WJEC MATHEMATICS

INTERMEDIATE

FRACTIONS, DECIMALS, AND PERCENTAGES

FRACTIONS AND PERCENTAGES OF AMOUNTS

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Contents

Credits

WJEC Question bank
http://www.wjec.co.uk/question-bank/question-search.html

Fractions of amounts - Non Calculator

To find a fraction of a number, divide that number by the denominator and multiply the result by the numerator.

Example 1

$\frac{2}{7}$ of 35

Step 1

$35 \div 7 =$ **5**

Step 2

5 $\times 2 = 10$

$= 10$

Example 2

$\frac{3}{5}$ of 55

Step 1

$55 \div 5 =$ **11**

Step 2

11 $\times 3 = 33$

$= 33$

Exercise N53

Calculate the following fractions of amounts without a calculator

a. $\frac{2}{5} of\ 45$

b. $\frac{3}{7} of\ 56$

c. $\frac{4}{9} of\ 63$

d. $\frac{8}{12} of\ 72$

e. $\frac{3}{4} of\ 36$

f. $\frac{1}{2} of\ 70$

g. $\frac{5}{6} of\ 36$

h. $\frac{7}{12} of\ 144$

i. $\frac{3}{13} of\ 65$

Fractions of amounts - Calculator

When you have a calculator let it do the hard work for you!

Key point!

'of' = multiply

Example

$$\frac{5}{12} of\ 156$$

On a calculator paper, this becomes

$$\frac{5}{12} \times 156 = 65$$

Use the fraction button on your calculator for this!

Exercise N54

Calculate the following fractions of amounts with a calculator

a. $\frac{7}{9} of\ 225$

b. $\frac{3}{7} of\ 175$

c. $\frac{4}{9} of\ 279$

d. $\frac{8}{12} of\ 300$

e. $\frac{3}{4} of\ 728$

f. $\frac{1}{2} of\ 449$

g. $\frac{5}{6} of\ 144$

h. $\frac{7}{12} of\ 408$

i. $\frac{3}{13} of\ 845$

Exam Questions N29

Try these questions without a calculator, then check using a calculator

1. *(b)* Find $\frac{4}{9}$ of 243. [2]

.....

.....

2. *(b)* Find $\frac{2}{11}$ of 242 g. [2]

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.....

3. *(e)* Calculate $\frac{3}{7}$ of 84. [2]

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*When completing these without a calculator you may need to practice bus stop division. See the book '*Four Operations and BIDMAS*'*

Percentages of Amounts - Non Calculator

To find percentages of amounts we need to know the following key facts:

To find 50%, half the number
To find 25%, divide the number by 4 (half and half again)
To find 10%, divide the number by 10
To find 1%, divide the number by 100

Example

Find 50%, 25%, 10% and 1% of 280

$\mathbf{50}\%\ of\ 280 = 280 \div 2 = 140$

$\mathbf{25}\%\ of\ 280 = 280 \div 4 = 70$

$\mathbf{10}\%\ of\ 280 = 280 \div 10 = 28$

$\mathbf{1}\%\ of\ 280 = 280 \div 100 = 2.8$

Exercise N55

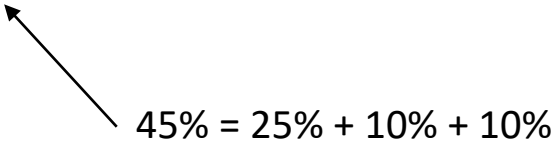
Find 50%, 25%, 10%, and 1% of the following numbers

a. 104 c. 1000 e. 500
b. 480 d. 160 f. 8

However, you may be asked to calculate a percentage other than the four above. To calculate these, we use the above four as building blocks

Example 1

Find 45% of 440



By breaking up 45% as seen above we can calculate the smaller percentages and add them together

$\mathbf{25}\%\ of\ 440 = 440 \div 4 = 110$

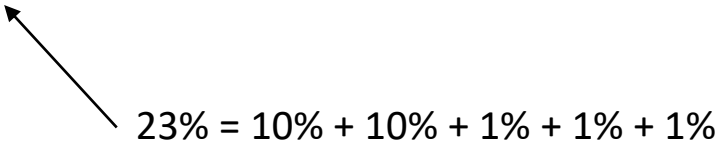
$\mathbf{10}\%\ of\ 440 = 440 \div 10 = 44$

