Frequency and intensity noise in an injection-locked, solid-state laser

Alejandro D. Farinas, Eric K. Gustafson, and Robert L. Byer

Edward L. Ginzton Laboratory, Stanford University, Stanford, California 94305-4085

Received May 9, 1994; accepted July 8, 1994; revised manuscript received August 22, 1994

We have calculated transfer functions for frequency and intensity fluctuations in an injection-locked solid-state laser. At modulation frequencies well below the locking frequency we find significant frequency-noise reduction, and at modulation frequencies above the locking frequency we find that the frequency noise is that of the free-running slave laser. Our intensity-noise theory predicts substantial damping of relaxation oscillations in the slave laser. To verify these results we have measured the frequency and intensity noise of a 5-W, injection-locked Nd:YAG laser.

1. INTRODUCTION

There are three approaches to obtaining narrow-linewidth, high-power laser radiation: stabilize a high-power oscillator, amplify a stable low-power oscillator, or injection lock a high-power oscillator with a stable low-power oscillator. Stabilizing a high-power oscillator requires intracavity elements to ensure single-axial-mode operation[1] and active noise suppression to reduce frequency noise. The intracavity elements increase the loss of the high-power oscillator and thereby lead to a reduction in efficiency and output power. Furthermore, the large linewidths typical of high-power oscillators require complex actuators to reduce the spectral density of frequency noise.[2] For these reasons, intracavity and active stabilization of high-power oscillators is often an unattractive approach.

Low-power Nd:YAG oscillators are less susceptible to the aforementioned difficulties. In particular, the monolithic nonplanar ring oscillator,[3] because of its monolithic design, has very low noise. Amplifying this radiation to high power levels is impractical at this time because of the low gain of currently available cw amplifiers. The third option is to injection lock a high-power oscillator with a stable, low-noise oscillator.[4] As applied to lasers, injection locking consists of injecting the output of a stable laser, the master oscillator, into the optical resonator of the laser to be stabilized, the slave oscillator. Typically the master-laser output power is a small fraction of the slave-laser output power. If the frequency difference between the master and slave oscillators is sufficiently small the free-running mode of the slave laser is extinguished, and the slave laser is frequency locked to the master laser. This frequency stabilization is accomplished without introduction of optical elements into the slave-laser cavity and requires a relatively simple feedback loop.

In our laboratory we have injection locked a lamp-pumped Nd:YAG laser[5] to produce up to 24 W of cw, single-frequency output power for nonlinear-optics experiments.[6] We have injection locked a laser-diode-pumped Nd:YAG miniature-slab laser[7] to explore laser design approaches that could meet the laser requirements of interferometric gravitational-wave detectors.[8] In this paper we present a small-signal theory of frequency and intensity modulation in an injection-locked laser. We measure the spectral density of frequency and intensity noise of the laser-diode-pumped miniature-slab laser and compare the experimental results with theory. The advantages of frequency-noise reduction by injection locking are demonstrated.

2. THEORY

A. Injection-Locking Model

Our injection-locking model is based on the laser equations described by Siegman.[9] We assume fields that vary slowly compared with the optical frequency and apply the slowly varying envelope approximation. Following the notation of Siegman, the equations describing the time evolution of the laser field, atomic polarization, and population inversion are given by

$$\frac{\mathrm{d}\tilde{E}(t)}{\mathrm{d}t} + [\gamma_c/2 + j(\omega - \omega_c)]\tilde{E}(t) = -j\,\frac{\omega}{2\epsilon}\,\tilde{P}(t) + \left(\frac{2\gamma_e}{\epsilon V_c}\right)^{1/2}\tilde{E}_e(t)\,, \quad \text{(1a)}$$

$$\frac{\mathrm{d}\tilde{P}(t)}{\mathrm{d}t} + [\Delta\omega_a/2 + j(\omega - \omega_a)]\tilde{P}(t) = -j\,\frac{\kappa}{2\omega V_c}\,\Delta N(t)\tilde{E}(t)\,, \quad \text{(1b)}$$

$$\frac{\mathrm{d}\Delta N(t)}{\mathrm{d}t} + \gamma_2\Delta N(t) + R_p(t) = j\,\frac{V_c}{4\hbar}\,[\tilde{E}(t)\tilde{P}^*(t) - \tilde{E}^*(t)\tilde{P}(t)]\,, \quad \text{(1c)}$$

where $j \equiv \sqrt{-1}$, ϵ is the dielectric permeability of the gain medium, $\tilde{E}$ is the complex cavity mode amplitude with frequency ω, $\tilde{E}_e$ is the amplitude of an external signal, $\tilde{P}$ is the atomic polarization, ΔN is the atomic population difference, and R_p is the pumping rate. The laser gain is

0740-3224/95/020328-07$06.00

centered at ω_a and has a linewidth of $\Delta\omega_a$. The cavity-signal-to-atomic-polarization coupling coefficient is κ, and the population decay rate is γ_2. The optical cavity is resonant at frequency ω_c, with cavity decay rate $\gamma_c \equiv \delta_c/\tau$, where δ_c is the total cavity loss and τ is the cavity roundtrip time. The external decay rate $\gamma_e \equiv \delta_e/\tau = -\ln(R_{oc})/\tau$ is the portion of the cavity decay rate that is due to output coupling R_{oc}. The fields occupy an effective mode volume V_c that we assume to be the same for both the cavity and the externally applied fields. The master laser is therefore assumed to be perfectly mode matched into the slave-laser's cavity.

