King Abdullah University of Science and Technology KAUST

# Freeform Architecture and Discrete Differential Geometry

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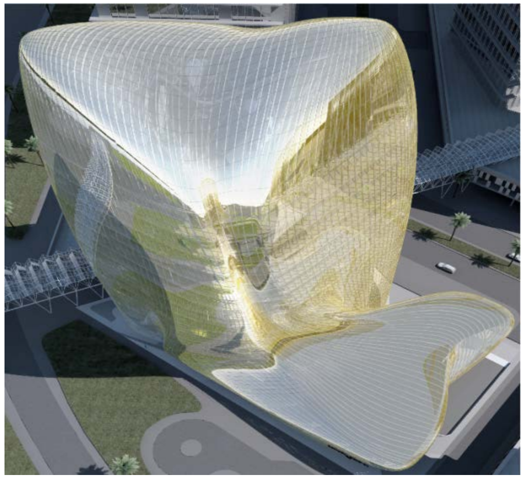
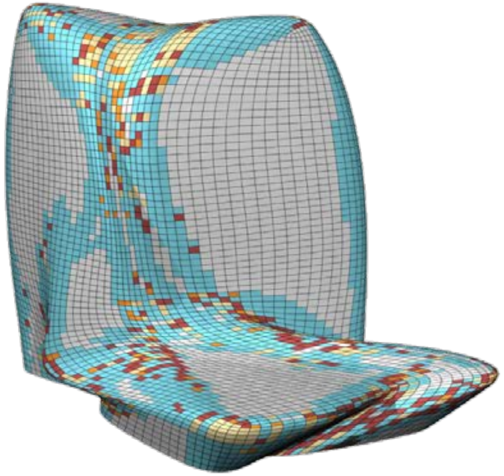
# Freeform Architecture

Motivation:

- Large scale architectural projects, involving complex freeform geometry
- Realization challenging and costly; available digital design technology is not adapted to the demands in this area.





# Research Goals

- Make geometrically complex architectural structures affordable through *novel computational tools by linking design, function and fabrication*
- Provide new methodology for computational design, especially through links to discrete & computational differential geometry and optimization
- Develop new tools to explore the variety of feasible / optimized designs through links to the geometry of shape spaces
- Advance the theory (discrete differential geometry, shape spaces, …) through novel concepts motivated by applications
- Contribute to and learn from real-world projects (Evolute GmbH)

# Overview

- Planar quad meshes
- Planar quads and beam layouts in real projects
- Single curved panels
- Paneling
- Design of self-supporting surfaces
- Future research

# Planar Quad Meshes

# quad meshes in architecture

- work by Schlaich & Schober

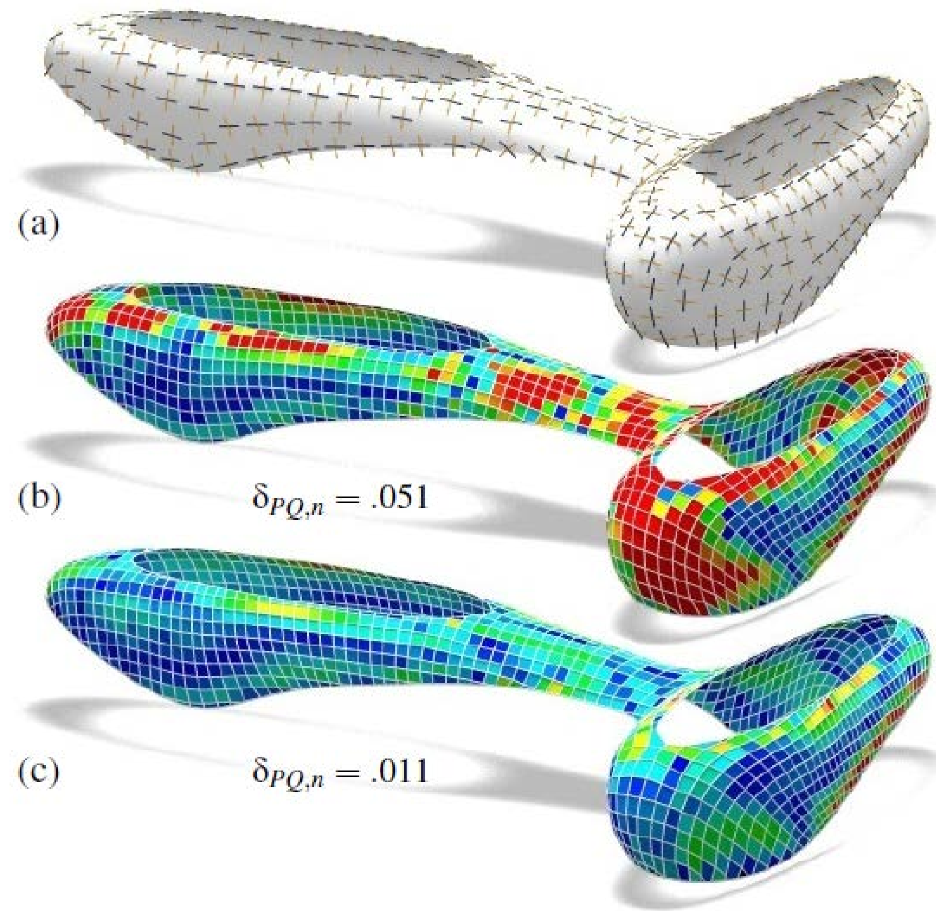

- quad meshes with planar faces (PQ meshes)
- only special shapes; what about freeform shapes?

# key insight on PQ meshes

- PQ meshes reflect curvature behavior
- relation to discrete differential geometry: PQ meshes are discrete versions of *conjugate curve networks*

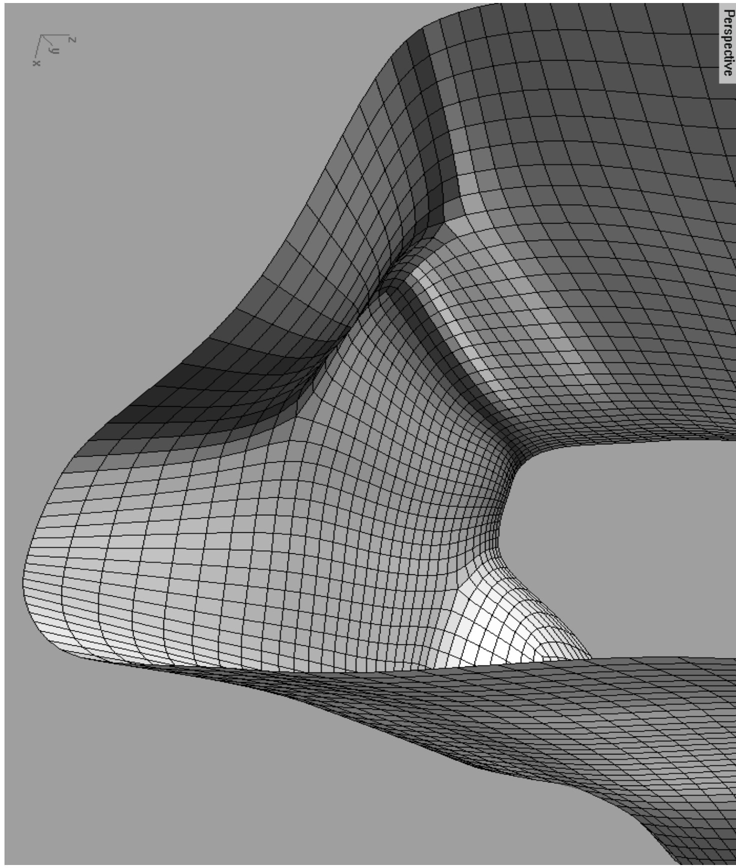
$$\mathbf{x}(u,v) \ \text{ with } \ \det(\mathbf{x}_u, \mathbf{x}_v, \mathbf{x}_{uv}) = 0$$

- any optimization has to be initialized respecting this fact



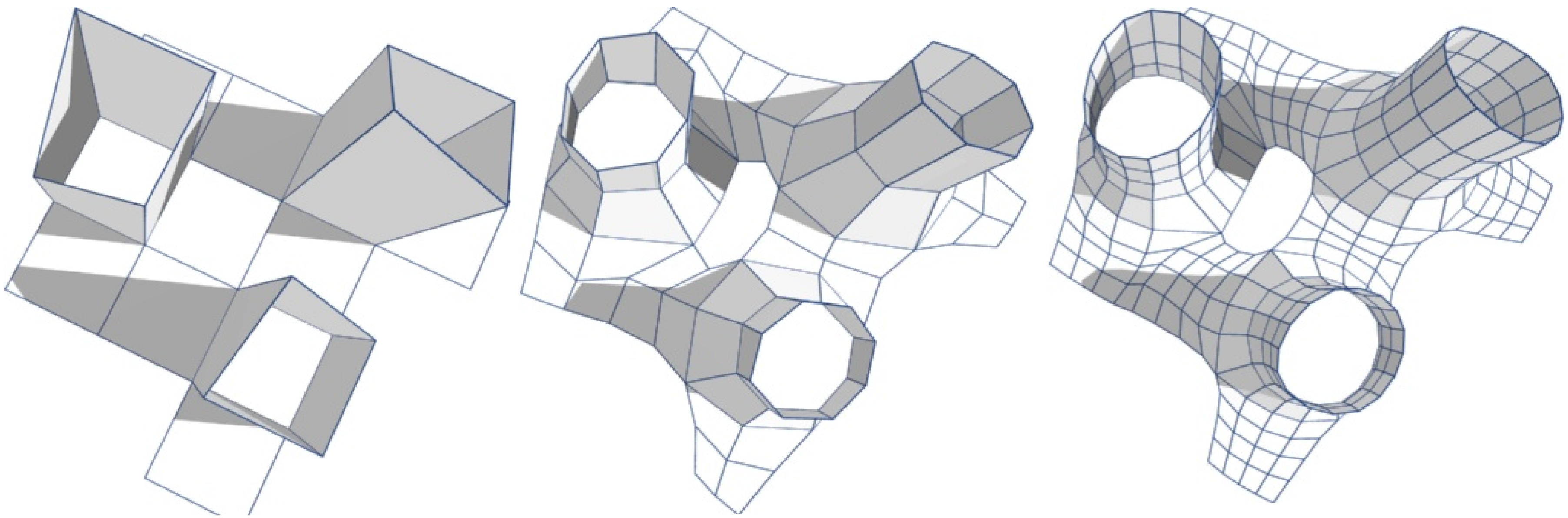
# Computing PQ meshes

- Computation of a PQ mesh is based on numerical optimization:
- Optimization criteria
  - planarity of faces
  - aesthetics (fairness of mesh polygons)
  - proximity to a given reference surface
- Requires *initial mesh*, found via a careful evaluation of the curvature behavior!

