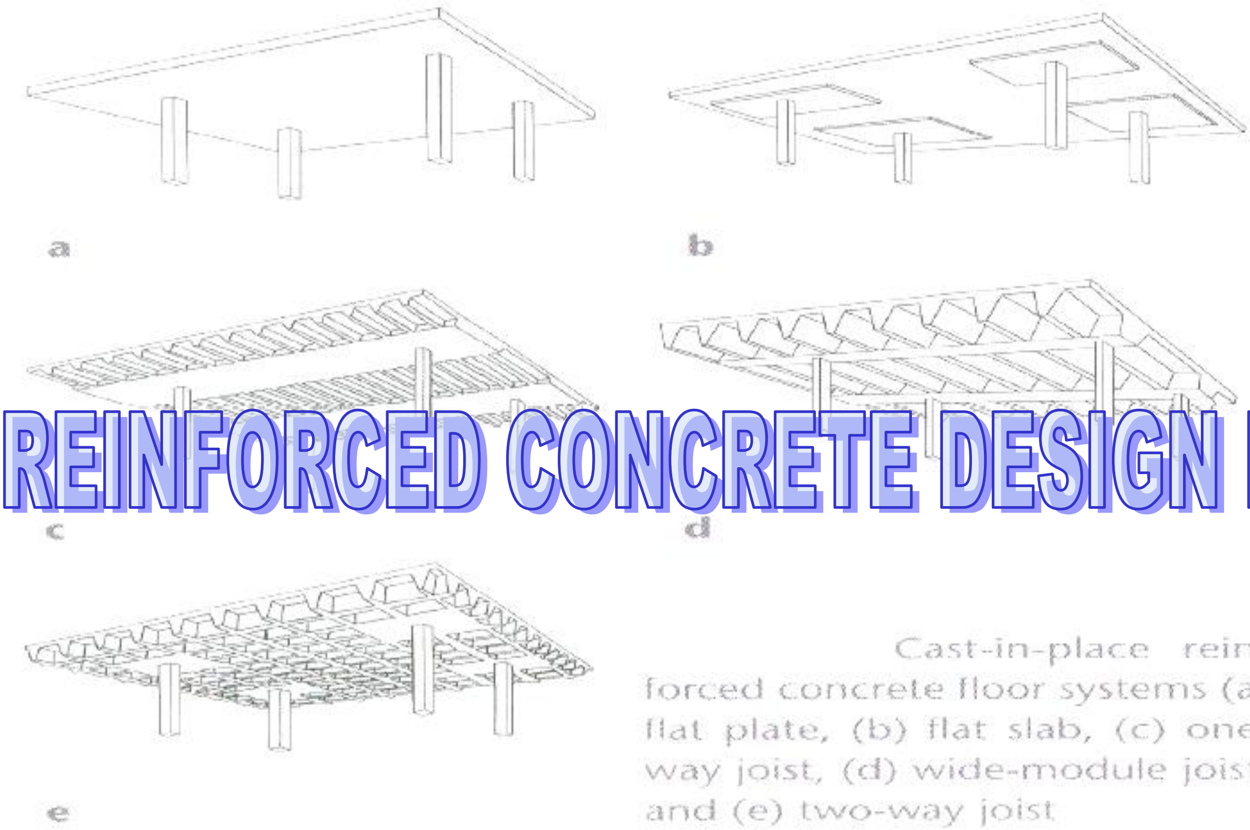
University of Baghdad

College of Engineering

Department of Civil Engineering

REINFORCED CONCRETE DESIGN II



Cast-in-place reinforced concrete floor systems (a) flat plate, (b) flat slab, (c) one-way joist, (d) wide-module joist, and (e) two-way joist

FOURTH YEAR CLASS

1

2018 - 2019

MHND

COURSE SYLLABUS

- Structural system and load paths
- Types of slabs
- Design of one-way slab
- Minimum slab thickness of two-way slabs
- Design of two-way slab
- General design concept of ACI Code
- Direct design method
- Total static moment in flat slab
- Equivalent frame method
- Shear in slab system with beams
- Shear strength in flat plate and flat slab (one way and punching shear)
- Transfer of moment at columns
- Yield line theory
- Prestressed concrete
- Design of stair case

TEXT BOOK AND REFERENCES

- Design of Concrete Structures; Arthur H. Nilson, David Darwin, and Charles W. Dolan.

- Reinforced Concrete Mechanics and Design; James K. Wight and James G. Macgregor.

- Building Design and Construction Handbook; Frederick S. Merritt and Jonathan T. Ricketts

- Structural details in Concrete; M. Y. H. Bangash.

- Manual for the Design of Reinforced Concrete Building Structures; the Institute of Structural Engineers.

- Reinforced Concrete Analysis and Design; S. S. Ray.

- ACI 318, Building Code Requirements for Structural Concrete (ACI 318M-14) and Commentary (ACI 318RM-14), ACI Committee 318, American Concrete Institute, Farmington Hills, MI, 2014.

- Shear Reinforcement for Slabs; Reported by ACI-ASCE Committee 421.

Structural system and load paths

The structural system shall include (a) through (g), as applicable:

(a) Floor construction and roof construction, including one-way and two-way slabs

(b) Beams and joists

(c) Columns

(d) Walls

(e) Diaphragms

(f) Foundations

(g) Joints, connections, and anchors as required to transmit forces from one component to another.

Load factors and combinations

According to ACI 318M-14, the required strength (U) shall be at least equal to the effects of factored loads in Table 5.3.1, with exceptions and additions in 5.3.3 through 5.3.12.

Table 5.3.1—Load combinations

Load combination	Equation	Primary load
$U = 1.4D$	(5.3.1a)	D
$U = 1.2D + 1.6L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1b)	L
$U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.5W)$	(5.3.1c)	L_r or S or R
$U = 1.2D + 1.0W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$	(5.3.1d)	W
$U = 1.2D + 1.0E + 1.0L + 0.2S$	(5.3.1e)	E
$U = 0.9D + 1.0W$	(5.3.1f)	W
$U = 0.9D + 1.0E$	(5.3.1g)	E

All members and structural systems shall be analyzed for the maximum effects of loads including the arrangements of live load in accordance with 6.4.

Strength

Design strength of a member and its joints and connections, in terms of moment, axial force, shear, torsion, and bearing, shall be taken as the nominal strength (S_n) multiplied by the applicable strength reduction factor (ø).

Structures and structural members shall have design strength at all sections (ø S_n) greater than or equal to the required strength (U) calculated for the factored loads and forces in such combinations as required by ACI-Code.

design strength $\geq$ required strength

ø $S_n \geq U$

Types of slabs

1. One-way slab: Slabs may be supported on two opposite sides only, in such case, the structural action of the slab is essentially "one-way", and the loads are carried by the slab in the direction perpendicular to the supporting beams, Figure (1-a).
2. Tow-way slab: Slabs have beam or support on all four sides. The loads are carried by the slab in two perpendicular directions to the supporting beams, Figure (1-b).
3. If the ratio of length to width of one slab panel is larger than 2, most of the load is carried by the short direction to the supporting beams, and one-way action is obtained in effect, even though supports are provided on all sides, Figure (1-c).
4. Concrete slab carried directly by columns, without the use of beams or girders, such slab is described by flat plates, and are commonly used where spans are not large and loads are not heavy, Figure (1-d).
5. Flat slabs are also beamless slab with column capitals, drop panels, or both, Figure (1-e).
6. Two–way joist systems (grid slab), to reduce the dead load of solid-slab, voids are formed in a rectilinear pattern through use of metal or fiberglass form inserts. A two–way ribbed construction results (waffle slab). Usually inserts are omitted near the columns, Figure (1-f).

One-way slabs: slabs reinforced to resist flexural stresses in only one direction.

Two-way slabs: reinforced for flexure in two directions.

Column capital: enlargement of the top of a concrete column located directly below the slab or drop panel that is cast monolithically with the column.

Drop panel: projection below the slab used to reduce the amount of negative reinforcement over a column or the minimum required slab thickness, and to increase the slab shear strength.

Panel: slab portion bounded by column, beam, or wall centerlines on all sides.

Column strip: a design strip with a width on each side of a column centerline equal to the lesser of 0.25 ℓ_2 and 0.25 ℓ_1. A column strip shall include beams within the strip, if present.

Middle strip: a design strip bounded by two column strips.

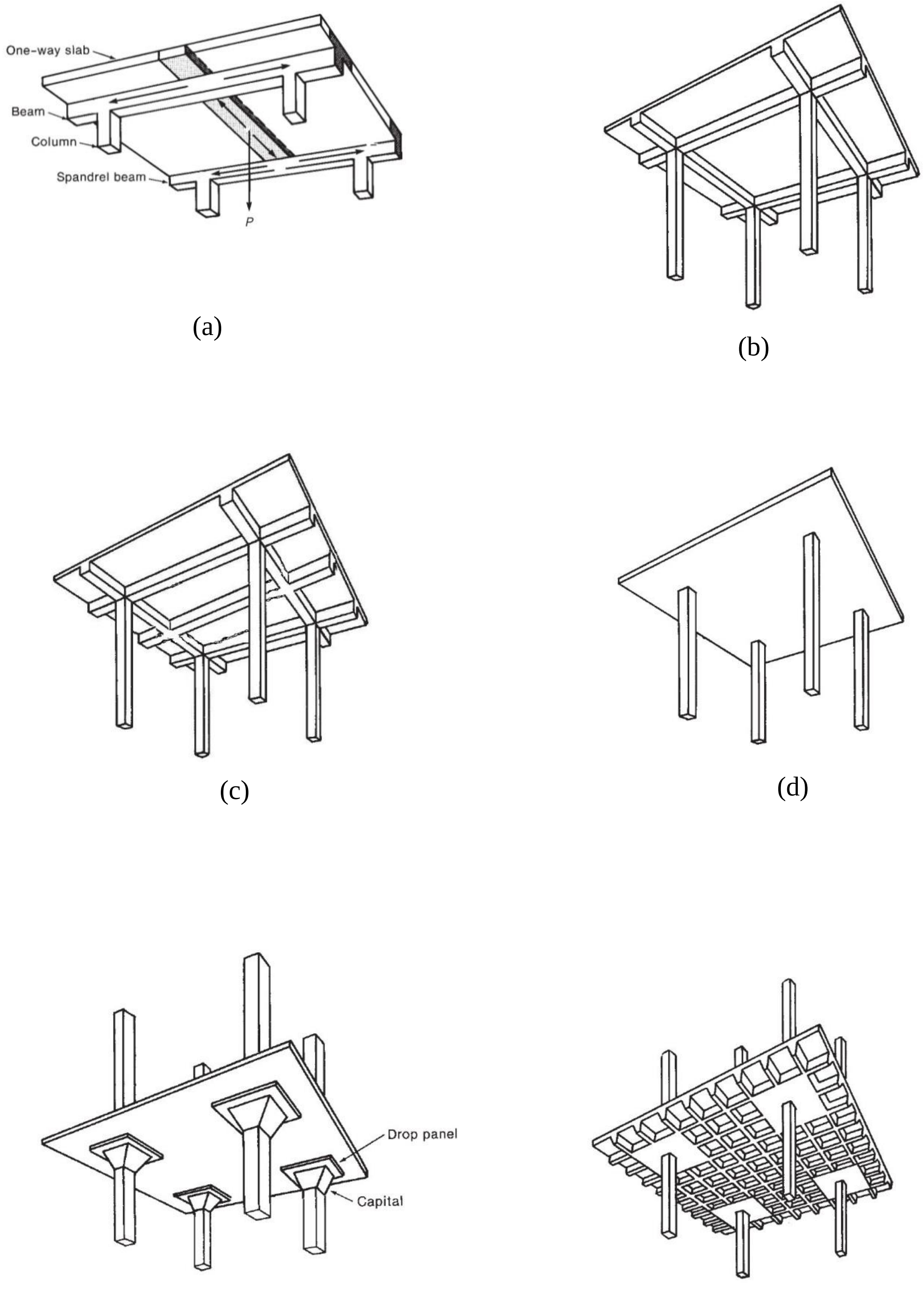
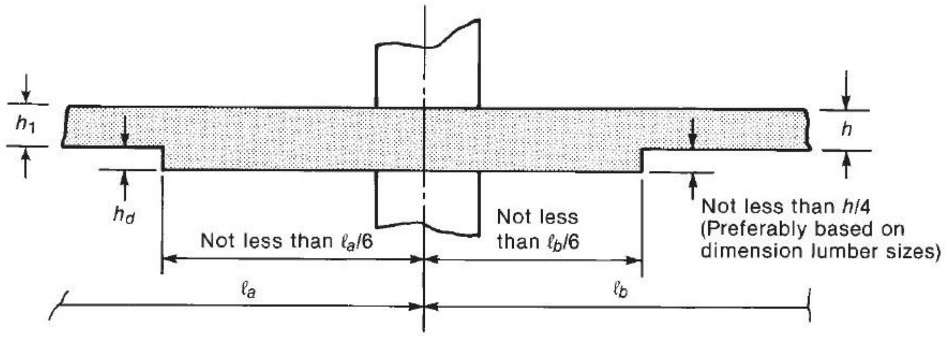


Figure (1) Types of slabs (a) one-way slab, (b) two-way slab, (c) one-way slab, (d) flat plate, (e) flat slab, (f) two–way joist

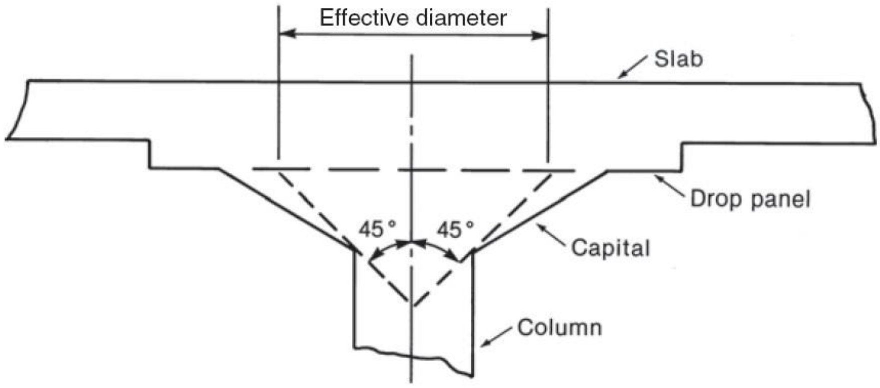
Size and projection of drop panel



Minimum size of drop panels

In computing required slab reinforcement, the thickness of drop panel below the slab shall not be assumed greater than one – quarter the distance from edge of drop panel to edge of column or column capital.

The column capital is normally 20 to 25% of the average span length.



(a) Effective diameter of column capital.

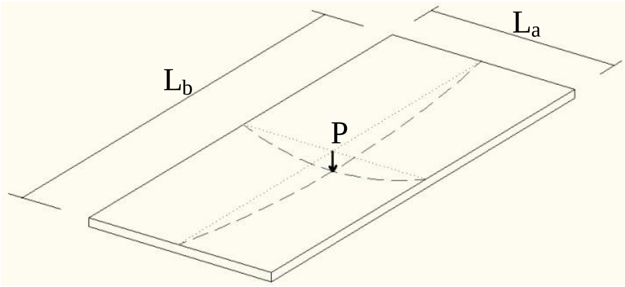
Design of one–way slab systems

At point of intersection (P) the deflection must be the same

$$\Delta = \frac{5\,w\,L^4}{384 E\,I} \Rightarrow \frac{5\,w_a\,{L_a}^4}{384\,E\,I} = \frac{5\,w_b\,{L_b}^4}{384\,E\,I}$$

$$\therefore \frac{w_a}{w_b} = \frac{{L_b}^4}{{L_a}^4}$$

$$\text{if } \frac{L_b}{L_a} = 2 \qquad \therefore \frac{w_a}{w_b} = 16 \qquad \Rightarrow w_a = 16\,w_b$$



For purposes of analysis and design a unit strip of such a slab cut out at right angles to the supporting beam may be considered as a rectangular beam of unit width (1.0 m) with a depth (h) equal to the thickness of the slab and a span (L_a) equal to the distance between supported edges.

Simplified method of analysis for one-way slabs

It shall be permitted to calculate M_u and V_u due to gravity loads in accordance with Section 6.5 for one-way slabs satisfying (a) through (e):

(a) Members are prismatic

(b) Loads are uniformly distributed

(c) $L \leq 3D$

(d) There are at least two spans

(e) The longer of two adjacent spans does not exceed the shorter by more than 20 percent

M_u due to gravity loads shall be calculated in accordance with Table 6.5.2. Moments calculated shall not be redistributed.

For slabs built integrally with supports, M_u at the support shall be permitted to be calculated at the face of support.

Floor or roof level moments shall be resisted by distributing the moment between columns immediately above and below the given floor in proportion to the relative column stiffnesses considering conditions of restraint.

Table 6.5.2—Approximate moments for nonprestressed continuous beams and one-way slabs

Moment	Location	Condition	M_u
Positive	End span	Discontinuous end integral with support	$w_u\ell_n^2/14$
		Discontinuous end unrestrained	$w_u\ell_n^2/11$
	Interior spans	All	$w_u\ell_n^2/16$
Negative[1]	Interior face of exterior support	Member built integrally with supporting spandrel beam	$w_u\ell_n^2/24$
		Member built integrally with supporting column	$w_u\ell_n^2/16$
	Exterior face of first interior support	Two spans	$w_u\ell_n^2/9$
		More than two spans	$w_u\ell_n^2/10$
	Face of other supports	All	$w_u\ell_n^2/11$
	Face of all supports satisfying (a) or (b)	(a) slabs with spans not exceeding 10 ft (b) beams where ratio of sum of column stiffnesses to beam stiffness exceeds 8 at each end of span	$w_u\ell_n^2/12$

[1]To calculate negative moments, ℓ_n shall be the average of the adjacent clear span lengths.

A minimum area of flexural reinforcement ($A_{s,min}$) shall be provided in accordance with Table 7.6.1.1.

Table 7.6.1.1—$A_{s,min}$ for nonprestressed one-way slabs

Reinforcement type	f_y (MPa)	$A_{s,min}$ (mm)	
Deformed bars	< 420	$0.0020\ A_g$	
Deformed bars or welded wire reinforcement	≥ 420	Greater of:	$\dfrac{0.0018 \times 420}{f_y}\ A_g$
			$0.0014\ A_g$

Reinforcement for shrinkage and temperature stresses normal to the principal reinforcement should be provided in a structural slab. ACI Code specifies the minimum ratios of reinforcement area to gross concrete area, as shown in Table 24.4.3.2.

Table 24.4.3.2—Minimum ratios of deformed shrinkage and temperature reinforcement area to gross concrete area

<table>
<tr><th>Reinforcement type</th><th>f_y
(MPa)</th><th colspan="2">Minimum reinforcement ratio</th></tr>
<tr><td>Deformed bars</td><td>< 420</td><td colspan="2">0.0020</td></tr>
<tr><td rowspan="2">Deformed bars or welded wire reinforcement</td><td rowspan="2">≥ 420</td><td rowspan="2">Greater of:</td><td>$\frac{0.0018 \times 420}{f_y}$</td></tr>
<tr><td>0.0014</td></tr>
</table>

The spacing of deformed shrinkage and temperature reinforcement shall not exceed the lesser of 5h and 450 mm.

V_u due to gravity loads shall be calculated in accordance with Table 6.5.4.

Table 6.5.4—Approximate shears for nonprestressed continuous beams and one-way slabs

Location	V_u
Exterior face of first interior support	$1.15w_u\ell_n/2$
Face of all other supports	$w_u\ell_n/2$

For slabs built integrally with supports, V_u at the support shall be permitted to be calculated at the face of support.

Minimum slab thickness

For solid nonprestressed slabs not supporting or attached to partitions or other construction likely to be damaged by large deflections, over all slab thickness (h) shall not be less than the limits in Table 7.3.1.1, unless the calculated deflection limits of 7.3.2 are satisfied.

Table 7.3.1.1—Minimum thickness of solid nonprestressed one-way slabs

Support condition	Minimum h[1]
Simply supported	$\ell/20$
One end continuous	$\ell/24$
Both ends continuous	$\ell/28$
Cantilever	$\ell/10$

For f_y other than 420 MPa, the expressions in Table 7.3.1.1 shall be modified.

For nonprestressed slabs not satisfying 7.3.1 and for prestressed slabs, immediate and time-dependent deflections shall be calculated in accordance with 24.2 and shall not exceed the limits in 24.2.2.

Table 24.2.2—Maximum permissible calculated deflections

Member	Condition		Deflection to be considered	Deflection limitation
Flat roofs	Not supporting or attached to nonstructural elements likely to be damaged by large deflections		Immediate deflection due to maximum of L_r, S, and R	$\ell/180$[1]
Floors			Immediate deflection due to L	$\ell/360$
Roof or floors	Supporting or attached to nonstructural elements	Likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements, which is the sum of the time-dependent deflection due to all sustained loads and the immediate deflection due to any additional live load[2]	$\ell/480$[3]
		Not likely to be damaged by large deflections		$\ell/240$[4]

[1]Limit not intended to safeguard against ponding. Ponding shall be checked by calculations of deflection, including added deflections due to ponded water, and considering time-dependent effects of sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[2]Time-dependent deflection shall be calculated in accordance with 24.2.4, but shall be permitted to be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be calculated on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

[3]Limit shall be permitted to be exceeded if measures are taken to prevent damage to supported or attached elements.

[4]Limit shall not exceed tolerance provided for nonstructural elements.

Design of two-way slab systems

Reinforced concrete slabs (R. C. Slabs) are usually designed for loads assumed to be uniformly distributed over on entire slab panel, bounded by supporting beam or column center-lines.

General design concept of ACI Code

1- Imagining vertical cuts are made through the entire building along lines midway between columns.
2- The cutting creates a series of frames whose width center lines lie along the column lines.
3- The resulting series of rigid frames, taken separately in the longitudinal and transverse directions of the building.
4- A typical rigid frame would consist of:
    a- The columns above and below the floor.
    b- The floor system, with or without beams, bounded laterally between the center lines of the two panels.
5- Two methods of design are presented by the ACI Code:
    a- Direct design method (DDM): An approximants method using moment and shear coefficients, Section 8.10 in ACI Code.
    b- Equivalent Frame method (EFM): More accurate using structural analysis after assuming the relative stiffness of the members, Section 8.11 in ACI Code.

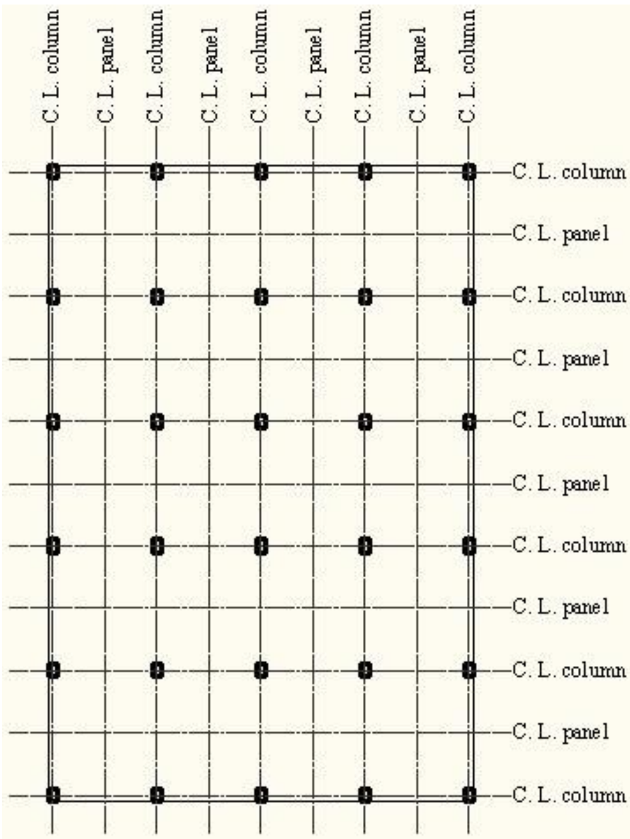


Figure (2) Floor plan.

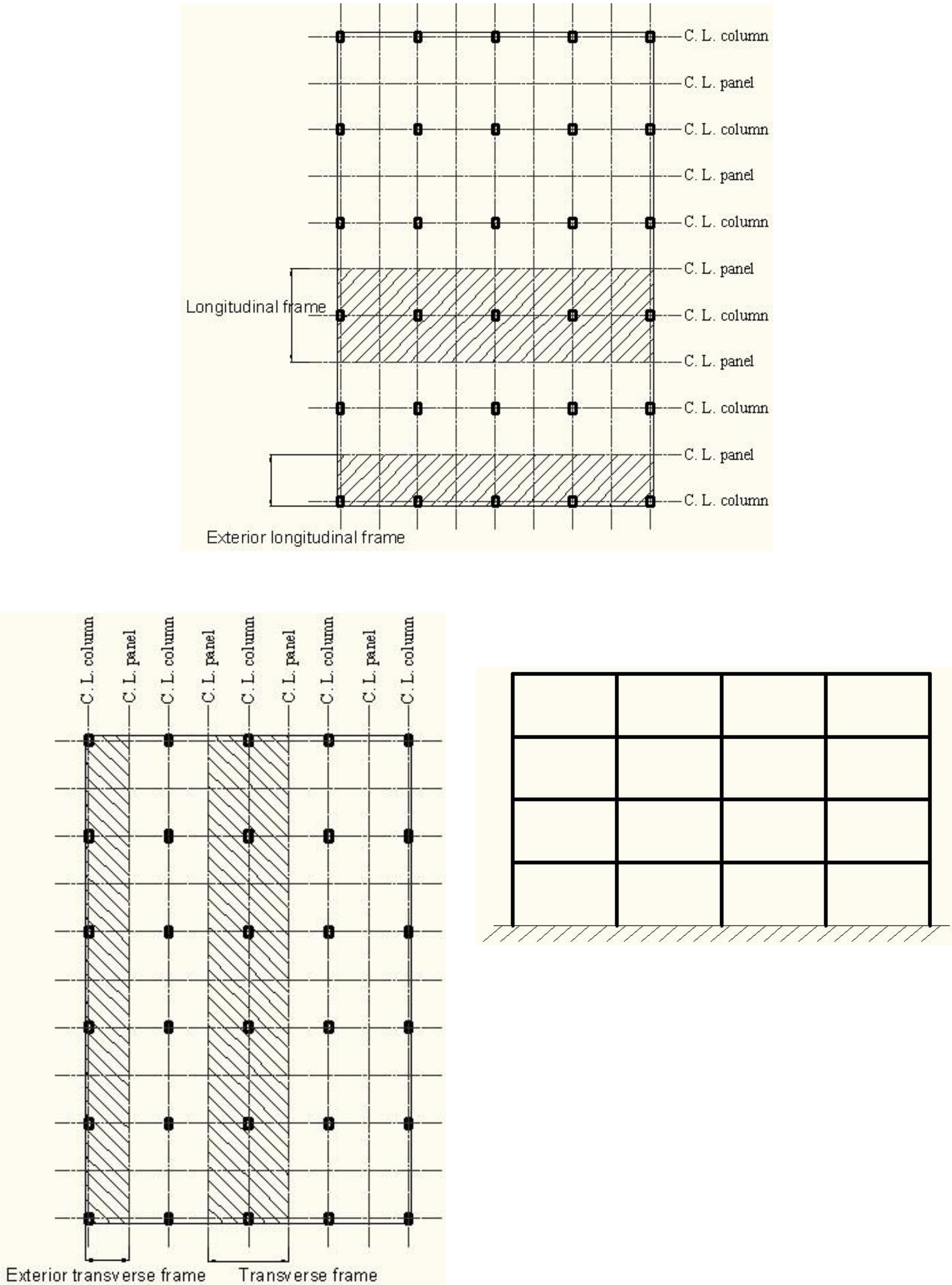


Figure (3) Location of longitudinal and transverse frames.

Direct design method (DDM)

Moments in two-way slabs can be found using a semi-empirical direct design method subject to the following Limitations:

1- There shall be at least three continuous spans in each direction.
2- Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span.
3- Panels shall be rectangular, with the ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2.
4- Column offset shall not exceed 10 percent of the span in direction of offset from either axis between centerlines of successive columns.
5- All loads shall be due to gravity only and uniformly distributed over an entire panel.
6- Unfactored live load shall not exceed two times the unfactored dead load.
7- For a panel with beams between supports on all sides, Eq. (8.10.2.7a) shall be satisfied for beams in the two perpendicular directions.

$$0.2 \leq \frac{\alpha_{f1}\ell_2^{\,2}}{\alpha_{f2}\ell_1^{\,2}} \leq 5.0 \qquad \text{(8.10.2.7a)}$$

ℓ_1: is defined as the span in the direction of the moment analysis, and
ℓ_2: as the span in lateral direction.
Spans ℓ_1& ℓ_2 are measured to column centerlines.
α_{f1} and α_{f2} are calculated by:

$$\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s}$$

The direct design method consists of a set of rules for distributing moments to slab and beam sections to satisfy safety requirements and most serviceability requirements simultaneously. Three fundamental steps are involved as follows:

(1) Determination of the total factored static moment (Section 8.10.3).
(2) Distribution of the total factored static moment to negative and positive sections (Section 8.10.4).
(3) Distribution of the negative and positive factored moments to the column and middle strips and to the beams, if any (Sections 8.10.5 and 8.10.6). The distribution of moments to column and middle strips is also used in the equivalent frame method (Section 8.11).

(1) Total static moment of factored loads (M_o)

M_o: Total static moment in a panel (absolute sum of positive and average negative factored moments in each direction).

$$M_o = \frac{q_u \ell_2 {\ell_n}^2}{8}$$

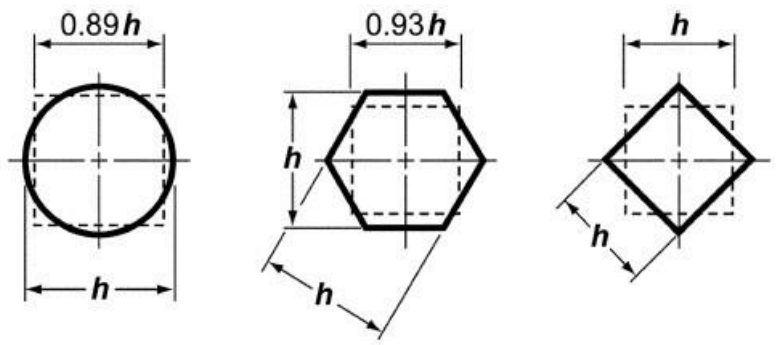
Where ℓ_n: Clear span in the direction of moment used.

ℓ_n is defined to extend from face to face of the columns, capitals, brackets, or walls but is not to be less than 0.65 ℓ_1.

M_o for a strip bounded laterally by the centerlines of the panel on each side of the centerline of support.

ℓ_2: Width of the frame.

Circular or regular polygon-shaped supports shall be treated as square supports with the same area.



(2) Longitudinal distribution of M_o

(a) Interior spans: M_o is apportioned between the critical positive and negative bending sections according to the following ratios:-

Neg. M_u = 0.65 M_o

Pos. M_u = 0.35 M_o

The critical section for a negative bending is taken at the face of rectangular supports, or at the face of an equivalent square support having the same sectional area.

(b) End span: In end spans, the apportionment of the total static moment (M_o) among the three critical moment sections (interior negative, positive, and exterior negative) depends upon the flexural restraint provided for the slab by the exterior column or the exterior wall and upon the presence or absence of beams on the column lines. End span, M_o shall be distributed in accordance with Table 8.10.4.2.

Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports		Exterior edge fully restrained
			Without edge beam	With edge beam	
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65

Note: At interior supports, negative moment may differ for spans framing into the common support. In such a case the slab should be designed to resist the larger of the two moments.

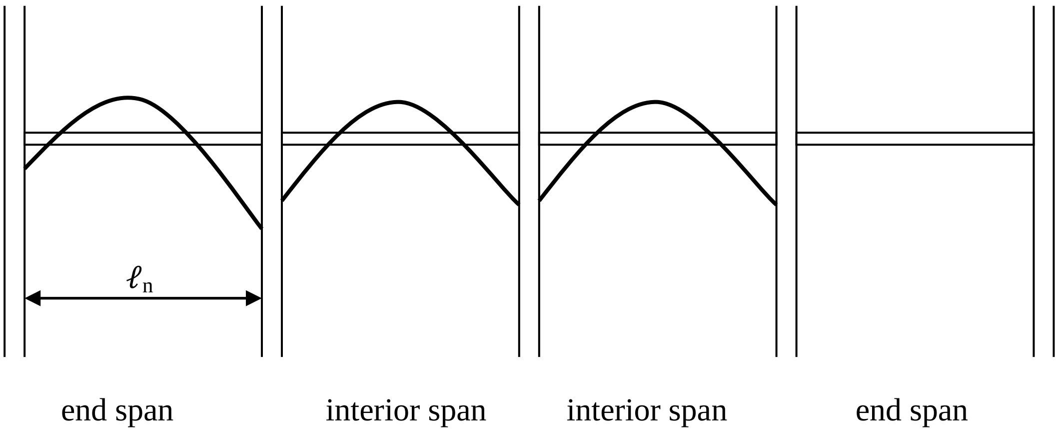


Figure (4) Longitudinal distribution of M_o

(3) Lateral distribution of moments

After the moment M_o distributed on long direction to the positive and negative moments, then these moments must distribute in lateral direction across the width, which consider the moments constant within the bounds of a middle strip or column strip. The distribution of moments between middle strips and column strip and beams depends upon:

1. The ratio ℓ_2/ℓ_1.
2. The relative stiffness of the beam and the slab.
3. The degree of torsional restraint provided by the edge beam.

The column strip shall resist the portion of interior negative M_u in accordance with Table 8.10.5.1.

Table 8.10.5.1—Portion of interior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1 0.5	1.0	2.0
0	0.75	0.75	0.75
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

The column strip shall resist the portion of exterior negative M_u in accordance with Table 8.10.5.2.

Table 8.10.5.2—Portion of exterior negative M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	β_t	ℓ_2/ℓ_1 0.5	1.0	2.0
0	0	1.0	1.0	1.0
	≥2.5	0.75	0.75	0.75
≥1.0	0	1.0	1.0	1.0
	≥2.5	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown. β_t is calculated using Eq. (8.10.5.2a), where C is calculated using Eq. (8.10.5.2b).

$$\beta_t = \frac{E_{cb}C}{2E_{cs}I_s} \qquad (8.10.5.2a)$$

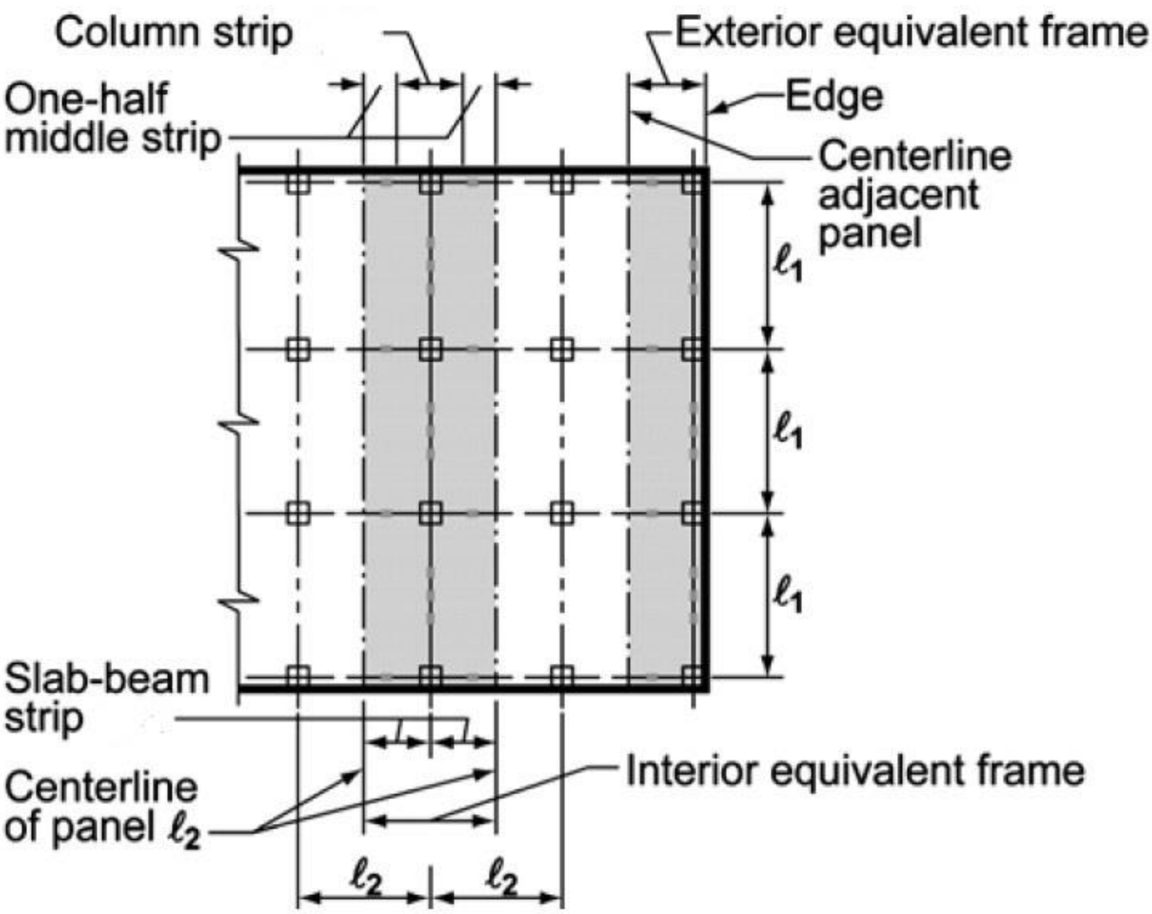
$$C = \Sigma\left(1-0.63\frac{x}{y}\right)\frac{x^3y}{3} \qquad (8.10.5.2b)$$

The column strip shall resist the portion of positive M_u in accordance with Table 8.10.5.5.

Table 8.10.5.5—Portion of positive M_u in column strip

$\alpha_{f1}\ell_2/\ell_1$	ℓ_2/ℓ_1		
	0.5	**1.0**	**2.0**
0	0.60	0.60	0.60
≥1.0	0.90	0.75	0.45

Note: Linear interpolations shall be made between values shown.

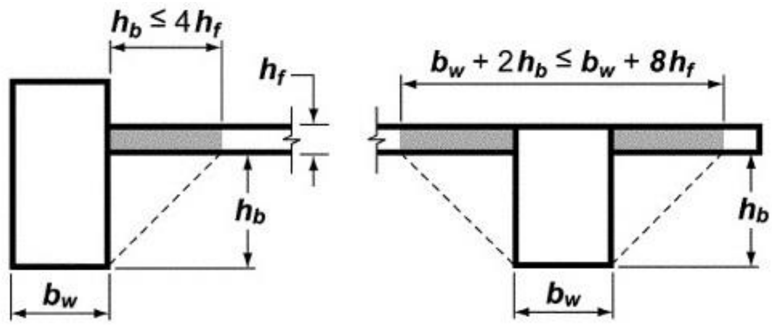


A convenient parameter defining the relative stiffness of the beam and slab spanning in either direction is:

$$\alpha_f = \frac{E_{cb}I_b}{E_{cs}I_s} = \frac{I_b}{I_s}$$

Where E_{cb}, E_{cs} are the moduli of elasticity of beam and slab concrete (usually the same), respectively. I_b and I_s are the moment of inertia of the effective beam and slab, respectively. The flexural stiffnesses of the beam and slab are based on the gross concrete section. Variation due to column capitals and drop panels are neglected (in applying DDM).

For monolithic or fully composite construction supporting two-way slabs, a beam includes that portion of slab, on each side of the beam extending a distance equal to the projection of the beam above or below the slab, whichever is greater, but not greater than four times the slab thickness.



The moment of inertia of flanged section

$$I_b = k\,\frac{b_w h^3}{12}$$

$$k \approx 1.0 + 0.2\left(\frac{b_E}{b_w}\right) \qquad \text{for} \qquad 2 < \frac{b_E}{b_w} < 4 \qquad \& \qquad 0.2 < \frac{h_f}{h} < 0.5$$

The relative restraint provided by the torsional resistance of the effective transverse edge beam is reflected by the parameter ß$_t$, defined by:

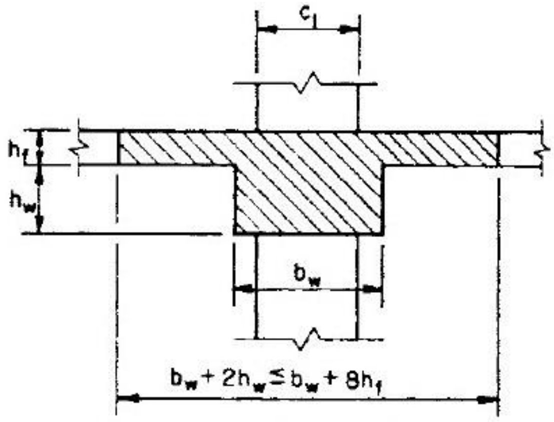
$$\beta_t = \frac{E_{cb} C}{2\,E_{cs} I_s} = \frac{C}{2\,I_s}$$

C: The torsional rigidity of the effective transverse beam, which is defined as the largest of the following three items:-

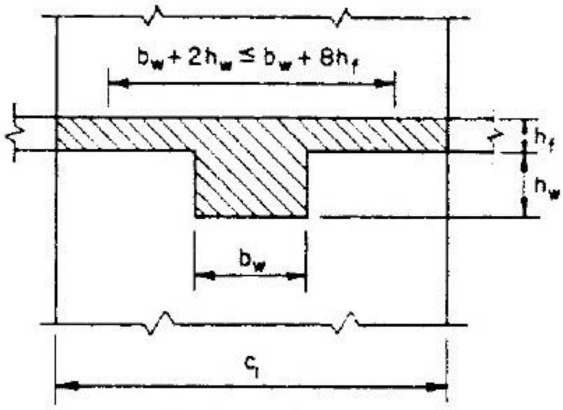
a- A portion of the slab having a width equal to that of the column, bracket, or capital in the direction in which moment are taken, c_1 (case of no actual beam).

b- The portion of the slab specified in (a) plus that part of any transverse beam above and below the slab.

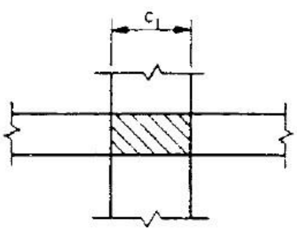
c- The transverse beam defined as before (in calculating α_f).



