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Fraction Sense!
Why? Fractions are Foundational!

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All students should leave elementary and middle school with a strong sense of number

What does that mean?

How do we do that?

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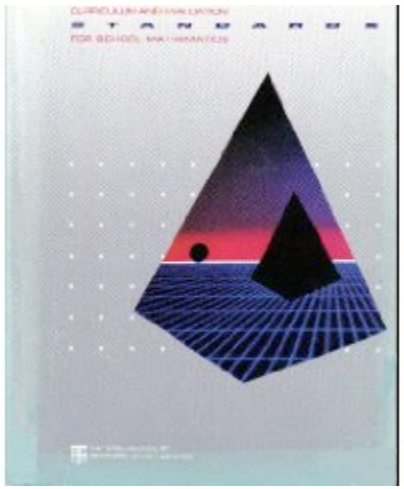
Number Sense - History

- Number Meaning
- Relationships
- Magnitude
- Operation Sense
- Real Life Number Sense - Applications



Howden, 1989

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Making Sense of Numbers…

1. Ability to compose and decompose numbers…
2. Ability to recognize the relative magnitude of numbers – including comparing and ordering.
3. Ability to deal with the absolute magnitude of numbers – realizing, for instance there are far fewer than 500 people in this session!
4. Ability to use benchmarks.
5. Ability to link numeration, operation, and relation symbols in meaningful ways.
6. Understanding the effects of operations on numbers.
7. The ability to perform mental computation through invented strategies that take advantages of numerical and operational properties.
8. Being able to use numbers flexibly to estimate numerical answers to computations, and to recognize when an estimate is appropriate.
9. A disposition towards making sense of numbers.

"It is possible to have good number sense for whole numbers, but not for fractions…"

Sowder, J. and Schappelle, Eds., 1989

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And more recently…

- In its most fundamental form, number sense entails an ability to immediately identify the numerical value associated with small quantities;
- …this more highly developed form of number sense should extend to numbers written in fraction, decimal, and exponential forms.
- …poor number sense interferes with learning algorithms and number facts and prevents use of strategies to verify if solutions to problems are reasonable.

NMAP, page 27

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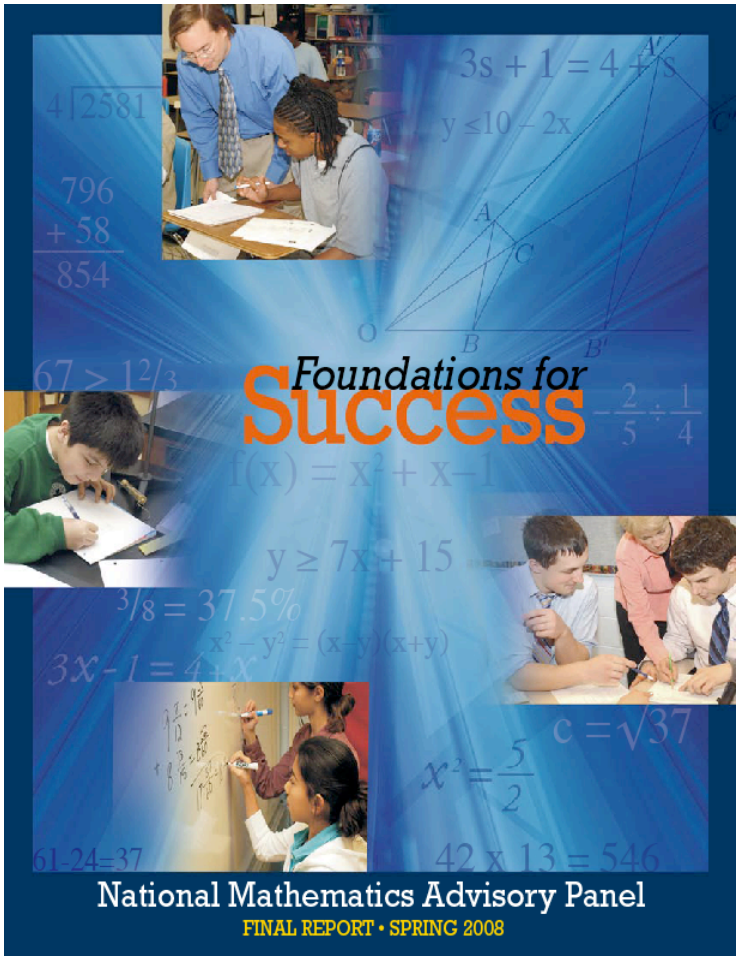
How did this get started…

Fraction issues…

- Conceptual Knowledge and Skills
- Learning Processes
- Assessment
- Survey of Algebra Teachers



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Do you have a sense of number?

- Is 7/8 > 5/6? How do you know?
- What is 6.2 x 12?
- If the restaurant bill was $119.23, how much of a tip should you leave?
- If a 10-year old is 5’ tall, how tall will the child be at age 20?
- Each MLB team plays is 162 games. Suggest an end-of-season wins to losses ratio for the Houston Astros
- At this writing the Astros had a record of 38 wins and 41 losses.

Policy and Political Issues

- Number sense is developed!
- Where and how does this fit in the TEKS?
- This is more than whole numbers!

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Curriculum Issues

Grade 1

- Equal Parts
- Halves – region model
- Thirds, Fourths – region model
- Fractions of a set – halves, thirds, fourths
- Non-unit fractions
  - Region and set models

Grade 3

- Dividing Regions into Equal Parts
- Identifying Fractions – region model
- Identifying Fractions – set model
- Fraction Benchmarks
- Equivalent Fractions – fraction bars
- Comparing Fractions – fraction bars
- Fractions on the Number Line
- Adding Fractions – Like denominators – fraction bars
- Subtracting Fractions - Like denominators – fraction bars
- Fractions and Decimals
- Money and Decimals
- Adding and Subtracting Money

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Curriculum Issues

Grade 5

- Identifying Fractions – region, set and number line models
- Fractions and sharing – multiple models
- Mixed Numbers and Improper Fractions
- Decimals – tenths and hundredths
- Decimals – thousandths
- Comparing and Ordering Fractions and Decimals on the Number Line.
- Equivalent Fractions – region model
- Equivalent Fractions – number line
- Simplest Form
- Comparing Fractions – fraction bars
- Comparing Fractions – denominators, numerators
- Ordering Fractions
- Comparing and Ordering Mixed Numbers

Grade 5 (cont.)