We assume that the atomic polarization relaxes much faster than the electric fields or the atomic population difference, which is a good approximation for solid-state lasers. In this linear susceptibility approximation, $\tilde{P}$ is replaced by the steady-state solution of Eq. (1b):

$$\tilde{P}(t) \approx -j\,\frac{\kappa}{\omega\Delta\omega_a V_c}\,\frac{1}{1 + 2j(\omega - \omega_a)/\Delta\omega_a}\,\Delta N(t)\tilde{E}(t)\,. \quad (2)$$

In Subsections 2.B and 2.C we analyze the frequency and intensity noise in an injection-locked laser, using Eq. (1) and relation (2). We assume uncorrelated frequency and intensity noise, which permits us to consider the two effects independently. Although this assumption ignores frequency- and intensity-noise coupling (for example through the Lorentzian line shape dependence of the rate-equation coupling coefficient, K), it is reasonable to use for small frequency and intensity modulation near line center.

B. Frequency Noise Analysis

Following the analysis presented by Siegman,[10] we expand the cavity and externally injected electric fields of Eq. (1) in phase-amplitude form, $\tilde{E}(t) \equiv E(t)\exp[\,j\phi(t)]$, and redefine the cavity-wave amplitude in the units of the external signal. We also assume the approximate value of the atomic polarization given in relation (2) and no change in the pumping rate. In this case Eq. (1) and relation (2) reduce to an equation for the time-varying amplitude of the cavity electric field:

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} + \frac{\gamma_c - \gamma_0}{2}\,E(t) = \gamma_e E_m(t)\cos[\phi(t) - \phi_m(t)]\,, \quad (3)$$

where the subscript m refers to the master (externally injected) laser and γ_0 represents the growth rate of the cavity field that is due to the (saturated) gain of the slave laser, and to an equation for the time-varying phase of the cavity electric field:

$$\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} + \omega_m - \omega_s(t) = -\gamma_e\,\frac{E_m(t)}{E(t)}\,\sin[\phi(t) - \phi_m(t)]\,, \quad (4)$$

where the subscript s refers to the free-running slave laser.

When the injected signal is small ($E_m \ll E_s$) we can assume that $E \approx E_s$, where E_s is the slave laser's free-running oscillation amplitude. In the limit of a small injected signal and no time variation in signal amplitudes, Eq. (4) reduces to

$$\frac{\mathrm{d}\phi(t)}{\mathrm{d}t} + \omega_m - \omega_s(t) = -\omega_{\mathrm{lock}}\,\sin[\phi(t) - \phi_m(t)]\,, \quad (5)$$

where $\omega_{\mathrm{lock}} \equiv \gamma_e E_m/E_s$. Equation (5), known as the Adler equation, was derived by Adler to describe injection locking of radio-frequency oscillators.[4] In steady state the Adler equation reduces to $\omega_m - \omega_s + \omega_{\mathrm{lock}}\sin\Delta\phi = 0$, where we assume that the slave-laser frequency is constant and $\Delta\phi$ is the steady-state phase difference between the injection-locked slave laser and the master laser. This equation can be solved for real $\Delta\phi$ only when $-\omega_{\mathrm{lock}} \le \omega_s - \omega_m \le \omega_{\mathrm{lock}}$. Therefore for the lasers to remain injection locked the difference in their free-running frequencies must be less than ω_{lock}, which can be written as

$$\omega_{\mathrm{lock}} \equiv \gamma_e\,\frac{E_m}{E_s} \approx T_{oc}\Delta f_{ax}\sqrt{P_m/P_s}\,, \quad (6)$$

where T_{oc} is the slave laser's output coupler transmission, Δf_{ax} is the slave laser's axial mode spacing, and P_m and P_s are the output power of the master laser and the free-running slave laser, respectively. The approximate expression is valid not only for small output coupling, $R_{oc} \approx 1$, and perfect mode matching of the external signal. For typical laser parameters the locking width is given by

$$\frac{\omega_{\mathrm{lock}}}{2\pi} = 1\ \mathrm{MHz}\left(\frac{T}{10\%}\right)\left(\frac{\Delta f_{ax}}{600\ \mathrm{MHz}}\right)\left(100\,\frac{P_m}{P_s}\right)^{1/2}\cdot \quad (7)$$

Inside this frequency range the steady-state output phase difference is given by

$$\Delta\phi \equiv \phi - \phi_m = \sin^{-1}\left(\frac{\omega_s - \omega_m}{\omega_{\mathrm{lock}}}\right)\cdot \quad (8)$$

To obtain the output frequency noise we first calculate the response of the output phase to small sinusoidal perturbations about the steady-state solution. Following an analysis performed for microwave oscillators[11] we calculate the response to changes in the master-laser and slave-laser phases separately and use superposition to obtain the response to multiple excitations.

We begin by considering a noisy master laser and a noise-free slave. Here $\omega_s - \omega_m$ is constant, and we assume a small perturbation on the master-laser phase, $\phi_m(t) = \phi_m\cos(\omega_{\mathrm{mod}}t)$. The output phase is assumed to vary as $\phi(t) = \Delta\phi + \mathrm{Re}[\hat{\phi}\exp(\,j\omega_{\mathrm{mod}}t)]$, where $\hat{\phi}$ is the small, complex amplitude of phase deviation from steady state. Substituting into the Adler equation and simplifying by expanding the sin term around $\Delta\phi$ gives the phase-modulation transfer function

$$H_m(\omega_{\mathrm{mod}}) \equiv \frac{\hat{\phi}}{\phi_m} = \frac{1}{1 + j\,\dfrac{\omega_{\mathrm{mod}}}{\omega_{\mathrm{lock}}\cos\Delta\phi}}\cdot \quad (9)$$

Figure 1 shows the magnitude of the phase-modulation transfer function plotted versus normalized angular frequency (solid curve). We see that frequency or phase fluctuations in the master laser are reproduced in the slave output at modulation frequencies well below the locking width, ω_{lock}. At modulation frequencies much greater than the locking width the phase perturbations of the master laser have no effect on the output phase of the injection-locked slave laser.

