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math HORIZONS

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Team Minnesota Sculpts Winning Mathematical Art

Stan Wagon
Macalester College

Rich Seeley

At the 2007 Breckenridge International Snow Sculpture Competition, the Minnesota team of David Chamberlain, Stan Wagon, Dan Schwalbe, Rich Seeley, and Beth Seeley won second place. A spokesperson for the team explained, "the most difficult artistic decision was whether to carve a rectangular box or a perfect cube with a base." Team Minnesota has a long and successful history of carving beautiful surfaces (see images to the right).

Stan Wagon teaches mathematics at Macalester College and his web page `stanwagon.com` has complete records of the team's snow sculpting work.

Stan Wagon

Rhapsody in White, Second Place, 2000 Artists' Choice, People's Choice Robert Longhurst, designer

Stan Wagon

A Twist in Time
Honorable Mention, 2002
Bathsheba Grossman, designer

Stan Wagon

Whirled White Web, Second Place, 2003 US Snow Sculpture of the Year
Brent Collins and Carlo Séquin, designers

Carl Scofield

Cool Jazz, Second Place, 2007
David Chamberlain, designer

Math Horizons is for undergraduates and others who are interested in mathematics. Its purpose is to expand both the career and intellectual horizons of students.

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In this Issue

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HORIZONS

From the Editors

Dear Readers,

Nobody reads the Letter from the Editors! So if you are reading this, then please stop reading it right now. We warned you.

If this issue of *Math Horizons* looks a little funny to you, it's because it was intended that way. Our theme is April Fools, and as a result, most of the content is satirical, paradoxical, or simply humorous. The centerpiece of this issue is our first issue of the tabloid *Mathematical Enquirer*, where everything is bogus.

So don't believe everything you read—including this letter. You've been warned!

Jenny and Art

Instructions for Authors

Math Horizons is intended primarily for undergraduates interested in mathematics. Thus, while we especially value and desire to publish high quality exposition of beautiful mathematics, we also wish to publish lively articles about the culture of mathematics. We interpret this quite broadly—we welcome stories of mathematical people, the history of an idea or circle of ideas, applications, fiction, folklore, traditions, institutions, humor, puzzles, games, book reviews, student math club activities, and career opportunities and advice. Manuscripts may be submitted electronically to Editors Arthur Benjamin, benjamin@hmc.edu, and Jennifer Quinn, jquinn@awm-math.org. If submitting by mail, please send two copies to Arthur Benjamin, Math Department, Harvey Mudd College, Claremont, CA, 91711.

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"Hemlock bartender. Bring me a glass of hemlock."

Pythagoras's Darkest Hour

Colin Adams
Williams College

"What the Hades is the matter with you?," Triangulus asked as he leaned over Pythagoras, whose face was buried in the wine-soaked sleeve of his toga, atop the stone bar. Pythagoras lifted his head groggily.

"What do you want? Leave me alone."

"Oh please, Pythagoras, it doesn't do you any good to drown your sorrows. You are taking this whole thing much too hard." Pythagoras waved him away.

"Hemlock, bartender. Bring me a glass of hemlock."

"Don't be so melodramatic. It's just one theorem."

"Oh, right. Just one theorem. I have been humiliated in front of the entirety of Greek civilization. I have been made a fool. My name will go down in history. They will call it the Pythagorean Folly. Hold it up to young children as an example of pure stupidity."

"You are exaggerating. In a week, no one will even remember."

"Right! Like they don't remember the Trojan Horse. Like they forgot Oedipus and his tiny mistake. I think you have a slightly warped view of Greek forbearance. Some poet will write an epic entitled the Pythagoriad, all about what a goat brain I am. It'll rise to the top ten and stay there forever."

Pythagoras began to suck on his wine-soaked sleeve.

"Hey, stop that. Remember who you are. You're the leader of the Pythagorean School of Mathematics."

"Yeah, right. The enrollments have been dropping like the Athenians in the Persian Wars. In another two days, the Pythagorean School of Mathematics will have an enrollment of one, and that's because you're my slave. By Zeus, I don't have a hope in Hades of getting out of this. What made me think I was a mathematician? Huh? I could have been perfectly happy doing something with my hands. A farmer maybe. Or a sandal maker. I always thought I had a good eye for fashion sandals. But no, I had to be a mathematician."

