Fractions: Number Meaning and Number Relationships Grade 3

Orcutt Union School District January 28, 2020



Today's Agenda and Outcomes

Objective:

□ Participants will unwrap the fraction standards to determine the concepts, procedures and type of problems students are expected to master. Participants will engage in learning activities that build meaning of fractions and fraction relationships using various tools and strategies.

□Agenda:

- Unwrap standards
- Building meaning of fractions using areas, number lines, and sets
- Make fraction equivalencies using areas and number lines
- Compare fractions using the same whole by comparing same numerators, same denominators, or benchmark values.
- Instructional Planning

Introductions and Outcomes

□Nametags

- Name (write this big)
- Site
- Yrs. in education

□Introduction

- What is your favorite number?
 - * Write it as a multiplication fact.

□ Talking Chips

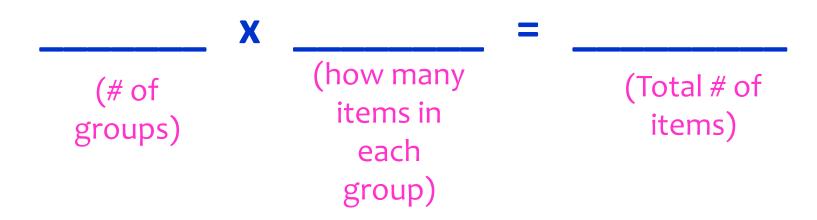
Take 4 coffee filters and put 3 chips in each one.

Michele D 29 yrs. (16 in PD) 2x4=4 or 1 x 4 = 4





Understanding Multiplication and Division



Multiplication

■What models are we asking students to use? How often?

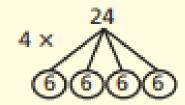




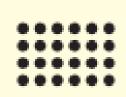
bags of lemons

 $4 \times 6 = 24$ $24 \div 4 = 6$

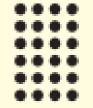
Equal Shares Drawing



Array Drawing

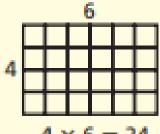


$$4 \times 6 = 24$$



$$6 \times 4 = 24$$

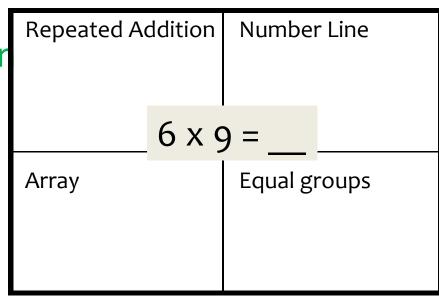


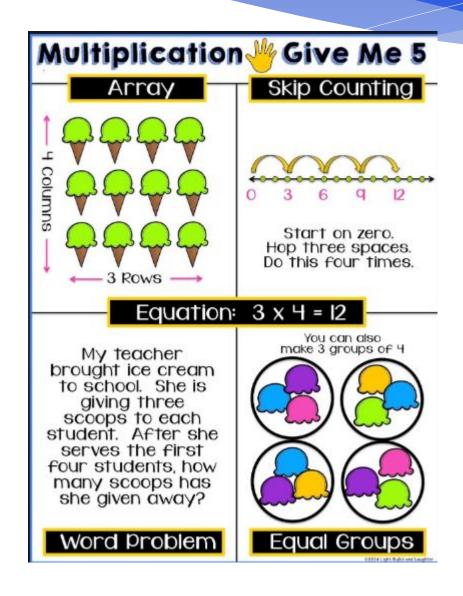


$$4 \times 6 = 24$$

4 Ways to Represent

- □ Fold your paper into 2 equal parts horizontally and then vertically to create 4 equal sections.
- ☐You are given a fact to display in 4 different ways.
 - Write your fact in the center
 - 1) repeated addition equation
 - 2) number line
 - 3)an array
 - 4) equal groups





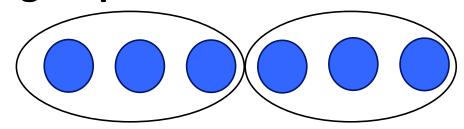
What is Division? How does it relate to multiplication?

What are fair shares? What does it mean to partition equally? (3.OA.2)

Division as an Operation

$$6 \div 2 =$$

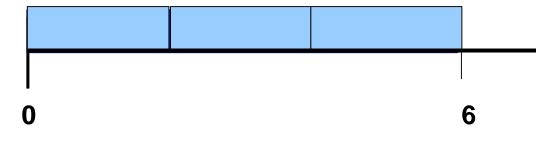
Partitive – Finding the size of the group



The question: How do you divide or partition 6 things into 2 groups?

There are 3 in each group.

Measurement – Finding the number of groups of a specific size



The question: How many groups of 2 units are in 6?

There are 3 groups of 2 in 6.

Strategies

□Counting on

□ Doubling

□Combining two known facts

Strategy 1: Start with 5 × 6, and count by 6 from there.

 $5 \times 6 = 30$, the next count by is 36. So, $6 \times 6 = 36$.

Strategy 2: Double a 3s multiplication.

 6×6 is twice 6×3 , which is 18. So, $6 \times 6 = 18 + 18 = 36$.

Strategy 3: Combine two multiplications you know.

 $4 \times 6 = 24$

4 sixes are 24.

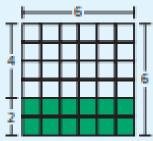
 $2 \times 6 = 12$

2 sixes are 12

 $6 \times 6 = 36$

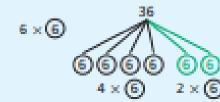
6 sixes are 36.

Here are two ways to show Strategy 3 with drawings.



unshaded area: $4 \times 6 = 24$ shaded area: $2 \times 6 = 12$

total area:



Explanation:

6 arougs of 6 is

4 groups of 6 plus

2 groups of 6.

 Strategy 4: Add 6 on to the 6s multiplication before or subtract 6 from the multiplication ahead.

5 x 6 = 30, add 6 more to get 36. So, 6 x 6 = 36.



Reflection

- □What have you implemented to support students build the concepts of multiplication and division?
 - What is working?
 - Where are students still getting stuck? With what specifically?
- □What are you still doing consistently to build fluency and to measure fluency?

What have you implemented to support students build the concepts of multiplication and division that IS working?

Top

What have you implemented to support students with multiplication and division yet they still remain stuck? on what specifically are they they stuck?

What are you doing consistently to build fluency?

A Building small sets of facts over time.

B connecting facts with the real world, ex: pairs or half dozen or days of the week

C Using story problems

D Having students use manipulatives, graph paper, other tools

E Using properties such as commutative, distributuve

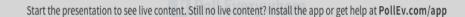
F. Using games

G. Students are charting their own growth

H. Measuring flexibility and accuracy but not yet effeciency

I. Timed tests







Critical Areas Identified



Number Sense • Number Meaning Number Relationships

- Number Magnitude
- Operations of **Numbers**



Problem Solving

- Problem Types
- Problem Structures
- Instructional Considerations for Solving Problem



Fluency

- Understanding Fluency
- Building Flexible Strategies
- Measured **Progress**

Critical Areas Identified

Grade	Operations and Algebraic Thinking		Numbers and Fractions		Measurement	
e 3	3.OA.1	3.OA.7	3.NF.1	3.NF.3	3.MD.2	3.MD.7
	3.OA.2	Multiply	Defining a	Equivalent	Solve	Concepts
	Interpret	and divide	fraction	fractions	problems	of area as
	products	within 100		and	of mass	it relates to
	and			comparing	and	multiplicati
	interpret			fractions	volume	on and
	quotients				using all	division.
					operations	



Standards Documents

Standards

□What do they say?