The second case is that of a noisy slave laser with a noise-free master laser. We assume that the master-

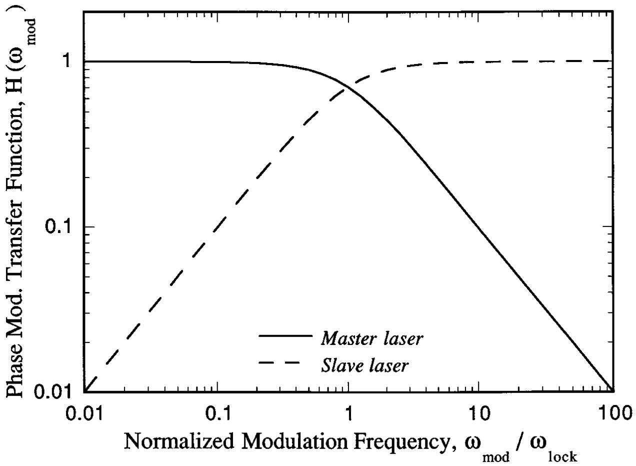

Fig. 1. Magnitude of the phase-modulation transfer functions. Response of the injection-locked laser to phase variation in the master laser, $H_m(\omega_{\mathrm{mod}})$, and phase variation in the free-running slave laser, $H_s(\omega_{\mathrm{mod}})$, plotted versus normalized frequency.

laser phase is zero, and the slave-laser phase is given by $\phi_s(t) = \phi_s \cos(\omega_{\mathrm{mod}} t)$. Therefore the time-varying frequency of the slave laser is given by $\omega_s(t) = \omega_s + \mathrm{d}\phi_s(t)/\mathrm{d}t = \omega_s - \phi_s \omega_{\mathrm{mod}} \sin(\omega_{\mathrm{mod}} t)$. Again the solution is of the form $\phi(t) = \Delta\phi + \mathrm{Re}[\hat{\phi} \exp(j\omega_{\mathrm{mod}} t)]$. The Adler equation now gives the phase-modulation transfer function

$$H_s(\omega_{\mathrm{mod}}) \equiv \frac{\hat{\phi}}{\phi_s} = \frac{j\,\dfrac{\omega_{\mathrm{mod}}}{\omega_{\mathrm{lock}} \cos\Delta\phi}}{1 + j\,\dfrac{\omega_{\mathrm{mod}}}{\omega_{\mathrm{lock}} \cos\Delta\phi}} \,. \tag{10}$$

In this case frequency, or phase, fluctuations of the slave laser are reproduced in the slave-laser output for modulation frequencies above the locking width, as shown in Fig. 1 (dashed curve). Free-running slave-laser phase perturbations well below the locking width are substantially attenuated. These equations are similar to those previously derived for microwave oscillators.[11]

These transfer functions can be used to predict the frequency noise of an injection-locked laser. If a master laser with a spectral density of frequency noise $S_{f,m}$ is used to injection lock a slave with spectral density of frequency noise $S_{f,s}$, and the noise sources are uncorrelated, the resultant spectral density of frequency noise of the injection-locked laser is given by

$$S_{f,il}(\omega_{\mathrm{mod}}) = [|H_m(\omega_{\mathrm{mod}})S_{f,m}(\omega_{\mathrm{mod}})|^2 + |H_s(\omega_{\mathrm{mod}})S_{f,s}(\omega_{\mathrm{mod}})|^2]^{1/2} \,. \tag{11}$$

Equation (11) predicts that the crossover point at which the output noise is no longer dominated by the master-laser noise is a function of several parameters, including the locking width and the relative magnitudes of the free-running noise spectral densities. This analysis becomes more complicated when, as is usually required, an electronic servo loop is used to stabilize the slave laser to keep it within the locking range. In this case, the output spectral density of frequency noise is also affected by the noise reduction in the electronic servo loop.[12]

Perhaps a more important consequence of the master-laser noise transfer function is that it limits the extent to which modulating the master laser affects the output of the injection-locked slave laser. Suppose, for example, that the output of the injection-locked laser is to be frequency locked to an external reference. In this situation the master laser would be the most logical place to apply the control signal. The phase-modulation transfer function given by Eq. (9) predicts a pole in the response of the system's output to this actuator. This pole must be considered when one is designing the control loop.

C. Intensity Noise Analysis

Intensity noise in solid-state lasers is dominated by relaxation oscillations and is typically analyzed by the use of rate equations.[13] As these rate equations do not include an external signal term, we rederive them. Following Siegman's analysis,[9] we define the cavity photon number $n(t) \equiv \epsilon V_c |\tilde{E}(t)|^2/(2\hbar\omega)$ and differentiate it with respect to time to obtain $\mathrm{d}n(t)/\mathrm{d}t = \epsilon V_c/2\hbar\omega[\tilde{E}(t)\mathrm{d}\tilde{E}^*(t)/\mathrm{d}t + \tilde{E}^*(t)\mathrm{d}\tilde{E}(t)/\mathrm{d}t]$. Substituting the results of Eq. (1a) and relation (2) into this equation, we obtain