"Pythagoras, I think it was a good idea. $x^2 + y^2 = z^2$. It has a ring to it."

"Yes, it does, but unfortunately, it isn't true."

"Well, no, I guess not. What made you think that the radius, circumference and area of a circle would be related like that?"

"Oh, I don't know. It's just such a pretty equation, that's all. You can always hope."

"Yes but you probably should have checked at least one example before announcing the theorem to the Assembly."

"Well, that's apparent now. But give me a break. We're living during the birth of mathematics here. Some of these things aren't so obvious."

"Well, maybe there's some way to save it."

"Yeah right."

"What if you kept the equation but tried it on some other object?"

"What do you mean? Like a pyramid? The Egyptians are smart. Don't you think that if it applied to a pyramid, they would have noticed by now?"

"True, true, but what about something simpler?"

"Like what?"

"Oh, I don't know. What about a square or a rectangle?"

"Nah, Triangulus, that's no good. There are only two side lengths and all the angles are identical right angles. And the area is just the product of the two side lengths. Nothing works there."

"Well, I guess you need something with three related quantities, maybe all lengths."

"You mean like a comb, with three teeth? No, the teeth would all have the same length. How about a family, with a mother and father and child? We could compare their heights."

"Seems an unlikely relation to hold, Pythagoras, at least for most families. How about we stick to more geometric objects, like a triangle?"

"Oh, I see where you are going. Make x, y and z the angles of a triangle. Not bad, not bad. Not true, but not bad."

"Actually, I was thinking more the side lengths."

"Yeah right, Triangulus, like that could work. It's not even true for an equilateral triangle."

"Hmmm, well, maybe we're just not coming at it from the right angle."

"Yeah, the right angle. That's what we need all right." Pythagoras began to suck on his sleeve again. Triangulus tugged it out of his mouth.

"Pythagoras, what about this? What numbers satisfy your equation?"

Cartoon by Brad Fitzpatrick

"Well, there's 3, 4, and 5. $3^2 + 4^2 = 5^2$."

"Can't you make a triangle with sides 3, 4, and 5?"

"Sure, there's a well-known triangle, actually a right triangle with side lengths 3, 4, and 5."

"Well, there."

"In fact, any constant multiple of 3, 4, and 5 satisfies the same equation, and that corresponds to scaling the triangle up or down."

"Okay."

"And 5, 12, and 13. They satisfy the equation. And there is a triangle with side lengths 5, 12, and 13! A right triangle!"

"There you go!"

"Triangulus. You are a genius. I think this is a theorem! After all, it works for at least two examples and all their multiples. Given a right triangle, with sides x, y and z where z is the hypotenuse, then $x^2 + y^2 = z^2$. That's it. I'm not sure how to prove it, but with some thought, we can figure that out later. Let's go announce it to the Assembly. My school is saved. My reputation is saved! And this theorem will go down in history as the Pythagoras Triangulus Theorem. And I will make you a free man."

"Oh, Pythagoras, that would be wonderful."

"Yes, yes of course…but on second thought, I don't know if the Pythagoras Triangulus Theorem is such a good name. It's a bit on the long side."

"Oh…..okay."

"And if I free you, then who would there be to pick up the togas at the laundry? And peel the grapes I eat? And figure out the bills? You know how terrible I am with arithmetic. Actually, Triangulus, I'm afraid I can't free you after all."

"I understand, Pythagoras."

"Now let's see. Getting back to my $x^2 + y^2 = z^2$ formula, it says that when x and y are both 1, z^2 must be 2. So an isosceles right triangle with legs of length 1 would have a hypotenuse whose square is 2. I've never seen a number whose square is 2, but since all quantities can be expressed as a fraction of integers, the numerator and denominator of this quantity must be pretty easy to find. I'll tell the Assembly that I have found a truly marvelous fraction whose square is 2, and challenge them to find it!"

"But are you sure such a fraction exists, Pythagoras?"

"Don't be irrational, Triangulus! Of course it does."

"Okay, but perhaps we should try to find it before we challenge others to do so."

"Why don't you work on that Triangulus, and in the mean time, I will try to find integers that satisfy the next equation $x^3 + y^3 = z^3$. That can't be much harder. And hey, if we just increase the exponent, this should be enough to keep us busy for the next two and a half millennia."

"I am sure it will, Pythagoras. I am sure it will."