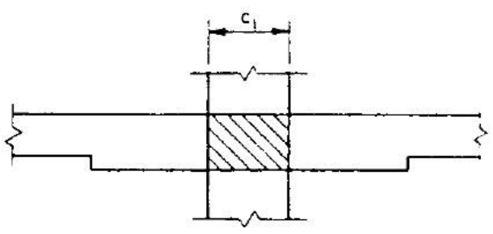
Condition (c)



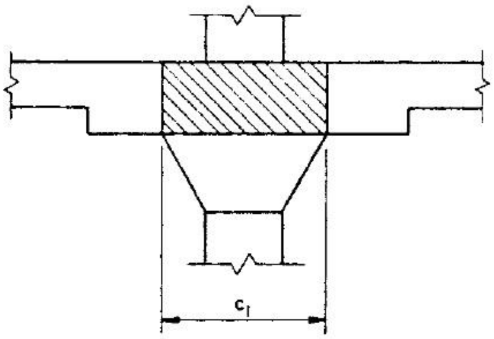
Condition (b)



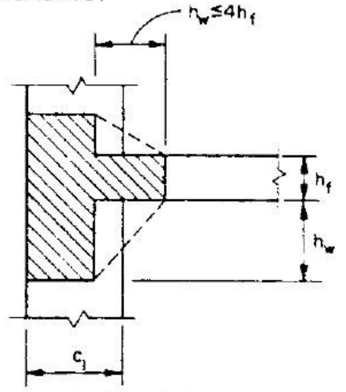
Condition (a)



Condition (a)



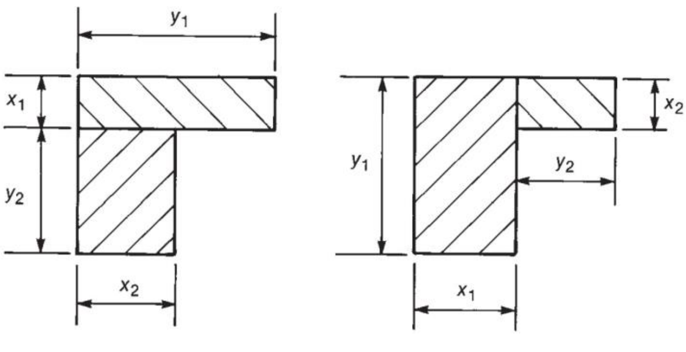
Condition (a)



Condition (c)

The constant C is calculated by dividing the section into its component rectangles, each having smaller dimension x and larger dimension y and summing the contributions of all the parts by means of the equation:

$$C = \sum\left(1 - 0.63\,\frac{x}{y}\right)\frac{x^3\,y}{3}$$



The subdivision can be done in such a way as to maximize C.

For slabs with beams between supports, the slab portion of column strips shall resist column strip moments not resisted by beams. Beams between supports shall resist the portion of column strip M_u in accordance with Table 8.10.5.7.1.

Table 8.10.5.7.1—Portion of column strip M_u in beams

$\alpha_{f1}\ell_2/\ell_1$	Distribution coefficient
0	0
≥1.0	0.85

Note: Linear interpolation shall be made between values shown.

The portion of the moment not resisted by the column strip is proportionately assigned to the adjacent half-middle strips. Each middle strip is designed to resist the sum of the moment assigned to its two half-middle strips. A middle strip adjacent and parallel to wall is designed for twice the moment assigned to the half-middle strip corresponding to the first row of interior support.

If the width of the column or wall is at least (¾)ℓ_2, negative M_u shall be uniformly distributed across ℓ_2.

Minimum flexural reinforcement in nonprestressed slabs, $A_{s,min}$, shall be provided near the tension face in the direction of the span under consideration in accordance with Table 8.6.1.1.

Table 8.6.1.1—$A_{s,min}$ for nonprestressed two-way slabs

Reinforcement type	f_y (MPa)	$A_{s,min}$ (mm)	
Deformed bars	< 420	0.0020 A_g	
Deformed bars or welded wire reinforcement	≥ 420	Greater of:	$\frac{0.0018 \times 420}{f_y} A_g$
			0.0014 A_g

Minimum spacing of reinforcement

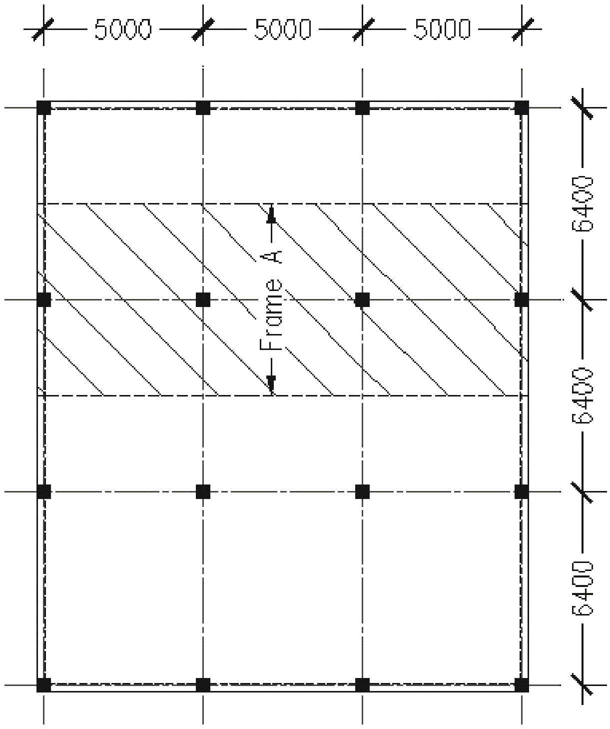
For parallel nonprestressed reinforcement in a horizontal layer, clear spacing shall be at least the greatest of 25 mm, d_b, and (4/3)$_{dagg}$.

For nonprestressed solid slabs, maximum spacing (s) of deformed longitudinal reinforcement shall be the lesser of 2h and 450 mm at critical sections, and the lesser of 3h and 450 mm at other sections.

Example 1

For the the longitudinal interior frame (Frame A) of the falt plate floor shown in Figure,by using the Direct Design Method, find:

- **a.** Longitudinal distribution of the total static moment at factored loads.
- **b.** Lateral distribution of the moment at exterior support.



Slab thickness = 200 mm, d = 165 mm

q_u = 15.0 kN/m^2

All edge beams = 250×500 mm

All columns = 500×500 mm

f_c' = 25 MPa, f_y = 400 MPa

Solution

a.)

for Frame A

ℓ_1 = 5000 mm

ℓ_2 = 6400 mm

$\ell_n = \ell_1 - 500 = 5000 - 500 = 4500$ mm

$$M_o = \frac{q_u \ell_2 \ell_n^{\,2}}{8}$$

$$M_o = \frac{15 \times 6.4 \times (4.5)^2}{8}$$

$= 243$ kN.m

Longitudinal distribution of total static moment at factored loads

Interior span:

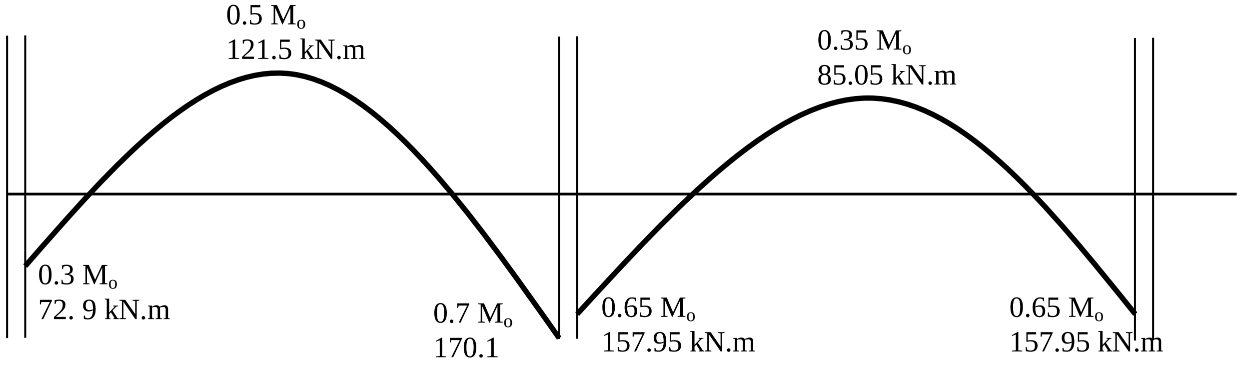
Neg. $M_u = 0.65\ M_o$

Pos. $M_u = 0.35\ M_o$

End span:

Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports: Without edge beam	Slab without beams between interior supports: With edge beam	Exterior edge fully restrained
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65

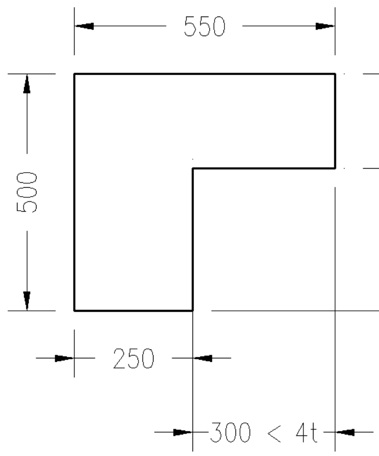


Longitudinal distribution of total static moment at factored loads

b.)

Negative moment at exterior support

Total moment = 72.9 kN.m



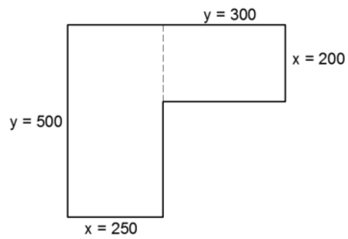
$\alpha_f = 0$

for edge beam

choose the section of edge beam

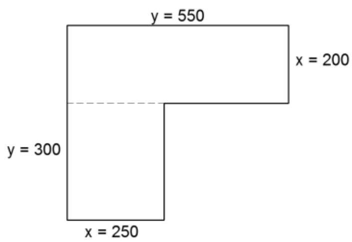
$$C = \sum\left(1 - 0.63 \times \frac{x}{y}\right) \cdot \frac{x^3 \cdot y}{3}$$

$$C_1 = \left(1 - 0.63 \times \frac{200}{300}\right) \times \frac{(200)^3 \times 300}{3} + \left(1 - 0.63 \times \frac{250}{500}\right) \times \frac{(250)^3 \times 500}{3}$$



$$C_1 = 2247854166667 \ \text{mm}^4$$

$$C_2 = \left(1 - 0.63 \times \frac{200}{550}\right) \times \frac{(200)^3 \times 550}{3} + \left(1 - 0.63 \times \frac{250}{300}\right) \times \frac{(250)^3 \times 300}{3}$$



$$C_2 = 1872854166667 \ \text{mm}^4$$

$$\therefore\ C = 2247854166667 \ \text{mm}^4$$

$$I_s = \frac{1}{12} \times \ell_2 \times t^3 = \frac{1}{12} \times 6400 \times (200)^3 = 4266666666.667 \ \text{mm}^4$$

$$\beta_t = \frac{E_{cb} \cdot C}{2 \times E_{cs} \cdot I_s} \quad ; \quad E_{cb} = E_{cs}$$

$$\beta_t = \frac{C}{2 \times I_s} = \frac{2247854166667}{2 \times 4266666666667} = 0.263$$

$$\frac{\ell_2}{\ell_1} = \frac{6.4}{5.0} = 1.28$$

ℓ_2/ℓ_1	1.0	1.28	2.0
$\beta_t = 0.0$	1.00	1.00	1.00
$\beta_t = 0.263$		**0.9737**	
$\beta_t \geq 2.5$	0.75	0.75	0.75

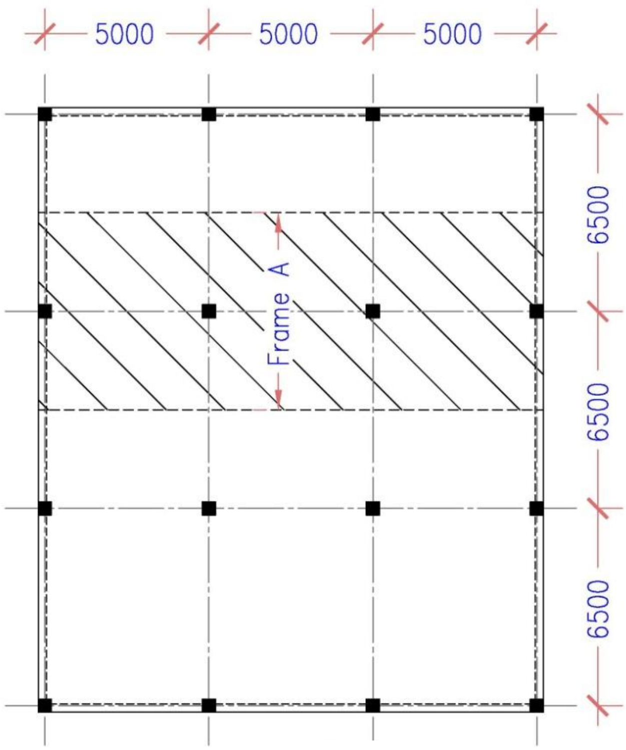
Negative moment at column strip = 72.9×0.9737 = 70.983 kN.m

Negative moment at middle strip = 72.9-70.983 = 1.917 kN.m

Example 2

For the the longitudinal interior frame of the falt plate floor shown in Figure, by using the Direct Design Method, find:

- **a.** Longitudinal distribution of total static moment at factored loads.
- **b.** Lateral distribution of moment at exterior panel.



Slab thickness = 180 mm, d = 150 mm

q_u = 14.0 kN/m^2

All edge beams = 250×500 mm

All columns = 400×400 mm

f_c' = 24 MPa, f_y = 400 MPa

<u>Solution</u>

a.)

for Frame A

ℓ_1 = 5000 mm

ℓ_2 = 6500 mm

$\ell_n = \ell_1 - 400 = 5000 - 400 = 4600$ mm

$$M_o = \frac{q_u \ell_2 \ell_n{}^2}{8}$$

$$M_o = \frac{14\times6.5\times(4.6)^2}{8}$$

$= 240.695$ kN.m

<u>Longitudinal distribution of total static moment at factored loads</u>

Interior span:

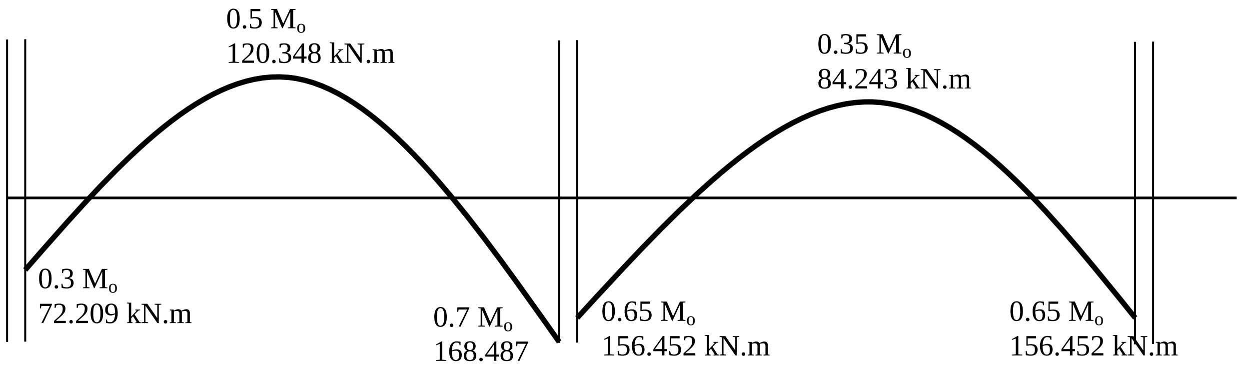
Neg. $M_u = 0.65\ M_o$

Pos. $M_u = 0.35\ M_o$

End span:

Table 8.10.4.2—Distribution coefficients for end spans

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports: Without edge beam	Slab without beams between interior supports: With edge beam	Exterior edge fully restrained
Interior negative	0.75	0.70	0.70	0.70	0.65
Positive	0.63	0.57	0.52	0.50	0.35
Exterior negative	0	0.16	0.26	0.30	0.65



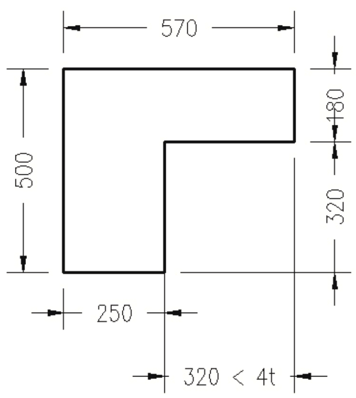
Longitudinal distribution of total static moment at factored loads

b.)

exterior panel

1- Negative moment at exterior support

Total moment = 72.209 kN.m



$\alpha_f = 0$

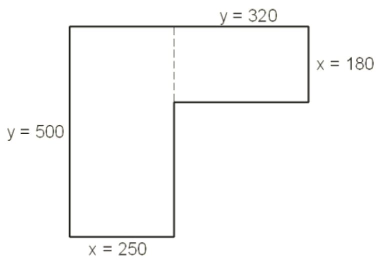
for edge beam

choose the section of edge beam

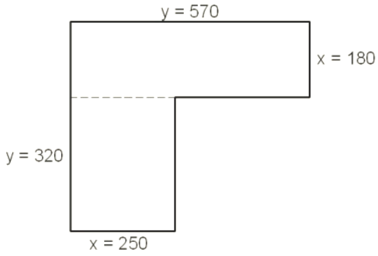
$$C = \sum\left(1-0.63\times\frac{x}{y}\right)\cdot\frac{x^3\cdot y}{3}$$

$$C_1 = \left(1-0.63\times\frac{180}{320}\right)\times\frac{(180)^3\times320}{3}+\left(1-0.63\times\frac{250}{500}\right)\times\frac{(250)^3\times500}{3}$$

$$C_1 = 21854845667\ \text{mm}^4$$



$$C_2 = \left(1 - 0.63 \times \frac{180}{570}\right) \times \frac{(180)^3 \times 570}{3} + \left(1 - 0.63 \times \frac{250}{320}\right) \times \frac{(250)^3 \times 320}{3}$$



$$C_2 = 17339845667 \quad mm^4$$

$$\therefore \quad C = 21854845667 \quad mm^4$$

$$I_s = \frac{1}{12} \times \ell_2 \times t^3 = \frac{1}{12} \times 6500 \times (180)^3 = 3159000000 \text{ mm}^4$$

$$\beta_t = \frac{E_{cb} \cdot C}{2 \times E_{cs} \cdot I_s} \quad ; \quad E_{cb} = E_{cs}$$

$$\beta_t = \frac{C}{2 \times I_s} = \frac{21854845667}{2 \times 3159000000} = 0.346$$

$$\frac{\ell_2}{\ell_1} = \frac{6.5}{5.0} = 1.3$$

ℓ_2/ℓ_1	1.0	1.3	2.0
$\beta_t = 0.0$	1.00	1.00	1.00
$\beta_t = 0.346$			
$\beta_t \geq 2.5$	0.75	0.75	0.75

Negative moment at column strip = 72.209 × = kN.m

Negative moment at middle strip = 72.209 - = kN.m

2- Positive moment

Total moment = 120.348 kN.m

$\alpha_f = 0$

3- Negative moment at interior support
Total moment = 168.487 kN.m

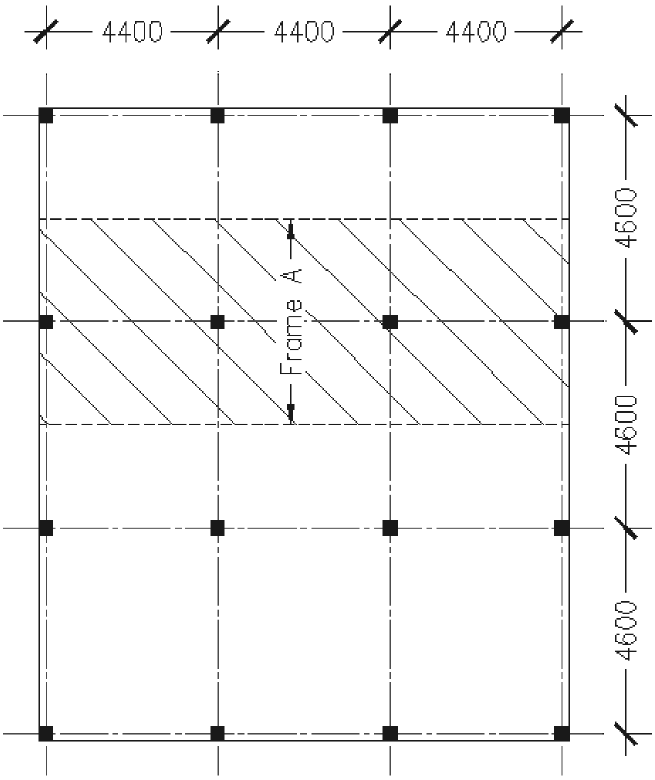
$\alpha_f = 0$

Example 3

For the the longitudinal interior frame (Frame A) of the falt plate floor shown in Figure, by using the Direct Design Method, find:

a- Longitudinal distribution of the total static moment at factored loads.

b- Lateral distribution of moment at interior panel (column and middle strip moments at negative and positive moments).



Slab thickness = 180 mm, d = 150 mm

$q_u = 15.0\,\text{kN/m}^2$

All columns = 400×400 mm

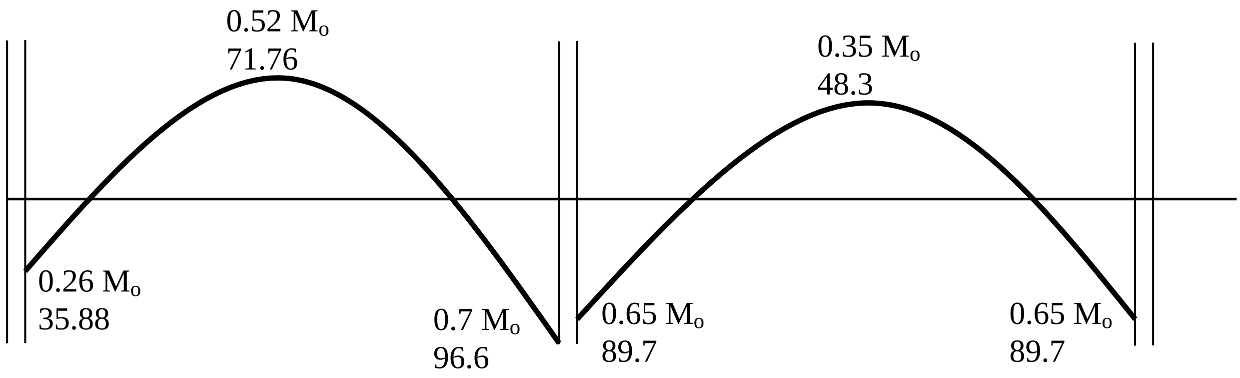
<u>Solution</u>

a.

$$L_n = L_1 - 0.5$$
$$= 4.4 - 0.4$$
$$= 4.0 \text{ m}$$

$$M_o = \frac{w_u \cdot L_2 \cdot L_n^2}{8}$$

$$M_o = \frac{15 \times 4.6 \times (4.0)^2}{8}$$
$$= 138 \text{ kN.m}$$



Longitudinal distribution of total static moment at factored loads

b.

interior panel

1) Negative moment

Total moment = 89.7 kN.m

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

$\alpha_1 = 0$

Negative moment at column strip = 89.7×0.75 = 67.275 kN.m

Negative moment at midlle strip = 89.7-67.275 = 22.425 kN.m

2) Positive moment

Total moment = 48.3 kN.m

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

$\alpha_1 = 0$

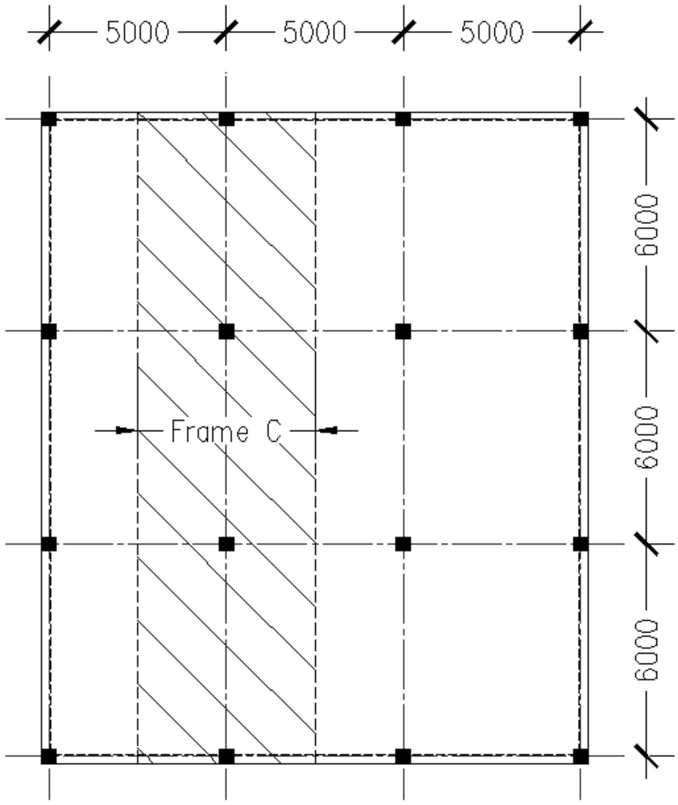
Negative moment at column strip = 48.3×0.60 = 28.98 kN.m

Negative moment at midlle strip = 48.3-28.98 = 19.32 kN.m

Example 4

For the the transverse interior frame (Frame C) of the flat plate floor with edge beams shown in Figure, by using the Direct Design Method, find:

1) Longitudinal distribution of total static moment at factored loads.
2) Lateral distribution of moment at interior panel (column and middle strip moments at negative and positive moments).
3) Lateral distribution of moment at exterior panel (column and middle strip moments at negative and positive moments).



Slab thickness $= 180$ mm, $d = 150$ mm

$q_u = 16.0\,kN/m^2$

All edge beams $= 250 \times 500$ mm

All columns $= 500 \times 500$ mm

Example 5

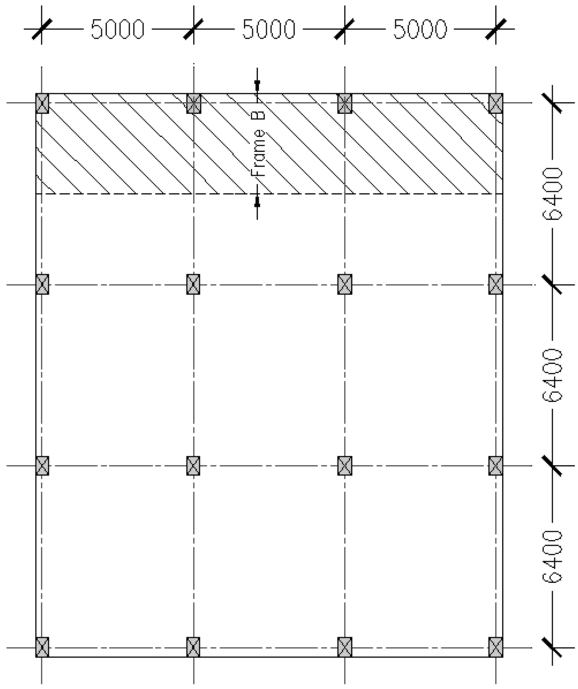
For the exterior longitudinal frame (Frame B) of the flat plate floor shown in figure, and by using the Direct Design Method, find:

- **a.** Longitudinal distribution of the total static moment at factored loads.
- **b.** Lateral distribution of moment at exterior panel (column and middle strip moments at exterior support)

Slab thickness $= 175$ mm, $d = 140$ mm

$q_u = 14.0\,kN/m^2$

All columns $= 600 \times 400$ mm



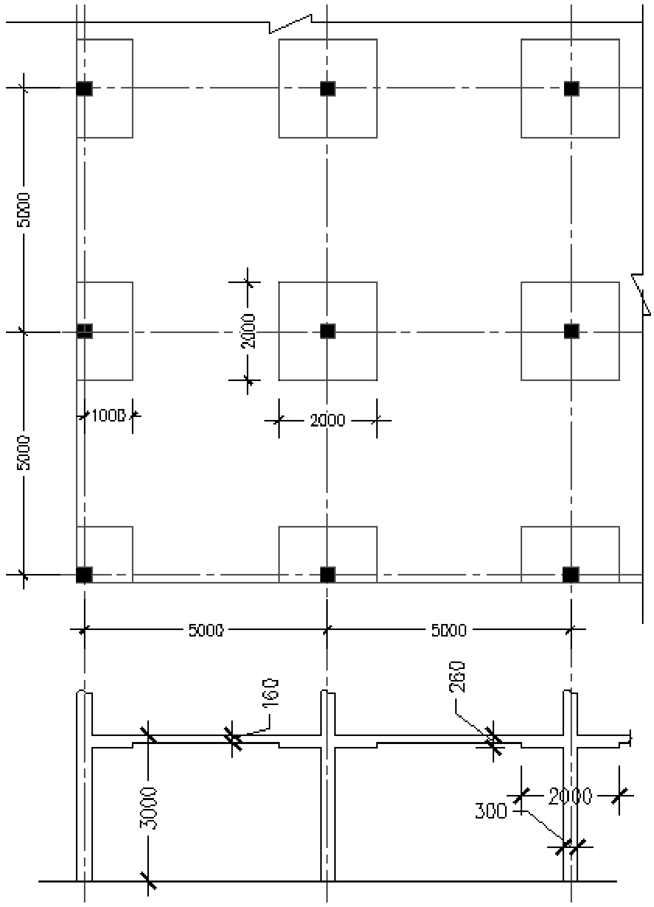
Example 6

For the exterior transverse frame of the flat slab floor shown in figure, and by using the Direct Design Method, find:

- **a.** Longitudinal distribution of the total static moment at factored loads.
- **b.** Lateral distribution of moment at exterior panel (column and middle strip moments at exterior support)

D = 6.5 kN/m^2

L = 5.0 kN/m^2



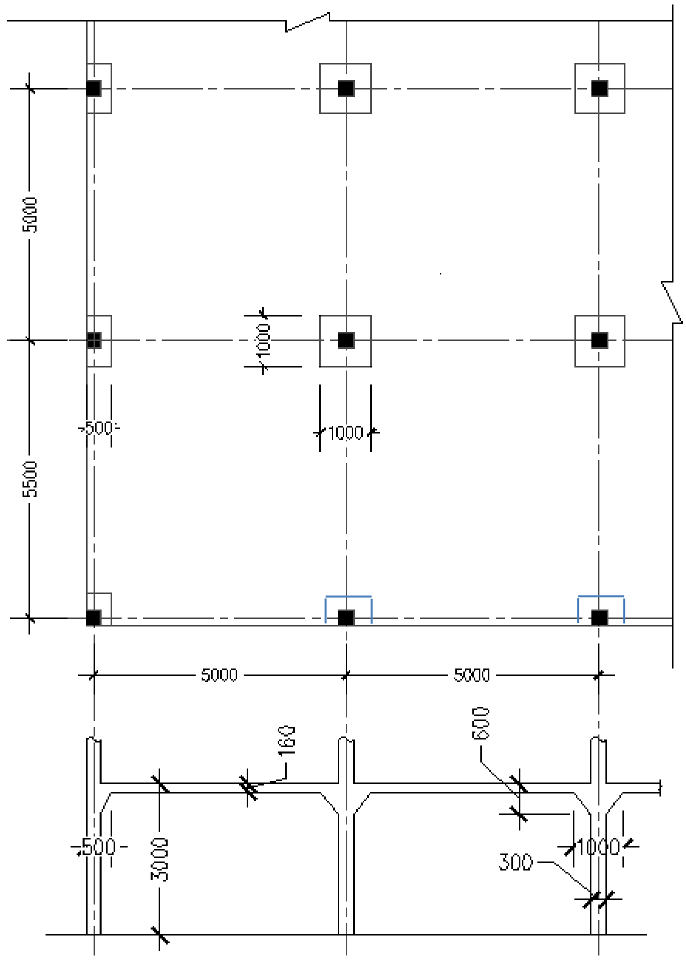
Example 7

For the transverse frame of the flat slab floor shown in figure, and by using the Direct Design Method, find:

- **a.** Longitudinal distribution of the total static moment at factored loads.
- **b.** Lateral distribution of moment at exterior panel (column and middle strip moments at exterior support)

D = 7.0 kN/m^2

L = 4.0 kN/m^2



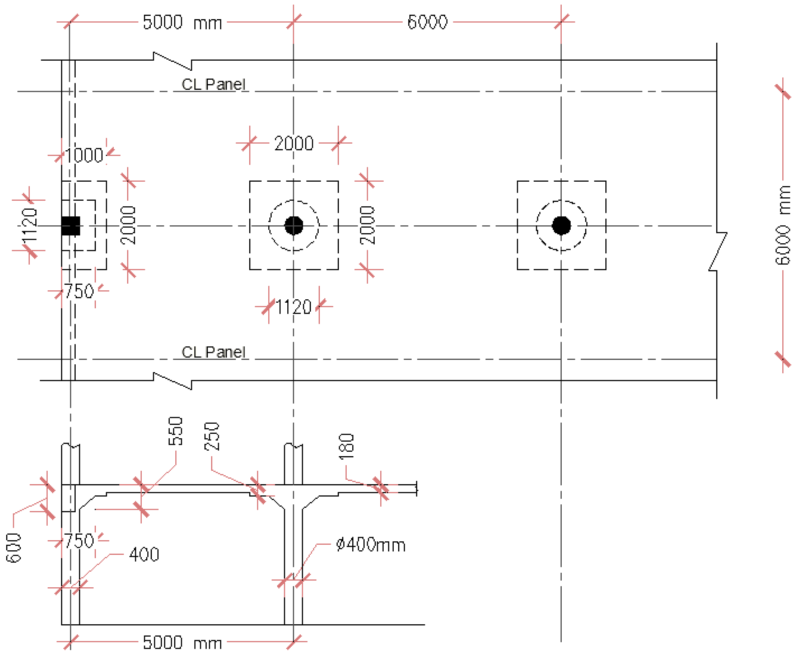
Example 8

For the longitudinal frame of the flat slab floor shown in figure, and by using the Direct Design Method, find:

- **a.** Longitudinal distribution of the total static moment at factored loads.
- **b.** Lateral distribution of moment at exterior panel (column and middle strip moments at exterior support)

$q_u = 18.0\ kN/m^2$

edge beams: 300×600 mm



Example 9

For the the transverse extiror frame (Frame D) of the falt plate floor, without edge beams, shown in Figure, and by using the Direct Design Method, find:

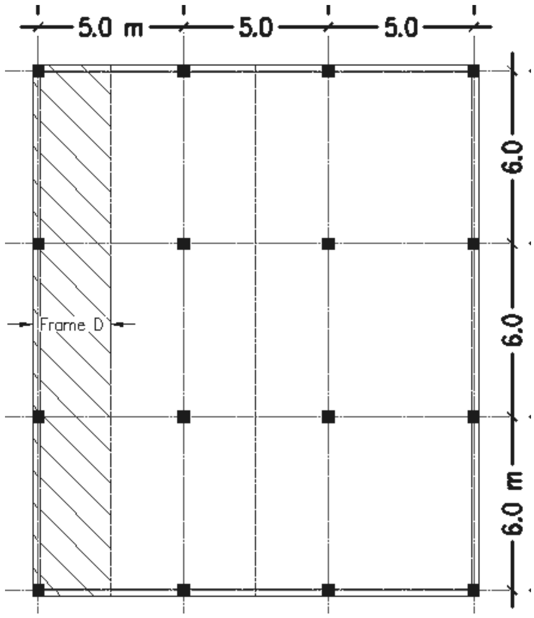
a. Longitudinal distribution of the total static moment at factored loads.

b. Lateral distribution of moment at interior panel (column and middle strip moments at negative and positive moments).

Slab thickness = 180 mm, d = 150 mm

q_u = 15.0 kN/m^2

All columns = 400×400 mm



Minimum slab thickness for two-way slabs

To ensure that slab deflection service will not be troublesome, the best approach is to compute deflections for the total load or load component of interest and to compare the computed deflections with limiting values.

Alternatively, deflection control can be achieved indirectly by adhering to more or less arbitrary limitations on minimum slab thickness, limitations developed from review of test data and study of the observed deflections of actual structures.

Simplified criteria are used to slabs without interior beams (provided by table), flat plates and flat slabs with or without edge beams. While equations are to be applied to slabs with beams spanning between the supports on all sides. In both cases, minimum thicknesses less than the specified value may be used if calculated deflections are within code specified limits.

Slab without interior beams (Flat plates and flat slabs with or without edge beams)

For nonprestressed slabs without interior beams spanning between supports on all sides, having a maximum ratio of long-to-short span of 2, overall slab thickness (h) shall not be less than the limits in Table 8.3.1.1, and shall be at least the value in (a) or (b), unless the calculated deflection limits of 8.3.2 (ACI 318) are satisfied:

(a) Slabs without drop panels as given in 8.2.4........... 125 mm.

(b) Slabs with drop panels as given in 8.2.4................ 100 mm.

Table 8.3.1.1—Minimum thickness of nonprestressed two-way slabs without interior beams (mm)[1]

f_y, MPa[2]	Without drop panels[3]			With drop panels[3]		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams[4]		Without edge beams	With edge beams[4]	
280	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$	$\ell_n/36$	$\ell_n/40$	$\ell_n/40$
420	$\ell_n/30$	$\ell_n/33$	$\ell_n/33$	$\ell_n/33$	$\ell_n/36$	$\ell_n/36$
520	$\ell_n/28$	$\ell_n/31$	$\ell_n/31$	$\ell_n/31$	$\ell_n/34$	$\ell_n/34$

[1]ℓ_n is the clear span in the long direction, measured face-to-face of supports (mm).

[2]For f_y between the values given in the table, minimum thickness shall be calculated by linear interpolation.

[3]Drop panels as given in 8.2.4.

[4]Slabs with beams between columns along exterior edges. Exterior panels shall be considered to be without edge beams if α_f is less than 0.8. The value of α_f for the edge beam shall be calculated in accordance with 8.10.2.7.

Slabs with beams on all sides:

For nonprestressed slabs with beams spanning between supports on all sides, overall slab thickness (h) shall satisfy the limits in Table 8.3.1.2, unless the calculated deflection limits of 8.3.2 are satisfied.

Table 8.3.1.2—Minimum thickness of nonprestressed two-way slabs with beams spanning between supports on all sides

<table>
<tr><th>α_{fm}[1]</th><th colspan="2">Minimum h, mm.</th><th></th></tr>
<tr><td>$\alpha_{fm} \le 0.2$</td><td colspan="2">8.3.1.1 applies</td><td>(a)</td></tr>
<tr><td rowspan="2">$0.2 < \alpha_{fm} \le 2.0$</td><td rowspan="2">Greater of:</td><td>$\dfrac{\ell_n\left(0.8+\dfrac{f_y}{1400}\right)}{36+5\beta\left(\alpha_{fm}-0.2\right)}$</td><td>(b)[2],[3]</td></tr>
<tr><td>125</td><td>(c)</td></tr>
<tr><td rowspan="2">$\alpha_{fm} > 2.0$</td><td rowspan="2">Greater of:</td><td>$\dfrac{\ell_n\left(0.8+\dfrac{f_y}{1400}\right)}{36+9\beta}$</td><td>(d)[2],[3]</td></tr>
<tr><td>90</td><td>(e)</td></tr>
</table>

[1]α_{fm} is the average value of α_f for all beams on edges of a panel and α_f shall be calculated in accordance with 8.10.2.7.

[2]ℓ_n is the clear span in the long direction, measured face-to-face of beams (mm).

[3]β is the ratio of clear spans in long to short directions of slab.

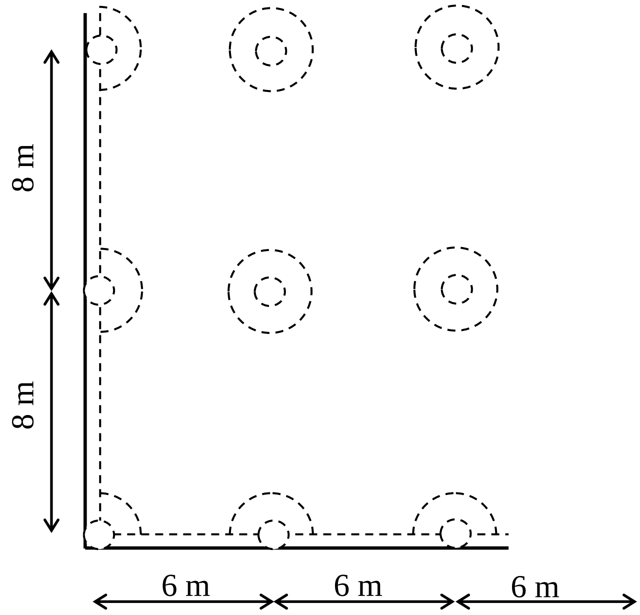
At discontinuous edges of slabs conforming to 8.3.1.2, an edge beam with $\alpha_f \ge 0.80$ shall be provided, or the minimum thickness required by (b) or (d) of Table 8.3.1.2 shall be increased by at least 10 percent in the panel with a discontinuous edge.

Example 1

Thickness of a flat slab with edge beams

Column capital diameter = 1000 mm

(f_y = 420 MPa)



Solution

exterior panel

$t = \ell_n/33$

$\ell_n = 8000 - 0.89 \times 1000 = 7110$ mm

$t = 7110/33 = 215.4$ mm $>$ 125 mm

interior panel

$t = \ell_n/33 = 215.4$ mm $>$ 125 mm

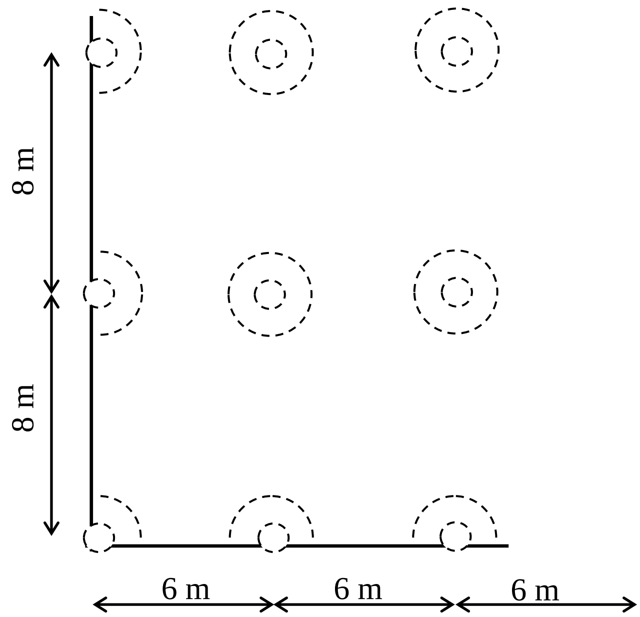
$\therefore$ use t = 220 mm

Example 2

Thickness of a flat slab without edge beams

Column capital diameter = 1000 mm

(f_y = 420 MPa)



Solution

exterior panel

$t = \ell_n/30$

$\ell_n = 8000 - 0.89 \times 1000 = 7110$ mm

$t = 7110/30 = 237.0$ mm $>$ 125 mm

interior panel

$t = \ell_n/33 = 215.4$ mm $>$ 125 mm

$\therefore$ use t = 240 mm

Example 3

Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use f_y = 350 MPa.

- **a-** Slab with beams (8.1 × 8.2) m clear span with **α_m = 2.3**
- **b-** Flat plate (4.4 × 4.6) m clear span.
- **c-** Flat slab with drop panels (6.2 × 6.2) m clear span.