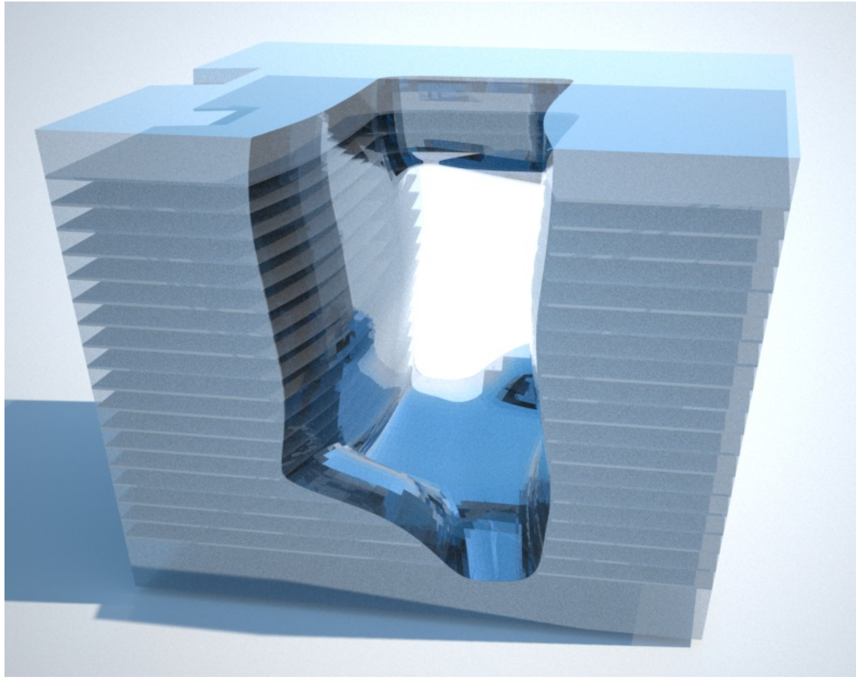


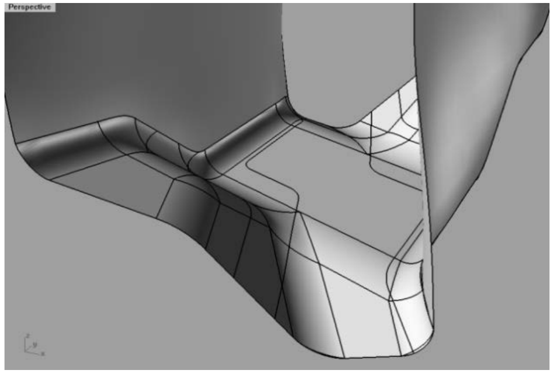
# subdivision & optimization

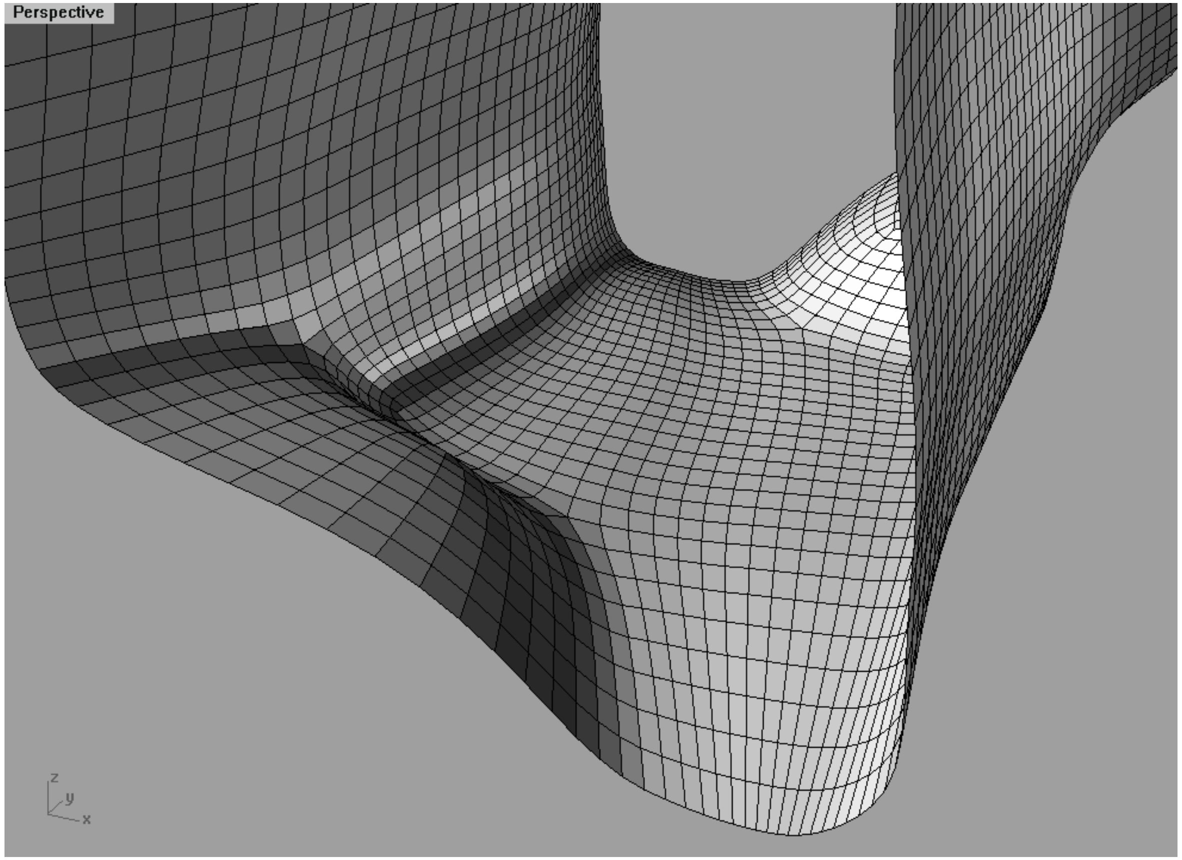
- refine a coarse PQ mesh by repeated application of subdivision and PQ optimization
- can be combined with surface fitting



# Opus (Zaha Hadid Architects)







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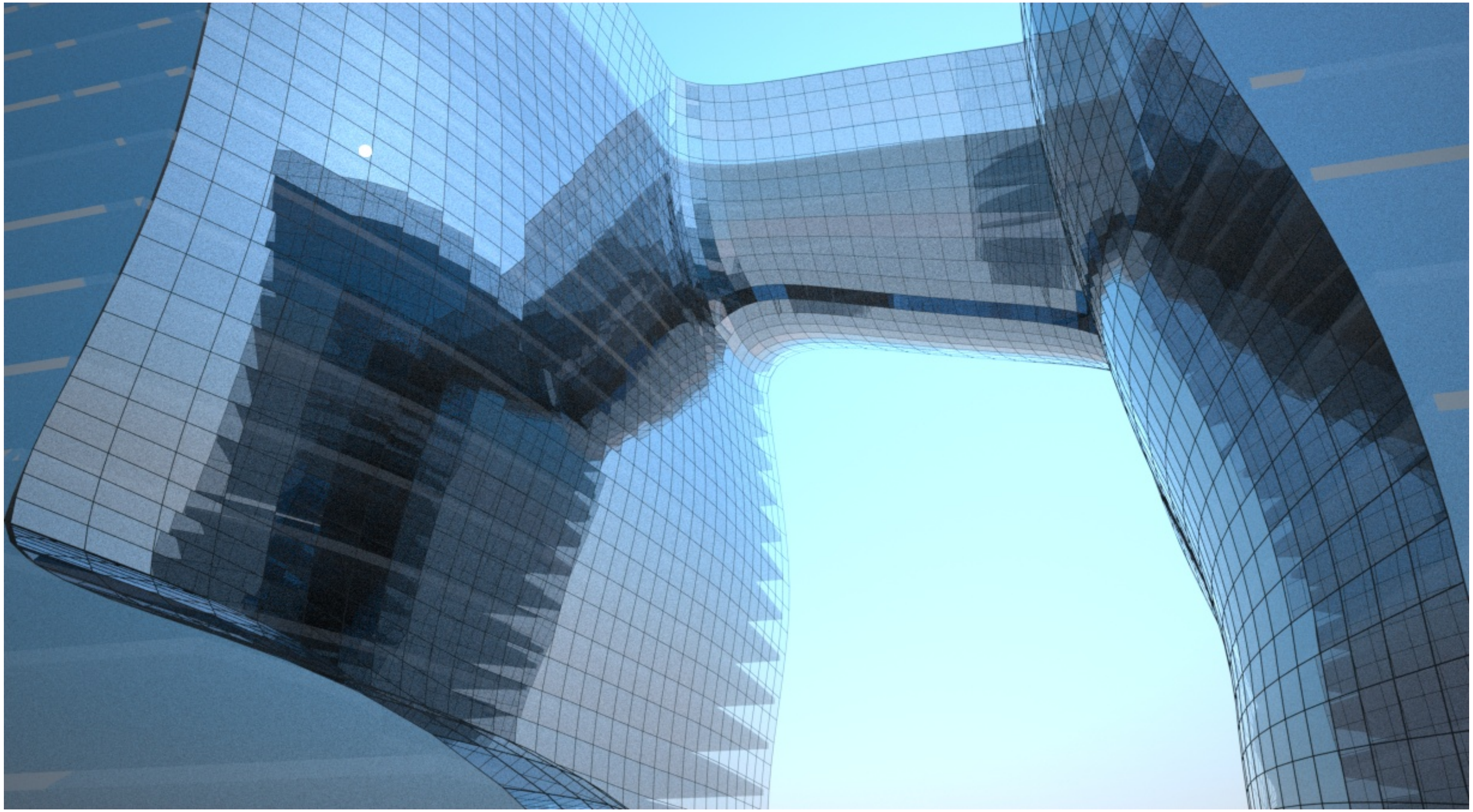
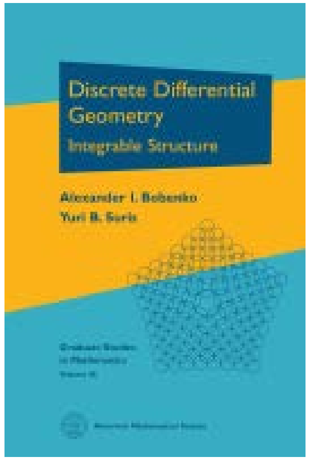