- Adding Fractions – Like denominators – fraction bars
- Subtracting Fractions - Like denominators – fraction bars
- Adding and Subtracting Fractions – Number Line
- Adding and Subtracting – Unlike Denominators
- **Applications in various topic areas**

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Curriculum Issues

Grade 7

- Adding and Subtracting Decimals
- Estimating Decimal Sums and Differences
- Multiplying Decimals – area model
- Dividing Decimals – tenth strips
- Simplifying Fractions
- Comparing and Ordering Fractions
- Mixed Numbers and Improper Fractions
- Fractions and Decimals – convert
- Comparing Rationals – Number Line
- Estimating with Fractions and Mixed Numbers – all operations
- Add and Subtract Fractions and Mixed Numbers
- Multiplying Fractions and Mixed Numbers
- Dividing Fractions and Mixed Numbers

Grade 7 (cont.)

- Ratios
- Unit Rates
- Proportions
- Similar Figures and Proportion
- Maps and Scale Drawings and Proportion
- Percents – Models (grid)
- Ratio and Percent
- Percents, Fractions, and Decimals
- Percents > 100% and < 1%
- Percent of a Number
- Percent and Proportion
- Percent and Equations
- Applications of Percent
- Percent of Change
- **Applications throughout – all other topics**

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What about research?

"…has a terrible time with basic skills. I mean, if we ever do anything with fractions, she's lost."*

*a lament by and about far too many…

Usher, …Middle School Students' Self-Efficacy in Mathematics, AERJ, 2009

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2nd Handbook Research on Mathematics Teaching & Learning

- Whole Number Concepts and Operations
  - Citations: 334
- Rational Numbers and Proportional Reasoning
  - Citations: 140
  - 2000's: 9 citations;
    - 109 in Whole Number Concepts and Operations

NCTM, 2007

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Another look…

NAEP, 2007 – Public Release Items

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Grade 4

What fraction of the group of umbrellas is closed?

A $\frac{1}{3}$

B $\frac{3}{7}$

C $\frac{4}{7}$

D $\frac{3}{4}$

Correct 80%
Incorrect 19%
Omit 1%

NAEP, 2007

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Grade 4

What fraction of the figure is shaded?

Correct: 77%
Incorrect: 22%
Omit: 1%

Answer: _____

Did you use the calculator on this question?

NAEP, 2007

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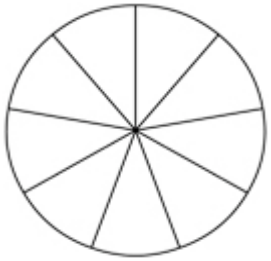
Grade 4

Luis wants to make a game spinner in which the chance of landing on blue will be twice the chance of landing on red. He is going to label each section either red (R) or blue (B). Show how he could label his spinner.



11% satisfactory
8% extended
14% partial
4% minimal
59% incorrect

Number of blues _____

Number of reds _____

NAEP, 2007

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Grade: 8

- In which of the following are the three fractions arranged from least to greatest?

A) $\frac{2}{7}, \frac{1}{2}, \frac{5}{9}$

B) $\frac{1}{2}, \frac{2}{7}, \frac{5}{9}$

C) $\frac{1}{2}, \frac{5}{9}, \frac{2}{7}$

D) $\frac{5}{9}, \frac{1}{2}, \frac{2}{7}$

E) $\frac{5}{9}, \frac{2}{7}, \frac{1}{2}$

49% correct
49% incorrect
1% omit

○ Yes ○ No

Did you use the calculator on this question?

NAEP, 2007

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In which of the following are the three fractions arranged in order from greatest to least?

A. 2/7, 1/2, 5/9

B. 1/2, 2/7, 5/9

C. 1/2, 5/9, 2/7

D. 5/9, 1/2, 2/7

E. 5/9, 2/7, 1/2

8th grade NAEP, 2007; 49% correct; 5th grade Kara's class – 68% correct

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Write these in order from greatest to least

0.66, 0.066, 06.6, 066.6, 0.666, 0.6

9 of 19, 47% correct, grade 5 Kara's class

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Grade 8

Tammy scored 52 out of 57 possible points on a quiz. Which of the following is closest to the percent of the total number of points that Tammy scored?

A) 0.91%
B) 1.10%
C) 52%
D) 91%
E) 95%

62% correct
37% incorrect
1% omit

Did you use the calculator on this question?

NAEP, 2007

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Lakers vs Nuggets

- Which player from the Lakers had the best shooting percentage
- Which player from the Lakers had the worst shooting percentage
- Same items for Nuggets
- Which players scored the most points, etc.

100% correct – 5th grade Kara's class

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- Fractions are a major area of study in upper elementary school mathematics. It is time to shift the emphasis and redefine the goal of fraction instruction from learning computation rules to developing fraction operation sense (Huinker, 2002).

- Do we do this?