> **"You might wonder exactly how many dots are hiding where there seems no place to hide…"**

Hide and Seek

Andy Martin
University of Kentucky

Imagine the real number line, stretching left and right, as a string of (dimensionless) dots and asterisks. The asterisks represent the rational numbers, and the dots the irrational. Now, for each rational number r in the open interval (0,1), choose a (nonzero) rational distance d_r, and change to asterisks ***all*** of the dots between the asterisks at $r - d_r$ and $r + d_r$. Thus the interval $[r - d_r, r + d_r]$ is now represented by a solid string of asterisks. There are infinitely many rationals, scattered densely throughout (0,1), each the midpoint of an interval of asterisks, and each such interval overlaps infinitely many others. It is clear that we have a proof that the entire interval (0,1) is now represented by a string of asterisks with no dots. There's no place for a dot to hide.

0*.*.*.*.*.*.*.*.*.*.*.*.*.1

Figure 1. If every irrational and rational number is represented by a * and dot respectively, and if we put a circle around each *, will every dot be covered?

Or is there? Suppose, for each rational number $r = p/q$ (in reduced form), we let the distance $d_r = 1/(6q^3)$. Thus, with $r = 1/2$, we would have $d_r = 1/48$, while for $r = 2/3$, we would have $d_r = 1/162$. Then, believe it or not, some real numbers will be uncovered. For example, the dot representing $1/\sqrt{2}$ did NOT change to an asterisk.

Indeed, if p/q is any rational in (0,1) with p and q relatively prime, then

$$\left|\frac{p}{q}-\frac{1}{\sqrt{2}}\right| = \left|\frac{\frac{p}{q}+\frac{1}{\sqrt{2}}}{\frac{p}{q}+\frac{1}{\sqrt{2}}}\left(\frac{p}{q}-\frac{1}{\sqrt{2}}\right)\right| =$$

$$\left|\frac{\frac{p^2}{q^2}-\frac{1}{2}}{\frac{p}{q}+\frac{1}{\sqrt{2}}}\right| = \frac{\frac{|2p^2-q^2|}{2q^2}}{\left(\frac{p}{q}+\frac{1}{\sqrt{2}}\right)} \geq \frac{1}{\left(\frac{p}{q}+\frac{1}{\sqrt{2}}\right)2q^2}$$

(This last follows from the fact that $\sqrt{2}$ is irrational, so that $2p^2 - q^2$ is a nonzero integer.) But now, noting that each term inside the parentheses is less than or equal to 1 and that $q \geq 2$, we can continue with the above being

$$\geq \frac{1}{(1+1)2q^2} = \frac{1}{(2)2q^2} \geq \frac{1}{(2)q^3} \geq \frac{1}{6q^3}$$

In fact, using the same argument, you can show that this covering also misses the numbers $1/\sqrt{3}$, $1/\sqrt{5}$, and $1/\sqrt{6}$, with a little extra work, it also misses $1/\sqrt[3]{2}$.

Now, you might wonder, exactly how many dots are hiding where there seems no place to hide? The answer: <u>A LOT</u>.

Theorem. There is an ***uncountable infinity*** of dots in (0,1) which did not change to asterisks.

Proof. Let $I_r = (p/q - 1/(6q^3), p/q + 1/(6q^3))$, so the length of I_r is $1/(3q^3)$. The total length L of these intervals obeys:

$$L = \sum_{r \in Q \cap (0,1)} I_r$$

$$\leq \sum_{q=1}^{\infty}\sum_{p=1}^{q}\frac{1}{3q^3} = \sum_{q=1}^{\infty}\frac{1}{3q^2} = \frac{1}{3}\sum_{q=1}^{\infty}\frac{1}{q^2}$$

$$= \left(\frac{1}{3}\right)\frac{\pi^2}{6} = \frac{\pi^2}{18}.$$

So the total length of all of the intervals of asterisks is no more than $\pi^2/18$ (slightly less than 0.55). Thus the set of dots has measure greater than .45, and so must be not only infinite, but uncountably infinite. So where are they all hiding? ■

Math Horizons / Editor Search

Do you have a vision for what *Math Horizons* could be? Current Editors Arthur Benjamin and Jennifer Quinn will soon complete their five-year term. The MAA has begun the search for the next Editor of *Math Horizons*. Interested potential candidates should see the full ad at www.maa.org.

