AP® Calculus AB

Free Response Questions
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Compiled by Kaye Autrey for face-to-face student instruction in the AP Calculus classroom

AP Calculus Free-Response Questions

1. Consider the following functions defined for all x:

$f_1(x) = x$

$f_2(x) = x\cos x$

$f_3(x) = 3e^{2x}$

$f_4(x) = x - |x|$

Answer the following questions (a, b, c, and d) about each of these functions. Indicate your answer by writing either yes or no in the appropriate space in the given rectangular grid. No justification is required but each blank space will be scored as an incorrect answer.

Questions			Functions	
	f_1	f_2	f_3	f_4
(a) Does $f(-x) = -f(x)$				
(b) Does the inverse function exist for all x?				
(c) Is the function periodic?				
(d) Is the function continuous at x = 0?				

2. A particle moves along the x-axis in such a way that its position at time t is given by $x = 3t^4 - 16t^3 + 24t^2$ for $-5 \le t \le 5.$

a. Determine the velocity and acceleration of the particle at time t.
b. At what values of t is the particle at rest?
c. At what values of t does the particle change direction?
d. What is the velocity when the acceleration is first zero?

3. Given $f(x) = \dfrac{1}{x} + \ln x,$ defined only on the closed interval $\dfrac{1}{e} \le x < e.$

a. Showing your reasoning, determine the value of x at which f has its
   (i) absolute maximum
   (ii) absolute minimum
b. For what values of x is the curve concave up?
c. On the coordinate axis provided, sketch the graph of f over the interval $\dfrac{1}{e} \le x < e.$
d. Given that the mean value (average ordinate) of f over the interval is

$\frac{2}{e-1}$, state in words a geometrical interpretation of this number relative to the graph.

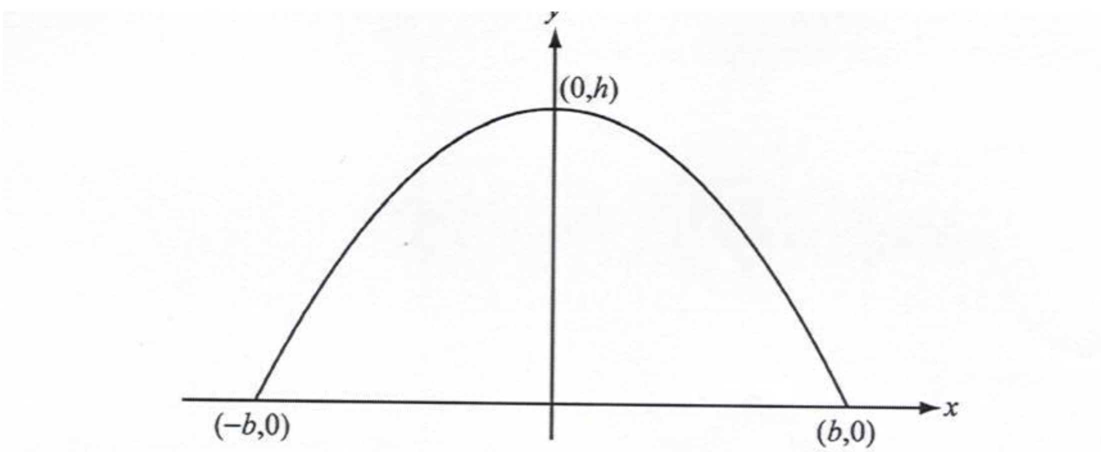
4. The number of bacteria in a culture at time t is given approximately by

$$y = 1000(25 + te^{\frac{-t}{20}}) \text{ for } 0 \le t \le 100.$$

a. Find the largest number and the smallest number of bacteria in the culture during the interval.
b. At what time during the interval is the rate of change in the number of bacteria a minimum?

5. Let R denote the region enclosed between the graph of $y = x^2$ and the graph of $y = 2x$.

a. Find the area of region R.
b. Find the volume of the solid obtained by revolving the region R about the y-axis.



6. An arched window with base width $2b$ and height h is set into a wall. The arch is to be either an arc of a parabola or a half-cycle of a cosine curve.

a. If the arch is an arc of a parabola, write an equation for the parabola relative to the coordinate system shown in the figure. (x-intercepts are $(-b, 0)$ and $(b, 0)$. y-intercept is $(0, h)$.)
b. If the arch is a half-cycle of a cosine curve, write an equation for the cosine curve relative to the coordinate system shown in the figure.
c. Of these two window designs, which has the greater area? Justify your answer.

7. a. On the coordinate axes provided, sketch the graph of $y=\frac{e^x+e^{-x}}{2}$.

b. Let R be a point on the curve and let the x-coordinate of R be r $(r\neq 0)$. The tangent line to the curve at R crosses the x-axis at a point Q. Find the coordinates of Q.

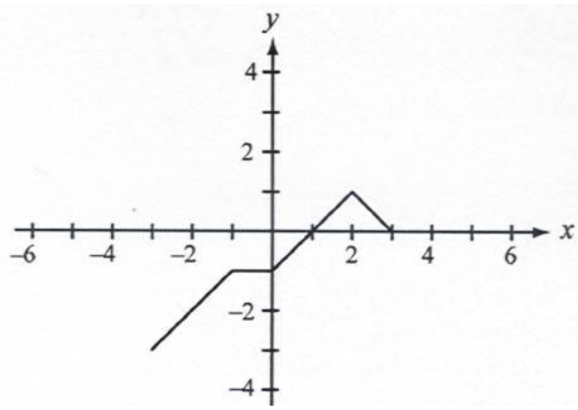
c. If P is the point $(r,0)$, find the length of PQ as a function of r and the limiting value of this length as r increases without bound.

8. Given the parabola $y=x^2-2x+3$:

a. Find an equation for the line L, which contains the point (2, 3) and is perpendicular to the line tangent to the parabola at (2, 3).

b. Find the area of that part of the first quadrant which lies below <u>both</u> the line L and the parabola.

9. A function f is defined on the closed interval from -3 to 3 and has the graph shown below.



a. Sketch the entire graph of $y=\left|f(x)\right|$.

b. Sketch the entire graph of $y=f(|x|)$.

c. Sketch the entire graph of $y=f(-x)$.

d. Sketch the entire graph of $y=f\left(\frac{1}{2}x\right)$.

e. Sketch the entire graph of $y=f(x-1)$.

10. Consider the function f given by $f(x)=x^{\frac{4}{3}}+4x^{\frac{1}{3}}$.

a. Find the coordinates of all points at which the tangent to the curve is a horizontal line.

b. Find the coordinates of all points at which the tangent to the curve is a vertical line.

c. Find the coordinates of all points at which the absolute maximum and absolute minimum occur.
d. For what values of x is this function concave down?
e. Sketch the graph of the function on this interval.

11. A right circular cone and a hemisphere have the same base, and the cone is inscribed in the hemisphere. The figure is expanding in such a way that the combined surface area of the hemisphere and its base is increasing at a constant rate of 18 square inches per second. At what rate is the volume of the cone changing at the instant when the radius of the common base is 4 inches? Show your work.

12. A particle moves along the x-axis in such a way that at time $t > 0$ its position coordinate is $x = \sin\left(e^t\right)$.

a. Find the velocity and acceleration of the particle at time t.
b. At what time does the particle first have zero velocity?
c. What is the acceleration of the particle at the time determined in part (b)?

13. A parabola P is symmetric to the y-axis and passes through (0, 0) and (b, e^{-b^2}) where $b > 0$.

a. Write an equation for P.
b. The closed region bounded by P and the line $y = e^{-b^2}$ is revolved about the y-axis to form a solid figure F. Compute the volume of F.
c. For what value of b is the volume of F a maximum? Justify your answer.

14. From the fact that $\sin t \le t \text{ for } t \ge 0$, use integration repeatedly to prove the following inequalities. Show your work.

$$1 - \frac{1}{2!}x^2 \le \cos x \le 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 \text{ for all } x \ge 0$$

15. Let f (x) = ln(x) for all x >0, and let g(x) = x^2 - 4 for all real x. Let H be the composition of f with g, that is, H(x) =f(g(x)). Let K be the composition of g with f, that is, K(x)= g(f(x)).

a. Find the domain of H.
b. Find the range of H.
c. Find the domain of K.
d. Find the range of K.
e. Find H′(7).

16. Let R be the region in the first quadrant that lies below both of the curves $y = 3x^2$ and $y = \frac{3}{x}$ and to the left of the line x = k, where k > 1.

a. Find the area of R as a function of k.
b. When the area of R is 7, what is the value of k?
c. If the area of R is increasing at the constant rate of 5 square units per second at what rate is k increasing when k = 15?

17. Consider $F(x)= \cos^2 x + 2\cos x$ over one complete period beginning with x = 0.

a. Find all values of x in this period at which F(x) = 0.
b. Find all values of x in this period at which the function has a minimum. Justify your answer.
c. Over what intervals in this period is the curve concave up?

18. Find the area of the largest rectangle (with sides parallel to the coordinate axes) that can be inscribed in the region enclosed by the graphs of $f(x) = 18 - x^2$ and $g(x) = 2x^2 - 9$.

19. Let R be the region of the first quadrant bounded by the x-axis and the curve $y = 2x - x^2$.

a. Find the volume produced when R is revolved around the x-axis.
b. Find the volume produced when R is revolved around the y-axis.

20. A particle starts at the point (5,0) at t=0 and moves along the x-axis in such a way that at time t > 0 its velocity v(t) is given by $v(t) = \frac{t}{1+t^2}$.

a. Determine the maximum velocity attained by the particle. Justify your answer.
b. Determine the position of the particle at t = 6.
c. Find the limiting value of the velocity as t increases without bound.
d. Does the particle ever pass the point (500,0)? Explain.

21. Let f be the function defined by $f(x) = |x| \; .5e^{-x^2}$ for all real numbers x.

a. Describe the symmetry of the graph of f.
b. Over what intervals of the domain is this function increasing?
c. Sketch the graph of f on the axes provided showing clearly:
   (i) behavior near the origin
   (ii) maximum and minimum points
   (iii) behavior for large |x|.

22. Let $f(x) = 4x^3 - 3x - 1$.

a. Find the x-intercepts of the graph of f.

b. Write an equation for the tangent line to the graph of f at x = 2.

c. Write an equation of the graph that is the reflection across the y-axis of the graph of f.

23. A particle starts at time t = 0 and moves on a number line so that its position at time t is given by $x(t) = (t - 2)^3 (t - 6)$.

a. When is the particle moving to the right?

b. When is the particle at rest?

c. When does the particle change direction?

d. What is the farthest to the left of the origin that the particle moves?

24. Let f(x) = k sin (kx), where k is a positive constant.

a. Find the area of the region bounded by one arch of the graph of f and the x-axis.

b. Find the area of the triangle formed by the x-axis and the tangent to one arch of f at the points where the graph of f crosses the x-axis.

25. A man has 340 yards of fencing for enclosing two separate fields, one of which is to be a rectangle twice as long as it is wide and the other a square. The square field must contain at least 100 square yards and the rectangular one must contain at least 800 square yards.

a. If x is the width of the rectangular field, what are the maximum and minimum possible values of x?

b. What is the greatest number of square yards that can be enclosed in the two fields? Justify your answer.

26. Let $y = 2e^{\cos(x).}$

a. Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b. If x and y both vary with time in such a way that y increases at a steady rate of 5 units per second, at what rate is x changing when x = $\frac{\pi}{2}$.

27. The shaded region R is bounded by the graphs of xy =1, x = 1, x = 2, and y = 0.

a. Find the volume of the solid figure generated by revolving the region R about the x-axis.

b. Find the volume of the solid figure generated by revolving the region R about the line x = 3.

28. A function f is defined for all real numbers and has the following three properties:

(i) $f(1) = 5$,

(ii) $f(3) = 21$, and

(iii) for all real values of a and b, $f(a + b) - f(a) = kab + 2b^2$ where k is a fixed real number independent of a and b.

a. Use a = 1 and b = 2 to find the value of k.

b. Find $f'(3)$.

c. Find $f'(x)$ and $f(x)$ for all real x.

29. Given $f(x) = x^3 - 6x^2 + 9x$ and $g(x) = 4$.

a. Find the coordinates of the points common to the graphs of f and g.

b. Find all the zeros of f.

c. If the domain of f is limited to the closed interval [0,2], what is the range of f? Show your reasoning.

30. A particle moves on the x-axis so that its acceleration at any time t > 0 is given by $a = \frac{t}{8} - \frac{1}{t^2}$. When t = 1, $v = \frac{9}{16}$, and $s = \frac{25}{48}$.

a. Find the velocity v in terms of t.

b. Does the numerical value of the velocity ever exceed 500? Explain.

c. Find the distance s from the origin at time t = 2.

31. Given the curve $x + xy + 2y^2 = 6$.

a. Find an expression for the slope of the curve at any point (x,y) on the curve.

b. Write an equation for the line tangent to the curve at the point (2,1).

c. Find the coordinate of all other points on this curve with slope equal to the slope at (2,1).

32.

a. What is the set of all values of b for which the graphs of $y = 2x + b$ and $y^2 = 4x$ intersect in two distinct points?

b. In the case b = -4, find the area of the region enclosed by $y = 2x - 4$ and $y^2 = 4x$.

c. In the case b = 0, find the volume of the solid generated by revolving about the x-axis the region bounded by $y = 2x$ and $y^2 = 4x$.

33.

a. Find the coordinate of the absolute maximum point for the curve $y = x e^{-kx}$ where k is a fixed positive number. Justify your answer.

b. Write an equation for the set of absolute maximum points for the curves $y = x e^{-kx}$ as k varies through positive values.

34. A manufacturer finds it costs him $x^2 + 5x + 7$ dollars to produce x tons of an item. At production levels above 3 tons, he must hire additional workers, and his costs increase by 3(x - 3) dollars on his total production. If the price he receives is $13 per ton regardless of how much he manufactures and if he has a plant capacity of 10 tons, what level of output maximizes his profits?

35.

a. Find the area A, as a function of k, of the region in the first quadrant enclosed by the y-axis and the graphs of y = tan x and y = k for k > 0.

b. What is the value of A when k = 1?

c. If the line y = k is moving upward at the rate of $\frac{1}{10}$ units per second, at what rate is A changing when k = 1?

36. Given $f(x) = |\sin x|$, $-\pi \le x \le \pi$, and $g(x) = x^2$ for all real x.

a. On the axes provided, sketch the graph of f.

b. Let H(x) = g(f(x)). Write an expression for H(x).

c. Find the domain and range of H.

d. Find an equation of the line tangent to the graph of H at the point where $x = \frac{\pi}{4}$.

37. Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$. The graph of y = P(x) is symmetric with respect to the y-axis, has a relative maximum at (0,1), and has an absolute minimum at (q, -3).

a. Determine the values of a, b, c, and d, and using these values write an expression for P(x).

b. Find all possible values for q.

38. Let $f(x) = kx^2 + c$.

a. Find x_o in terms of k such that the tangent lines to the graph of f at $(x_o, f(x_o))$ and $(-x_o, f(-x_o))$ are perpendicular.

b. Find the slopes of the tangent lines mentioned in a.

c. Find the coordinates, in terms of k and c, of the point of intersection of the tangent lines mentioned in a.

39. Let f be a function defined for all x > -5 and having the following properties.

(i) $f''(x) = \dfrac{1}{3\sqrt{x+5}}$ for all x in the domain of f.

(ii) The line tangent to the graph of f at (4,2) has an angle of inclination of 45°. Find an expression for f(x).

40. A ball is thrown from the origin of a coordinate system. The equation of its path is $y = mx - \dfrac{1}{1000}e^{2m}x^2$, where m is positive and represents the slope of the path of the ball at the origin.

a. For what value of m will the ball strike the horizontal axis at the greatest distance from the origin? Justify your answer.

b. For what value of m will the ball strike at the greatest height on a vertical wall located 100 feet from the origin?

41. Given two functions f and g defined by f(x) = tan (x) and g(x) = $\sqrt{2}$ cos x.

a. Find the coordinates of the point of intersection of the graphs of f and g in the interval $0 < x < \dfrac{\pi}{2}$.

b. Find the area of the region enclosed by the y-axis and the graphs of f and g.

42. The rate of change in the number of bacteria in a culture is proportional to the number present. In a certain laboratory experiment, a culture had 10,000 bacteria initially, 20,000 bacteria at time t_1 minutes, and 100,000 bacteria at (t_1 + 10) minutes.

a. In terms of t only, find the number of bacteria in the culture at any time t minutes, $t \geq 0$.

b. How many bacteria were there after 20 minutes?

c. How many minutes had elapsed when the 20,000 bacteria were observed?

43. Let R be the region bounded by the graphs of y = ln x, the line x = e, and the x-axis.

a. Find the volume generated by revolving R about the x-axis.

b. Find the volume generated by revolving R about the y-axis.

44. Given the function f defined by f(x) = ln (x^2 - 9).

a. Describe the symmetry of the graph of f.

b. Find the domain of f.

c. Find all values of x such that f(x) = 0.

d. Write a formula for $f^{-1}(x)$, the inverse function of f, for x > 3.

45. A particle moves along the x-axis in such a way that its position at time t for $t \geq 0$ is given by $x = \frac{1}{3}t^3 - 3t^2 + 8t$.

a. Show that at time t= 0 the particle is moving to the right.
b. Find all values of t for which the particle is moving to the left.
c. What is the position of the particle at time t = 3?
d. When t = 3, what is the total distance the particle has traveled?

46. Given the function f defined for all real numbers by $f(x) = 2|x-1|x^2$.

a. What is the range of the function?
b. For what values of x is the function continuous?
c. For what values of x is the derivative of f(x) continuous?
d. Determine the $\int_0^1 f(x)dx$.

47. Given the function defined by y = x + sin(x) for all x such that $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$.

a. Find the coordinates of all maximum and minimum points on the given interval. Justify your answers.
b. Find the coordinates of all points of inflection on the given interval. Justify your answers.
c. On the axes provided, sketch the graph of the function.

48. The line x = c where c > 0 intersects the cubic $y = 2x^3 + 3x^2 - 9$ at point P and the parabola $y = 4x^2 + 4x + 5$ at point Q.

a. If a line tangent to the cubic at point P is parallel to the line tangent to the parabola at point Q, find the value of c where c > 0.
b. Write the equations of the two tangent lines described in a.

49. Let R be the region in the first quadrant bounded by the graphs of $\frac{x^2}{9} + \frac{y^2}{81} = 1$ and 3x + y = 9.

a. Set up but <u>do not integrate</u> an integral representing the area of R. Express the integrand as a function of a single variable.
b. Set up but <u>do not evaluate</u> an integral representing the volume of the solid generated when R is rotated about the x-axis. Express the integrand as a function of a single variable.
c. Set up but <u>do not evaluate</u> an integral representing the volume of the solid generated when R is rotated about the y-axis. Express the integrand

as a function of a single variable.

50. Given a function f with the following properties:

(i) $f(x + h) = e^h f(x) + e^x f(h)$ for all real numbers x and h.
(ii) f(x) has a derivative for all real numbers x.
(iii) $f'(0) = 2$.

a. Show that f(0) = 0.

b. Using the definition of $f'(0)$, find $\lim_{x \to 0} \frac{f(x)}{x}$.

c. Prove there exists a real number p such that $f'(x) = f(x) + pe^x$ for all real numbers x.

d. What is the value of the number p that is described in c?

51. Let f be a real-valued function defined by f(x) = $\sqrt{1+6x}$

a. Give the domain and range of f.
b. Determine the slope of the line tangent to the graph of f at x = 4.
c. Determine the y-intercept of the line tangent to the graph of f at x = 4.
d. Give the coordinate of the point on the graph of f where the tangent line is parallel to y = x + 12.

52. Given the two functions f and h such that $f(x) = x^3 - 3x^2 - 4x + 12$ and

h(x) = {$h(x) = \frac{f(x)}{x-3}$ for $x \neq 3$, and p for x = 3}

a. Find all zeros of the function f.
b. Find the value of p so that the function h is continuous at x = 3. Justify your answer.
c. Using the value of p found in b, determine whether h is an even function. Justify your answer.

53. Let R be the region bounded by the curves f(x) = $\frac{4}{x}$ and $g(x) = (x - 3)^2$.

a. Find the area of R.
b. Find the volume of the solid generated by revolving R about the x-axis.

54.

a. A point moves on the hyperbola $3x^2 - y^2 = 23$ so that its y-coordinate is increasing at a constant rate of 4 units per second. How fast is the x-coordinate changing when
x = 4?

b. For what values of k will the line 2x + 9y + k= 0 be normal to the hyperbola $3x^2 - y^2 = 23$?

55. Given the function defined by y = $e^{\sin(x)}$ for all x such that -□ $\le$ x $\le$ 2 □.

a. Find the x- and y-coordinates of all maximum and minimum points on the given interval. Justify your answers.

b. On the axes provided, sketch the graph of the function.

c. Write an equation for the axis of symmetry of the graph

56. a. Given $5x^3 + 40 = \int_c^x f(t)dt$.

(i) Find f(x).