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Number Meaning - Critical Issues

Whole Numbers:

- Number Meaning
- Counting - Counting on, Counting back
- Composing and Decomposing

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And for fractions…

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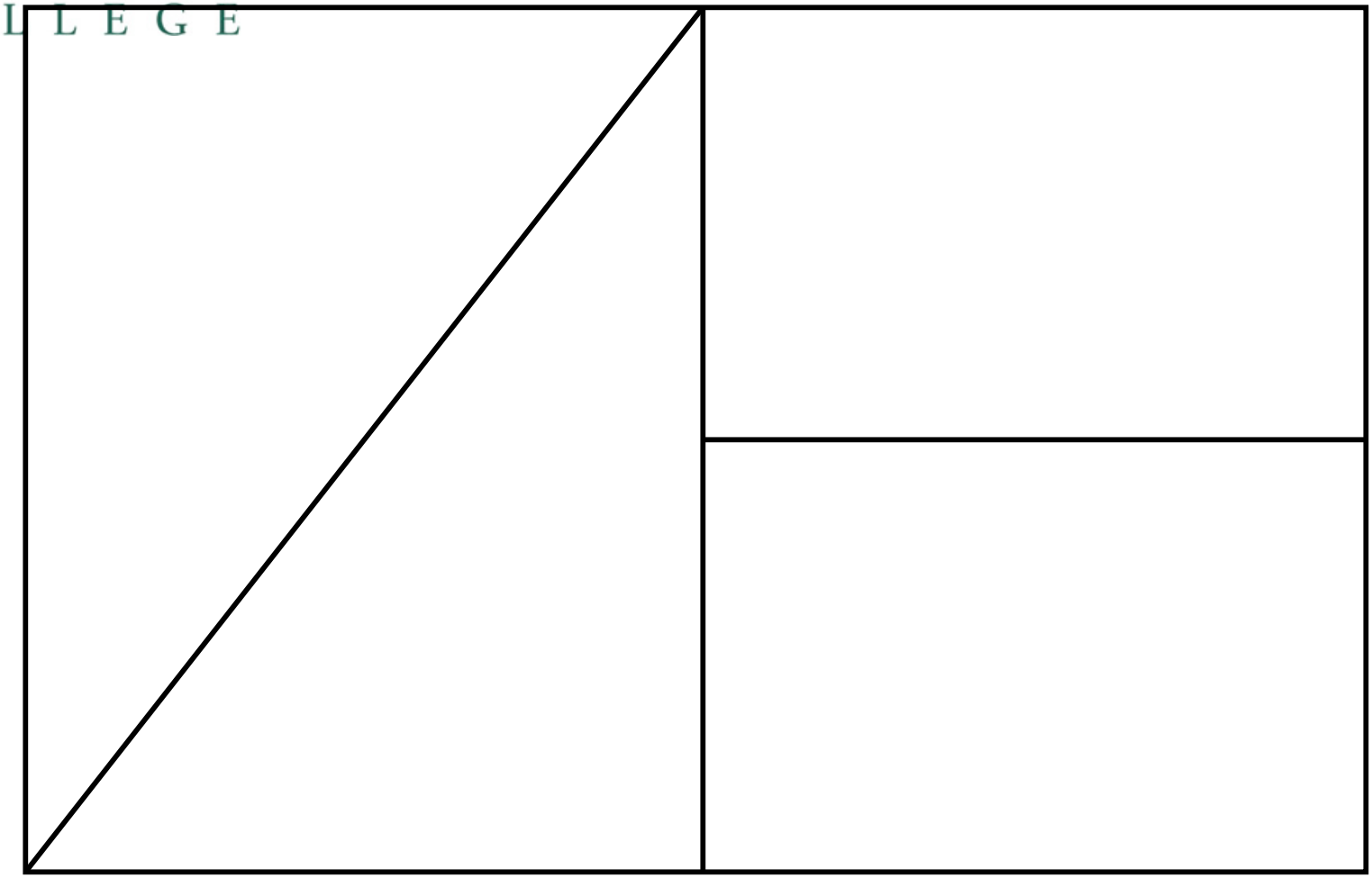

When the cake is cut, size of each piece?

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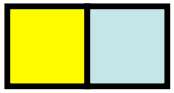
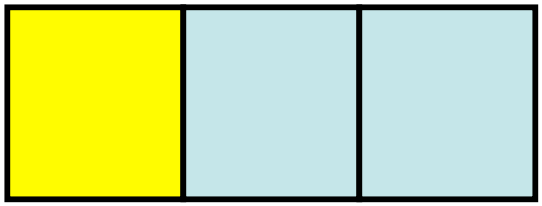
Fraction beginnings…

- Which one is larger, 1/2 or 1/3?





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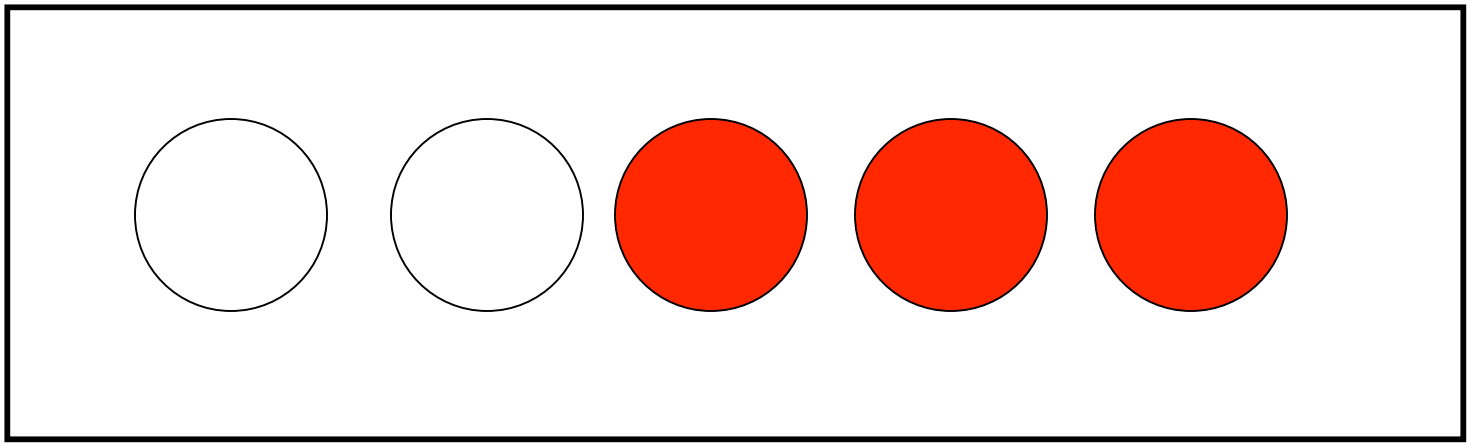