- How are you making sense of the expectations of the standards?
- How will the standards guide your instruction?

CONCEPTS What concepts do students need to understand?	PROCEDURES What procedures do students need to master?
PROBLEM SOLVING What types of problem solving are students expected to do?	TOOLS What tools should students use to making meaning of this concept?

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"Current instruction that gives a brief introduction through part-whole fractions and then proceeds to introduce computation procedures does not give children the time they need to construct important ideas and ways of thinking."

(Lamon, 1999; emphasis added)

Confusion results when students try to apply their understanding of whole numbers to fractions.

The textbook approach to teaching:

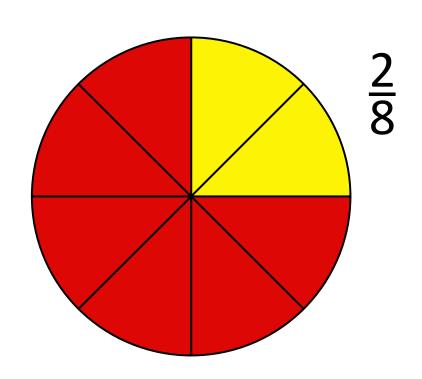
- Moves too quickly over the major concepts
- Rushes to have students practice procedures, resulting in simplistic, mechanical problem solving
- Has limited representation, weakening conceptual understanding

Fallout from Whole Numbers

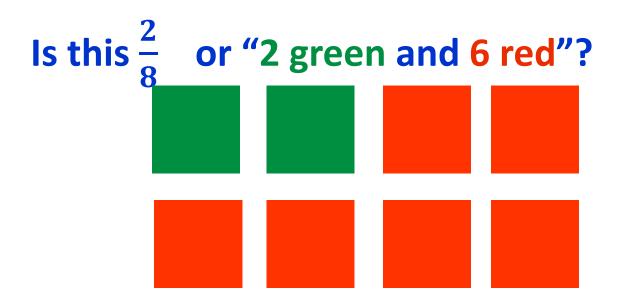
What do you see?

- 2 and 6
- 2 and 8

Is there a relationship?



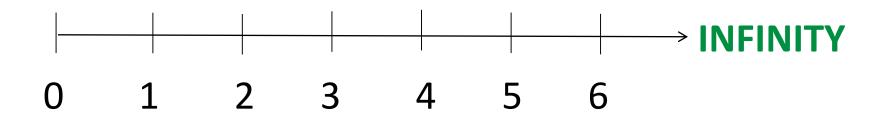
Some Models Can Be Confusing

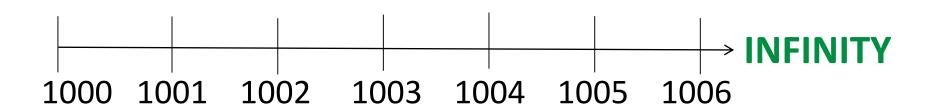


Say: "Two-eighths"

not "2 over 8" or "2 out of 8."

☐ Fractions and the Concept of Infinity



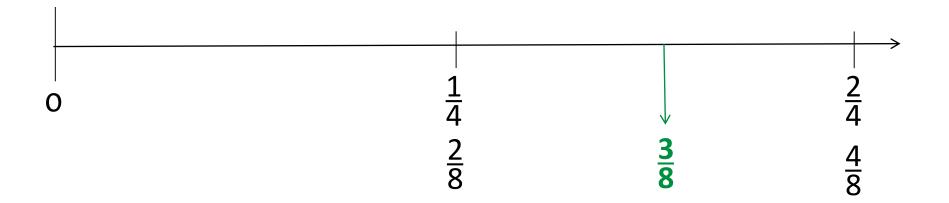


☐ Fractions and the Concept of Infinity

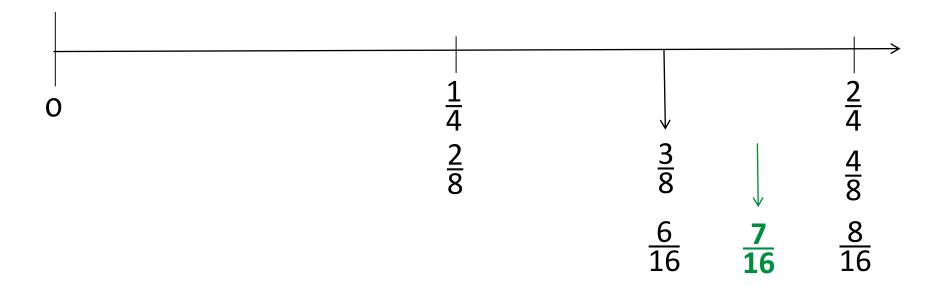




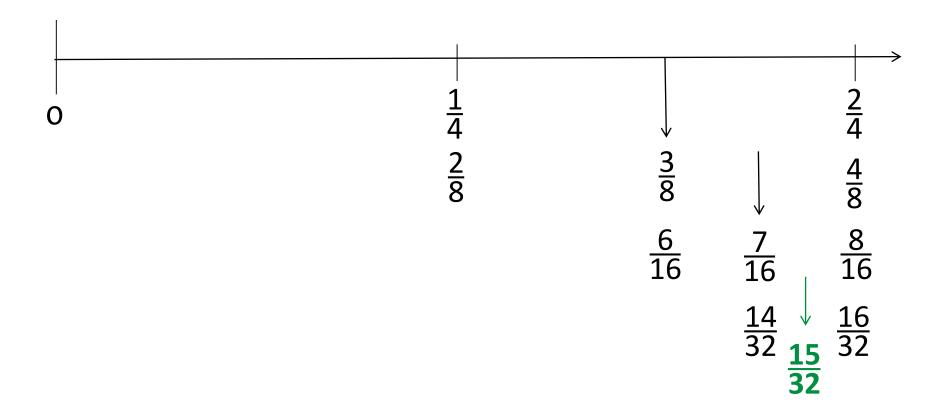
An Infinite Number of Fractions?



An Infinite Number of Fractions?



An Infinite Number of Fractions?



- ☐ Take the time to develop the concept of fractions.
 - Foundational to decimals, percentage, ratios, and proportions
- ☐ Use visual representations and classroom talk.
- □ Develop a robust understanding of part-whole relationships.
 - Avoid simplistic instruction
 - Before teaching operations on fractions

□Students need to learn how to:

- Represent a fraction multiple ways.
- Show the value of a fraction based on different definitions of the whole.
- Work with equivalent fractions.
- Understand that comparisons can only occur when the wholes are equivalent.
- Use the meaning of numerator and denominator to compare fractions and reason about the size of a fraction.