(ii) Find the value of c.

b. If $F(x) = \int_x^3 \sqrt{1+t^{16}}dt$, find F′(x).

57. For a differentiable function f, let $f^*(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x-h)}{h}$.

a. Determine $f^*(x)$ for f(x) = x^2 + x.

b. Determine $f^*(x)$ for f(x) = cos(x).

c. Write an equation that expresses the relationship between the functions f^* and f' where f' denotes the usual derivative of f. (The BC exam requires justification of the answer)

58. Let f(x) = cos(x) for 0 $\le$ x $\le$ 2 □, and let g(x) = ln(x) for all x > 0. Let S be the composition of g with f, that is, S(x) = g(f(x)).

a. Find the domain of S.

b. Find the range of S.

c. Find the zeros of S.

d. Find the slope of the line tangent to the graph of S at x = $\dfrac{\pi}{3}$.

59. Consider the function f defined by f(x) = $(x^2 - 1)^3$ for all real numbers x.

a. For what values of x is the function increasing?

b. Find the x- and y-coordinates of the relative maximum and minimum points. Justify your answer.

c. For what values of x is the graph of f concave upward?

d. Using the information found in parts a, b, and, c, sketch the graph of f on the axes provided.

60. Given the function f defined for all real numbers by f(x) = $e^{\frac{x}{2}}$.

a. Find the area of the region R bounded by the line y = e, the graph of f, and the y-axis.

b. Find the volume of the solid generated by revolving R, the region in a, about the x-axis.

61. Let f and g and their inverses f^{-1} and g^{-1} be differentiable functions and let the values of f, g, and the derivatives f′ and g′ at x = 1 and x = 2 be given by the table below:

x	1	2
$f(x)$	2	3
$g(x)$	2	π
$f'(x)$	5	6
$g'(x)$	4	7

Determine the value of each of the following:

a. The derivative of f + g at x = 2.

b. The derivative of fg at x = 2.

c. The derivative of f/g at x = 2.

d. h'(1) where h(x) = f(g(x)).

e. The derivative of g^{-1} at x = 2.

62. A particle moves along the x-axis with acceleration given by

$$a(t) = 2t - 10 + \frac{12}{t}$$

for t $\geq$ 1.

a. Write an expression for the velocity v(t), given that v(1) = 9.

b. For what values of t, 1 $\leq$ t $\leq$ 3, is the velocity a maximum?

c. Write an expression for the position x(t), given that x(1) = -16.

63. A rectangle has a constant area of 200 square meters and its length L is increasing at the rate of 4 meters per second.

a. Find the width W at the instant the width is decreasing at the rate of 0.5 meters per second.

b. At what rate is the diagonal D of the rectangle changing at the instant when the width is 10 meters.

64. Let f be the *real-valued function defined by f(x) = sin³(x) + sin³*|x|.

a. Find $f'(x)$ for x > 0.

b. Find $f'(x)$ for $x < 0$.
c. Determine whether f(x) is continuous at x = 0. Justify your answer.
d. Determine whether the derivative of f(x) exists at x = 0. Justify your answer.

65. Given the function f defined by $f(x) = x^3 - x^2 - 4x + 4$.
a. Find the zeros of f.
b. Write an equation of the line tangent to the graph of f at x = -1.
c. The point (a,b) is on the graph of f and the line tangent to the graph at (a,b) passes through the point (0,-8) which is not on the graph of f. Find the value of a and b.

66. Let $f(x) = (1 - x)^2$ for all real numbers x, and let g(x) = ln(x) for all x > 0. Let $h(x) = (1 - \ln(x))^2$.
a. Determine whether h(x) is the composition f(g(x)) or the composition g(f(x)).
b. Find h′ (x).
c. Find h″(x).
d. On the axes provided, sketch the graph of h.

67. Given the function f defined by $f(x) = \dfrac{2x-2}{x^2+x-2}$.
a. For what values of x is f(x) discontinuous?
b. At each point of discontinuity found in part a, determine whether f(x) has a limit and, if so, give the value of the limit.
c. Write an equation for each vertical and horizontal asymptote to the graph of f. Justify your answer.
d. A rational function $g(x) = \dfrac{a}{b+x}$ is such that g(x) = f(x) wherever f is defined. Find the value of a and b.

68. A particle moves on the x-axis so that its velocity at any time t is given by v(t) = sin(2t). At t = 0, the particle is at the origin.
a. For $0 \le t \le \pi$, find all values of t for which the particle is moving to the left.
b. Write an expression for the position of the particle at any time t.
c. For $0 \le t \le \dfrac{\pi}{2}$, find the average value of the position function determined in part b.

69. Given the curve $x^2 - xy + y^2 = 9$.
a. Write a general expression for the slope of the curve.
b. Find the coordinates of the points on the curve where the tangents are

vertical.

c. At the point (0, 3) find the rate of change in the slope of the curve with respect to x.

70. Given the function f defined by $f(x) = e^{-x^2}$.

a. Find the maximum area of the rectangle that has two vertices on the x-axis and two on the graph of f. Justify your answer.

b. Let R be the region in the first quadrant bounded by the x- and y-axes, the graph of f, and the line x = k. Find the volume of the solid generated by revolving R about the y-axis.

c. Evaluate the limit of the volume determined in part b as k increases without bound.

71. Let g and h be any two twice-differentiable functions that are defined for all real numbers and that satisfy the following properties for all x:

(i) $(g(x))^2 + (h(x))^2 = 1$

(ii) $g'(x) = (h(x))^2$

(iii) $h(x) > 0$

(iv) $g(0) = 0$

a. Justify that $h'(x) = -g(x)h(x)$ for all x.

b. Justify that h has a relative maximum at x = 0.

c. Justify that the graph of g has a point of inflection at x = 0.

72. Given the function f defined by $f(x) = 2x^3 - 3x^2 - 12x + 20$.

a. Find the zeros of f.

b. Write an equation of the line normal to the graph of f at x = 0.

c. Find the x- and y-coordinates of all points on the graph of f where the line tangent to the graph is parallel to the x-axis.

73. A function f is defined by $f(x) = x\,e^{-2x}$ with domain $0 \leq x \leq 10$.

a. Find all values of x for which the graph of f is increasing and all values of x for which the graph decreasing.

b. Give the x- and y-coordinates of all absolute maximum and minimum points on the graph of f. Justify your answers.

74. Find the maximum volume of a box that can be made by cutting out squares from the corners of an 8-inch by 15-inch rectangular sheet of cardboard and folding up the sides. Justify your answer.

75. A particle moves along a line so that at any time t its position is given by $x(t) = 2\pi t + \cos 2\pi t$.

a. Find the velocity at time t.

b. Find the acceleration at time t.
c. What are all values of t, $0 \leq t \leq 3$, for which the particle is at rest?
d. What is the maximum velocity?

76. Let R be the region bounded by the graph of y = (1/x) ln(x), the x-axis, and the line x = e.

a. Find the area of the region R.
b. Find the volume of the solid formed by revolving the region R about the y-axis.

77. Given the function f where $f(x) = x^2 - 2x$ for all real numbers x.

a. On the axes provided, sketch the graph of $y = |f(x)|$.
b. Determine whether the derivative of $|f(x)|$ exists at x = 0. Justify your answer.
c. On the axes provided, sketch the graph of $y = f(|x|)$.
d. Determine whether $f(|x|)$ is continuous at x = 0. Justify your answer.

78. Let f be the function defined by f(x) $x^3 + ax^2 + bx + c$ and having the following properties:

(i) The graph of f has a point of inflection at (0,-2).
(ii) The average (mean) value of f(x) on the closed interval [0,2] is -3.
a. Determine the values of a, b, and c.
b. Determine the value of x that satisfies the conclusion of the Mean Value Theorem for f on the closed interval [0,3].

79. Let R be the region enclosed by the graphs of $y = x^3$ and $y = \sqrt{x}$.

a. Find the area of R.
b. Find the volume of the solid generated by revolving R about the x-axis.

80. A rectangle ABCD with sides parallel to the coordinate axes is inscribed in the region enclosed by the graph of $y = -4x^2 + 4$ and the x-axis.

a. Find the x- and y-coordinates of C so that the area of the rectangle ABCD is a maximum.
b. The point C moves along the curve with its x-coordinate increasing at the constant rate of 2 units per second. Find the rate of change of the area of rectangle ABCD when $x = \frac{1}{2}$.

81. Let $f(x) = \ln(x^2)$ for $x > 0$ and $g(x) = e^{2x}$ for $x \geq 0$. Let H be the composition of f with g, that is H(x) = f(g(x)), and let K be the composition of g with f, that is, K(x) = g(f(x)).

a. Find the domain of H and write an expression for H(x) that does not contain the exponential function.
b. Find the domain of K and write an expression for K(x) that does not contain the exponential function.
c. Find an expression for $f^{-1}(x)$, where f^{-1} denotes the inverse function of f, and find the domain of f^{-1}.

82. The acceleration of a particle moving along a straight line is given by $a = 10\,e^{2t}$.
a. Write an expression for the velocity v, in terms of time t, if v = 5 when t = 0.
b. During the time when the velocity increases from 5 to 15, how far does the particle travel?
d. Write an expression for the position s, in terms of time t, of the particle if s = 0 when t = 0.

83. Given the function f defined by $f(x) = \cos(x) - \cos^2 x$ for $-\pi \le x \le \pi$.
a. Find the x-intercepts of the graph of f.
b. Find the x- and y-coordinates of all relative maximum points of f. Justify your answer.
c. Find the intervals on which the graph of f is increasing.
d. Using the information found in parts a, b, and c, sketch the graph of f on the axes provided.

84. Let y = f(x) be the continuous function that satisfies the equation $x^4 - 5x^2y^2 + 4y^4 = 0$ and whose graph contains the points (2,1) and (-2,-2). Let $\underline{\ell}$ be the line tangent to the graph of f at x = 2.
a. Find an expression for y′.
b. Write an equation for line $\underline{\ell}$.

c. Give the coordinates of a point that is on the graph of f but is not on line $\underline{\ell}$.
d. Give the coordinates of a point that is on line $\underline{\ell}$ but is not on the graph of f.

85. Let p and q be real numbers and let f be the function defined by:

$$f(x) = \begin{cases} 1 + 2p(x-1) + (x-1)^2, & \text{for } x \le 1 \\ qx + p, & \text{for } x > 1 \end{cases}$$

a. Find the value of q, in terms of p, for which f is continuous at x = 1.
b. Find the values of p and q for which f is continuous at x = 1.
c. If p and q have the values determined in part b, is f'' a continuous

function? Justify your answer.

86. Let f be the function defined by f(x) = $x^4 - 3x^2 + 2$.
a. Find the zeros of f.
b. Write an equation of the line tangent to the graph of f at the point where x = 1.
c. Find the x-coordinate of each point at which the line tangent to the graph of f is parallel to the line y = -2x + 4.

87. Let R be the region in the first quadrant enclosed by the graphs of y = $4 - x^2$, y = 3x, and the y-axis.
a. Find the area of the region R.
b. Find the volume of the solid formed by revolving the region R about the x-axis.

88. Let f be the function defined by f(x) = $12x^{(2/3)} - 4x$.
a. Find the intervals on which f is increasing.
b. Find the x- and y-coordinates of all relative maximum points.
c. Find the x- and y-coordinates of all relative minimum points.
d. Find the intervals on which f is concave downward.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

89. Let f be the function defined by f(x) = $5^{\sqrt{2x^2-1}}$.
a. Is f an even or odd function? Justify your answer.
b. Find the domain of f.
c. Find the range of f.
d. Find $f'(x)$.

90. Let f be a function defined by $f(x) = \begin{Bmatrix} 2x+1, \text{ for } x \le 2 \\ \frac{1}{2}x^2 + k, \text{ for } x > 2 \end{Bmatrix}$.

a. For what values of k will f be continuous at x = 2? Justify your answer.
b. Using the value of k found in part a, determine whether f is differentiable at x = 2. Use the definition of the derivative to justify your answer.
c. Let k = 4. Determine whether f is differentiable at x = 4. Justify your answer.

91. A particle moves along the x-axis so that at time t its position is given by $x(t) = \sin(\pi t^2)$ for $-1 \le t \le 1$.

a. Find the velocity at time t.
b. Find the acceleration at time t.
c. For what values of t does the particle change direction?
d. Find all values of t for which the particle is moving to the left.

92. Let f be a continuous function that is defined for all real numbers x and that has the following properties.

(i) $\int_1^3 f(x) = \frac{5}{2}$

(ii) $\int_1^5 f(x) = 10$

a. Find the average (mean) value of f over the closed interval [1,3].
b. Find the value of $\int_3^5 (2f(x)+6)dx$
c. Given that f(x) = ax + b, find the values of a and b.

93. A particle moves along the x-axis in such a way that its acceleration at time t for t > 0 is given by $a(t) = \frac{3}{t^2}$. When t = 1, the position of the particle is 6 and the velocity is 2.

a. Write an equation for the velocity, v(t), of the particle for all t > 0.
b. Write an equation for the position, x(t), of the particle for all t > 0.
c. Find the position of the particle when t = e.

94. Given that f is the function defined by $f(x) = \frac{x^3 - x}{x^3 - 4x}$.

a. Find the $\lim_{x \to 0} f(x)$.
b. Find the zeros of f.
c. Write an equation for each vertical and each horizontal asymptote to the graph of f.
d. Describe the symmetry of the graph of f.
e. Using the information found in parts a, b, c, and d, sketch the graph of f on the axes provided.

95. Let R be the region in the first quadrant that is enclosed by the graph of y = tan(x), the x-axis, and the line x = $\frac{\pi}{3}$.

a. Find the area of R.
b. Find the volume of the solid formed by revolving R about the x-axis.

96. A ladder 15 feet long is leaning against a building so that the end X is on level ground and end Y is on the wall. X is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.

a. Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.
b. Find the rate of change in square feet per second of the area of the triangle XOY when X is 9 feet from the building.

97. Let f be the function defined by $f(x) = (x^2 + 1)e^{-x}$ for $-4 \le x \le 4$.

a. For what value of x does f reach its absolute maximum? Justify your answer.
b. Find the x-coordinates of all points of inflection of f. Justify your answer.

98. A tank with a rectangular base and rectangular sides is to be open at the top. It is to be constructed so that its width is 4 meters and volume is 36 cubic meters. If building the tank costs $10 per square meter for the base and $5 per square meter for the sides, what is the cost of the least expensive tank?

99. For all real numbers x, f is a differentiable function such that f(-x) = f(x). Let f(p) = 1 and $f'(p) = 5$ for some p > 0.

a. Find $f'(p)$.
b. Find $f'(0)$.
c. If l_1 and l_2 are lines tangent to the graph of f at (-p,1) and (p,1), respectively, and if l_1 and l_2 intersect at point Q, find the x- and y-coordinates of Q in terms of p.

100. Let f be the function defined by $f(x) = -2 + \ln(x^2)$.

a. For what real numbers x is f defined?
b. Find the zeros of f.
c. Write an equation for the line tangent to the graph of f at x = 1.

101. A particle moves along the x-axis so that at time t its position is given by $x(t) = t^3 - 6t^2 + 9t + 11$.

a. What is the velocity of the particle at t = 0?
b. During what time intervals is the particle moving to the left?
c. What is the total distance traveled by the particle from t = 0 to t = 2?

102. Let f be the function defined for $\frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$ by f(x) = x + $\sin^2 x$.

a. Find all values of x for which $f'(x) = 1$.
b. Find the x-coordinates of <u>all</u> minimum points of f. Justify your answer.
c. Find the x-coordinates of <u>all</u> inflection points of f. Justify your answer.

103. Let R be the shaded region between the graph of $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 2$ and the x-axis from x = 0 to x = 1.

a. Find the area of R by setting up and integrating a definite integral.
b. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid formed by revolving the region R about the x-axis.
c. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid formed by revolving the region R about the line x = 1.

104. At time t = 0, a jogger is running at a velocity of 300 meters per minute. The jogger is slowing down with a negative acceleration that is directly proportional to time t. This brings the jogger to a stop in10 minutes.

a. Write an expression for the velocity of the jogger at time t.
b. What is the total distance traveled by the jogger in that 10-minute interval?

105. Let R be the region in the first quadrant bounded by the graph of $y = 8 - x^{3/2}$, the x-axis, and the y-axis.

a. Find the area of the region R.
b. Find the volume of the solid generated when R is revolved about the x-axis.
c. The vertical line x = k divides the region R into two regions such that when these two regions are revolved about the x-axis, the generate solids with equal volumes. Find the value of k.

106. A particle moves along the x-axis so that, at any time t $\geq$ 0, its acceleration is given by a(t) = 6 t + 6. At time t = 0, the velocity of the particle is -9, and its position is -27.

a. Find v(t), the velocity of the particle at any time t $\geq$ 0.
b. For what values of t $\geq$ 0 is the particle moving to the right?
c. Find x(t), the position of the particle at any time t $\geq$ 0.

107. Let f be the function defined by $f(x)=\dfrac{x+\sin x}{\cos x}$ for $\dfrac{-\pi}{2}$ < x < $\dfrac{\pi}{2}$.

a. State whether f is an even function or an odd function. Justify your answer.
b. Find $f'(x)$.
c. Write an equation of the line tangent to the graph of f at the point where x = 0.

108. Let R be the region enclosed by the x-axis, the y-axis, the line x = 2, and the y = $2e^x$ + 3x.

a. Find the area of R by setting up and evaluating the definite integral. Your work must include an antiderivative.
b. Find the volume of the solid generated by revolving R about the <u>y-axis</u> by setting up and evaluating a definite integral. Your work must include an antiderivative.

109. A function f is continuous on the closed interval [-3,3] such that f(-3) = 4 and f(3) = 1. The functions $f'(x)$ and $f''(x)$ have the properties given in the table below.

x	$f'(x)$	$f''(x)$
$-3<x<-1$	Positive	Positive
$x=-1$	Fails to exist	Fails to exist
$-1<x<1$	Negative	Positive
$x=1$	Zero	Zero
$1<x<3$	Negative	negative

a. What are the x-coordinates of all absolute maximum and minimum points of f on the interval [-3,3]? Justify your answer.
b. What are the x-coordinates of all points of inflection on the interval [-3,3]? Justify your answer.
c. On the axes provided, sketch a graph that satisfies the given properties of f.

110. The volume V of a cone is increasing at the rate of 28 □ cubic inches per second. At the instant when the radius r on the cone is 3 units, its volume is 12 □ cubic units and the radius is increasing at $\frac{1}{2}$ unit per second.

a. At the instant when the radius of the cone is 3 units, what is the rate of change of the area of its base?
b. At the instant when the radius of the cone is 3 units, what is the rate of

change of its height h?

c. At the instant when the radius of the cone is 3 units, what is the instantaneous rate of change of the area of its base with respect to its height h?

111. Let f be the function given by $f(x) = \dfrac{2x-5}{x^2-4}$.

a. Find the domain of f.
b. Write an equation for each vertical and each horizontal asymptote for the graph of f.
c. Find $f'(x)$.
d. Write an equation for the line tangent to the graph of f at the point (0,f(0)).

112. A particle moves along the x-axis with acceleration given by a(t) = cos(t) for t $\geq$ 0. At t = 0 the velocity v(t) of the particle is 2 and the position x(t) is 5.

a. Write an expression for the velocity v(t) of the particle.
b. Write an expression for the position x(t).
c. For what values of t is the particle moving to the right? Justify your answer.
d. Find the total distance traveled by the particle from t = 0 to t = $\frac{\pi}{2}$□

113. Let R be the region enclosed by the graphs of y = e^{-x}, y = e^{x} and x = ln 4.

a. Find the area of R by setting up and evaluating a definite integral.
b. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume generated when the region R is revolved about the x-axis.
c. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume generated when the region R is revolved about the y-axis.

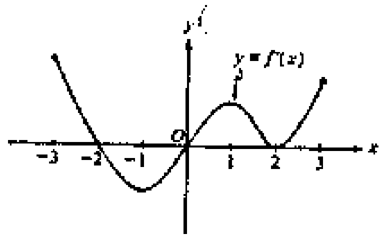
114. Let f(x) = $(14\,\pi)x^2$ and $g(x) = k^2 \sin \dfrac{\pi x}{2k}$ for k > 0.

a. Find the average value of f on [1,4].
b. For what value of k will the average value of g on [0,k] be equal to the average value of f on [1,4]?

115. A balloon in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of 261□ cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is 144□ cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters per minute.

a. At this instant, what is the height of the cylinder?

b. At this instant, how fast is the height of the cylinder increasing?



116. The figure above shows the graph of f′, the derivative of a function f. The domain of the function f is the set of all x such that $-3 \le x \le 3$.

a. For what values of x, $-3 < x < 3$, does f have a relative maximum? A relative minimum? Justify your answer.
b. For what values of x is the graph of f concave up? Justify your answer.
c. Use the information found in parts (a) and (b) and the fact that f(-3) = 0 to sketch a possible graph of f on the axes provided.

117. Let f be the function defined by $f(x) = 7 - 15x + 9x^2 - x^3$ for all real numbers x.
a. Find the zeros of f.
b. Write an equation of the line tangent to the graph of f at x = 2.
c. Find the x-coordinates of all points of inflection of f. Justify your answer.

