From Randomness to Probability

Chapter 13

Dealing with Random Phenomena

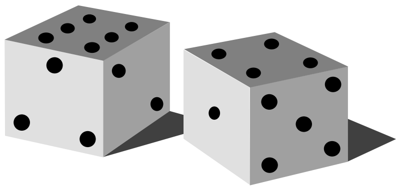
A random phenomenon is a situation in which we know what outcomes could happen, but we don't know which particular outcome did or will happen.

In general, each occasion upon which we observe a random phenomenon is called a trial.

At each trial, we note the value of the random phenomenon, and call it an outcome.

When we combine outcomes, the resulting combination is an event.

The collection of *all possible outcomes* is called the sample space.



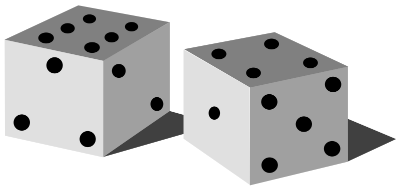
Definitions

Probability is the mathematics of chance.

It tells us the relative frequency with which we can expect an event to occur

The greater the probability the more likely the event will occur.

It can be written as a fraction, decimal, percent, or ratio.

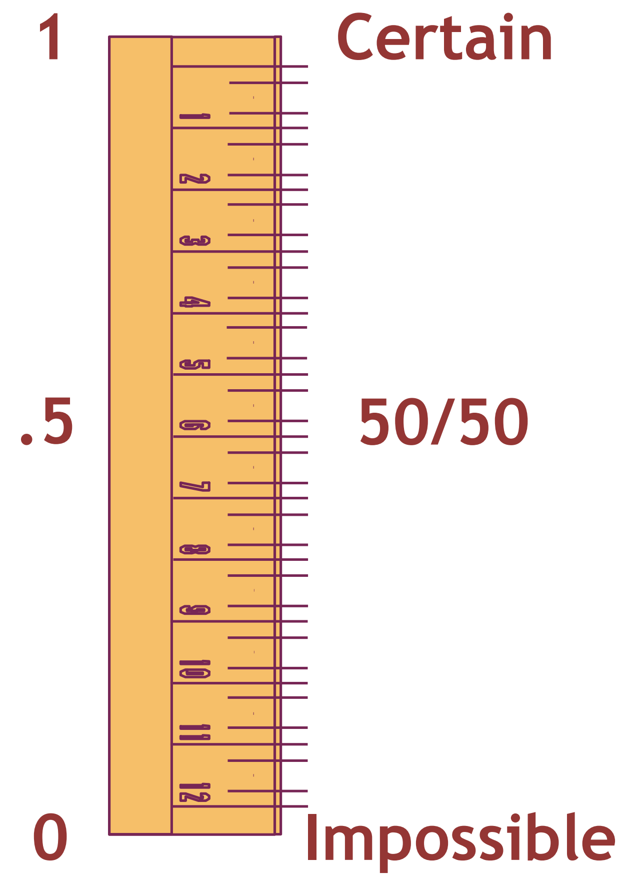


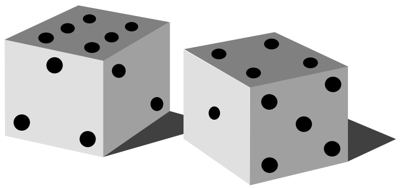
Definitions

Probability is the numerical measure of the likelihood that the event will occur.

Value is between 0 and 1.

Sum of the probabilities of all events is 1.



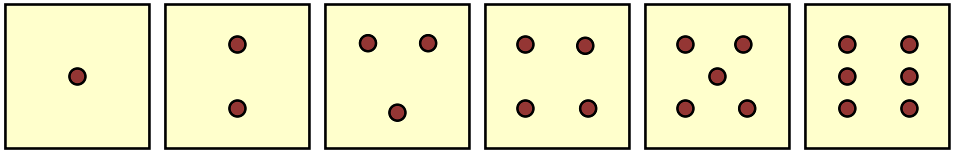


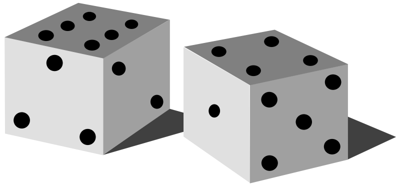
Definitions

A probability experiment is an action through which specific results (counts, measurements, or responses) are obtained.