"Although Newton was able to unravel the mathematical secrets of the laws of nature, the laws of human nature seem to have eluded his grasp."

When Lions Battle

Nicholas Tasaday
Piltdown University

"When lions battle, jackals flee." So wrote Isaac Newton to Gottfried Leibniz as their public and vitriolic feud over priority in discovering calculus began. If this quotation sounds unfamiliar to historians of science, it is because it comes from a collection of letters recently discovered in a London estate sale that is already having a tectonic effect on our current understanding of the Newton-Leibniz dispute. Indeed, the date on the previously quoted letter makes it clear that Newton and Leibniz were in fact *discussing matters with each other* as early as 1677, a turn of events that no one has previously postulated. And this, as we shall see, is only the very tip of the iceberg. The battle over priority in the discovery of calculus is arguably the most well-studied and bitter scientific dispute in history. The debate continued for centuries after the original disputants' deaths with charges and recriminations and bitterness flying back and forth across the English Channel as British mathematicians repudiated the calumnies of the Leibnizian Continentals and hurled brickbats of their own. It is only in the past thirty or so years that a consensus view on the three-century-old conflict has developed. (See Hall [3].) Our discovery shatters that consensus and suggests a shocking new explanation of events in the calculus priority war.

The Consensus View

The current consensus holds that in 1665–66, his *annus mirabilis*, Isaac Newton working alone and not telling anyone what he'd done worked out the details of differentiation, integration, and the inverse relation between them. He recognized the inherent difficulty of integration and developed series methods for approximating definite integrals. By no later than October 1666 he was essentially in possession of the ideas and techniques that comprise the first two semesters of the college-level calculus course. (See Westfall [4].) Leibniz traveled a similar path in the years 1673-76, at least as regards differentiation and integration. In 1676 Newton, in response to a request from Leibniz, wrote him two well-known letters containing some hints about differentiation and integration, but mostly concerned series manipulations and representations. Also, during a 1676 visit to London, Leibniz examined letters and draft publications about calculus written years earlier by Newton and shown to Leibniz by Newton's correspondent, John Collins. Leibniz's access to these documents and letters formed the basis for the charges leveled against him many years later that he had plagiarized the calculus from Newton.

Leibniz published first, in 1684 and 1686. Newton was at that time fully engaged in producing his masterwork, the *Principia*. Not eager to enter a priority dispute with Leibniz, but equally unwilling to forego his portion (which he counted the lion's share) of the credit, he inserted a comment into the *Principia* stating that he had told Leibniz ten years previously about his calculus discoveries. And there matters might have rested had not John Wallis and then Nicholas Fatio de Duillier taken it into their heads to publicly pick a fight with Leibniz, asserting not only Newton's priority, but also the inherent superiority of Newton's methods. Leibniz responded in print with others, most especially Johann Bernoulli, coming to his defense. Eventually Leibniz and Newton strayed from their initial positions of publicly recognizing the other's independent discovery and each accused the other of outright plagiary. The conflict lasted beyond the deaths of the main antagonists and English mathematicians scorned Continentals (most of whom were Leibniz supporters) and vice versa for a century. The accepted modern view is that Leibniz and Newton each came to his respective understandings of calculus independently of the other, but even as the opinions of most scholars have converged on this version of events, nagging questions remain. To what degree were the subordinates (e.g., Bernoulli, Wallis) campaigning with their masters' consent? And how was it that both Newton and Leibniz moved so far from their early positions of mutual respect to ones of such reckless animosity?

Priority disputes between seventeenth-century scientists were common as a result of the structure of scientific practice at the time. In the Middle Ages one gained scientific prestige by publicly posing problems to stump others and, conversely, solving the challenges posed by others. It was an advantage to keep one's methods to oneself. University positions were awarded to winners of public problem-solving competitions. As the Scientific Revolution took root, practice moved towards today's model of journal publication of ideas, methods, and discoveries, but in Newton's day the scholarly world was still in transition and nearly every scientist was