# Discrete Differential Geometry

- Develops discrete equivalents of notions and methods of classical differential geometry
- The latter appears as limit of the refinement of the discretization
- Basic structures of DDG related to the theory of integrable systems
- A. Bobenko, Y. Suris: *Discrete Differential Geometry: Integrable Structure*, AMS, 2008



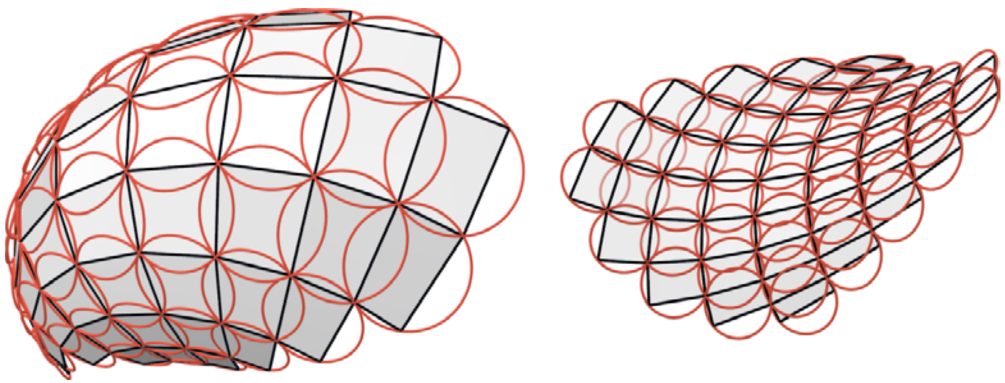

- Discretize the theory, not the equations!
- Several discretizations; which one is the best?

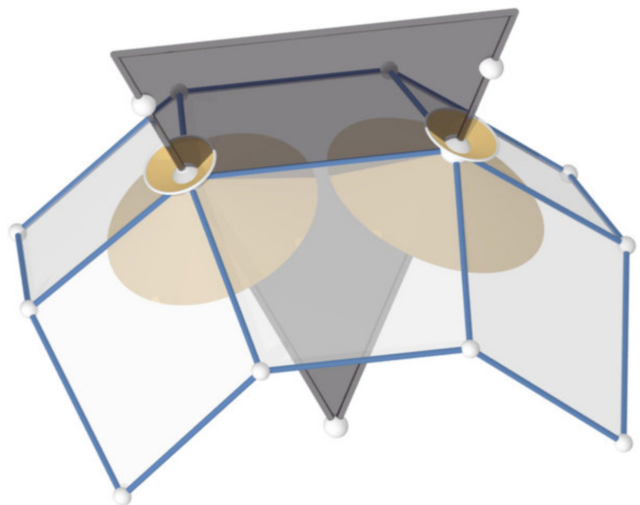
# Conical meshes

- panels as rectangular as possible
- a discrete counterpart of network of principal curvature lines
- *circular mesh*
- for architecture, even better:

  **conical mesh**

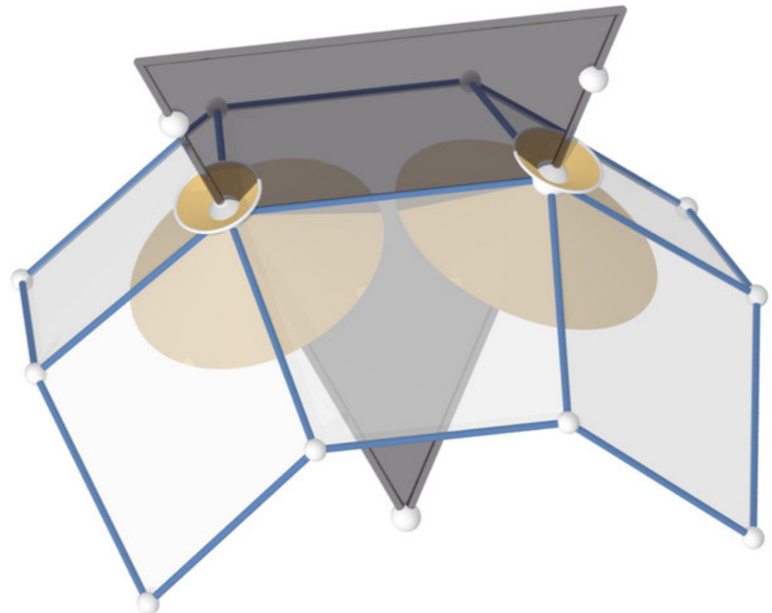
- PQ mesh is conical if all vertices of valence 4 are conical: incident oriented face planes are tangent to a right circular cone

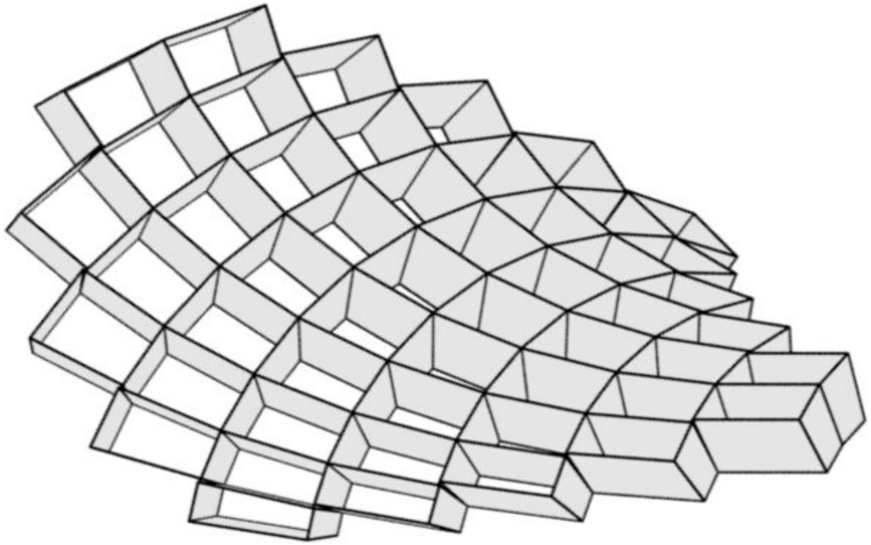


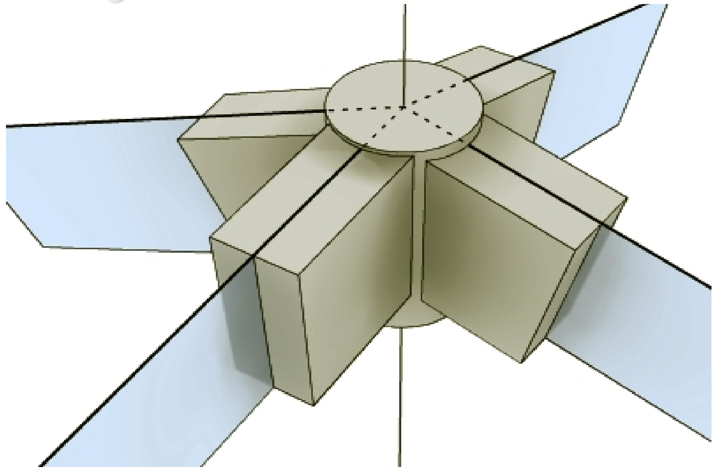


# normals of a conical mesh

- neighboring cone axes (discrete normals) are coplanar
- conical mesh has *precise offsets* and a *torsion-free support structure*