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = KN(t)n(t) - \gamma_c n(t) + \left(\frac{\epsilon V_c}{2\hbar\omega}\right)\left(\frac{2\gamma_e}{\epsilon V_c}\right)^{1/2} \times [\tilde{E}_e(t)\tilde{E}^*(t) + \tilde{E}_e^*(t)\tilde{E}(t)] \,. \tag{12}$$

Equation (12) is equal to the usual photon-number equation when $\tilde{E}_e = 0$. We can simplify the cross product of the external signal field and the cavity electric field by assuming that the two signals are at the same frequency. When the laser is injection locked, the two fields differ only by the steady-state phase shift $\Delta\phi$, which is typically kept near zero by the electronic feedback loop used to keep the lasers within the injection-locking range. With this assumption, the photon-number equation can be expressed as

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = KN(t)n(t) - \gamma_c n(t) + 2\Delta f_{\mathrm{ax}} \cos\Delta\phi[n(t)n_e(t)]^{1/2} \,, \tag{13}$$

where $n_e(t) = T_{\mathrm{oc}}|\tilde{E}_e(t)|^2/(\Delta f_{\mathrm{ax}}\hbar\omega)$ is the number of external signal photons inside the cavity. The atomic population equation remains

$$\frac{\mathrm{d}N(t)}{\mathrm{d}t} = R_p(t) - \gamma_2 N(t) - KN(t)n(t) \,. \tag{14}$$

First we obtain the steady-state solutions to the rate equations, treating the external signal as a small perturbation ($n_e \ll n$). In steady state with no external signal, the population inversion is clamped at the threshold value $N_{\mathrm{th}} = \gamma_c/K$, and the photon number is $n_{\mathrm{ss}} = (r-1)\gamma_2/K$, where $r \equiv R_p/\gamma_2 N_{\mathrm{th}}$ is the number of times above threshold. Adding a small external signal yields

$$N_0 = N_{\mathrm{th}} - 2\,\frac{\Delta f_{\mathrm{ax}}}{K} \cos\Delta\phi\sqrt{n_e/n_{\mathrm{ss}}} \tag{15}$$

for the steady-state population and

$$n_0 = n_{\mathrm{ss}} + 2r\,\frac{\gamma_2}{\gamma_c}\,\frac{\Delta f_{\mathrm{ax}}}{K} \cos\Delta\phi\sqrt{n_e/n_{\mathrm{ss}}} \tag{16}$$

for the steady-state photon number. Addition of the external signal reduces the population inversion and increases the photon number, as expected.

As in Subsection 2.B, the responses of the injection-locked laser to modulations of the pumping rate and of the external signal amplitude are calculated separately. First we consider pump modulation; hence the external signal amplitude is assumed to be constant and the pumping rate is assumed to vary sinusoidally as $R_p(t) \equiv R_p + R_{p1}\cos(\omega_{\mathrm{mod}}t)$. The solutions take the form of a time-varying photon number $n(t) = n_0 + \mathrm{Re}[\hat{n}\exp(j\omega_{\mathrm{mod}}t)]$ and population inversion $N(t) = N_0 + \mathrm{Re}[\hat{N}\exp(j\omega_{\mathrm{mod}}t)]$. Substituting these expressions into Eqs. (13) and (14) and dropping second-order small quantities gives the pump-modulation transfer function

$$G_p(\omega_{\mathrm{mod}}) \equiv \frac{\hat{n}_1}{R_{p1}} = \frac{1}{\gamma_c}\,\frac{{\omega_{\mathrm{sp}}}^2}{{\omega_{\mathrm{sp}}}^2 - {\omega_{\mathrm{mod}}}^2 + 2j\omega_{\mathrm{mod}}\gamma'_{\mathrm{sp}}}\,, \quad (17)$$

where the spiking frequency is given by

$$\begin{aligned}{\omega_{\mathrm{sp}}}^2 &= (r-1)\gamma_2\gamma_c + 2(r+1)\Delta f_{\mathrm{ax}}\gamma_2\cos\Delta\phi\sqrt{n_e/n_{\mathrm{ss}}}\\ &\approx (r-1)\gamma_2\gamma_c\,, \end{aligned} \quad (18)$$

which is essentially unchanged by the injected signal, while the spiking decay rate is given by

$$\begin{aligned}\gamma'_{\mathrm{sp}} &= \frac{r\gamma_2}{2} + \Delta f_{\mathrm{ax}}\left(r\frac{\gamma_2}{\gamma_c} + 1\right)\cos\Delta\phi\sqrt{\frac{n_e}{n_{\mathrm{ss}}}}\\ &\approx \frac{r\gamma_2}{2} + \gamma_e\left(r\frac{\gamma_2}{\gamma_c} + 1\right)\cos\Delta\phi\sqrt{\frac{P_m}{P_s}} \end{aligned} \quad (19)$$

and is a strong function of the injected signal value. This classic damped second-order transfer function is responsible for the relaxation-oscillation peak observed in solid-state lasers. Figure 2 shows the pump-modulation transfer function plotted for three values of the master–slave power ratio, assuming a Nd:YAG laser pumped three times above threshold, with total cavity losses of 17% and a 750-MHz axial mode spacing. The effect of the externally injected signal is to damp the relaxation-oscillation peak. The damping ratio, $\zeta = \gamma_{\mathrm{sp}}'/\omega_{\mathrm{sp}}$, is plotted in Fig. 3 for a wide range of injection-locking power ratios. The relaxation oscillations are at least critically damped, $\zeta \geq 1$, for all practical power ratios.