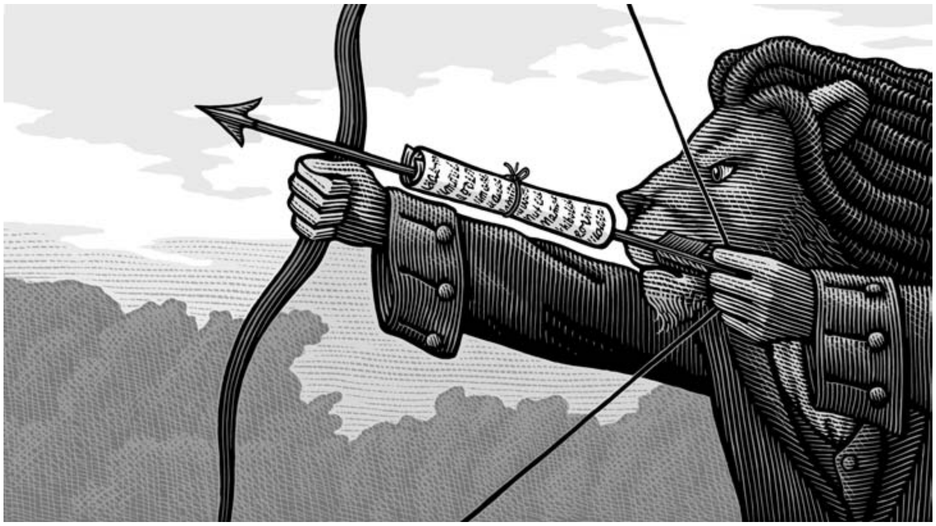


Illustration by Greg Nemec

is reasonable to surmise that they intentionally hid them from public view after Newton's death.

The packet contains thirty-four letters, all addressed to Isaac Newton. The author of the majority of the letters is Leibniz, although two are signed by Leibniz's famous bulldog, Johann Bernoulli. The first letter is dated January 1677 and was written by Leibniz to thank Newton for his two letters of 1676. The final letter is dated just two days before Leibniz's death in 1716. A book [1] containing reproductions and translations of all thirty-four letters will appear soon, as will a more thorough article [2]. Although scholarly etiquette suggests restraint, the explosive contents of these letters demands that we offer at least a preview of the radical new interpretation of events that is surely to emerge. We begin with an excerpt from the very first letter (translated from the original Latin):

involved in one or more bitter conflicts. Newton, for example, famously battled Flamsteed, Hooke, and Huygens, in addition to Leibniz. (Newton is perhaps not a reasonable example, being nearly as exceptional for his pugnacity as for his genius.) The important point here is that priority disputes were common enough that it would have been clear to both Newton and Leibniz that they were a hindrance to scientific progress, leading us again to the question of how these masters let such a thing happen. As we shall explain, it appears that although Newton was able to unravel the mathematical secrets of the laws of nature, the laws of human nature seem to have eluded his grasp.

The Missing Letters

The description for item 56AZ1/CHB-02 at a December Christie's auction was innocently labeled as "Early 18th Century Gamebooks and Seasonal Almanacs" and attributed to the library of one Lord Roswell Stephens of Sussex. As it turned out, Stephens, an avid sportsman who was known to walk with a limp due to the loss of several toes in two separate hunting accidents, was the brother-in-law of Hans De Berger, accountant and minor partner in the London accounting firm of Sokal and Conduit. Although spelled differently, this Conduit was indeed the same family as Catherine Conduitt, the niece and, for twenty years, housemate of none other than Sir Issac Newton. Conduitt and her husband John took care of Newton in his old age and, in fact, were alone with him when he died. After his death they purchased Newton's papers from his estate. Most of those papers eventually ended up in the Cambridge University Library. Item 56AZ1 is indeed a small box full of eighteenth-century gamebooks (records of game shot on the Stephens estate), but one item tucked inconspicuously near the bottom was in fact a packet of correspondence that had been mislabeled. Given the contents of these letters and the esteem of the Conduitts for Newton, it

January 13, 1677
My Dearest Newton,

I must express my profound gratitude for your letters of June and October sent on to me by Oldenburg. I have as yet only scratched the surface of the wondrous mysteries whose depths are revealed in them. I am most eager to apply myself to a thorough study of your wonderful ideas, but I felt that I must stop, take pen in hand and acknowledge your generosity. Too, I wish to express my gratitude in a more substantial fashion by explaining to you some of my own notions regarding tangents and quadratures. I suspect, from hints I discern from my first perusal of your letters, that some of these ideas are already known to you.