Solution

a- Slab with beams (8.1 × 8.2) m clear span with **α_m = 2.3**

$\alpha_m = 2.3 > 2.0$

$$\Rightarrow\ t_{min} = \frac{l_n\left(0.8+\dfrac{f_y}{1400}\right)}{36+9\beta} \quad ; \quad \beta = \frac{L_n}{S_n} = \frac{8.2}{8.1} = 1.012$$

$$t_{min} = \frac{8200\times\left(0.8+\dfrac{350}{1400}\right)}{36+9\times1.012} = 190.875\text{ mm} \quad > 90\text{ mm O.K.}$$

Use t = 200 mm

b- Flat plate (4.4 × 4.6) m clear span.

From table

For f_y = 280 $\quad t = \frac{L_n}{36} = \frac{4600}{36} = 127.778$ mm

For f_y = 420 $\quad t = \frac{L_n}{33} = \frac{4600}{33} = 139.394$ mm

For f_y = 350 (by linear interpolation)

$$t = \frac{127.778+139.394}{2} = 133.586\text{ mm} \quad > 125\text{ mm O.K.}$$

Use t = 140 mm

c- Flat slab with drop panels (6.2 × 6.2) m clear span.

From table

For f_y = 280 $\quad t = \frac{L_n}{40} = \frac{6200}{40} = 155$ mm

For f_y = 420 $\quad t = \frac{L_n}{36} = \frac{6200}{36} = 172.222$ mm

For f_y = 350 (by linear interpolation) $\quad t = \frac{155+172.222}{2} = 163.611$ mm $\quad >$ 100 mm O.K.

Use t = 170 mm

Example 4

Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use f_y = 420 MPa. (60000 psi).

- **a-** Slab with beams (8.2 × 7.7) m clear span with **α_m= 2.3**
- **b-** Slab without drop panels (5.4 × 4.8) m clear span with **α_m= 0.18**
- **c-** Flat plate (4.2 × 4.6) m clear span.
- **d-** Flat slab with drop panels (6.0 × 6.2) m clear span.
- **e-** Slab with beams (5.8 × 5.8) m clear span with **α_m= 1.5**

<u>Solution</u>

a- Slab with beams (8.2 × 7.7) m clear span with **α_m= 2.3**

$$\alpha_m = 2.3 > 2.0$$

$$\Rightarrow\ t_{min} = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} \quad ; \quad \beta = \frac{L_n}{S_n} = \frac{8.2}{7.7} = 1.065$$

$$t_{min} = \frac{8200 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times 1.065} = 197.872\ \text{mm} \quad > 90\ \text{mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 200 mm

b- Slab without drop panels (5.4 × 4.8) m clear span with **α_m= 0.18**

$$\alpha_m = 0.18 < 0.2$$

From table $t = \frac{L_n}{33} = \frac{5400}{33} = 163.636\ \text{mm} \qquad > 125\ \text{mm} \quad \text{O.K.}$

$\Rightarrow$ Use t = 170 mm

c- Flat plate (4.2 × 4.6) m clear span.

From table $t = \frac{L_n}{33} = \frac{4600}{33} = 139.394\ \text{mm} \qquad > 125\ \text{mm} \quad \text{O.K.}$

$\Rightarrow$ Use t = 140 mm

d- Flat slab with drop panels (6.0 × 6.2) m clear span.

From table $t = \frac{L_n}{36} = \frac{6200}{36} = 172.222\ \text{mm} \qquad > 100\ \text{mm} \quad \text{O.K.}$

$\Rightarrow$ Use t = 175 mm

e- Slab with beams (5.8 × 5.8) m clear span with **α_m= 1.5**

$$0.2 < \alpha_m = 1.5 < 2.0$$

$$\Rightarrow\ t_{min} = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_m - 0.2)}$$

$$\beta = \frac{L_n}{S_n} = \frac{5.8}{5.8} = 1.0$$

$$t_{min} = \frac{5800 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 5 \times 1.0 \times (1.5 - 0.2)} = 150.118\ \text{mm} \quad > 125\ \text{mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 160 mm

Example 5

Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use f_y = 420 MPa.

- **a-** Flat slab with drop panels (7.0 × 5.6) m clear span.
- **b-** Slab with beams (5.0 × 6.3) m clear span with **α_m= 2.3**
- **c-** Slab with beams (5.0 × 5.5) m clear span with **α_m= 1.7**
- **d-** Flat plate (4.2 × 4.5) m clear span.
- **e-** Flat slab without drop panels (5.9 × 4.2) m clear span.

Solution

a) Flat slab with drop panels (7.0 × 5.6) m clear span.

From table

$$t = \frac{\ell_n}{36} = \frac{7000}{36} = 194.444\ \text{mm} \qquad > 100\ \text{mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 200 mm

b) Slab with beams (5.0 × 6.3) m clear span with **α_m= 2.3**

$\alpha_m = 2.3 > 2.0$

$$\Rightarrow\ t_{min} = \frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta}$$

$$\beta = \frac{\ell_n}{s_n} = \frac{6.3}{5.0} = 1.26$$

$$t_{min} = \frac{6300 \times \left(0.8 + \frac{420}{1400}\right)}{36 + 9 \times 1.26} = 146.388\ \text{mm} \quad > 90\ \text{mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 150 mm

c) Slab with beams (5.0 × 5.5) m clear span with **α_m= 1.7**

$0.2 < \alpha_m = 1.7 < 2.0$

$$\Rightarrow\ t_{min} = \frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_m - 0.2)}$$

$$\beta = \frac{\ell_n}{s_n} = \frac{5.5}{5.0} = 1.10$$

$$t_{min} = \frac{5500\times\left(0.8 + \frac{420}{1400}\right)}{36 + 5\times1.1\times(1.7 - 0.2)} = 136.723\ \text{mm} \quad > 125\ \text{mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 140 mm

d) Flat plate (4.2 × 4.5) m clear span.

From table

$$t = \frac{\ell_n}{33} = \frac{4500}{33} = 136.364\ \text{mm} \qquad > 125\ \text{mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 140 mm

e) Flat slab without drop panels (5.9 × 4.2) m clear span.

From table

$$t = \frac{\ell_n}{33} = \frac{5900}{33} = 178.788\ \text{mm} \qquad > 125\ \text{mm} \quad \text{O.K.}$$

$\Rightarrow$ Use t = 180 mm

Example 6

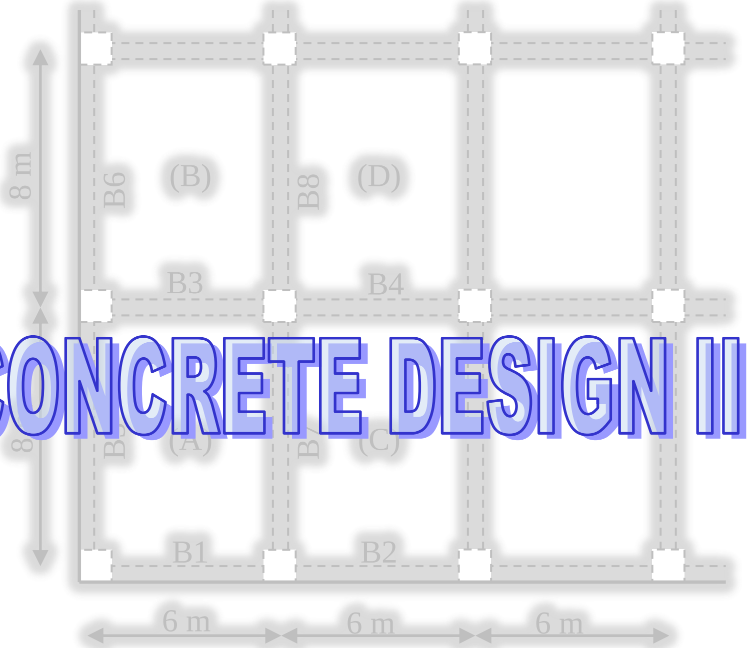
Find the minimum thickness of a slab for an interior panels due to deflection control for the following: Use **f_y = 420 MPa**. (60000 psi).

- **a)** Flat slab with drop panels (6.4 × 6.0) m clear span.
- **b)** Flat plate (4.4 × 4.0) m clear span.
- **c)** Slab with beams (5.8 × 5.6) m clear span with **α_m = 1.7**
- **d)** Slab with beams (8.0 × 6.5) m clear span with **α_m = 3.4**
- **e)** Slab without drop panels (5.5 × 4.8) m clear span with **α_m = 0.19**

University of Baghdad

College of Engineering

Department of Civil Engineering



REINFORCED CONCRETE DESIGN II

FOURTH YEAR CLASS

2

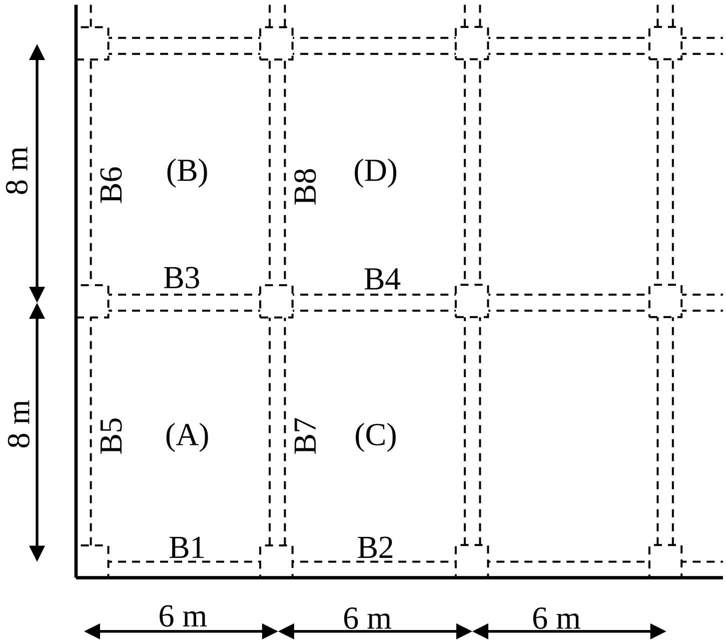
2018 - 2019

MHND

General Example 1

Slab with beams

- All interior beams are 300 × 600 mm
- B1 & B2 are 300 × 600 mm
- B5 & B6 are 300 × 700 mm
- All columns are 600 × 600 mm
- Slab thickness = 180 mm
- Live load = 4.25 kN/m^2
- $\gamma_{concrete}$ = 25 kN/m^3



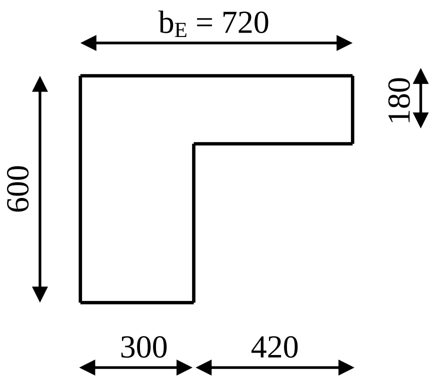
Solution

(1) Computing α_f

Compute the ratio of the flexural stiffness of the longitudinal beams to that of the slab (α_f) in the equivalent rigid frame, for all beams around panels A, B, C, and D.

Beam sections

B1 and B2



$$\left.\begin{aligned} 2 &< \frac{b_E}{b_w} = \frac{720}{300} = 2.40 < 4 \\ 0.2 &< \frac{h_f}{h} = \frac{180}{600} = 0.3 < 0.5 \end{aligned}\right\} \text{O.K.}$$

$$k = 1 + 0.2\,\frac{b_E}{b_w} = 1 + 0.2\,(2.4) = 1.48$$

$$I_b = k\,\frac{b_w h^3}{12} = 1.48\left(\frac{300\,(600)^3}{12}\right) = 7.992 \times 10^9 \text{mm}^4$$

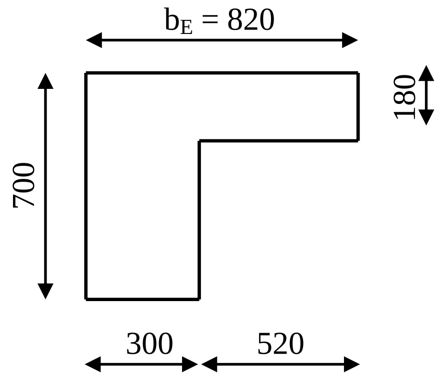
$$I_s = \frac{1}{12}\,b\,t^3 = \frac{1}{12} \times 4300(180)^3 = 2.090 \times 10^9 \text{mm}^4$$

$$b = \frac{8000}{2} + 300 = 4300 \text{ mm}$$

$$\alpha_{fB1} = \alpha_{fB2} = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{I_b}{I_s} = \frac{7.992 \times 10^9}{2.090 \times 10^9} = 3.823$$

Where $E_{cb} = E_{cs}$

<u>B5 and B6</u>



$$\left.\begin{array}{l} 2 < \dfrac{b_E}{b_w} = \dfrac{820}{300} = 2.73 < 4 \\ 0.2 < \dfrac{t}{h} = \dfrac{180}{700} = 0.26 < 0.5 \end{array}\right\} \text{O.K.}$$

$$k = 1 + 0.2\,\frac{b_E}{b_w} = 1 + 0.2\,(2.73) = 1.546$$

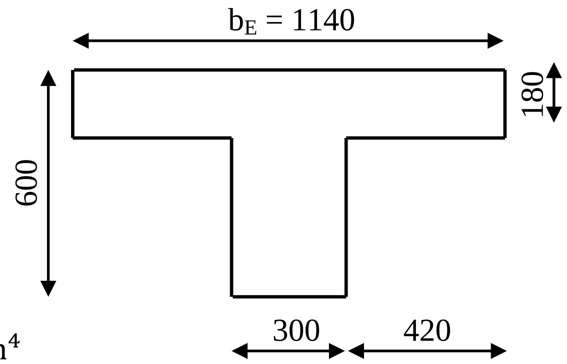
$$I_b = k\,\frac{b_w h^3}{12} = 1.546\left(\frac{300\,(700)^3}{12}\right) = 13.26 \times 10^9 \text{mm}^4$$

$$I_s = \frac{1}{12}\,b\,t^3 = \frac{1}{12} \times 3300(180)^3 = 1.604 \times 10^9 \text{mm}^4$$

$$b = \frac{6000}{2} + 300 = 3300 \text{ mm}$$

$$\alpha_{fB5} = \alpha_{fB6} = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{I_b}{I_s} = \frac{13.26 \times 10^9}{1.604 \times 10^9} = 8.267$$

<u>B3 and B4</u>



$$\left.\begin{array}{l} 2 < \dfrac{b_E}{b_w} = \dfrac{1140}{300} = 3.8 < 4 \\ 0.2 < \dfrac{t}{h} = \dfrac{180}{600} = 0.3 < 0.5 \end{array}\right\} \text{O.K.}$$

$$k = 1 + 0.2\,\frac{b_E}{b_w} = 1 + 0.2\,(3.8) = 1.76$$

$$I_b = k\,\frac{b_w h^3}{12} = 1.76\left(\frac{300\,(600)^3}{12}\right) = 9.504 \times 10^9 \text{mm}^4$$

$$I_s = \frac{1}{12}\,b\,t^3 = \frac{1}{12} \times 8000(180)^3 = 3.888 \times 10^9 \text{mm}^4$$

$$b = 8000 \text{ mm}$$

$$\alpha_{fB3} = \alpha_{fB4} = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{I_b}{I_s} = \frac{9.504 \times 10^9}{3.888 \times 10^9} = 2.444$$

<u>B7 and B8</u>

$I_b = 9.504 \times 10^9$ same as B_3 and B_4

$$I_s = \frac{1}{12}\,b\,t^3 = \frac{1}{12} \times 6000 \times (180)^3 = 2.916 \times 10^9 \text{mm}^4$$

b = 6000 mm

$$\alpha_{fB7} = \alpha_{fB8} = \frac{E_{cb} I_b}{E_{cs} I_s} = \frac{I_b}{I_s} = \frac{9.504 \times 10^9}{2.916 \times 10^9} = 3.259$$

<u>Note</u>: for slab without beams, α_f = zero.

To use the DDM, first checking the seven limitations

Limitations 1 to 5 are satisfied by inspections.

Limitation 6:- L.L. shall not exceed 2 times D.L.

D.L. of the slab = 0.18 × 25 = 4.50 kN/m^2

D.L. of tiles = 0.10 × 20 = 2.00 kN/m^2

D.L. of partition = 1.00 kN/m^2

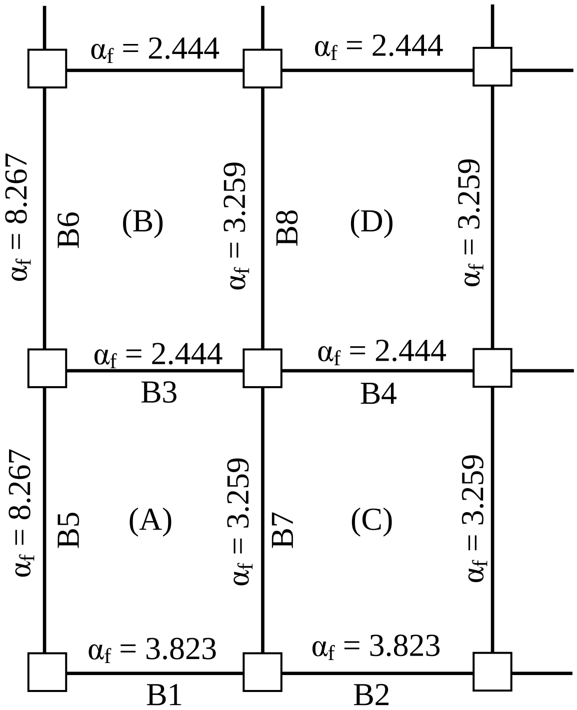
D.L. of fall ceiling = 0.08 kN/m^2

7.58 kN/m^2

$$\frac{\mathrm{L.L.}}{\mathrm{D.L.}} = \frac{4.25}{7.58} = 0.56 \ < \ 2.0 \qquad \mathrm{O.K.}$$

Limitation 7:- For each panel

$$0.2 \ \le \ \frac{\alpha_{f1}\ell_2{}^2}{\alpha_{f2}\ell_1{}^2} \ \le \ 5.0$$



<u>Panel A</u>

$$\frac{\alpha_{f1}\ell_2{}^2}{\alpha_{f2}\ell_1{}^2} = \frac{\frac{1}{2}(\alpha_{fB1} + \alpha_{fB3}) \times (8000)^2}{\frac{1}{2}(\alpha_{fB5} + \alpha_{fB7}) \times (6000)^2} = \frac{\frac{1}{2}(3.823 + 2.444) \times (8000)^2}{\frac{1}{2}(8.267 + 3.259) \times (6000)^2} = 0.97$$

$0.2 \ < 0.97 < 5.0 \qquad \mathrm{O.K.}$

<u>Panel B</u>

$$\frac{\alpha_{f1}\ell_2{}^2}{\alpha_{f2}\ell_1{}^2} = \frac{\frac{1}{2}(\alpha_{fB3} + \alpha_{fB3}) \times (8000)^2}{\frac{1}{2}(\alpha_{fB6} + \alpha_{fB8}) \times (6000)^2} = \frac{\frac{1}{2}(2.444 + 2.444) \times (8000)^2}{\frac{1}{2}(8.267 + 3.259) \times (6000)^2} = 0.754$$

$0.2 \ < 0.754 < 5.0 \qquad \mathrm{O.K.}$

Panel C

$$\frac{\alpha_{f1}\ell_2{}^2}{\alpha_{f2}\ell_1{}^2} = \frac{\frac{1}{2}(\alpha_{fB2} + \alpha_{fB4}) \times (8000)^2}{\frac{1}{2}(\alpha_{fB7} + \alpha_{fB7}) \times (6000)^2} = \frac{\frac{1}{2}(3.823 + 2.444) \times (8000)^2}{\frac{1}{2}(3.259 + 3.259) \times (6000)^2} = 1.71$$

$0.2 < 1.71 < 5.0 \qquad \text{O.K.}$

Panel D

$$\frac{\alpha_{f1}\ell_2{}^2}{\alpha_{f2}\ell_1{}^2} = \frac{\frac{1}{2}(\alpha_{fB4} + \alpha_{fB4}) \times (8000)^2}{\frac{1}{2}(\alpha_{fB8} + \alpha_{fB8}) \times (6000)^2} = \frac{\frac{1}{2}(2.444 + 2.444) \times (8000)^2}{\frac{1}{2}(3.259 + 3.259) \times (6000)^2} = 1.333$$

$0.2 < 1.333 < 5.0 \qquad \text{O.K.}$

Computing α_{fm}

Panel A

$$\alpha_{fmA} = \frac{1}{4}(\alpha_{fB1} + \alpha_{fB3} + \alpha_{fB5} + \alpha_{fB7}) = \frac{1}{4}(3.823 + 2.444 + 8.267 + 3.259) = 4.448$$

α_{fmB} = 4.104

α_{fmC} = 3.196

α_{fmD} = 2.852

Computing or checking slab thickness

Panel A

ℓ_n = 8000 – 600 = 7400 mm ; S_n = 6000 – 600 = 5400 mm

$$\beta = \frac{\ell_n}{S_n} = \frac{7400}{5400} = 1.37$$

α_{mA} = 4.448 ; here α_m > 2.0 ; use Eq. (2)

$$t = \frac{\ell_n\ (0.8 + \frac{f_y}{1400})}{36 + 9\beta} = \frac{7400 \times\ (0.8 + \frac{350}{1400})}{36 + 9 \times 1.37} = 158.2\ \text{mm} \qquad \text{say } 160\ \text{mm} > 90\ \text{mm}$$

Edge beam (B1 and B5) have α > 0.8 ∴ t = 160 mm

Summary of required slab thickness

A	B	C	D
160	160	160	160

t = 160 mm > 90 mm ∴ O.K. t_{min} = 160 mm

t_{actual} = 180 mm > 160 mm ∴ O.K.

Computing C

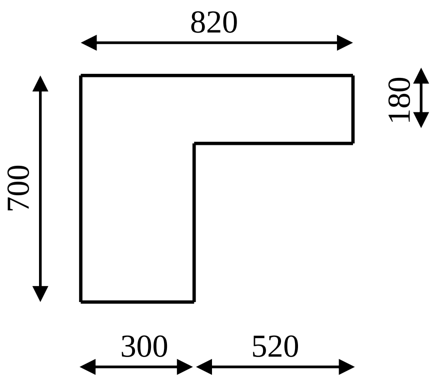
For B5 and B6

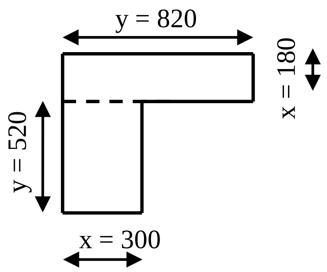
$$C = \sum \left(1 - 0.63\ \frac{x}{y}\right)\frac{x^3\ y}{3}$$

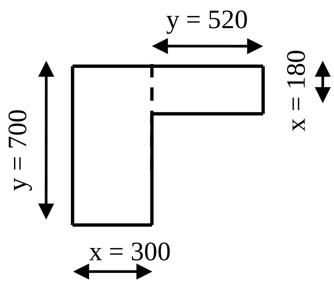
$$C_1 = \left(1 - 0.63\ \times\ \frac{180}{820}\right)\frac{(180)^3 \times 820}{3} + \left(1 - 0.63\ \times\ \frac{300}{520}\right)\frac{(300)^3 \times 520}{3}$$
$$= 4.353 \times 10^9\ \ \mathrm{mm}^4$$

$$C_2 = \left(1 - 0.63\ \times\ \frac{300}{700}\right)\frac{(300)^3 \times 700}{3} + \left(1 - 0.63\ \times\ \frac{180}{520}\right)\frac{(180)^3 \times 520}{3}$$
$$= 5.191 \times 10^9\ \ \mathrm{mm}^4$$

∴ For beam B5 and B6 C = 5.191× 10^9 mm^4





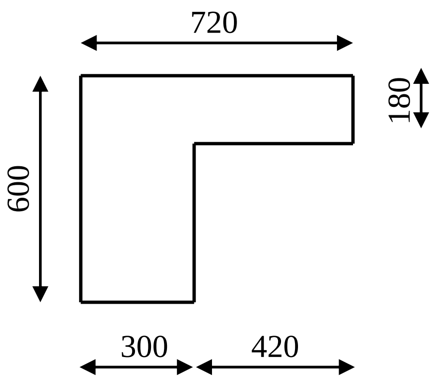


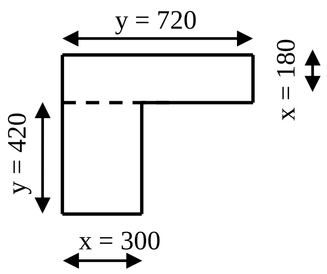
B1 and B2

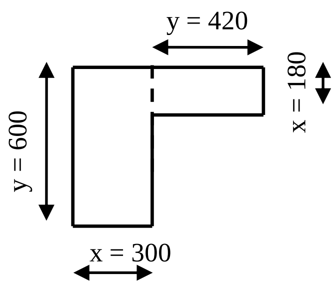
$$C_1 = \left(1 - 0.63\ \times\ \frac{180}{720}\right)\frac{(180)^3 \times 720}{3} + \left(1 - 0.63\ \times\ \frac{300}{420}\right)\frac{(300)^3 \times 420}{3}$$
$$= 3.258 \times 10^9\ \ \mathrm{mm}^4$$

$$C_2 = \left(1 - 0.63\ \times\ \frac{180}{420}\right)\frac{(180)^3 \times 420}{3} + \left(1 - 0.63\ \times\ \frac{300}{600}\right)\frac{(300)^3 \times 600}{3}$$
$$= 4.295 \times 10^9\ \ \mathrm{mm}^4$$

∴ For beam B1 and B2   C = 4.295 × 10^9 mm^4







Computing β_t

$$\beta_t = \frac{E_{cb}C}{2\,E_{cs}I_s} = \frac{C}{2\,I_s} \qquad ; \qquad E_{cb} = E_{cs}$$

For B5 and B6

$$I_s = \frac{1}{12}\ell_2 t^3 = \frac{1}{12} \times 8000 \times (180)^3 = 3.888 \times 10^9 \text{ mm}^4$$

$$\beta_t = \frac{C}{2\,I_s} = \frac{5.191 \times 10^9}{2 \times 3.888 \times 10^9} = 0.693$$

For beam B1 and B2

$$I_s = \frac{1}{12} \times 6000 \times (180)^3 = 2.916 \times 10^9 \text{ mm}^4$$

$$\beta_t = \frac{4.295 \times 10^9}{2 \times 2.916 \times 10^9} = 0.736$$

<u>Exterior longitudinal frame</u>

D.L. = 4.5 (slab) + 2.0 (tiles) + 1.0 (partition) + 0.08 (fall ceiling) = 7.58 kN/m^2

L.L. = 4.25 kN/m^2

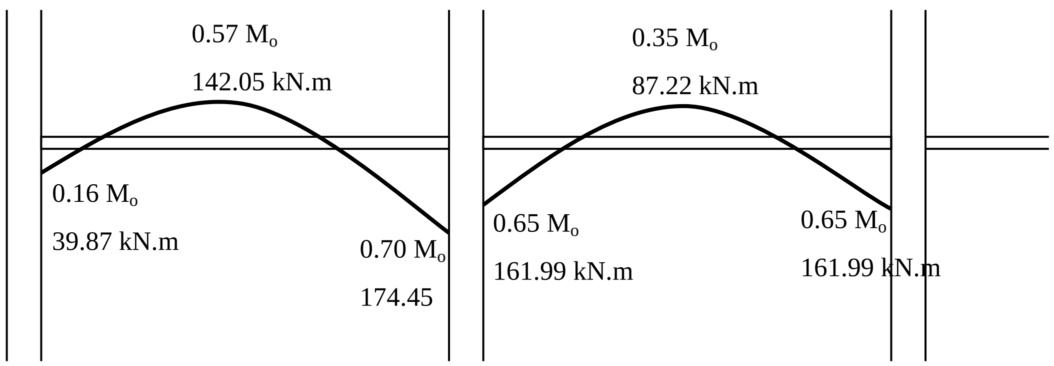
q_u = 1.2×7.58 + 1.6× 4.25 = 15.9 kN/m^2

$$\ell_2 = \frac{8000}{2} + \frac{600}{2} = 4300 \text{ mm}$$

$$\ell_n = 6000 - 600 = 5400 \text{ mm}$$

$$M_o = \frac{1}{8} q_u \ell_2 \ell_n^2 = \frac{1}{8} \times 15.9 \times 4.3 \times (5.4)^2 = 249.21 \text{ kN.m}$$

<u>Longitudinal distribution of moments</u>:



Transverse distribution of longitudinal moments

End span

Negative moment at exterior support (total = -0.16 M_o= -39.87 kN.m)

need $\frac{\alpha_{f1}\ell_2}{\ell_1}$, β_t , and $\frac{\ell_2}{\ell_1}$

Here $\alpha_{f1} = \alpha_{fB1} = 3.823$, $\ell_2 = 8000$ mm, $\ell_1 = 6000$ mm

$$\frac{\ell_2}{\ell_1} = \frac{8000}{6000} = 1.333 \quad \& \quad \frac{\alpha_1\ell_2}{\ell_1} = \frac{3.823 \times 8000}{6000} = 5.10 > 1.0$$

$\beta_t = \beta_{tB5} = 0.693 \approx 0.69$

ℓ_2/ℓ_1		1.0	1.333	2.00
$\frac{\alpha_{f1}\ell_2}{\ell_1} > 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t = 0.69$		**90.34**	
	$\beta_t \geq 2.5$	75	65	45

$$\frac{y}{0.667} = \frac{30}{1} \rightarrow \quad y = 20$$

∴ Neg. moment in column strip = 39.87 × 0.903 = 36.02 kN.m

Neg. moment in beam = 36.02 × 0.85 = 30.62 kN.m

Neg. moment in column strip slab = 36.02 – 30.62 = 5.4 kN.m

Neg. moment in middle strip = 39.87 - 36.02 = 3.85 kN.m

Positive moments (total = 0.57 M_o = 142.05 kN.m)

ℓ_2/ℓ_1	1.0	1.333	2.0
$\frac{\alpha_{f1}\ell_2}{\ell_1} > 1.0$	75	**65**	45

Moment in column strip = 142.05 × 0.65 = 92.33 kN.m

Moment in beam = 92.33 × 0.85 = 78.48 kN.m

Moment in column strip slab = 92.33 – 78.48 = 13.85 kN.m

Moment in middle strip = 142.05 – 92.33 = 49.72 kN.m

Interior negative moment (total = 0.70 M_o = -174.45 kN.m)

ℓ_2/ℓ_1	1.0	1.333	2.0
$\frac{\alpha_{f1}\ell_2}{\ell_1} > 1.0$	75	65	45

Moment in column strip = 174.45 × 0.65 = -113.39 kN.m
Moment in beam = 113.39 × 0.85 = -96.38 kN.m
Moment in column strip slab = 113.39 – 96.38 = -17.01 kN.m
Moment in middle strip = 174.45 – 113.39 = -61.06 kN.m

Interior span
Negative moment (total = -0.65 M_o = -161.99 kN.m)
Negative moment in column Strip = 161.99 × 0.65 = 105.29 kN.m
Negative moment in beam = 105.29 × 0.85 = 89.50 kN.m
Negative moment in column strip slab = 105.29 – 89.5 = 15.79 kN.m
Negative moment in middle strip = 161.99 – 105.29 = 56.7 kN.m

Positive moment (total = 0.35 M_o = 87.22 kN.m)
Moment in column strip = 87.22 × 0.65 = 56.69 kN.m
Moment in beam = 56.69 × 0.85 = 48.19 kN.m
Moment in column strip slab = 56.69 – 48.19 = 8.5 kN.m
Moment in middle strip = 87.22 – 56.69 = 30.53 kN.m

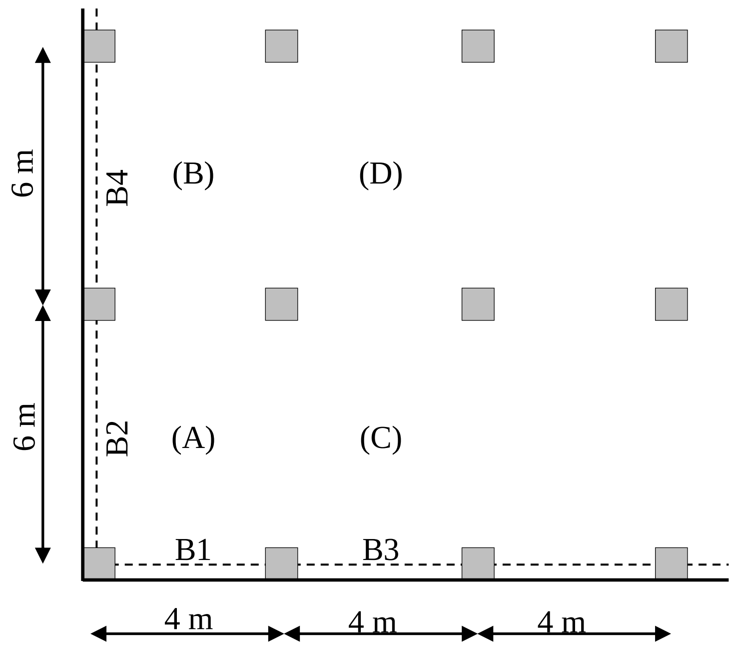
Moments in Exterior longitudinal frame
Total width = 4.3 m, column strip width = 1.8 m, & half middle strip width = 2.5 m.

	Exterior span			Interior span	
	Exterior negative	Positive	Interior negative	Negative	Positive
Total moment (kN.m)	-39.87	+142.05	-174.45	-161.99	+87.22
Moment in beam (kN.m)	-30.62	+78.48	-96.38	-89.50	+48.19
Moment in column strip slab (kN.m)	-5.4	+13.85	-17.01	-15.79	+8.50
Moment in middle strip slab (kN.m)	-3.85	+49.72	-61.06	-56.70	+30.53

General Example 2

Flat plate with edge beams

- Edge beams are 250 × 500 mm
- All columns are 500 × 500 mm
- Slab thickness = 200 mm
- Live load = 4.0 kN/m^2
- $\gamma_{concrete}$ = 24 kN/m^3



Solution

(1) Computing α_f

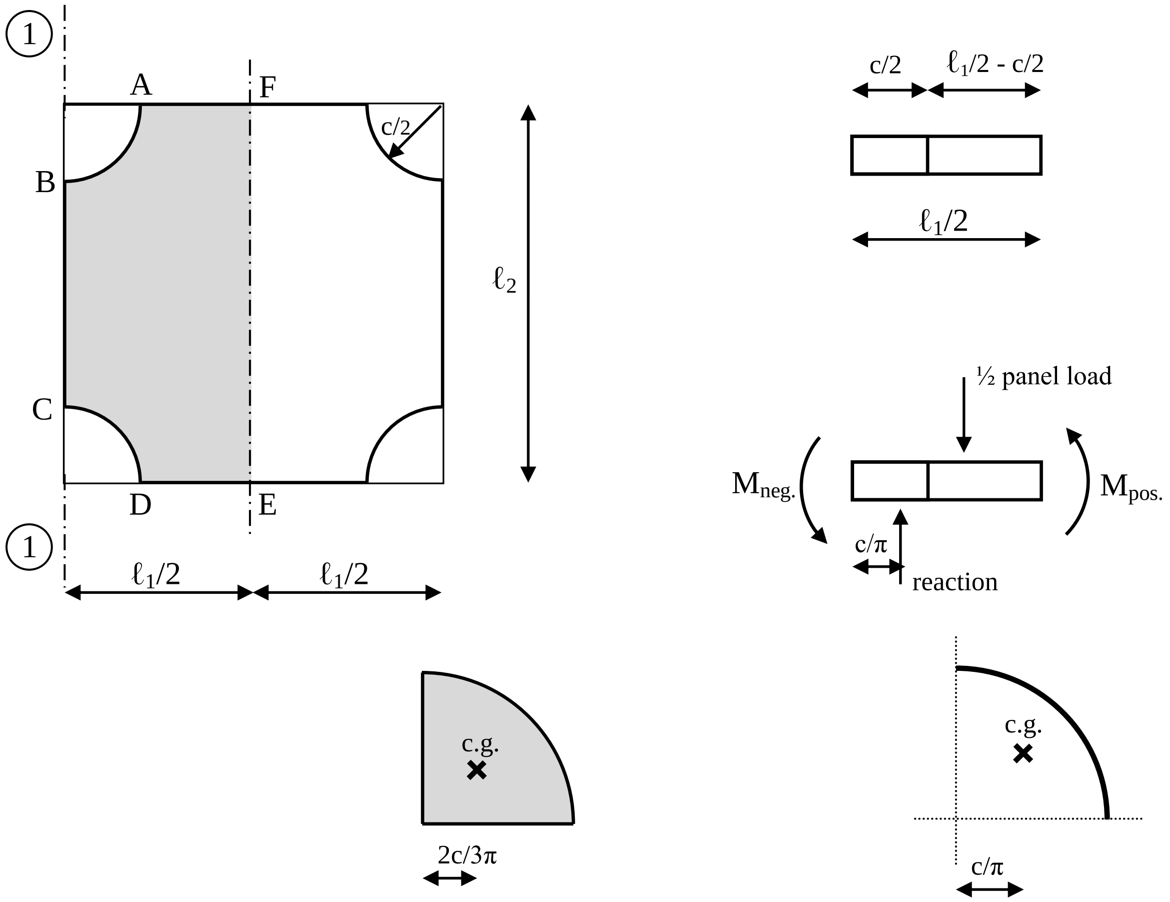
Compute the ratio of the flexural stiffness of the longitudinal beams to that of the slab (α_f) in the equivalent rigid frame, for all edge beams.

Beam sections

B1 and B3

Total static moment in flat slab

c = diameter of column capital



Sum of reactions on arcs AB and CD = load on area ABCDEF

$$= q_u \left\{ \ell_2 \frac{\ell_1}{2} - 2\left(\frac{1}{4}\pi\left(\frac{c}{2}\right)^2\right)\right\}$$

$$= q_u \left\{ \frac{\ell_2\,\ell_1}{2} - \frac{\pi\,c^2}{8}\right\}$$

No shear along lines AF, BC, DE, EF

$\sum M_{1\text{-}1} = 0$

$$M_{neg.} + M_{pos.} + q_u \left\{ \frac{\ell_2\,\ell_1}{2} - \frac{\pi\,c^2}{8}\right\}\frac{c}{\pi} - \frac{q_u\,\ell_2\,\ell_1}{2}\left(\frac{\ell_1}{4}\right) + q_u \times 2\left(\frac{1}{4}\frac{\pi\,c^2}{4} \times \frac{2\,c}{3\,\pi}\right) = 0$$

previously $\qquad M_o = \dfrac{q_u\,\ell_2\,\ell_n^{\,2}}{8} \qquad \ldots\ldots(1)$

Letting $M_o = M_{neg.} + M_{pos.}$

$$M_o = \frac{q_u\,\ell_2\,\ell_1^{\,2}}{8}\left(1 - \frac{4\,c}{\pi\,\ell_1} + \frac{c^3}{3\,\ell_2\,\ell_1^{\,2}}\right)$$

$$M_o \approx \frac{q_u\,\ell_2\,\ell_1^{\,2}}{8}\left(1 - \frac{2\,c}{3\,\ell_1}\right)^2 \qquad \ldots\ldots(2)$$

Eq. (1) is useful for flat plate floor or two – way slab with beams, while Eq. (2) is more suitable for flat slab, where in round column capitals are used.

Example:

Compute the total factored static moment in the long and short directions for an interior panel in flat slab 6 × 7 m, given $q_u = 15$ kN/m^2, column capital = 1.40 m.

Solution:-

a- In long direction

$$M_o = \frac{q_u\, \ell_2\, {\ell_1}^2}{8}\left(1 - \frac{2\,c}{3\,\ell_1}\right)^2 = \frac{15\ \times 6\ \times\ (7)^2}{8}\left(1 - \frac{2\ \times 1.4}{3\ \times 7}\right)^2 = 414\ \ \mathrm{kN.m}$$

b- In short direction

$$M_o = \frac{15\ \times 7\ \times\ (6)^2}{8}\left(1 - \frac{2\ \times 1.4}{3\ \times 6}\right)^2 = 337\ \ \mathrm{kN.m}$$

To compare with previous method:-

a- In long direction

$\ell_n = 7.0 - 0.89 \times 1.4 = 5.754$ m

$$M_o = \frac{q_u\, \ell_2\, {\ell_n}^2}{8} = \frac{15\ \times 6\ \times\ (5.754)^2}{8} = 372.4\ \ \mathrm{kN.m}$$

b- In short direction

$\ell_n = 6.0 - 0.89 \times 1.4 = 4.754$ m

$$M_o = \frac{15\ \times 7\ \times\ (4.754)^2}{8} = 296.6\ \ \mathrm{kN.m}$$

	Eq. 1 (kN.m)	Eq. 2 (kN.m)	Error (%)
long direction	414	372.4	10
short direction	337	296.4	12

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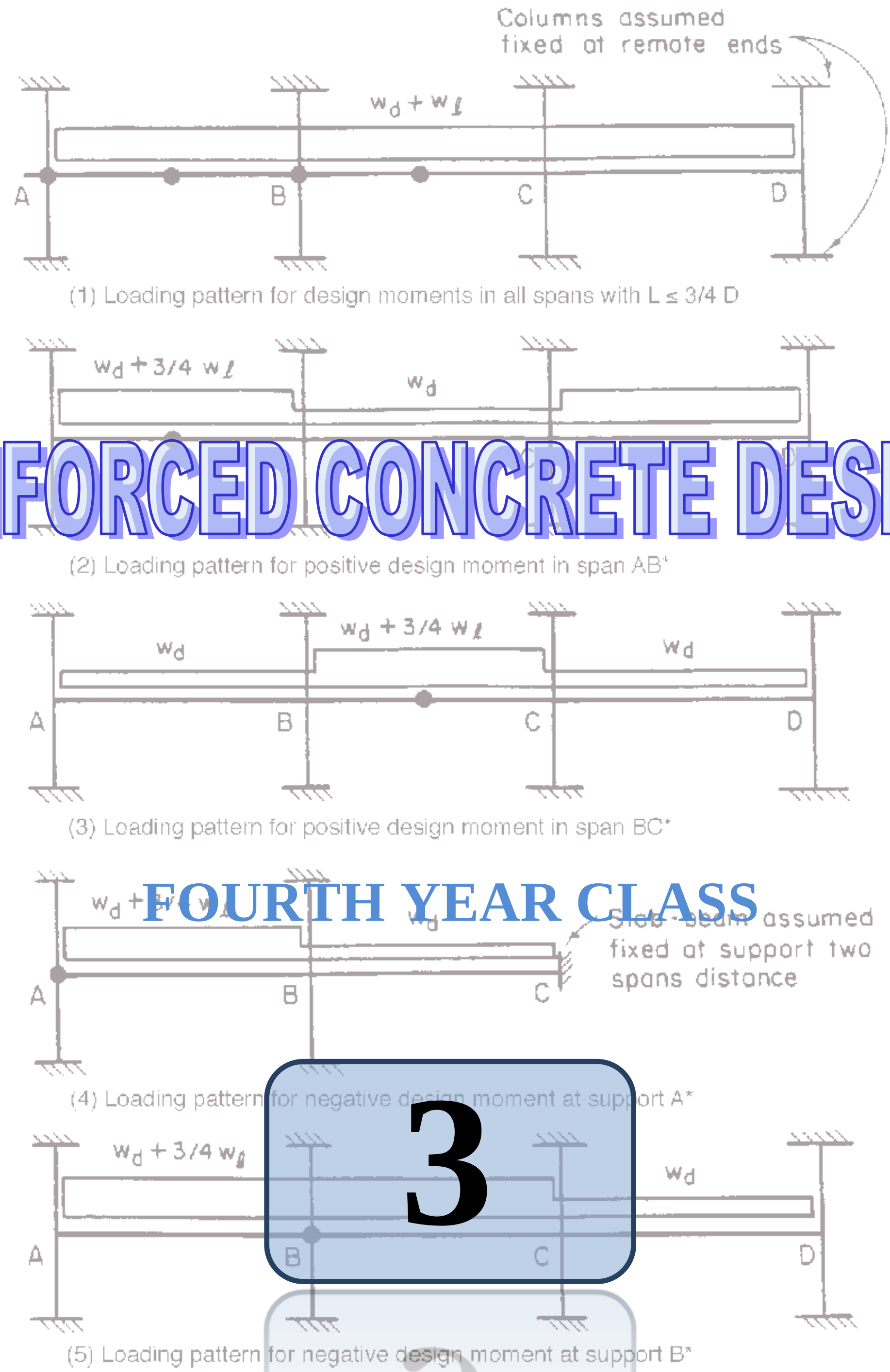
REINFORCED CONCRETE DESIGN II



FOURTH YEAR CLASS

3

2018 - 2019

MHND

Equivalent frame method (EFM)

The equivalent frame method involves the representation of the three-dimensional slab system by a series of two-dimensional frames that are then analyzed for loads acting in the plane of the frames. The negative and positive moments so determined at the critical design sections of the frame are distributed to the slab sections (column strip, middle strip and beam).