Building Number Sense of Fractions

- **□**Number Meaning
 - Counting, Writing, Naming, and Representing
- **■**Number Relationships

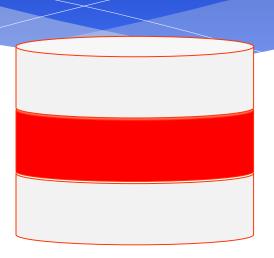
■Building a Sense of Operation

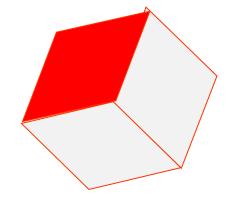
■Number Magnitude and Size

Building Meaning of Fractions

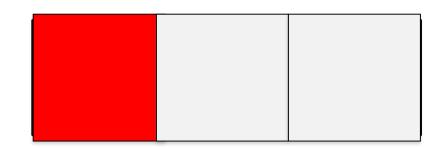
Area Shapes



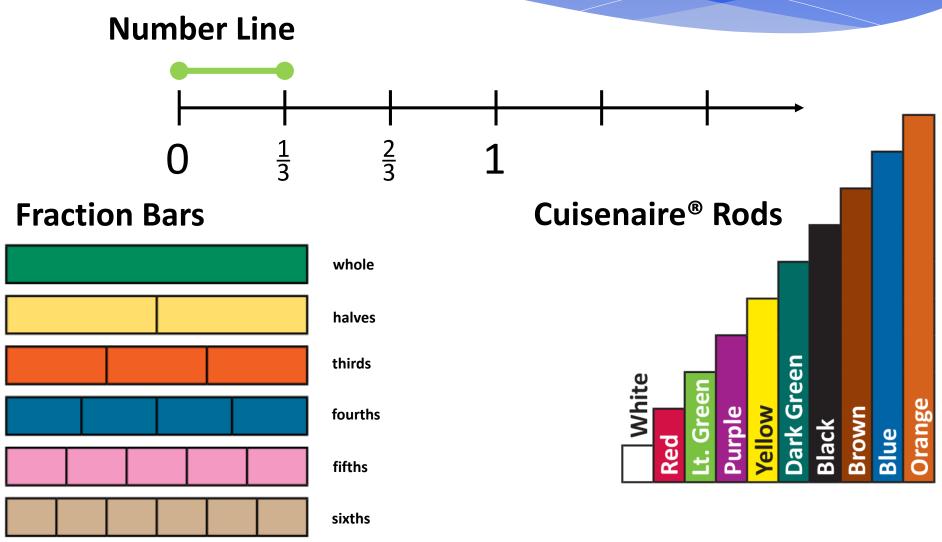




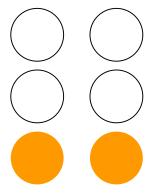
What does 1/3 of this rectangle look like?



Length Models



Set Models



Symbolic Notation

These words are Level 3 vocabulary and don't have meaning or any common reference for children. The words themselves will NOT help students understand their meaning.

Numerator — 2

(# of equal parts)

Means enumeration

or counting

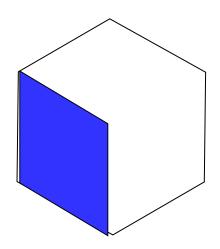
Denominator

(# of equal parts in the whole)
Means name of the class or
type of thing being
represented like a
denomination of money
being counted.

The Meaning of a/b

We have established the meaning of 1/3

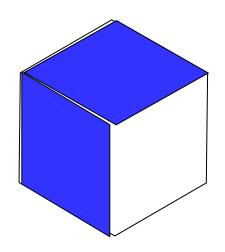
A whole is partitioned into three equal sized pieces. When we want to reference only 1 of the 3 pieces, we call the size of 1/3 of the whole.



The Meaning of a/b

What does 2/3 really mean?

There are 2 pieces in blue. They are each 1/3 of the whole. So two 1/3 pieces is the same as 2/3 of the whole.



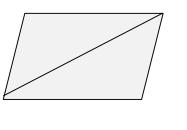
- □Partitioning is a central mathematical idea, and it is central to equal shares
 - Begins in 1st grade

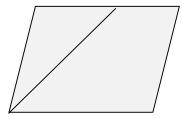
- DEqual shares are not easy for students to draw, but they are essential for understanding (especially for addition and subtraction)
- ☐ The idea of relational is essential

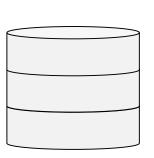
Area Models are Particularly Useful for Showing Fair Shares

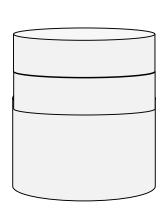
Fair Share Examples





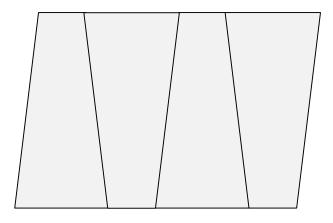


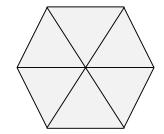




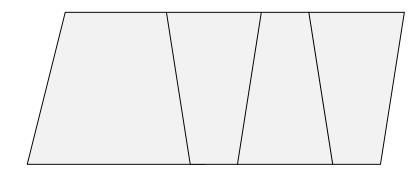
Area Models are Particularly Useful for Showing Fair Shares

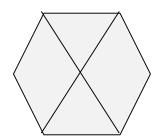
Fair Share Examples



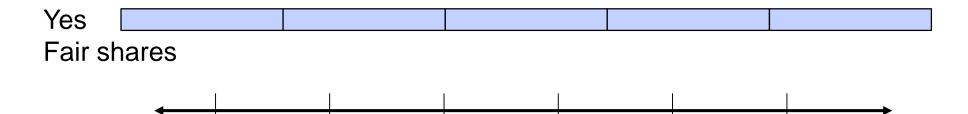


Non-Examples of Fair Shares

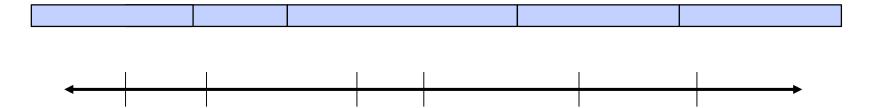


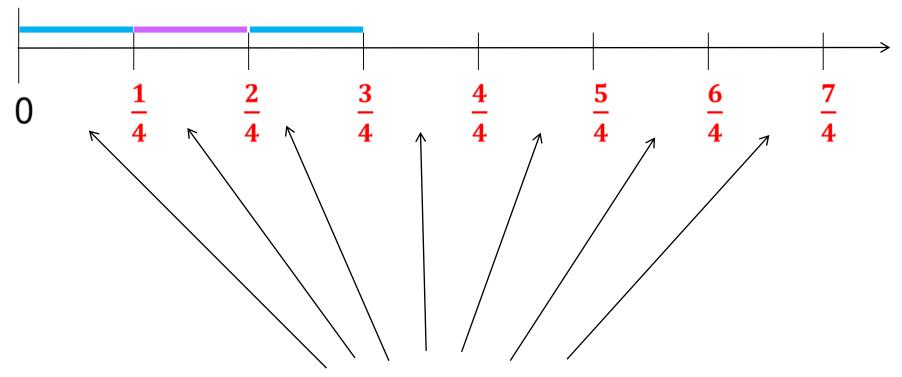


Length Models and Fair Shares



Non-examples of fair shares



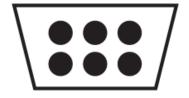


Each segment is equal in size so breaking up the number line this way shows fair shares.

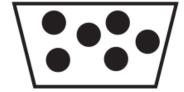
Why are these called fourths?

What does equal shares look like in a set model?