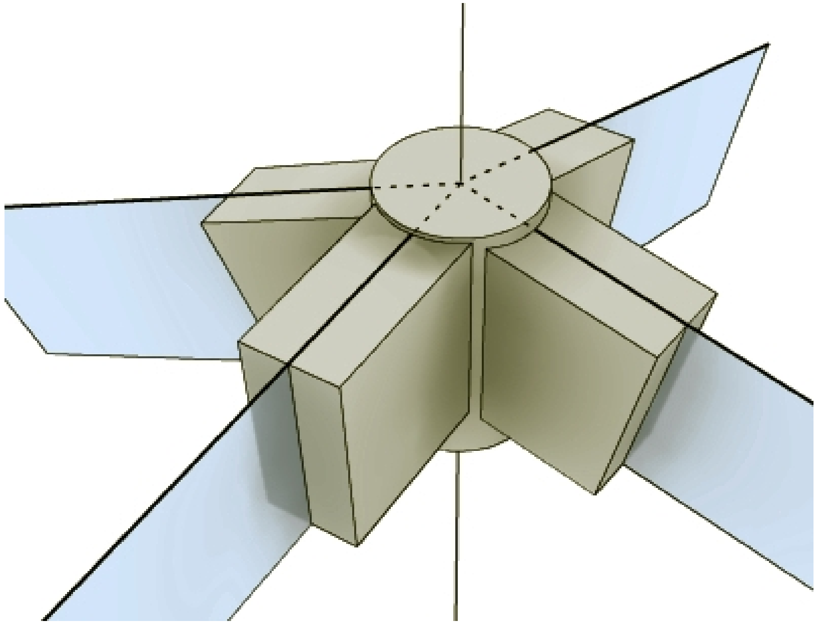






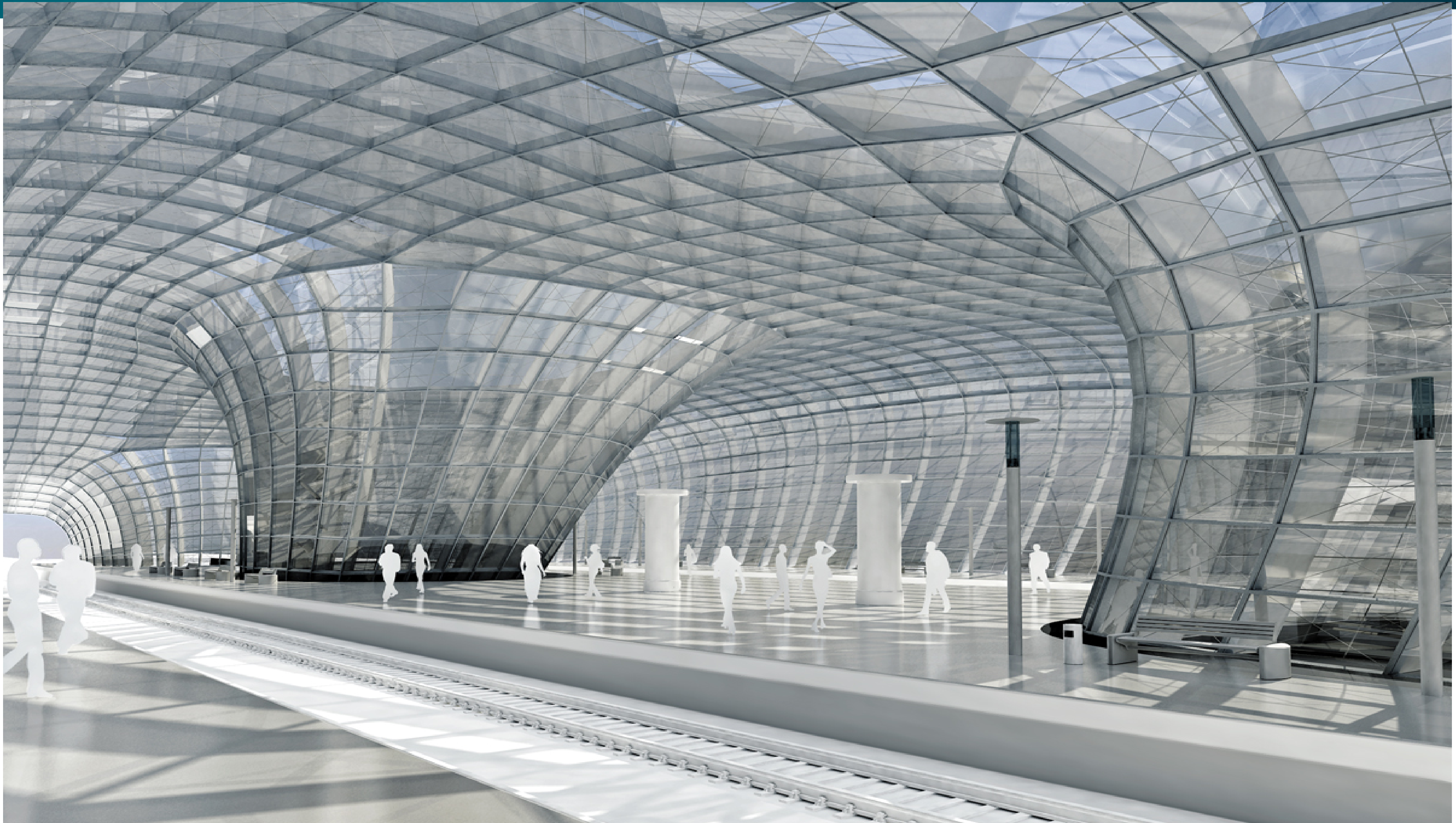
# nodes in the support structure

- triangle mesh: generically nodes of valence 6; `torsion´: central planes of beams not co-axial



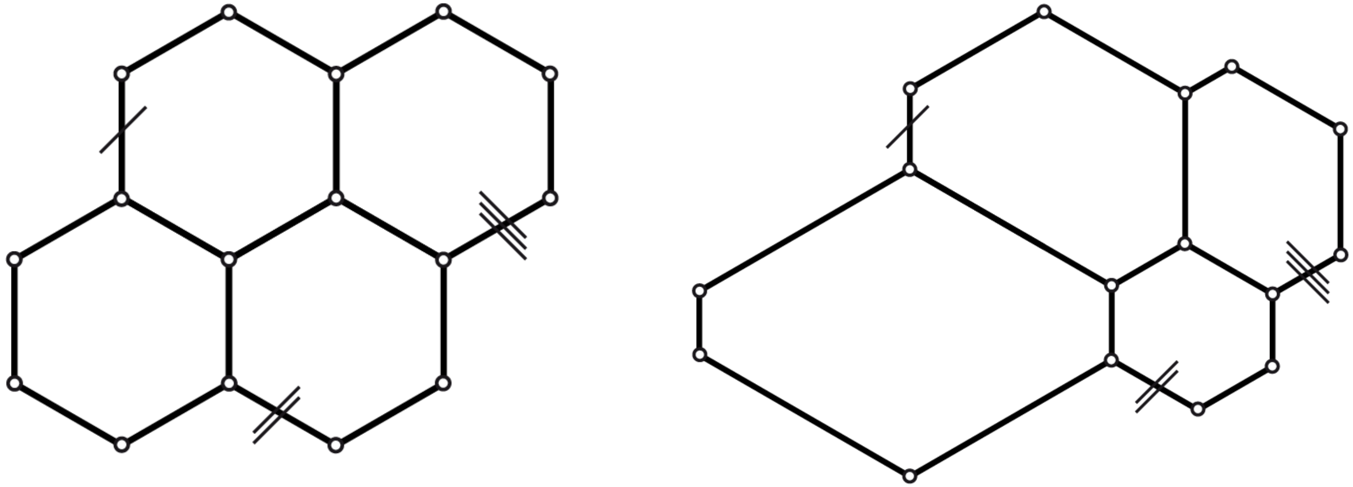
torsion-free node

# Conical mesh



# Discrete curvature theory

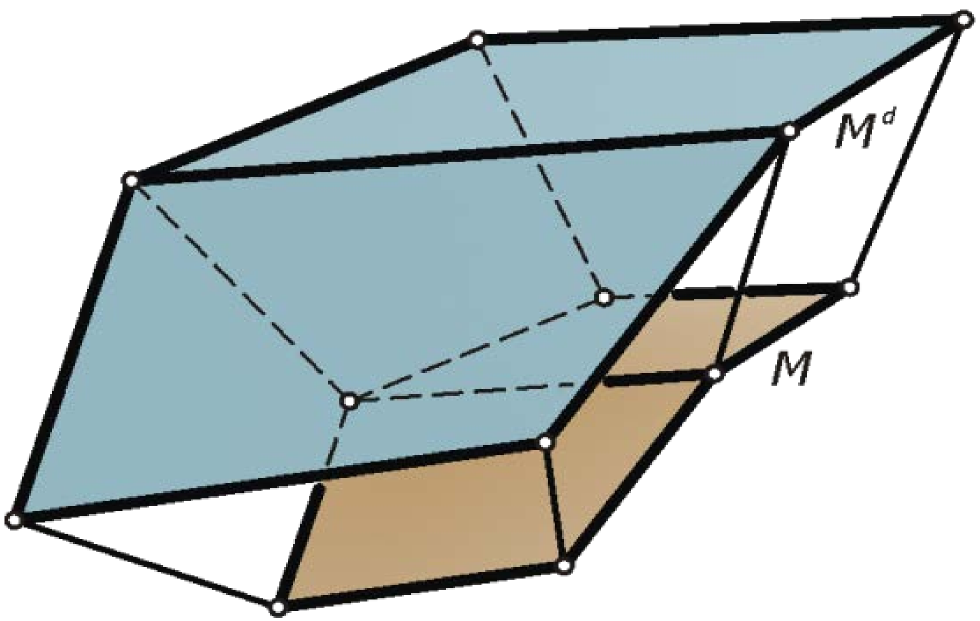
- The study of meshes with offsets led to a new curvature theory for discrete surfaces based on parallel meshes (Bobenko, P., Wallner, Math. Annalen, 2010)

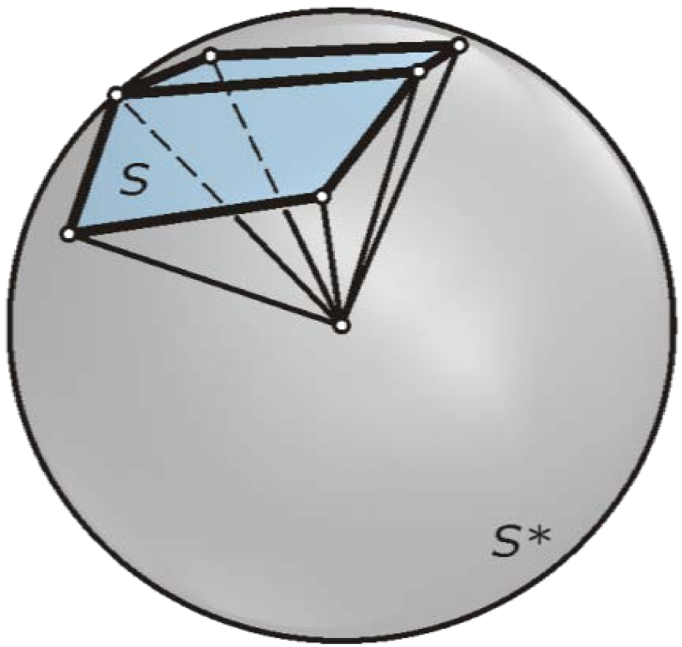


  - meshes *M*, *M** with planar faces are parallel if they are *combinatorially equivalent* and *corresponding edges are parallel*

# Discrete curvature theory

- Gaussian image mesh *S* of *M* is parallel to *M* and approximates the unit sphere
- Offset mesh at distance *d*: *M*+*d S*