118. Let f be the function given by $f(x) = \frac{9x^2 - 36}{x^2 - 9}$.
a. Describe the symmetry of the graph of f.
b. Write an equation for each vertical and each horizontal asymptote of f
c. Find the intervals on which f is increasing.
d. Using the results found in parts (a), (b), and (c), sketch the graph of f.

119. A particle moving along the x-axis so that at any time $t \ge 1$ its acceleration is given by $a(t) = \frac{1}{t}$. At time t = 1, the velocity of the particle is v(1) = -2 and its position is x(1) = 4.
a. Find the velocity v(t) for $t \ge 1$.
b. Find the position x(t) for $t \ge 1$.
c. What is the position of the particle when it is farthest to the left?

120. Let f be the function defined as follows:

$$f(x) = \begin{cases} |x-1| + 2, & \text{for } x < 1 \\ ax^2 + bx, & \text{for x} \ge 1, \text{ where a and b are constants} \end{cases}$$

a. If a = 2 and b = 3, is f continuous of all x? Justify your answer.
b. Describe all values of a and b for which f is a continuous function.
c. For what values of a and b if f both continuous and differentiable?

121. Let A(x) be the area of the rectangle inscribed under the curve $y = e^{-2x^2}$ with vertices at (-x, 0) and (x, 0), x $\geq$ 0.

a. Find A(1).
b. What is the greatest value of A(x)? Justify your answer.
c. What is the average value of A(x) on the interval 0 $\leq$ x $\leq$ 2?

122. The region enclosed by the graphs of y = $\tan^2 x$, y = $\frac{1}{2}\sec^2 x$, and the y-axis.

a. Find the area of the region R.
b. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid formed by revolving the region about the x-axis.

123. A particle moves along the x-axis so that its acceleration at any time t is given by a(t) = 6t - 18. At time t = 0 the velocity of the particle is v(0) = 24, and at time t = 1, its position is x(1) = 20.

a. Write an expression for the velocity v(t) of the particle at any time t.
b. For what values of t is the particle at rest?
c. Write an expression for the position x(t) of the particle at any time t.
d. Find the total distance traveled by the particle from t = 1 to t = 3.

124. Let f(x) = $\sqrt{1-\sin x}$.

a. What is the domain of f?
b. Find $f'(x)$.
c. What is the domain of $f'(x)$?
d. Write an equation for the line tangent to the graph of f at x = 0.

125. Let R be the region enclosed by the graphs of $\sqrt[4]{64x}$ and y = x.

a. Find the volume of the solid generated when region R is revolved about the x-axis.
b. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable the volume of the solid generated when the region R is revolved about the y-axis.

126. Let f be the function given by $f(x) = 2\ln(x^2 + 3) - x$ with domain $-3 \leq x \leq 5$.

a. Find the x-coordinate of each relative maximum point and each relative minimum point of f. Justify your answer.

b. Find the x-coordinate of each inflection point of f.

c. Find the absolute maximum value of f(x).

127. A trough is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t, let h be the depth and V be the volume of water in the trough.

a. Find the volume of water in the trough when it is full.

b. What is the rate of change in h at the instant when the trough is $\frac{1}{4}$ full by volume?

c. What is the rate of change in the area of the surface of the water at the instant when the trough is $\frac{1}{4}$ full by volume?

128. Let f be a function such that $f(x) < 1$ and $f'(x) < 0$ for all x.

a. Suppose that $f(b) = 0$ and $a < b < c$. Write an expression involving integrals for the area of the region enclosed by the graph of f, the lines $x = a$ and $x = c$, and the x-axis.

b. Determine whether $g(x) = \frac{1}{f(x) - 1}$ is increasing or decreasing. Justify your answer.

c. Let he be a differentiable function such that $h'(x) < 0$ for all x. Determine whether $F(x) = H(f(x))$ is increasing or decreasing. Justify your answer.

129. Let f be the function given by $f(x) = \sqrt{x^4 - 16x^2}$.

a. Find the domain of f.

b. Describe the symmetry, if any, of the graph of f.

c. Find $f'(x)$.

d. Find the slope of the line normal to the graph of f at $x = 5$.

130. A particle moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

a. Find the acceleration a(t) of the particle at any time t.

b. Find all values of t, $0 \leq t \leq 2$, for which the particle is at rest.

c. Find the position x(t) of the particle at any time t if $x(0) = 0$.

131. Let R be the region in the first quadrant enclosed by the hyperbola $x^2 - y^2 = 9$, the x-axis, and the line $x = 5$.

a. Find the volume of the solid generated by revolving R about the x-axis.
b. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the line $x = -1$.

132. Let f be the function defined by $f(x) = 2xe^{-x}$ for all real numbers x.

a. Write an equation of the horizontal asymptote for the graph of f.
b. Find the x-coordinate of each critical point of f. For each such x, determine whether f(x) is a relative maximum, a relative minimum, or neither.
c. For what values of x is the graph of f concave down?
d. Using the results found in parts a, b, and c, sketch the graph of $y = f(x)$ in the xy-plane provided.

133. Let R be the region in the first quadrant under the graph of $y = \dfrac{x}{x^2+2}$ for $0 \le x \le \sqrt{6}$.

a. Find the area of R.
b. If the line $x = k$ divides R into two regions of equal area, what is the value of k?
c. What is the average value of $y = \dfrac{x}{x^2+2}$ on the interval $0 \le x \le \sqrt{6}$?

134. Let f be a differentiable function, defined for all real numbers x, with the following properties.

(i) $f'(x) = ax^2 + bx$
(ii) $f'(1) = 6$ and $f''(1) = 18$
(iii) $\int_1^2 f(x) = 18.$

Find f(x). Show your work.

135. Let f be the function given by $f(x) = x^3 - 7x + 6$.

a. Find the zeros of f.
b. Write an equation of the line tangent to the graph of f at $x = -1$.
c. Find the number c that satisfies the conclusion of the Mean Value Theorem for f on the closed interval [1,3].

136. Let R be the region in the first quadrant enclosed by the graph of $y = \sqrt{6x+4}$, the line y = 2x, and the y-axis.

a. Find the area of R.

c. Set up, but do not integrate, an integral expression in terms of a single variable for
the volume of the solid generated when R is revolved about the x-axis.

d. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

137. A particle moves along the x-axis in such a way that its acceleration at time t for t = 0 is given by a(t) = 4 cos(2t). At time t = 0, the velocity of the particle is v(0) = 1 and its position is x(0) = 0.

a. Write an equation for the velocity v(t) of the particle.

b. Write an equation for the position x(t) of the particle.

c. For what values of t, $0 \le t \le \pi$, is the particle at rest?

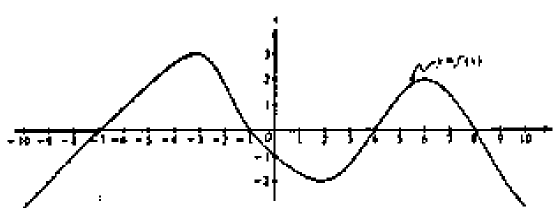
138. Let f be the function given by $f(x) = \dfrac{x}{\sqrt{x^2-4}}$.

a. Find the domain of f.

b. Write an equation for each vertical asymptote to the graph of f.

c. Write an equation for each horizontal asymptote to the graph of f.

d. Find $f'(x)$.



139.
The figure above shows the graph of f′, the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \le x \le 10$.

a. For what values of x does the graph of f have a horizontal tangent?

b. For what values of x in the interval (-10, 10) does f have a relative maximum?

c. For what values of x is the graph of f concave downward?

140. Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, $\frac{dy}{dt} = ky$, where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

a. Write an equation for y, the amount of oil remaining in the well at any time t.
b. At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?
c. In order not to lose money, at what time t should oil no longer be pumped from the well?

141. A particle, initially at rest, moves along the x-axis so that its acceleration at any time $t \geq 0$ is given by $a(t) = 12t^2 - 4$. The position of the particle when t=1 is x(1) = 3.

a. Find the values of t for which the particle is at rest.
b. Write an expression for the position x(t) of the particle at any time $t \geq 0$.
c. Find the total distance traveled by the particle from t = 0 to t = 2.

142. Let f be the function given by $\ln \frac{x}{x-1}$.

a. What is the domain of f?
b. Find the value of the derivative of f at x = -1.
c. Write an expression for $f^{-1}(x)$, where f^{-1} denotes the inverse function of f.

143. Let R be the region enclosed by the graphs of $y = e^x$, $y = (x - 1)^2$, and the line x = 1.

a. Find the area of R.
b. Find the volume of the solid generated when R is revolved about the x-axis.
c. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

144. The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second.

a. At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?
b. At the time when the volume of the sphere is 36□ cubic centimeters,

what is the rate of increase of the area of a cross section through the center of the sphere?

c. At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

145. Let f be the function defined by f(x) = $\sin^2 x - \sin(x)$ for $0 \le x \le \frac{3\pi}{2}$.

a. Find the x-intercepts of the graph of f.
b. Find the intervals on which f is increasing.
c. Find the absolute maximum value and the absolute minimum value of f. Justify your answer.

146. Let f be the function that is given by $f(x) = \frac{ax+b}{x^2-c}$ and that has the following properties.

i) The graph of f is symmetric with respect to the y-axis.
ii) $\lim_{x \to 2+} f(x) = +\infty$
iii) $f'(1) = -2$.

a. Determine the values of a, b, and c.
b. Write an equation for each vertical and each horizontal asymptote of the graph of f.
c. Sketch the graph of f in the xy-plane.

147. Let f be the function that is defined for all real numbers x and that has the following properties.

i) $f''(x) = 24x - 18$
ii) $f'(1) = -6$
iii) $f(2) = 0$

a. Find each x such that the line tangent to the graph of f at (x,f(x)) is horizontal.
b. Write an expression for f(x).
c. Find the average value of f on the interval $1 \le x \le 3$.

148. Let R be the region between the graphs of y = 1 + sin□□x) and y = x^2 from x = 0 to x = 1.

a. Find the area of R.
b. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the x-axis.

c. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

149. Let f be the function defined by f(x) = $(1 + \tan x)^{1.5}$ for $\frac{\pi}{4} < x < \frac{\pi}{2}$

a. Write an equation for the line tangent to the graph of f at the point where x = 0.
b. Using the equation found in part a, approximate f(0.02).
c. Let $f^{-1}(x)$ denote the inverse function of f. Write an expression that gives $f^{-1}(x)$ for all x in the domain of f^{-1}.

150. Let f be the function given by $f(x) = \frac{|x|-2}{x-2}$.

a. Find all the zeros of f.
b. Find $f'(1)$.
c. Find $f'(-1)$.
d. Find the range of f.

151. Let f be a function that is even and continuous on the closed interval [-3,3]. The function f and its derivatives have the properties indicated in the table below.

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$
$f(x)$	1	Positive	0	Negative	-1	Negative
$f'(x)$	$\varnothing$	Negative	0	Negative	$\varnothing$	Positive
$f''(x)$	$\varnothing$	Positive	0	Negative	$\varnothing$	negative

a. Find the x-coordinate of each point at which f attains an absolute maximum value or an absolute minimum value. For each coordinate you give, state whether f attains an absolute maximum or an absolute minimum.
b. Find the x-coordinate of each point of inflection on the graph of f. Justify your answer.
c. In the xy-plane provided, sketch the graph of a function with all the given characteristics of f.

152. A tightrope is stretched 30 feet above the ground between the Jay and Tee buildings which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A.

a. How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)
b. How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)
c. How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee Building when she is 10 feet from point B? (Indicate units.)

153. Let f be the function defined by $f(x) = 3x^5 - 5x^3 + 2$.

a. On what interval is f increasing?
b. On what intervals is the graph of f concave upward?
c. Write an equation on each horizontal tangent line to the graph of f.

154. A particle moves along the x-axis so that its velocity at time t, $0 \le t \le 5$, is given by $v(t) = 3(t-1)(t-3)$. At time t = 2, the position of the particle is $x(2)=0$.

a. Find the minimum acceleration of the particle.
b. Find the total distance traveled by the particle.
c. Find the average velocity of the particle over the interval $0 \le t \le 5$.

155. Let f be the function given by $f(x) = \ln \left| \frac{x}{1+x^2} \right|$.

a. Find the domain of f.
b. Determine whether f is an even function, an odd function, or neither. Justify your conclusion.
c. At what values of x does f have a relative maximum or a relative minimum? For each such x, use the first derivative test to determine whether f(x) is a relative maximum or a relative minimum.
d. Find the range of f.

156. Consider the curve defined by the equation $y + \cos(y) = x + 1$ for $0 \le y \le 2\pi$.

a. Find $\frac{dy}{dx}$ in terms of y.
b. Write an equation for each vertical tangent to the curve.
c. Find $\frac{d^2y}{dx^2}$ in terms of y.

157. Let f be the function given by $f(x) = e^{-x}$, and let g be the function given by $g(x) = kx$, where k is the nonzero constant such that the graph of f is tangent to the graph of g.

a. Find the x-coordinate of the point of tangency and the value of k.

b. Let R be the region enclosed by the y-axis and the graph of f and g. Using the results found in part a, determine the area of R.
c. Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid generated by revolving the region R, given in part b, about the x-axis.

158. At time t, $t \geq 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At t = 0, the radius of the sphere is 1 and at t = 15, the radius is 2.

a. Find the radius of the sphere as a function of t.
b. At what time t will the volume of the sphere be 27 times its volume at t = 0?

159. Let f be the function given by $f(x)= x^3 -5x^2 + 3x + k$, where k is a constant.

a. On what interval is f increasing?
b. On what intervals is the graph of f concave downward?
c. Find the value of k for which f has 11 as its relative minimum.

160. A particle moves on the x-axis so that its position at any time $t \geq 0$ is given by $x(t) = 2te^{-t}$.

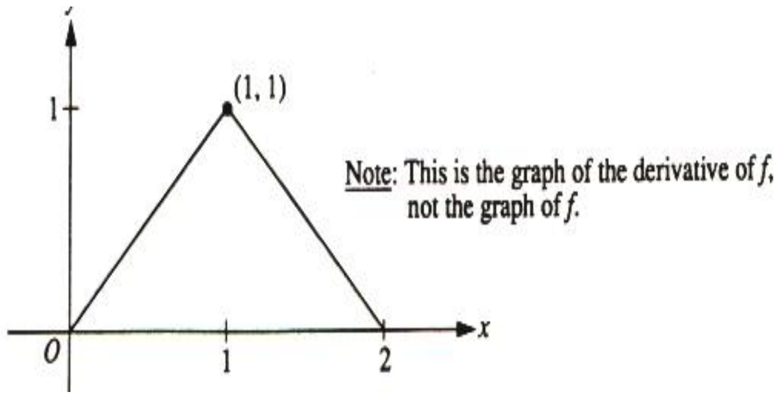
a. Find the acceleration of the particle at t = 0.
b. Find the velocity of the particle when its acceleration is 0.
c. Find the total distance traveled by the particle from t = 0 to t = 5.

161. Consider the curve $y^2 = 4 + x$ and chord AB joining points A(-4,0) and B(0,2) on the curve.

a. Find the x- and y-coordinates of the points on the curve where the tangent line is parallel to chord AB.
b. Find the area of the region R enclosed by the curve and the chord AB.
c. Find the volume of the solid generated when the region R, defined in part b, is revolved about the x-axis.

162. Let f be the function defined by $f(x) = \ln(2 + \sin x)$ for □ ≤ x ≤ 2□.

a. Find the absolute maximum value and the absolute minimum value of f. Show the analysis that leads to your conclusion.
b. Find the x-coordinate of each inflection point on the graph of f. Justify your answer.



163. The figure above shows the graph of $f'(x)$, the derivative of a function f. The domain of f is the set of all x such that 0 < x < 2.

a. Write an expression of $f'(x)$ in terms of x.

b. Given that f(1) = 0, write an expression for f(x) in terms of x.

c. Sketch the graph of y = f(x).

164. Let P(t) represent the number of wolves in a population at time t years, when t $\geq$0. The population P(t) is increasing at a rate proportional to 800 - P(t), where the constant of proportionality is k.

a. If P(0) = 500, find P(t) in terms of t and k.

b. If P(2) = 700, find k.

c. Find the $\lim_{x\to\infty} P(t)$.

165. Let f be the function given by f(x) = $3x^4 + x^3 - 21x^2$.

a. Write an equation of the line tangent to the graph of f at the point (2, -28)

b. Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

c. Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

166. Let R be the region enclosed by the graphs of y = e^x, y = x, and the lines x = 0 and x = 4.

a. Find the area of R.

b. Find the volume of the solid generated when R is revolved about the x-axis.

c. Set up, but do not integrate, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the y-axis.

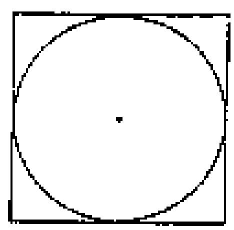
167. Consider the curve defined by $x^2 + xy + y^2 = 27$.

a. Write an expression for the slope of the curve at any point (x,y).

b. Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel.
c. Find the points on the curve where the lines tangent to the curve are vertical.

168. A particle moves along the x-axis so that at any time $t > 0$ its velocity is given by $v(t) = t \ln t - t$.
a. Write an expression for the acceleration of the particle.
b. For what values of t is the particle moving to the right?
c. What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

169.



A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency.
a. Find the rate at which the <u>perimeter</u> of the square is increasing. Indicate units of measure.
b. At the instant when the area of the circle is 25π square inches, find the rate of increase in the <u>area enclosed between</u> the circle and the square. Indicate units of measure.

170. Let $F(x) = \int_0^x \sin t^2 \, dt$ for $0 \le x \le 3$.
a. Use the Trapezoidal Rule with four equal subdivisions of the closed interval [0, 1] to approximate F(1).
b. On what intervals is F increasing?
c. If the average rate of change of F on the closed interval [1, 3] is k, find the $\int_0^3 \sin t^2 \, dt$ in terms of k.

171. Let f be the function given by $f(x) = \frac{2x}{\sqrt{x^2 + x + 1}}$.

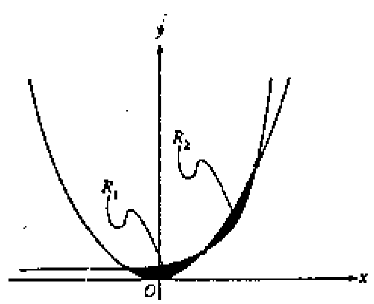
a. Find the domain of f. Justify your answer.
b. In the viewing window [-5, 5] x [-3, 3], sketch the graph of f.
c. Write an equation for each horizontal asymptote of the graph of f.
d. Find the range of f. Use $f'(x)$ to justify your answer.

172. A particle moves along the y-axis so that its velocity at any time $t \geq 0$ is given by v(t) = t cos(t). At time t = 0, the position of the particle is y = 3.

a. For what values of t, $0 \leq t \leq 5$, is the particle moving upward?
b. Write an expression for the acceleration of the particle in terms of t.
c. Write an expression for the position y(t) of the particle.
d. For t > 0, find the position of the particle the first time the velocity of the particle is zero.

173. Consider the curve defined by $-8x^2 + 5xy + y^3 = -149$.

a. Find $\frac{dy}{dx}$.
b. Write an equation for the line tangent to the curve at the point (4,-1).
c. There is a number k so that the point (4.2, k) is on the curve. Using the tangent line found in part b, approximate the value of k.
d. Write an equation that can be solved to find the actual value of k so that the point (4.2, k) is on the curve.
e. Solve the equation found in part d for the value of k.

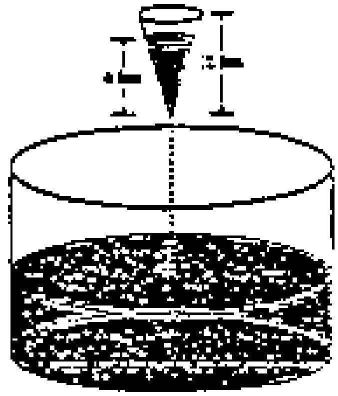


174. The shaded regions R_1 and R_2 shown above are enclosed by the graphs of $f(x) = x^2$ and $g(x) = 2^x$.

a. Find the x- and y-coordinates of the three points of intersection of the graphs of f and g.
b. Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the graphs of f and g. Do not evaluate.
c. Without using absolute value, set up an expression involving one or more

integrals that gives the volume of the solid generated by revolving the region R_1 about the line $y = 5$. Do not evaluate.

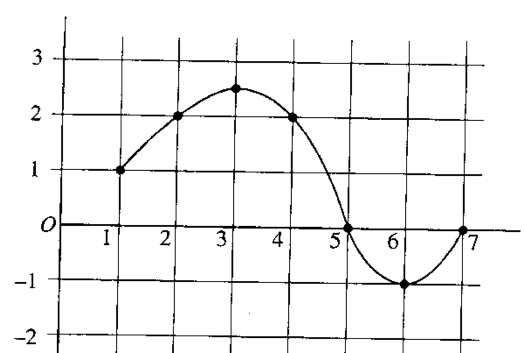
175.