Equal Shares

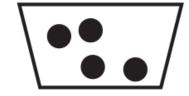


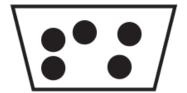


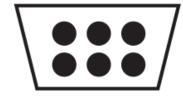


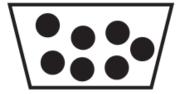


Non-Equal Shares

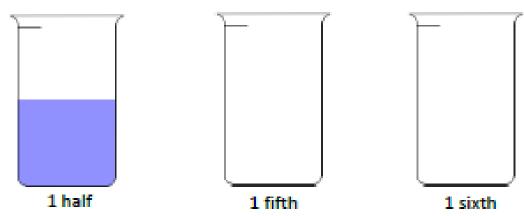




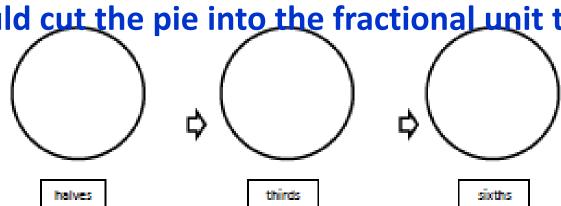




□A beaker is considered full when the liquid reaches the fill line shown near the top. Estimate the amount of water in the beaker by shading the drawing to the indicated amount. The first one is done for you.



□ Each circle represents 1 whole pie. Estimate to show how you would cut the pie into the fractional unit that is given.



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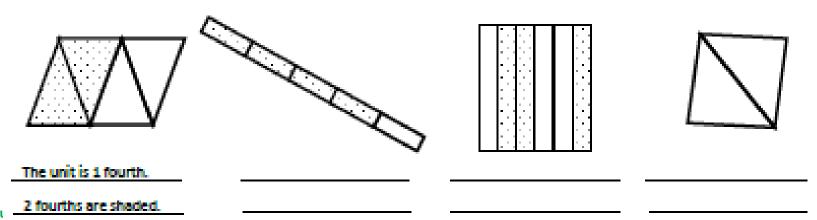
☐ Circle the figures that show fair shares.



☐ Each shape is a whole divided into equal parts.

Name the fractional unit. Then determines how

many of those units are shaded.

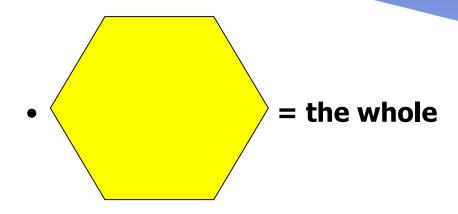




☐What is the value of

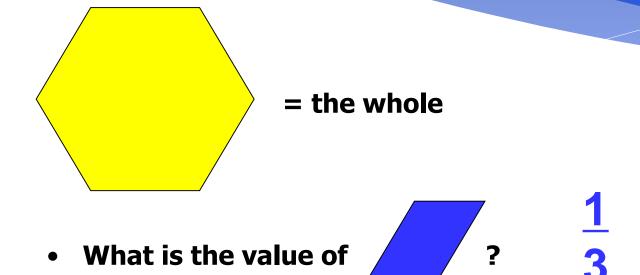
1 2

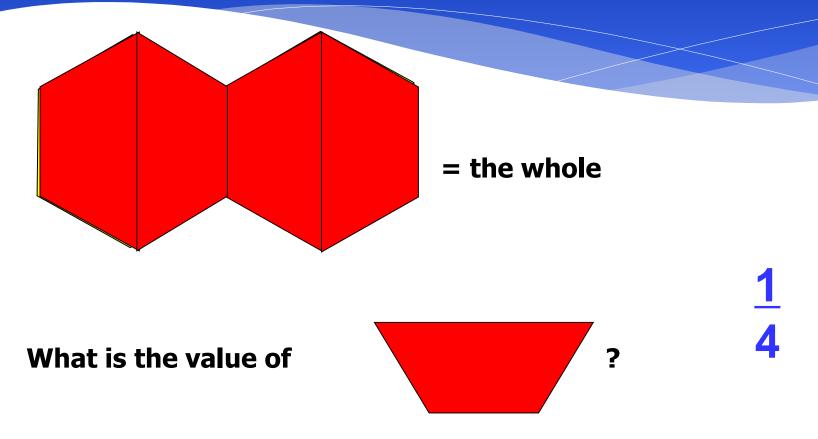
It takes two of the trapezoids to make the whole so one of them would be one-half of the whole.

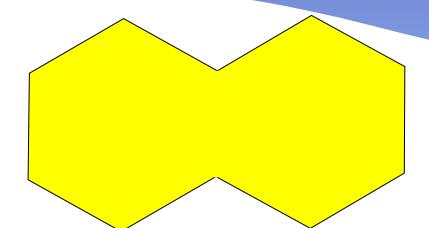


What is the value of ?

<u>1</u>







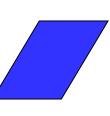
= the whole

What is the value of



? <u>1</u> 12

What is the value of

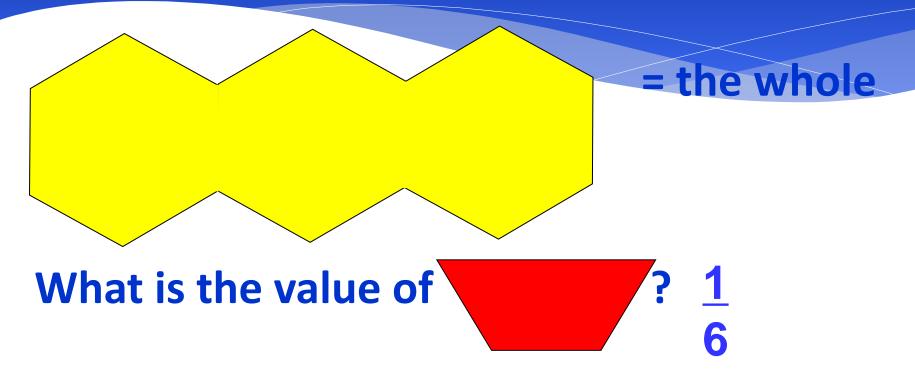


<u>1</u>

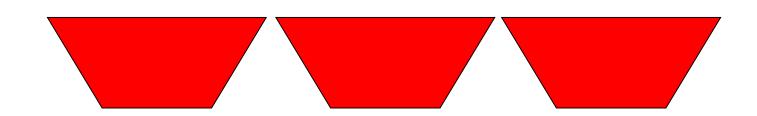
What is the value of



 $\frac{2}{6} = \frac{1}{3}$



What shape(s) will show ½ of this unit?



- □ Partitioning is a central mathematical idea, and it is central to equal shares
- DEqual shares are not easy for students to draw, but they are essential for understanding (especially for addition and subtraction)
- Let's stop to work on how to build the idea of equal shares with areas using different shapes and questions.

The Meaning of a/b

Lengths or numbers lines can also be used. Think about the red pieces. They show units of ¼ of the whole.

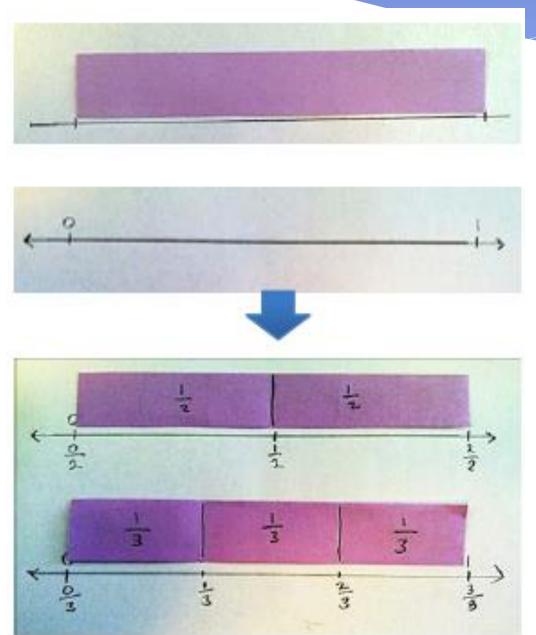
☐ Use these pieces as a reference to show a length that is 2/4 or 3/4. Why can they be called these lengths?