So,

$$45\% = 25\% + 10\% + 10\% = 110 + 44 + 44$$
$$= 198$$

Example 2

Find 23% of 120



$\mathbf{10}\%\ of\ 120 = 120 \div 10 = 12$

$\mathbf{1}\%\ of\ 120 = 120 \div 100 = 1.2$

So,

$$23\% = 10\% + 10\% + 1\% + 1\% + 1\%$$
$$= 12 + 12 + 1.2 + 1.2 + 1.2$$
$$= 27.6$$

Exercise N56

Calculate the percentages of the following amounts

a. 35% of 150	d. 31% of 420	g. 64% of 580
b. 65% of 480	e. 81% of 500	h.39% of 444
c. 75% of 120	f. 99% of 350	i. 1.5% of 200

Percentages of amounts - Calculator

Finding percentages of amounts with a calculator requires two steps;

1. Convert the percentage to decimals. (See 'Converting between FDP' booklet for more help with this)
2. Change the 'of' to a multiply sign

Example

Find 56% of 260

As a decimal is 0.56

Becomes,

$$0.56 \times 260 = 145.6$$

This is called the **multiplier**

Exercise N57

Calculate the percentages of the following amounts

a. 26% of 189	d. 54% of 484	g. 24% of 8461
b. 94% of 846	e. 68% of 1659	h.79% of 465.5
c. 27% of 645	f. 32% of 5497	i. 31.9% of 0.54

Exam Questions N30

Try these without a calculator, then check your answers with a calculator

1. *(c)* Find 60% of 70.

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.....

[2]

2. *(c)* Find 40% of 70.

.....

.....

[2]

3. *(a)* Find 23% of £52. [2]

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.....

4. *(a)* Find 67% of £234. [2]

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5. *(b)* Calculate 38% of £56.

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[2]

Percentage Change - Without a calculator

GCSE questions often ask you to increase or decrease a number by a percentage.

> To increase by a percentage - add the percentage onto the original amount
> To decrease by a percentage - subtract the percentage from the original amount

Example 1

Increase £460 by 35%

35% = 10% + 10% + 10% + 5%

Half of 10%

$\mathbf{10}\%\ of\ 460 = 460\ \div 10 = 46$

$\mathbf{5}\%\ of\ 460 = 46\ \div 2 = 23$

So,

$$35\% = 10\% + 10\% + 10\% + 5\%$$
$$= 46 + 46 + 46 + 23$$
$$= 161$$

So, to increase, add this to the original amount

$$460 + 161 = £621$$

Example

Decrease £460 by 35%

35% = 10% + 10% + 10% + 5%

Half of 10%

$\mathbf{10}\%\ of\ 460 = 460\ \div 10 = 46$

$\mathbf{5}\%\ of\ 460 = 46\ \div 2 = 23$

So,

$$35\% = 10\% + 10\% + 10\% + 5\%$$
$$= 46 + 46 + 46 + 23$$
$$= 161$$

So, to decrease, subtract this from the original amount

$$460 - 161 = £299$$

Exercise N58

Increase:

a. 450 by 40%
b. 240 by 35%
c. 840 by 23%
d. 550 by 15%
e. 880 by 86%
f. 400 by 64%

Decrease:

a. 120 by 30%
b. 440 by 65%
c. 80 by 75%
d. 680 by 64%
e. 800 by 73%
f. 40 by 31%

Percentage change - With a calculator

To increase by a percentage - multiply by **one plus the decimal**
To decrease by a percentage - multiply by **one minus the decimal**

Example 1

Increase 432 by 45%

Decimal is 0.45

$$432 \times (1 + 0.45) =$$
$$432 \times 1.45 = 626.4$$

Example 2

Decrease 954 by 31%

Decimal is 0.31

$$954 \times (1 - 0.31) =$$
$$954 \times 0.69 = 658.26$$

Important!

These numbers are known as the **multipliers**

Exercise N59

Increase:

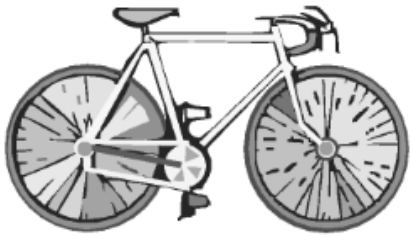
a. 59 by 40%
b. 94 by 35%
c. 596 by 23%
d. 845 by 15%
e. 1652 by 86%
f. 2265 by 64%

Decrease:

a. 165 by 51%
b. 845 by 13%
c. 61 by 54%
d. 12 by 48%
e. 9451 by 54%
f. 164 by 13%

Exam Questions N31

1. Daniel wants to buy a new bicycle. It is priced at £480.



Daniel can either
- pay £480 immediately, or
- pay a 15% deposit, followed by 24 monthly payments of £22.