Could the drawing above represent:

a. 5/3 of something
b. 3/5 of something
c. 1 ÷ 3/5 = 1 2/3
d. 5 ÷ 3 = 1 2/3

Thompson and Saldanha, NCTM, 2003, page 107

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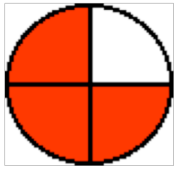
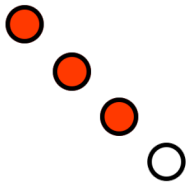
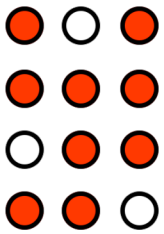
Thinking about ¾…

a) 

b) 

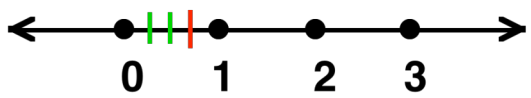
c) 

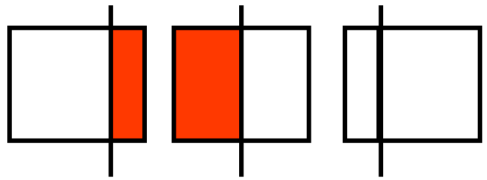
d) How many 4's are there in 3?

e) 18 crayons out of a box of 24

f) .75

g) I want to share 3 bottles of soda equally among 4 people. How much will each person get?

h) 

i) 

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Name…

- A fraction between:
  - 0 and 1
  - ¼ and ½
  - 1/3 and 2/3
  - 5/6 and 1
  - 0 and 1/8
- A decimal between:
  - 1.1 and 1.2
  - 1.10 and 1.11
- …

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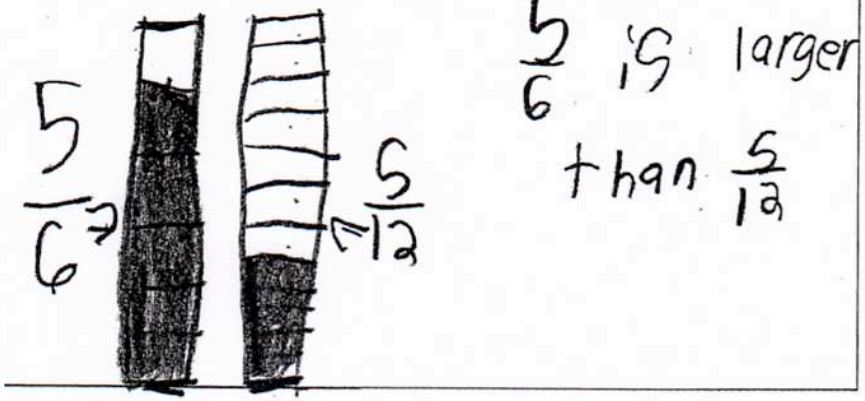
Which is more?

$\frac{5}{6}$ or $\frac{5}{12}$ of a chocolate candy bar?

How do you know?



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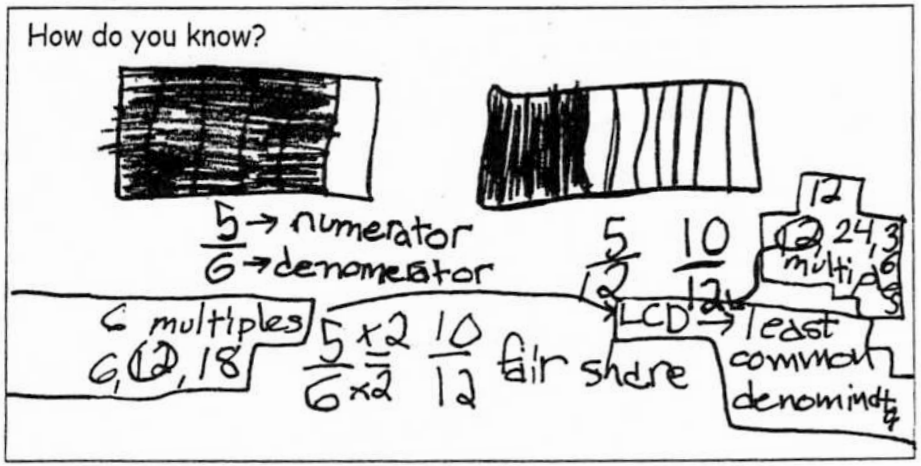