Next we consider the effect of intensity noise in the master laser. The pumping rate is now assumed to be constant and the injection signal to vary as $n_e(t) \equiv n_e + n_{e1}\cos(\omega_{\mathrm{mod}}t)$. The time-dependent cavity photon-number and population-inversion equations are identical to those defined in the preceding case. Substituting these quantities into Eqs. (13) and (14) yields the master-laser modulation transfer function

$$G_m(\omega_{\mathrm{mod}}) \equiv \frac{\Delta P/P}{\Delta P_m/P_m} = r\gamma_2\gamma_e\cos\Delta\phi\sqrt{\frac{P_m}{P_s}} \times \frac{\left(1 + j\,\dfrac{\omega_{\mathrm{mod}}}{r\gamma_2}\right)}{{\omega_{\mathrm{sp}}}^2 - {\omega_{\mathrm{mod}}}^2 + 2j\omega_{\mathrm{mod}}\gamma'_{\mathrm{sp}}}\,, \quad (20)$$

which includes the same resonant denominator as the pump modulation transfer function. Figure 4 shows this transfer function plotted for two power ratios for the Nd:YAG laser already mentioned. Although the effects of pump modulation are always strongest near the relaxation-oscillation frequency, the damping effect of the injected signal keeps this noise contribution small.

As with the frequency-noise transfer functions derived in Subsection 2.C, the pump-modulation and the master-laser modulation transfer functions can be used to predict the intensity noise of the injection-locked laser. The spectral density of relative intensity noise of the injection-locked laser is given by

$$\begin{aligned}S_{\mathrm{RIN},il}(\omega_{\mathrm{mod}}) = [&|G_p(\omega_{\mathrm{mod}})S_{N,p}(\omega_{\mathrm{mod}})|^2\\ &+ |G_m(\omega_{\mathrm{mod}})S_{\mathrm{RIN},m}(\omega_{\mathrm{mod}})|^2]^{1/2}\,,\end{aligned} \quad (21)$$

where $S_{N,p}(\omega_{\mathrm{mod}})$ is the spectral density of the pumping rate fluctuation, $S_{\mathrm{RIN},m}$ is the spectral density of the relative intensity noise of the master laser, and we assume that all the relative intensity noise of the free-running slave laser is due to pump power fluctuations.

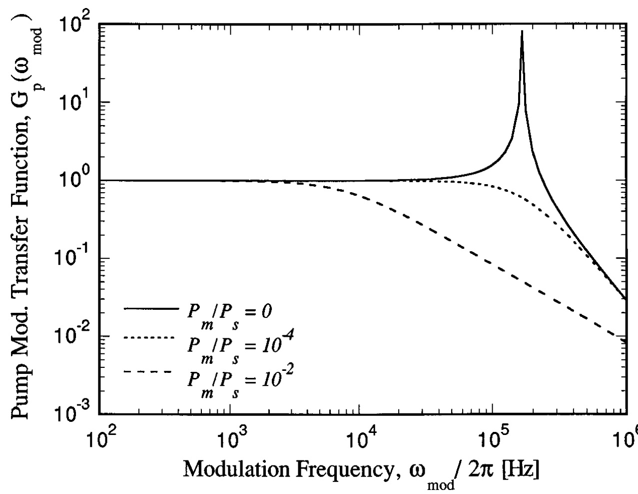

Fig. 2. Magnitude of the pump modulation transfer function $G_p(\omega_{\mathrm{mod}})$ plotted for three master–slave power ratios, assuming a Nd:YAG laser pumped three times above threshold, with total cavity losses of 17% and 750-MHz axial mode spacing. The presence of the injected signal damps the relaxation-oscillation peak.

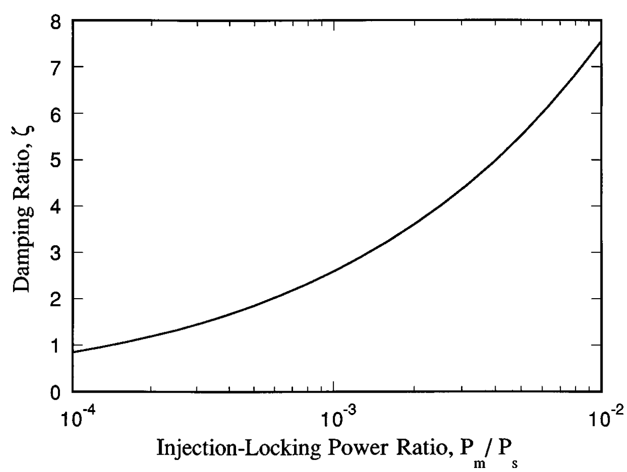

Fig. 3. Damping ratio ζ of the relaxation oscillations plotted versus the injection-locking power ratio. The system is over-damped for all practical injection-locking power ratios.

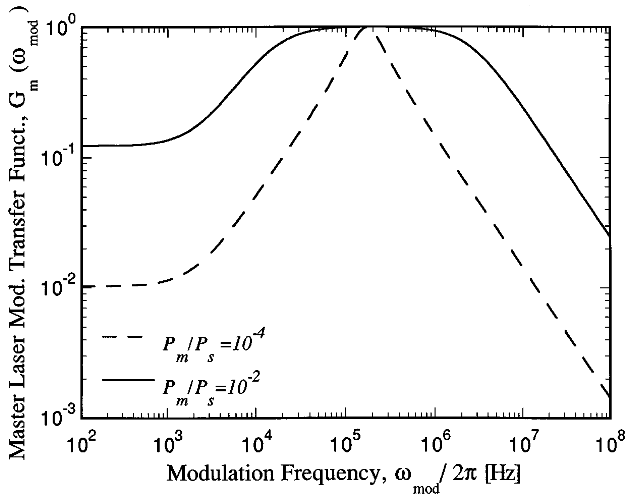

Fig. 4. Magnitude of the master-laser modulation transfer function $G_m(\omega_{\text{mod}})$ plotted for two master–slave power ratios. The effect of intensity noise in the master laser is greatest near the slave laser's relaxation-oscillation frequency but is kept small by the damping effect of the injected signal.