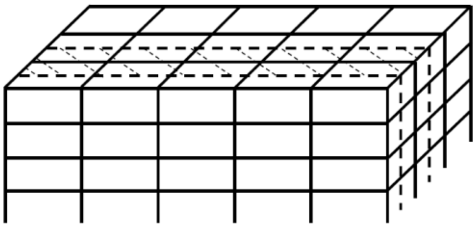
Limitations:

1) Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2.
2) Live load shall be arranged in accordance with arrangement of live loads.
3) Complete analysis must include representative interior and exterior equivalent frames in both the longitudinal and transverse directions of the floor.

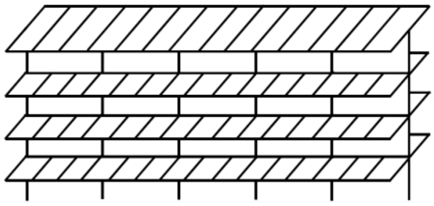
Procedure:-

1- Divide the structure into longitudinal and transverse frames centered on column and bounded by panels.
2- Each frame shall consist of a row of columns and slab-beam strips, bounded laterally by of panels.
3- Columns shall be assumed to be attached to slab-beam strips by torsional members transverse to the direction of the span for which moment are being determined.
4- Frames adjacent and parallel to an edge shall be bounded by that edge and the centerline of adjacent panel.
5- The slab–beam may be assumed to be fixed at any support two panels distance from the support of the span where critical moments are being obtained, provided the slab is continuous beyond that point.

Selected frame in 3-D building

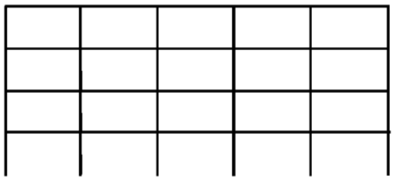


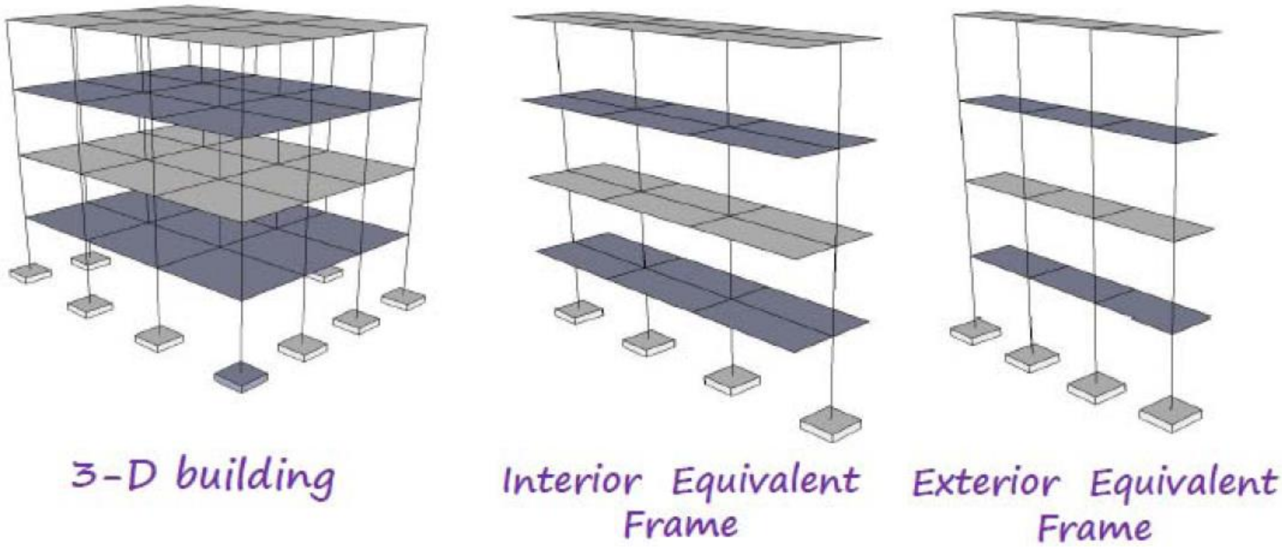
The detached frame alone



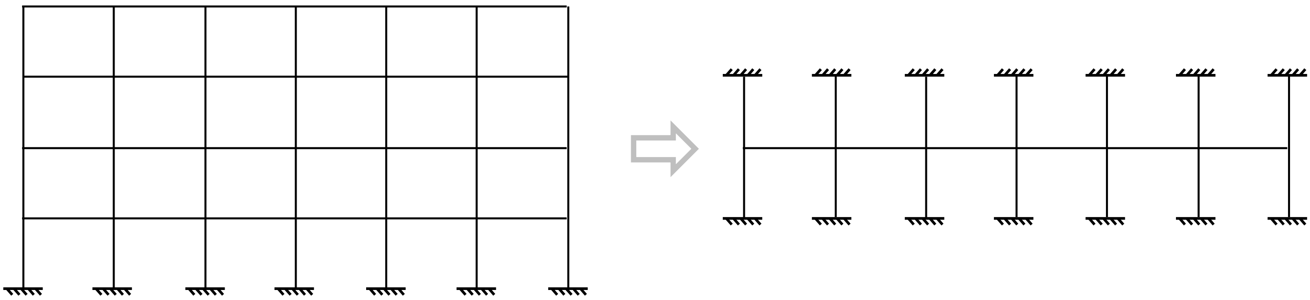
The width of the frame is same as mentioned in DDM. The length of the frame extends up to full length of 3-D system and the frame extends the full height of the building.

2-D frame

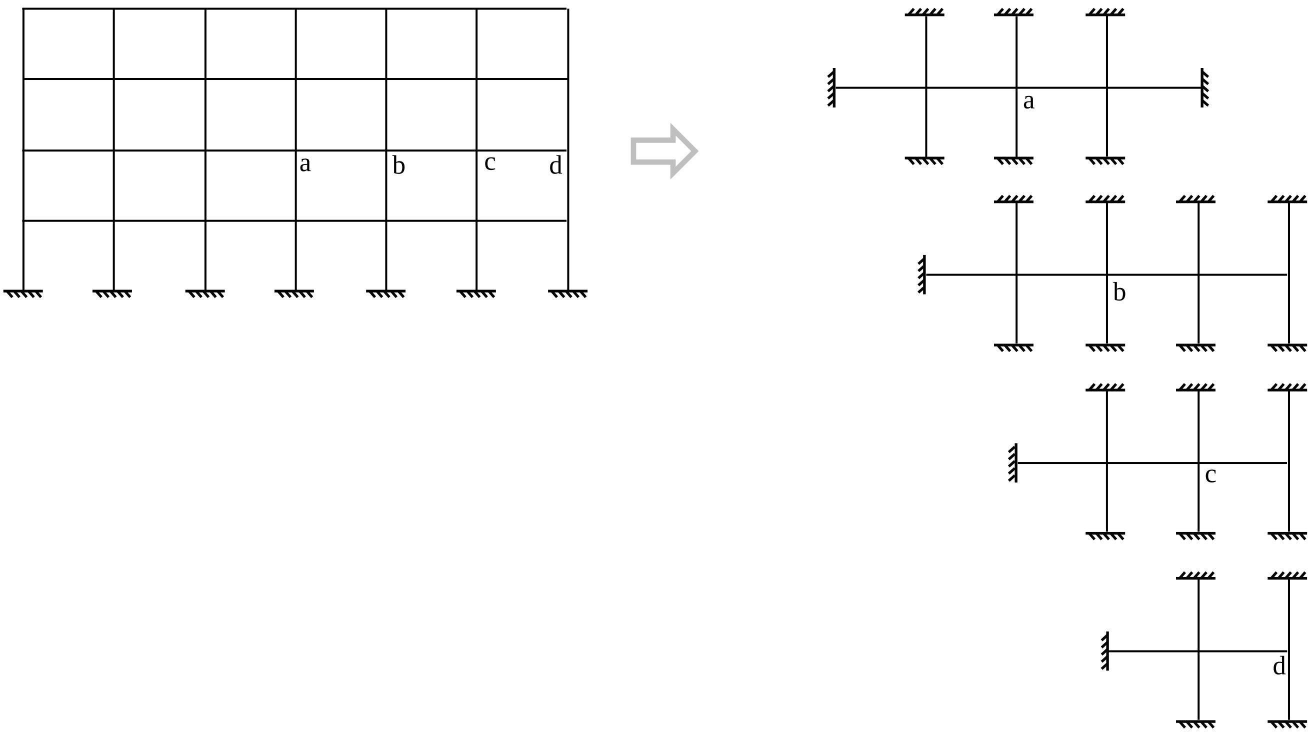




Analysis of each equivalent frame in its entirety shall be permitted. Alternatively, for gravity loading, a separate analysis of each floor or roof with the far ends of columns considered fixed is permitted.

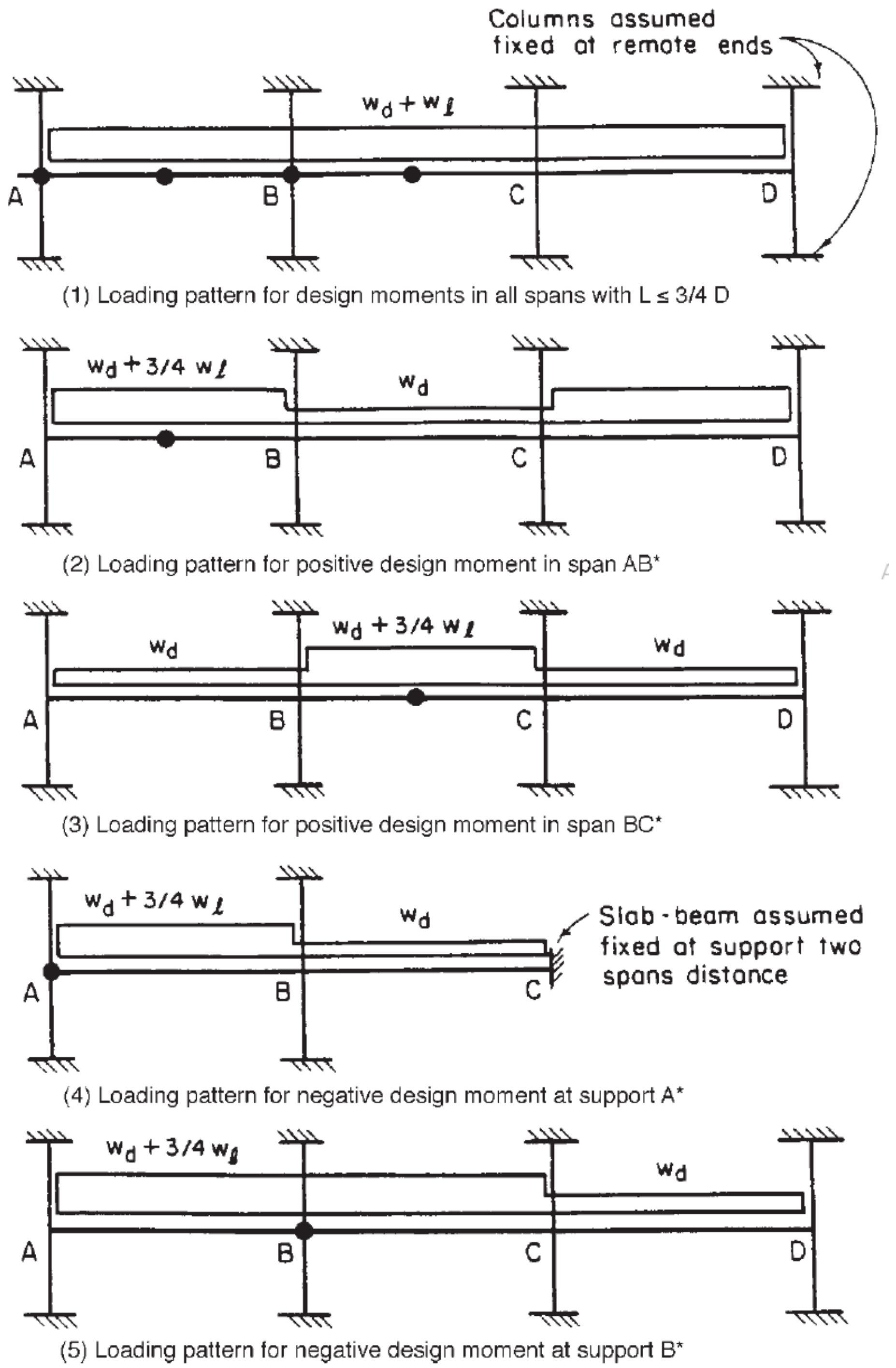


If slab-beams are analyzed separately, it shall be permitted to calculate the moment at a given support by assuming that the slab-beam is fixed at supports two or more panels away, provided the slab continues beyond the assumed fixed supports.



Arrangement of live loads:

1- If the arrangement of L is known, the slab system shall be analyzed for that arrangement.
2- If all panels will be loaded with L, the slab system shall be analyzed when full factored L on all spans.
3- If the arrangement of L is unknown:
   a- $L \leq 0.75\ D \Rightarrow$ Maximum factored moment when full factored L on all spans.
   b- $L > 0.75\ D \Rightarrow$ Pattern live loading using 0.75(factored L) to determine maximum factored moment.



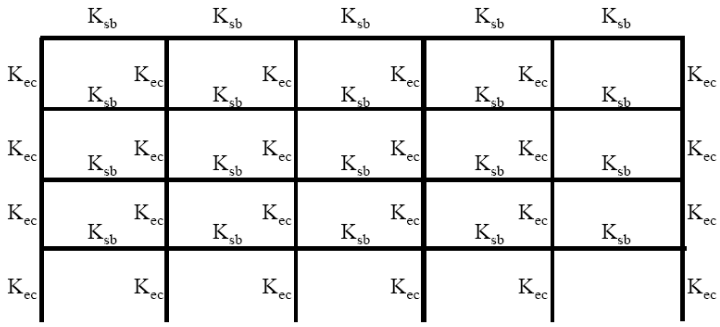
Partial frame analysis for vertical loading

Stiffness calculation:

- Stiffness of Slab-Beam Member
- Stiffness of Equivalent Column

    Stiffness of Column

    Stiffness of Torsional Member



K_{sb} represents the combined stiffness of slab and longitudinal beam (if any).

K_{ec} represents the modified column stiffness. The modification depends on lateral members (slab, beams etc.) and presence of column in the story above.

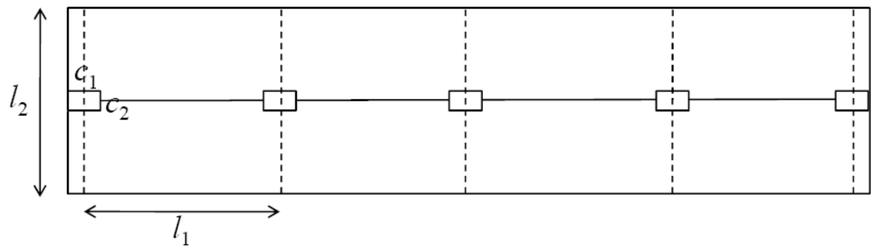
Once a 2-D frame is obtained, the analysis can be done by any method of 2-D frame analysis.

Stiffness of slab beam member (K_{sb}):

The stiffness of slab beam ($K_{sb}= kEI_{sb}/\ell$) consists of combined stiffness of slab and any longitudinal beam present within.

For a span, the k factor is a direct function of ratios c_1/ℓ_1 and c_2/ℓ_2.

Tables are available in literature for determination of k for various conditions of slab systems.

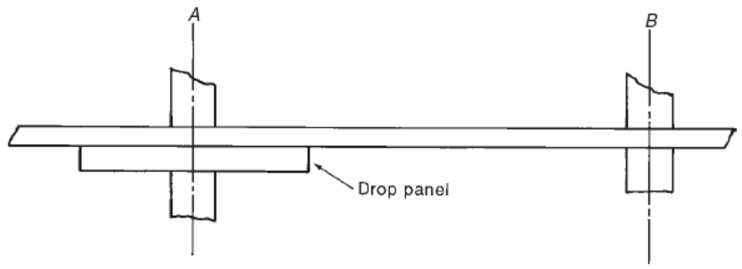


In the moment-distribution method, it is necessary to compute *flexural stiffnesses, K; carryover factors, COF; distribution factors, DF*; and *fixed-end moments, FEM,* for each of the members in the structure. For a prismatic member fixed at the far end and with negligible axial loads, the flexural stiffness is:

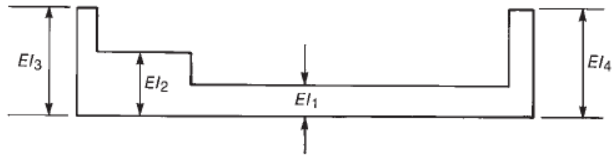
$$K = k\,\frac{E\,I}{l}$$

where k = 4 and the carryover factor is 0.5, the sign depending on the sign convention used for moments. For a prismatic, uniformly loaded beam, the fixed-end moments are $w\ell^2/12$.

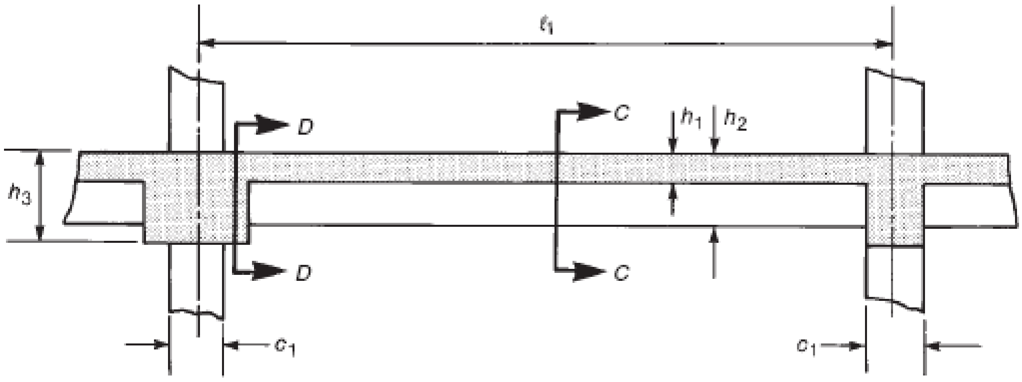
In the equivalent-frame method, the increased stiffness of members within the column–slab joint region is accounted for, as is the variation in cross section at drop panels. As a result, all members have a stiffer section at each end, as shown in Figure. If the *EI* used is that at the midspan of the slab strip, *k* will be greater than 4; similarly, the carryover factor will be greater than 0.5, and the fixed-end moments for a uniform load (w) will be greater than $w\ell^2/12$.



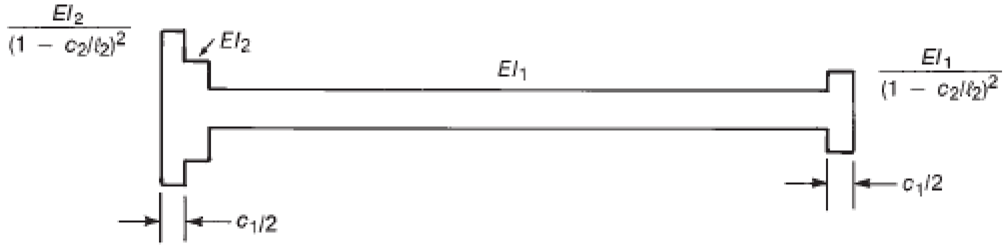
(a) Slab *A–B*.



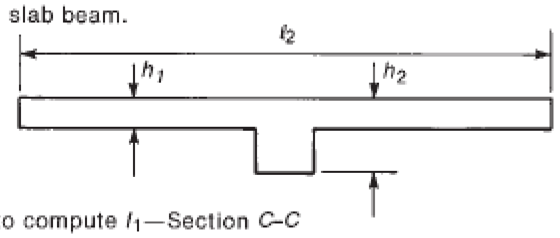
(b) Distribution of *EI* along slab.



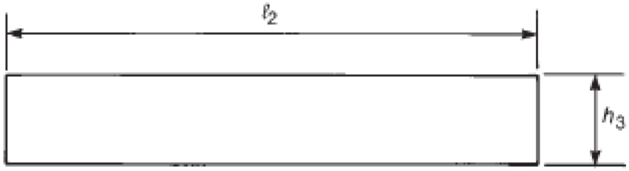
(a) Slab with beams in two directions.



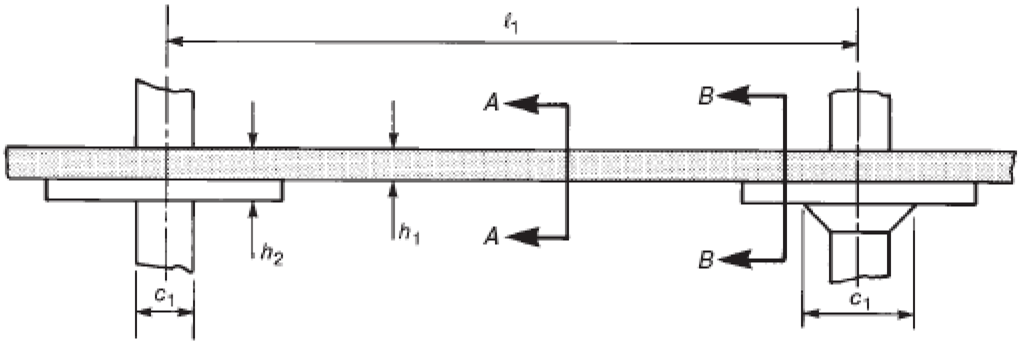
(b) Variation in *EI* along slab beam.



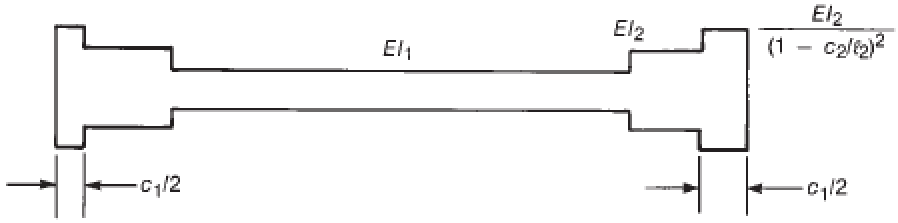
(c) Cross section used to compute I_1—Section *C–C*



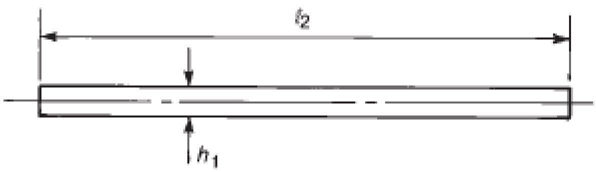
(d) Cross section used to compute I_2—Section *D–D*



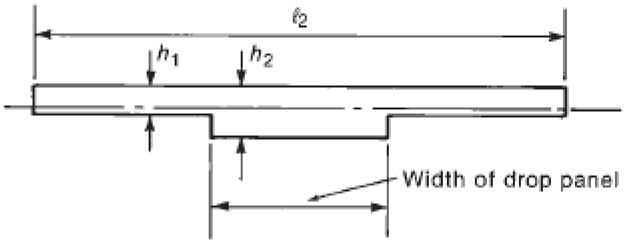
(a) Slab with drop panels.



(b) Variation in EI along slab-beam.



(c) Cross section used in compute I_1—Section A–A.



(d) Cross section used to compute I_2—Section B-B.

Several methods are available for computing values of *k*, *COF*, and *FEM*. Originally; these were computed by using the column analogy.

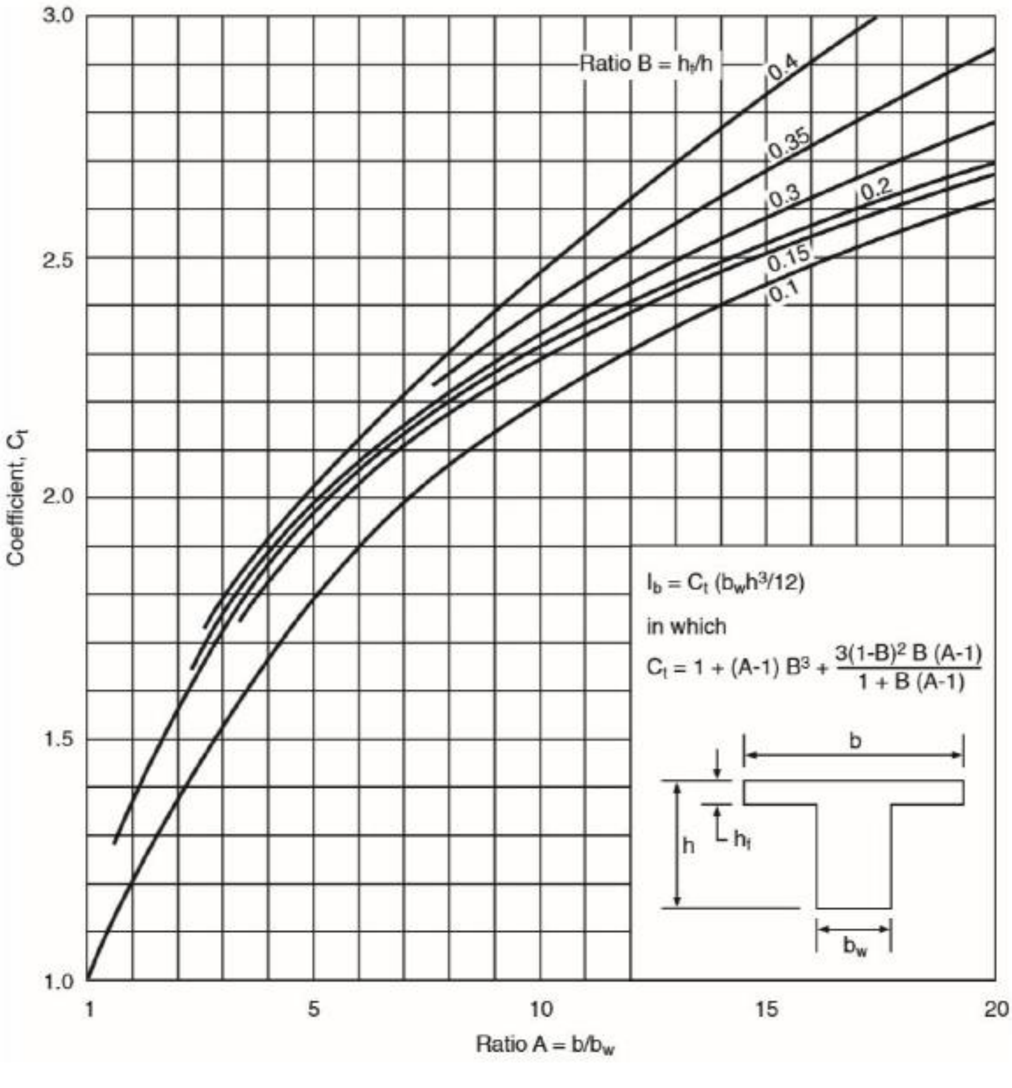
Properties of Slab–Beams

The horizontal members in the equivalent frame are referred to as *slab-beams.* These consist of either only a slab, or a slab and a drop panel, or a slab with a beam running parallel to the equivalent frame.

It shall be permitted to use the gross cross-sectional area of concrete to determine the moment of inertia of slab-beams at any cross section outside of joints or column capitals.

The moment of inertia of the slab-beams from the center of the column to the face of the column, bracket, or capital shall be taken as the moment of inertia of the slab-beam at the face of the column, bracket, or capital divided by the quantity $(1 - c_2/\ \ell_2)^2$, where ℓ_2 is the transverse width of the equivalent frame and c_2 is the width of the support parallel to ℓ_2.

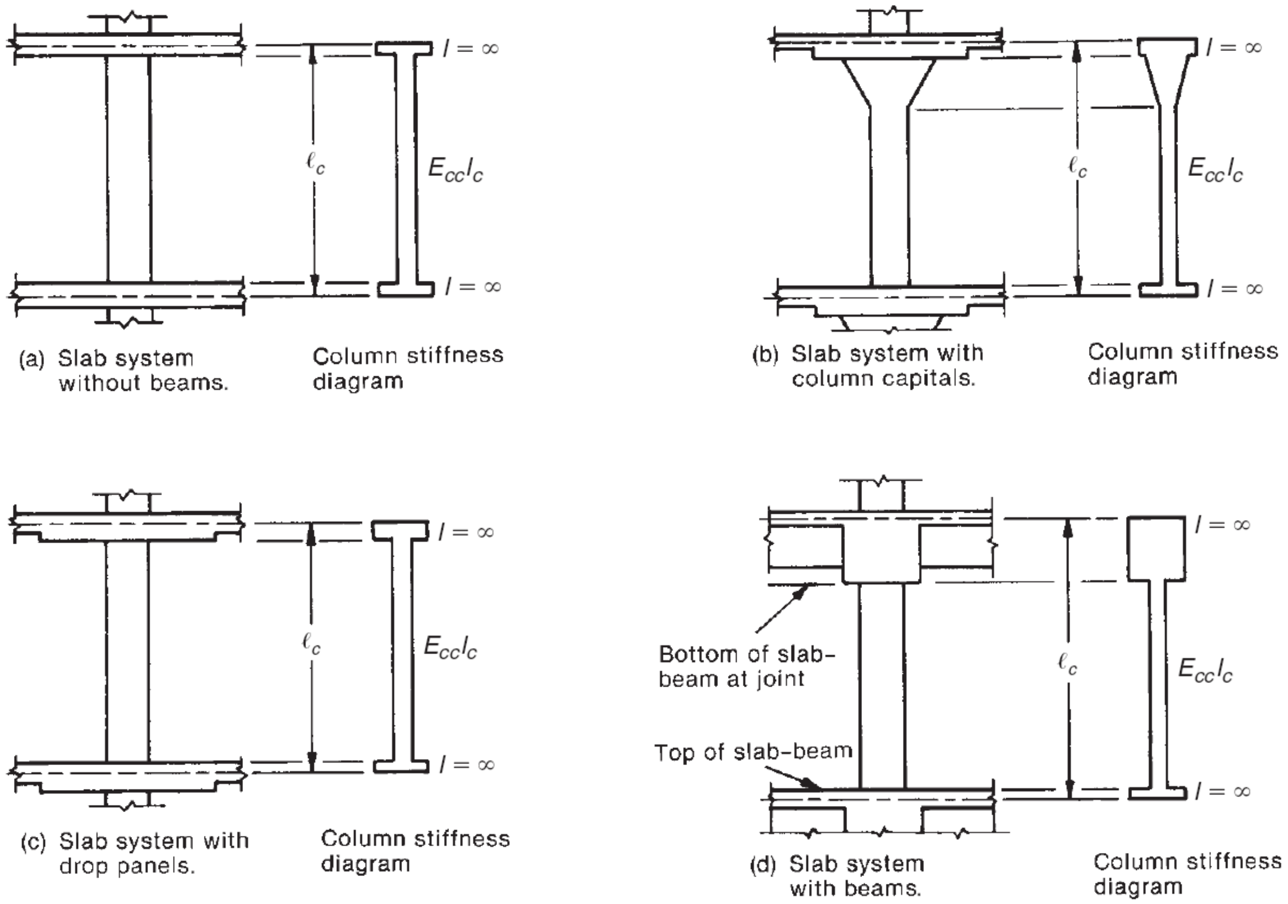
Moment of inertia of the slab-beam strip can be calculated from the following figure or equation:



Properties of Columns

The moment of inertia of columns at any cross section outside of the joints or column capitals may be based on the gross area of the concrete.

The moment of inertia of columns shall be assumed to be infinite within the depth of the slab-beam at a joint.



Sections for the calculations of column stiffness (K_c)

ℓ_c is the overall height and ℓ_u is the unsupported or clear height.

$$K_t = \sum \frac{9\ E_{cs}\ C}{\ell_2 \left(1 - \frac{c_2}{\ell_2}\right)^3}$$

where ℓ_2 refers to the transverse spans on each side of the column. For a corner column, there is only one term in the summation.

If a beam parallel to the ℓ_1 direction, multiply K_t by the ratio I_{sb}/I_s, where I_{sb} is the moment of inertia of the slab and beam together and I_s is the moment of inertia of the slab neglecting the beam stem.

$$\frac{1}{K_{ec}} = \frac{1}{\sum K_c} + \frac{1}{K_t}$$

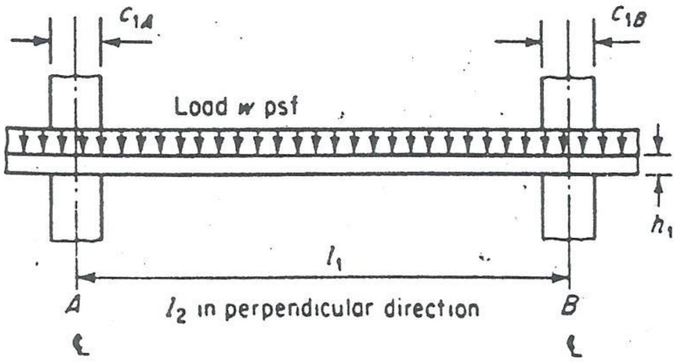
Factored moments

At interior supports, the critical section for negative M_u in both column and middle strips shall be taken at the face of rectilinear supports, but not farther away than $0.175\ell_1$ from the center of a column.

At exterior supports without brackets or capitals, the critical section for negative M_u in the span perpendicular to an edge shall be taken at the face of the supporting element.

At exterior supports with brackets or capitals, the critical section for negative M_u in the span perpendicular to an edge shall be taken at a distance from the face of the supporting element not exceeding one-half the projection of the bracket or capital beyond the face of the supporting element.

Table A.13a Coefficients for slabs with variable moment of inertia†

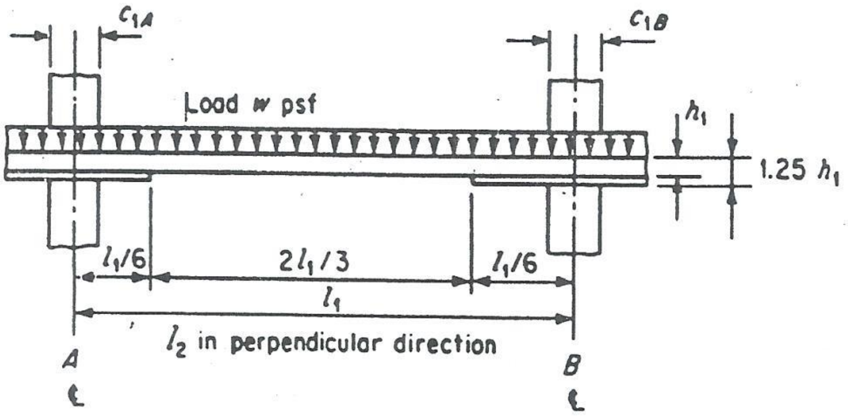


Column dimension		Uniform load FEM = coeff. $(wl_2 l_1^2)$		Stiffness factor‡		Carryover factor	
$\frac{c_{1A}}{l_1}$	$\frac{c_{1B}}{l_1}$	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
0.00	0.00	0.083	0.083	4.00	4.00	0.500	0.500
	0.05	0.083	0.084	4.01	4.04	0.504	0.500
	0.10	0.082	0.086	4.03	4.15	0.513	0.499
	0.15	0.081	0.089	4.07	4.32	0.528	0.498
	0.20	0.079	0.093	4.12	4.56	0.548	0.495
	0.25	0.077	0.097	4.18	4.88	0.573	0.491
0.05	0.05	0.084	0.084	4.05	4.05	0.503	0.503
	0.10	0.083	0.086	4.07	4.15	0.513	0.503
	0.15	0.081	0.089	4.11	4.33	0.528	0.501
	0.20	0.080	0.092	4.16	4.58	0.548	0.499
	0.25	0.078	0.096	4.22	4.89	0.573	0.494
0.10	0.10	0.085	0.085	4.18	4.18	0.513	0.513
	0.15	0.083	0.088	4.22	4.36	0.528	0.511
	0.20	0.082	0.091	4.27	4.61	0.548	0.508
	0.25	0.080	0.095	4.34	4.93	0.573	0.504
0.15	0.15	0.086	0.086	4.40	4.40	0.526	0.526
	0.20	0.084	0.090	4.46	4.65	0.546	0.523
	0.25	0.083	0.094	4.53	4.98	0.571	0.519
0.20	0.20	0.088	0.088	4.72	4.72	0.543	0.543
	0.25	0.086	0.092	4.79	5.05	0.568	0.539
0.25	0.25	0.090	0.090	5.14	5.14	0.563	0.563

† Applicable when $c_1/l_1 = c_2/l_2$. For other relationships between these ratios, the constants will be slightly in error.

‡ Stiffness is $K_{AB} = k_{AB} E(l_2 h_1^3/12 l_1)$ and $K_{BA} = k_{BA} E(l_2 h_1^3/12 l_1)$.

Table A.13b Coefficients for slabs with variable moment of inertia†

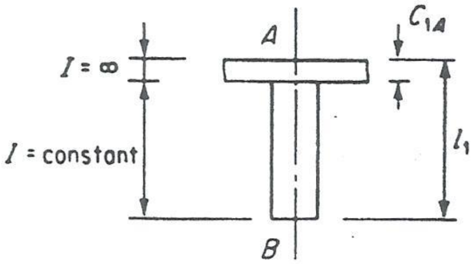


Column dimension		Uniform load FEM = coeff. $(wl_2 l_1^2)$		Stiffness factor‡		Carryover factor	
$\frac{c_{1A}}{l_1}$	$\frac{c_{1B}}{l_1}$	M_{AB}	M_{BA}	k_{AB}	k_{BA}	COF_{AB}	COF_{BA}
0.00	0.00	0.088	0.088	4.78	4.78	0.541	0.541
	0.05	0.087	0.089	4.80	4.82	0.545	0.541
	0.10	0.087	0.090	4.83	4.94	0.553	0.541
	0.15	0.085	0.093	4.87	5.12	0.567	0.540
	0.20	0.084	0.096	4.93	5.36	0.585	0.537
	0.25	0.082	0.100	5.00	5.68	0.606	0.534
0.05	0.05	0.088	0.088	4.84	4.84	0.545	0.545
	0.10	0.087	0.090	4.87	4.95	0.553	0.544
	0.15	0.085	0.093	4.91	5.13	0.567	0.543
	0.20	0.084	0.096	4.97	5.38	0.584	0.541
	0.25	0.082	0.100	5.05	5.70	0.606	0.537
0.10	0.10	0.089	0.089	4.98	4.98	0.553	0.553
	0.15	0.088	0.092	5.03	5.16	0.566	0.551
	0.20	0.086	0.094	5.09	5.42	0.584	0.549
	0.25	0.084	0.099	5.17	5.74	0.606	0.546
0.15	0.15	0.090	0.090	5.22	5.22	0.565	0.565
	0.20	0.089	0.094	5.28	5.47	0.583	0.563
	0.25	0.87	0.097	5.37	5.80	0.604	0.559
0.20	0.20	0.092	0.092	5.55	5.55	0.580	0.580
	0.25	0.090	0.096	5.64	5.88	0.602	0.577
0.25	0.25	0.094	0.094	5.98	5.98	0.598	0.598

† Applicable when $c_1/l_1 = c_2/l_2$. For other relationships between these ratios, the constants will be slightly in error.

‡ Stiffness is $K_{AB} = k_{AB}E(l_2 h_1^3/12l_1)$ and $K_{BA} = k_{BA}E(l_2 h_1^3/12l_1)$.