Mirra, Grade 3-5 Focal Points, NCTM, 2008

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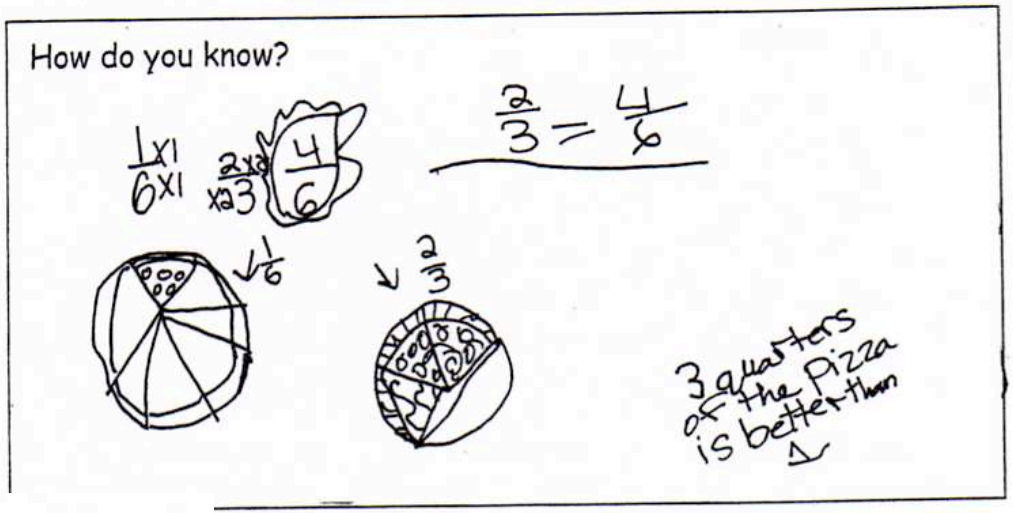
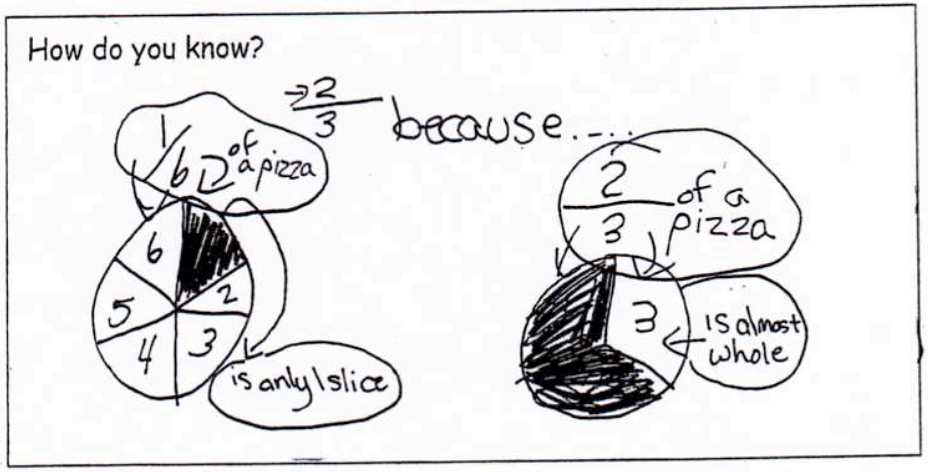
Which is more?
$\frac{1}{6}$ or $\frac{2}{3}$ of a pizza?



Which is more?
$\frac{1}{6}$ or $\frac{2}{3}$ of a pizza?



Mirra, Grade 3-5 Focal Points, NCTM, 2008

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Math Wall Activities

2%

100%

3/4

3.11

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Introducing fractions using sharing

- Four children want to share 10 cookies. Each child gets the same amount. How many cookies does each child get?
- Why?
  - Sharing is intuitive
  - Solution combines wholes and fractions (2½)
  - Sharing and repeated halving

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Fraction Sorting

- Sort the fractions below as near: 0, ½, or 1

4/7	1/7	8/9	3/5
2/3	1/10	4/8	6/11
4/5	2/12	9/12	5/12
1/8	3/8	4/9	7/14

- What's alike about all fractions near 1? Near 0?

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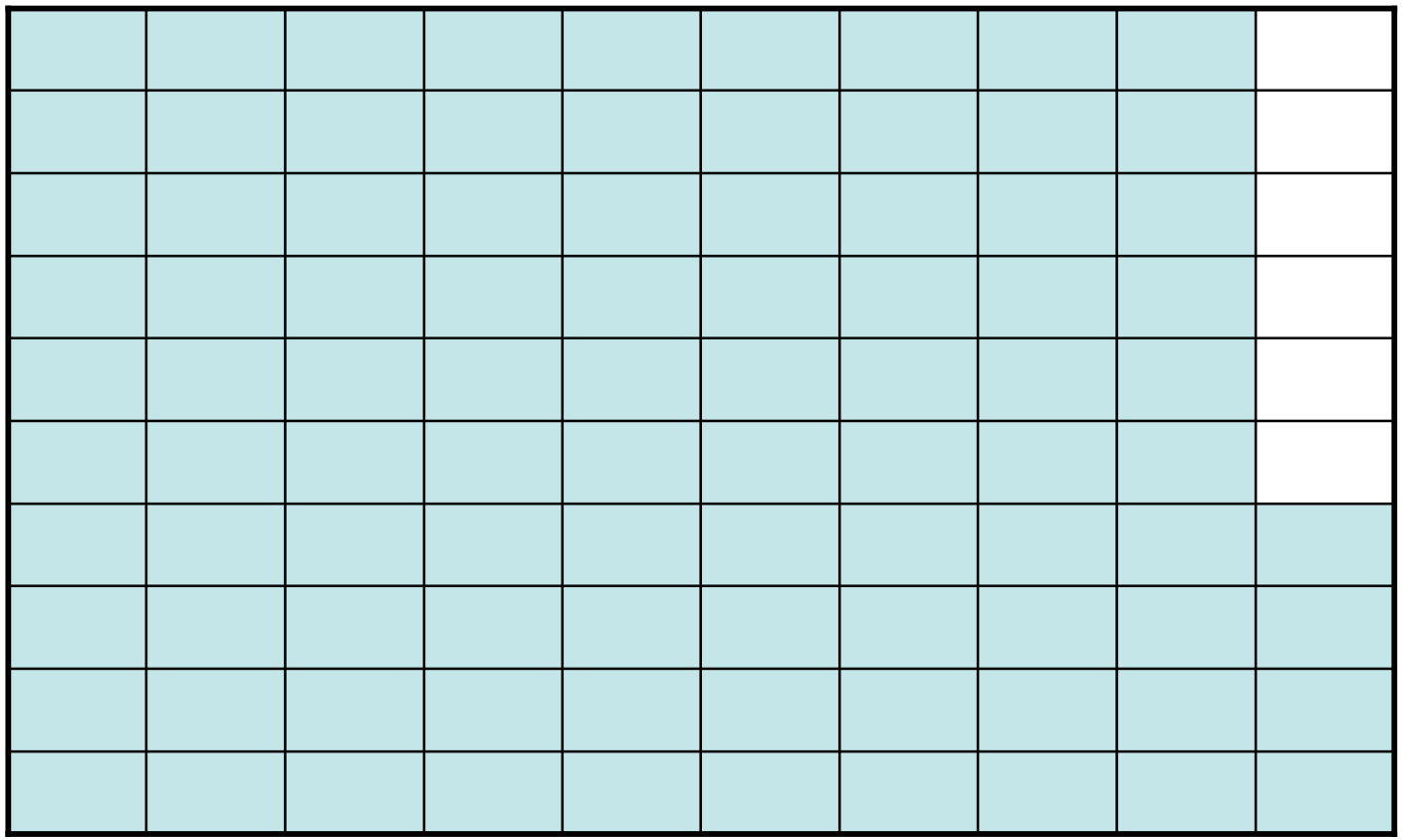