Imagine a vanishingly small increment of x, which we will call the differential of x, and the corresponding increment in y. …

After this follows a surprisingly modern sounding explanation of differentiation. The next several letters discuss Leibniz's discoveries in calculus and contain essentially everything one learns in a standard first course. Several of the letters refer to letters of Newton; e.g.,

June 12, 1677
My Dearest Newton,

Yes, it seems that your fluxions are identical to my ratio of differentials. As you say, I was confused by your notation, even *more was I confused by your language. You seem to be conceiving of these curves as being generated by moving points while my methods dispense with that notion and treat the curve as a static object. …*

Much of the rest of this letter, and large portions of the next five, concern the relative advantages and disadvantages of the two different notations they developed. Eventually they apparently agreed to disagree, each preferring his own notation. It is fascinating to observe this discussion (or at least Leibniz's half of it) because it reveals the differences between their intuitions, which are hinted at in the passage above. Newton had a movie running in his head of a particle traversing a path and the tangent was the direction the particle would fly off were it not constrained to the path. Leibniz had no such dynamical intuition, or at least did not exploit one in his exposition, which reads very much like that found in most modern textbooks. It is also clear from these letters, all written long before the public controversy began, that Leibniz acknowledges that Newton was in possession of the calculus long before he was and there is no hint that Newton believes anything other than that Leibniz was an independent, but second, discoverer. The air of mutual respect between these two seventeenth-century geniuses is unmistakable; but it was about to change.

Letter twelve is one of the few letters not from Leibniz, and contains the first clues as to why the recipient of all of these letters went to such lengths to conceal their existence. It was penned by Johann Bernoulli. Bernoulli, of course, would eventually make his reputation applying and defending the calculus he learned from Leibniz, but at the time of this writing he was a mere 23 years old and living obscurely in the shadow of his established brother Jakob.

February 12, 1690
Dear Sir,

I do not need to tell you how great is your reputation as a geometer and philosopher, with your recent publication Philosophiae Naturalis Principia Mathematica being only the latest evidence. Perhaps the greatest compliment I can pay you is to say that you are held in the highest esteem by the great Leibniz, my own teacher, friend and mentor. And this is why I humbly write to you with a request for some assistance with the famous problem of Galileo on the shape of the hanging chain...

Originally posed by Galileo, it is widely known that the problem of finding the proper equation of the catenary curve (as it had come to be called, *catena* is Latin for "chain") was something of an obsession for the older Jakob Bernoulli. Apparently Newton obliged the request because it was later that year, in what is now a famous effort to one-up his brother, that Johann burst onto the intellectual scene by publishing the correct solution as his own. Any doubts that Newton was indeed the legitimate author are put to rest in Bernoulli's follow-up letter where he begs for Newton's "eminent indulgence" to let the deception persist a bit longer in the cause of what amounted to a fraternal practical joke. From a June 18, 1690 letter, the younger Bernoulli writes that

…in my more mathematically naïve days, my older brother persuaded me of the convergence of the harmonic series, a "fact" that I publicly put forth on many occasions to his hysterical delight and my later embarrassment. Thus it is as a form of brotherly revenge that I have created the charade of easily solving the problem of the hanging chain that has vexed poor Jakob for so many years, and, on my most profound honor as a gentleman and a philosopher, I certainly promise to expose the true author of the solution in a timely way.

Yours most humbly,
Johann Bernoulli

But the promised announcement did not come—or at least did not come quickly enough for the ornery English mathematician—and the reason for this may be the same reason why Johann Bernoulli did not go to his mentor Leibniz for help in the first place. Put simply, Bernoulli most likely never had any intention of revealing the truth on this matter. Having smelled a rat, Newton hatched his own plan for a very particular kind of justice—a plan that would require the unknowing assistance of Gottfried Leibniz. The next letter is from Leibniz and reads with a tone of caution and confusion.