3. MEASUREMENTS

A. Frequency Noise

We have experimentally tested the predictions of the frequency-modulation transfer-function theory, using the injection-locked Nd:YAG laser shown in Fig. 5. The laser-diode-pumped miniature-slab laser[7] is injection locked by a 300-mW single-frequency Nd:YAG laser (Lightwave Electronics Model 122-1064-300-F). The phase modulator and Detector 1 are used in a Pound–Drever–Hall locking scheme[14] to generate Error Signal 1. This error signal is a function of the frequency difference between the master laser and an axial mode of the slave laser. A servo actuator generates Control Signal 1, which keeps the two lasers inside the locking range. This feedback loop has a unity-gain frequency of 30 kHz. In this system the master-laser power at the output coupler is 150 mW, the slave-laser power is 5 W, the output coupling is 12.5%, and the cavity path length is 40 cm; therefore the locking width is $2\pi \times 2.7$ MHz.

To measure the frequency noise of the injection-locked laser we have built an optical phase-locked loop.[15] As shown in Fig. 6, a portion of the output of the injection-locked laser is combined with the output of a 40-mW single-frequency Nd:YAG laser (Lightwave Electronics Model 120), the local oscillator. The lasers are mode matched and attenuated to produce a beat signal with a contrast ratio of 30% on Detector 2. This phase detector signal is used by a servo controller to generate Control Signal 2, which is applied to the fast frequency (piezoelectric) actuator on the local oscillator. If the open-loop gain is much greater than 1, this control signal is an accurate measure of the frequency noise of the injection-locked laser.[12] The unity-gain frequency of our type II phase-locked loop[16] is 300 kHz, which permits accurate measurement of frequency noise below 30 kHz. Similarly, the frequency noise of the free-running slave laser can be measured at control point 1.

We have measured the spectral density of the frequency noise of the 300-mW master laser as well as that of the 5-W slave oscillator in both free-running and injection-locked operation. Figure 7 shows the spectral density of the frequency noise, $S_f(\omega_{\text{mod}})$, measured in Hz/$\sqrt{\text{Hz}}$,

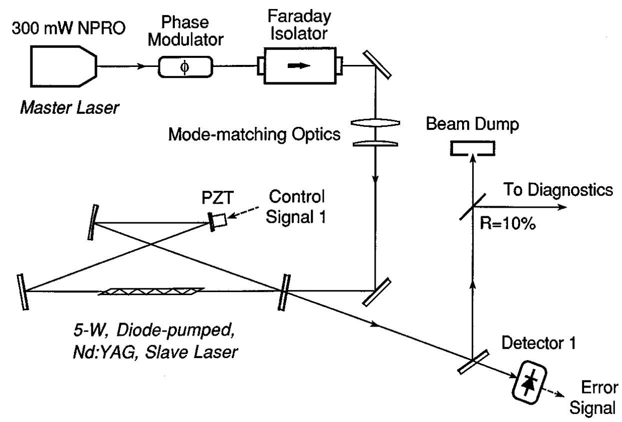

Fig. 5. Schematic of the injection-locked laser. The 300-mW nonplanar ring oscillator (NPRO) is used to injection lock the 5-W laser-diode-pumped ring laser. The optical isolator prevents perturbation of the master laser by the slave laser. A piezoelectric transducer (PZT) is used to vary the free-running frequency of the slave laser.

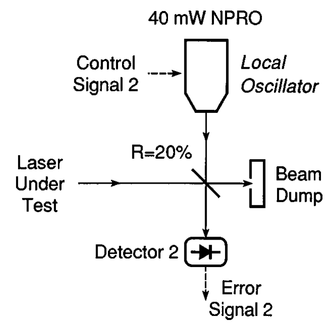

Fig. 6. Schematic of the laser diagnostics. The laser diagnostic output is used as the transmitter in a phase-locked loop. The control signal applied to the local oscillator, the 40-mW nonplanar ring oscillator (NPRO), is a measure of the frequency noise of the transmitter.

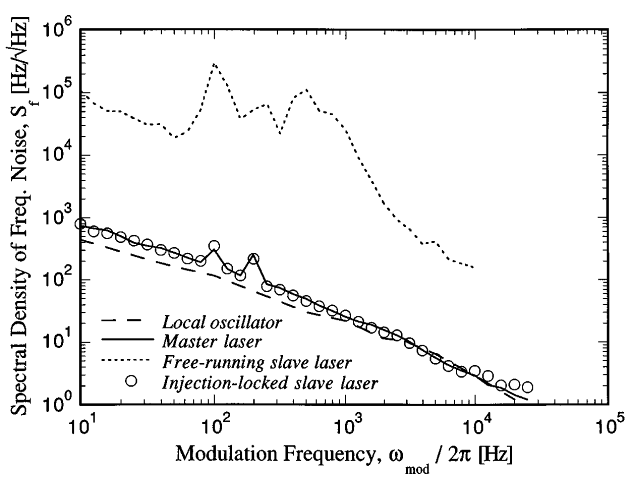

Fig. 7. Measured spectral density of frequency noise. The frequency noise of the master laser is reproduced in the injection-locked output below 1 kHz. The frequency noise of the local oscillator, which sets a resolution limit on the experiment, dominates above 1 kHz.