The result of a single **trial** in a probability experiment is an **outcome**.

The set of all possible outcomes of a probability experiment is the **sample space**, denoted as S.
e.g. All 6 faces of a die: S = { 1 , 2 , 3 , 4 , 5 , 6 }

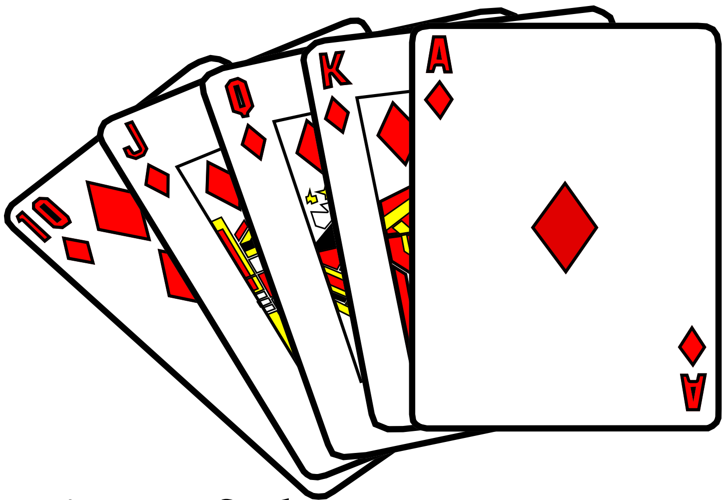




Definitions

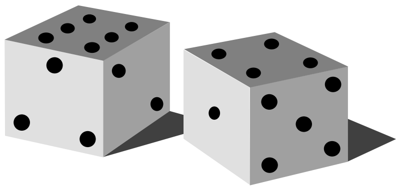
Other Examples of Sample Spaces may include:

Lists
Lattice Diagrams
Venn Diagrams
Tree Diagrams



May use a combination of these.

Where appropriate always display your sample space.

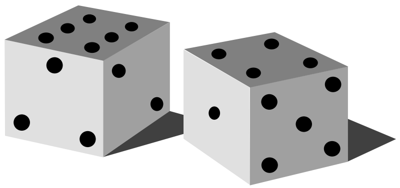


Definitions

An **event** consists of one or more outcomes and is a subset of the sample space.

Events are often represented by uppercase letters, such as A, B, or C.

Notation: The probability that event E will occur is written P(E) and is read “the probability of event E.”



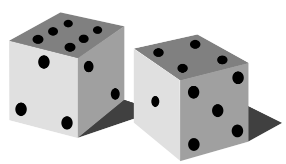
Definitions

- The Probability of an Event, E:

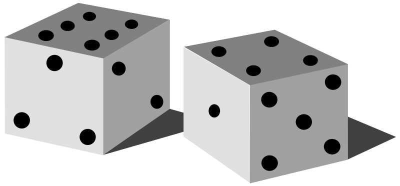
$$P(E) = \frac{\text{Number of Event Outcomes}}{\text{Total Number of Possible Outcomes in S}}$$

Consider a pair of Dice

- Each of the Outcomes in the Sample Space are random and equally likely to occur.

e.g. P() $= \frac{2}{36} = \frac{1}{18}$

(There are 2 ways to get one 6 and the other 4)



Definitions

There are three types of probability

1. Theoretical Probability

Theoretical probability is used when each outcome in a sample space is equally likely to occur.

$$P(E) = \frac{\text{Number of Event Outcomes}}{\text{Total Number of Possible Outcomes in S}}$$

The Ultimate probability formula ☺

Theoretical Probability

Probabilities determined using mathematical computations based on *possible* results, or outcomes.

This kind of probability is referred to as theoretical probability.

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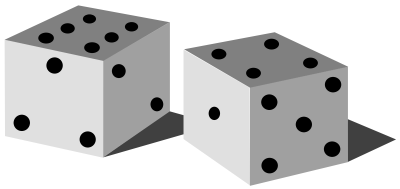

Definitions

There are three types of probability

2. Experimental Probability (or Empirical Probability)

Experimental probability is based upon observations obtained from probability experiments.

$$P(E) = \frac{\text{Number of Event Occurrences}}{\text{Total Number of Observations}}$$

The experimental probability of an event E is the relative frequency of event E

Experimental Probability

Probabilities determined from repeated experimentation and observation, recording results, and then using these results to predict expected probability.