Close to 1? How much?

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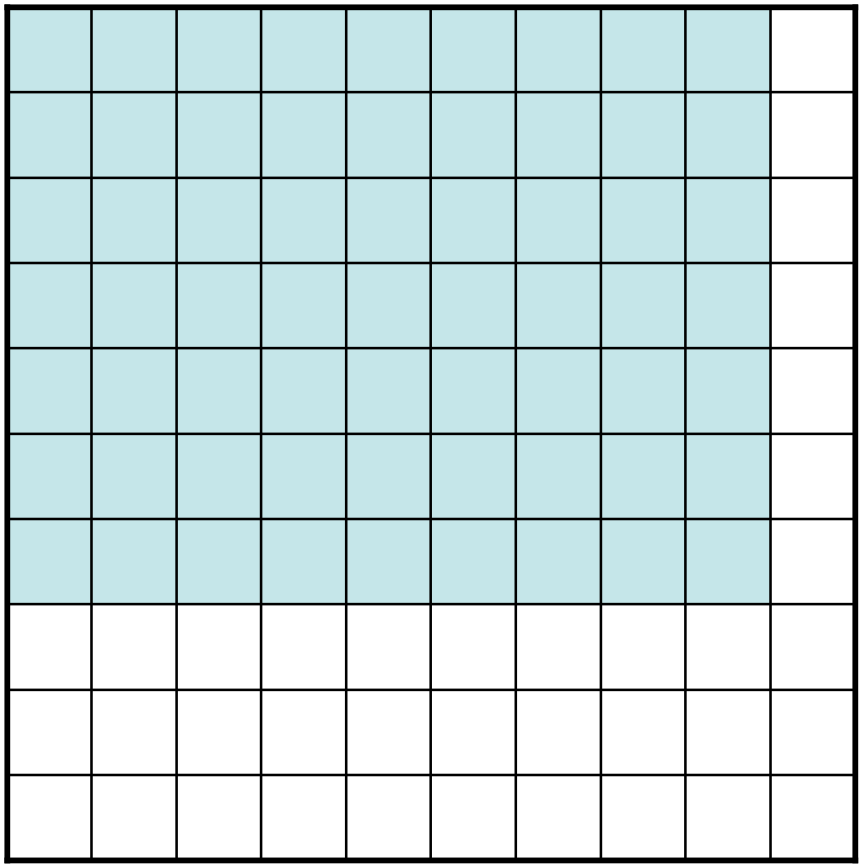

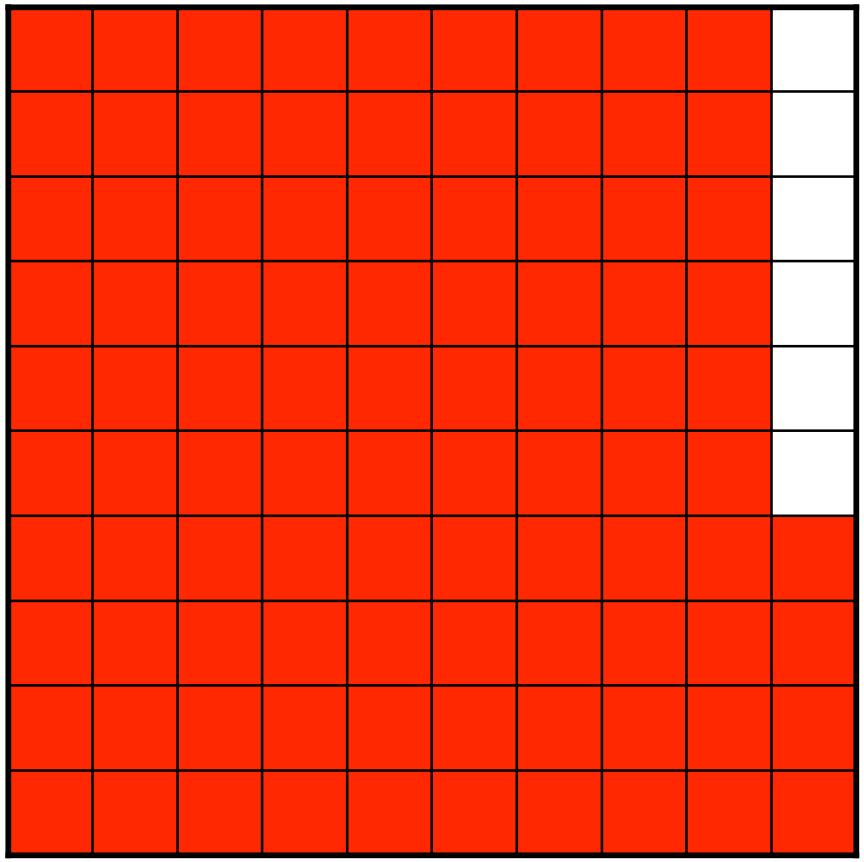

How much?