As shown in the figure below, water is draining from a conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400□ square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h - 12)$ feet per minute.

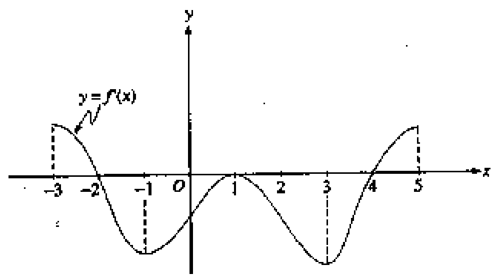
a. Write an expression for the volume of water in the conical tank as a function of h.
b. At what rate is the volume of water in the conical tank changing when $h = 3$? Indicate the units of measure.
c. Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

176. The graph of a differentiable function f on the closed interval [1,7] is shown. Let $h(x) = \int f(t)\,dt$ for $1 \le x \le 7$.



a. Find h(1).
b. Find h′ (4).

c. On what interval or intervals is the graph of h concave upward? Justify your answer.
d. Find the value of x at which h has its minimum on the closed interval [1,7]. Justify your answer.



Note: This is the graph of the derivative of f, not the graph of f.

177. The figure above shows the graph of $f'(x)$, the derivative of a function f.
The domain of f is the set of all real numbers x such that -3 < x < 5.
a. For what values of x does f have a relative maximum? Why?
b. For what values of x does f have a relative minimum? Why?
c. On what intervals is the graph of f concave upward? Use f' to justify your answer.

d. Suppose that f(1) = 0. In the xy-plane provided, draw a sketch that shows the general shape of the graph of the function f on the open interval 0 < x < 2.

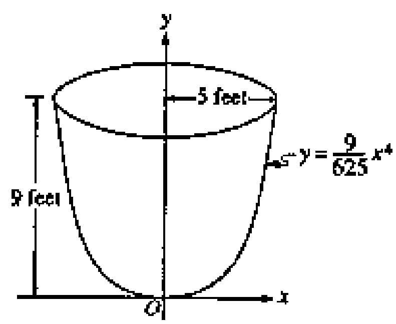
178. Let R be the region in the first quadrant under the graph of y = $\frac{1}{\sqrt{x}}$ for $4 \le x \le 9$.
a. Find the area of R.
b. If the line x = k divides the region R into two regions of equal area, what it is the value of k?
d. Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.

179. The rate of consumption of cola in the United States is given by $S(t) = C e^{kt}$, where S is measured in billions of gallons per year and t is measured in years from the beginning of 1980.
a. The consumption rate doubles every 5 years and the consumption rate at the beginning of 1980 was 6 billion gallons per year. Find C and k.
b. Find the average rate of consumption of cola over the 10-year time period beginning January 1,1983. Indicate units of measure.
c. Use the trapezoidal rule with four equal subdivisions to estimate the integral from 5 to 7 of S(t) dt.

d. Using correct units, explain the meaning of the integral from 5 to 7 of S(t) dt in terms of cola consumption.

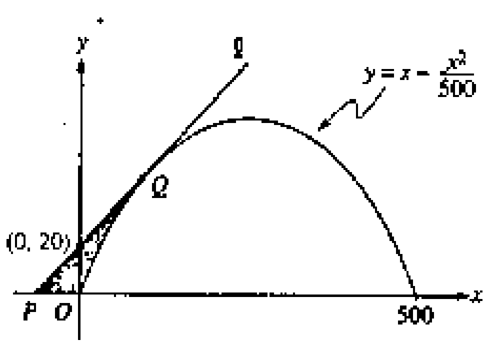
180. This problem deals with functions defined by f(x) = x + b sin x, where b is positive and constant and [- 2π, 2π].

a. Sketch the graphs of two of these functions, y = x + sin(x) and y = x + 3 sin(x), as indicated below.
b. Find the x-coordinates of all points, [-2□, 2□], where the line y = x + b is tangent to the graph of f(x) = x + b sin(x).
c. Are the points of tangency described in part (b) relative maximum points of f ? Why?
d. For all values of b > 0, show that all inflection points of the graph of f lie on the line y = x.



181. An oil storage tank has the shape shown above, obtained by revolving the curve y = $\frac{9}{625}x^4$ from x = 0 to x = 5 about the y-axis, where x and y are measured in feet. Oil flows into the tank at the constant rate of 8 cubic feet per minute.

a. Find the volume of the tank. Indicate units of measure.
b. To the nearest minute, how long would it take to fill the tank if the tank was empty initially?
e. Let h be the depth, in feet, of oil in the tank. How fast is the depth of the oil in the tank increasing when h = 4? Indicate units of measure.

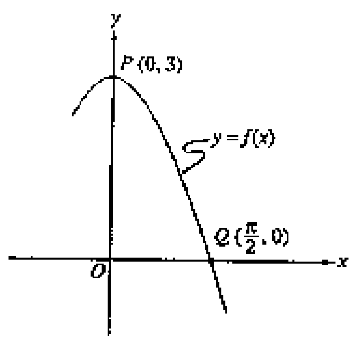


182. Let *l* is tangent to the graph of y = x - $\frac{x^2}{500}$ at the point Q, as shown in the figure above.

a. Find the x-coordinate of point Q.
b. Write an equation for line ℓ.
c. Suppose the graph of y = x - $\frac{x^2}{500}$ shown in the figure, where x and y are measured in feet, represents a hill. There is a 50-foot tree growing vertically at the top of the hill. Does a spotlight at point P directed along the line *l* shine on any part of the tree? Show the work that leads to your conclusion.

183. A particle moves along the x-axis so that its velocity at any time t $\geq$ 0 is given by v(t) = $3t^2$ - 2t - 1. The position x(t) is 5 for t = 2.

a. Write a polynomial expression for the position of the particle at any time t $\geq$ 0.
b. For what values of t, 0 $\leq$ t $\leq$ 3, is the particle's instantaneous velocity the same as its average velocity on the closed interval [0, 3]?
c. Find the total distance traveled by the particle from time t= 0 until time t = 3.



184. Let f be the function given by f(x) = 3 cos(x). The graph of f crosses the y-axis at point P and the x-axis at point Q.

a. Write an equation for the line passing through points P and Q.
b. Write an equation for the line tangent to the graph of f at point Q. Show the analysis that leads to your equation.
c. Find the x-coordinate of the point on the graph of f, between points P and Q, at which the line tangent to the graph of f is parallel to line PQ.
d. Let R be the region in the first quadrant bounded by the graph of f and line segment PQ. Write an integral expression for the volume of the solid generated by revolving the region R about the x-axis. Do not evaluate.

185. Let f be the function given by f(x) = $\sqrt{x-3}$.

a. Sketch the graph of f and shade the region R enclosed by the graph of f, the x-axis, and the vertical line x = 6.

b. Find the area of the region R described in part (a).

c. Rather than using the line x = 6 as in part (a), consider the line x = w, where w can be any number greater than 3. Let A(w) be the area of the region enclosed by the graph of f, the x-axis, and the vertical line x = w. Write an integral expression for A(w).

d. Let A(w) be as described in part (c). Find the rate of change of A with respect to w when w = 6.

186. Let f be the function given by f(x) = $x^3 - 6x^2 + p$, where p is an arbitrary constant.

a. Write an expression for f′(x) and use it to find the relative maximum and minimum values of f in terms of p. Show the analysis that leads to your conclusion.

b. For what values of the constant p does f have 3 distinct real roots?

c. Find the value of p such that the average value of f over the closed interval [-1, 2] is 1.

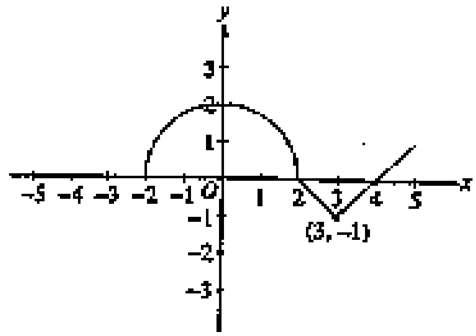
187. The graph of a function f consists of a semicircle and two line segments as shown below. Let g be the function given by $\int_0^x f(t)dt$

a. Find g(3).

b. Find all values of x on the open interval (-2,5) at which g has a relative maximum. Justify your answer.

c. Write an equation for the line tangent to the graph of g at x = 3.

d. Find the x-coordinate of each point of inflection of the graph of g on the open interval (-2,5). Justify your answer.



188. Let v(t) be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dy}{dx}$ = -2v - 32, with initial condition v(0) = -50.

a. Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

b. Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

c. It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

189. Let R be the region bounded by the x-axis, the graph of $y = \sqrt{x}$, and the line x= 4.

a. Find the area of the region R.

b. Find the value of h such that the vertical line x= h divides the region R into two regions of equal area.

c. Find the volume of the solid generated when R is revolved about the x-axis.

d. The vertical line x=k divides the region R into two regions such that when these two regions are revolved about the x-axis, they generate solids with equal volumes. Find the value of k.

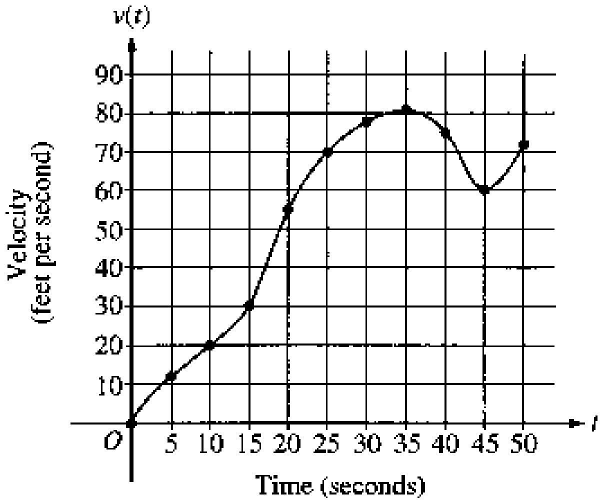
190. Let f be the function given by $f(x) = 2x\,e^{2x}$.

a. Find $\lim_{x\to-\infty} f(x)$ and $\lim_{x\to\infty} f(x)$.

b. Find the absolute minimum value of f. Justify that your answer is an absolute minimum.

c. What is the range of f?

d. Consider the family of functions defined by $y = bx\,e^{bx}$, where b is a nonzero constant. Show that the absolute minimum value of $bx\,e^{bx}$ is the same of all nonzero values of b.



t (seconds)	v(t) (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

191. The graph of the velocity v(t), in ft/sec, of a car traveling on a straight road, for $0 \leq t \leq 50$, is shown above. A table of values for v(t), at 5-second intervals of time t, is shown to the right of the graph.

a. During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
b. Find the average acceleration of the car, in ft/sec^2, over the interval $0 \leq t \leq 50$.
c. Find one approximation for the acceleration of the car in ft/sec^2, at t = 40. Show the computations you used to arrive at your answer.
d. Approximate the integral from 0 to 50 of v(t) dt with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

192. Let f be the function with f(1) = 4 such that for all points (x,y) on the graph of the slope is given by $\frac{3x^2+1}{2y}$.

a. Find the slope of the graph of f at the point where x = 1.
b. Write an equation for the line tangent to the graph of f at x = 1 and use it to approximate f(1.2).
c. Find f(x) by solving the separable differential equation $\frac{dy}{dx} = \frac{3x^2+1}{2y}$ with the initial condition f(1) = 4.
d. Use your solution from part (c) to find f(1.2).

193. The temperature outside a house during a 24-hour period is given by $F(t) = 80 - 10\cos\frac{\pi t}{12}$, $0 \le t \le 24$, where F(t) is measured in degrees Fahrenheit and t is measured in hours.

a. Sketch the graph of F on the grid provided.
b. Find the average temperature, to the nearest degree Fahrenheit, between t = 6 and t = 14.
c. An air conditioner cooled the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of t was the air conditioner cooling the house?
d. The cost of cooling the house accumulates at the rate of $0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

194. Consider the curve defined by $2y^3 + 6x^2y - 12x^2 + 6y = 1$.

a. Show that $\frac{dy}{dx} = \frac{4x - 2xy}{x^2 + y^2 + 1}$.
b. Write an equation of each horizontal tangent line to the curve.
c. The line through the origin with slope -1 is tangent to the curve at point P. Find the x- and y-coordinates of point P.

195. A particle moves along the y-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \ge 0$.

a. In which direction (up or down) is the particle moving at time t = 1.5? Why?
b. Find the acceleration of the particle at time t = 1.5. Is the velocity of the particle increasing at t = 1.5? Why or why not?
c. Given that y(t) is the position of the particle at time t and the y(0) = 3, Find y(2).
d. Find the total distance traveled by the particle from t = 0 to t = 2.

196. The shaded region, R, is bounded by the graph of $y = x^2$ and the line y = 4.

a. Find the area of R.
b. Find the volume of the solid generated by revolving R about the x-axis.
c. There exists a number k, k > 4, such that when R is revolved about the line y = k, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

197. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table shows the rate as measured every 3 hours for a 24-hour period.

t	R(t)
(hours)	(gallons per hour)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

a. Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate from 0 to 24 of R(t) dt. Using correct units, explain the meaning of your answer in terms of water flow.

b. Is there some time t, 0 < t < 24, such that R'(t) = 0? Justify your answer.

c. The rate of water flow R(t) can be approximated by Q(t) = $\frac{1}{79}$ (768 + 23 t – t^2). Use Q(t) to approximate the average rate of water flow during the 24-hour time period.

198. Suppose that the function f has a continuous second derivative for all x, and that $f(0)=2,\ f'(0)=-3,\ \text{and}\ f''(0)=0.$ Let g be a function whose derivative is given by $g'(x)=e^{-2x}(3f(x)+2f'(x)$ for all x.

a. Write an equation of the line tangent to the graph of f at the point where x = 0.

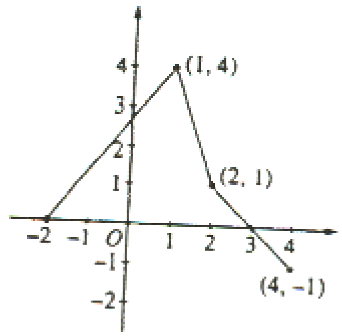
b. Is there sufficient information to determine whether or not the graph of f has a point of inflection where x = 0? Explain your answer.

c. Given that g(0) = 4, write an equation of the line tangent to the graph of g at the point where x = 0.

d. Show that $g''(x)=e^{-2x}(-6f(x)-f'(x)+2f''(x)$. Does g have a local maximum at x = 0? Justify your answer.

199. The graph of the function f, consisting of three line segments, is given below. Let

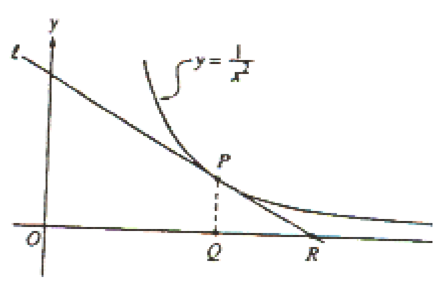
$$g(x) = \int_1^x f(t)dt \ .$$



a. Compute g(4) and g(-2).
b. Find the instantaneous rate of change of g, with respect to x, at x = 1.
c. Find the absolute minimum value of g on the closed interval [-2,4]. Justify your answer.
d. The second derivative of g is not defined at x = 1 and x = 2. How many of these values are x-coordinates of points of inflection of the graph of f? Justify your answer.

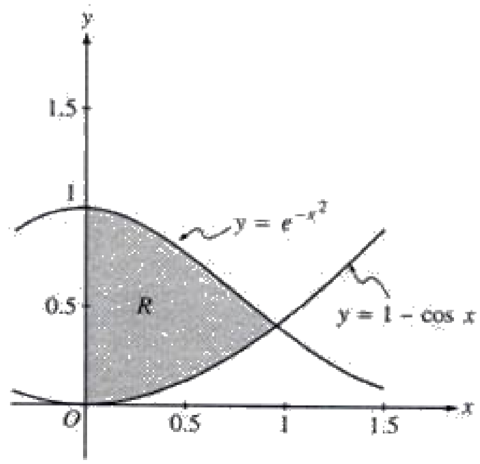
200.

In the figure below, line l is tangent to the graph of y = $\frac{1}{x^2}$ at point P, with coordinates (w, $\frac{1}{w^2}$), where w > 0. Point Q has coordinates (w,0). Line l crosses the x-axis at the point R, with coordinates (k, 0).



a. Find the value of k when w = 3.
b. For all w > 0, find k in terms of w.
c. Suppose that w is increasing at the constant rate of 7 units per second. When w = 5, what is the rate of change of k with respect to time?
d. Suppose that w is increasing at the constant rate of 7 units per second. When w = 5, what is the rate of change of the area of triangle PQR with respect to time? Determine whether the area

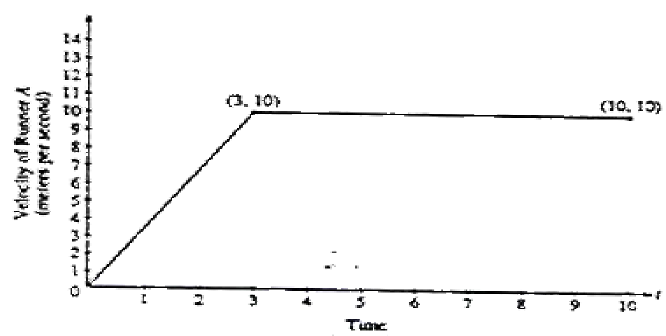
is increasing or decreasing at this instant.



201. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, y = 1 – cos(x), and the y-axis.

a. Find the area of the region R.
b. Find the volume of the solid generated when the region R is revolved about the x-axis.
c. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

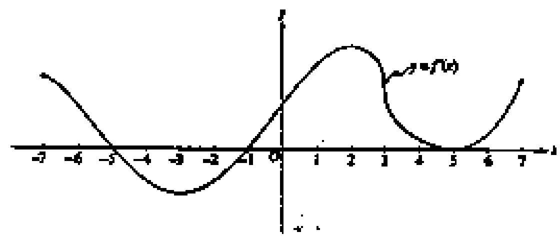
202. Two runners, A and B, run on a straight racetrack for $0 \leq t \leq 10$ seconds. The graph below, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.



a. Find the velocity of Runner A and the velocity of Runner B at time t =

2 seconds. Indicate units of measure.

b. Find the acceleration of Runner A and the acceleration of Runner B at time
   t = 2 seconds. Indicate units of measure.
c. Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.



203. The figure above shows the graph of f f', the derivative of the function f, for $-7 \le x \le 7$.
The graph of f' has horizontal tangent lines at x = -3, x = 2, and x = 5, and a vertical tangent line at x = 3.

a. Find all values of x, for $-7 \le x \le 7$, at which f attains a relative minimum. Justify your answer.
b. Find all values of x, for $-7 \le x \le 7$, at which f attains a relative maximum. Justify your answer.
c. Find all values of x, for $-7 \le x \le 7$, at which $f''(x) < 0$.
d. At what value of x, for $-7 \le x \le 7$, does f attain its absolute maximum? Justify your answer.

204. Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per
minute, for $0 \le t \le 120$ minutes. At time t = 0, the tank contains 30 gallons of water.

a. How many gallons of water leak out of the tank from time t= 0 to
   t = 3 minutes?
b. How many gallons of water are in the tank at time t = 3 minutes?
c. Write an expression for A(t), the total number of gallons of water in the tank at time t.
d. At what time t, for $0 \le t \le 120$, is the amount of water in the tank a

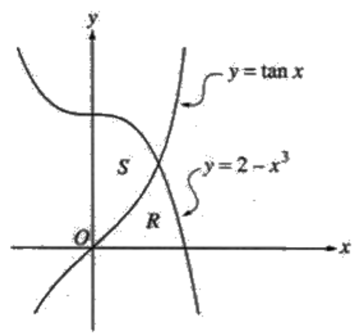
maximum? Justify your answer.

205. Consider the curve given by $xy^2 - x^3y = 6$.

a. Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.
b. Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.
c. Find the x-coordinate of each point on the curve where the tangent line is vertical.

206. Consider the differential equation $\frac{dy}{dx} = \frac{3x^2}{e^{2y}}$.

a. Find a solution y = f(x) to the differential equation satisfying f(0) = $\frac{1}{2}$.
b. Find the domain and range of the function f found in part (a).