This kind of probability is referred to as <u>experimental probability</u>.

Also known as Relative Frequency.

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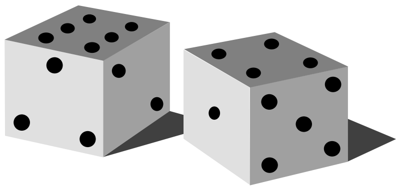


Definitions

There are three types of probability

3. Personal Probability

Personal probability is a probability measure resulting from intuition, educated guesses, and estimates.

Therefore, there is no formula to calculate it.

Usually found by consulting an expert.

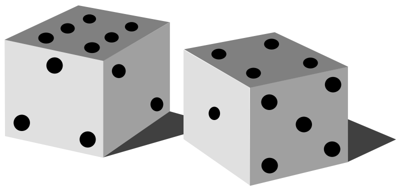
Theoretical vs. Experimental Probability

Related by **The Law of Large Numbers.**

The Law of Large Numbers: States that the long-run relative frequency (experimental probability) of repeated independent events gets closer and closer to the theoretical probability as the number of trials increases.

Independent - Roughly speaking, this means that the outcome of one trial doesn’t influence or change the outcome of another.

For example, coin flips are independent.



Definitions

Law of Large Numbers

As an experiment is repeated over and over, the experimental probability of an event approaches the theoretical probability of the event.

The greater the number of trials the more likely the experimental probability of an event will equal its theoretical probability.

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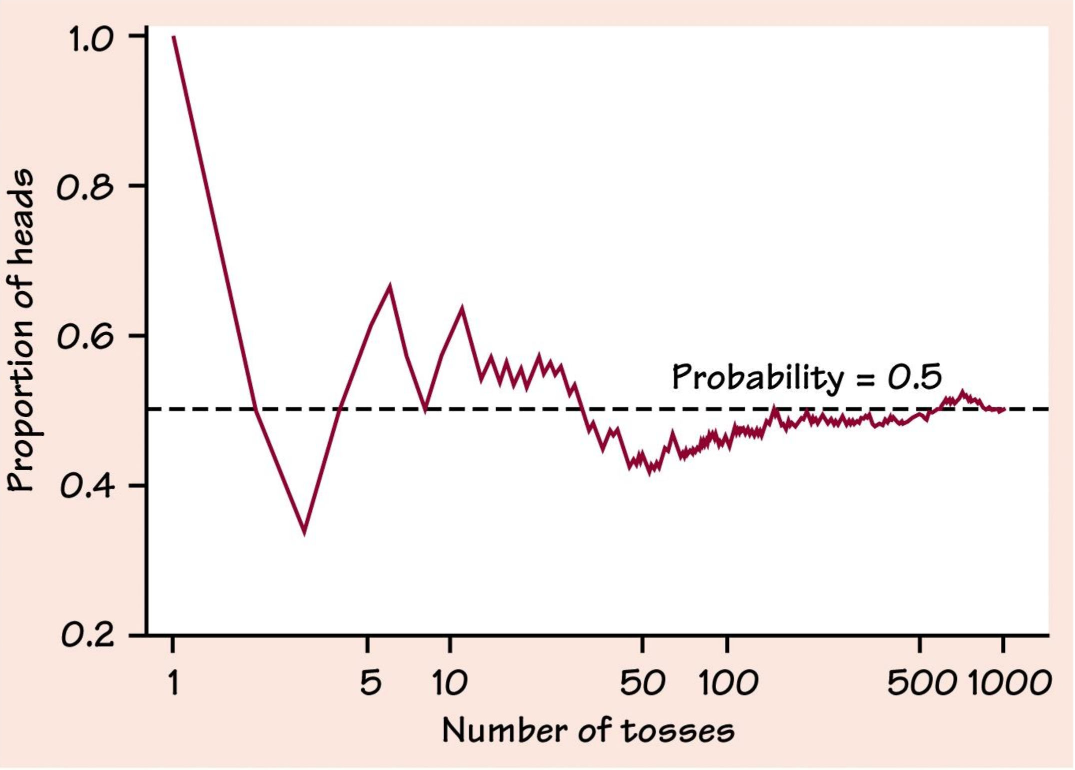
Example: Theoretical vs. Experimental Probability

In the long run, if you flip a coin many times, heads will occur about ½ of the time.