The Meaning of a/b



 \square ³/₄ is the same an 3 units of $\frac{1}{4}$ in length.

$$\square^{3}/_{4} = 1/_{4} + 1/_{4} + 1/_{4}$$



□Creating Paper Strips

- How do we use them to name fractions?
- How do they help students to make sense of part to whole relationships?

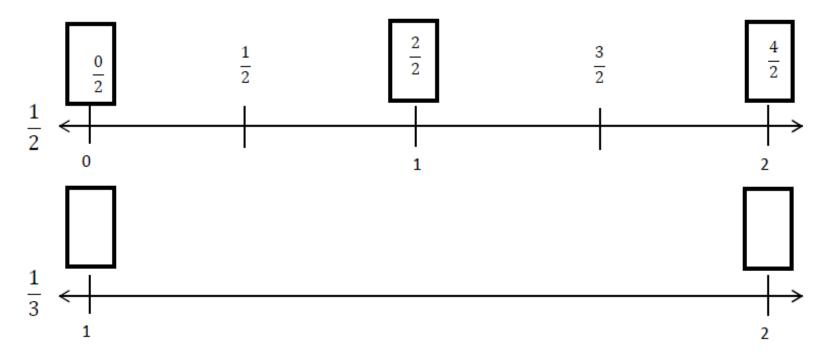
☐ Use Paper Strips to build number lines

White board

- Draw 5-8 lines of different lengths.
- * Choose one line. Fold your write board to match the endpoints of your line. Where is the middle? Open it up and mark this point on the line. It shows two equal segments or ½ of the line.
- * Repeat with two other lines.
- * Now "eye-spy" on the next line where you think will be the point to represent ½ of the line. Fold to see how close you are. REPEAT this until you find you are very close to finding the ½ point for a given line.

	Total Number of Equal Parts		Unit Fraction	Fraction Shaded
Sample:	6	5	<u>1</u>	<u>5</u>
b.				

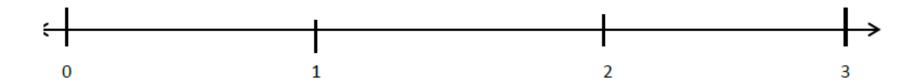
1. Estimate to equally partition and label the unit fractions on the number line. Label the wholes as fractions and box them. The first one is done for you.





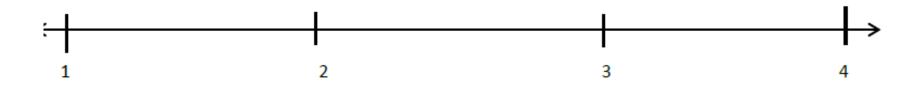
Locate and label the following fractions on the number line.

<u>0</u>

Locate and label the following fractions on the number line.

 $\frac{4}{4}$



□Stations for Making Meaning of Fractions

- □ Each station will show the given fraction using each of the following:
 - 1 meter of yarn
 - Maroonish rectangular piece
 - Green or blue rectangular piece
 - brown square piece
 - A number of cups

□Stations for Making Meaning of Fractions

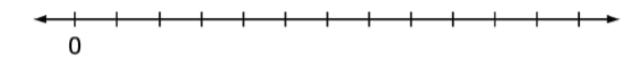
□Museum Walk

- Answer the questions for the different shapes.
 - * What is the unit fraction? How do the units relate to each other from one station to the next?
 - * What is the same at each station?
 - * What do you notice about a fractional unit within a given station?
 - * What surprised you when you were looking at these fractional units?

Shade $\frac{4}{6}$ of the rectangle below. Use the line tool to divide the rectangle by creating horizontal and vertical lines.

Use the line tool to separate the rectangle below into 8 equal parts.				
Use the shading tool to shade one part of the whole rectangle.				
What fraction of the whole rectangle is represented by the part of the rectangle you shaded? Type your answer in the space provided.				

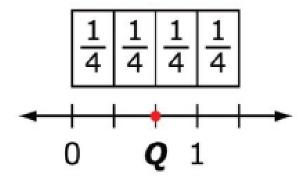
The number line below is divided into equal parts. The zero (0) is already placed on the number line.



Place each of the following numbers in the proper place on number line. To place a number, click on a tile and then click on the number line. You may move and clear numbers as needed.

 $\begin{array}{|c|c|c|c|}\hline 1 & & & \\ \hline \hline \frac{3}{5} & & \\ \hline \end{array}$

• Eva thinks that Q shows $\frac{2}{4}$ on the number line. Eva labeled the number line with unit fractions to show how she determined her answer.



 Is Eva's drawing correct? Explain your reasoning in words, numbers, and /or pictures.

Reflection

□What does it mean for students to have number meaning of fractions?

- What is it?
- What do you already do to support students construct number meaning for fractions?
- What do you need to add to your instruction and learning opportunities to add to their understanding of number meaning?

Fraction Relationships

How will you build the important concepts for students to compare fractions and find equivalency?

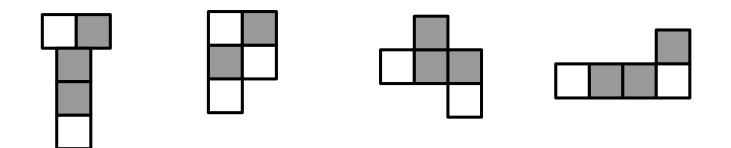
Building Number Sense of Fractions and Decimals

- Number Meaning
 - Counting, Writing, Naming, and Representing
- □ Number Relationships and Number Magnitude and Size
 - How are fractions expressed as equivalent values?
 - How do we compare fractions?
- Building a Sense of Operation

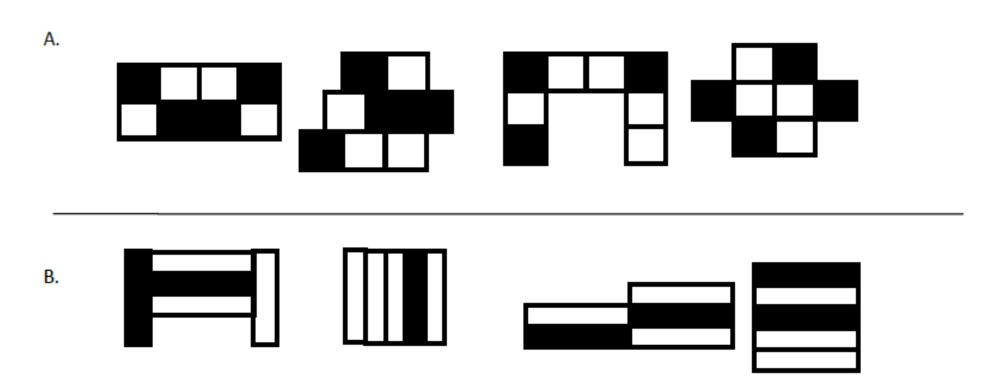
Fraction Relationships: Equivalency

1.