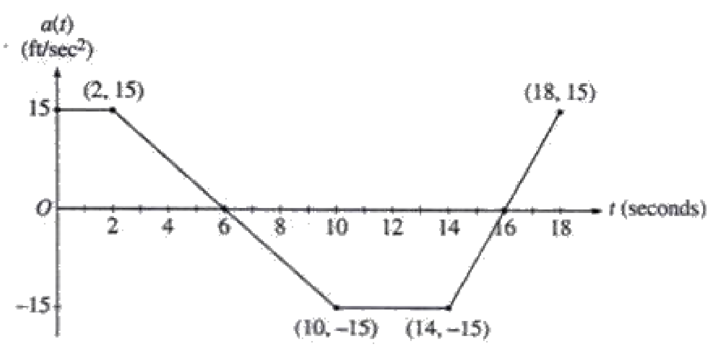
207. Let R and S be the regions in the first quadrant shown in the figure above. The region R is bounded by the x-axis and the graphs of y = 2 – x^3 and y = tan x. The region S is bounded by the y-axis and the graphs of y = 2 – x^3 and y = tan x.

a. Find the area of R.
b. Find the area of S.
c. Find the volume of the solid generated when S is revolved about the x-axis

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

208. The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

a. Use data from the table to find an approximation for W′(12). Show the computations that lead to your answer. Indicate units of measure.
b. Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \leq t \leq 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.
c. A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and P(t) is measured in degrees Celsius. Find P′(12). Using appropriate units, explain the meaning of your answer in terms of water temperature.
d. Use the function P defined in part c to find the average value, in degrees Celsius, of P(t) over the time interval $0 \leq t \leq 15$ days.



209. A car traveling on a straight road with velocity 55 ft/sec at time t = 0. For $0 \leq t \leq 18$ seconds, the car's acceleration a(t), in ft/sec^2, is the piecewise linear function defined by the graph above.

a. Is the velocity of the car increasing at t = 2 seconds? Why or why not?
b. At what time in the interval $0 \leq t \leq 18$, other than t = 0, is the velocity of

the car 55 ft/sec? Why?

c. On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.
d. At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

210. Let h be a function defined for all $x \neq 0$ such that h(4) = -3 and the derivative of h is given by $h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$.

a. Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
b. On what intervals, if any, is the graph of h concave up? Justify your answer.
c. Write an equation for the line tangent to the graph of h at x = 4.
d. Does the line tangent to the graph of h at x = 4 lie above or below the graph of h for x > 4? Why?

211. A cubic polynomial function f is defined by $f(x) = 4x^3 + ax^2 + bx + k$ where a, b, and k are constants. The function f has a local minimum at x = -1, and the graph of f has a point of inflection at x = -2.

a. Find the values of a and b.
b. If $\int_0^1 f(x)dx = 32$, what is the value of k?

212. The function f is differentiable for all real numbers. The point $(3, \frac{1}{4})$ is on the graph of y = f(x), and the slope at each point (x,y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

a. Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $(3, \frac{1}{4})$.
b. Find y = f(x) by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the initial condition $f(3) = \frac{1}{4}$.

213. Let f and g be the functions given by $f(x) = e^x$ and g(x)= ln x.

a. Find the area of the region enclosed by the graphs of f ad g between

$x = \frac{1}{2}$ and x = 1.

b. Find the volume of the solid generated when the region enclosed by the graphs of f and g between $x = \frac{1}{2}$ and x= 1 is revolved about the line y = 4.

c. Let h be the function given by h(x) = f(x) – g(x). Find the absolute minimum value of h(x) on the closed interval $\frac{1}{2} \le x \le 1$, and find the absolute maximum value of h(x) on the closed interval $\frac{1}{2} \le x \le 1$. Show the analysis that leads to your answers.

214. The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E = \frac{15600}{\left(t^2 - 24t + 160\right)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L = \frac{9890}{\left(t^2 - 38t + 370\right)}.$$

Both E(t) and L(t) are measured in people per hour and time t is measured in hours after midnight. These functions are valid $9 \le t \le 23$, the hours during which the park is open. At time t = 9, there are no people in the park.

a. How many people have entered the park by 5:00 p.m. (t = 17)? Round answer to the nearest who number.

b. The price of admission to the park is \$15 until 5:00 p.m. (t = 17). After 5:00 p.m., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

c. Let $H(t) = \int_0^1 \left(E(x) - L(x)dx\right.$ for $9 \le t \le 23$. The value of H(17) to the nearest whole number is 3725. Find the value of H'(17) and explain the meaning of H(17) and H'(17) in the context of the park.

d. At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

215. An object moves along the x-axis with initial position x(0) = 2. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

a. What is the acceleration of the object at time t = 4?

b. Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.

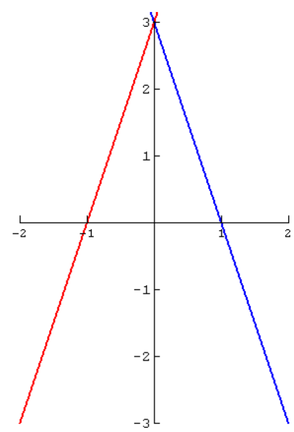
Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

c. What is the total distance traveled by the object over the time interval $0 \leq t \leq 4$?

d. What is the position of the object at time t = 4?

216. The graph of the function f shown consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)dt$.

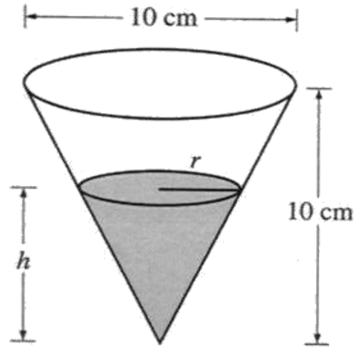


a. Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

b. For what values of x in the open interval (-2, 2) is g increasing? Explain your reasoning.

c. For what values of x in the open interval (-2, 2) is the graph of g concave down? Explain your reasoning.

d. On the axes provided, sketch the graph of g on the closed interval [-2, 2].



217. A container has the shape of an open right circular cone. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

a. Find the volume V of water in the container when h = 5 cm. Indicate units of measure.
b. Find the rate of change of the volume of water in the container, with respect to time, when h = 5 cm. Indicate units of measure.
c. Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

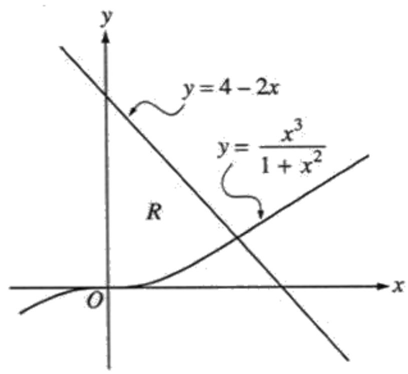
218.

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

a. Evaluate $\int_0^{1.5} \left(3f'(x)+4\right)dx$. Show the work that leads to your answer.
b. Write an equation of the line tangent to the graph of f at the point where x = 1. Use this line to approximate the value of f(1.2). Is this approximation greater than or less than the actual value of f(1.2)?
c. Find a positive real number r having the property that there must exist a value c with 0 < c < 0.5 and f''(c)=r. Give a reason for your answer.
d. Let g be the function given by g(x) = $\begin{cases} 2x^2 - x - 7 \text{ for } x < 0 \\ 2x^2 + x - 7 \text{ for } x \ge 0 \end{cases}$.

The graph of g passes through each of the points (x, f(x)) given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.



219. Let R be the region bounded by the y-axis and the graphs of $y = \dfrac{x^3}{1+x^2}$ and $y = 4 - 2x$.

a. Find the area of R.
b. Find the volume of the solid generated when R is revolved about the x-axis.
c. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

220. The number of gallons, P(t), of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time t = 0. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

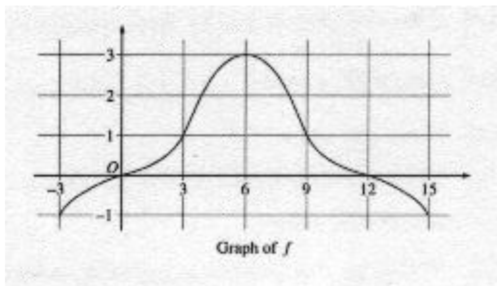
a. Is the amount of pollutant increasing at time t = 9? Why or why not?
b. For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.
c. Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.
d. An investigator uses the tangent line approximation to P(t) at t = 0 as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

221. A particle moves along the x-axis so that its velocity v at any time t, for $0 \le t \le 16$, is given by $v(t) = e^{2 \sin t} - 1$. At time t = 0, the particle is at the origin.

a. On the axes provided, sketch the graph of v(t) for $0 \le t \le 16$.

b. During what intervals of time is the particle moving to the left? Give a reason for your answer.
c. Find the total distance traveled by the particle from t = 0 to t = 4.
d. Is there any time t, $0 \leq t \leq 16$, at which the particle returns to the origin? Justify your answer.

222. The graph of a differentiable function f on the closed interval [-3,15] is shown.



The graph of f has a horizontal tangent line at x = 6. Let

$g(x) = 5 + \int_6^x f(t)dt$ for $-3 \leq x \leq 15$.

a. Find $g(6), g'(6),$ and $g''(6)$.
b. On what intervals is g increasing? Justify your answer.
c. On what intervals is the graph of g concave down? Justify your answer.
d. Find a trapezoidal approximation of $\int_{-3}^{15} f(t)dt$ using six subintervals of length $\Delta t = 3$.

223. Consider the differential equation $\frac{dy}{dx} = \frac{3-x}{y}$.

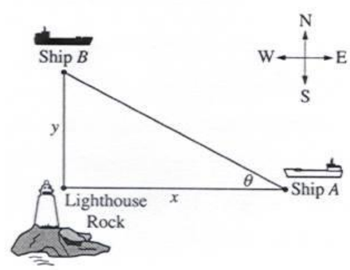
a. Let y = f(x) be the particular solution to the given differential equation for 1 < x < 5 such that the line y = -2 is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at the point. Justify your answer.
b. Let y = g(x) be the particular solution to the given differential equation for -2 < x < 8, with the initial condition g(6) = -4. Find y = g(x).

224. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour. Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t,and let y be the distance between Ship B and Lighthouse Rock at time t, as shown in the figure below.

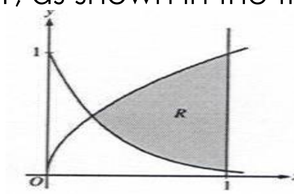
a. Find the distance, in kilometers, between Ship A and Ship B when x = 4 km and y = 3 km.

b. Find the rate of change, in km/hr, of the distance between the two ships when x = 4 km and y = 3 km.

c. Let θ be the angle shown in the figure. Find the rate of change of θ, in radians per hour, when x = 4 km and y = 3 k



225. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line x = 1, as shown in the figure below.



a. Find the area of R.

b. Find the volume of the solid generated when R is revolved about the horizontal line y = 1.

c. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

226. A particle moves along the x-axis so that its velocity at time t is given by $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$. At time t = 0, the particle is at position x = 1.

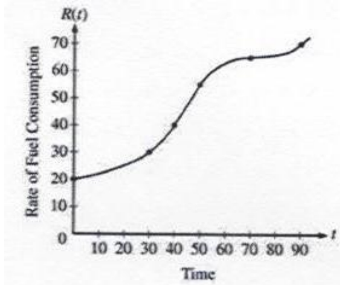
a. Find the acceleration of the particle at time t= 2. Is the speed of the particle increasing at t= 2? Why or why not?

b. Find all times t in the open interval 0 < t < 3 when the particle changes direction. Justify your answer.

c. Find the total distance traveled by the particle from time t = 0 until time t = 3.

d. During the time interval 0 $\leq$ t $\leq$ 3, what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

227. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by the twice-differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of R(t), for the time interval 0 $\leq$ t $\leq$ 90 minutes, are shown below



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

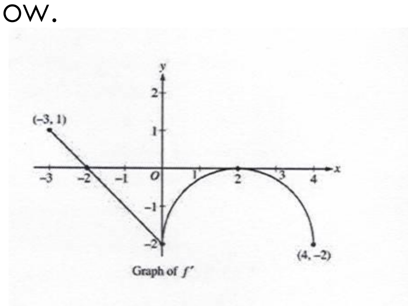
a. Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

b. The rate of fuel consumption is increasing fastest at time t = 45 minutes. What is the value of $R''(45)$? Explain your reasoning.

c. Approximate the value of $\int_0^{90} R(t)dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)dt$? Explain your reasoning.

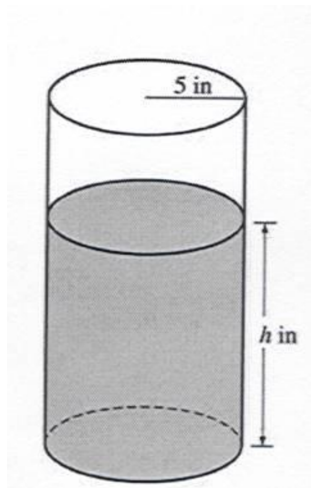
d. For 0 < b < 90 minutes, explain the meaning of $\int_0^{b} R(t)dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{b} R(t)dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

228. Let f be a function defined on the closed interval -3 $\leq$ x $\leq$ 4 with f(0) = 3. The graph of f', the derivate of f, consists of one line segment and a semicircle, as shown below.



a. On what intervals, if any, is f increasing? Justify your answer.

b. Find the x-coordinate of each point of inflection of the graph of f on the open interval -3 < x < 4. Justify your answer.
c. Find an equation for the line tangent to the graph of f at the point (0,3).
d. Find f (-3) and f (4). Show the work that leads to your answers.



229. A coffeepot has the shape of a cylinder with radius 5 inches, as shown in the figure above. Let h be the depth of the coffee in the pot, measured in inches, where h is a function of time t, measured in seconds. The volume V of coffee in the pot is changing at the rate of $-5\pi\sqrt{h}$ cubic inches per second.

a. Show that $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$.
b. Given that h = 17 at time t = 0, solve the differential equation $\frac{dh}{dt} = -\frac{\sqrt{h}}{5}$. for h as a function of t.
c. At what time t is the coffeepot empty?

230. Let f be the function defined by

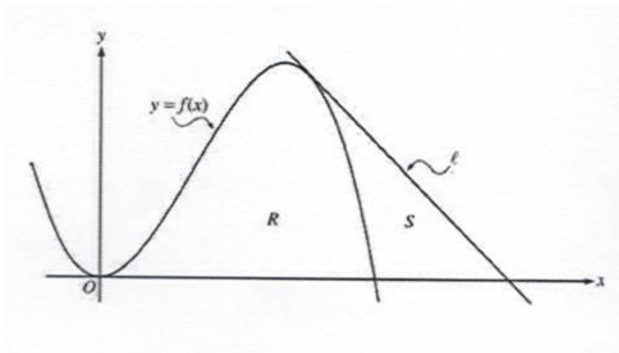
$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5-x & \text{for } 3 < x < 5 \end{cases}$$

a. Is f continuous at x = 3? Explain why or why not.
b. Find the average value of f (x) on the closed interval $0 \le x \le 5$.
c. Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx+2 & \text{for } 3 < x < 5 \end{cases}$$

where k and m are constants. If g is differentiable at x = 3, what are the values of k and m?

231. Let f be the function given by $f(x)=4x^2-x^3$, and let l be the line y = 18 – 3x,where l is tangent to the graph of f. Let R be the region bounded by the graph of f and the x-axis, and let S be the region bounded by the graph of f, the line l, and the x-axis, as shown below



a. Show that l is tangent to the graph of y = f (x) at the point x = 3.
b. Find the area of S.
c. Find the volume of the solid generated when R is revolved about the x – axis.

232. A tank contains 125 gallons of heating oil at time t = 0. During the time interval 0 ≤ t ≤ 12 hours, heating oil is pumped into the tank at the rate

$H(t)=2+\dfrac{10}{(1+\ln(t+1))}$ gallons per hour. During the same time interval, heating oil is removed from the tank at the rate $R(t)=12\sin\left(\dfrac{t^2}{47}\right)$ gallons per hour.

a. How many gallons of heating oil are pumped into the tank during the time interval 0 ≤ t ≤ 12 hours?
b. Is the level of heating oil in the tank rising or falling at time t = 6 hours? Give a reason for your answer.
c. How many gallons of heating oil are in the tank at time t = 12 hours?
d. At what time t, for 0 ≤ t ≤ 12, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

Distance x (mm)	0	60	120	180	240	300	360
Diameter $B(x)$ (mm)	24	30	28	30	26	24	26

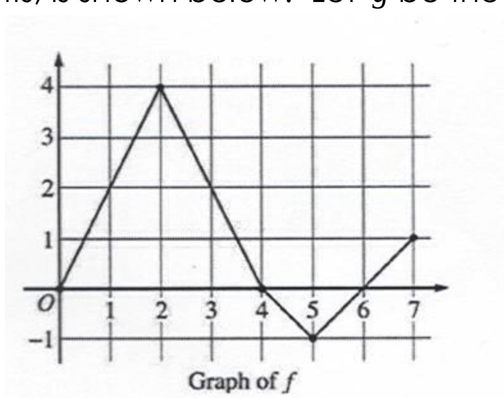
233.
A blood vessel is 360 millimeters (mm) long with circular cross sections of varying diameter. The table above gives the measurements of the diameter of the blood at selected points along the length of the blood vessel, where x represents the distance from one end of the blood vessel and B(x) is a twice-differentiable function that represents the diameter at that point.

a. Write an integral expression in terms of B(x) that represents the average radius, in mm, of the blood vessel between x = 0 and x = 360.
b. Approximate the value of your answer from part (a) using the data from the table and a midpoint Riemann sum with three subintervals of equal length. Show the computations that lead to your conclusion;
c. Using correct units, explain the meaning of $\pi \int_{125}^{275} \left(\frac{B(x)}{2} \right)^2 dx$ in terms of the blood vessel.
d. Explain why there must be at least one value x, for 0 < x < 360, such that $B''(x) = 0$.

234.
A particle moves along the x-axis with velocity at time t $\geq$ 0 given by $v(t) = -1 + e^{1-t}$.

a. Find the acceleration of the particle at time t = 3.
b. Is the speed of the particle increasing at time t = 3? Give a reason for your answer.
c. Find all values of t at which the particle changes directions. Justify your answer.
d. Find the total distance traveled by the particle over the time interval 0 $\leq$ t $\leq$ 3.

235. Let f be a function defined on the closed interval [0, 7]. The graph of f consisting of four line segments, is shown below. Let g be the function given



Graph of f

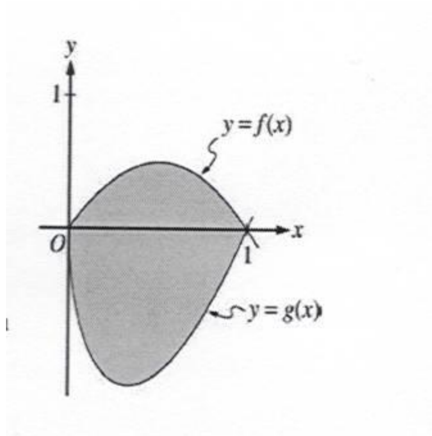
by $g(x) = \int_2^x f(t)dt$.

a. Find $g(3),\ g'(3),\ \text{and}\ g''(3).$
b. Find the average rate of change of g on the interval $0 \le x \le 3$.
c. For how many values c, where $0 < c < 3$, is $g'(c)$ equal to the average rate found in part b? Explain your reasoning.
d. Find the x-coordinate of each point of inflection of the graph of g on the interval $0 < x < 7$. Justify your answer.

236.
Let f be the function satisfying $f'(x) = x\sqrt{f(x)}$ for all real numbers x, where $f(3) = 25.$
a. Find $f''(3).$
b. Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = x\sqrt{y}$ with the initial condition $f(3) = 25.$

237.
Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by $F(t) = 82 + 4\sin\left(\frac{t}{2}\right)$ for $0 \le t \le 30,$ where F(t) is measured in cars per minute and t is measured in minutes.
a. To the nearest whole number, how many cars pass through the intersection over the 30-minute period?
b. Is the traffic flow increasing or decreasing at t = 7? Give a reason for your answer.
c. What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.
d. What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

238.
Let f and g be the functions given by $f(x) = 2x(1-x)$ and $g(x) = 3(x-1)\sqrt{x}$ for $0 \le x \le 1.$ The graphs of f and g are shown in the figure below.



a. Find the area of the shaded region enclosed by the graphs of f and g.

b. Find the volume of the solid generated when the shaded region enclosed by the graphs of f and g is revolved about the horizontal line y = 2.

c. Let h be the function given by $h(x) = kx(1-x) \text{ for } 0 \le x \le 1.$ For each k > 0, the region (not shown) enclosed by the graphs of h and g is the base of a solid with square cross sections perpendicular to the x-axis. There is a value of k for which the volume of this solid is equal to 15. Write, but do not solve, an equation involving an integral expression that could be used to find the value of k.

239.

A particle moves along the y-axis so that its velocity v at time t $\geq$ 0 is given by $v(t) = 1 - \tan^{-1}(e^t).$ At time t = 0, the particle is at y = -1.

a. Find the acceleration of the particle at time t = 2.

b. Is the speed of the particle increasing or decreasing at time t = 2? Give a reason for your answer.

c. Find the time t $\geq$ 0 at which the particle reaches its highest point. Justify your answer.

d. Find the position of the particle at time t = 2. Is the particle moving toward the origin or away from the origin at time t = 2? Justify your answer.