Theoretical vs. Experimental Probability

<u>Theoretical Probability</u> – What *should* occur or happen.

<u>Experimental Probability</u> – What *actually* occurred or happened (relative frequency).

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The Nonexistent Law of Averages

The LLN says nothing about short-run behavior.

Relative frequencies even out *only in the long run*, and this long run is *really* long (*infinitely* long, in fact).

The so called Law of Averages (that an outcome of a random event that hasn't occurred in many trials is "due" to occur) doesn't exist at all.

The First Three Rules of Working with Probability

We are dealing with probabilities now, not data, but the three rules don't change.

- Make a picture.
- Make a picture.
- Make a picture.

The First Three Rules of Working with Probability

All the pictures we use help us indentify the sample space. Once all possible outcomes have been indentified, calculating the probability of an event is:

$$P(E) = \frac{\text{Number of Event Outcomes}}{\text{Total Number of Possible Outcomes in S}}$$

Pictures:

- **Lists**
- **Lattice Diagrams**
- **Venn Diagrams**
- **Tree Diagrams**

Formal Probability (Laws of Probability)

1. Two requirements for a probability:

A probability is a number between 0 and 1.

For any event **A**, $0 \leq P(\mathbf{A}) \leq 1$.

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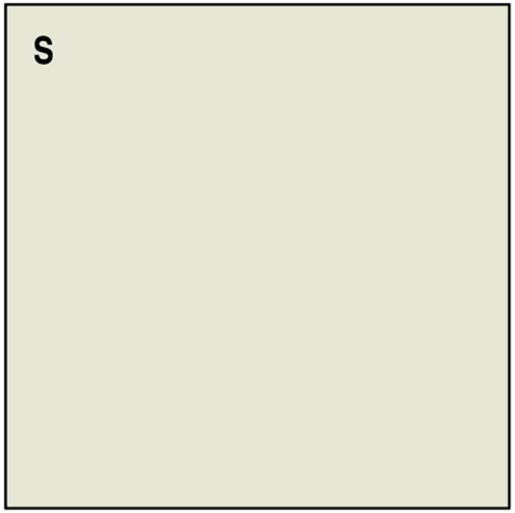
Formal Probability

2. Probability Assignment Rule:

The probability of the set of all possible outcomes of a trial must be 1.

P(**S**) = 1 (**S** represents the set of all possible outcomes.)

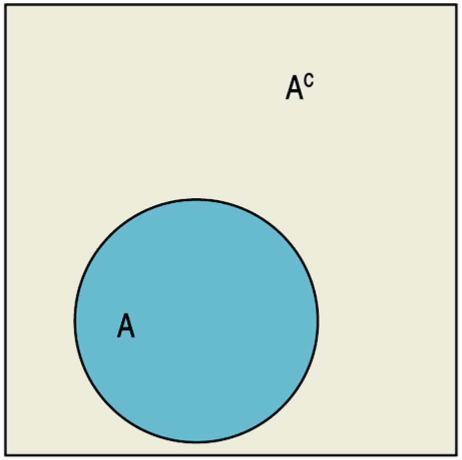


The sample space **S.**

Formal Probability

3. Complement Rule:

- The set of outcomes that are *not* in the event **A** is called the complement of **A**, denoted $\mathbf{A^C}$.
- The probability of an event occurring is 1 minus the probability that it doesn't occur: $P(A) = 1 - P(A^C)$



The set **A** and its complement.

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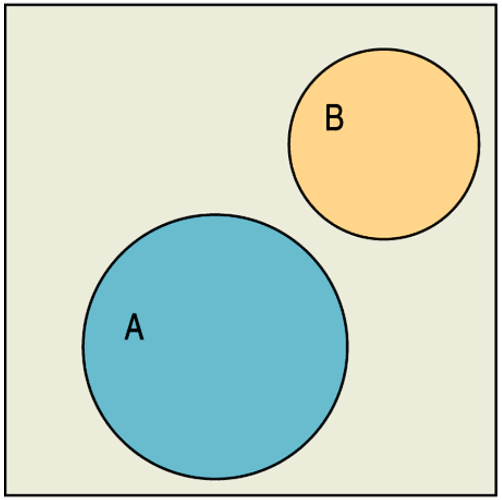
Formal Probability

4. Addition Rule:

Events that have no outcomes in common (and, thus, cannot occur together) are called disjoint (or mutually exclusive).