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Sense Making

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- How can we share **eleven** hoagies (aka subs) among four people?

- How can we share **eleven** hoagies (aka subs) among five people?

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Ballparking

- About, around, close to:

  ½

  ¼

  25%

  10%

  1/3

  2/3

  75%

  200%

1. **Provide fraction or % and have students provide one close to that amount.**
2. **Provide a context and have them write about that amount/context using one of the ballpark amounts.**

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True or False – 5/6

- More than ½?
- Less than 0.75?
- More than 80%?
- The ratio of boys to girls in our class?
- Between ½ and 1?
- Between ¾ and 1?
- Less than 1

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$\frac{3}{4}$

- What happens to the value of the fraction if the numerator is increased by 1?
- What happens to the value of the fraction if the denominator is decreased by 1?
- What happens to the value of the fraction if the denominator is increased?

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Ordering Fractions

Write these fractions in order from least to greatest. Tell how you decided.

- 5/3 5/6 5/5 5/4 5/8
- 7/8 2/8 10/8 3/8 1/8

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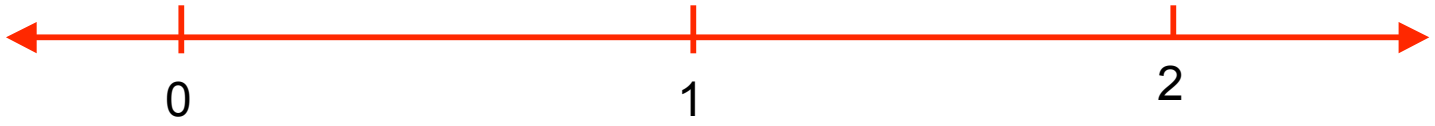
- Tell me about where 2/3 + 1/6 would be on this number line (Cramer, Henry, 2002).



Sense Making:

“2/3 is almost 1, 1/6 is a bit more, but the sum is < 1”

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- A student said that 3/4 and 5/6 are the same size because they both have one part missing – 3 is one less than 4 and 5 is one less than 6.

- Agree? Why or why not? How can you show the difference?

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Missing Numbers

- What's my number?
- 2x + 7 = y
- Rule: Double the number and add 7. What's the number if x =

10

100

0.1

0.01

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Use Percent – Don't Wait!

- Put 2/3; 0.5 and 3/4 in order from smallest to largest.
- It's easy, 0.5 is 50% and 2/3 is 66%, and so it goes first 0.5, then 2/3 and then ¾ because that's 75%.*

*response by Andy in New Approaches to Teaching the Rational Number System by Joan Moss in How Students Learn: Mathematics in the Classroom, NRC, 2005.

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You can't make this stuff up!

- The weather reporter on WCRB (a Boston radio station) said there was a 30% chance of rain. The host of the show asked what that meant. The weather reporter said ``It will rain on 30% of the state." ``What are the chances of getting wet if you are in that 30% of the state?" ``100%."

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You can't make this stuff up

- Gettysburg Outlets – July 3, 2009. 50% off sale on all purchases at the Izod store. Sign indicates 50% off the all-store sale.
  - Patron – "well that means it's free."
  - Clerk – "no sir, it's 50% off the 50% off sale."
  - Patron – "well, 50% + 50% is 100% so that means it should be free."
  - This went on for a while. AND, there was a sign indicating 70% off for some items, meaning 70% off the 50% off original sale, which our patron would interpret as the item being free and 20% in cash!

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Operations and Sense Making

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More than or less than ONE

- 1/12 + 2/3
- 5/6 + 1/3
- ½ + ¼
- 1 ½ - 7/8
- 2/3 x 2/3
- 5/6 x 7/8
- 4/5 ÷ 2/3
- 9/10 – 1/12

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7/8 – 1/8 = ?

- Interviewer: Melanie these two circles represent pies that were each cut into eight pieces for a party. This pie on the left had seven pieces eaten from it. How much pie is left there?
- **Melanie:** ***One-eighth, writes 1/8*****.**
- Interviewer: The pie on the right had three pieces eaten from it. How much is left of that pie?
- **Melanie:** ***Five-eighths, writes 5/8.***
- Interviewer: If you put those two together, how much of a pie is left?
- **Melanie:** ***Six-eighths, writes 6/8.***
- Interviewer: Could you write a number sentence to show what you just did?
- **Melanie:** ***Writes 1/8 + 5/8 = 6/16.***
- Interviewer: That's not the same as you told me before. Is that OK?
- **Melanie:** ***Yes, this is the answer you get when you add fractions.***

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What Happens Here?

- 1/2 x 3/4 **< or >** 3/4
- 3/4 x 1/2 **< or >** 1/2

- 1/2 ÷ 3/4 **< or >** 1/2
- 3/4 ÷ 1/2 **< or >** 3/4

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The quotient is 0.5, what is the divisor and dividend?

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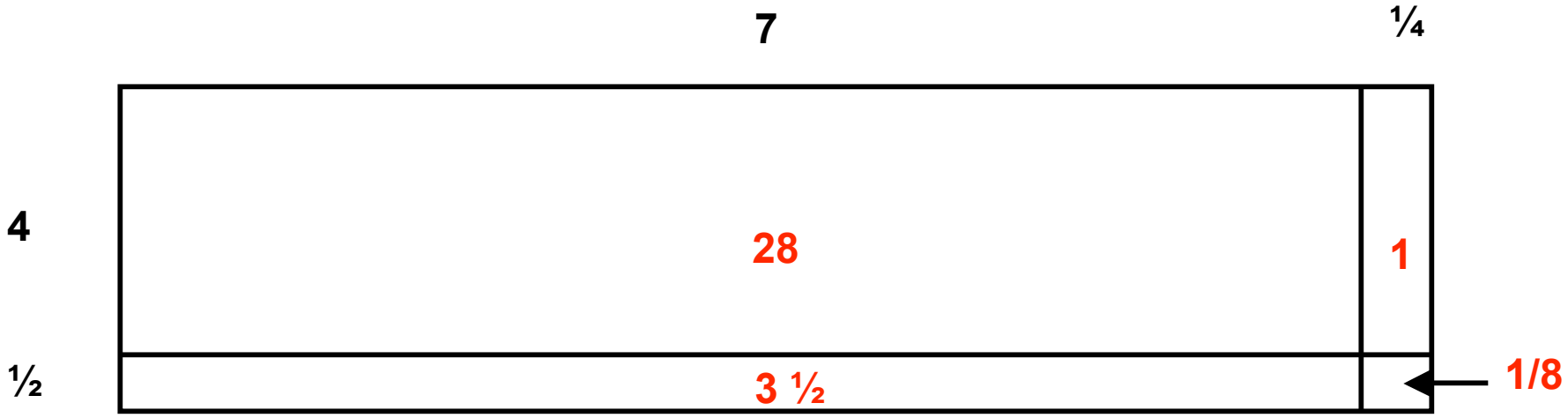
Boxes to multiply…