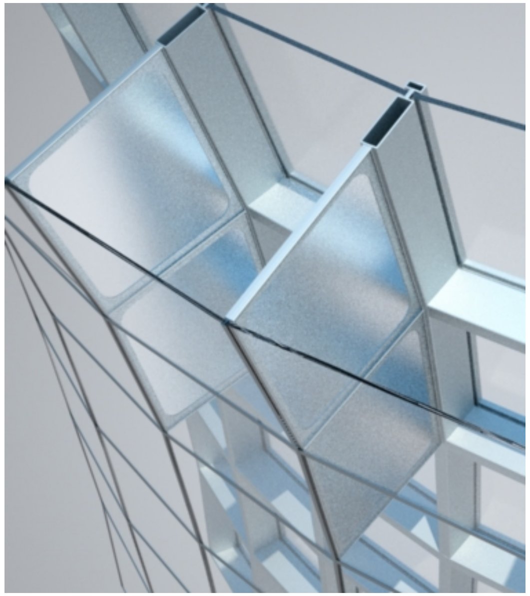


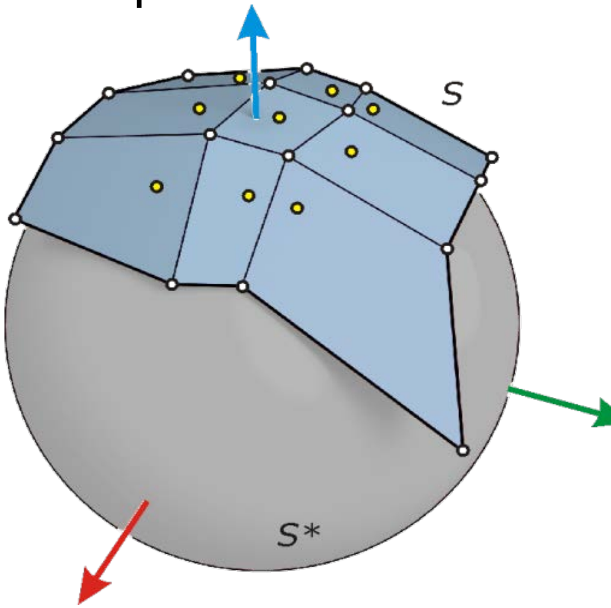



# Discrete curvature theory

- Examples:
- conical mesh: faces of the Gaussian image are tangent to the unit sphere

  offsets at constant face-face distance






- circular mesh: vertices of Gaussian image lie on unit sphere; corresponding vertices of base mesh and offset at constant distance

# Discrete curvature theory

- surface area of the offset $M^d = M + dS$ of the mesh *M* relative to the Gauss image $S = \sigma(M)$

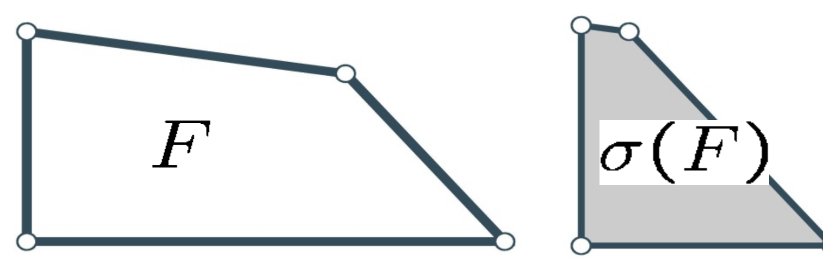
$$\text{area}(M^d) = \sum_{F:\text{face of } M} (1 - 2dH_F + d^2 K_F)\,\text{area}(F)$$

- analogous to *Steiner`s formula*
- define *curvatures in face F*

$$K_F = \frac{\text{area}(\sigma(F))}{\text{area}(F)}$$

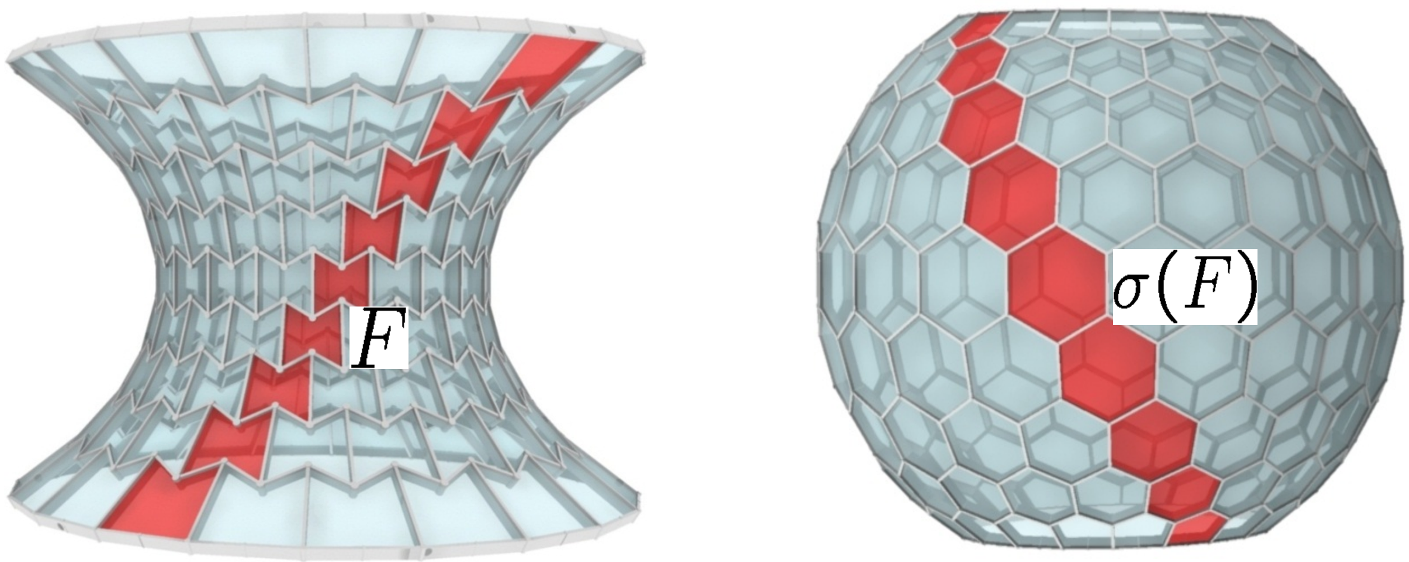
$$H_F = -\frac{\text{area}(F,\sigma(F))}{\text{area}(F)}$$

*mixed area*



# Discrete curvature theory

- Discrete minimal surface: $H_F = 0 \iff \mathrm{area}(F, \sigma(F)) = 0$




- valid for polyhedral surfaces (different from triangle meshes)
- extends to relative differential geometry, where Euclidean sphere is replaced by another convex surface

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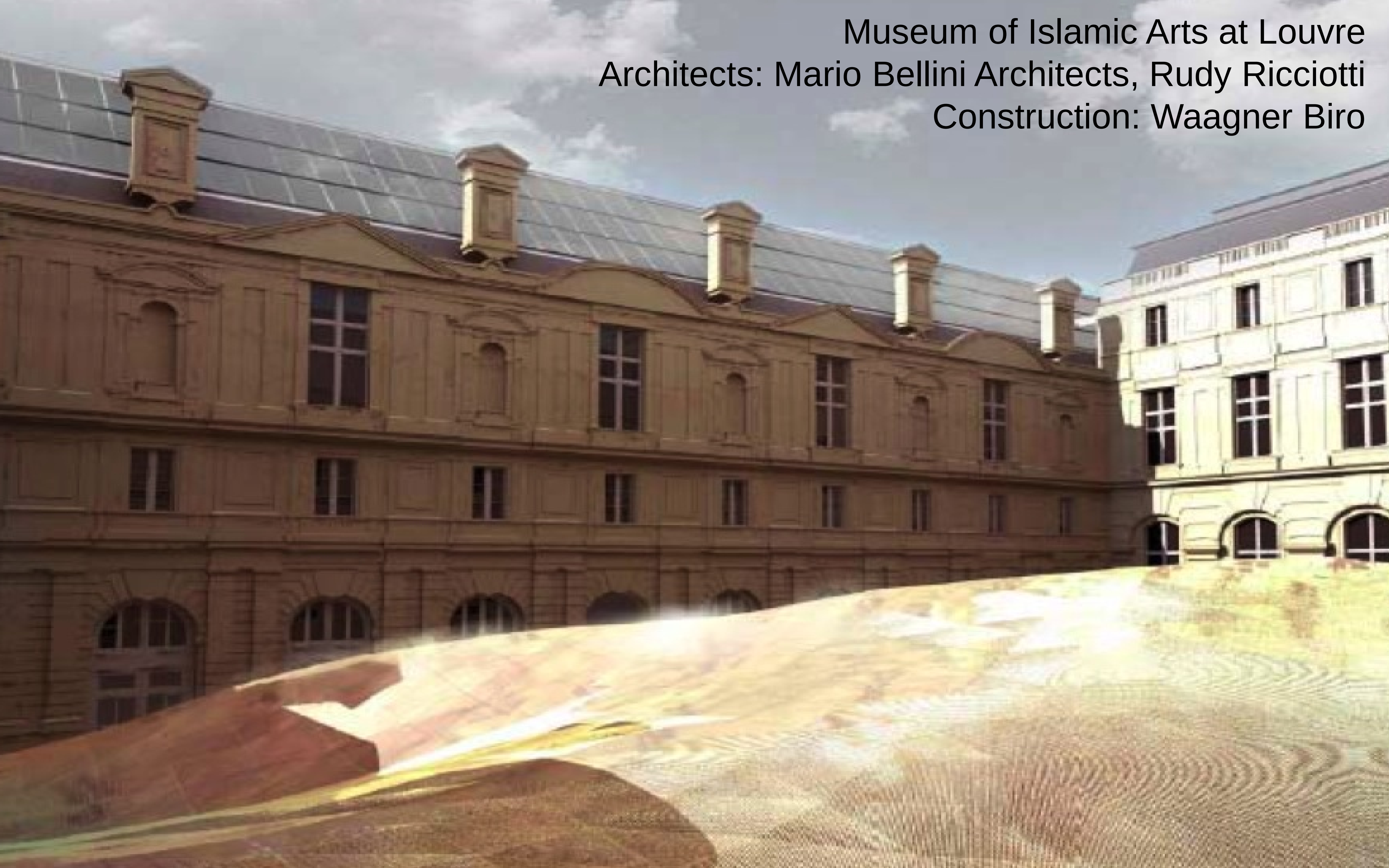
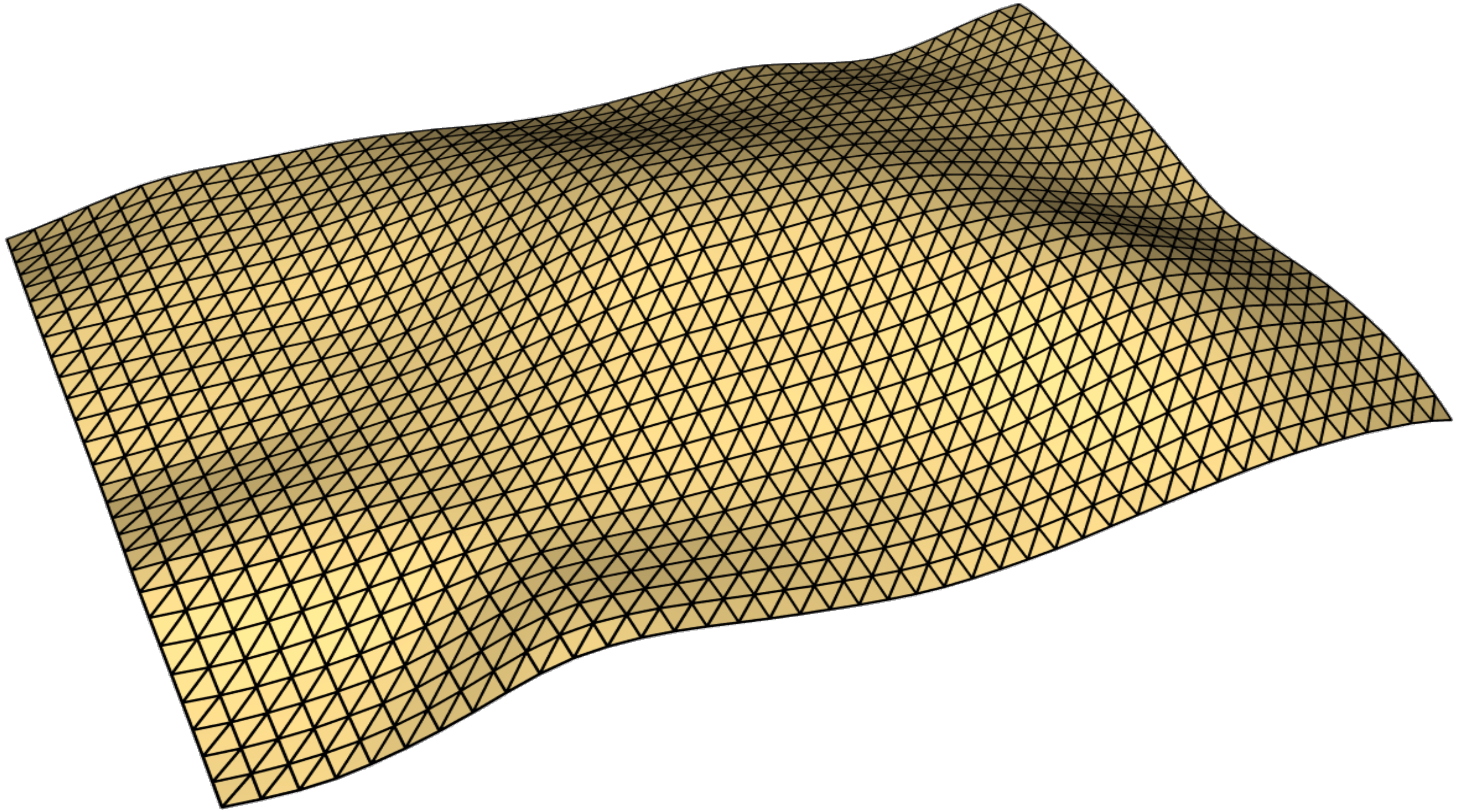
# Planar quads and beam layouts in real projects