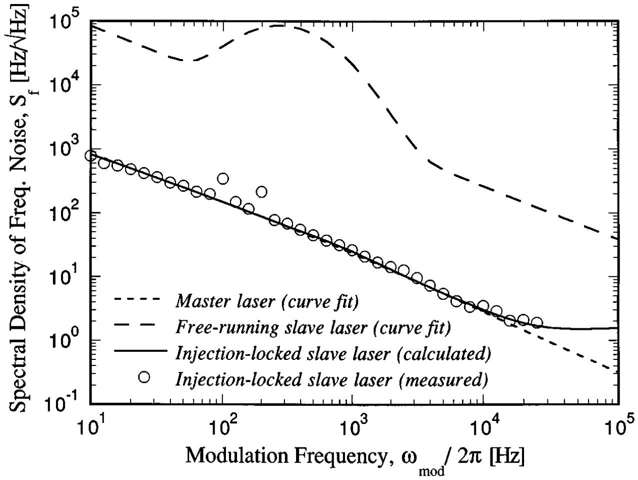

Fig. 8. Calculated spectral density of frequency noise. Phase-modulation transfer functions are used to model the spectral density of frequency noise of the injection-locked slave laser, given the measured master-laser and free-running slave-laser frequency noise. Low frequency noise in the free-running slave laser is suppressed.

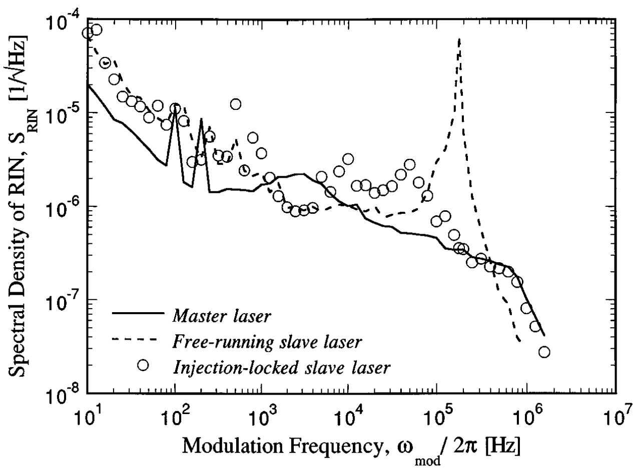

Fig. 9. Measured relative intensity noise, RIN, in the master, the free-running slave, and the injection-locked slave lasers. The free-running slave laser's intensity-noise peak (near 100 kHz) is eliminated in the injection-locked slave laser.

together with an independent measurement[12] of the frequency noise of the 40-mW local oscillator. These results show that the frequency noise of the injection-locked laser is the same as that of the master laser below 1 kHz. Above 1 kHz the measured frequency noise is comparable with that measured for the local oscillator, which sets the sensitivity limit for this experiment. Injection locking leads to a hundredfold reduction in the frequency noise of the power oscillator over the measured frequency range.

The measured spectral density of frequency noise can be compared with that predicted by the theory of Subsection 2.B. Equation (11) predicts the spectral density of frequency noise of the injection-locked laser given the measured noise in the master laser and the free-running slave laser. Figure 8 shows the value of the analytical expressions used to estimate the measured spectral density of frequency noise of the master and the free-running slave lasers as well as the calculated and measured frequency noise in the injection-locked slave laser. The low frequency noise in the free-running slave laser is reduced by injection locking and ultimately limited by the frequency noise of the master laser.

B. Intensity Noise

The predictions of the intensity-modulation theory were tested in the same apparatus shown in Fig. 6. By shutting off the local oscillator, we use Detector 2 to measure the intensity noise of the master, the free-running slave, or the injection-locked slave lasers. Figure 9 shows the spectral density of relative intensity noise, $S_{\mathrm{RIN}}(\omega_{\mathrm{mod}})$, measured in $1/\sqrt{\mathrm{Hz}}$. The measurement reveals a large noise peak in the free-running slave laser, owing to its relaxation oscillations. Absence of a similar peak in the master laser is due to an electronic feedback system[17] built into the device. Relaxation oscillations in the injection-locked slave laser are not observed. Excess intensity noise in the injection-locked slave laser near 10 and 50 kHz is due to the action of the slave's intracavity piezoelectric transducer, imperfections of which lead to frequency-noise-to-amplitude-noise conversion.

The measured spectral density of relative intensity noise can be compared with that predicted by the theory of Subsection 2.C. Equation (21) predicts the spectral density of relative intensity noise of the injection-locked laser, given the measured noise in the master laser and the free-running slave laser. Figure 10 shows the value of the analytical expressions used to estimate the measured spectral density of the relative intensity noise of the master and the free-running slave lasers as well as the calculated and measured frequency noise in the injection-locked slave laser. The slave laser's intensity noise is relatively unchanged by injection locking, but its relaxation-oscillation noise peak is eliminated.