Table A.13c Coefficients for columns with variable moment of inertia



Slab depth c_{1A}/l_1	Uniform load FEM = coeff. $(wl_2\,l_1^2)$ M_{AB}	M_{BA}	Stiffness factors k_{AB}	k_{BA}	Carryover factors COF_{AB}	COF_{BA}
0.00	0.083	0.083	4.00	4.00	0.500	0.500
0.05	0.100	0.075	4.91	4.21	0.496	0.579
0.10	0.118	0.068	6.09	4.44	0.486	0.667
0.15	0.135	0.060	7.64	4.71	0.471	0.765
0.20	0.153	0.053	9.69	5.00	0.452	0.875
0.25	0.172	0.047	12.44	5.33	0.429	1.000

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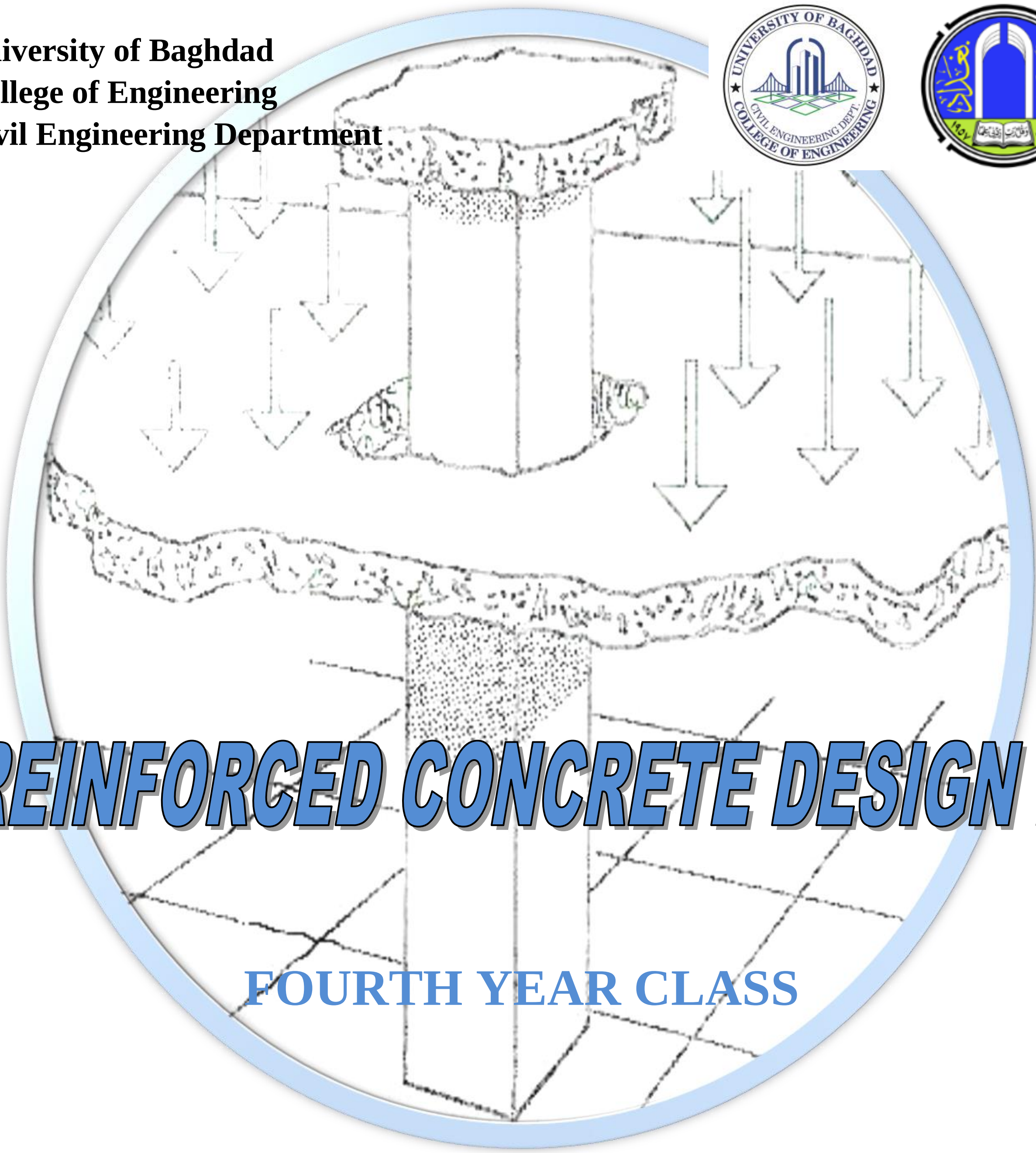
College of Engineering
Civil Engineering Department



REINFORCED CONCRETE DESIGN II

FOURTH YEAR CLASS

4

2018 - 2019

MHND

Shear in slabs

One-way shear or beam-action shear: involves an inclined crack extending across the entire width of the panel.
Two-way shear or punching shear: involves a truncated cone or pyramid-shaped surface around the column

For each applicable factored load combination, design strength shall satisfy:
- $\phi V_n \geq V_u$ at all sections in each direction for one-way shear.
- $\phi v_n \geq v_u$ at the critical sections for two-way shear.

Interaction between load effects shall be considered.

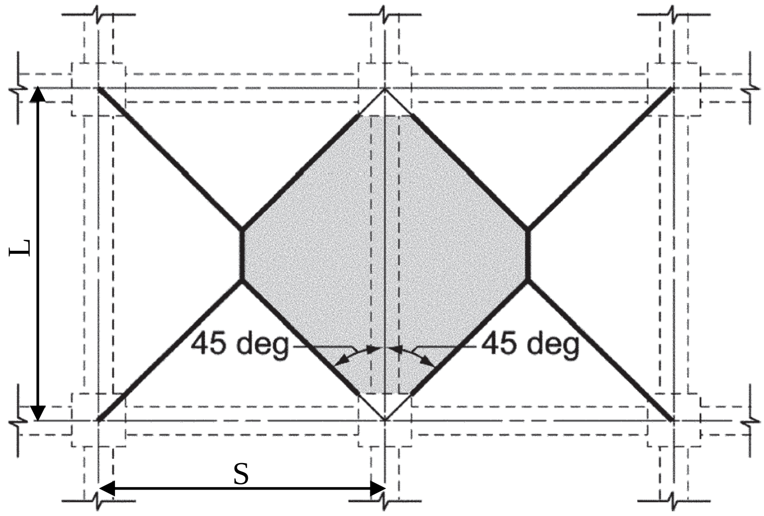
$V_n = V_c + V_s$
$v_n = v_c$ (nominal shear strength for two-way members without shear reinforcement).
$v_n = v_c + v_s$ (nominal shear strength for two-way members with shear reinforcement other than shearheads).

$\phi = 0.75$
V_u is the factored shear force at the slab section considered.
V_n is the nominal shear strength.
V_c is the nominal shear strength provided by concrete.
V_s is the nominal shear strength provided by shear reinforcement.
v_n is the equivalent concrete stress corresponding to nominal two-way shear strength of slab.
v_u is the maximum factored two-way shear stress calculated around the perimeter of a given critical section.
v_{ug} is the factored shear stress on the slab critical section for two-way action due to gravity loads without moment transfer.

shear cap: a projection below the slab used to increase the slab shear strength. It shall project below the slab soffit and extend horizontally from the face of the column a distance at least equal to the thickness of the projection below the slab soffit.

Shear in slab with beams

Shear shall be checked at a distance **d** from the face of the support (beam).



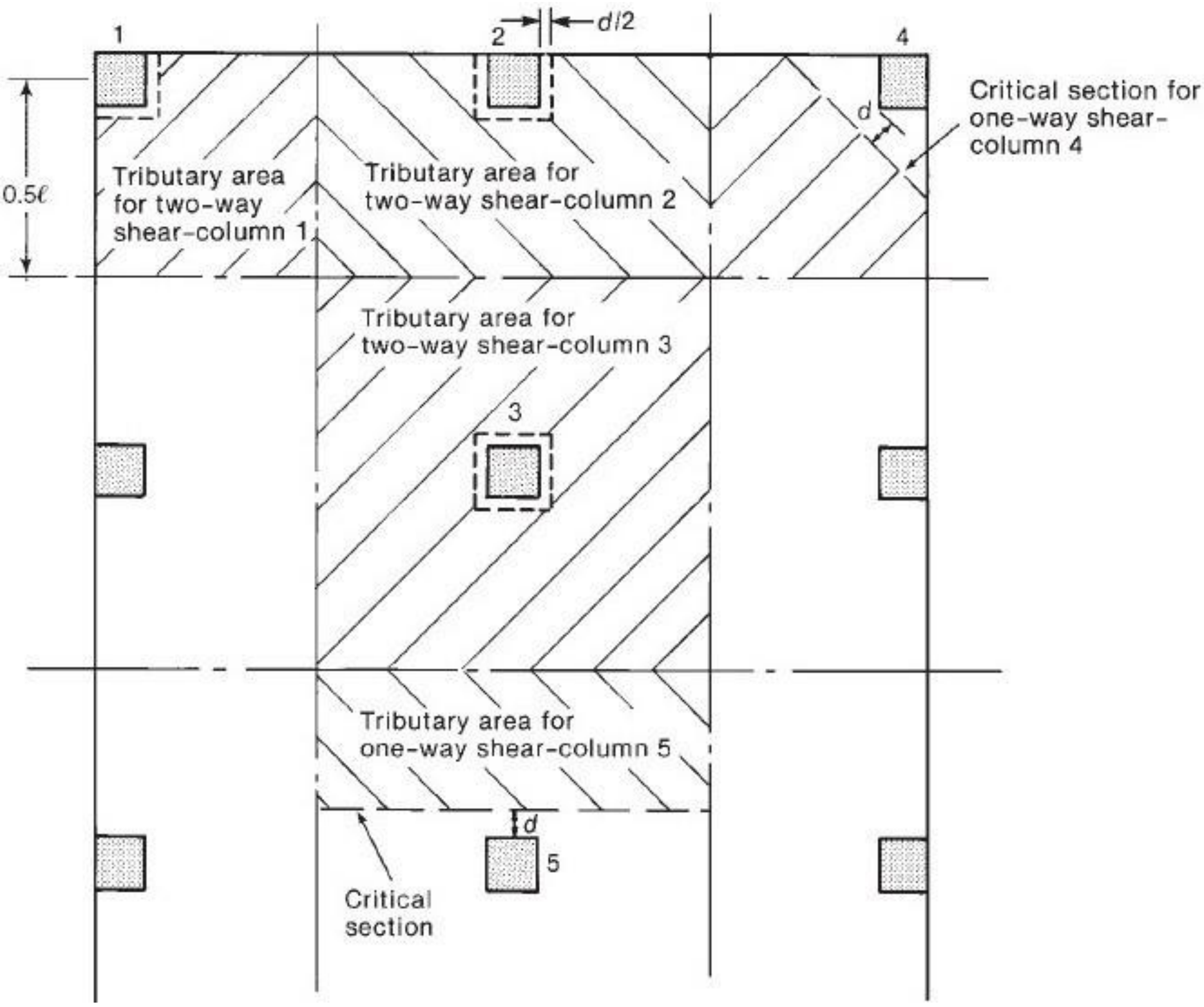
Tributary area for shear on an interior beam.

Shear in flat plate and flat slab

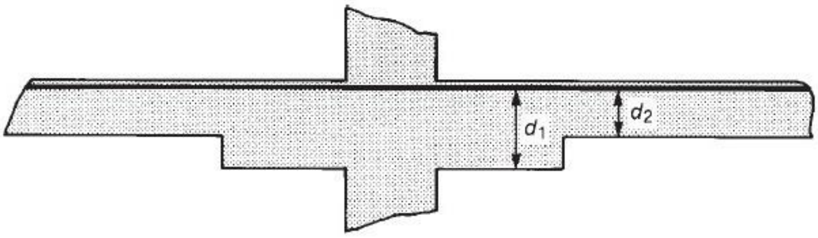
Types:-

1- Beam action (one – way shear action)

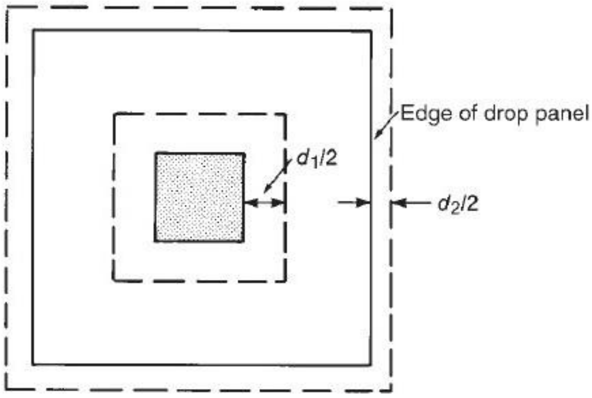
2- Punching shear (two – way shear action)



Critical sections and tributary areas for shear in flat plate.

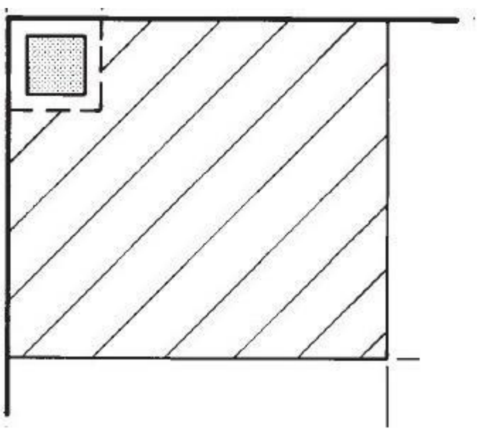


(a) Section through drop panel.

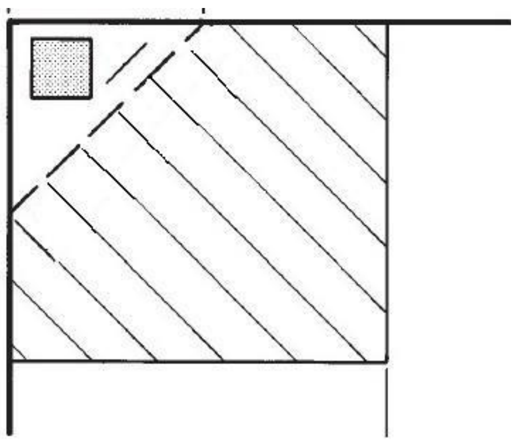


(b) Critical sections.

Critical sections in a slab with drop panels.



(a) Two-way shear.



(b) One-way shear.

Critical shear perimeters and tributary areas for corner column.

Factored one-way shear

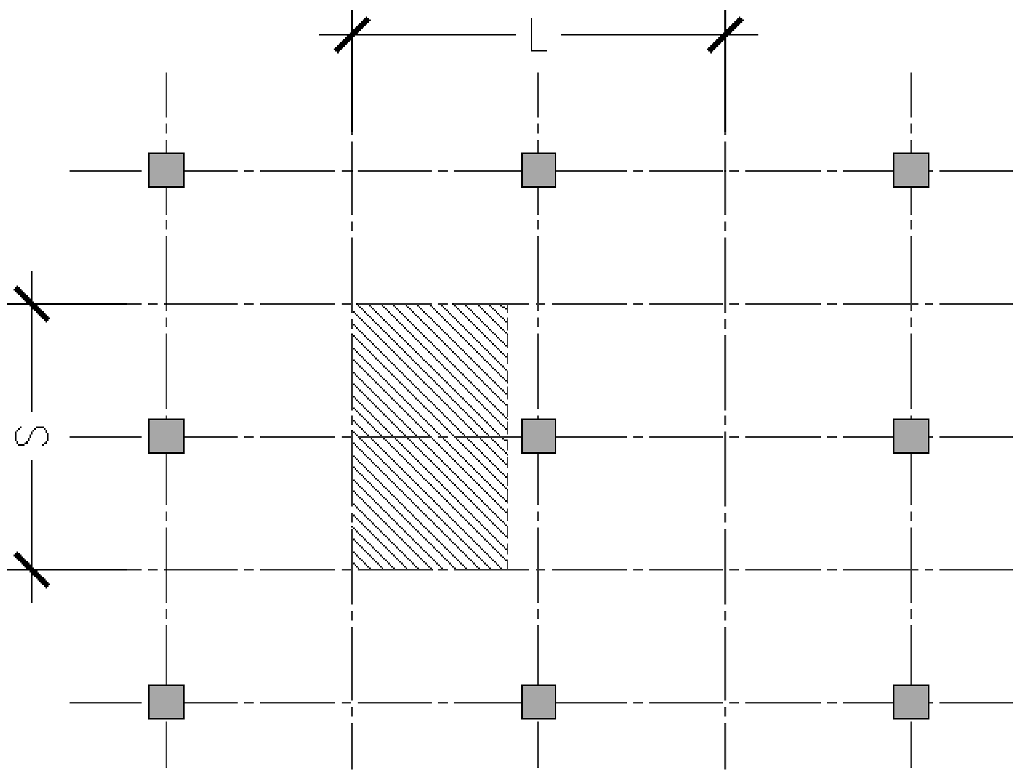
For slabs built integrally with supports, V_u at the support shall be permitted to be calculated at the face of support.

Sections between the face of support and a critical section located a distance **d** from the face of support for nonprestressed slabs shall be permitted to be designed for $\mathbf{V_u}$ at that critical section if (a) through (c) are satisfied:

(a) Support reaction, in direction of applied shear, introduces compression into the end regions of the slab.

(b) Loads are applied at or near the top surface of the slab.

(c) No concentrated load occurs between the face of support and critical section.



One-way shear strength

Nominal one-way shear strength at a section (**V_n**) shall be calculated by:

$V_n = V_c + V_s$

Cross-sectional dimensions shall be selected to satisfy:

$$V_u \le \phi\left(V_c + 0.66\sqrt{f_c{'}}\, b_w d\right)$$

For nonprestressed members without axial force, V_c shall be calculated by:

$$V_c = 0.17\sqrt{f_c'}\ b\ d$$

unless a more detailed calculation is made in accordance with Table 22.5.5.1.

Table 22.5.5.1 - Detailed method for calculating V_c

V_c		
Least of (a), (b), and (c):	$\left(0.16\sqrt{f_c'} + 17\,\rho_w \dfrac{V_u d}{M_u}\right) b\ d$	(a)
	$\left(0.16\sqrt{f_c'} + 17\,\rho_w\right) b\ d$	(b)
	$0.29\sqrt{f_c'}\ b\ d$	(c)

M_u occurs simultaneously with V_u at the section considered.

Effect of any openings in members shall be considered in calculating V_n.

At each section where $V_u > \phi V_c$, transverse reinforcement shall be provided such that the equation

$$V_s \ge \frac{V_u}{\phi} - V_c$$

is satisfied.

The critical section extending across the entire width at a distance d from:-

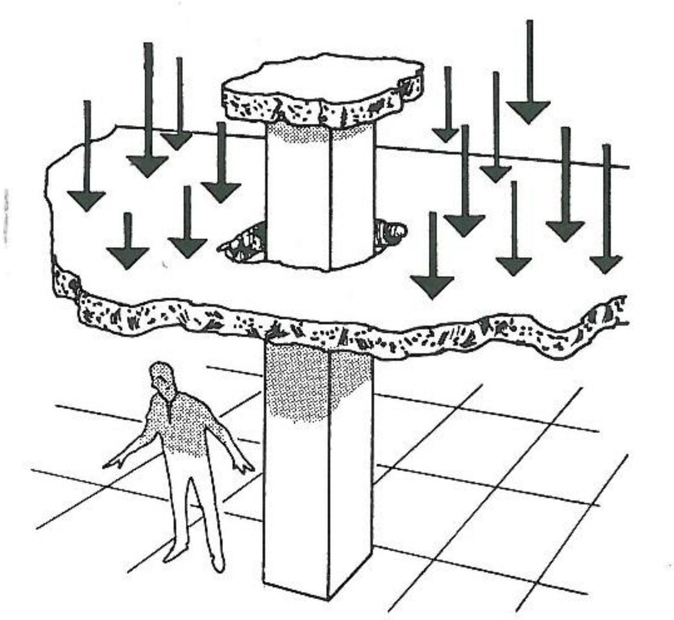
1- The face of the rectangular column in flat plate.

2- The face of the equivalent square column capital or from the face of drop panel, if any in flat slab.

The short direction is controlling because it has a wider area and short critical section:-

$$V_u = q_u \cdot S \cdot \left[\frac{L}{2} - \left(\frac{c}{2} + d\right)\right] \qquad ; \qquad v_n = \frac{V_n}{b \cdot d} = \frac{V_n}{S \cdot d}$$

Factored two-way shear (punching)



Critical section:

Slabs shall be evaluated for two-way shear in the vicinity of columns, concentrated loads, and reaction areas at critical sections.

Two-way shear shall be resisted by a section with a depth (**d**) and an assumed critical perimeter (b_o).

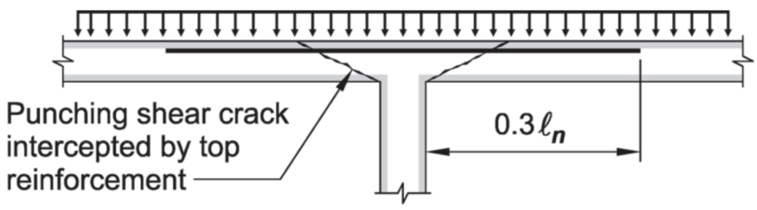
For calculation of $\mathbf{v_c}$ and $\mathbf{v_s}$ for two-way shear, **d** shall be the average of the effective depths in the two orthogonal directions.

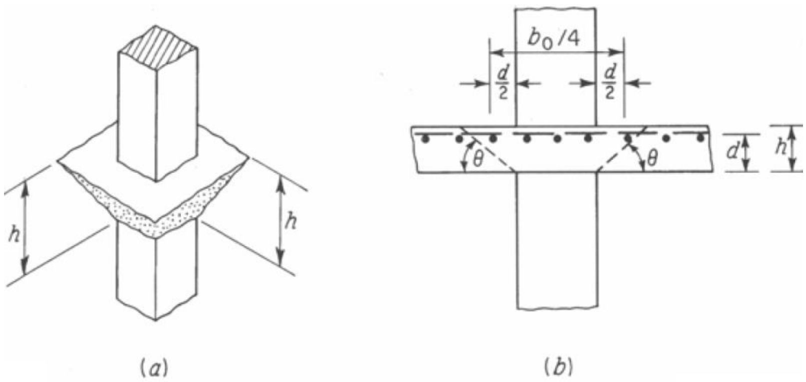
For two-way shear, critical sections shall be located so that the perimeter (b_o) is a minimum but need not be closer than **d/2** to (a) and (b):

(a) Edges or corners of columns, concentrated loads, or reaction areas.

(b) Changes in slab or footing thickness, such as edges of capitals, drop panels, or shear caps.

For a circular or regular polygon-shaped column, critical sections for two-way shear shall be permitted to be defined assuming a square column of equivalent area.





Failure surface defined by punching shear

Nominal shear strength for two-way members without shear reinforcement shall be calculated by:

$v_n = v_c$

v_c for two-way shear shall be calculated in accordance with Table 22.6.5.2.

Table 22.6.5.2 - Calculation of v_c for two-way shear

v_c		
Least of (a), (b), and (c):	$0.33\sqrt{f'_c}$	(a)
	$0.17\left(1+\frac{2}{\beta}\right)\sqrt{f'_c}$	(b)
	$0.083\left(2+\frac{\alpha_s d}{b_o}\right)\sqrt{f'_c}$	(c)

Note: β is the ratio of long side to short side of the column, concentrated load, or reaction area.

α_s = 40 for interior columns

= 30 for edge columns

= 20 for corner columns

Nominal shear strength for two-way members with shear reinforcement other than shearheads shall be calculated by:

$v_n = v_c + v_s$

For two-way members with shear reinforcement, v_c shall not exceed the limits:

$v_c = 0.17\sqrt{f'_c}$

For two-way members with shear reinforcement, effective depth shall be selected such that v_u calculated at critical sections does not exceed the value:

$v_u \le \phi 0.5\sqrt{f'_c}$

For two-way members reinforced with headed shear reinforcement or single- or multi-leg stirrups, a critical section with perimeter b_o located d/2 beyond the outermost peripheral line of shear reinforcement shall also be considered. The shape of this critical section shall be a polygon selected to minimize b_o.

<u>Effective depth</u>

For calculation of v_c and v_s for two-way shear, d shall be the average of the effective depths in the two orthogonal directions.

<u>Two-way shear strength provided by single- or multiple-leg stirrups:</u>

Single- or multiple-leg stirrups fabricated from bars or wires shall be permitted to be used as shear reinforcement in slabs and footings satisfying (a) and (b):

(a) d is at least 150 mm.

(b) d is at least $16d_b$, where d_b is the diameter of the stirrups.

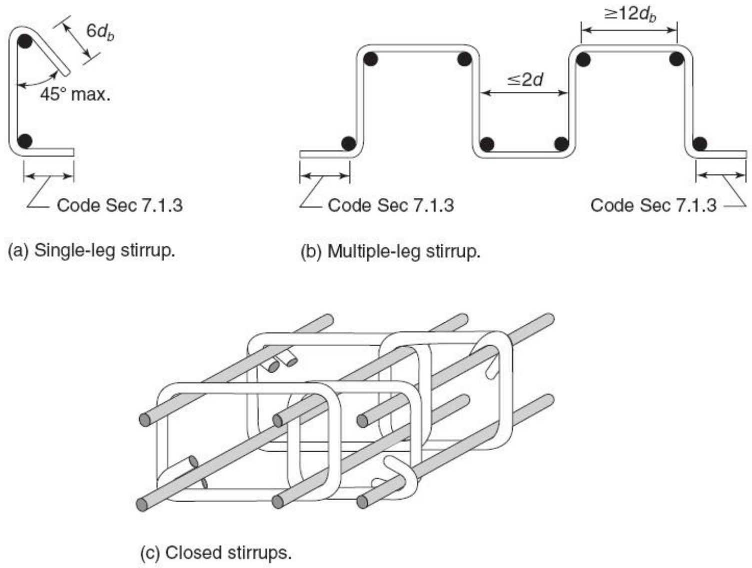
For two-way members with stirrups, v_s shall be calculated by:

$$v_s = \frac{A_v\, f_y}{b_o\, s}$$

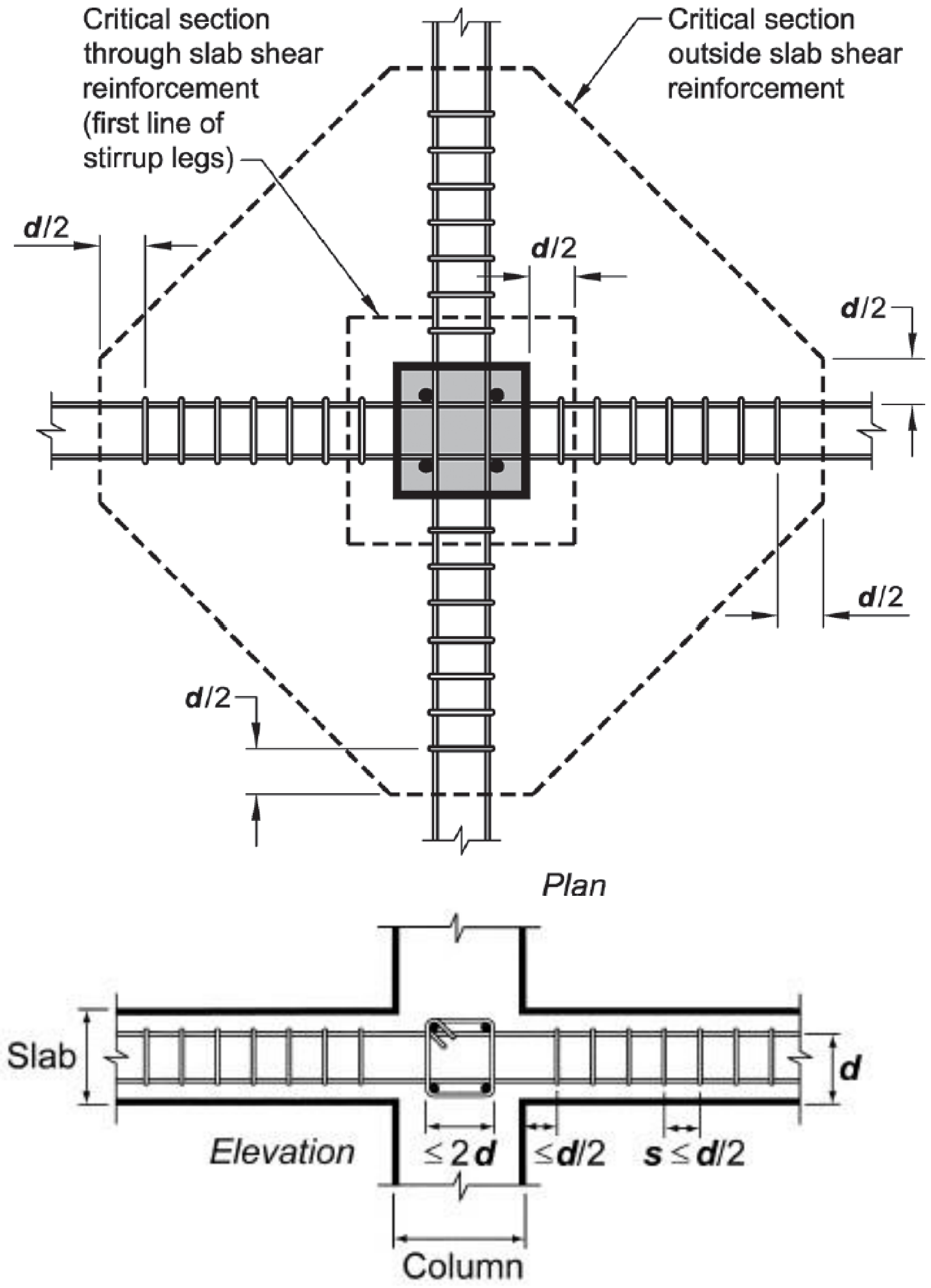
Where

A_v: is the sum of the area of all legs of reinforcement on one peripheral line that is geometrically similar to the perimeter of the column section.

s: is the spacing of the peripheral lines of shear reinforcement in the direction perpendicular to the column face.

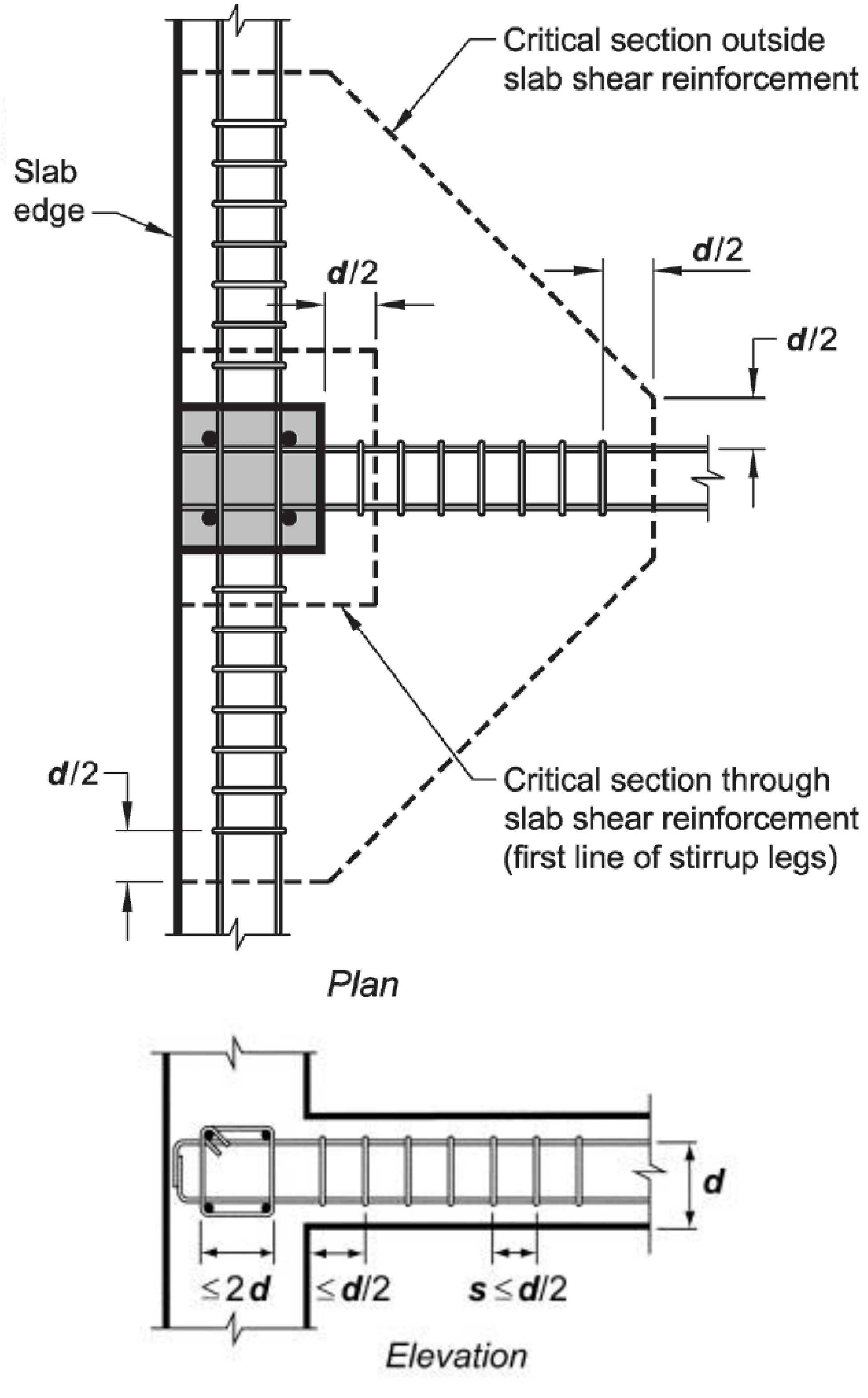


Shear reinforcement.



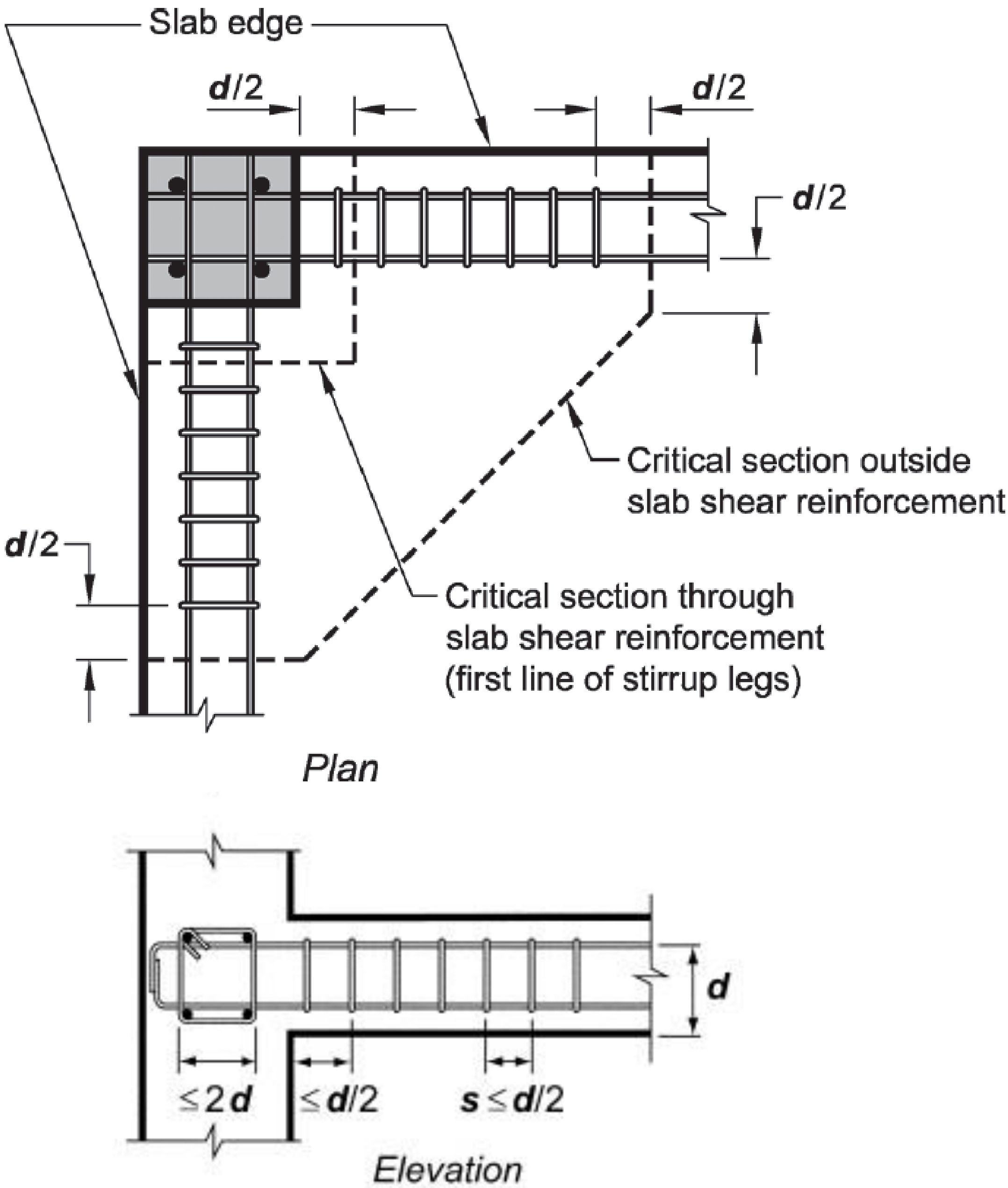
Arrangement of stirrup shear reinforcement, interior column.

Critical sections for two-way shear in slab with shear reinforcement at interior column.



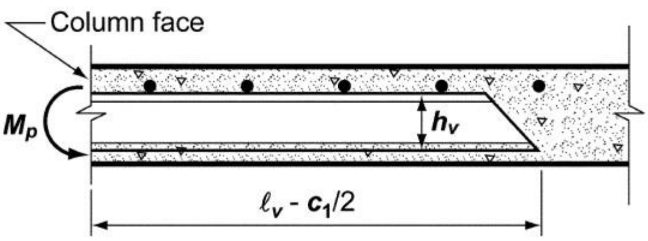
Arrangement of stirrup shear reinforcement, edge column.

Critical sections for two-way shear in slab with shear reinforcement at edge column.

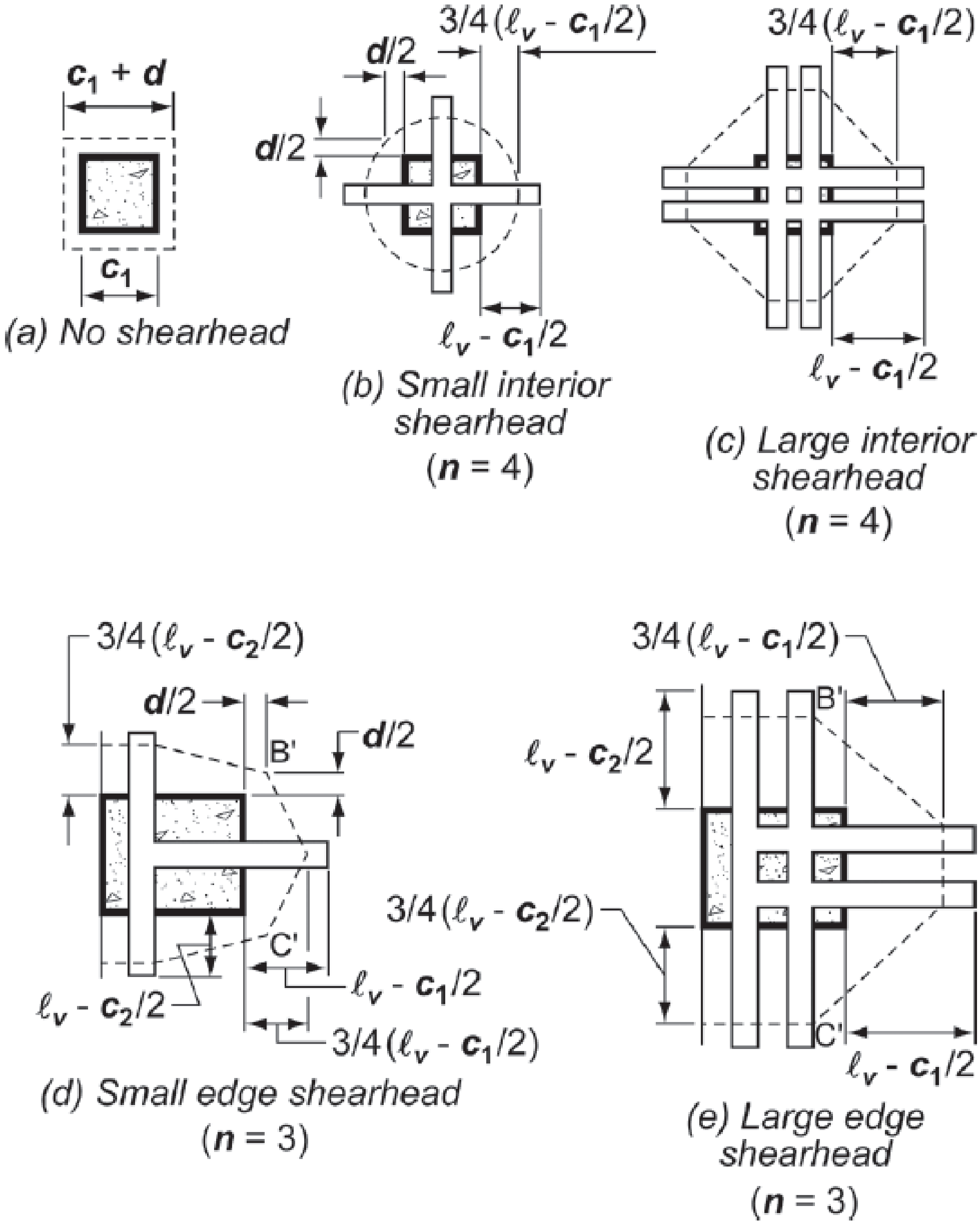


Arrangement of stirrup shear reinforcement, corner column.

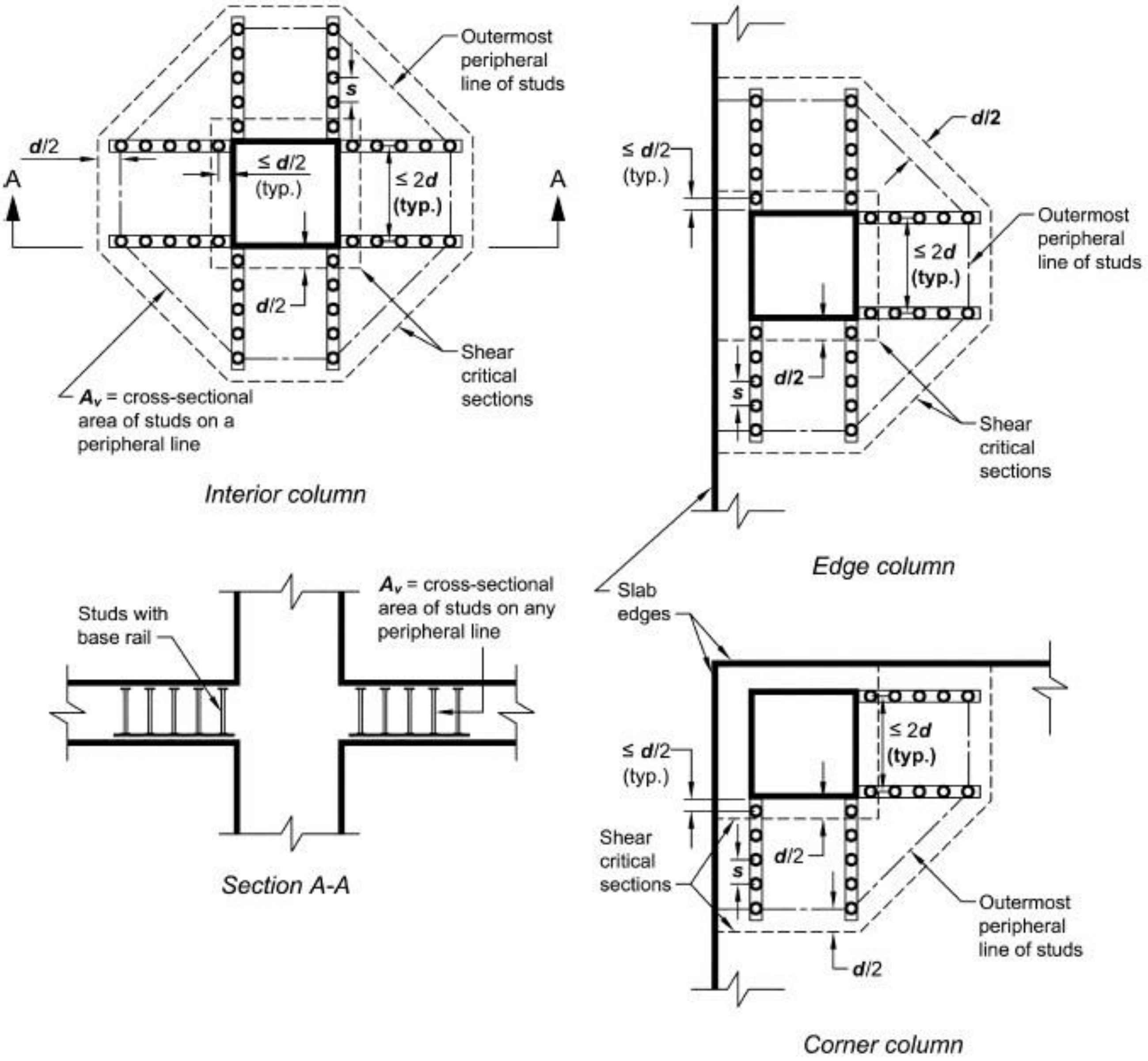
Critical sections for two-way shear in slab with shear reinforcement at corner column.