Museum of Islamic Arts at Louvre
Architects: Mario Bellini Architects, Rudy Ricciotti
Construction: Waagner Biro

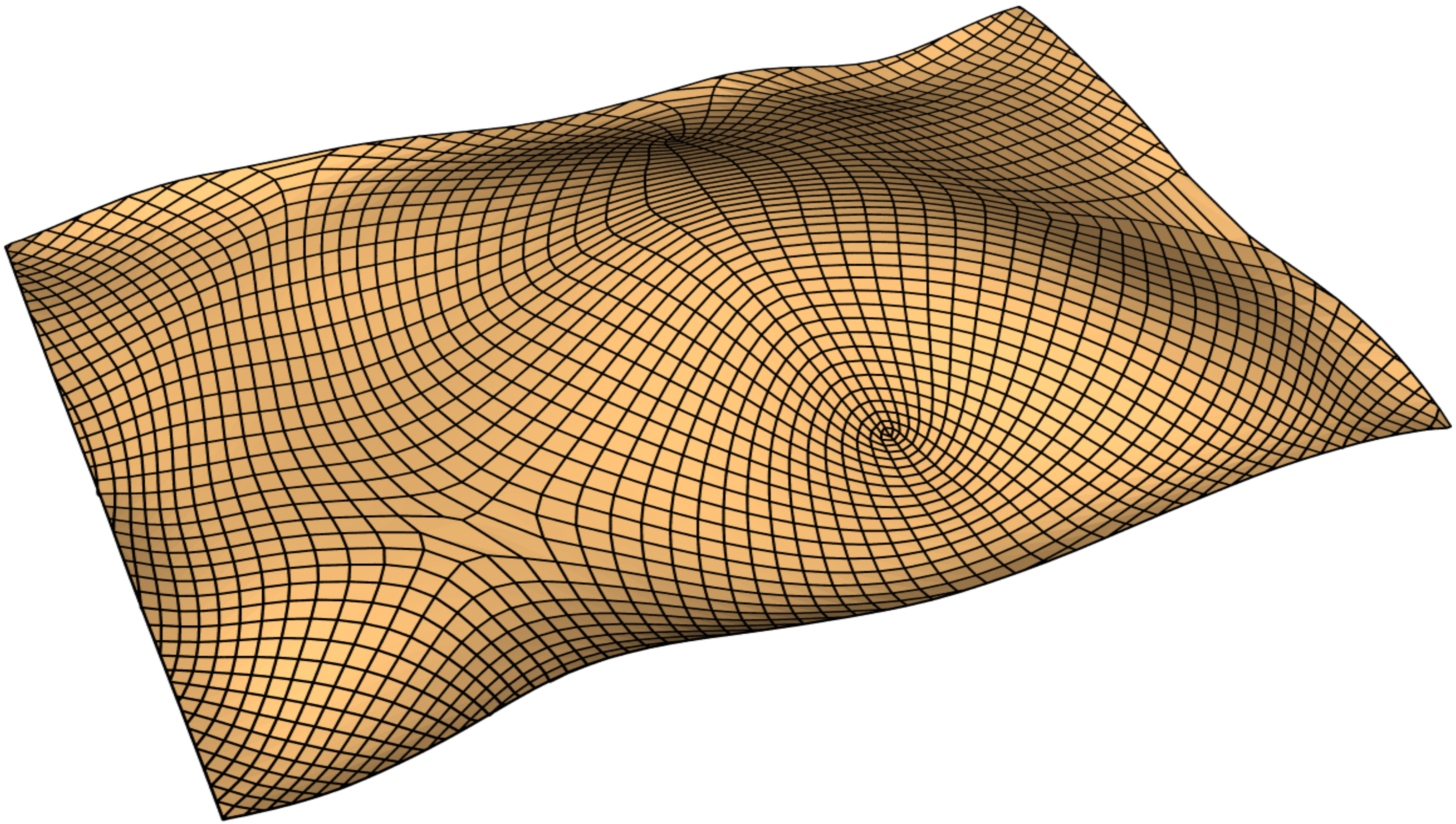


# Museum of Islamic Arts

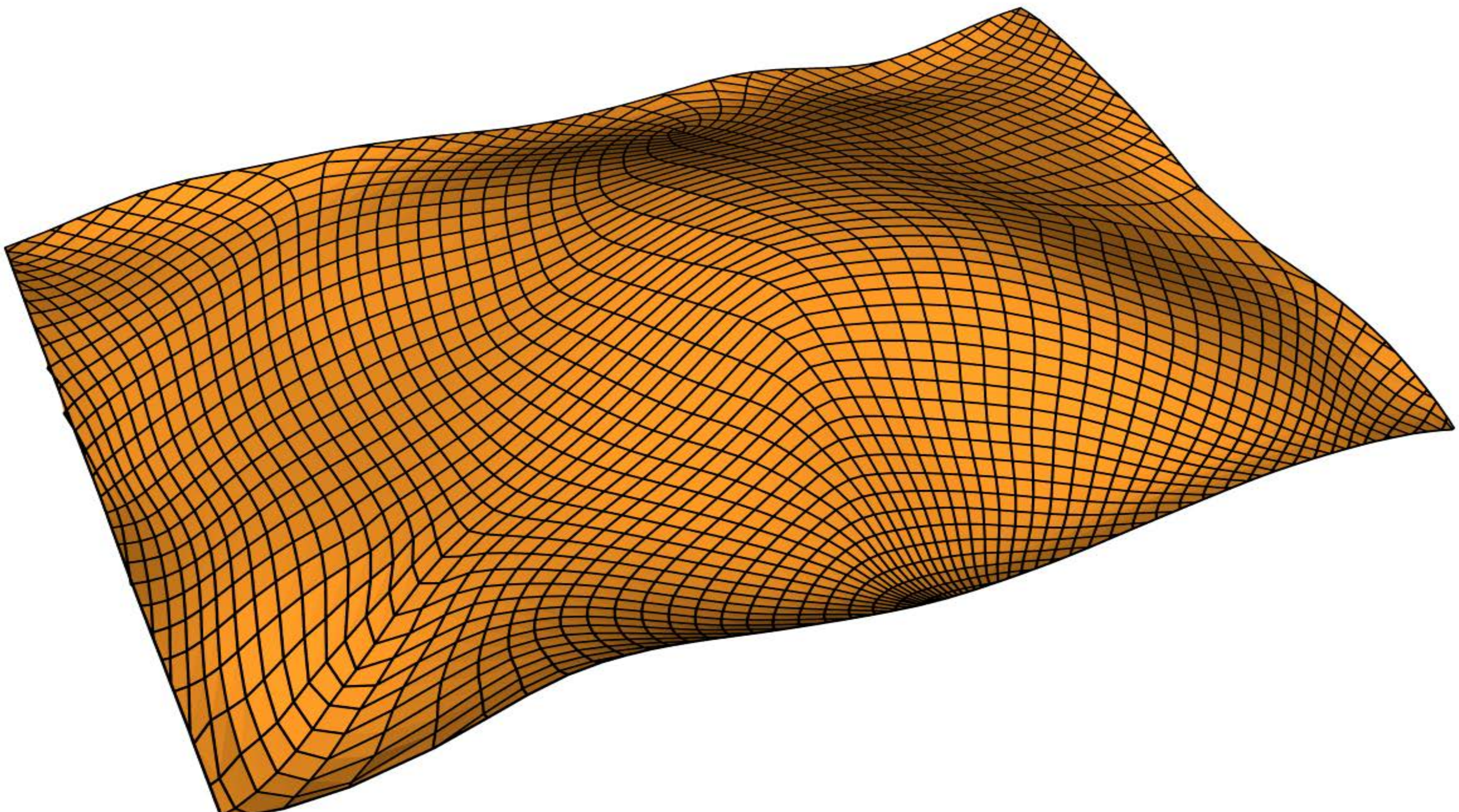
# triangle mesh



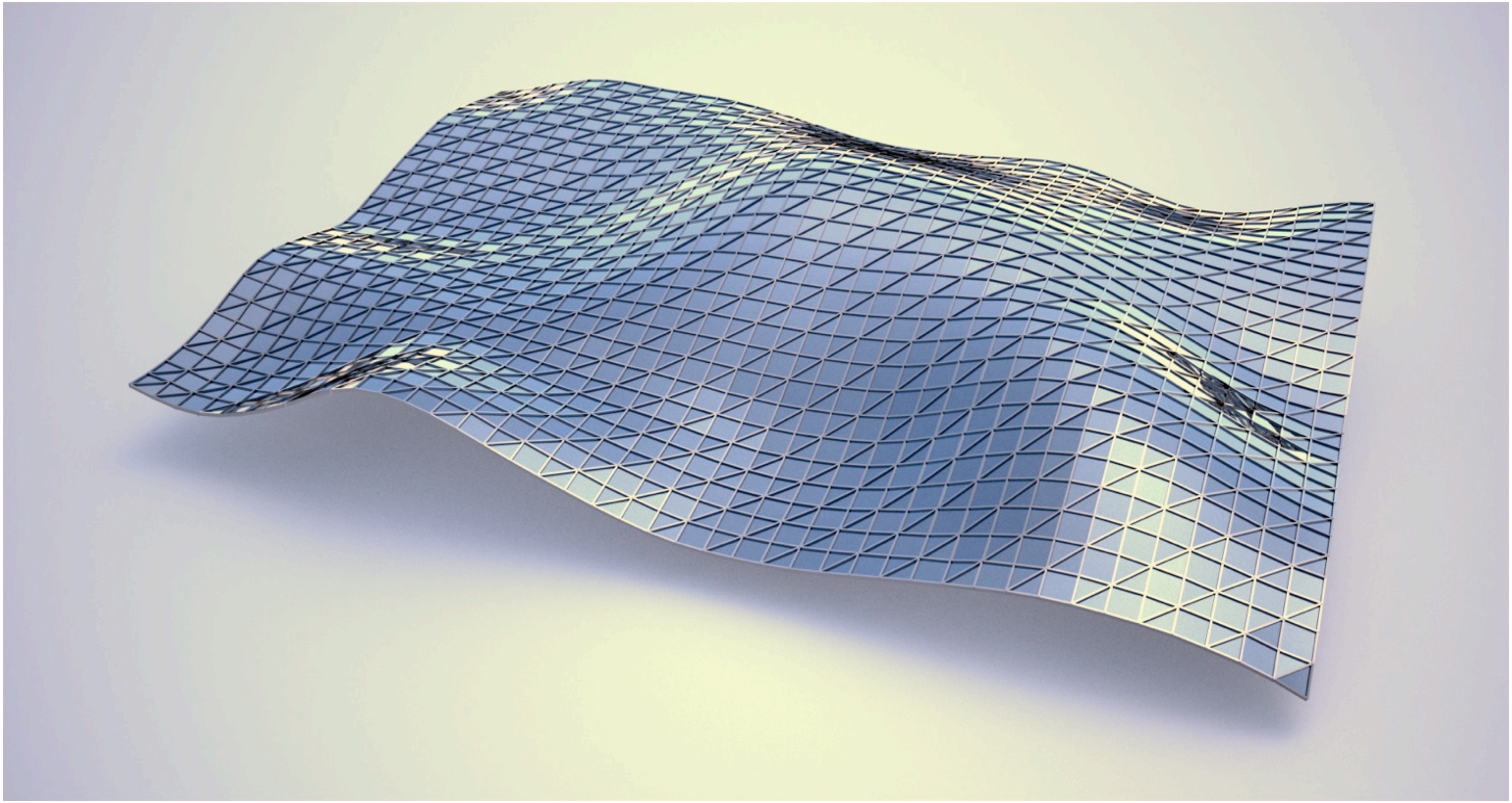
# planar quad mesh for Louvre



# another planar quad mesh



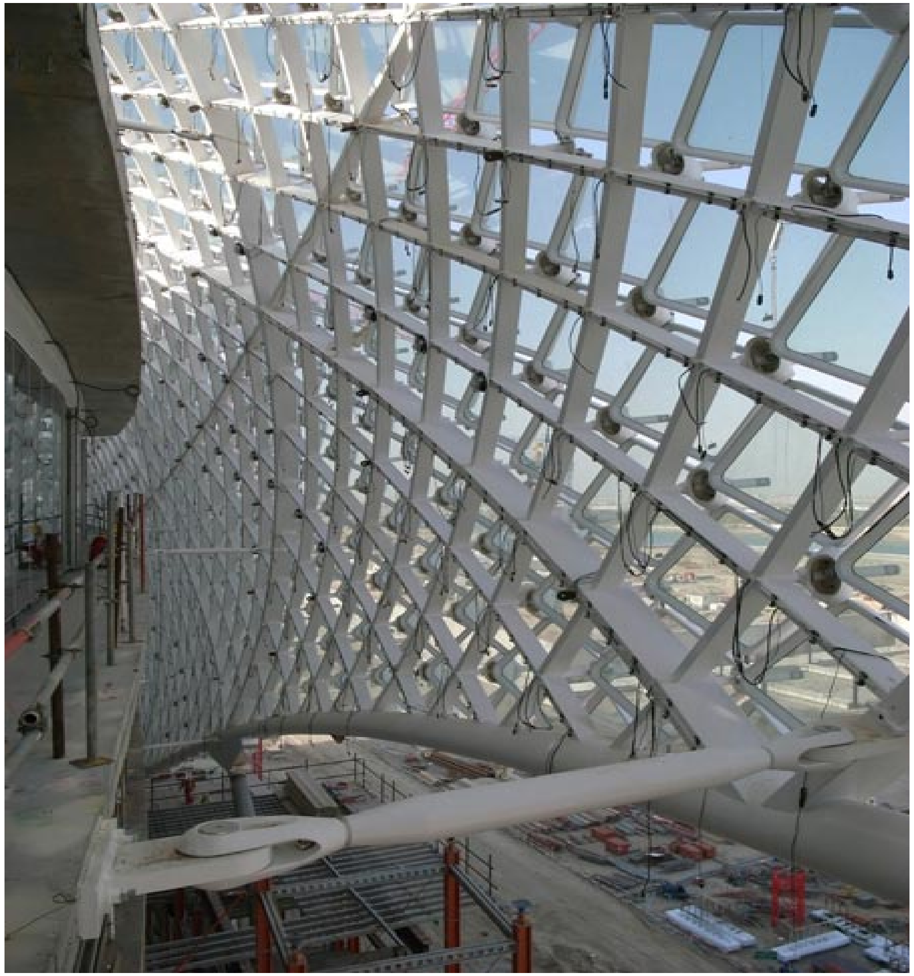
# Solution: hybrid mesh from planar quads and triangles



Yas Island Marina Hotel
Abu Dhabi
Architect: Asymptote Architecture
Steel/glass construction: Waagner Biro

# steel beam layout

- Faces non-planar: there is no elegant exact solution
- node axes should be nearly normal to surface
- *node axes as solution of an optimization problem*