- Draw a rectangle to show 4 ½ x 7 ¼ =



4 ½ x 7 ¼ = (4 x 7) + (½ x 7) + (4 x ¼) + (1/2 x 1/8)

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- How might you represent 7 x 2/3 and would you think of 2/3 x 7 differently?

- If you shared 7 doughnuts among 3 people, how could you use this to help determine 2/3 x 7?

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Decimals - What Happens?

Number	x 0.05	x 0.48	x 0.9
100			
60			
24			
?			

- In general, what happens when you multiply a whole number by: 0.05; 0.48; 0.9?
- Begin thinking of 0.05 as 5% or nickel:dollar, etc.

Reys (ed.), 1991

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Mental Math

- (25 x 16) x ¼
- 3 1/8 x 24
- 4/5 x 15

- 68 x 0.5
- 0.25 x 48
- 0.2 x 375
- 0.05 x 280
- 56 x 0.125
- 0.75 x 72

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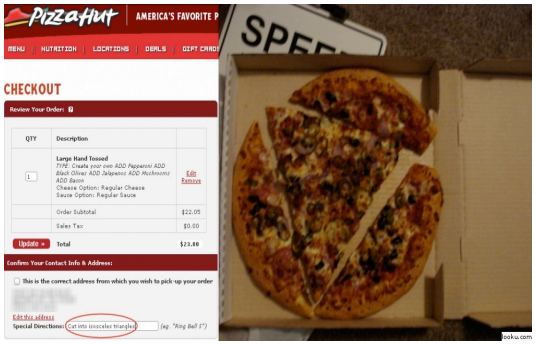
Context is good



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Favorites

- Write **3** numbers that have some significance to your life – these must be fractions, decimals, percents.
- Exchange lists. Provide random clues for the numbers.
- Guess which numbers fit the clues.

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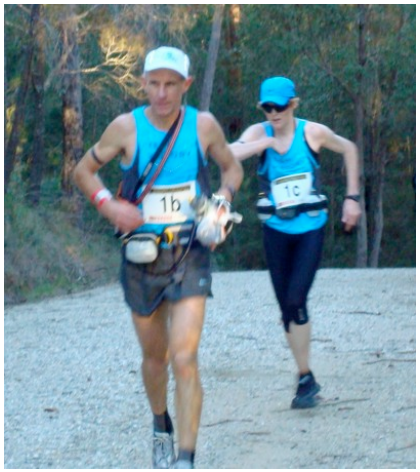
For your next Ultra…

Trail Mix (for 6)



- ½ cup raisins
- ¾ cup peanuts
- 2/3 cup granola
- ½ cup dried fruit
- 2 tablespoons sunflower seeds
- ¼ cup M&Ms

Mix for the whole club – 30 runners
Make that 40 runners…

Adaptation of Bresser, Holtzman, 1999)

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- On a scale 1” = 12 miles. If two places are 4” apart, how far are they away from each other in miles?

1”	12 miles
4”	

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Now what?

- There are 25 students in our class. Each student will get ¼ of a pizza. Your job is to find out how many pizzas we should order. Be sure to show your work.

- How many pizzas should we order?

Percent Benchmarks

0%

100% **50%** **< 10%**

~25% **~75%** **~90%**

> 50% **< 50%**

- Lefthanders in the room or class
- Once lived in **New Jersey**
- Been involved in education > 10 years
- People who were born in **Texas**

Reys, et al, 1991

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Rational Number Sense

- Using representations of fractions, decimals, and percents interchangeably;
- Comparing and ordering fractions, decimals and percents;
- Using benchmarks to estimate when comparing and ordering and in determining sums, differences, products, and quotients.
- (Adapted from Moss, 2002)

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Concluding Thoughts

- Number sense is elusive
- Number sense should be nurtured – every day!
- A sense of number breeds confidence.
- Fractions – all of ‘em - are numbers too!
- Decimals and inconsistency…
- Probability – now!

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Plastics…

A major goal for K-8 mathematics should be proficiency with fractions…

NMAP, 2008

Home Project Programs Resources Certification Research Contacts

Elementary Mathematics Specialists and Teacher Leaders Project

McDANIEL COLLEGE

We need elementary school mathematics specialists, elementary classroom teachers who know and understand mathematics and can effectively

About the Project

mathspecialists.org

The major goals of the McDaniel College EMS&TL Project are to fully establish a graduate program leading to an MS degree in elementary mathematics teacher leadership, to develop a clearinghouse relative to elementary mathematics specialist programs nationally, and to ensure the continuing professional development and mentoring of a cadre of mathematics teacher-leaders and elementary school mathematics specialists in Maryland. Additionally, the project seeks to determine the impact of mathematics specialists and mathematics teacher specialists on student achievement and school and school district improvement regionally, statewide, and potentially, nationally.

This is a proposed multi-year project, with the formal establishment of the