Two disjoint sets, **A** and **B.**

Formal Probability

4. Addition Rule (cont.):

For two disjoint events **A** and **B**, the probability that one *or* the other occurs is the sum of the probabilities of the two events.

$P(\mathbf{A} \cup \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B})$, provided that **A** and **B** are disjoint.

Formal Probability

5. Multiplication Rule:

For two independent events **A** and **B**, the probability that *both* **A** and **B** occur is the product of the probabilities of the two events.

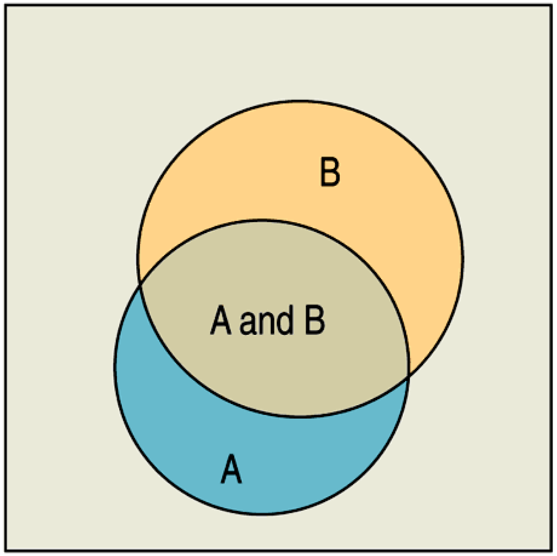
$P(\mathbf{A} \cap \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$, provided that **A** and **B** are independent.

**Independent - ** the outcome of one trial doesn't influence or change the outcome of another.

Formal Probability

5. Multiplication Rule (cont.):

Two independent events **A** and **B** are not disjoint, provided the two events have probabilities greater than zero:



Two sets **A** and **B** that are not disjoint. The event (**A** and **B**) is their intersection.

Formal Probability

5. Multiplication Rule (cont.):

Many Statistics methods require an **Independence Assumption**, but *assuming* independence doesn’t make it true.

Always *Think* about whether that assumption is reasonable before using the Multiplication Rule.

Formal Probability - Notation

Notation alert:

In this text we use the notation $P(\mathbf{A} \cup \mathbf{B})$ and $P(\mathbf{A} \cap \mathbf{B})$.

In other situations, you might see the following:

- $P(\mathbf{A} \text{ or } \mathbf{B})$ instead of $P(\mathbf{A} \cup \mathbf{B})$
- $P(\mathbf{A} \text{ and } \mathbf{B})$ instead of $P(\mathbf{A} \cap \mathbf{B})$

Putting the Rules to Work

In most situations where we want to find a probability, we'll use the rules in combination.

A good thing to remember is that it can be easier to work with the *complement* of the event we're really interested in.

What Can Go Wrong?

- Beware of probabilities that don't add up to 1.
  - To be a legitimate probability distribution, the sum of the probabilities for all possible outcomes must total 1.
- Don't add probabilities of events if they're not disjoint.
  - Events must be disjoint to use the Addition Rule.

What Can Go Wrong?

- Don’t multiply probabilities of events if they’re not independent.
  - The multiplication of probabilities of events that are not independent is one of the most common errors people make in dealing with probabilities.
- Don’t confuse disjoint and independent—disjoint events *can’t* be independent.

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What have we learned?

Probability is based on long-run relative frequencies.

The Law of Large Numbers speaks only of long-run behavior.

- Watch out for misinterpreting the LLN.

What have we learned?

There are some basic rules for combining probabilities of outcomes to find probabilities of more complex events. We have the:

- Probability Assignment Rule
- Complement Rule
- Addition Rule for disjoint events
- Multiplication Rule for independent events

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Assignment

Ch-14, pg.338 – 341: #8 -13 all, 15 – 19 all, 21 – 25 odd, 31

Read Ch-15, pg. 342 -360