Structural shearheads.

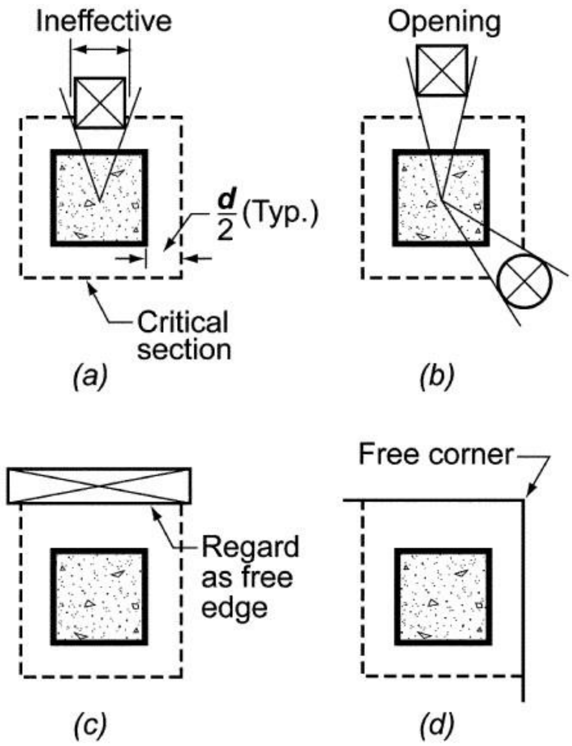


Location of critical section without and with shearheads.



Typical arrangements of headed shear stud reinforcement and critical sections.

Effect of any openings and free edges in slab shall be considered in calculating v_n



Effect of openings and free edges (effective perimeter shown with dashed lines).

Example:

The flat plate slab of 200 mm total thickness and 160 mm effective depth is carried by 300 mm square column 4.50 m on centers in each direction. A factored load of 580 kN must be transmitted from the slab to a typical interior column. Determine if shear reinforcement is required for the slab, and if so, design integral beams with vertical stirrups to carry the excess shear. Use f_y = 414 MPa, f_c' = 30 MPa.

Solution:-

Shear perimeter (b_o) = (300 + 160) × 4 = 1840 mm

V_u = 580 kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{580 \times 10^3}{1840 \times 160} = 1.970 \text{ MPa}$$

i) without shear reinforcement

The design shear strength of the concrete alone at the critical section d/2 from the face of column is

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{30} = 1.807 \text{ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1}\right) \times \sqrt{30} = 2.793 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{40 \times 160}{1840}\right) \times \sqrt{30} = 2.49 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{300}{300} = 1$$

$\therefore \; v_c = 1.807$ MPa

$v_n = v_c$

$\phi v_n = 0.75 \times 1.807 = 1.355$ MPa $< v_u = 1.97$ MPa   not O.K.

$\therefore$ Shear reinforcement is required

ii) with shear reinforcement

$v_n = v_c + v_s$

For two-way members with shear reinforcement, effective depth shall be selected such that v_u calculated at critical sections does not exceed the value:

$v_u \le \phi 0.5\sqrt{f_c'}$

$v_u = v_{ug} = 1.97$ MPa $< \phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{30} = 2.054$ MPa   O.K.

$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{30} = 0.931$ MPa

Let $\phi v_n = v_u = 1.97$ MPa

$\phi\,(v_c + v_s) = v_u$

$$\Rightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.97}{0.75} - 0.931 = 1.696 \text{ MPa}$$

$$v_s = \frac{A_v\, f_y}{b_o\, s}$$

$$\Rightarrow \quad A_v = \frac{v_s\, b_o\, s}{f_y} = \frac{1.696 \times 1840 \times 80}{414} = 603 \text{ mm}^2 \qquad s = \frac{d}{2} = 80 \text{ mm}$$

The required area of vertical shear reinforcement = 603 mm^2

For trial, ø10 mm vertical closed hoop stirrups will be selected and arranged along four integral beams.

effective depth = 160 mm = 16 × 10 (d is at least $16d_b$). O.K.

A_v provided is 4 × 2 × 78.5 = 628 mm^2 at the first critical section, at distance d/2 = 80 mm from the column face.

The required perimeter of the second critical section, at which the concrete alone can carry the shear, is found from the controlling equation as follows:

$$v_u = \phi v_n = \phi v_c = \phi\, 0.17 \sqrt{f_c'} = 0.75 \times 0.17 \times \sqrt{30} = 0.698 \text{ MPa}$$

$$v_u = v_{ug} = 0.698 = \frac{580 \times 10^3}{b_o \times 160} \quad \Rightarrow \quad b_o = 5193.4 \text{ mm}$$

5193.4 = 4 × (3d + y)

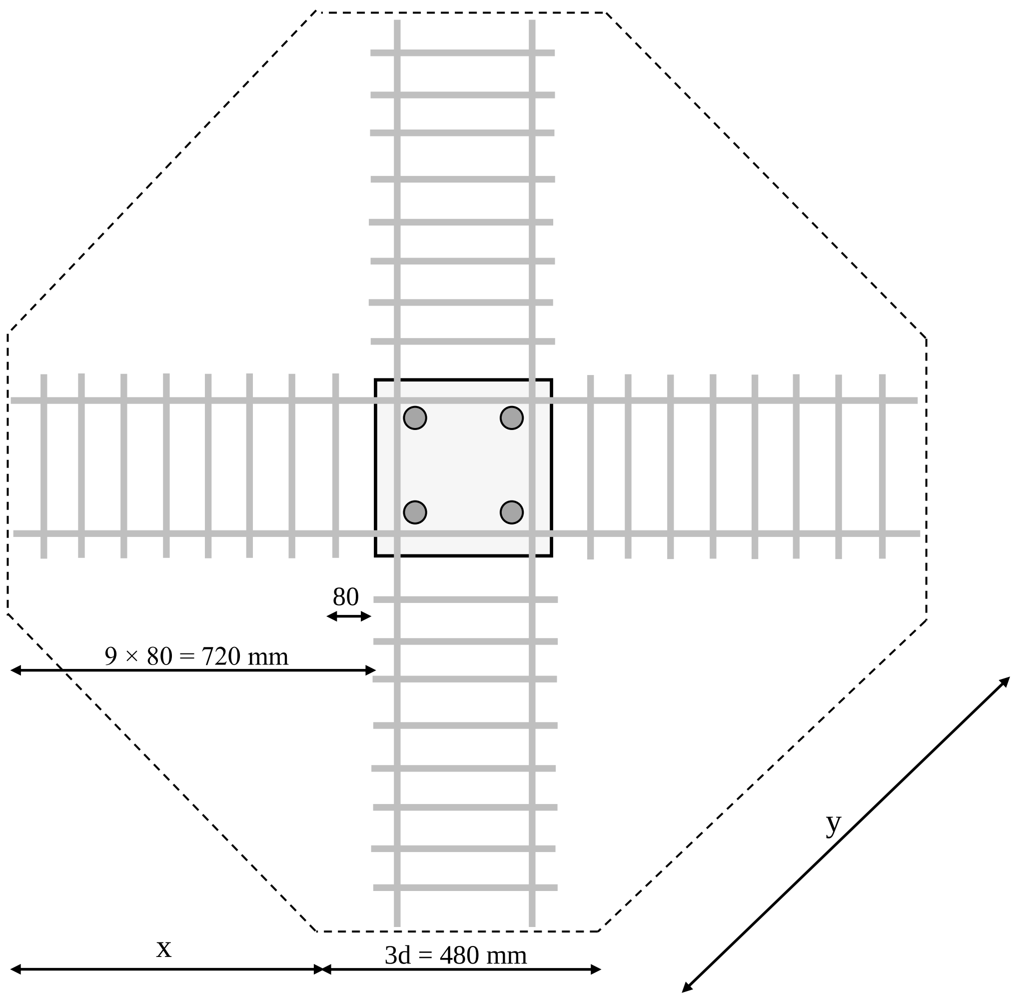
⇒ y = 818.35 mm

x = 818.35 × sin 45 = 578.7 mm

8 stirrups at constant 80 mm spacing will be sufficient, the first placed at 80 mm from the column face, this provides a shear perimeter (b_o) at second critical section of:

9 × 80 + 150 = 870 mm > x + 240 = 818.7 mm O.K.

It is essential that this shear reinforcement engage longitudinal reinforcement at both the top and bottom of the slab, so 4 longitudinal ø16 bars will be provided inside the corners of each closed hoop stirrup. Alternatively, the main slab reinforcement could be used.


80
9 × 80 = 720 mm
y
x
3d = 480 mm

Example:

Check the two way shear action (punching shear) only around an edge column (400×400) mm in a flat plate floor of a span (6.0 × 6.0) m. Find the area of vertical shear reinforcement if required. Assume d = 158 mm. Total q_u = 16.0 kPa (including slab weight), f_c' = 25 MPa, f_y = 400 MPa.

Solution:-

Shear perimeter (b_o) = (400 + 79) × 2 + (400 + 158) = 1516 mm

V_u = 16 × (6 × 3.2 – 0.558 × 0.479) = 302.923 kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{302.923 \times 10^3}{1516 \times 158} = 1.265 \text{ MPa}$$

i) without shear reinforcement

The design shear strength of the concrete alone at the critical section d/2 from the face of column is

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{25} = 1.65 \text{ MPa} \\ 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1+\frac{2}{1}\right)\times\sqrt{25} = 2.55 \text{ MPa} \\ 0.083\left(2+\frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2+\frac{30 \times 158}{1516}\right)\times\sqrt{25} = 2.128 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{400}{400} = 1$$

$\therefore$ $v_c = 1.65$ MPa

$v_n = v_c$

$\phi v_n = 0.75 \times 1.65 = 1.238$ MPa $<$ $v_u = 1.265$ MPa not O.K.

$\therefore$ Shear reinforcement is required

ii) with shear reinforcement

$v_n = v_c + v_s$

For two-way members with shear reinforcement, effective depth shall be selected such that v_u calculated at critical sections does not exceed the value:

$v_u \leq \phi 0.5\sqrt{f_c'}$

$v_u = v_{ug} = 1.265$ MPa $<$ $\phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{25} = 1.875$ MPa O.K.

$$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{25} = 0.85 \text{ MPa}$$

Let $\phi v_n = v_u = 1.265$ MPa

$\phi\,(v_c + v_s) = v_u$

$$\Rightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.265}{0.75} - 0.85 = 0.837 \text{ MPa}$$

$$v_s = \frac{A_v f_y}{b_o s}$$

$$\Rightarrow \quad A_v = \frac{v_s\, b_o\, s}{f_y} = \frac{0.837\ \times 1516\ \times 79}{400} = 250.6\ \text{mm}^2 \qquad s = \frac{d}{2}\ = 79\ \text{mm}$$

The required area of vertical shear reinforcement = 250.6 mm^2

To design the integral beams with the vertical stirrups to carry the excess shear:

For trial, ø8 mm vertical closed hoop stirrups will be selected and arranged along three integral beams.
effective depth = 158 mm > 16 × 8 = 128 mm (d is at least $16d_b$). O.K.

A_v provided is 3 × 2 × 50.2 = 301 mm^2 at the first critical section, at distance d/2 ≈ 75 mm from the column face.

The required perimeter of the second critical section, at which the concrete alone can carry the shear, is found from the controlling equation as follows:

$$v_u = \phi v_n = \phi v_c = \phi\ 0.17\sqrt{f'_c} = 0.75\ \times\ 0.17\ \times\ \sqrt{25} = 0.638\ \text{MPa}$$

$$v_u = v_{ug} = 0.638 = \frac{302.923\ \times 10^3}{b_o \times 158} \quad \Rightarrow \quad b_o\ =\ 3005.1\ \text{mm}$$

Example:

Check the two way shear action (punching shear) only around a corner column (400×400) mm in a flat plate floor of a span (6.0×6.0) m. Find the area of vertical shear reinforcement if required. Assume d =158 mm. Total q_u = 19.0 kPa (including slab weight), f_c'= 25 MPa, f_y = 400 MPa.

Solution:-

Shear perimeter (b_o) = (400 + 79) × 2 = 958 mm

V_u = 19 × (3.2 × 3.2 – 0.479 × 0.479) = 190.201 kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{190.201 \times 10^3}{958 \times 158} = 1.257 \text{ MPa}$$

i) without shear reinforcement

The design shear strength of the concrete alone at the critical section d/2 from the face of column is

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{25} = 1.65 \text{ MPa} \\ 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1+\frac{2}{1}\right)\times\sqrt{25} = 2.55 \text{ MPa} \\ 0.083\left(2+\frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2+\frac{20 \times 158}{958}\right)\times\sqrt{25} = 2.199 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{400}{400} = 1$$

$\therefore \; v_c = 1.65$ MPa

$v_n = v_c$

$\phi v_n = 0.75 \times 1.65 = 1.238$ MPa $< v_u = 1.257$ MPa not O.K.

$\therefore$ Shear reinforcement is required

ii) with shear reinforcement

$v_n = v_c + v_s$

For two-way members with shear reinforcement, effective depth shall be selected such that v_u calculated at critical sections does not exceed the value:

$v_u \le \phi 0.5\sqrt{f_c'}$

$v_u = v_{ug} = 1.257 \text{ MPa} < \phi 0.5\sqrt{f_c'} = 0.75 \times 0.5 \times \sqrt{25} = 1.875 \text{ MPa}$ O.K.

$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{25} = 0.85 \text{ MPa}$

Let $\phi v_n = v_u = 1.257$ MPa

$\phi\,(v_c + v_s) = v_u$

$$\Rightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.257}{0.75} - 0.85 = 0.826 \text{ MPa}$$

$$v_s = \frac{A_v\, f_y}{b_o\, s}$$

$$\Longrightarrow \quad A_v = \frac{v_s\, b_o\, s}{f_y} = \frac{0.826 \times 958 \times 75}{400} = 148.4 \text{ mm}^2 \qquad s = 75 \text{ mm} < \frac{d}{2} = 79 \text{ mm}$$

The required area of vertical shear reinforcement = 148.4 mm^2

Example:

Check the two way shear action (punching shear) only around an interior column (450×450) mm in a flat plate floor of a span (5.8×5.6) m. Find the area of vertical shear reinforcement if required. Assume d =150 mm. Total q_u = 17.5 kPa (including slab weight), f_c'= 32 MPa, f_y = 420 MPa.

Example:

Check the two way shear action (punching shear) only around an interior column (400×500) mm in a flat plate floor of a span (5.6×5.6) m. Find the area of vertical shear reinforcement if required. Assume d =170 mm. Total q_u = 18.0 kPa (including slab weight), f_c'= 30 MPa, f_y = 420 MPa.

Solution:-

Shear perimeter (b_o) = (400 + 170) × 2 + (500 + 170) × 2 = 2480 mm

V_u = 18 × (5.6 × 5.6 – 0.57 × 0.67) = 557.606 kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{557.606 \times 10^3}{2480 \times 170} = 1.323 \text{ MPa}$$

i) without shear reinforcement

The design shear strength of the concrete alone at the critical section d/2 from the face of column is

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{30} = 1.807 \text{ MPa} \\ 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1+\frac{2}{1.25}\right)\times\sqrt{30} = 2.421 \text{ MPa} \\ 0.083\left(2+\frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2+\frac{40 \times 170}{2480}\right)\times\sqrt{30} = 2.156 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{500}{400} = 1.25$$

$\therefore\ v_c = 1.807$ MPa

$v_n = v_c$

ϕv_n = 0.75 × 1.807 = 1.355 MPa > v_u = 1.323 MPa not O.K.

$\therefore$ Shear reinforcement is not required

Example:

Check the two way shear action (punching shear) only around an edge column (300×300) mm in a flat plate floor of a span (4.0×4.0) m. Find the area of vertical shear reinforcement if required. Assume d =165 mm. Total q_u = 17.6 kPa (including slab weight), f_c'= 35 MPa, f_y = 420 MPa.

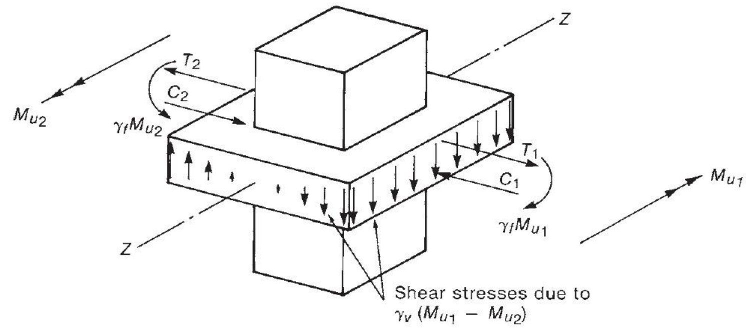
Transfer of moments to columns

The above analysis for punching shear in slabs assumed that the shear force (V_u) was uniformly distributed around the perimeter of the critical section (b_o), at distance d/2 from the face of supporting column and resisted by concrete shear strength (v_c), which was given by the minimum of three equations. If significant moment is to be transferred from the slab to the column, the shear stress on the critical section is no longer uniformly distributed. The situation is shown in figures below.

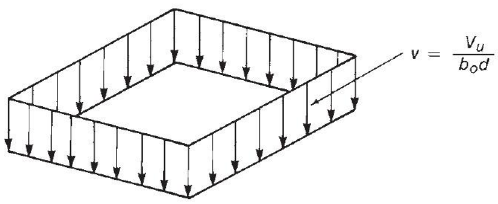
V_u represents the total vertical reaction to be transferred to the column.

M_u ($\gamma_v M_{sc}$) represents the unbalanced moment to be transferred by shear.

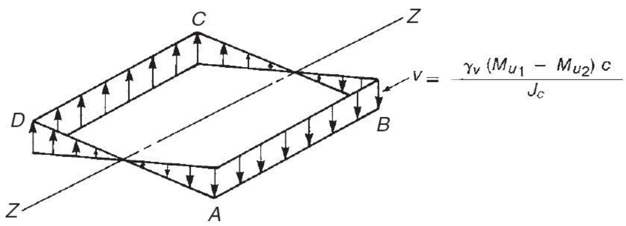
V_u causes shear stress distributed uniformly around the perimeter of the critical section, which acting downward. M_u causes additional loading, which add to shear stresses in one side and subtract to the other side.



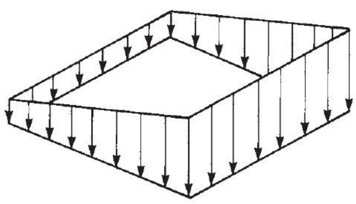
(a) Transfer of unbalanced moments to column.



(b) Shear stresses due to V_u.

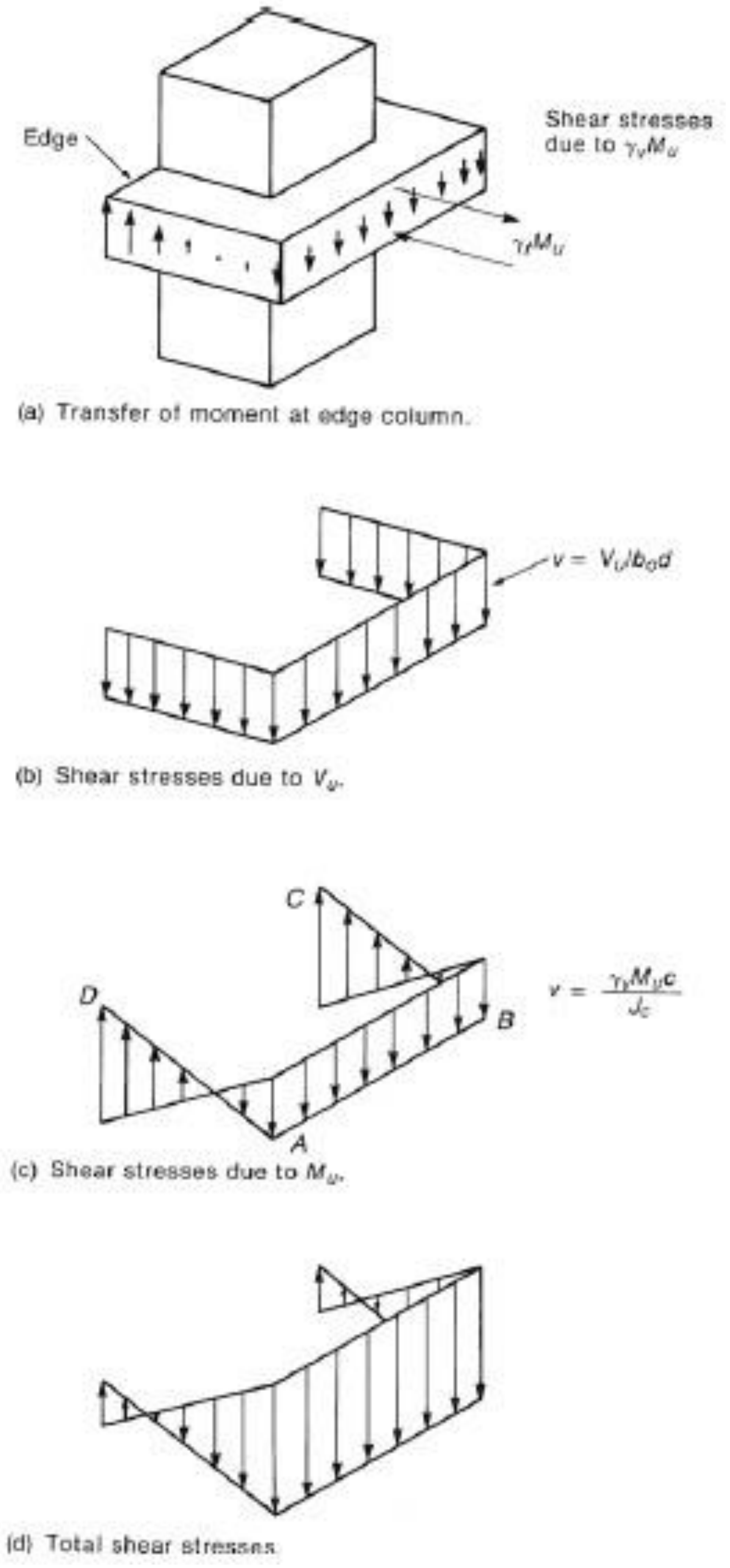


(c) Shear due to unbalanced moment.

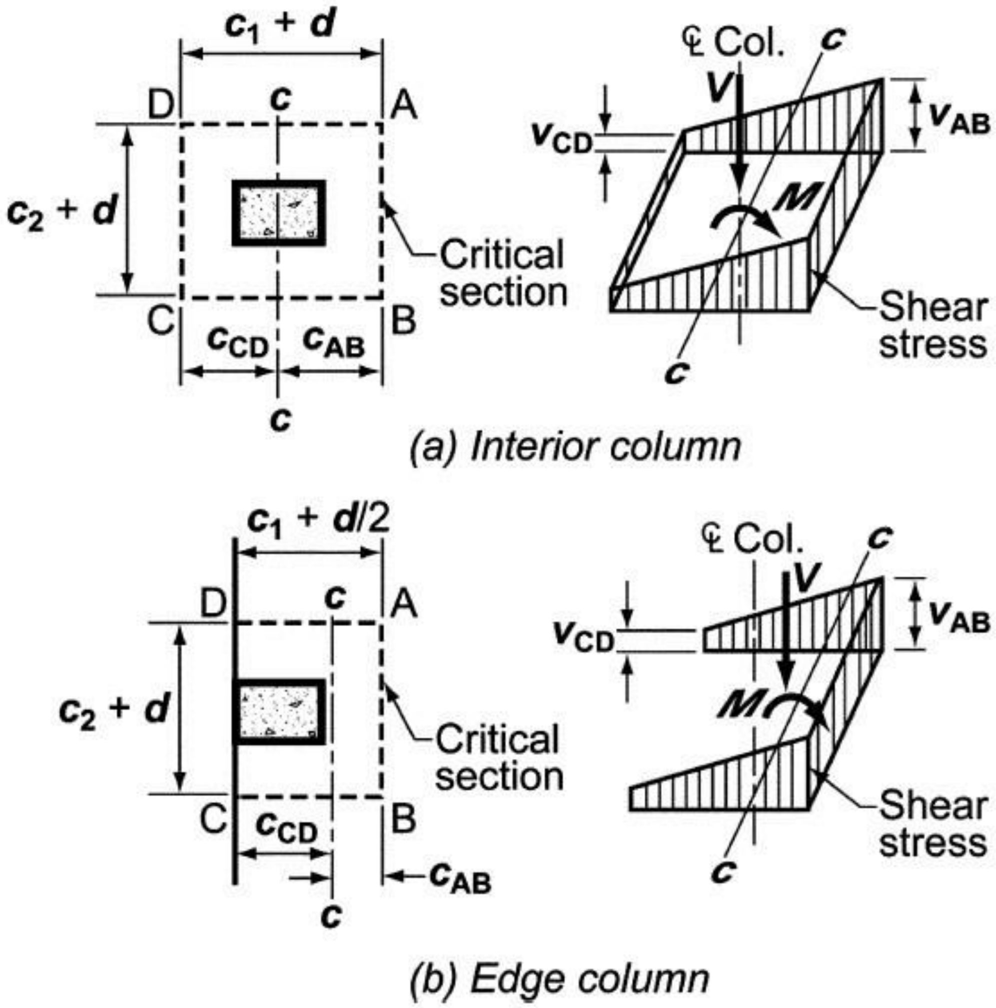


(d) Total shear stresses.

Shear stresses due to shear and moment transfer at an interior column.



Shear stresses due to shear and moment transfer at an edge column.



Assumed distribution of shear stress.

If there is a transfer of moment between the slab and column, a fraction of M_{sc}, the factored slab moment resisted by the column at a joint, shall be transferred by flexure ($\gamma_f M_{sc}$), where γ_f shall be calculated by:

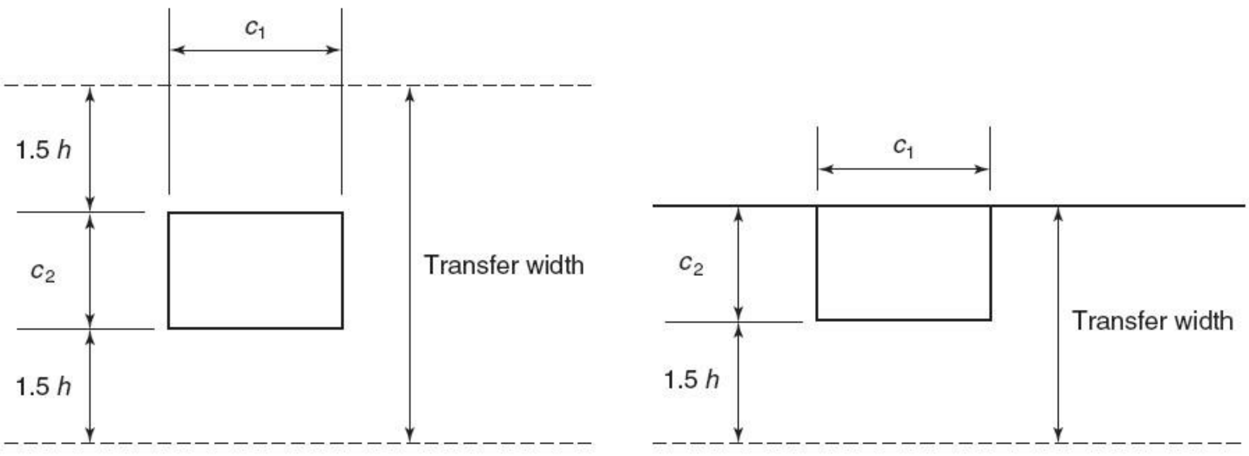
$$\gamma_f = \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}}$$

For nonprestressed slabs, where the limitations on v_{ug} and ε_t in Table 8.4.2.3.4 are satisfied, γ_f shall be permitted to be increased to the maximum modified values provided in Table 8.4.2.3.4, where v_c is calculated in accordance with Table 22.6.5.2, and v_{ug} is the factored shear stress on the slab critical section for two-way action due to gravity loads without moment transfer.

Table 8.4.2.3.4—Maximum modified values of γ_f for nonprestressed two-way slabs

Column location	Span direction	v_{ug}	ε_t (within b_{slab})	Maximum modified γ_f
Corner column	Either direction	$\leq 0.5\phi v_c$	≥ 0.004	1.0
Edge column	Perpendicular to the edge	$\leq 0.75\phi v_c$	≥ 0.004	1.0
	Parallel to the edge	$\leq 0.4\phi v_c$	≥ 0.010	$\dfrac{1.25}{1+\left(\dfrac{2}{3}\right)\sqrt{\dfrac{b_1}{b_2}}} \leq 1.0$
Interior column	Either direction	$\leq 0.4\phi v_c$	≥ 0.010	$\dfrac{1.25}{1+\left(\dfrac{2}{3}\right)\sqrt{\dfrac{b_1}{b_2}}} \leq 1.0$

The effective slab width (b_{slab}) for resisting $\gamma_f M_{sc}$ shall be the width of column or capital plus **1.5 h** of slab or drop panel on either side of column or capital.



(a) Interior column.
(b) Exterior column with moment transferred parallel to the edge.

The fraction of M_{sc} not calculated to be resisted by flexure shall be assumed to be resisted by eccentricity of shear.

For two-way shear with factored slab moment resisted by the column, factored shear stress (v_u) shall be calculated at critical sections. v_u corresponds to a combination of v_{ug} and the shear stress produced by $\gamma_v M_{sc}$.

The fraction of M_{sc} transferred by eccentricity of shear ($\gamma_v M_{sc}$) shall be applied at the centroid of the critical section, where:
$\gamma_v = 1 - \gamma_f$

The stress distribution is assumed as illustrated in Figure above for an interior or exterior column. The perimeter of the critical section, ABCD, is determined. The factored shear stress (v_{ug}) and factored slab moment resisted by the column (M_{sc}) are determined at the centroidal axis c-c of the critical section. The maximum factored shear stress may be calculated from:

$$v_{u,AB} = v_{ug} + \frac{\gamma_v\, M_{sc}\, c_{AB}}{J_c} \qquad ; \qquad v_{u,CD} = v_{ug} - \frac{\gamma_v\, M_{sc}\, c_{DB}}{J_c}$$

J_c = property of assumed critical section analogous to polar moment of inertia

Interior column:
$$J_c = \frac{d(c_1+d)^3}{6} + \frac{(c_1+d)d^3}{6} + \frac{d(c_2+d)(c_1+d)^2}{2}$$
or
$$J_c = 2\left(\frac{b_1 d^3}{12} + \frac{d{b_1}^3}{12}\right) + 2(b_2 d)\left(\frac{b_1}{2}\right)^2$$

Edge column:
In case of moment about an axis parallel to the edge:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}}$$
$$c_{AB} = \frac{2(b_1 d)\left(\frac{b_1}{2}\right)}{2(b_1 d) + b_2 d}$$
$$J_c = 2\left[\frac{b_1 d^3}{12} + \frac{d{b_1}^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right] + (b_2 d)c_{AB}^2$$
In case of moment about an axis perpendicular to the edge:
$$J_c = \left(\frac{b_2 d^3}{12} + \frac{d{b_2}^3}{12}\right) + 2(b_1 d)\left(\frac{b_2}{2}\right)^2$$

Corner column:

$$c_{AB} = \frac{(b_1 d)\left(\frac{b_1}{2}\right)}{b_1 d + b_2 d}$$

$$J_c = \left[\frac{b_1 d^3}{12} + \frac{d {b_1}^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right] + (b_2 d) c_{AB}^2$$

At an interior support, columns or walls above and below the slab shall resist the factored moment calculated by the equation below in direct proportion to their stiffnesses unless a general analysis is made.

$$M_{sc} = 0.07\left[(q_{Du} + 0.5\, q_{Lu})\, \ell_2\, {\ell_n}^2 - q_{Du}'\, \ell_2'\, (\ell_n')^2\right]$$

where q_{Du}', ℓ_2', and ℓ_n' refer to the shorter span.

The gravity load moment to be transferred between slab and edge column shall not be less than $0.3M_o$.

Calculation of factored shear strength v_u (ACI 421.1R-4)

The maximum factored shear stress v_u at a critical section produced by the combination of factored shear force V_u and unbalanced moments M_{ux} and M_{uy}, is:

$$v_u = \frac{V_u}{A_c} + \frac{\gamma_{vx} M_{ux}\, y}{J_x} + \frac{\gamma_{vy} M_{uy}\, x}{J_y}$$

A_c: area of concrete of assumed critical section.

x, y: coordinate of the point at which v_u is maximum with respect to the centroidal principal axes x and y of the assumed critical section.

M_{ux}, M_{uy}: factored unbalanced moments transferred between the slab and the column about the centroidal axes x and y of the assumed critical section, respectively

γ_{ux}, γ_{uy}: fraction of moment between slab and column that is considered transferred by eccentricity of shear about the axes x and y of the assumed critical section. The coefficients γ_{ux} and γ_{uy} are given by:

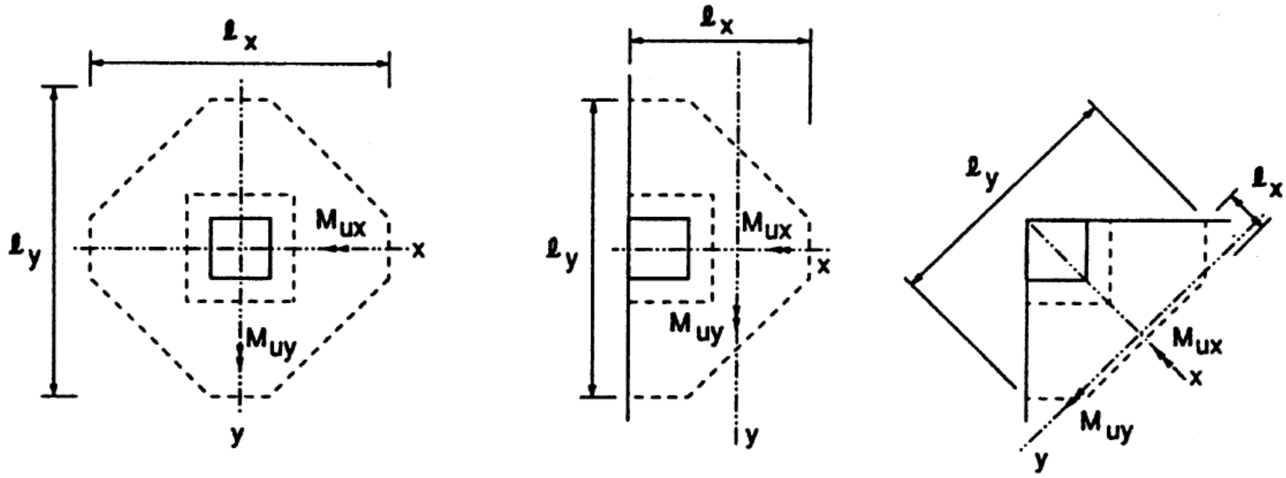
$$\gamma_{vx} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\ell_{y1}/\ell_{x1}}}$$

$$\gamma_{vy} = 1 - \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\ell_{x1}/\ell_{y1}}}$$

where ℓ_{x1} and ℓ_{y1} are lengths of the sides in the x and y directions of the critical section at d/2 from column face.

J_x , J_y: property of assumed critical section, analogous to polar amount of inertia about the axes x and y, respectively. In the vicinity of an interior column, J_y for a critical section at d/2 from column face is:

$$J_y = d\left[\frac{{\ell_{x1}}^3}{6} + \frac{\ell_{y1}{\ell_{x1}}^2}{2}\right] + \frac{\ell_{x1} d^3}{6}$$



$\gamma_{vx} = 1 - \dfrac{1}{1 + \frac{2}{3}\sqrt{\ell_y/\ell_x}}$

γ_{vx} = same as for interior column

$\gamma_{vx} = 0.4$

$\gamma_{vy} = 1 - \dfrac{1}{1 + \frac{2}{3}\sqrt{\ell_x/\ell_y}}$

$\gamma_{vy} = 1 - \dfrac{1}{1 + \frac{2}{3}\sqrt{(\ell_x/\ell_y) - 0.2}}$

(but $\gamma_{vy} = 0$ when $(\ell_x/\ell_y) < 0.2$)

γ_{vy} = same as for edge column

Equations for γ_{vx} and γ_{vy} applicable for critical sections at d/2 from column face and outside shear-reinforced zone. Note: ℓ_x and ℓ_y are projections of critical sections on directions of principal x and y axes.

Properties of critical sections of general shape

This section is general; it applies regardless of the type of shear reinforcement used. Figure below shows the top view of critical sections for shear in slab in the vicinity of interior column. The centroidal x and y axes of the critical sections, V_u, M_{ux}, and M_{uy} are shown in their positive directions. The shear force V_u is acting at the column centroid; V_u, M_{ux}, and M_{uy} represent the effects of the column on the slab.

v_u for a section of general shape, the parameters J_x and J_y may be approximated by the second moments of area I_x and I_y given below. The coefficients γ_{vx} and γ_{vy} are given in Figure, which is based on finite element studies.

The critical section perimeter is generally composed of straight segments. The values of A_c, I_x, and I_y can be determined by summation of the contribution of the segments:

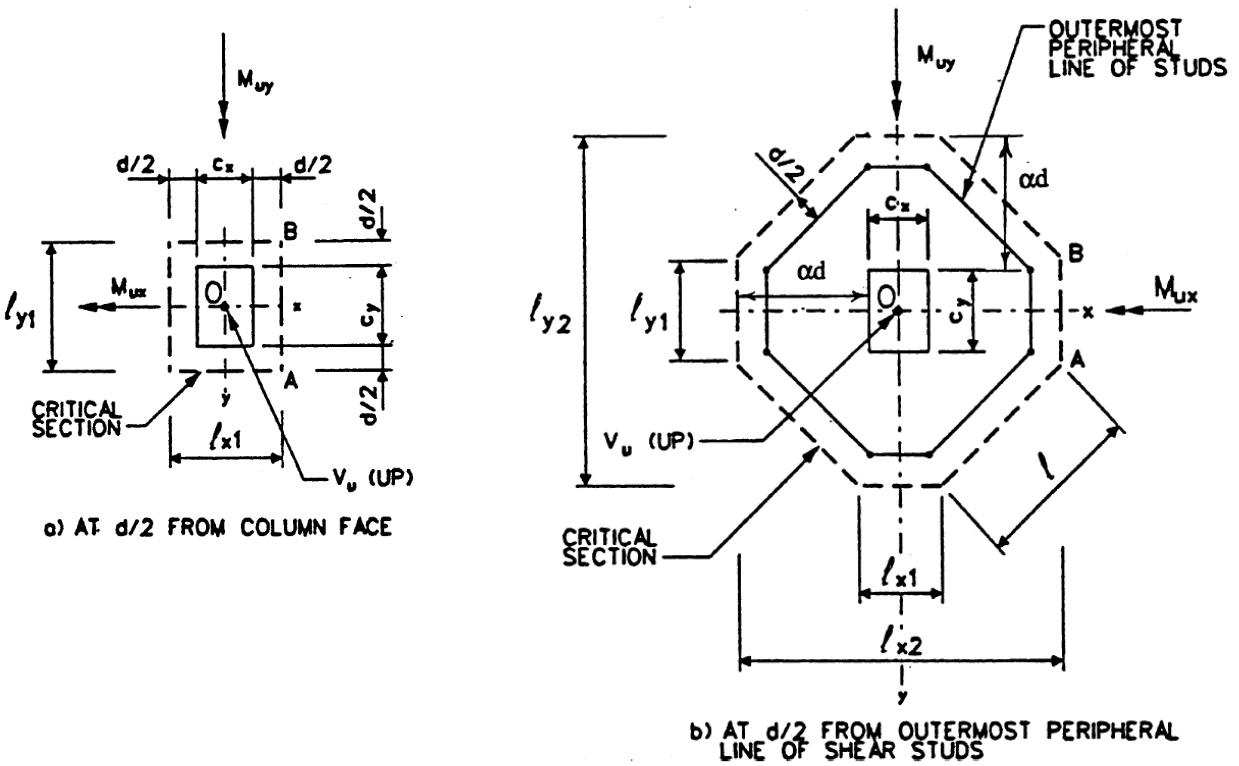
$$A_c = d\sum \ell$$

$$I_x = d\sum\left[\frac{\ell}{3}\left(y_i^2 + y_i y_j + y_j^2\right)\right]$$

$$I_y = d\sum\left[\frac{\ell}{3}\left(x_i^2 + x_i x_j + x_j^2\right)\right]$$

where x_i , y_i , x_j , and y_j are coordinates of Points i and j at the extremities of the segment whose length is ℓ.

When the maximum v_u occurs at a single point on the critical section, rather than on a side, the peak value of v_u does not govern the strength due to stress redistribution. In this case, v_u may be investigated at a point located at a distance 0.4d from the peak point. This will give a reduced v_u value compared with the peak value; the reduction should not be allowed to exceed 15%.



Critical sections for shear in slab in vicinity of interior column.

Example:-

Check combined shear and moment transfer at an edge column 400 mm square column supporting a flat plate slab system. Use $f_c' = 28$ MPa , $f_y = 420$ MPa

Overall slab thickness (t) = 190 mm, (d = 154 mm).