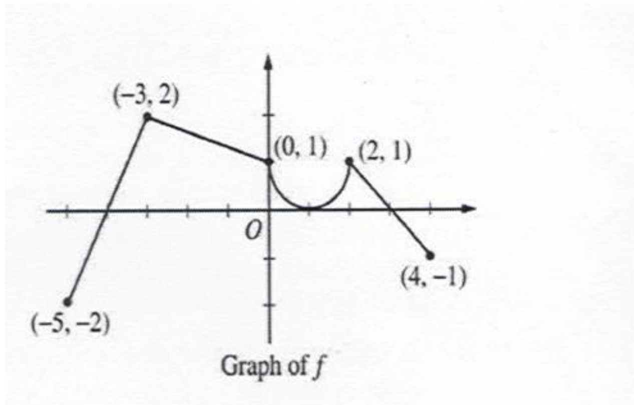
240.

Consider the curve given by $x^2 + 4y^2 = 7 + 3xy.$

a. Show that $\dfrac{dy}{dx} = \dfrac{3y - 2x}{8y - 3x}$.

b. Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

c. Find the value of $\frac{d^2y}{dx^2}$ at the point P found in part b. Does the curve have a local maximum, a local minimum, or neither at the point P? Justify your answer.



241.

The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)dt.$

a. Find $g(0)$ and $g'(0).$

b. Find all values of x in the open interval (-5, 4) at which g attains a relative maximum. Justify your answer.

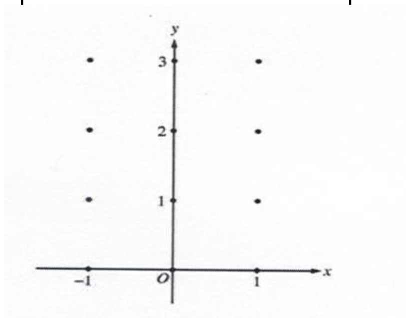
c. Find the absolute minimum value of g on the closed interval [-5, 4]. Justify your answer.

d. Find all values of x in the open interval (-5, 4) at which the graph of g has a point of inflection.

242.

Consider the differential equation $\frac{dy}{dx} = x^2(y-1).$

a. On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



b. While the slope field in part a is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

c. Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.

243.

Let R be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line x = 10, and the x-axis.

a. Find the area of R.
b. Find the volume of the solid generated when R is revolved about the horizontal line y = 3.
c. Find the volume of the solid generated when R is revolved about the vertical line x = 10.

244.

For 0 ≤ t ≤ 31, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time t = 0.

a. Show that the number of mosquitoes is increasing at time t = 6.
b. At time t = 6, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.
c. According to the model, how many mosquitoes will be on the island at time t= 31? Round your answer to the nearest whole number.
d. To the nearest whole number, what is the maximum number of mosquitoes for 0 ≤ t ≤ 31? Show the analysis that leads to your conclusion.

245.

A test plane flies in a straight line with positive velocity v(t), in miles per minute at time t minutes, where v is a differential function of t. Selected values of v(t) for 0 ≤ t ≤ 40 are shown in the table below.

t (min)	0	5	10	15	20	25	30	35	40
$v(t)$ (mpm)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

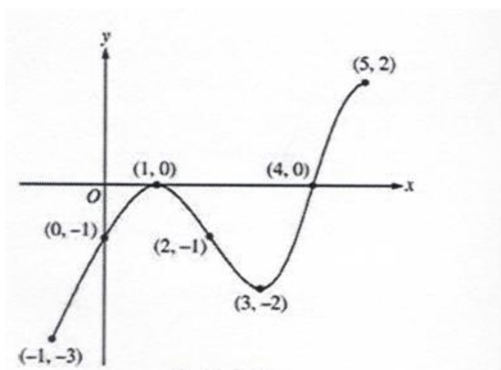
a. Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)dt$. Show the computations that lead to your answer. Using correct units explain the meaning of

$\int_0^{40} v(t)dt.$ in terms of the plane's flight.

b. Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval 0 < t < 40? Justify your answer.

c. The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right),$ is used to model the velocity of the plane, in miles per minute, for 0 $\leq$ t $\leq$ 40. According to this model, what is the acceleration of the plane at t = 23? Indicates units of measures.

d. According to the model f, given in part c, what is the average velocity of the plane, in miles per minute, over the time interval 0 $\leq$ t $\leq$ 40?



Graph of f'

246.

The figure above shows the graph of f', the derivative of the function f, on the closed interval -1 $\leq$ x $\leq$ 5. The graph of f' has horizontal tangent lines at x = 1 and x =3. The function f is twice differentiable with $f(2) = 6.$

a. Find the x-coordinate of each of the points of inflection of the graph of f. Give a reason for your answer.

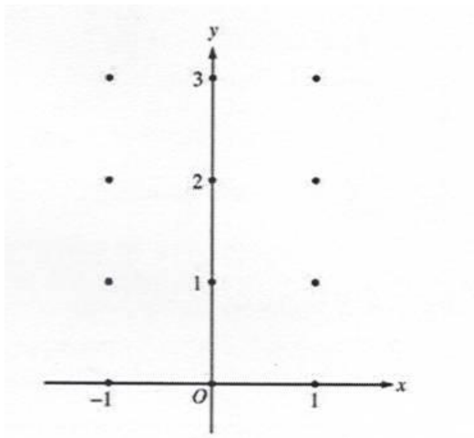
b. At what value of x does f attain its absolute minimum value on the closed interval -1 $\leq$ x $\leq$ 5? At what value of x does f attain its absolute maximum value on the closed interval -1 $\leq$ x $\leq$ 5? Show the analysis that leads to your answers.

c. Let g be the function defined by $g(x) = xf(x).$ Find an equation for the line tangent to the graph of g at x = 2.

247.

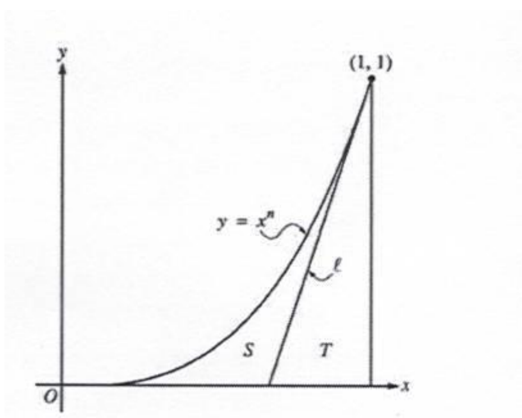
Consider the differential equation $\frac{dy}{dx} = x^4(y-2)$.

a. On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



b. While the slope field in part a is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are negative.

c. Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 0$.



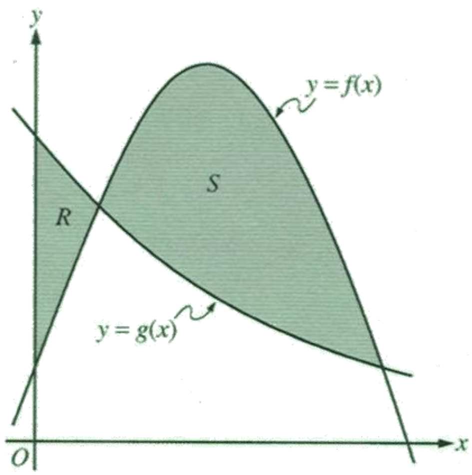
248.

Let l be the line tangent to the graph of $y = x^n$ at the point (1, 1), where n > 1, as
shown above.

a. Find $\int_0^1 x^n dx$ in terms of n.

b. Let T be the triangular region bounded by l , the x-axis, and the line x = 1. Show that the area of T is $\frac{1}{2n}$.

c. Let S be the region bounded by the graph of $y = x^n$, the line l, and the x-axis. Express the area of S in terms of n and determine the value of n that maximizes the area of S.



249.

Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$ (as shown above). Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of f and g, and Let S be the shaded region in the first quadrant enclosed by the graphs of f and g, as shown in the figure above.

a. Find the area of R.
b. Find the area of S.
c. Find the volume of the solid generated when S is revolved about the horizontal line y = -1.

250.

The tide moves sand from Sandy Point Beach at a rate modeled by the function R, given by $R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$. A pumping station adds sand to the beach at a rate modeled by the function S, given by $S(t) = \frac{15t}{1+3t}$. Both R(t) and S(t) have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time t = 0, the beach contains 2500 cubic yards of sand.

a. How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

b. Write an expression for Y(t), the total number of cubic yards of sand on the beach at time t.

c. Find the rate at which the total amount of sand on the beach is changing at time t = 4.

d. For $0 \leq t \leq 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

251.

Distance x (cm)	0	1	5	6	8
Temperature T(x) (°C)	100	93	70	62	55

A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature T(x), in degrees Celsius (°C), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.

a. Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.

b. Write an integral expression in terms of T(x) for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.

c. Find $\int_0^8 T'(x)dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x)dx$ in terms of the temperature of the wire.

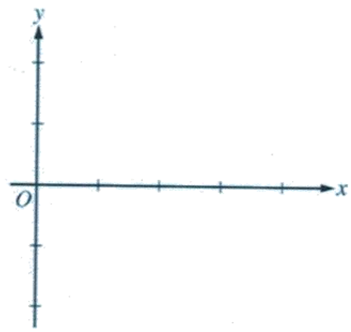
d. Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval 0 < x < 8? Explain your answer.

252.

X	0	0 < x < 1	1	1 < x < 2	2	2 < x < 3	3	3 < x <4
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	positive

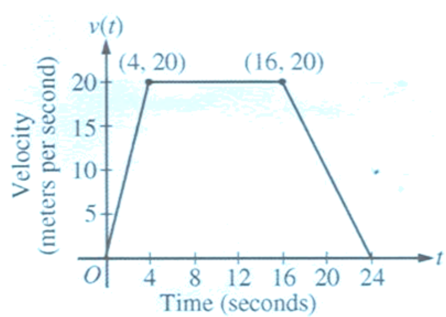
Let f be a function that is continuous on the interval [0,4). The function f is twice differentiable at x = 2. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at x = 2.

a. For 0 < x < 4, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.
b. On the axes provided, sketch the graph of a function that has all the characteristics of f.



c. Let g be the function defined by $g(x) = \int_1^x f(t)dt$ on the open interval (0, 4). For 0 < x < 4, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.
d. For the function g defined in part c, find all values of x, for 0 < x < 4, at which the graph of g has a point of inflection. Justify your answer.

253.



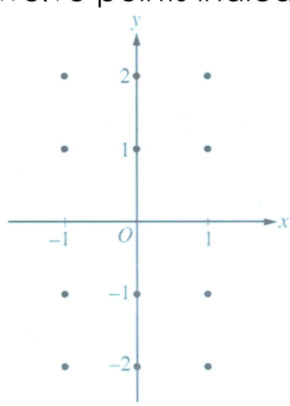
A car is traveling on a straight road. For $0 \le t \le 24$ seconds, the car's velocity v(t), in meters per second, is modeled by the piecewise-linear function defined by the graph above.

a. Find $\int_0^{24} v(t)dt$. Using correct units, explain the meaning of $\int_0^{24} v(t)dt$.
b. For each $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.
c. Let a(t) be the car's acceleration at time t, in meters per second per second. For 0 < t < 24, write the piecewise-defined function for a(t).
d. Find the average rate of change of v over the interval 8 $\leq$ t $\leq$ 20. Does the Mean Value Theorem guarantee a value of c, for 8 < t < 20, such that $v'(c)$ is equal to this average rate of change? Why or why not?

254.

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

a. On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



b. Let y = f(x) be the particular solution to the differential equation with the initial condition f(1) = -1. Write an equation for the line tangent to the graph of f at (1, -1) and use it to approximate f(1.1)
c. Find the particular solution y = f(x) to the given differential equation with the initial condition f(1) = -1.

255.

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$.

a. Find the area of R.
b. Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

c. Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.

256.

At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t}\sin^2\left(\frac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours.

a. To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.
b. Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.
c. Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

257.

The graph of the function f shown consists of six line segments. Let g be the function given by $g(x) = \int_0^x f(t)dt.$

a. Find $g(4), g'(4),$ and $g''(4).$
b. Does g have a relative minimum, a relative maximum, or neither at x = 1? Justify your answer.
c. Suppose that f is defined for all real numbers x and is periodic with a period of length 5. The graph shows two periods of f . Given that $g(5) = 2$, find $g(10)$ and write an equation for the line tangent to the graph of g at x = 108.

258.

t (seconds)	0	10	20	30	40	50	60	70	80
$v(t)$ (feet per second)	5	14	22	29	35	40	44	47	49

Rocket A has positive velocity $v(t)$ after being launched upward from an initial height of 0 feet at time $t = 0$ seconds. The velocity of the rocket is recorded for selected values of t over the interval $0 \le t \le 80$ seconds, as shown in the table above.

a. Find the average acceleration of rocket A over the time interval $0 \le t \le 80$ seconds. Indicate units of measure.

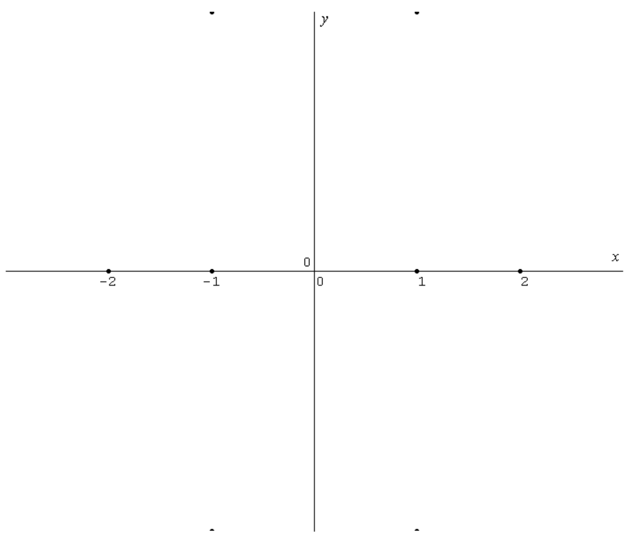
b. Using correct units, explain the meaning of $\int_{10}^{70} v(t)dt$ in terms of the rocket's flight. Use a midpoint Riemann sum with 3 subintervals of equal length to approximate $\int_{10}^{70} v(t)dt$.

c. Rocket B is launched upward with an acceleration of $a(t) = \frac{3}{\sqrt{t+1}}$ feet per second per second. At time $t = 0$ seconds, the initial height of the rocket is 0 feet, and the initial velocity is 2 fee per second. Which of the two rockets is traveling faster at time $t = 80$ seconds? Explain your answer.

259.

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0.$

a. On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



b. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

260.

The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2,\ f'(0) = -4,\ \text{and } f''(0) = 3.$$

a. The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a.
b. The function h is given by $h(x) = \cos(kx) f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

261.

Let R be the region in the first and second quadrants bounded above by the graph of $y = \dfrac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

a. Find the area of R.
b. Find the volume of the solid generated when R is rotated about the x-axis.
c. The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles. Find the volume of this solid.

262.
The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \le t \le 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is
$f(t) = 100t^2 \sin\left(\sqrt{t}\right)$ gallons per hour for $0 \le t \le 7$

(ii) The rate at which water leaves the tank is
$g(t) = \begin{cases} 250 \text{ for } 0 \le t < 3 \\ 2000 \text{ for } 3 < t \le 7 \end{cases}$ gallons per hours.

The graphs of f and g, which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

a. How many gallons of water enter the tank during the time interval $0 \le t \le 7$? Round your answer to the nearest gallon.
b. For $0 \le t \le 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
c. For $0 \le t \le 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

263.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	**6**	**4**	**2**	**5**
2	**9**	**2**	**3**	**1**
3	**10**	**-4**	**4**	**2**
4	**-1**	**3**	**6**	**7**

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by
$h(x) = f(g(x)) - 6.$

a. Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5.$
b. Explain why there must be a value c for $1 < r < 3$ such that $h'(r) = -5.$
c. Let w be the function given by $w(x) = \int_1^{g(x)} f(t)dt.$ Find the value of $w'(3).$

d. If g^{-1} is the inverse of g, write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2.$

264.

A particle moves along the x-axis with position at time t given by $x(t) = e^{-t}\sin t$ for $0 \le t \le 2\pi.$

a. Find the time t at which the particle is farthest to the left. Justify your answer.
b. Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi.$

265.

t (minutes)	0	2	5	7	11	12
$r'(t)$ (ft/ min)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t),$ of the radius of the balloon over the time interval $0 \le t \le 12.$ The radius of the balloon is 30 feet when $t = 5.$ (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3.$)

a. Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5.$ Is your estimate greater than or less than the true value? Give a reason for your answer.
b. Find the rate of change of the volume of the balloon with respect to time when $t = 5.$ Indicate units of measure.
c. Use a right Riemann Sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt.$ Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt.$ in terms of the radius of the balloon.

d. Is your approximation in part c greater than or less than $\int_0^{12} r'(t)dt$?

Give a reason for your answer.

266.

Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0,$ where k is a positive constant.

a. Find $f'(x)$ and $f''(x).$
b. For what value of the constant k does f have a critical point at $x = 1.$ For this value of k, determine whether f has a relative minimum, relative maximum, or neither at $x = 1.$ Justify your answer.
c. For a certain value of the constant k, the graph of f has a point of inflection on the x-axis Find this value of k.

267.

Let R be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2,$ and let S be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$and $y = 2$.

a. Find the area of R.
b. Find the area of S.
c. Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 1.$

268.

A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = \sin(t^2).$ The graph of v is shown above for $0 \leq t \leq \sqrt{5\pi}.$ The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5.$

a. Find the acceleration of the particle at time $t = 3.$
b. Find the total distance traveled by the particle from time $t = 0$ to $t = 3.$
c. Find the position of the particle at time $t = 3.$
d. For $0 \leq t \leq \sqrt{5\pi}$, find the time t at which the particle is farthest to theer right. Explain your answer.

269.

The wind chill is the temperature, in degrees Fahrenheit $\left({}^{0}F\right)$, a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is $32^{0}F$, then the wind chill is given by $W(v)=55.6-22.1v^{0.16}$ and is valid for $5\le v\le 60.$

a. Find $W'(20).$ Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.
b. Find the average rate of change of W over the interval $5\le v\le 60.$ Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5\le v\le 60.$
c. Over the time interval $0\le t\le 4$ hours, the air temperature is a constant $32^{0}F$. At time $t=0$, the wind velocity is $v=20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t=3$ hours? Indicate units of measure.

270.

Let f be a function defined on the closed interval $-5\le x\le 5$ with $f(1)=3.$ The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

a. For $-5<x<5$, find all values of x at which f has a relative maximum. Justify your answer.
b. For $-5<x<5$, find all values of x at which f has a point of inflection. Justify your answer.
c. Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
d. Find the absolute minimum value of $f(x)$ over the closed interval $-5\le x\le 5$. Explain your reasoning.

271.

Consider the differential equation $\dfrac{dy}{dx}=\dfrac{1}{2}x+y-1.$

a. On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

b. Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

c. Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

d. Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

272.

Let f be a twice-differentiable function such that $f(2) = 5$ and $f(5) = 2$. Let g be the function given by $g(x) = f(f(x))$.

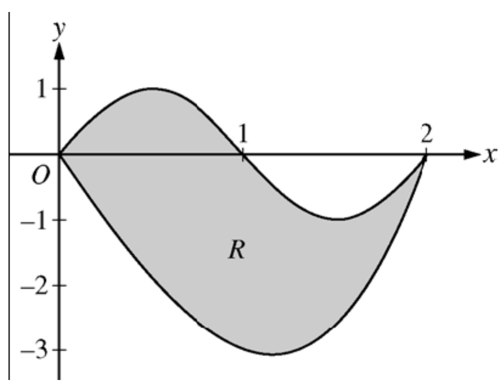
a. Explain why there must be a value c for $2 < c < 5$ such that $f'(c) = -1$.

b. Show that $g'(2) = g'(5)$. Use this result to explain why there must be a value k for $2 < k < 5$ such that $g''(k) = 0$.

c. Show that if $f''(x) = 0$ for all x, then the graph of g does not have a point of inflection.

d. Let $h(x) = f(x) - x$. Explain why there must be a value of r for $2 < r < 5$ such that $h(r) = 0$.

273.



Let R be the region bounded by the graphs of $y = \sin(\pi x)$ and $y = x^3 - 4x$, as shown in the figure above.

a. Find the area of R.

b. The horizontal line $y=-2$ splits the region R into two parts. Write, but do not integrate, an integral expression for the area of the part of R that is below this horizontal line.
c. The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.
d. The region R models the surface of a small pond. At all points R at a distance x from the y-axis, the depth of the water is given by $h(x)=3-x$. Find the volume of water in the pond.

274.

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ($(t=0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0\le t\le 9.$ Values of $L(t)$ at various times t are shown in the table above.

a. Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t=5.5)$. Show the computations that lead to your answer. Indicate units of measure.
b. Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
c. For $0\le t\le 9,$ what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.
d. The rate at which tickets were sold for $0\le t\le 9$ is modeled by $r(t)=550te^{\frac{-t}{2}}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t=3)$, to the nearest whole number?