 Label each fraction and then circle the fractions that are equal.



- **□**Which shapes show equivalent areas?
- □What fraction would you use to describe each shape? Which ones are equivalent?

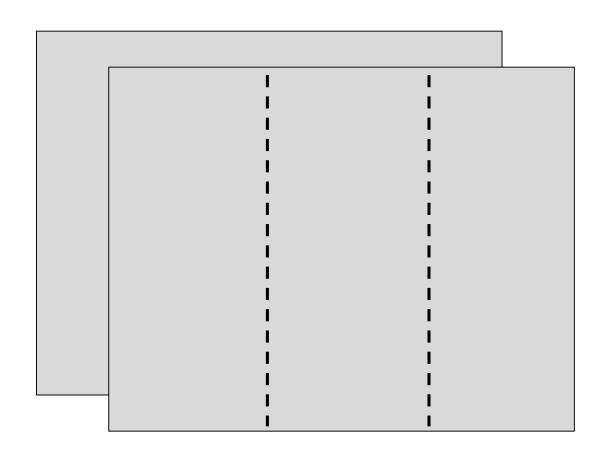


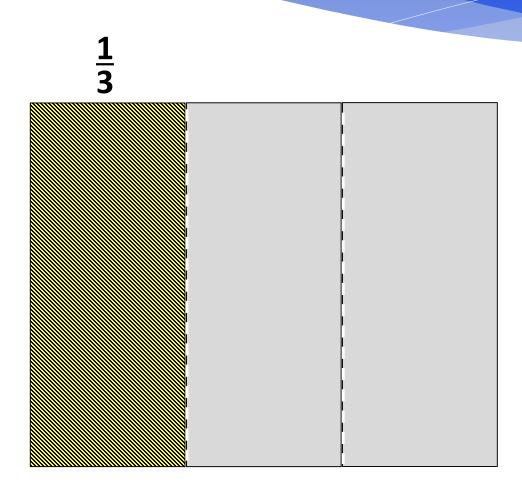
• Katlynn and Ryan are measuring the length of a spider. Ryan measures and says the spider is $1\frac{2}{4}$ inches long. Katlynn measure the same spider and says it is $1\frac{1}{2}$ inches long.

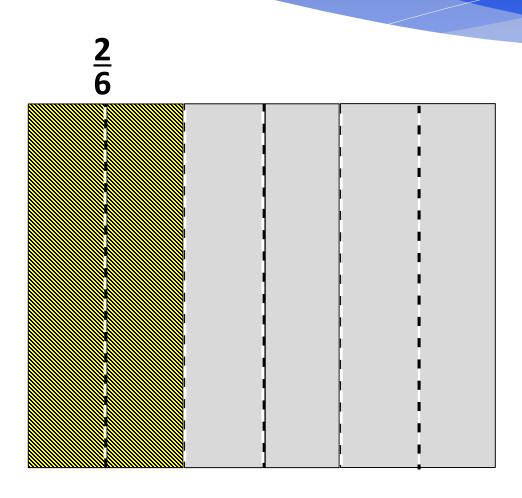
- Who is correct? Why?
- Explain your answer using words, pictures, and numbers.

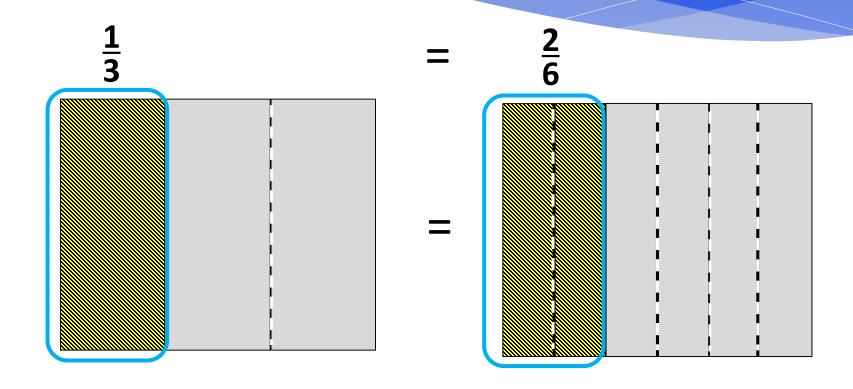
- Katlynn walks $\frac{1}{2}$ a mile to school.
- Ryan says he walks $\frac{2}{4}$ of a mile to school.
- JC walks $\frac{4}{8}$ a mile to school.
 - JC claims he walked the furthest to school. Do you agree or disagree? Why? Why not?
 - Explain your answer using words, pictures, and numbers.

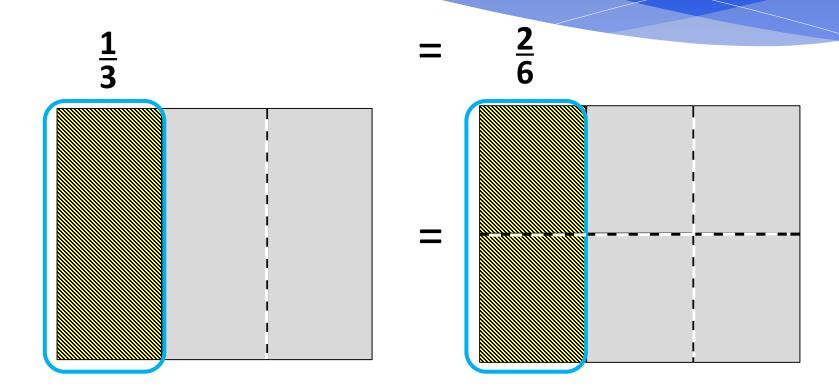
Showing Equivalence with Area Models











☐Get out your pattern blocks.

• If a hexagon is the whole, what are all the ways to make the whole using the same shape? What does this tell us?

• How could we describe a whole using the paper strips?

☐Get out your pattern blocks.

• If a hexagon is the whole, what is the half? Is there another way to make this half?

• How can we make one-third? Is there more than one way?

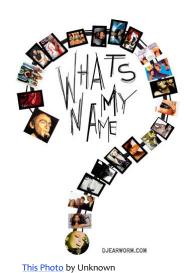
- * What equivalencies can we show using the Cuisenaire rods?
 - If the brown is the whole, then ….
 - What statements can you make that will show equivalent lengths?

- * What equivalencies can we show using the Cuisenaire rods?
 - If the blue is the whole, then ….
 - What statements can you make that will show equivalent lengths?

- Let's look at your fraction strips and the number line.
 - Which strips show equivalence? Why?

What does it mean for these strips to be equivalent? How is it the same as the pattern blocks? How is it different?

Equivalence

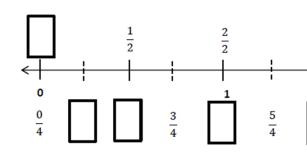


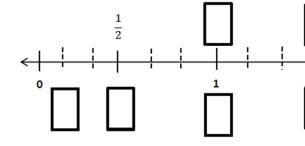
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□ Which of these are another name for 1? Convince me with a picture or number line.

U I	HUHHDEI	IIIIC.		
8		8		
$\frac{8}{8}$	4	1	4	
	- 5		8	
$\frac{8}{4}$		7		
4		8		
	2		2	
				

* Before students work with number lines on paper, have students construct number lines of fractions to see equivalence.