Consider two loading conditions:

1- Total factored shear force $V_u = 125$ kN, the factored slab moment resisted by the column (M_{sc}) = 35 kN.m, and $\varepsilon_t = 0.004$
2- $V_u = 250$ kN , $M_{sc} = 70$ kN.m, and $\varepsilon_t > 0.004$

Solution:

$$b_1 = c_1 + \frac{d}{2} = 400 + \frac{154}{2} = 477 \text{ mm}$$

$b_2 = c_2 + d = 400 + 154 = 554$ mm

$b_o = 2\,b_1 + b_2 = 2 \times 477 + 554 = 1508$ mm

Edge column:

In case of moment about an axis parallel to the edge:

$$c_{AB} = \frac{2(b_1 d)\left(\frac{b_1}{2}\right)}{2(b_1 d) + b_2 d} = \frac{(b_1)^2}{2\,b_1 + b_2} = \frac{(477)^2}{2 \times 477 + 554} = 150.9 \text{ mm}$$

$$J_c = 2\left[\frac{b_1 d^3}{12} + \frac{d {b_1}^3}{12} + (b_1 d)\left(\frac{b_1}{2} - c_{AB}\right)^2\right] + (b_2 d) c_{AB}^2$$

$$J_c = 2\left[\frac{477 \times (154)^3}{12} + \frac{154 \times (477)^3}{12} + (477 \times 154)\left(\frac{477}{2} - 150.9\right)^2\right] + (554 \times 154)(150.9)^2$$

$$= 6146105085.12 \text{ mm}^4$$

$A_c = (2\,b_1 + b_2)\,d = (2 \times 477 + 554) \times 154 = 232232$ mm^2

A_c: area of critical section.

The design shear strength of the concrete alone (without shear reinforcement) at the critical section d/2 from the face of the column is:

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{28} = 1.746 \text{ MPa} \\ 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \frac{2}{1}\right) \times \sqrt{28} = 2.699 \text{ MPa} \\ 0.083\left(2 + \frac{\alpha_s d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \frac{30 \times 154}{1508}\right) \times \sqrt{28} = 2.224 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{400}{400} = 1$$

$\therefore\ v_c = 1.746$ MPa

$\phi v_c = 0.75 \times 1.746 = 1.31$ MPa

Loading condition (1) $V_u = 125$ kN, $M_{sc} = 35$ kN.m, and $\varepsilon_t = 0.004$

$$v_{ug} = \frac{V_u}{A_c} = \frac{125 \times 10^3}{232232} = 0.538 \text{ MPa}$$

Span direction is perpendicular to the edge

$0.75\phi v_c = 0.75 \times 1.31 = 0.983$ MPa $>$ $v_{ug} = 0.538$ MPa & $\varepsilon_t = 0.004$ $\Rightarrow$ $\gamma_f = 1.0$

Therefore, all of the factored slab moment resisted by the column (M_{sc}) may be considered to be transferred by flexure (i.e $\gamma_f = 1.0$ and $\gamma_v = 0$).

Check shear strength of the slab without shear reinforcement. Shear stress along inside face of the critical section.

$v_n = v_c$

$\phi v_n = 1.31$ MPa $>$ $v_u = v_{ug} = 0.538$ MPa O.K.

$\therefore$ Shear reinforcement is not required

Loading condition (2) $V_u = 250$ kN, $M_{sc} = 70$ kN.m, and $\varepsilon_t > 0.004$

$$v_{ug} = \frac{V_u}{A_c} = \frac{250 \times 10^3}{232232} = 1.077 \text{ MPa}$$

Span direction is perpendicular to the edge

$0.75\phi v_c = 0.983$ MPa $<$ $v_{ug} = 1.077$ MPa $\Rightarrow$ $\gamma_f < 1.0$

$$\gamma_f = \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} = \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{477}{554}}} = 0.618$$

$\gamma_v = 1 - \gamma_f = 1 - 0.618 = 0.382$

Check shear strength of the slab without shear reinforcement. Shear stress along inside face of the critical section.

$v_n = v_c$

$$v_{u,AB} = v_{ug} + \frac{\gamma_v M_{sc} c_{AB}}{J_c} = 1.077 + \frac{0.382 \times 70 \times 10^6 \times 150.9}{6146105085.12} = 1.734 \text{ MPa}$$

$\phi v_n = 1.31$ MPa $<$ $v_u = 1.734$ MPa not O.K.

$\therefore$ Shear reinforcement is required to carry excess shear stress.

Check maximum shear stress permitted with shear reinforcement.

$v_u \le \emptyset\, 0.5 \sqrt{f_c'}$

$v_u = 1.734 \text{ MPa} < 0.75 \times 0.5\sqrt{28} = 1.984 \text{ MPa}$ O.K.

$v_c = 0.17\sqrt{f_c'} = 0.17 \times \sqrt{28} = 0.9$ MPa

Let $\phi v_n = v_u = 1.734$ MPa

$\phi\,(v_c + v_s) = v_u$

$$\Longrightarrow \quad v_s = \frac{v_u}{\phi} - v_c = \frac{1.734}{0.75} - 0.9 = 1.412 \text{ MPa}$$

$$v_s = \frac{A_v\, f_y}{b_o\, s}$$

$$\Longrightarrow \quad A_v = \frac{v_s\, b_o\, s}{f_y} = \frac{1.412 \times 1508 \times 75}{420} = 380.2 \text{ mm}^2$$

$$\text{here } s = \frac{d}{2} = \frac{154}{2} = 77 \approx 75 \text{ mm}$$

The required area of vertical shear reinforcement = 380.2 mm^2

For trial, 3ø8 mm vertical single-leg stirrups will be selected and arranged along three integral beams.

effective depth = 154 mm > 16 × 8 = 128 mm (d is at least $16d_b$). O.K.

A_v provided is 3 × 3 × 50.2 = 451.8 mm^2 at the first critical section, at distance d/2 ≈ 75 mm from the column face.

Example:

A flat plate floor has a thickness equals to 220 mm, and supported by 500 mm square columns spaced 6.0 m on center each way. Check the adequacy of the slab in resisting punching shear at a typical interior column, and provide shear reinforcement, if needed. The floor will carry a total factored load of 17.0 kN/m^2 and the factored slab moment resisted by the column is 40 kN.m.
Use effective depth = 170 mm, f_y = 420 MPa, and f_c' = 28.0 MPa.

Solution:-

The first critical section for punching shear is at distance d/2 = 85 mm from the column face.

$b_1 = c_1 + d = 500 + 170 = 670$ mm

$b_2 = c_2 + d = 500 + 170 = 670$ mm

Shear perimeter (b_o) = 2 b_1 + 2 b_2 = 2 × 670 + 2 × 670 = 2680 mm

The design shear strength of the concrete alone (without shear reinforcement) at the critical section d/2 from the face of the column is:

$$v_c = \min.\begin{cases} 0.33\sqrt{f_c'} = 0.33\sqrt{28} = 1.746 \text{ MPa} \\ 0.17\left(1 + \dfrac{2}{\beta}\right)\sqrt{f_c'} = 0.17\left(1 + \dfrac{2}{1}\right) \times \sqrt{28} = 2.699 \text{ MPa} \\ 0.083\left(2 + \dfrac{\alpha_s\, d}{b_o}\right)\sqrt{f_c'} = 0.083\left(2 + \dfrac{40 \times 170}{2680}\right) \times \sqrt{28} = 1.993 \text{ MPa} \end{cases}$$

$$\beta_c = \frac{500}{500} = 1$$

$\therefore \; v_c = 1.746$ MPa

$\phi v_c = 0.75 \times 1.746 = 1.31$ MPa

$V_u = 17.0 \times [\,(6.0)^2 - (0.67)^2\,] = 604.369$ kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{604.369 \times 10^3}{2680 \times 170} = 1.327 \text{ MPa}$$

$$v_{u,AB} = v_{ug} + \frac{\gamma_v \cdot M_{sc} \cdot c_{AB}}{J_c}$$

$$\gamma_f = \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{b_1}{b_2}}} = \frac{1}{1 + \left(\frac{2}{3}\right)\sqrt{\frac{670}{670}}} = 0.6$$

$\gamma_v = 1 - \gamma_f = 1 - 0.6 = 0.4$

Example:

The flat plate slab of 200 mm total thickness and 160 mm effective depth is carried by 300 mm square column 4.50 m on centers in each direction. A factored load of 370 kN and a factored slab moment resisted by the column is 44 kN.m must be transmitted from the slab to a typical interior column. Determine if shear reinforcement is required for the slab, and if so, design integral beams with vertical stirrups to carry the excess shear. Use f_y = 420 MPa, f_c' = 30 MPa.

Solution:-

The first critical section for punching shear is at distance d/2 = 80 mm from the column face.

$b_1 = c_1 + d =$

$b_2 = c_2 + d =$

Shear perimeter (b_o) = 2 b_1 + 2 b_2 =

The design shear strength of the concrete alone (without shear reinforcement) at the critical section d/2 from the face of the column is:

$$v_c = \min. \begin{cases} 0.33\sqrt{f_c'} = \\ 0.17\left(1+\dfrac{2}{\beta}\right)\sqrt{f_c'} = \\ 0.083\left(2+\dfrac{\alpha_s\, d}{b_o}\right)\sqrt{f_c'} = \end{cases}$$

$$\beta_c = \frac{300}{300} = 1$$

V_u = 370 kN

$$v_{ug} = \frac{V_u}{b_o \cdot d} = \frac{370 \times 10^3}{1840 \times 160} = 1.257 \text{ MPa}$$

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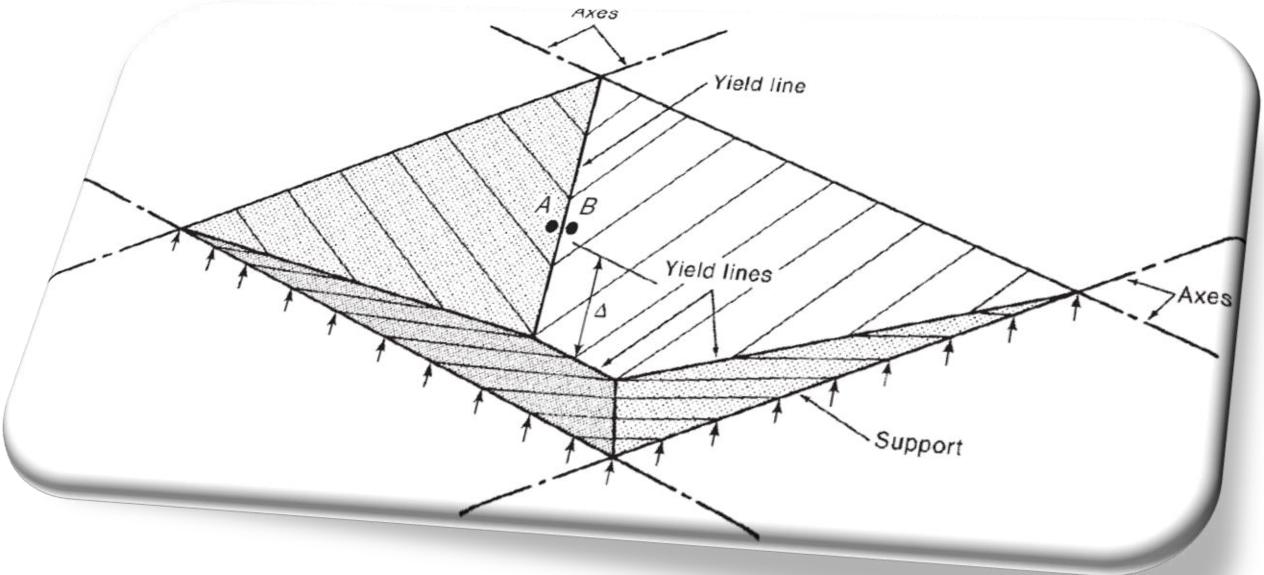
College of Engineering
Civil Engineering Department



REINFORCED CONCRETE DESIGN II

FOURTH YEAR CLASS

5

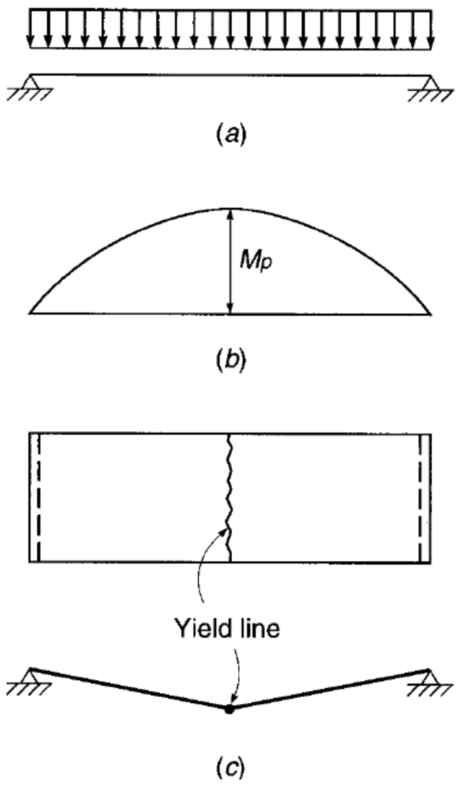
2018 - 2019

MHND

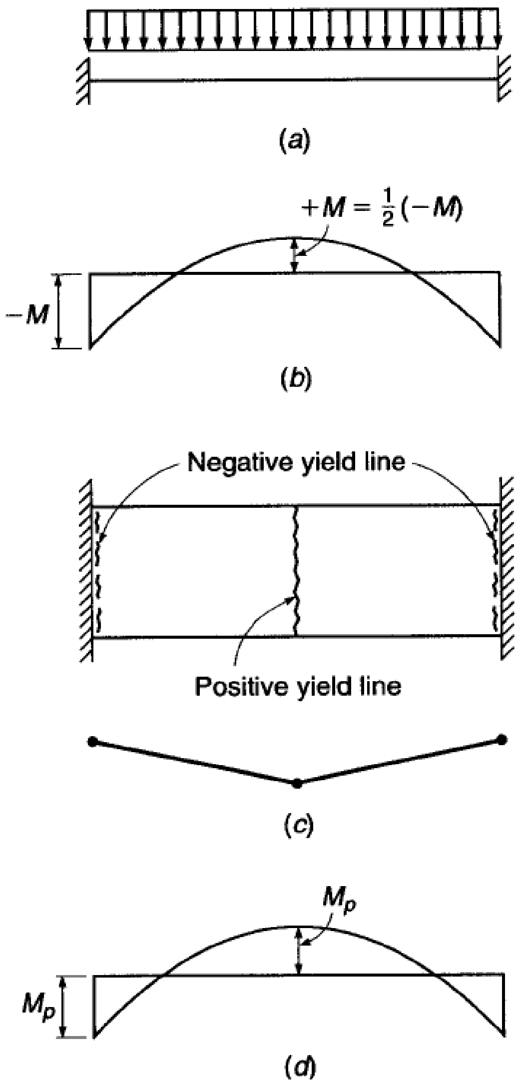
Yield Line Analysis for Slabs

In a slab failing in flexure, the reinforcement will yield first in a region of high moment. When that occurs, this portion of the slab acts as a plastic hinge, only able to resist its hinging moment. When the load is increased further, the hinging region rotates plastically, and the moments due to additional loads are redistributed to adjacent sections, causing them to yield. The bands in which yielding has occurred are referred to as yield lines and divide the slab into a series of elastic plates. Eventually, enough yield lines exist to form a plastic mechanism in which the slab can deform plastically without an increase in the applied load.

In the yield-line method for slabs, the loads required to develop a plastic mechanism are compared directly to the plastic resistance (nominal strength) of the member.



Simply supported uniformly loaded one-way slab.



Fixed-end uniformly loaded one-way slab.

<u>Axes of rotations</u>:

Yield lines form in regions of maximum moment and divide the slab into a series of elastic plate segments. When the yield lines have formed, all further deformations are concentrated at the yield lines, and the slab deflects as a series of stiff plates joined together by long hinges. The pattern of deformation is controlled by axes that pass along support lines, over columns, and by the yield lines. Because the individual plates rotate about the axes and/or yield lines, these axes and lines must be straight.

Location of Axes of rotations and yield-lines:

a- Axes of rotation generally lie along lines of support (the support line may be a real hinge as in simple supported, or it may establish the location of a yield line, which acts as a plastic hinge and in continuous or fixed support).

b- Axes of rotation pass over any columns.

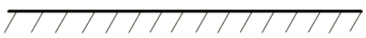
c- The slab segments can be considered to rotate as right bodies in space about these axes of rotation.

d- Yield lines are generally straight.

e- A yield line passes through the intersection of the axes of rotation of adjacent slab.

f- A yield line passes under the point load (concentrated force).

<u>Notations</u>:

— · — · — · — · — · — · —	Axis of rotation
	Positive yield line
	Negative yield line
	Simply supported
	Fixed or continuous support
———————————	Free edge
= = = = = = = = = = = = :	Beam
● ■	Column
⊗	Point load (concentrated force)
::	Line load

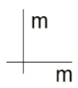
Isotropic slab: The slab is reinforced identically in all directions. The resisting moment, is the same along any line regardless of its location and orientation.
Orthotropic slab: The resisting moments are different in two perpendicular directions.

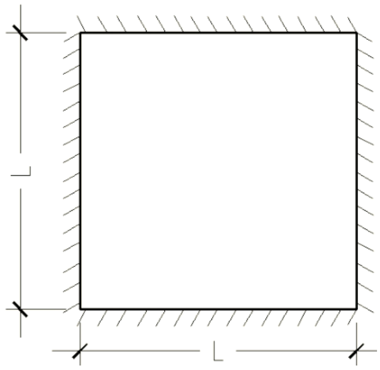
<u>Methods of solution</u>:
Once the general pattern of yielding and rotation has been established by applying the guid lines the location and the orientation of axes of rotation and the failure load for the slab can be established by either of two methods.

- Equilibrium method.
- Virtual-work method.

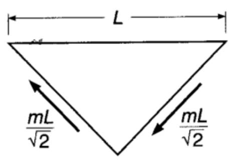
Equilibrium method:
By this method, the correct axes of rotation and the collapse load for the correspond mechanism can be found considering equilibrium of the slab segments. Each segment, studied as a free body, must be in equilibrium under the action of the applied load, the moments along the yield lines, and the reactions or shear force along the support line. Zero shear force and twisting moment along the positive yield line, and only moment per linear length (m) is considered in writing equilibrium equation.

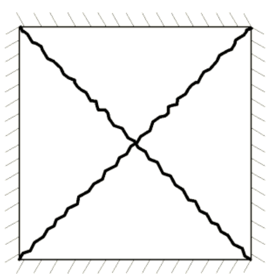
Example
A square slab is simply supported along all sides and is to be isotropically reinforced. Determine the ultimate resisting moment (m) per linear meter required just to sustain a uniformly distributed load (q) in kN/m^2.





Solution
Conditions of symmetry indicate the yield line pattern as shown.





Consider the moment equilibrium of any one of the identical slab segments about its support:

$$\sum \mathrm{M} = 0$$

$$\mathrm{q} \times \mathrm{L} \times \frac{\mathrm{L}}{2} \times \frac{1}{2} \times \frac{1}{3} \times \frac{\mathrm{L}}{2} = \frac{\mathrm{m\,L}}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \times 2$$

$$\mathrm{m} = \frac{\mathrm{q\,L^2}}{24}$$

Virtual-work method:

Since the moment and load are in equilibrium when the yield line pattern has formed, an increase in load will cause the structure to deflect further. The external work done by the loads to cause a small arbitrary virtual deflection must equal the internal work done as the slab rotates at the yield line to accommodate this deflection.

External work done by loads:

External work (EW or W_e) equals to the product of external load and the distance through which the point of application of the load moves. If the load is distributed over a length or an area rather than concentrated, the work can be calculated as the product of the total load and the displacement of the point of application of its resultant.

More complicated shapes may always be subdivided into components of triangles and rectangles. The total external work calculated by summing the work done by loads on the individual point of the failure mechanism.

Internal work done by resisting moment:

The internal work (IW or W_i) done during the assigned virtual displacement is found by summing the products of bending moment per unit length of yield line (m), the length of the yield line, and the angle change at that yield line corresponding to the virtual displacement (θ).

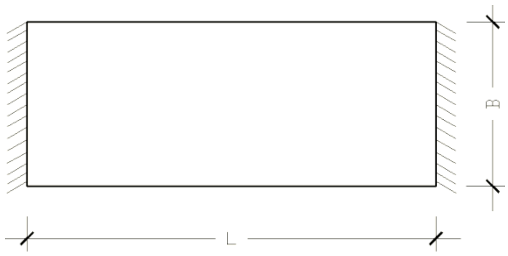
$$\mathrm{IW} = \sum [\mathrm{m}\,\ell\,\theta]$$

For orthotropic slab ($m_x \neq m_y$) it is necessary to choose the axes of moment parallel to the edges if possible

$$\mathrm{IW} = \sum \left[(\mathrm{m_x}\,\ell_\mathrm{x}\,\theta_\mathrm{x}) + \left(\mathrm{m_y}\,\ell_\mathrm{y}\,\theta_\mathrm{y}\right)\right]$$

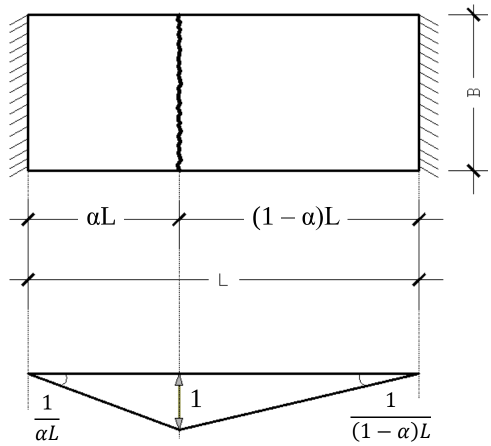
Example

Find the ultimate moment for the slab shown using the yield line theory. The slab is one way and simply supported of length (L) and normally loaded by a uniformly distributed load (w).



Solution

$$W_e = w \times B \times \alpha L \times \tfrac{1}{2} + w \times B \times (1-\alpha)L \times \tfrac{1}{2}$$



$$W_e = \frac{w\,B\,L}{2} \times [\alpha + (1-\alpha)]$$

$$W_e = \frac{w\,B\,L}{2}$$

$$W_i = \left[m \times B \times \frac{1}{\alpha L}\right] + \left[\left(m \times B \times \frac{1}{(1-\alpha)L}\right)\right]$$

$$W_i = m\,B\left[\frac{1}{\alpha L} + \frac{1}{(1-\alpha)L}\right]$$

$$W_e = W_i$$

$$\frac{w\,B\,L}{2} = m\,B\left[\frac{1}{\alpha L} + \frac{1}{(1-\alpha)L}\right]$$

$$w = \frac{2\,m}{L^2}\left[\frac{1}{\alpha} + \frac{1}{(1-\alpha)}\right]$$

to find the value of α, drive w with respect to α and equate the result to zero

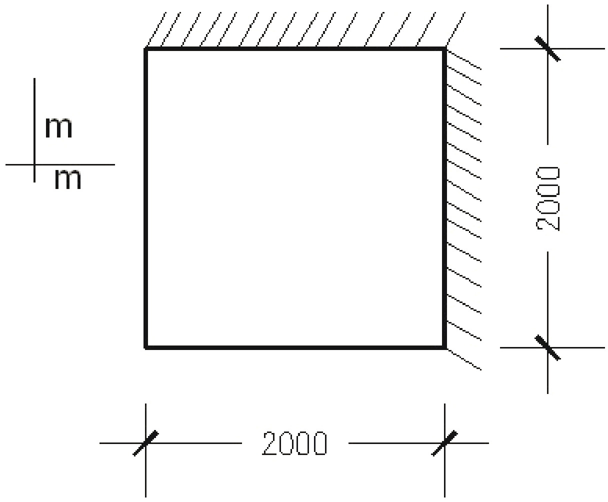
$$\frac{d\,w}{d\,\alpha} = \frac{2\,m}{L^2}\left[\frac{-1}{\alpha^2} + \frac{1}{(1-\alpha)^2}\right] = 0$$

$$\Rightarrow \quad \alpha = 0.5$$

$$\therefore\ w = \frac{2\,m}{L^2}\left[\frac{1}{0.5} + \frac{1}{(1-0.5)}\right] \quad \rightarrow \quad m = \frac{w\,L^2}{8}$$

Example

By using the yield line theory, determine the moment (m) for an isotropic reinforced concrete two-way slab shown in figure under a uniformly distributed load (w).



Solution

$$EW=\left(w\times 2.0\times 2.0\times \frac{1}{2}\times \frac{1}{3}\right)\times 2=\frac{4w}{3}$$

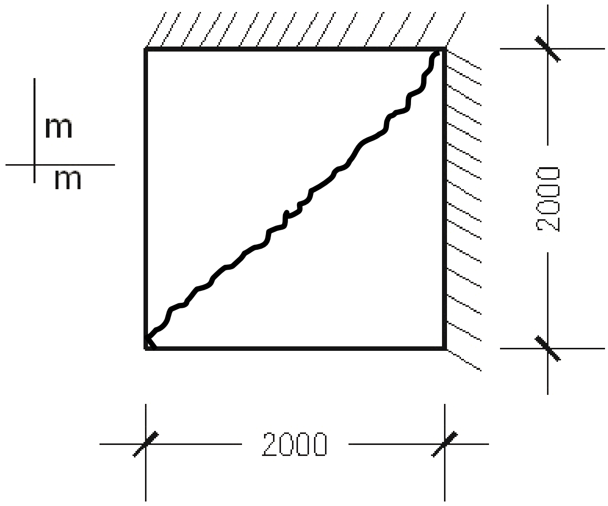
$$IW=\left(m\times 2\times \frac{1}{2}\right)\times 2=2m$$

$$EW=IW$$

$$\frac{4w}{3}=2\,m$$

$$\Rightarrow\ m=\frac{2w}{3}$$

$$m=0.667\,w$$



Example

By using the yield line theory, determine the moment (m) for an isotropic reinforced concrete two-way slab shown in figure under a consentrated force (P) on the free corner.

Solution

$$EW=P\times 1=P$$

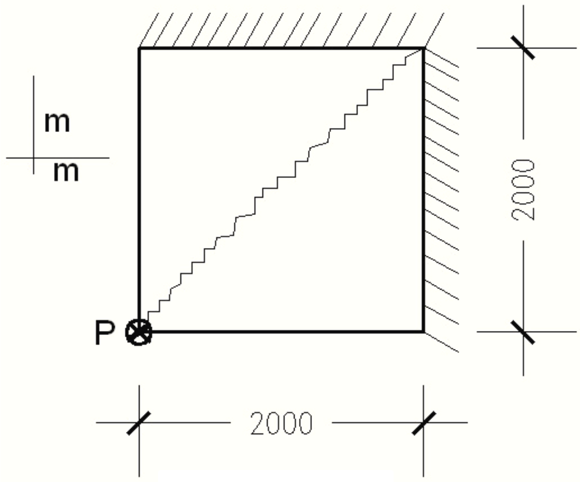
$$IW=\left(m\times 2\times \frac{1}{2}\right)\times 2=2m$$

$$EW=IW$$

$$P=2\,m$$

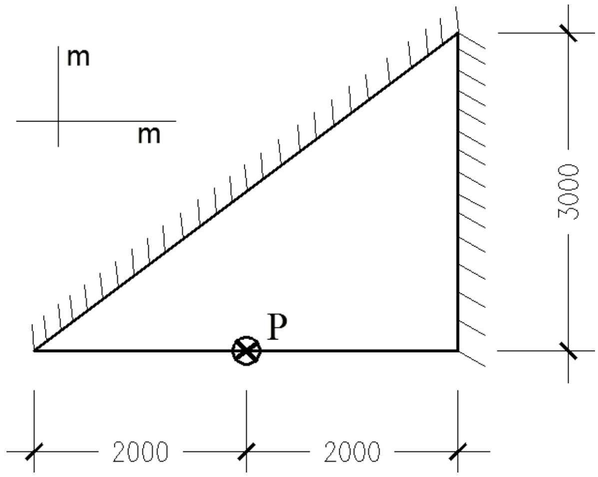
$$\Rightarrow\ m=\frac{P}{2}$$

$$m=0.5\,P$$



Example

By using the yield line theory, determine the moment (m) for an isotropic reinforced concrete two-way slab shown in figure under the load (P) (all dimensions are in mm).



Solution

$$W_e = P \times 1 = P$$

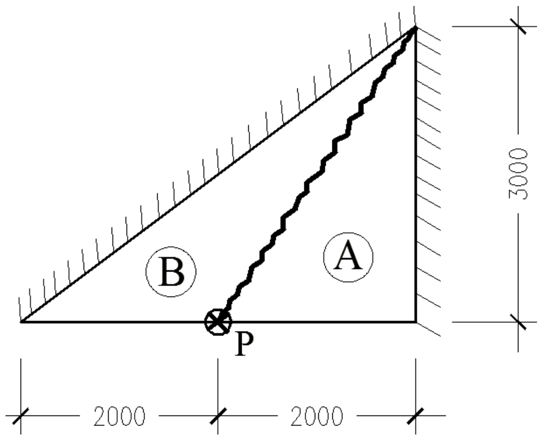
$$W_i = \left[m \times 3 \times \frac{1}{2}\right] + \left[\left(m \times 2 \times \frac{1}{1.5}\right) + \left(m \times 3 \times \frac{1}{2}\right)\right]$$

$$W_i = \left[\frac{3\text{ m}}{2}\right] + \left[\left(\frac{4\text{ m}}{3}\right) + \left(\frac{3\text{ m}}{2}\right)\right]$$

$$W_i = \frac{26\text{ m}}{6}$$

$$W_i = \frac{13\text{ m}}{3}$$

$$W_i = 4.333\text{ m}$$



$$W_i = W_e$$

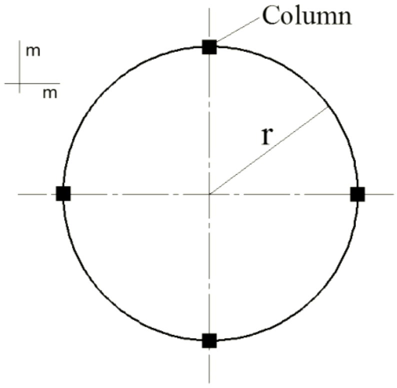
$$\frac{13\text{ m}}{3} = P$$

$$m = \frac{3\text{ P}}{13}$$

$$m = 0.231\text{ P}$$

Example

The circular slab of radius r supported by four columns, as shown in figure, is to be isotropically reinforced. Find the ultimate resisting moment (m) per linear meter required just to sustain a concentrated factored load of P kN applied at the center of the slab.

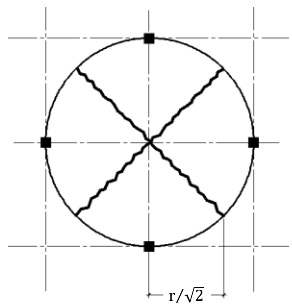


Solution

$$W_e = P \times 1 = P$$

$$W_i = \left(m \times \frac{r}{\sqrt{2}} \times 2 \times \frac{1}{r}\right) \times 4 = \sqrt{2}\, m$$

$$W_i = W_e$$

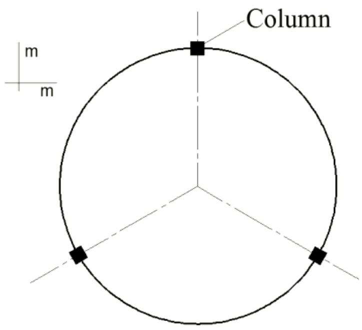


$$\sqrt{2}\, m = P$$

$$m = \frac{P}{\sqrt{2}}$$

Example

The circular slab of radius 2 m supported by three columns, as shown in figure, is to be isotropically reinforced. Find the ultimate resisting moment per linear meter (m) required just to sustain a uniformly distributed load (q) equals 16 kN/m^2.



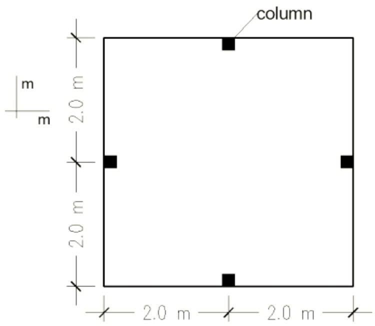
Solution

$W_e =$

$W_i =$

Example

By using the yield line theory, determine the ultimate resisting moment (m) for an isotropic reinforced concrete two-way slab shown in figure under a uniform load (q).



Solution

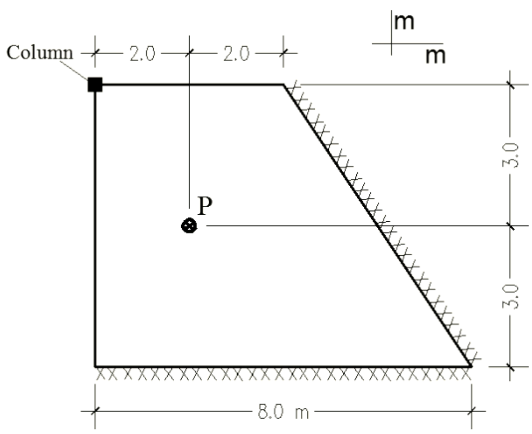
$W_e =$

$W_i =$

$W_e = W_i$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way slab to sustain a concentrated factored load of P kN applied as shown in figure.



Solution

$W_e = P \times 1 = P$

$W_i =$

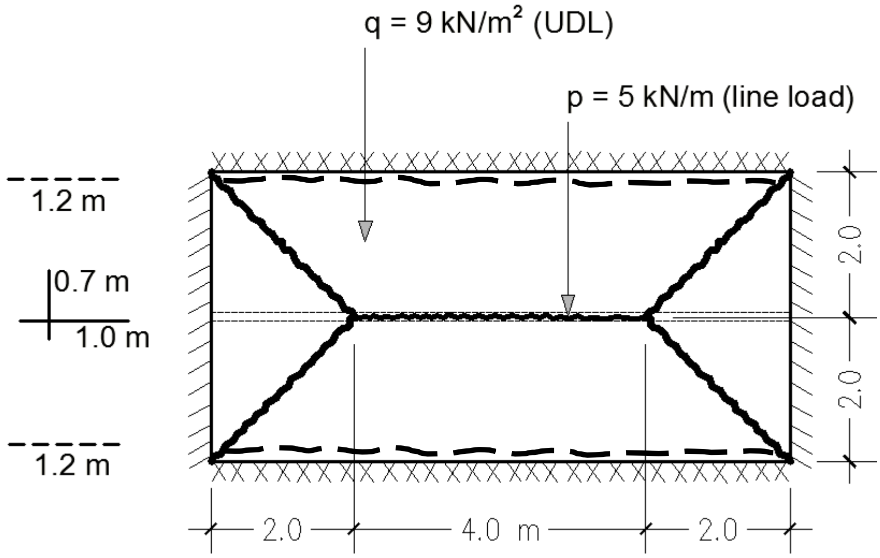
$W_e = W_i$

$P = 11.333\ m$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an orthotropic reinforced concrete two-way slab to sustain a uniformly distributed load and line load applied as shown in figure.

Solution



$$W_e = 9 \times \left[\left(2 \times 2 \times \frac{1}{2} \times \frac{1}{3} \times 8\right) + \left(4 \times 2 \times \frac{1}{2} \times 2\right)\right] + 5 \times \left[\left(2 \times \frac{1}{2} \times 2\right) + (4 \times 1)\right]$$

$$W_e = 150 \ \text{kN.m}$$

$$W_i = \left[0.7\,m \times 4 \times \frac{1}{2}\right] \times 2 + \left[\left(m \times 8 \times \frac{1}{2}\right) + \left(1.2\,m \times 8 \times \frac{1}{2}\right)\right] \times 2$$

$$W_i = 20.4\,m$$

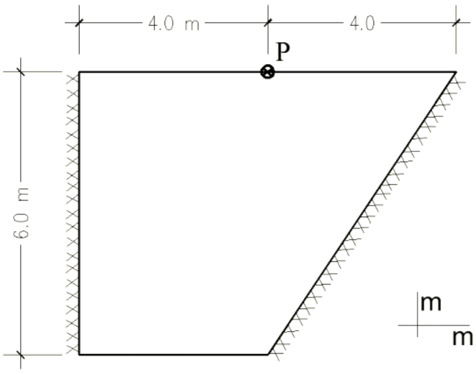
$$W_e = W_i$$

$$150 = 20.4\,m$$

$$m = 7.353 \ \text{kN.m/m}$$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way slab to sustain a concentrated factored load of P kN applied as shown in figure.

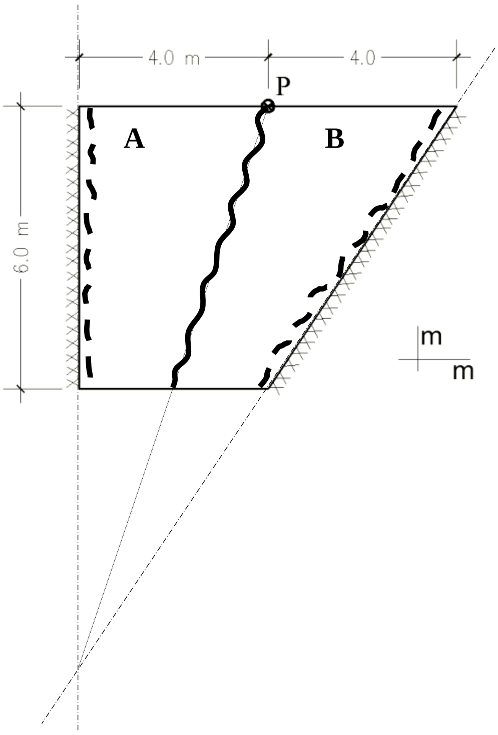


Solution

$W_e = P \times 1 = P$

$$W_i = \left[\left(m \times 6 \times \frac{1}{4}\right) + \left(m \times 6 \times \frac{1}{4}\right)\right] + \left[\left(m \times 2 \times \frac{1}{6} + m \times 6 \times \frac{1}{4}\right) + \left(m \times 4 \times \frac{1}{6} + m \times 6 \times \frac{1}{4}\right)\right]$$

$W_i = 7\,m$

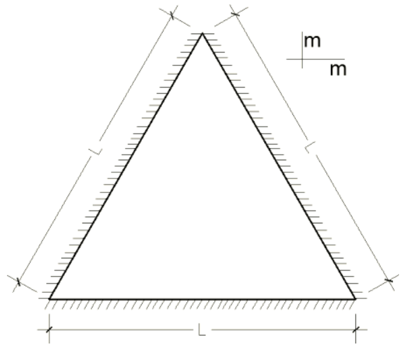


$W_e = W_i$

$P = 7\,m$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way simply supported triangle slab shown in figure under a uniform load (q).



Solution

$$W_e = q \times \left(L \times x \times \frac{1}{2} \times \frac{1}{3}\right) \times 3 \qquad x = \frac{L}{3}$$

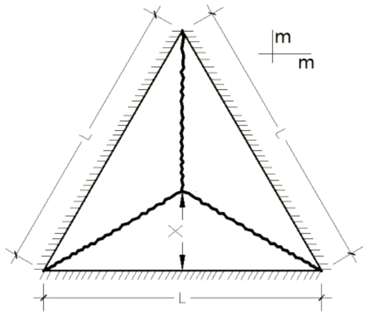
$$W_e = \frac{q\,L\,x}{2}$$

$$W_i = \left(m \times L \times \frac{1}{x}\right) \times 3 = \frac{3\,m\,L}{x}$$

$$W_e = W_i$$

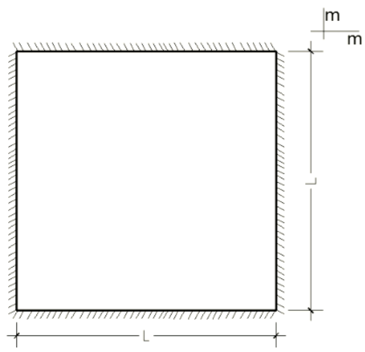
$$\frac{q\,L\,x}{2} = \frac{3\,m\,L}{x}$$

$$m = \frac{q\,x^2}{6} = \frac{q\,L^2}{54}$$



Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way simply supported square slab shown in figure under a uniform load (q).



Solution

$$W_e = q \times \left(L \times x \times \frac{1}{2} \times \frac{1}{3}\right) \times 4 \qquad x = \frac{L}{2}$$

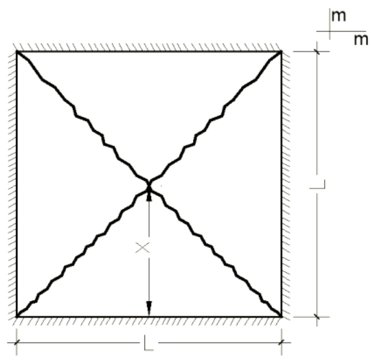
$$W_e = \frac{2\,q\,L\,x}{3}$$

$$W_i = \left(m \times L \times \frac{1}{x}\right) \times 4 = \frac{4\,m\,L}{x}$$

$$W_e = W_i$$

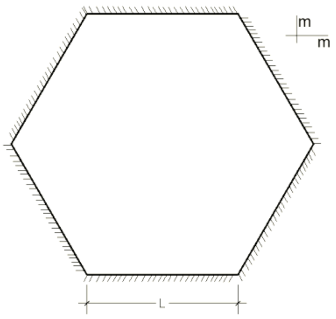
$$\frac{2\,q\,L\,x}{3} = \frac{4\,m\,L}{x}$$

$$m = \frac{q\,x^2}{6} = \frac{q\,L^2}{24}$$



Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way simply supported polygon slab shown in figure under a uniform load (q).

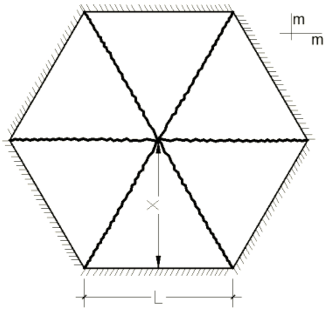


Solution

$$W_e = q \times \left(L \times x \times \frac{1}{2} \times \frac{1}{3}\right) \times 6 \qquad x = \frac{\sqrt{3}\,L}{2}$$

$$W_e = q\,L\,x$$

$$W_i = \left(m \times L \times \frac{1}{x}\right) \times 6 = \frac{6\,m\,L}{x}$$



$$W_e = W_i$$

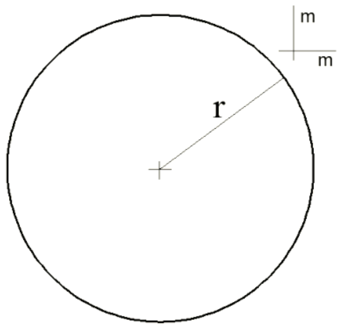
$$q\,L\,x = \frac{6\,m\,L}{x}$$

$$m = \frac{q\,x^2}{6} = \frac{q\,L^2}{8}$$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way simply supported circular slab shown in figure under a uniform load (q).

Solution



$$W_e =$$

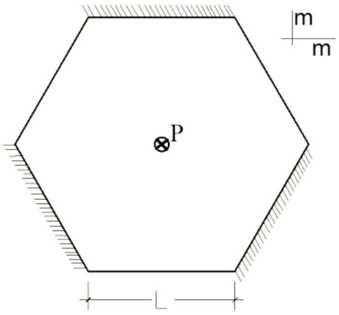
$$W_i =$$

$$W_e = W_i$$

$$m = \frac{q\,r^2}{6}$$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way simply supported polygon slab shown in figure under a concentrated factored load of P.