275. Oil is leaking from a pipeline on the surface of a lake and forms an oil slick whose volume increases at a constant rate of 2000 cubic centimeters per minute. The oil slick takes the form of a right circular cylinder with both its radius and height changing with time. (Note: The volume V of a right circular cylinder with radius r and height h is given by $V=\pi r^2 h.$

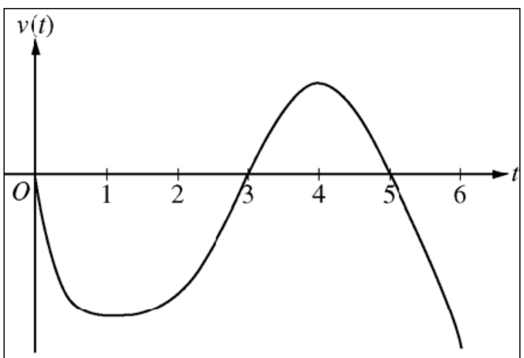
a. At the instant when the radius of the oil slick is 100 centimeters and the height is 0.5 centimeter, the radius is increasing at the rate of 2.5

centimeters per minute. At this instant, what is the rate of change of the height of the oil slick with respect to time, in centimeters per minute?

b. A recovery device arrives on the scene and begins removing oil. The rate at which oil is removed is $R(t) = 400\sqrt{t}$ cubic centimeters per minute, where t is the time in minutes since the device began working. Oil continues to leak at the rate of 2000 cubic centimeters per minute. Find the time t when the oil slick reaches its maximum volume. Justify your answer.

c. By the time the recovery device began removing oil, 60,000 cubic centimeters of oil had already leaked. Write, but do not evaluate, an expression involving an integral that gives the volume of oil at the time found in part b.

276.



Graph of v

A particle moves along the x-axis so that its velocity at time t, for $0 \le t \le 6,$ is given by a differentiable function v whose graph is shown above. The velocity is 0 at $t = 0,\ t = 3,$ and $t = 5,$ and the graph has horizontal tangents at $t = 1$ and $t = 4.$ The areas of the regions bounded by the t-axis and the graph of v on the intervals [0,3], [3,5], and [5,6] are 8, 3, and 2, respectively. At time $t = 0,$ the particle is at $x = -2.$

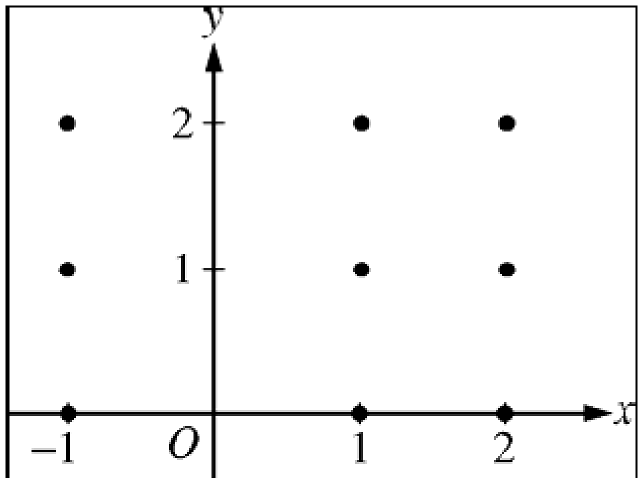
a. For $0 \le t \le 6,$ find both the time and the position of the particle when the particle is farthest to the left. Justify your answer.
b. For how many values of t, where $0 \le t \le 6,$ is the particle at $x = -8?$ Explain your reasoning.
c. On the interval $2 < t < 3$, is the speed of the particle increasing or decreasing? Give a reason for your answer.

d. During what time intervals, if any, is the acceleration of the particle negative? Justify your answer.

277.

Consider the differential equation $\frac{dy}{dx} = \frac{y-1}{x^2}$, where $x \neq 0$.

a. On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.



b. Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(2) = 0$.

c. For the particular solution $y = f(x)$ described in part b, find $\lim_{x \to \infty} f(x)$.

278.

Let f be the function given by $f(x) = \frac{\ln x}{x}$ for all $x > 0$. The derivative of f is given by $f'(x) = \frac{1 - \ln x}{x^2}$.

a. Write an equation for the line tangent to the graph of f at $x = e^2$.

b. Find the x-coordinate of the critical point of f. Determine whether this point is a relative minimum, a relative maximum, or neither for the function f. Justify your answer.

c. The graph of the function f has exactly one point of inflection. Find the x-coordinate of this point.

d. Find $\lim_{x \to 0^+} f(x)$.

279.

Let R be the region in the first quadrant bounded by the graphs of $y=\sqrt{x}$ and $y=\frac{x}{3}$.

a. Find the area of R.
b. Find the volume of the solid generated when R is rotated about the vertical line $x=-1$.
c. The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

280.

For time $t \geq 0$ hours, let $r(t)=120\left(1-e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel x kilometers is modeled by $g(x)=0.05x\left(1-e^{-\frac{x}{2}}\right)$.

a. How many kilometers does the car travel during the first 2 hours?
b. Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t=2$ hours. Indicate units of measure.
c. How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

281.

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by $v(t)=16+2\sin(\sqrt{t+10})$ for $0 \leq t \leq 120$ minutes.

a. Use a trapezoidal sum with four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.

b. The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.

c. The scientist proposes the function f, given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.

d. Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using you answer from part (c), find the average value of the volumetric flow during the time interval $40 \le t \le 60$ minutes. Does this value indicate that the water must be diverted?

282.

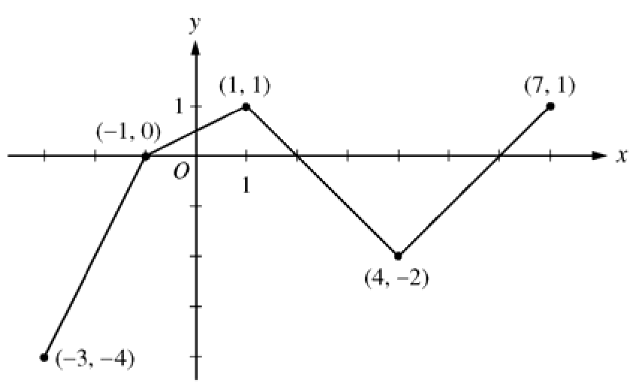
The functions f and g are given by $f(x) = \int_0^{3x} \sqrt{4+t^2}\,dt$ and $g(x) = f(\sin x)$.

a. Find $f'(x)$ and $g'(x)$.

b. Write an equation for the line tangent to the graph of $y = g(x)$ at $x = \pi$.

c. Write, but do not evaluate, an integral expression that represents the maximum value of g on the interval $0 \le x \le \pi$. Justify your answer.

283.



Graph of g'

Let g be a continuous function with $g(2)=5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

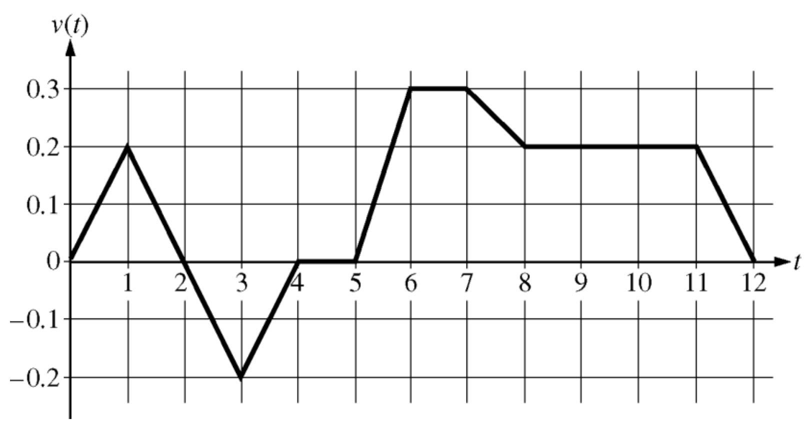
a. Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 \le x \le 7$. Justify your answer.
b. Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.
c. Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.
d. Find the average rate of change of g' on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

284.

Consider the closed curve in the xy – plane given by $x^2 + 2x + y^4 + 4y = 5$.

a. Show that $\dfrac{dy}{dx} = \dfrac{-(x+1)}{2(y^3+1)}$.
b. Write an equation for the line tangent to the curve at the point (-2,1).
c. Find the coordinates of the two points on the curve where the line tangent to the curve is vertical.
d. Is it possible for this curve to have a horizontal tangent at points where it intersects the x – axis? Explain your reasoning.

285.



Caren rides her bicycle along a straight road from home to school, starting at home at time t = 0 minutes and arriving at school at time t = 12 minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per hour, is modeled by the piecewise-linear function whose graph is shown above.

a. Find the acceleration of Caren's bicycle at time t = 7.5 minutes. Indicate units of measure.

b. Using correct units, explain the meaning of $\int_0^{12} |v(t)| dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)| dt$.

c. Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

d. Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$ where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

286.

The rate at which people enter an auditorium for a rock concert is modeled by the function *R* given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; *R*(*t*) is measured in people per hour. No one is in the auditorium at time *t* = 0, when the doors open. The doors close and the concert begins at time *t* = 2.

a. How many people are in the auditorium when the concert begins?

b. Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

c. The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function *w* models the total wait time for all the people who enter the auditorium before time *t*. The derivative of *w* is given by $w'(t) = (2-t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time *t* = 1.

d. On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

287.

Mighty Cable Company manufactures cable that sells for \$120 per meter. For a cable of fixed length, the cost of producing a portion of the cable varies with its distance from the beginning of the cable. Mighty reports that the cost to produce a portion of a cable that is x meters from the beginning of the cable is

$6\sqrt{x}$ dollars per meter. (Note: Profit is defined to be the difference between the amount of money received by the company for selling the cable and the company's cost of producing the cable.)

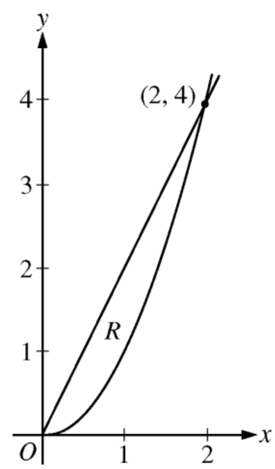
a. Find Mighty's profit on the sale of a 25-meter cable.

b. Using correct units, explain the meaning of $\int_{25}^{30} 6\sqrt{x}dx$ in the context of this problem.

c. Write an expression, involving an integral, that represents Mighty's profit on the sale of a cable that is *k* meters long.

d. Find the maximum profit that Mighty could earn on the sale of one cable. Justify your answer.

288.



Let *R* be the region in the first quadrant enclosed by the graphs of $y = 2x$ and $y = x^2$, as shown in the figure above.

a. Find the area of *R*.

b. The region *R* is the base of a solid. For this solid, at each x the cross section perpendicular to the x-axis has area $A(x) = \sin\left(\frac{\pi}{2}x\right)$. Find the volume of the solid.

c. Another solid has the same base *R*. For this solid, the cross sections perpendicular to the *y*-axis are squares. Write, but do not evaluate, an integral expression for the volume of the solid.

289.

x	2	3	5	8	13
$f(x)$	1	4	-2	3	6

Let f be a function that is twice differentiable for all real numbers. The table Above gives values of f for the selected points in the closed interval $2 \le x \le 13$.

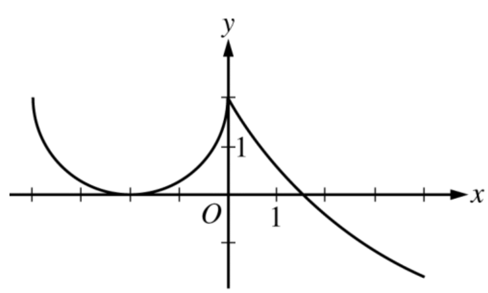
a. Estimate $f'(4)$. Show the work that leads to your answer.

b. Evaluate $\int_2^{13} (3 - 5f'(x))\,dx$. Show the work that leads to your answer.

c. Use a Left Riemann sum with subintervals indicated by the data in the table to approximate $\int_2^{13} f(x)\,dx$. Show the work that leads to your answer.

d. Suppose $f'(5) = 3$ and $f''(x) < 0$ for all x in the closed interval $5 \le x \le 8$. Use the line tangent to the graph of f at $x = 5$ to show that $f(7) \le 4$. Use the secant line for the graph of f on $5 \le x \le 8$ to show that $f(7) \ge \frac{4}{3}$.

290.



Graph of f'

The derivative of a function f is defined by $f'(x) = \begin{cases} g(x) & \text{for } -4 \le x \le 0 \\ 5e^{-\frac{x}{3}} - 3 & \text{for } 0 < x \le 4 \end{cases}$

The graph of the continuous function f', shown in the figure above, has x-intercepts at $x = -2$ and $x = 3\ln\left(\frac{5}{3}\right)$. The graph of g on $-4 \le x \le 0$ is a semicircle, and $f(0) = 5$.

a. For $-4 < x < 4$, find all values of x at which the graph of f has a point of inflection. Justify your answer.

b. Find $f(-4)$ and $f(4)$.

c. For $-4 \le x \le 4$, find the value of x at which f has an absolute maximum. Justify your answer.

291.

At a certain height, a tree trunk has a circular cross section. The radius *R*(*t*) of that cross section grows at a rate modeled by the function $\dfrac{dR}{dt} = \dfrac{1}{16}\left(3 + \sin\left(t^2\right)\right)$ centimeters per year for $0 \le t \le 3$, where time *t* is measured in years. At time *t* = 0, the radius is 6 centimeters. The area of the cross section at time *t* is denoted by *A*(*t*).

a. Write an expression, involving an integral, for the radius *R*(*t*) for $0 \le t \le 3$. Use your expression to find *R*(3).

b. Find the rate at which the cross-sectional area *A*(*t*) is increasing at time *t* = 3 years. Indicate units of measure.

c. Evaluate $\int_0^3 A'(t)\,dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

292.

A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, *t* hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \dfrac{1}{2\sqrt{t}} - \sin t$.

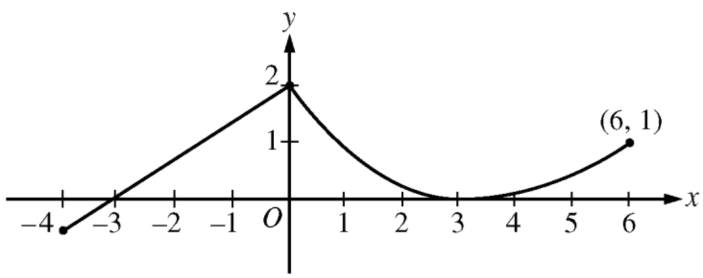
a. What was the distance between the road and the edge of the water at the end of the storm?

b. Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.

c. At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.

d. After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of *g*(*p*) meters per day, where *p* is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

293.



Graph of f

A continuous function *f* is defined on the closed interval $-4 \le x \le 6.$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at x = 3, as shown in the figure above. On the interval 0 < x < 6, the function f is twice differentiable, with $f''(x) > 0.$

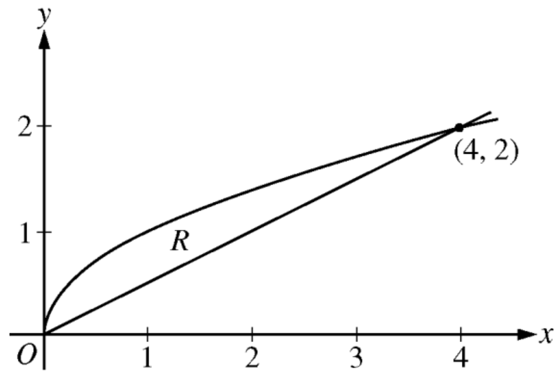
a. Is *f* differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

b. For how many values of a, $-4 \le a \le 6$, is the average rate of change of f on the interval [a , 6] equal to 0 ? Give a reason for your answer.

c. Is there a value of a, $-4 \le a \le 6$, for which the Mean Value Theorem, applied to the interval [a , 6], guarantees a value c, a < c < 6, at which $f'(c) = \frac{1}{3}$? Justify your answer.

d. The function g is defined by $g(x) = \int_0^x f(t)dt$ for $-4 \le x \le 6.$ On what intervals contained in [-4, 6] is the graph of *g* concave up? Explain your reasoning.

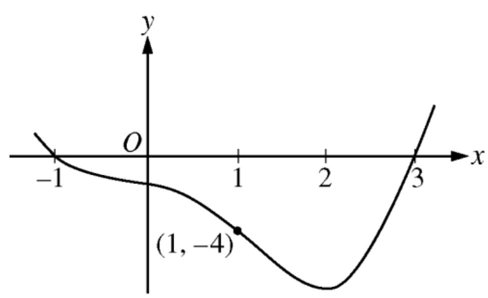
294.



Let *R* be the region bounded by the graphs of $y = \sqrt{x}$ and $y = \dfrac{x}{2}$, as shown in the figure above.

a. Find the area of *R*.
b. The region *R* is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are squares. Find the volume of this solid.
c. Write, but do not evaluate, an integral expression for the volume of the solid generated when *R* is rotated about the horizontal line *y* = 2.

295.



Graph of f'

Let f be a twice-differentiable function defined on the interval $-1.2 < x < 3.2$ with $f(1) = 2.$ The graph of f', the derivative of f, is shown above. The graph of f' crosses the x-axis at $x = -1$ and $x = 3$ and has a horizontal tangent at $x = 2.$ Let g be the function given by $g(x) = e^{f(x)}.$

a. Write an equation for the line tangent to the graph of g at $x = 1.$
b. For $-1.2 < x < 3.2,$ find all values of x at which g has a local maximum. Justify your answer.
c. The second derivative of g is $g''(x) = e^{f(x)}\left[\left(f'(x)\right)^2 + f''(x)\right].$ Is $g''(-1)$

positive, negative, or zero? Justify your answer.

d. Find the average rate of change of g', the derivative of g, over the interval $[1,3]$.

296.

t (seconds)	0	8	20	25	32	40
$v(t)$ m/sec	3	5	-10	-8	-4	7

The velocity of a particle moving along the x-axis is modeled by a differentiable function v, where the position x is measured in meters. And time t is measured in seconds. Selected values of $v(t)$ are given in the table above. The particle is at position $x = 7$ meters when $t = 0$ seconds.

a. Estimate the acceleration of the particle at $t = 36$ seconds. Show the computations that lead to your answer. Indicate units of measure.

b. Using correct units, explain the meaning of $\int_{20}^{40} v(t)dt$ in the context of this problem. Use a trapezoidal sum with three subintervals indicated by the data in the table to approximate $\int_{20}^{40} v(t)dt$.

c. For $0 \le t \le 40$, must the particle change direction in any of the subintervals indicated by the data in the table? Is so, identify the subintervals and explain your reasoning. If not, explain why not.

d. Suppose that the acceleration of the particle is positive for $0 < t < 8$ seconds. Explain why the position of the particle at $t = 8$ seconds must be greater than $x = 30$ meters.

297.

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. (t = 6). The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time *t* hours after midnight is modeled by $g(t)=\begin{Bmatrix} 0 & \text{for } 0\le t<6 \\ 125 & \text{for } 6\le t<7 \\ 108 & \text{for } 7\le t\le 9 \end{Bmatrix}$.

a. How many cubic feet of snow have accumulated on the driveway by 6 A.M.?
b. Find the rate of change of the volume of snow on the driveway at 8 A.M.
c. Let *h*(*t*) represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time *t* hours after midnight. Express *h* as a piecewise-defined function with domain $0\le t\le 9.$.
d. How many cubic feet of snow are on the driveway at 9 A.M.?

298.

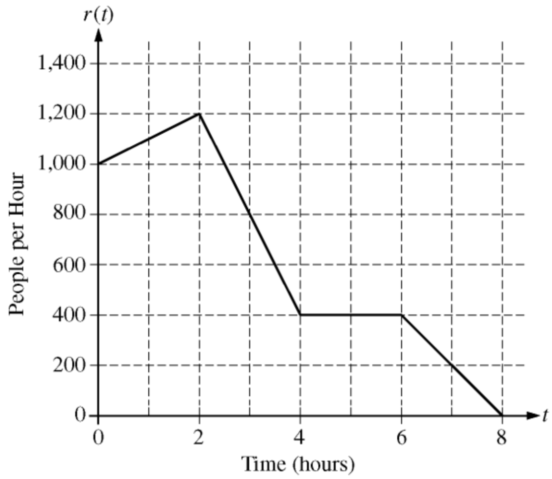
t (hours)	0	2	5	7	8
$E(t)$ (hundreds of entries)	0	4	13	21	23

A zoo sponsored a one-day contest to name a new baby elephant. Zoo visitors deposited entries in a special box between noon (*t* = 0) and 8 P.M. (*t* = 8). The number of entries in the box t hours after noon is modeled by a differentiable function E for $0\le t\le 8.$ Values of $E(t)$, in hundreds of entries, at various times *t* are shown in the table above.

a. Use the data in the table to approximate the rate, in hundreds of entries per hour, at which entries were being deposited at time $t=6.$ Show the computations that lead to your answer.
b. Use a trapezoidal sum with the four subintervals given by the table to approximate the value of $\frac{1}{8}\int_0^8 E(t)dt.$ Using correct units, explain the meaning of $\frac{1}{8}\int_0^8 E(t)dt.$ in terms of the number of entries.
c. At 8 P.M., volunteers began to process the entries. They processed the entries at a rate modeled by the function $P,$ where $P(t)=t^3-30t^2+298t-976$ hundreds of entries per hour for $8\le t\le 12.$ According to the model, how many entries had not yet been processed by midnight ($t=12$) ?

d. According to the model from part (c), at what time were the entries being processed most quickly? Justify your answer.