(a) Calculate the total amount Daniel would pay using the deposit and monthly payments method. [3]
You must show all your working.

2. **Show clearly** whether the following statement is true or false. [4]

'If you increase a positive number by 10% and then decrease that new value by 10%, you get back to your original number.'

.....

3. Suzanne has a credit card on which she owes £1000. She decides not to make any further purchases using this card.
Each month interest at 1·5% is added on before any payment is made.
Each month the minimum repayment she has to make is 3% of the amount owed after the interest has been added on.
Starting with the amount owed of £1000, calculate, to the nearest penny, how much she will owe after paying the minimum payment for 2 months.

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4. **12.** *(a)* A dress, which costs £60 to make, is sold at a profit of 45%. What is the selling price of the dress?

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.....

Reverse Percentage Change

Some questions will give you the price of something **after** the percentage increase / decrease has been applied.

Example 1

An item in a shop has been increased by 15% and is now £460. How much was the item to begin with

Remember, to calculate a percentage increase/decrease

$$Original\ Cost \times multiplier = New\ cost$$

For increase, the multiplier is 1 **plus** the percentage as a decimal

For decrease, the multiplier is 1 **minus** the percentage as a decimal

So, for this example

$$Original\ cost \times 1.15 = 460$$

We can rearrange this. (See the booklet '*Rearranging*' for help with this)

$$Original\ cost = \frac{460}{1.15}$$

$$= £400$$

Example 2

A car costs 34% less now than it was when new. It now costs £29700. How much was the car new?

$$Original\ cost \times multiplier = New\ cost$$

$$Original\ cost \times 0.66 = 29700$$

This is a percentage **decrease** question so the multiplier is 1-0.34

Rearrange:

$$Original\ cost = \frac{29700}{0.66}$$

$$= £45000$$

Exercise N60

1. An item in a shop was increased by 22% and sold for £549. What was the original cost?

2. An item in a shop was increased by 78% and sold for £40.05. What was the original cost?

3. An item in a shop was decreased by 54% and sold for £455.40. What was the original cost?

4. An item in a shop was decreased by 19% and sold for £421.20. What was the original cost?

Exam Questions N32

1. In a competition, Dewi threw the javelin 69·93 metres.
This was an improvement of 8% on his previous best throw.

Calculate the length of his previous best throw. [3]

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2. Complete the table below.

Original amount	After a decrease of	
	40%	2%
£	£492	£ .....

3. A shop has reduced the price of a bicycle by 40% of its original price.

The sale price of the bicycle is £192.

Calculate the original price of the bicycle.

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4. The bill for repairing a washing machine came to £151.68, inclusive of VAT at 20%.
What was the cost before VAT was added? [3]

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5. *(a)* A measurement has been increased by 26%.
After the increase the measurement is 57·96 cm.
Calculate the original measurement.

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One number as a percentage of another

Non Calculator

You may be asked to calculate one number as a percentage of another.

Example

Jamie scores 21 out of 25 on a test what percentage is this?

So, we need to calculate 21 as a percentage of 25.

> Remember: 'percent' means out of 100
> Write the fraction, then get the denominator to be 100

As a fraction

$$\frac{21}{25} = \frac{?}{100}$$

For the denominator to change from 25 to 100 we must multiply by 4.

So we do the same to the numerator

$$21 \times 4 = 84\%$$

Example 2

Write 180 as a fraction of 300

Again, start with fraction and get the denominator to be 100

$$\frac{180}{300} = \frac{?}{100}$$

To get the denominator to be 100 we need to divide by 3. We do the same to the numerator

$$180 \div 3 = 60\%$$

Example N61

1. Write 14 as a percentage of 20
2. Write 22 as a percentage of 50
3. Write 86 as a percentage of 200
4. Write 150 as a percentage of 300
5. Write 150 as a percentage of 500

With a calculator

When you have a calculator, multiply the fraction by 100 to calculate the percentage.

Example 1

Write 45 as a fraction of 80

$$\frac{45}{80} \times 100 = 56.25\%$$

Example 2

Write 47 as a percentage of 121 (to one decimal place)

$$\frac{47}{121} \times 100 = 38.8\%$$

Exercise N62

1. Write 37 as a percentage of 55 (to one decimal place)
2. Write 46 as a percentage of 58 (to one decimal place)
3. Write 165 as a percentage of 241 (to one decimal place)

Exam Questions N33

1. *(a)* What is 84 out of 300 as a percentage?

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[2]

2. *(a)* What is 120 out of 300 as a percentage?

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[2]