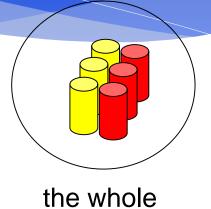




- * https://apps.mathlearningcente r.org/number-line/
- * https://clotheslinemath.com/

A Six Pack of Cola

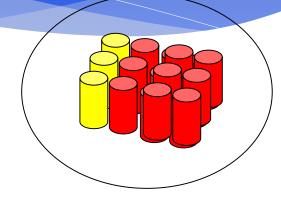
□ The kids drank 3 cans of soda. What fraction of the six pack did they drink?



$$\frac{3}{6} = \frac{1}{2}$$

Soda Packaged as a Carton

There are two six packs in a carton. The kids drank 3 cans of soda. What fraction of the carton did they drink?



the whole

$$\frac{3}{12} = \frac{1}{4}$$

Use the tape diagram to model equivalent fractions. Fill in the blanks and answer the following questions.





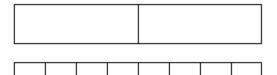
4 sixths is equal to _____ thirds.

$$\frac{4}{6} = \frac{1}{3}$$

The whole stays the same.

What happened to the size of the equal parts when there were less equal parts?

What happened to the number of equal parts when the equal parts became larger?



1 half is equal to _____ eighths.

$$\frac{1}{2} = \frac{1}{8}$$

The whole stays the same.

What happened to the size of the equal parts when there were more equal parts?

What happened to the number of equal parts when the equal parts became smaller?

Reflection

□What does it mean for students to have number relationships (equivalence) of fractions?

- What is it?
- What do you already do to support students construct number meaning for fractions?
- What do you need to add to your instruction and learning opportunities to add to their understanding of number meaning?

Fraction Relationships: Comparing Fractions

Comparing Fractions

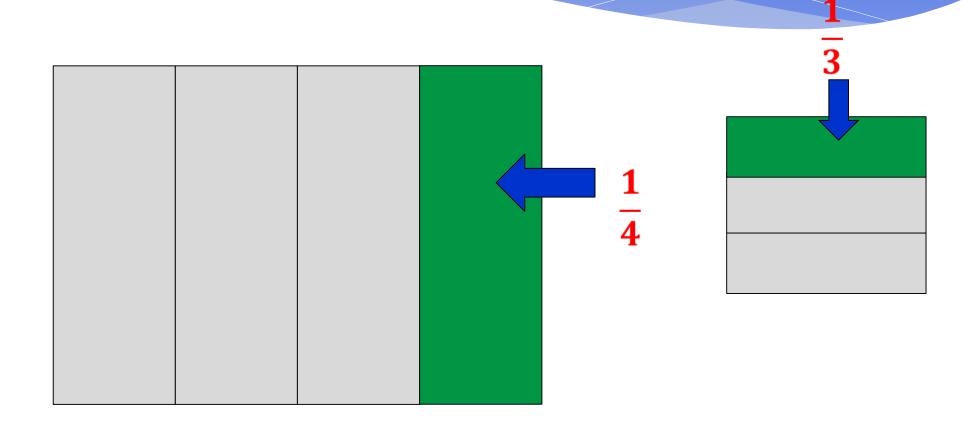
□ Katlynn and Rylan have Valentine's day chocolate bars. Katlynn has eaten 1/3 of her chocolate bar. Rylan has eaten 3/6 of his chocolate bar.

- Who has eaten more?
- Explain your answer using words, pictures, and numbers.

What do students need to understand to answer this question?

- □ First, comparing fractions is ONLY possible if the whole is the same.
 - ½ of a king size candy bar is NOT equivalent to ½ of the mini bar given out at Halloween
 - 1/3 of your kids allowance is NOT equal to 1/3 of your superintendent's salary
- □ Comparing fraction instruction should scaffold
 - Comparing like size pieces
 - Comparing equal number of pieces
 - Comparing to benchmark values

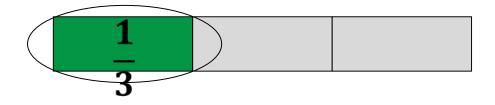
Comparing Fractions: Area Model



Area Models Can Be Deceptive when the whole is not the same.

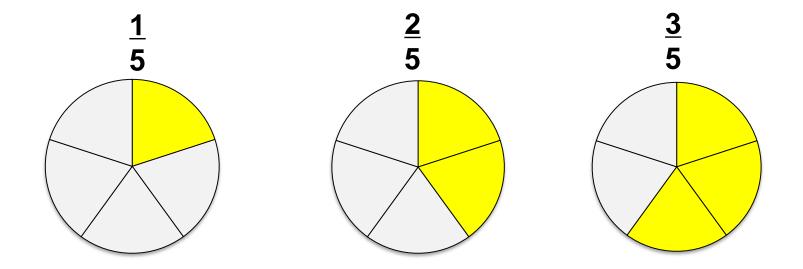
Comparing Fractions: Linear Model





Length Models Can Also Be Deceptive when the whole is not the same.

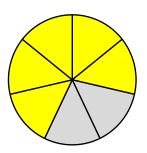
☐ Step 1: Comparing same size pieces



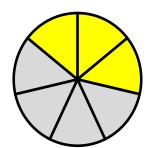
Comparing when the size of the parts are the same is a matter of knowing how many pieces are in one versus the other.

Which is greater in value? $\frac{5}{7}$ or $\frac{3}{7}$

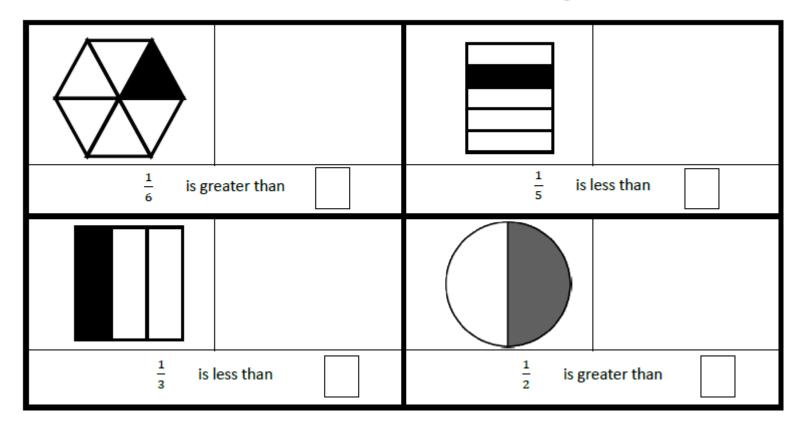
$$\frac{5}{7}$$
 or $\frac{3}{7}$



$$\frac{5}{7} > \frac{3}{7}$$



Fill in the blank with a fraction to make the statement true and draw a matching model.

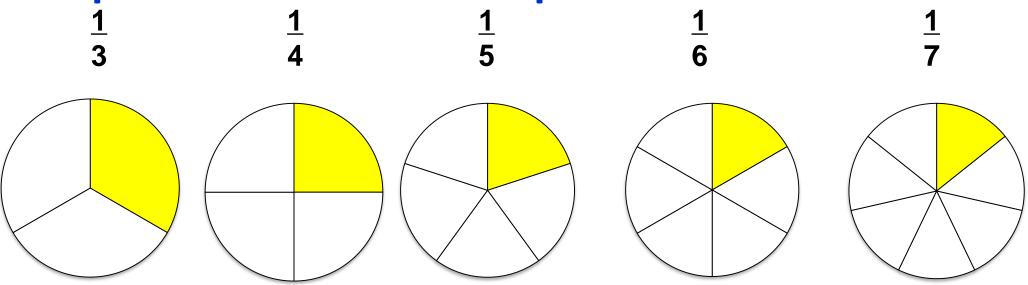


Comparing Fractions

□What order will these values appear on a number line?