4. SUMMARY

Injection locking is a powerful technique for reducing the frequency noise of a high-power laser oscillator. We have derived transfer functions for frequency and intensity modulation in an injection-locked laser. The phase-

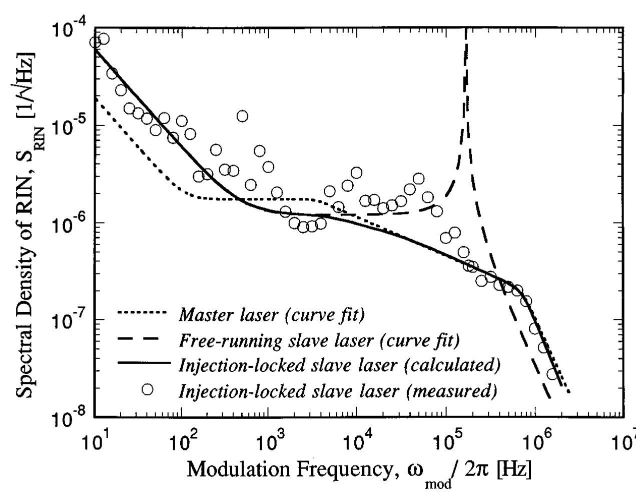

Fig. 10. Calculated relative intensity noise, RIN, in the three lasers. Intensity-noise transfer functions are used to model the intensity noise in the injection-locked slave laser, given the measured intensity noise in the master and the free-running slave lasers.

modulation transfer functions predict that the frequency noise of the master laser will dominate the frequency noise of the injection-locked laser at modulation frequencies below the injection-locking frequency. A stable master laser makes possible large reductions in the frequency noise of the slave laser with a simple feedback loop and no intracavity elements. The free-running frequency noise of the slave laser dominates at high frequencies; hence the lower Schawlow–Townes noise[18] of the higher-power slave laser is not affected by the presence of the master laser. Our measurements of the spectral density of frequency noise of an injection-locked miniature-slab laser show that the frequency noise of the injection-locked laser is dominated by master-laser noise below 1 kHz, as predicted. The intensity-modulation theory predicts damping of the relaxation oscillations in the slave laser and sensitivity to master-laser intensity noise in a broad band near the relaxation-oscillation frequency. Our intensity-noise measurements confirm this prediction. Relaxation-oscillation damping combined with electronic feedback of the master laser's pump produces a high-power laser with no characteristic relaxation-oscillation intensity-noise peak. The predictions of our analysis can be applied to optimizing cavity parameters to minimize frequency and intensity noise in injection-locked solid-state lasers.

ACKNOWLEDGMENTS

We thank Bob Shine, Tony Alfrey, and Tim Day for many useful technical discussions. This work has been funded by the National Science Foundation, the Sony Corporation, and the U.S. Office of Naval Research.

REFERENCES

1. W. Koechner, *Solid State Laser Engineering*, 3rd ed. (Springer-Verlag, New York, 1992), p. 79.
2. M. Zhu and J. L. Hall, "Stabilization of optical phase/frequency of a laser system: application to a commercial dye laser with an external stabilizer," J. Opt. Soc. Am. B **10,** 802 (1993).
3. T. J. Kane and R. L. Byer, "Monolithic, unidirectional single-mode Nd:YAG ring laser," Opt. Lett. **10,** 65 (1985).
4. R. Adler, "A study of locking phenomena in oscillators," Proc. IRE **34,** 351 (1946).
5. C. D. Nabors, A. D. Farinas, T. Day, S. T. Yang, E. K. Gustafson, and R. L. Byer, "Injection locking of a 13-W cw Nd:YAG ring laser," Opt. Lett. **14,** 1189 (1989).
6. S. T. Yang, C. C. Pohalski, E. K. Gustafson, R. L. Byer, R. S. Feigelson, R. J. Raymakers, and R. K. Route, "6.5-W, 532-nm radiation by cw resonant external-cavity second-harmonic generation of an 18-W Nd:YAG laser in $LiNb_3O_5$," Opt. Lett. **16,** 1493 (1991).
7. A. D. Farinas, E. K. Gustafson, and R. L. Byer, "Design and characterization of a 5.5-W, cw, injection-locked, fiber-coupled laser-diode pumped Nd:YAG miniature-slab laser," Opt. Lett. **19,** 114 (1994).
8. A. Abramovici, W. E. Althouse, R. W. P. Drever, Y. Gürsel, S. Kawamura, F. Raab, D. Shoemaker, L. Sievers, R. E. Spero, K. S. Thorne, R. E. Voght, R. Weiss, S. Whitcomb, and M. E. Zucker, "LIGO: the laser interferometer gravitational-wave observatory," Science **256,** 325 (1992).
9. A. E. Siegman, *Lasers* (University Science, Mill Valley, Calif., 1986), p. 923.
10. Ref. 9, p. 1129.
11. M. E. Hines, J. R. Collinet, and J. G. Ondria, "FM noise suppression of an injection phase-locked oscillator," IEEE Trans. Microwave Theory Tech. **MTT-16,** 738 (1968).
12. T. Day, E. K. Gustafson, and R. L. Byer, "Sub-hertz relative frequency stabilization of two-diode laser-pumped Nd:YAG lasers locked to a Fabry–Perot interferometer," IEEE J. Quantum Electron. **28,** 1106 (1992).
13. Ref. 9, p. 954.
14. R. W. P. Drever, J. L. Hall, F. V. Kawalski, J. Hough, G. M. Ford, A. J. Munley, and H. Ward, "Laser phase and frequency stabilization using an optical resonator," Appl. Phys. B **31,** 97 (1983).
15. T. Day, A. D. Farinas, and R. L. Byer, "Demonstration of a low bandwidth 1.06 μm optical phase-locked loop for coherent homodyne communication," IEEE Photon. Technol. Lett. **2,** 294 (1990).
16. J. Smith, *Modern Communications Circuits* (McGraw-Hill, New York, 1986), p. 295.
17. T. J. Kane, "Intensity noise in diode-pumped single-frequency Nd:YAG lasers and its control by electronic feedback," IEEE Photon. Technol. Lett. **2,** 244 (1990).
18. A. L. Schawlow and C. H. Townes, "Infrared and optical masers," Phys. Rev. **112,** 1940 (1958).