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# Single Curved Panels

# developable surfaces in architecture

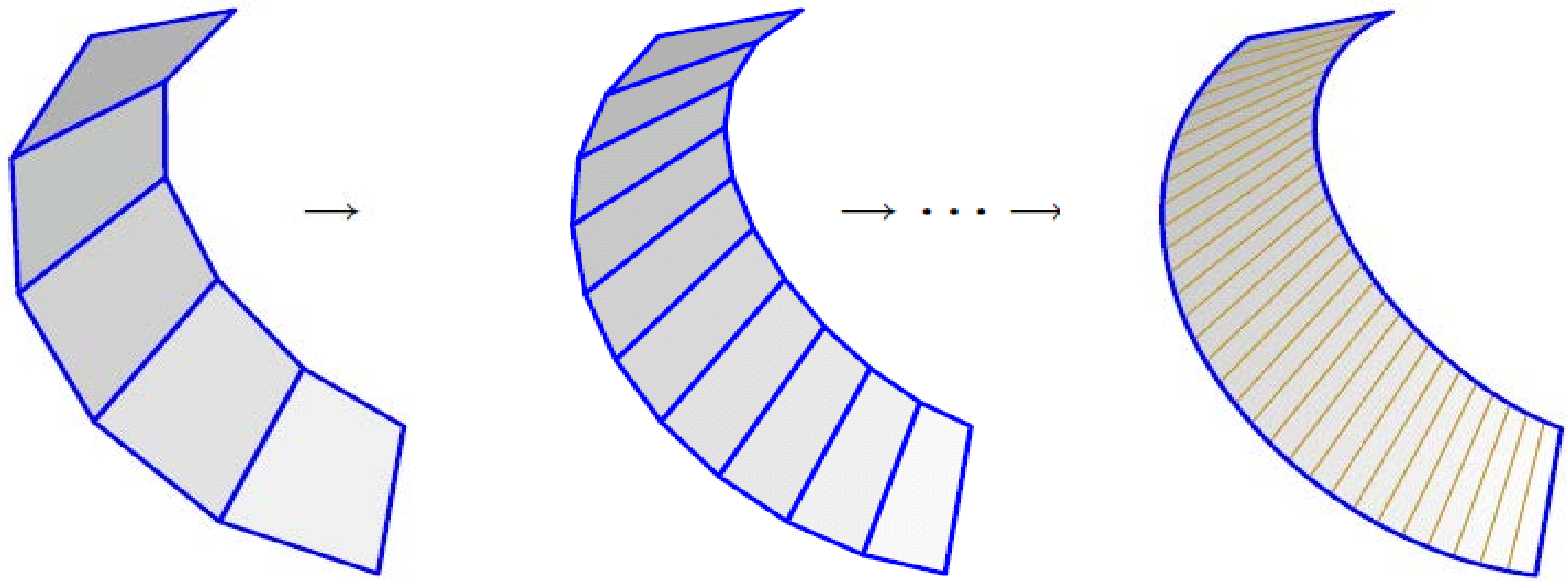
- (nearly) developable surfaces

F. Gehry, Guggenheim Museum, Bilbao

F. Gehry,
Walt Disney
Concert Hall,
Los Angeles

# developable surface strips

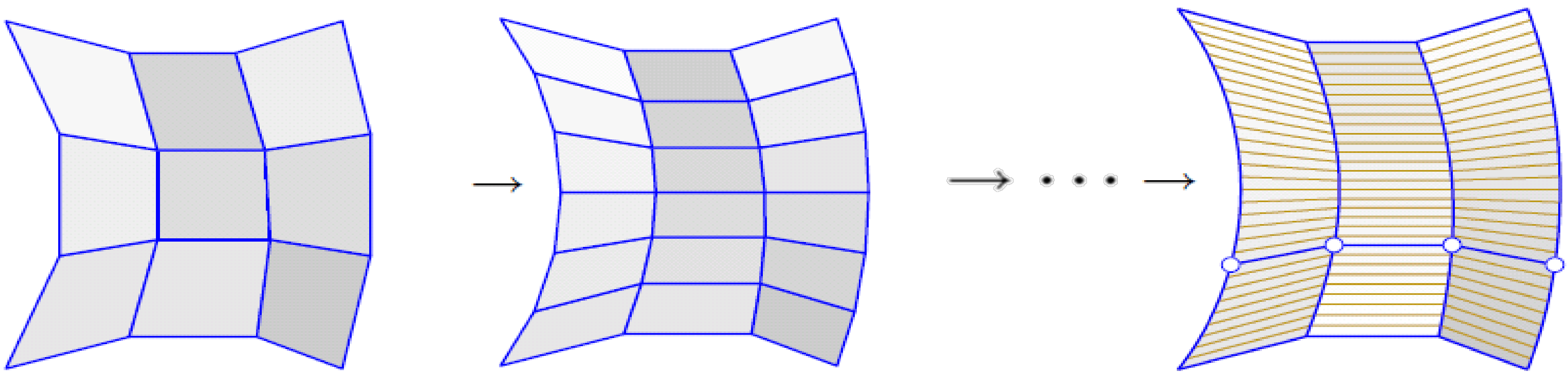
- Refinement of a PQ strip (iterate between subdivision and PQ optimization)



Limit: developable surface strip

# D-strip models

One-directional limit of a PQ mesh:

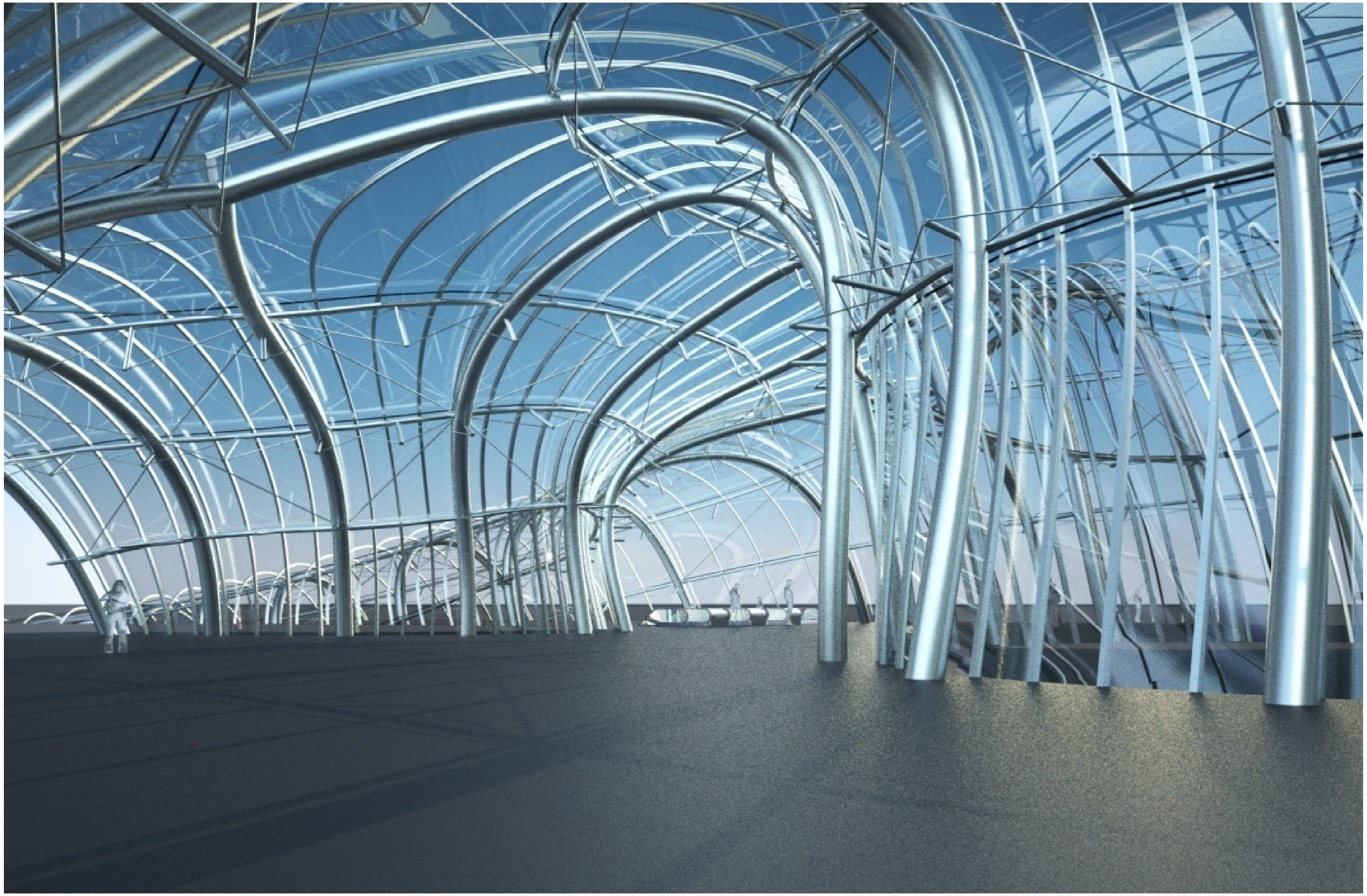


developable strip model (D-strip model)

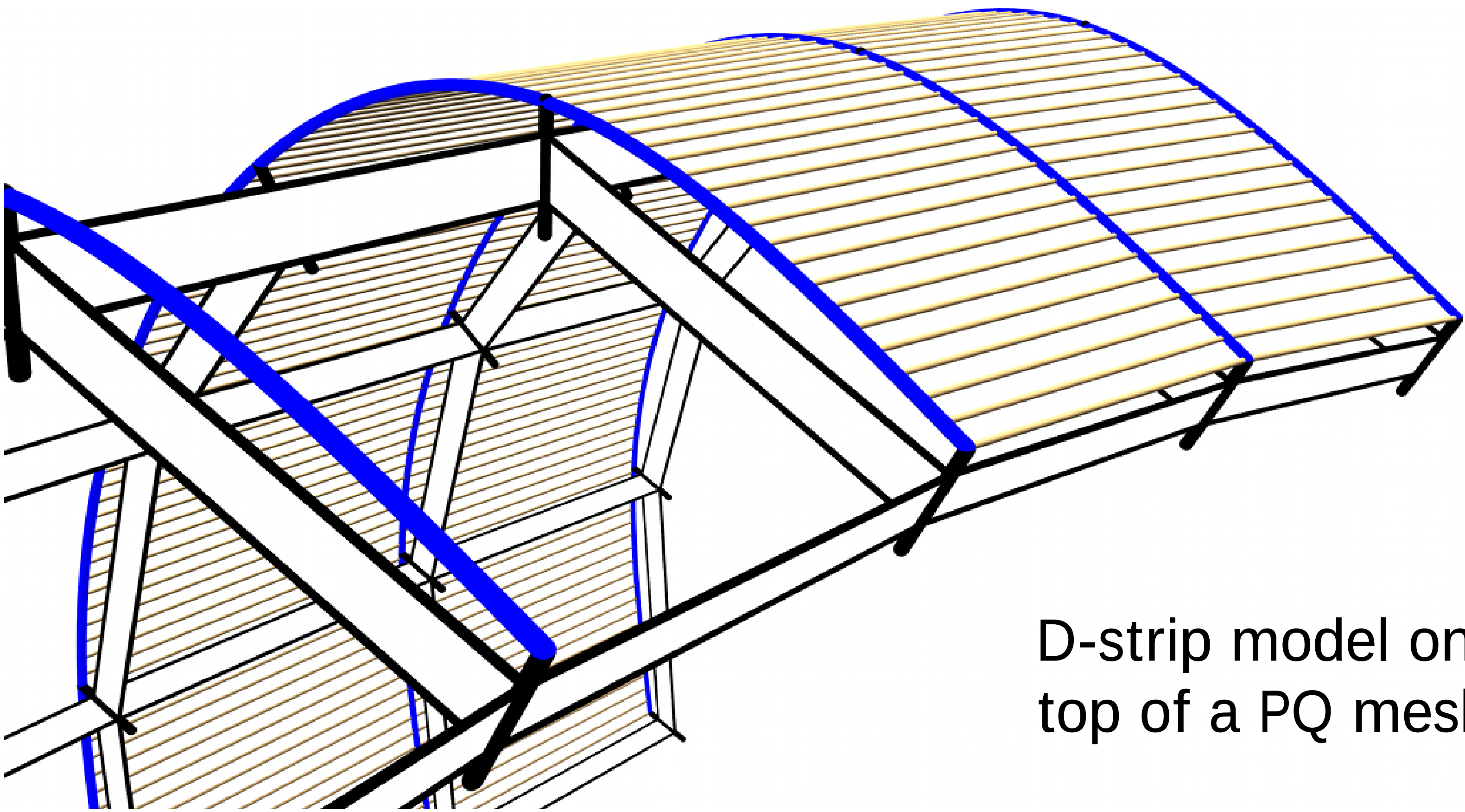
semi-discrete surface representation

initiated research on semi-discrete surface representations

# Design from single-curved panels based on subdivision modeling



# Multi-layer structure



D-strip model on
top of a PQ mesh