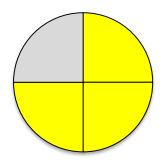
■ B.
$$\frac{5}{7}$$
 $\frac{3}{7}$ $\frac{6}{7}$ $\frac{10}{7}$

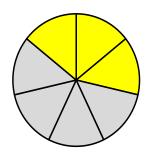
Step 2: Comparing same number of pieces. The equal share area gets smaller as a whole is partitioned into more pieces.



Comparing unit fractions or fractions with the same number of pieces means understanding the size of each part.

Which is greater in value? $\frac{3}{4}$ or $\frac{3}{7}$





$$\frac{3}{4} > \frac{3}{7}$$

Comparing fractions with the same number of pieces means understanding the size of each part.

Comparing Fractions

Compare unit fractions and write >, <, or =.

- a. 1 fourth
- \bigcirc
- 1 eighth

- b. 1 seventh
- \bigcirc
- 1 fifth

- c. 1 eighth
- \bigcirc

1 8

- d. 1 twelfth
- \bigcirc

 $\frac{1}{10}$

e. $\frac{1}{15}$

- \bigcirc
- 1 thirteenth

- f. 3 thirds
- \bigcirc
- 1 whole

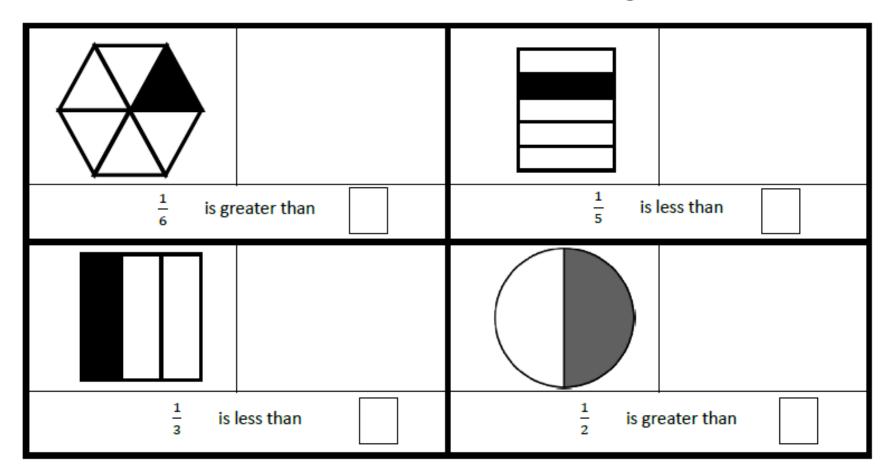
What order will these values appear on a number line?

• A.
$$\frac{4}{8}$$

■ B.
$$\frac{4}{10}$$

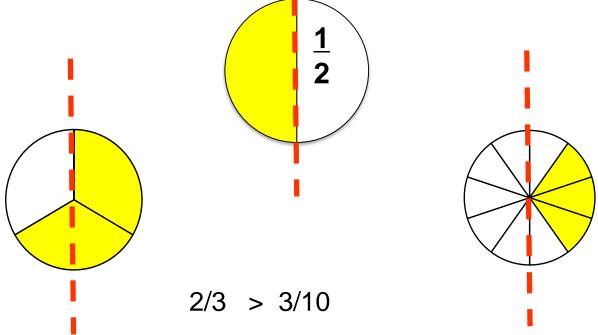
Do we ask these types of comparative questions?

Fill in the blank with a fraction to make the statement true and draw a matching model.



□Step 3: Comparing to benchmarks: 0, ½, 1

 $\frac{2}{3}$ or $\frac{3}{10}$ Which is greater in value? Why?



Comparing when the a benchmark is a matter of knowing if a fraction is closer to one than another one.

Comparing Fractions

Which method can be used to compare the following sets of fractions? Do some have multiple ways they can be compared?

$$\frac{6}{4}$$
 $\frac{9}{4}$

$$\frac{3}{2}$$
 $\frac{5}{2}$

$$\frac{19}{8}$$
 $\left(\right)$ $\frac{16}{8}$

$$\frac{16}{8}$$
 $\frac{3}{2}$

$$\frac{9}{4}$$
 $\frac{19}{8}$

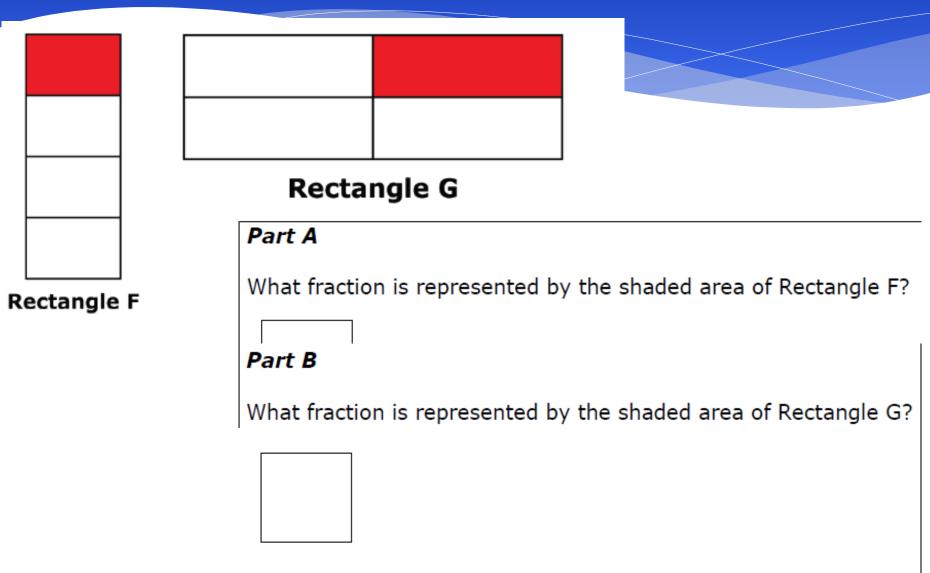
$$\frac{4}{2}$$
 $\frac{16}{8}$

$$\frac{6}{4}$$
 $\frac{16}{8}$

$$\frac{5}{2}$$
 $\frac{9}{4}$

$$\frac{24}{8}$$
 $\frac{11}{4}$

An View from SBAC



Part C

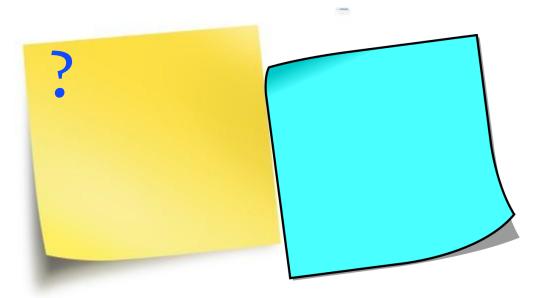
Is the shaded area of Rectangle F equal to the shaded area of Rectangle G? Explain your thinking. Use what you know about the **area** of Rectangle F and Rectangle G to explain.

Developing A Plan

□What should instruction look like for students to develop the meaning and relationships of fractions?

□What will success look like?

Questions and Next Steps



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