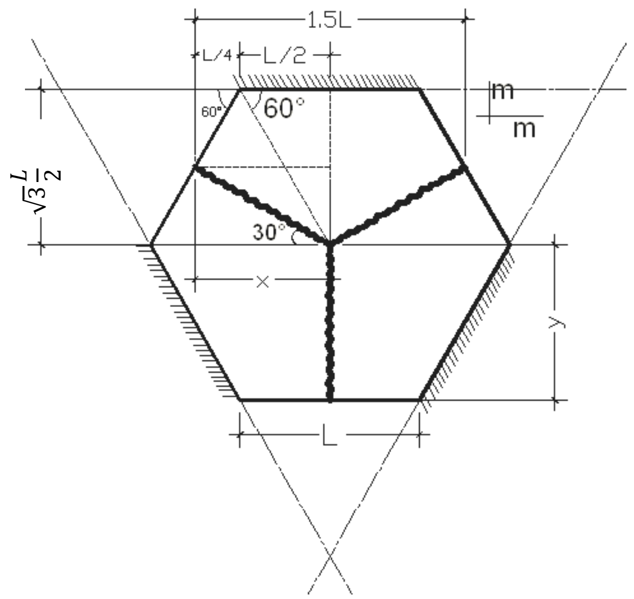


Solution

$W_e = P \times 1 = P$

$y = \sqrt{3}\,\frac{L}{2}$

$x = y \cdot \cos 30 = \frac{\sqrt{3}}{2}\,y$

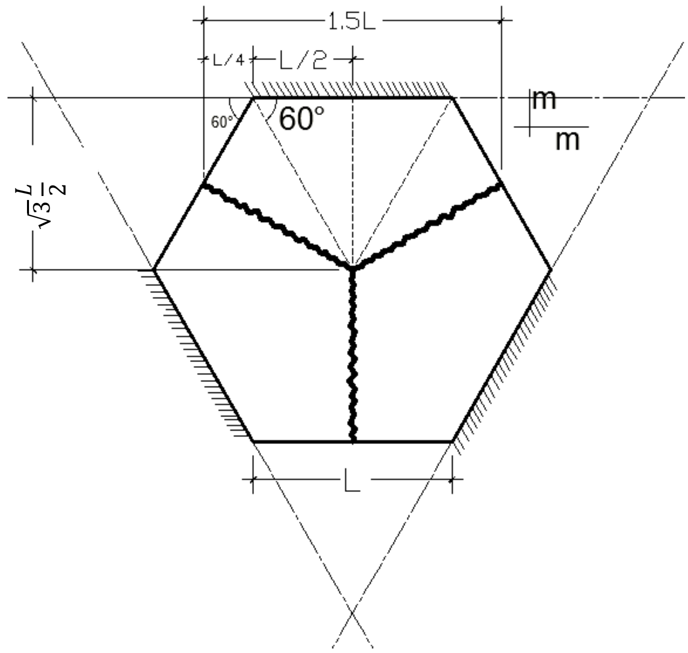


$$W_i = \left(m \times 1.5\,L \times \frac{1}{\sqrt{3}\frac{L}{2}} \right) \times 3 = 3\sqrt{3}\,m$$

$W_e = W_i$

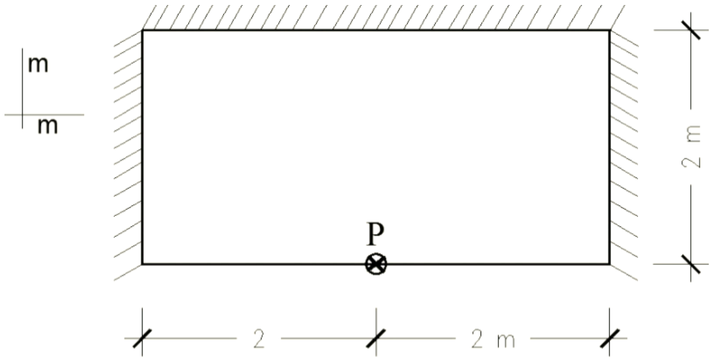
$P = 3\sqrt{3}\,m$

$m = \frac{P}{3\sqrt{3}} = 0.192\,P$



Example

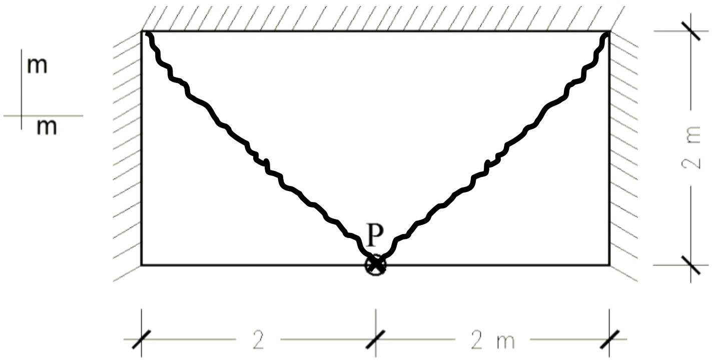
By using the yield line theory, determine the moment (**m**) for an isotropic reinforced concrete two-way slab shown in Figure under a concentrated factored load of **P**.



Solution

$$WE = P \times 1 = P$$

$$WI = \left(2 \times m \times \frac{1}{2}\right) \times 2 + \left(4 \times m \times \frac{1}{2}\right) = 4m$$



$$WE = WI$$

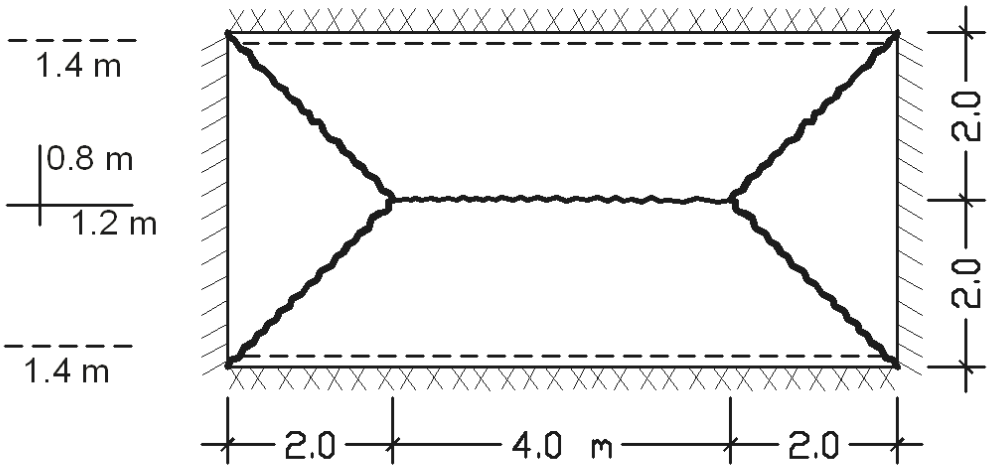
$$P = 4m$$

$$\Rightarrow \ m = \frac{P}{4}$$

$$m = 0.25P$$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an orthotropic rectangulare reinforced concrete two-way slab, shown in Figure, to sustain a uniformly distributed load equals 12 kN/m^2. Use the proposed positions for the positive and negative yield lines as shown in Figure.



Solution

$$W_e = 12 \times \left[\left(2 \times 2 \times \frac{1}{2} \times \frac{1}{3} \times 8\right) + \left(4 \times 2 \times \frac{1}{2} \times 2\right)\right]$$

$$W_e = 160 \text{ kN.m}$$

$$W_i = \left[0.8\text{ m} \times 4 \times \frac{1}{2}\right] \times 2 + \left[\left(1.2\text{ m} \times 8 \times \frac{1}{2}\right) + \left(1.4\text{ m} \times 8 \times \frac{1}{2}\right)\right] \times 2$$

$$W_i = 24\text{ m}$$

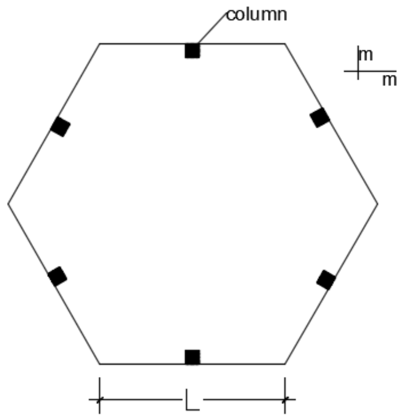
$$W_e = W_i$$

$$160 = 24\text{ m}$$

$$\text{m} = 6.667 \text{ kN.m/m}$$

Example

By using the yield line theory, determine the ultimate resisting moment per linear meter (m) for an isotropic reinforced concrete two-way polygon slab shown in figure under a uniform load (q).



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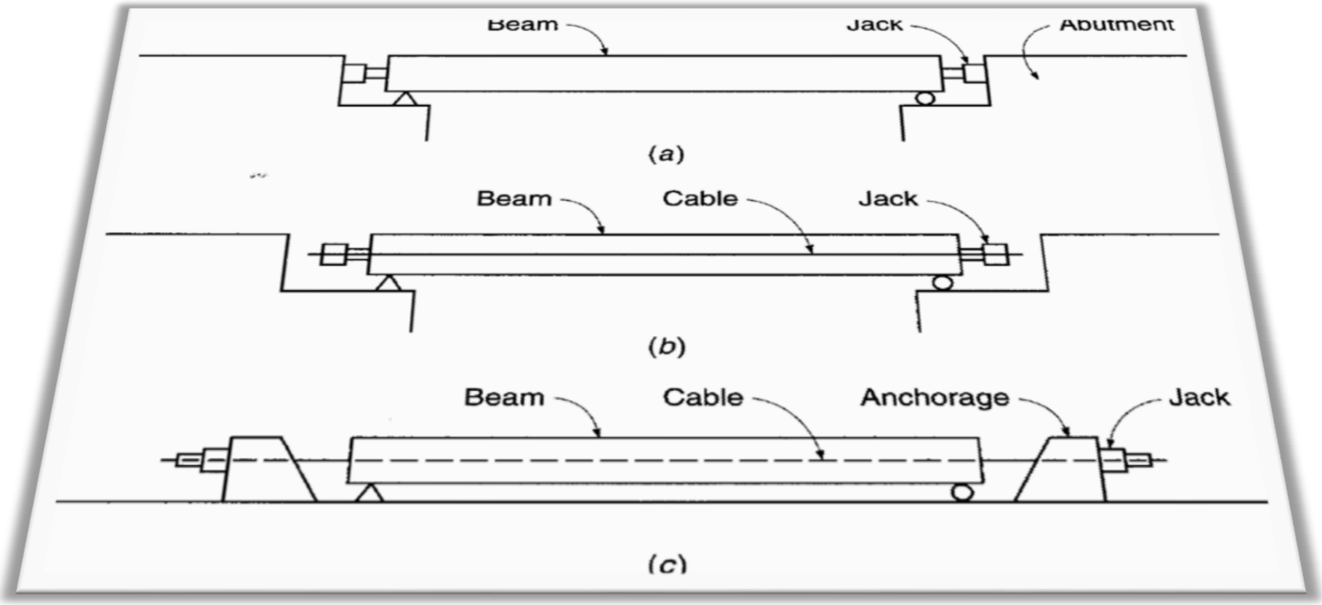
College of Engineering
Civil Engineering Department



REINFORCED CONCRETE DESIGN II

FOURTH YEAR CLASS

6

2018 - 2019

MHND

Prestressed Concrete

Prestressed concrete member can be defined as one in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from the given external loading are counteracted to a designed degree.

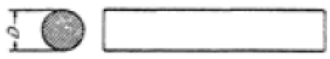
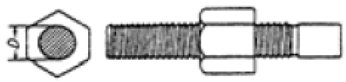
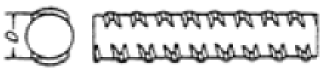
Advantage of prestressed concrete

1- High strength steel and concrete.
2- Eliminated cracks in concrete.
3- Prestressed concrete more suitable for structure of long span and those carrying heavy loads.
4- Under dead load, the deflection is reduced, owing to the cambering effected of prestress (useful for bridges and long cantilevers).

Disadvantage of prestressed concrete

1- Higher cost of materials.
2- More complicated formwork may be necessitated.
3- End anchorages and bearing plates are usually required.
4- Labor costs are greater.

<u>Tendon</u>: A stretched element used in a concrete member of structure to impart prestress to the concrete. Generally, high tensile steel wires, bars, cables or strands are used as tendons.
<u>Strand</u>: A group of wires (7 wires).
<u>Wires</u>: individually drawn wires of 7 mm diameter;
<u>Bar</u>: a specially formed bar of high strength steel of greater than 20 mm diameter
<u>Anchorage</u>: A device generally used to enable the tendon to impart and maintain prestress the concrete.

Type	Size (Diameter)		Shape
	mm	in.	
Plain round wire	2.0 ~ 9.0	0.06 ~ 0.360	
Indented wire	5.0 ~ 7.0	0.200 ~ 0.276	
Sumi - Twist	7.3 ~ 13.0	0.276 ~ 0.512	
Two-wire strand	2.9 x 2	0.114 x 2	
Seven-wire strand	6.2 ~ 15.2	0.250 ~ 0.600	
Nineteen-wire strand	17.8 ~ 21.8	0.700 ~ 0.860	
Round bar	9.2 ~ 32.0	0.362 ~ 1.260	
Threaded bar (Dywidag)	23.0 ~ 32.0	0.906 ~ 1.260	

Classifications and types

a- Externally and internally prestressed

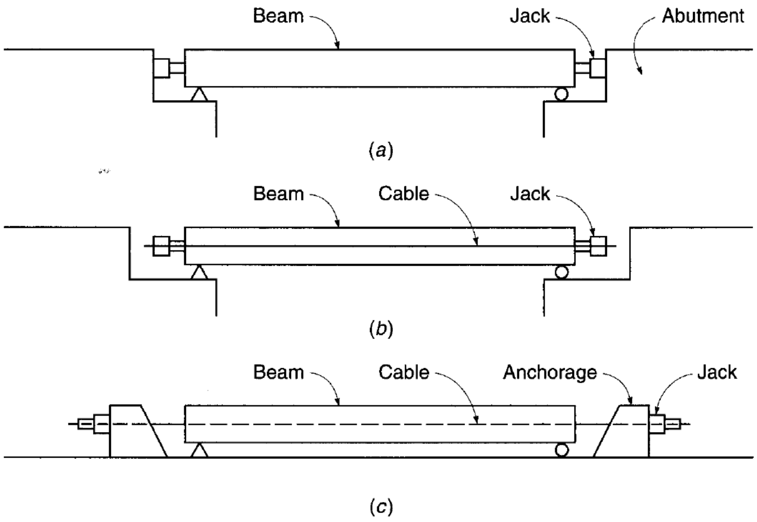
- Externally by jacking against abutments, this cannot be accomplished in practice, because even if abutment is stiff, shrinkage and creep in concrete y completely offset the strain.
- Internally accomplished by pretensioiny of steel.

b- Linear and circular prestressing

- Linear for beam and slabs, can be curved.
- Circular used for round tanks, silos, and pipes.

c- Pretensioning and postensioning

- Pretensioning: tendons tensioned before the concrete is placed, used in prestressing plants where permanent beds are provided for such tensioning.
- Posttensioning: tendons are tensioned after the concrete has hardened.



Prestressing methods: (a) post-tensioning by jacking against abutments; (b) post-tensioning with jacks reacting against beam; (c) pretensioning with tendon stressed between fixed external anchorages.

d- End-anchored and non-end-anchored tendons

- End-anchored: used in post tensioned, the tendons are anchored at their ends by means of mechanical devices to transmit the prestress to the concrete.
- Non-end-anchored: used in pretensioned where the tendons have their perstress transmitted to the concrete by their bond action near the ends. This type is limited to wires and strand of small size.

e- Bounded and unbounded tendons

Bounded: denote those bounded throughout their length to the surrounding concrete.

Non-end-anchored: tendons may be either bounded or unbounded to the concrete by grouting.

f- Precast, cast-in-place, composite construction

- Precasting: involves the placing of concrete away from its final position. This permits better control on mass production, and it is economical.
- Cast-in-place: concrete requires more form and false work.
- Composite: to precast pant of a member, erect it, casting the remaining portion in place.

g- Partial and full prestressing

- Full prestressing: the member is designed, so that, under working loads (service) there are no tensile stresses in it.
- Partial: tension is produced under working load. Addition, mild steel bars are provided to reinforce the tension zone.

Stages of Loading:

1- Initial stage: the member is under prestress, but is not subjected to any superimposed external loads.
2- Intermediate stage: during transportation and erection.
3- Final (service) stage: when the actual working load come on the structure.

Concrete:

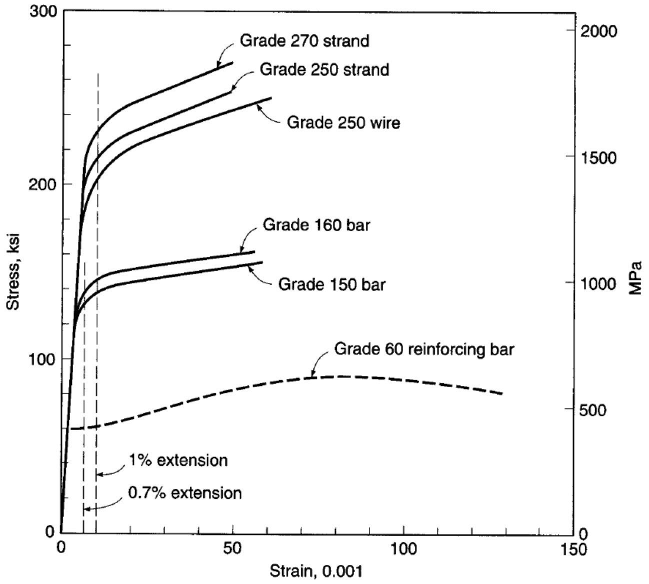
High strength concrete is used ($f_c' > 40$ MPa) for the following reasons:

1- High bearing stresses needed at end anchorage in post-tensioned.
2- High bond offered by high strength concrete in pretension.
3- A smaller cross sectional area can be used to carry a given load.
4- Higher modulus of elasticity, this means a reduction in initial elastic strain under application of prestress force and a reduction in creep strain. This results in a reduction in loss of prestress.

Steel:

The tensile strengths of prestressing steels range from about 2.5 to 6 times the yield strengths of commonly used reinforcing bars. The grade designations correspond to the minimum specified tensile strength in ksi (MPa). For the widely used seven-wire strand, two grades are recognized in ASTM A416: Grade 250 ksi (1725 MPa) and Grade 270 ksi (1860 MPa). For alloy steel bars, two grades are used: Grade 150 ksi (regular) and Grade 160 ksi. Round wires may be obtained in Grades 235, 240, and 250 ksi.

High strength steel must be used due to the low prestressing force obtained by using ordinary steel is quickly lost due to shrinkage and creep.



Typical stress-strain curves for prestressing steels.

Losses in prestressing force

The magnitude of prestress force will gradually decrease. The most significant causes are:-

1- Elastic shortening of concrete.
2- Concrete creep under sustained load.
3- Concrete shrinkage.
4- Relaxation of stress in steel.
5- Friction loss between the tendons and the concrete during stressing operation.
6- Loss due to slip of steel strands.

Summary of losses:

Pretensioned beam		**Post-tensioned beam**	
a- Before transfer			
- Shrinkage	3%	————	
b- At transfer			
- Elastic shortening	3%	- Elastic shortening	1%
		- Anchor slip	2%
		- Friction	2%
c- After transfer			
- Shrinkage	4%	- Shrinkage	4%
- Creep	7%	- Creep	4%
- Steel relation	3%	- Steel relation	3%
total	20%		16%

Analysis: to determine the stresses in the steel and concrete when the shape and size of a section are already given or assumed.

Design: to determine a suitable section for a given loading and stresses.

The analysis is a simpler operation than design.

The f_{pu} is the ultimate strength of the steel and f_{py} is the yield strength.

Stages of investigation of prestressed beam:

<u>Initial stage</u>

Initial force (P_i) plus beam weight (w_g):

Stress at top	$f_{ti} = \frac{-P_i}{A} + \frac{P_i \cdot e \cdot c_t}{I} - \frac{M_g \cdot c_t}{I}$
Stress at bottom	$f_{bi} = \frac{-P_i}{A} - \frac{P_i \cdot e \cdot c_b}{I} + \frac{M_g \cdot c_b}{I}$

<u>Service stage</u>

The beam under effective prestressing force (P_e) plus weight of the beam plus service load (live load plus weight of cast-in-situ concrete):

Stress at top	$f_{ts} = \frac{-P_e}{A} + \frac{P_e \cdot e \cdot c_t}{I} - \frac{M_g \cdot c_t}{I} - \frac{M_s \cdot c_t}{I}$
Stress at bottom	$f_{bs} = \frac{-P_e}{A} - \frac{P_e \cdot e \cdot c_b}{I} + \frac{M_g \cdot c_b}{I} + \frac{M_s \cdot c_b}{I}$

Permissible stresses in prestressed concrete flexural members

For calculation of stresses at transfer of prestress, at service loads, and at cracking loads, elastic theory shall be used with assumptions (a) and (b):

(a) Strains vary linearly with distance from neutral axis.

(b) At cracked sections, concrete resists no tension.

Classification of prestressed flexural members

Prestressed flexural members shall be classified as Class U, T, or C in accordance with Table 24.5.2.1, based on the extreme fiber stress in tension $\mathbf{f_t}$ in the precompressed tension zone calculated at service loads assuming an uncracked section.

Table 24.5.2.1 - Classification of prestressed flexural members based on $\mathbf{f_t}$

Assumed behavior	**Class**	**Limits of $\mathbf{f_t}$**
uncracked	U	$f_t \le 0.62\sqrt{f'_c}$
Transition between uncracked and cracked	T	$0.62\sqrt{f'_c} < f_t \le 1.0\sqrt{f'_c}$
cracked	C	$f_t > 1.0\sqrt{f'_c}$

Prestressed two-way slabs shall be designed as Class U

Three classes of behavior of prestressed flexural members are defined. Class U members are assumed to behave as uncracked members. Class C members are assumed to behave as cracked members. The behavior of Class T members is assumed to be in transition between uncracked and cracked. The serviceability requirements for each class are summarized in Table R24.5.2.1. For comparison, Table R24.5.2.1 also shows corresponding requirements for nonprestressed members.

Table R24.5.2.1—Serviceability design requirements

	Prestressed			Nonprestressed
	Class U	Class T	Class C	
Assumed behavior	Uncracked	Transition between uncracked and cracked	Cracked	Cracked
Section properties for stress calculation at service loads	Gross section 24.5.2.2	Gross section 24.5.2.2	Cracked section 24.5.2.3	No requirement
Allowable stress at transfer	24.5.3	24.5.3	24.5.3	No requirement
Allowable compressive stress based on uncracked section properties	24.5.4	24.5.4	No requirement	No requirement
Tensile stress at service loads 24.5.2.1	$\leq 0.62\sqrt{f'_c}$	$0.62\sqrt{f'_c} < f_t \leq 1.0\sqrt{f'_c}$	No requirement	No requirement
Deflection calculation basis	24.2.3.8, 24.2.4.2 Gross section	24.2.3.9, 24.2.4.2 Cracked section, bilinear	24.2.3.9, 24.2.4.2 Cracked section, bilinear	24.2.3, 24.2.4.1 Effective moment of inertia
Crack control	No requirement	No requirement	24.3	24.3
Computation of Δf_{ps} or f_s for crack control	—	—	Cracked section analysis	$M/(A_s \times$ lever arm), or $2/3f_y$
Side skin reinforcement	No requirement	No requirement	9.7.2.3	9.7.2.3

For Class U and T members, stresses at service loads shall be permitted to be calculated using the uncracked section.

For Class C members, stresses at service loads shall be calculated using the cracked transformed section.

Permissible concrete stresses at transfer of prestress

Calculated extreme concrete fiber stress in compression immediately after transfer of prestress, but before time-dependent prestress losses, shall not exceed the limits in Table 24.5.3.1.

Table 24.5.3.1—Concrete compressive stress limits immediately after transfer of prestress

Location	Concrete compressive stress limits
End of simply-supported members	$0.70f_{ci}'$
All other locations	$0.60f_{ci}'$

Calculated extreme concrete fiber stress in tension immediately after transfer of prestress, but before time-dependent prestress losses, shall not exceed the limits in Table 24.5.3.2, unless permitted by 24.5.3.2.1.

Table 24.5.3.2—Concrete tensile stress limits immediately after transfer of prestress, without additional bonded reinforcement in tension zone

Location	Concrete tensile stress limits
Ends of simply-supported members	$0.50\sqrt{f'_{ci}}$
All other locations	$0.25\sqrt{f'_{ci}}$

Permissible concrete compressive stresses at service loads

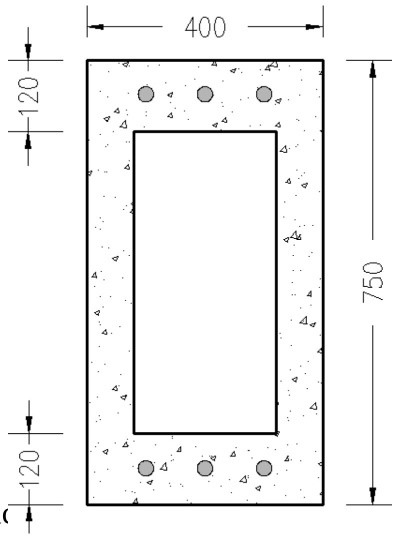
For Class U and T members, the calculated extreme concrete fiber stress in compression at service loads, after allowance for all prestress losses, shall not exceed the limits in Table 24.5.4.1.

Table 24.5.4.1—Concrete compressive stress limits at service loads

Load condition	Concrete compressive stress limits
Prestress plus sustained load	$0.45f_c'$
Prestress plus total load	$0.60f_c'$

Example

A prestress rectangular box beam post-tensioned by straight high tensile steel wires of total area A_s mm^2, equally divided between the top and bottom flanges and placed on center of flanges. The forces are initially stressed to 850 N/mm^2 and the total losses of prestress is 15%. The beam is required to carry a uniformly distributed superimposed load of 4.5 kN/m in addition to its own weight, over a span of 15 m. If the concrete stresses are not to exceed 17.5 N/mm^2 in compression and 1 N/mm^2 in tension (during the prestressing operation and working load). Calculate the max. and min. A_s of steel, which may be used. Use γ_c = 25 kN/m^3



Solution

$A = 400 \times 750 - 240 \times 510 = 177600\ mm^2$

$$I = \frac{400 \times (750)^3 - 240 \times (510)^3}{12} = 1.140948 \times 10^{10}\ mm^4$$

$w_g = 177600 \times 10^{-6} \times 25 = 4.44\ kN/m$

Note:

a- check compressive stress at initial stage.

b- check compressive stress at top and tensile stress at bottom at servic

a- Immediately after prestressing

prestressing force before losses = 850 A_s

initial compressive stress =

$$f_{ti} = f_{bi} = \frac{P_i}{A}$$

$$-17.5 \le \frac{-850 \times A_s}{177600} \quad \Rightarrow \quad A_s \le 3656.5\ mm^2$$

b- Service stage (final stage)

1- Top fiber

Prestressing stress after losses =

Final stress @ top =

2- Bottom fiber:

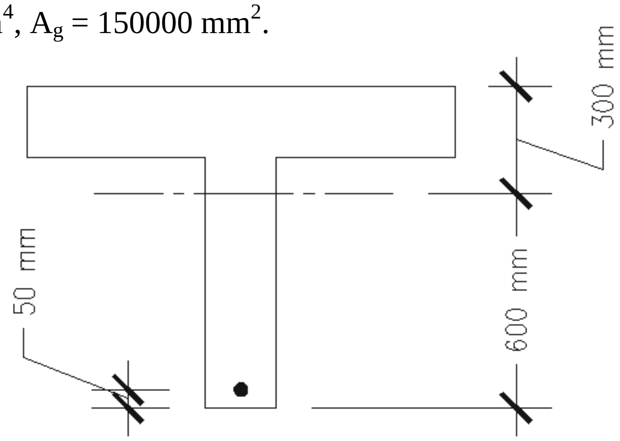
Example

A simply supported prestressed beam, of span 8 m and its cross section is shown in Figure, is carrying a live load equals to 12 kN/m. Compute the required prestressing forces for:

a) Tob fiber stress equals to zero under beam weight plus prestressing force only.

b) Bottom fiber stress equals to zero under full load.

Use γ_c = 25 kN/m^3, I = 120 × 10^8 mm^4, A_g = 150000 mm^2.



Solution

$c_t = 300\text{ mm} \qquad c_b = 600\text{ mm} \qquad e = 550\text{ mm}$

$$w_g = A \cdot \gamma = 150000 \times 10^{-6} \times 25 = 3.75\text{ kN/m}$$

$$M_g = \frac{w_g \cdot L^2}{8} = \frac{3.75 \times (8)^2}{8} = 30.0\text{ kN.m}$$

a) Top fiber stress equals to zero under beam weight plus prestressing force only at top fiber

$$f_{ti} = \frac{-P_e}{A} + \frac{P_e \cdot e \cdot c_t}{I} - \frac{M_g \cdot c_t}{I}$$

$$0 = \frac{-P \times 10^3}{150000} + \frac{P \times 10^3 \times 550 \times 300}{120 \times 10^8} - \frac{30 \times 10^6 \times 300}{120 \times 10^8}$$

$$0 = \frac{-P}{15} + \frac{P \times 11}{80} - \frac{30}{4}$$

$$\Rightarrow \quad P = 7.5 \times 17 = 105.882\text{ kN}$$

b) Bottom fiber stress equals to zero under full load

$$M_s = \frac{w_s \cdot L^2}{8} = \frac{12 \times (8)^2}{8} = 96\text{ kN.m}$$

$$f_{bs} = \frac{-P_e}{A} - \frac{P_e \cdot e \cdot c_b}{I} + \frac{M_g \cdot c_b}{I} + \frac{M_s \cdot c_b}{I}$$

$$0 = \frac{-P \times 10^3}{150000} - \frac{P \times 10^3 \times 550 \times 600}{120 \times 10^8} + \frac{30 \times 10^6 \times 600}{120 \times 10^8} + \frac{96 \times 10^6 \times 600}{120 \times 10^8}$$

$$0 = \frac{-P}{15} - \frac{P \times 11}{40} + 15 + 48$$

$$\Rightarrow \quad P = \frac{63 \times 120}{41} = 194.390\text{ kN}$$

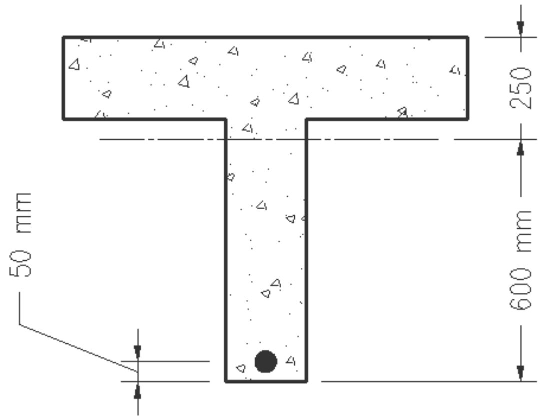
Example

A simply supported prestressed beam, of span 8 m and its cross section is shown in Figure, is carrying a live load equals to 10 kN/m. Compute the required prestressing forces for:

a) Top fiber stress equals to zero under beam weight plus prestressing force only.

b) Bottom fiber stress equals to zero under full load.

Use γ_c = 25 kN/m^3, I = 10 × 10^9 mm^4, A_g = 100000 mm^2.



Solution

$c_t = 250\text{ mm} \qquad c_b = 600\text{ mm} \qquad e = 550\text{ mm}$

$$w_g = A\cdot\gamma = 100000\times10^{-6}\times25 = 2.5\text{ kN/m}$$

$$M_g = \frac{w_g\cdot L^2}{8} = \frac{2.5\times(8)^2}{8} = 20.0\text{ kN.m}$$

a) Top fiber stress equals to zero under beam weight plus prestressing force only at top fiber

$$f_{ti} = \frac{-P_e}{A} + \frac{P_e\cdot e\cdot c_t}{I} - \frac{M_g\cdot c_t}{I}$$

$$0 = \frac{-P\times10^3}{100000} + \frac{P\times10^3\times550\times250}{10\times10^9} - \frac{20\times10^6\times250}{10\times10^9}$$

$$0 = \frac{-P}{100} + \frac{P\times1.375}{100} - \frac{5}{10}$$

$$\Rightarrow \quad P = 133.333\text{ kN}$$

b) Bottom fiber stress equals to zero under full load

$$M_s = \frac{w_s\cdot L^2}{8} = \frac{10\times(8)^2}{8} = 80\text{ kN.m}$$

$$f_{bs} = \frac{-P_e}{A} - \frac{P_e\cdot e\cdot c_b}{I} + \frac{M_g\cdot c_b}{I} + \frac{M_s\cdot c_b}{I}$$

$$0 = \frac{-P\times10^3}{100000} - \frac{P\times10^3\times550\times600}{10\times10^9} + \frac{20\times10^6\times600}{10\times10^9} + \frac{80\times10^6\times600}{10\times10^9}$$

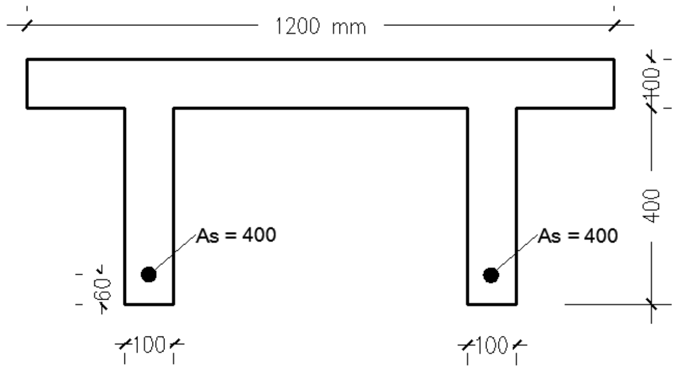
$$0 = \frac{-P}{100} - \frac{P\times3.3}{100} + \frac{12}{10} + \frac{48}{10}$$

$$\Rightarrow \quad P = 139.535\text{ kN}$$

Example

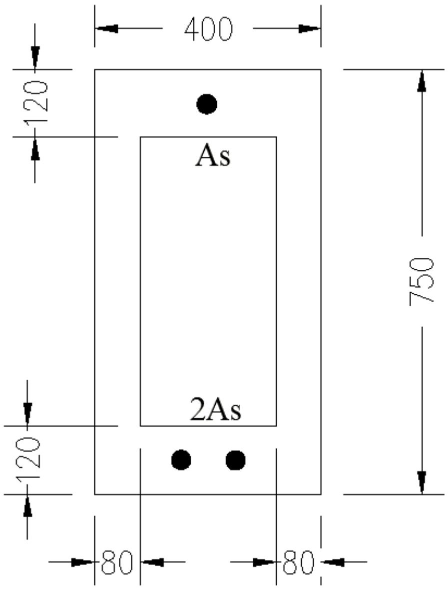
A double-T simply supported concrete beam its cross section is shown in Figure, is prestressed with 2 tendons each 400 mm^2. Determine the allowable service load.

Use span = 12 m, f_{se} = 1300 MPa, f_c' = 40 MPa, γ_c = 25 kN/m^3.



Example

A prestressed simply supported 15 m span beam with rectangular box section is post-tensioned by straight high tensile steel wires as shown in Figure. The prestressing wires are placed at the center line of the flanges and initially stressed to 850 N/mm^2. The beam is required to carry a uniformly distributed superimposed load of 4.5 kN/m in addition to its weight. If the concrete stresses are not to exceed 17 MPa in compression and 1 MPa in tension at service stage, calculate the range of the total prestressing wires area required. Ignore prestressing force losses in your answer. (γ_c = 24 kN/m^3).



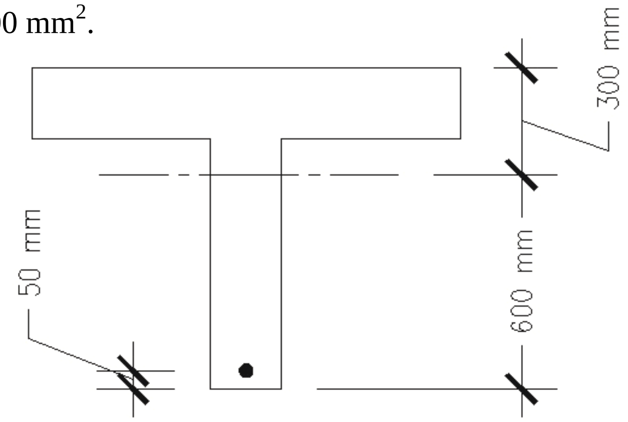
Example

A simply supported prestressed beam, of span 10 m and its cross section is shown in Figure, is carrying a live load equals to 10 kN/m. Compute the required prestressing forces for:

a) Top fiber stress equals to zero under beam weight plus prestressing force only.

b) Bottom fiber stress equals to zero under full load.

Use γ_c = 25 kN/m^3, I = 150 × 10^8 mm^4, A_g = 100000 mm^2.

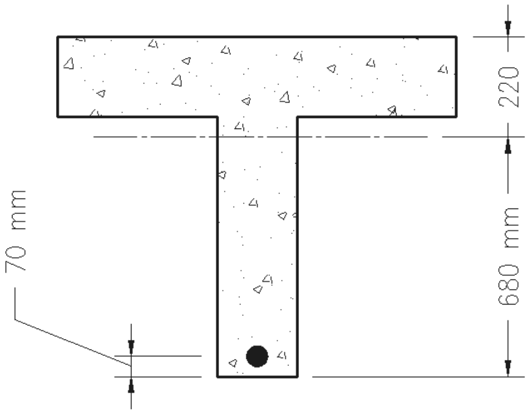


Example

A simply supported prestressed concrete beam, of span 10 m and its cross section is shown in Figure, is carrying a service load equals to 12 kN/m. Compute the required prestressing forces for:

a) Top fiber stress equals to zero under beam weight plus prestressing force only.
b) Bottom fiber stress equals to zero under full loads.

Use γ_c= 24 kN/m^3, I = 12×10^9 mm^4, A_g = 120000 mm^2.



<u>Solution</u>

$c_t = 220$ mm $\qquad c_b = 680$ mm $\qquad e = 610$ mm

$$w_g = A \cdot \gamma = 100000 \times 10^{-6} \times 24 = 2.4 \text{ kN/m}$$

$$M_g = \frac{w_g \cdot L^2}{8} = \frac{2.4 \times (10)^2}{8} = 30.0 \text{ kN.m}$$

a) Top fiber stress equals to zero under beam weight plus prestressing force only at top fiber

$$f_{ti} = \frac{-P}{A} + \frac{P \cdot e \cdot c_t}{I} - \frac{M_g \cdot c_t}{I}$$

$$0 = \frac{-P \times 10^3}{120000} + \frac{P \times 10^3 \times 610 \times 220}{12 \times 10^9} - \frac{30 \times 10^6 \times 220}{12 \times 10^9}$$

$$0 = \frac{-P}{120} + \frac{P \times 1.342}{120} - \frac{6.6}{12}$$

$\Rightarrow \quad P = 192.982$ kN

b) Bottom fiber stress equals to zero under full loads

$$M_s = \frac{w_s \cdot L^2}{8} = \frac{12 \times (10)^2}{8} = 150 \text{ kN.m}$$

$$f_{bs} = \frac{-P_e}{A} - \frac{P_e \cdot e \cdot c_b}{I} + \frac{M_g \cdot c_b}{I} + \frac{M_s \cdot c_b}{I}$$

$$0 = \frac{-P \times 10^3}{120000} - \frac{P \times 10^3 \times 610 \times 680}{12 \times 10^9} + \frac{30 \times 10^6 \times 680}{12 \times 10^9} + \frac{150 \times 10^6 \times 680}{12 \times 10^9}$$

$$0 = \frac{-P}{120} - \frac{P \times 4.148}{120} + \frac{20.4}{12} + \frac{102}{12}$$

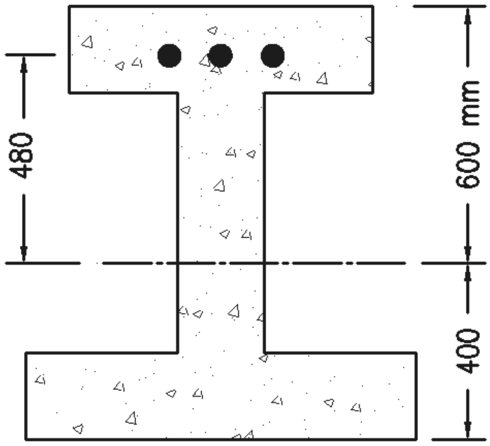
$\Rightarrow \quad P = 237.762$ kN

Example

A cantilever prestressed concrete beam, of span 6 m and its cross section is shown in Figure, is carrying a service load equals to 12 kN/m. Compute the required prestressing forces for:

a) Bottom fiber stress equals to zero under beam weight plus prestressing force only.
b) Top fiber stress equals to zero under full loads.

Use γ_c= 25 kN/m^3, I = 18×10^9 mm^4, A_g = 120000 mm^2.



Solution

$c_t = 600$ mm $\qquad c_b = 400$ mm $\qquad e = 480$ mm

$$w_g = A \cdot \gamma = 120000 \times 10^{-6} \times 25 = 3.0 \text{ kN/m}$$

$$M_g = \frac{w_g \cdot L^2}{2} = \frac{3.0 \times (6)^2}{2} = 54.0 \text{ kN.m}$$

a) Bottom fiber stress equals to zero under beam weight plus prestressing force only at bottom fiber

$$f_{bi} = \frac{-P}{A} + \frac{P \cdot e \cdot c_b}{I} - \frac{M_g \cdot c_b}{I}$$

$$0 = \frac{-P \times 10^3}{120000} + \frac{P \times 10^3 \times 480 \times 400}{18 \times 10^9} - \frac{54 \times 10^6 \times 400}{18 \times 10^9}$$

$$\Rightarrow \quad P = 514.286 \text{ kN}$$

b) Top fiber stress equals to zero under full loads

$$M_s = \frac{w_s \cdot L^2}{2} = \frac{12 \times (6)^2}{2} = 216 \text{ kN.m}$$

$$f_{ts} = \frac{-P}{A} - \frac{P \cdot e \cdot c_t}{I} + \frac{M_g \cdot c_t}{I} + \frac{M_s \cdot c_t}{I}$$

$$0 = \frac{-P \times 10^3}{120000} - \frac{P \times 10^3 \times 480 \times 600}{18 \times 10^9} + \frac{54 \times 10^6 \times 600}{18 \times 10^9} + \frac{216 \times 10^6 \times 600}{18 \times 10^9}$$

$$\Rightarrow \quad P = 369.863 \text{ kN}$$

Example

A simply supported rectangular prestressed concrete beam, of span 13 m and its cross section as shown in figure, is carrying a live load equals to 30 kN/m in addition to its weight, compute the following stresses and compare it with ACI allowable stress:

a) Bottom fiber stress at support in initial stage.
b) Top fiber stress at mid span in final stage.

Use $\gamma_c = 24\,\mathrm{kN/m^3}$, $\mathrm{As} = 600\ \mathrm{mm^2}$, initial stress of the prestressed steel = 1200 MPa, total losses is 20%, $\mathrm{f_{ci}} = 22$ MPa, and $\mathrm{f_c}' = 28$ MPa