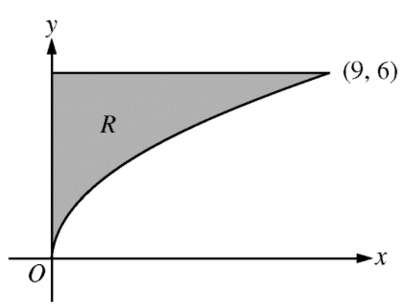
299.



There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time *t* is measured in hours from the time the ride begins operation.

a. How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.
b. Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.
c. At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.
d. Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time *t* at which there is no longer a line for the ride.

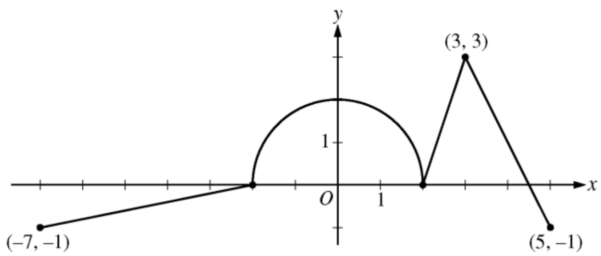
300.



Let *R* be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal line $y = 6$, and the *y*-axis, as shown in the figure above.

a. Find the area of *R*.
b. Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.
c. Region R is the base of a solid. For each *y*, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the *y*-axis is a rectangle whose height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.

301.



Graph of g'

The function *g* is defined and differentiable on the closed interval [-7, 5] and satisfies $g(0) = 5$. The graph of $y = g'(x)$, the derivative of g, consists of a semicircle and three line segments, as shown in the figure above.

a. Find $g(3)$ and $g(-2)$.
b. Find the x-coordinate of each point of inflection of the graph of $y = g(x)$ on the interval $-7 < x < 5$. Explain your reasoning.
c. The function h is defined by $h(x) = g(x) - \frac{1}{2}x^2$. Find the x-coordinate of each critical point of h, where $-7 < x < 5$, and classify each critical point as the location of a relative minimum, relative maximum, or neither a minimum nor a maximum. Explain your reasoning.

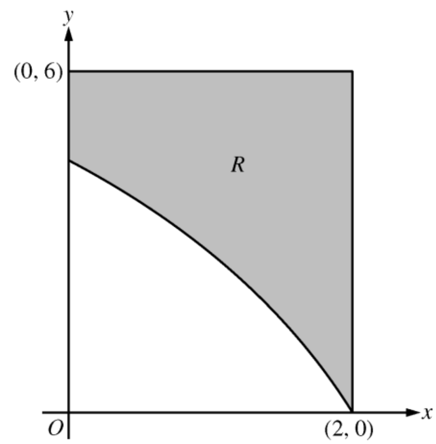
302.

Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3(1 + 3x^2y^2)$.

Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

a. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.
b. Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.
c. Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

303.



In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3-x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

a. Find the area of R.
b. Find the volume of the solid generated when *R* is revolved about the horizontal line $y = 8$..
c. The region *R* is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

304.

The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and

$$g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right).$$

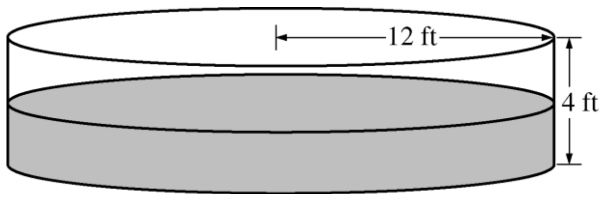
a. Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.
b. On what subintervals of $(0.12, 1)$, if any, is the graph of *g* concave down?

Justify your answer.

c. Write an equation for the line tangent to the graph of g at $x = 0.3$.

d. Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

305.



t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63

The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0.$ During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$.

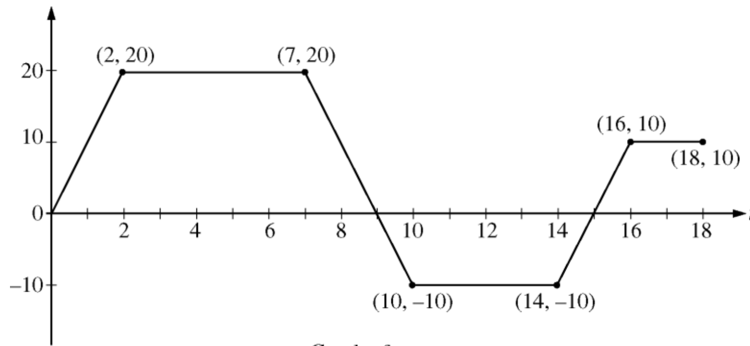
a. Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

b. Calculate the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours.

c. Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

d. Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

306.



Graph of v

A squirrel starts at building *A* at time $t = 0$ and travels along a straight, horizontal wire connected to building *B*. For $0 \le t \le 18,$ the squirrel's velocity is modeled by the piecewise-linear function defined by the graph above.

a. At what times in the interval $0 \le t \le 18,$, if any, does the squirrel change direction? Give a reason for your answer.

b. At what time in the interval $0 \le t \le 18$ is the squirrel farthest from building *A* ? How far from building *A* is the squirrel at that time?

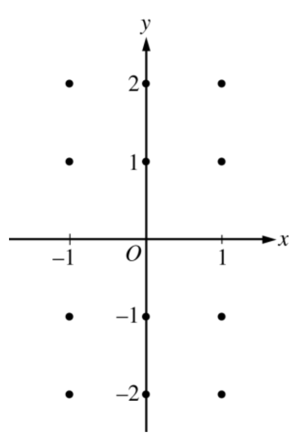
c. Find the total distance the squirrel travels during the time interval $0 \le t \le 18$.

d. Write expressions for the squirrel's acceleration $a(t)$, velocity $v(t)$, and distance $x(t)$ from building *A* that are valid for the time interval $7 < t < 10$.

307/

Consider the differential equation $\dfrac{dy}{dx} = \dfrac{x+1}{y}.$

a. On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0, -1).$



b. While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0.$ Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1.$

c. Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2.$

308.

Two particles move along the x-axis. For $0 \le t \le 6$, the position of particle P at time t is given by $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$, while the position of particle R at time t is given by $r(t) = t^3 - 6t^2 + 9t + 3..$

a. For $0 \le t \le 6$ find all times *t* during which particle R is moving to the right.

b. For $0 \le t \le 6$, find all times *t* during which the two particles travel in opposite directions.

c. Find the acceleration of particle P at time $t = 3$. Is particle P speeding up, slowing down, or doing neither at time $t = 3$? Explain your reasoning.

d. Write, but do not evaluate, an expression for the average distance between the two particles on the interval $1 \le t \le 3.$

309.

For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{\frac{t}{4}}\right) + 1.$

The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{\frac{t}{4}}\cos\left(\frac{t}{4}\right)$ and $x(0) = 2.$

a. Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

b. Find the average velocity of the particle for the time period $0 \le t \le 6$.

c. Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

d. For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

310.

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

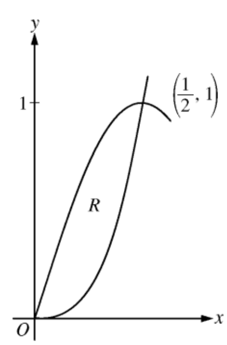
a. Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

b. Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)dt$ in context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)dt$.

c. Evaluate $\int_0^{10} H'(t)dt$. Using correct units, explain the meaning of the expression in the context of this problem.

d. At time $t = 0$, biscuits with temperature $100^\circ C$ were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

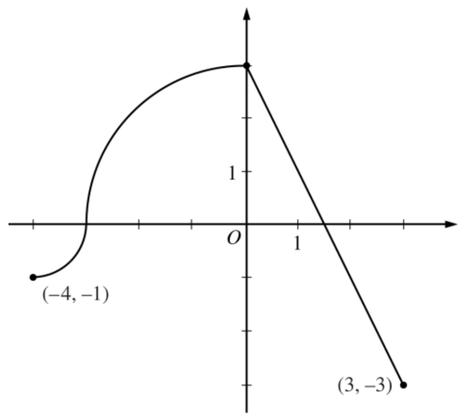
311.



Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.

a. Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.
b. Find the area of R.
c. Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

312.



Graph of f

The continuous function *f* is defined on the interval $-4 \le x \le 3.$. The graph of *f* consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)dt$.

a. Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.
b. Determine the x-coordinate of the point at which *g* has an absolute maximum on the interval $-4 \le x \le 3.$ Justify your answer.
c. Find all values of x on the interval $-4 < x < 3.$ for which the graph of g has a point of inflection. Give a reason for your answer.
d. Find the average rate of change of *f* on the interval $-4 \le x \le 3.$ There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

313.

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

a. Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

b. Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

c. Find the particular solution $W = W(t)$ to the differential equation e with initial condition $W(0) = 1400$.

314.

Let f be a function defined by $f(x) = \begin{cases} 1 - 2\sin x & \text{for } x \le 0 \\ e^{-4x} & \text{for } x > 0 \end{cases}$.

a. Show that f is continuous at $x = 0$.

b. For $x \ne 0$, express $f'(x)$ as a piecewise-defined function. Find the value of x for which $f'(x) = -3$.

c. Find the average value of f on the interval $[-1, 1]$.

315.

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and *t* is measured in days for $0 \le t \le 60$. The rate at

which the height of the water is rising in the can is given by
$S'(t) = 2\sin(0.03t) + 1.5.$

a. According to the model, what is the height of the water in the can at the end of the 60-day period?

b. According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

c. Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

d. During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M , where $M(t) = \frac{1}{400}(3t^3 - 30t^2 + 330t).$ The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t).$ Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time *t*, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

316.

A 12,000-liter tank of water is filled to capacity. At time $t = 0$, water begins to drain out of the tank at a rate modeled by $r(t)$, measured in liters per hour, where r is given by the piecewise-defined function

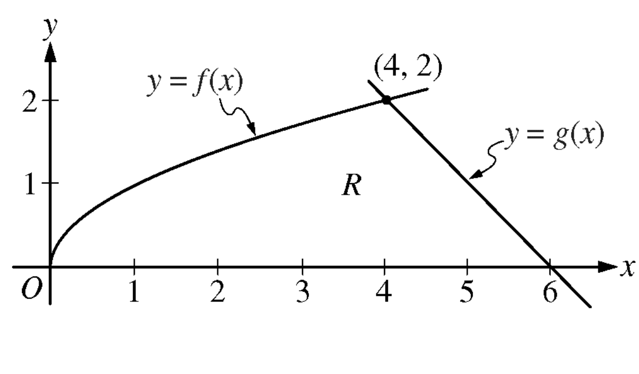
a. Is r continuous at $t = 5$? Show the work that leads to your answer.

b. Find the average rate at which water is draining from the tank between time $t = 0$ and time $t = 8$ hours.

c. Find $r'(3)$. Using correct units, explain the meaning of that value in the context of this problem.

d. Write, but do not solve, an equation involving an integral to find the time *A* when the amount of water in the tank is 9000 liters.

317.



The functions f and g are given by $f(x)=\sqrt{x}$ and $g(x)=6-x.$ Let R be the region bound by the x-axis and the graphs of f and g, as shown in the figure above.

a. Find the area of R.
b. The region R is the base of a solid. For each y, where $0 \le y \le 2$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose base lies in R and whose height is $2y$. Write, but do not evaluate, an integral expression that gives the volume of the solid.
c. There is a point P on the graph of f at which the line tangent to the graph of f is perpendicular to the graph of g. Find the coordinates of point P.

318.

Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x)=(4-x)x^{-3}$ for $x>0.$

a. Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.
b. Find all intervals on which the graph of f is concave down. Justify your answer.
c. Given that $f(1)=2$, determine the function f.

319.

t (seconds)	0	10	40	60
$B(t)$ (meters)	100	136	9	49
$v(t)$ (meters per second)	2.0	2.3	2.5	4.6

Ben rides a unicycle back and forth along a straight east-west track. The twice-differentiable function B models Ben's position on the track, measured in meters from the western end of the track, at time t, measured in seconds from the start of the ride. The table above gives values for $B(t)$ and Ben's velocity, $v(t)$, measured in meters per second, at selected times $t.$

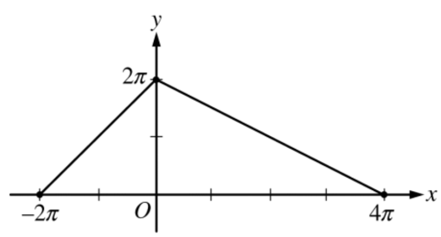
a. Use the data in the table to approximate Ben's acceleration at time $t=5$ seconds. Indicate units of measure.

b. Using correct units, interpret the meaning of $\int_0^{60} |v(t)|\, dt$ in the context of this problem. Approximate $\int_0^{60} |v(t)|\, dt$ using a left Riemann sum with the subintervals indicated by the data in the table.

c. For $40 \le t \le 60$, must there be a time *t* when Ben's velocity is 2 meters per second? Justify your answer.

d. A light is directly above the western end of the track. Ben rides so that at time t, the distance $L(t)$ between Ben and the light satisfies $(L(t))^2 = 12^2 + (B(t))^2$. At what rate is the distance between Ben and the light changing at time $t = 40$?

320.



Graph of g

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

a. Find $\int_{-2\pi}^{4\pi} f(x)dx.$ Show the computations that lead to your answer.

b. Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

d. Let $h(x) = \int_0^{3x} g(t)dt.$ Find $h'\left(-\frac{\pi}{3}\right)$.

321.

t (minutes)	0	4	9	15	20
W(t) (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where W(t) is measured in degrees Fahrenheit and t is measured in minutes. At time $t=0$, the temperature of the water is 55° F. The water is heated for 30 minutes, beginning at time $t=0$. Values of W(t) at selected times t for the first 20 minutes are given in the table above.

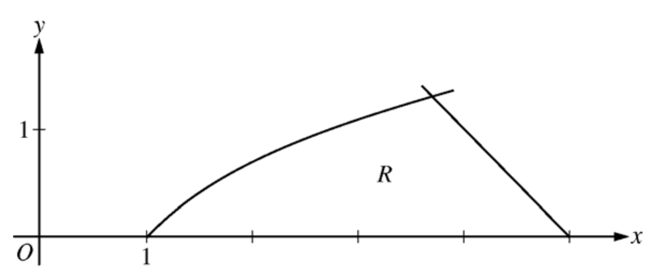
a. Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

b. Use the data in the table to evaluate $\int_0^{20} W'(t)dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)dt$ in the context of this problem.

c. For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W'(t)dt.$ Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W'(t)dt.$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

d. For $20 \le t \le 25,$ the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

322.



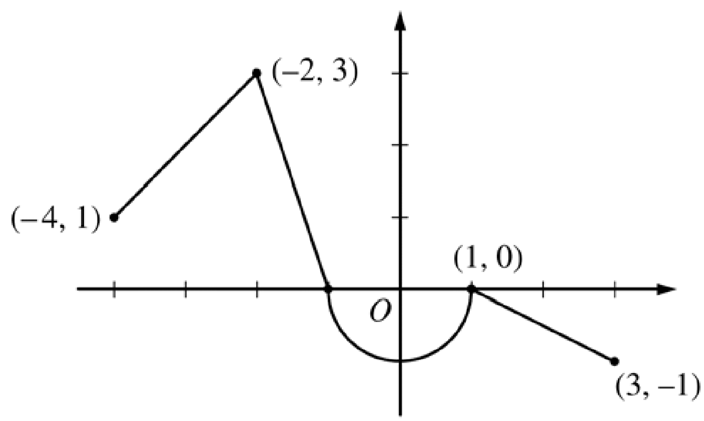
Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln(x)$ and $y = 5 - x$, as shown in the figure above.

a. Find the area of R.

b. Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

c. The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

323.



Graph of f

Let f be the continuous function defined on [-4, 3] whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)dt.$

a. Find the values of $g(2)$ and $g(-2)$.

b. For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

c. Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

d. For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

324.

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

a. Find $f'(x)$.

b. Write an equation for the line tangent to the graph of f at $x = -3$.

c. Let g be the function defined by $g(x)=\begin{cases} f(x) \text{ for } -5 \le x \le -3 \\ x+7 \text{ for } -3 < x \le 5. \end{cases}$

Is g continuous at $x=-3$? Use the definition of continuity to explain your answer.

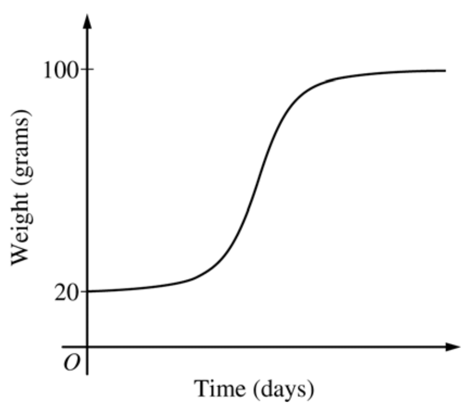
d. Find the value of $\int_0^5 x\sqrt{25-x^2}\,dx.$

325.

The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t=0$, when the bird is first weighed, its weight is 20 grams. If B(t) is the weight of the bird, in grams, at time t days after it is first weighed, then $\frac{dB}{dt}=\frac{1}{5}(100-B).$ Let $y=B(t)$ be the solution to the differential equation above with initial condition $B(0)=20.$

a. Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

b. Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



c. Use separation of variables to find $y=B(t)$, the particular solution to the differential equation with initial condition $B(0)=20.$

326.

For $0 \le t \le 12,$ a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t)=\cos\left(\frac{\pi}{6}t\right).$ The particle is at position $x=-2$ at time $t=0.$

a. For $0 \le t \le 12,$ when is the particle moving to the left?

b. Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from $t = 0$ to time $t = 6$.
c. Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.
d. Find the position of the particle at time $t = 4$.

327.
On a certain workday, the rate, in tons per hours, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

a. Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.
b. Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.
c. Is the amount of unprocessed gravel at the plant increasing or decreasing at $t = 5$ hours? Show work that leads to your answer.
d. What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

328.
A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{\frac{6}{5}} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

a. Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.
b. Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.
c. Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.
d. Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

329.

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t, $0 \le t \le 6$, is given by a differentiable function C, where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

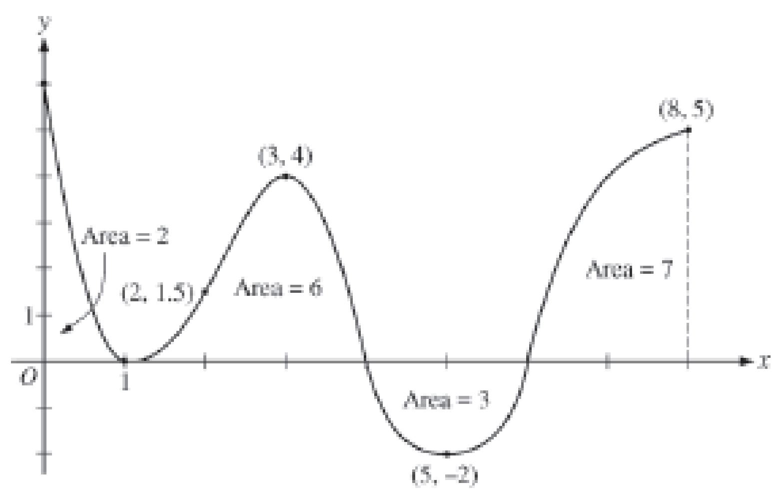
a. Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.

b. Is there a time t, $2 \le t \le 4$, at which $C'(t) = 2$? Justify your answer.

c. Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6}\int_0^6 C(t)dt$ in the context of the problem.

d. The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

330.



Graph of f'

The figure above shows the graph of f', the derivate of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4.$

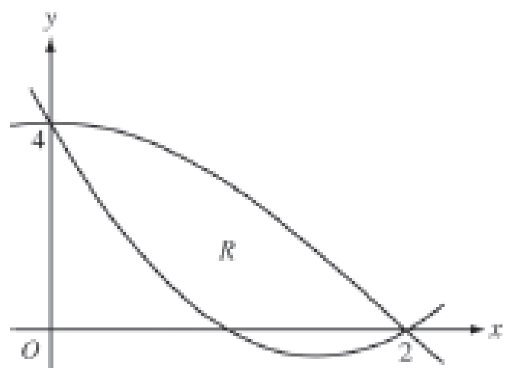
a. Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

b. Determine the absolute minimum value of f on the closed interval $0 \le x \le 8.$ Justify your answer.

c. On what open interval contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

d. The function g is defined by $g(x) = \left(f(x)\right)^3.$ If $f(3) = -\dfrac{5}{2},$ find the slope of the line tangent to the graph of g at $x = 3.$

331.



Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right).$ Let R be the region bounded by the graphs of f and $g,$ as shown in the figure above.

a. Find the area of $R.$

b. Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4.$

c. The region R is the base of a solid. For this solid, each cross sectin perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

332.

Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

a. Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

b. Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.