April 1, 1691
My Dearest Newton,

Yes, I do agree that not all of our colleagues appear to understand the significance of our discoveries on tangents, quadrature and series. And I also agree that it is unfortunate the degree to which the philosophical community wastes its time bickering over priority. However, I'm not sure I understand your suggested solution to these problems. Is it true that you are suggesting that we engage in a faux public dispute in order to foster a wide and vigorous dissemination of our techniques that might simultaneously convey a gentle lesson about priority disputes? Is that what you mean by "When lions battle, jackals flee?" I do agree that you and I are in a position to give some direction to our colleagues and I do feel a duty to do my best to diminish the occurrence of controversies between philosophers, but what you appear to be suggesting seems to me to at least have the potential to make things much worse. Do you really think that a messy *public battle between us will have the desired effect? Do please write more clearly about what you intend. I'm puzzled and anxious, I remain*

Your most affectionate and honored friend,
Leibniz

Over the course of the next few letters Newton and Leibniz work out their scheme with Leibniz, at first reluctant, eventually becoming convinced and enthusiastically encouraging Newton. The plan calls for Newton to convince John Wallis to "stir the coals" with a strongly worded nod to Newton's priority in his forthcoming *Algebra*. Then Leibniz is to counter by enlisting his most outspoken student to come to his defense. "I would agree with your suggestion," Leibniz writes, "that the younger Bernoulli is an excellent candidate for the part." Thus, the greatest scientific priority dispute in history was born of an object lesson gone awry and orchestrated by the author of the *Principia* himself. However, the *Principia* and its author did not take into account the principles of thermodynamics, a mistake that would prove nearly fatal as events began to heat up.

Shortly after Wallis's opening salvo appeared in 1693 Newton received the following formal sounding letter signed by his friend and containing a gift of "mutual respect."

March 15, 1693
Dear Sir,

As these arguments about priority begin to proliferate, it seems important to reaffirm our mutual respect and admiration, both for the truths hidden in nature and for the intellectual integrity you and I have exhibited for each other in our respective pursuits. In this spirit, and knowing of your ongoing researches in alchemy and related matters, I have enclosed a vial of a remarkable metal that I think you will find worthy of more investigations. I offer it to you along with my utmost esteem and regard. Know, Sir, that I am,

Yours most sincerely,
Leibniz

The content of this letter becomes chilling when one ponders the date. Newton's ongoing researches into alchemy are well-documented, and so is his mental breakdown of 1693. Speculations of a relationship between the two are common as it is known that Newton would often ingest different alchemical ingredients as part of his experiments, but this disclosure surely heightens every suspicion. Was Newton unwittingly poisoned? Or, if we adopt a more sinister mindset, was this a *deliberate* attempt at foul play? To make sense of what follows one has to imagine events as they appeared to Newton and Leibniz at this point. The arrangement these two had made had been to start a faux dispute about *priority*, but Wallis implicitly made the debate one about *plagiarism*, a charge that Newton must have assumed Leibniz was not prepared for. Leibniz gives no indication of his thinking in his subsequent letters, content to carry on in the role of willing accomplice. Newton, meanwhile, was mentally incapacitated for much of this year and was unable to engage in any sort of meaningful correspondence.

Newton recovered his health early the following year, and, as we shall see, determined to his own satisfaction that it was indeed some ingredient contained in the alchemical peace offering that had been his undoing. Taking stock of the situation, Newton saw that the budding calculus priority debate was still gaining momentum, and after a quick recalculation, he recognized a golden opportunity. Whereas Johann Bernoulli was the original target of his retribution, Newton readjusted his vengeful sights on none other than the celebrated Leibniz and adapted his scheme accordingly. In 1695 Wallis in the preface to his *Mathematical Works* states, much more plainly than he had in his *Algebra* only two years before, that Newton preceded Leibniz and, in fact, had helped the latter achieve his results. In 1699 Newton's young protégé Nicholas Fatio de Duillier openly questioned (in a volume published by the Royal Society) whether Leibniz was a plagiarist or merely "a second inventor." He also pointedly contrasted Leibniz's "eager zeal" for credit with Newton's "modesty." The gloves had most definitely come off. Experts have long wondered how much influence Newton exerted over Fatio's publication; it seems clear now that it was considerable.

Leibniz, unaware that Newton is orchestrating the new more pointed attacks, begins expressing reservations, and he urges Newton to reconsider their plan and to stop "making goats of our friends and defenders." Newton agrees—or at least pretends to—and as a final gesture proposes that he and Leibniz enter the fray and join the battle in person. The next part of the scheme, and Leibniz's growing doubts about it, are clearly laid out in a letter dated August 1703.

August 3, 1703
My Dearest Newton,

Very well then, we agree. You will assert your priority in your upcoming On Quadratures and I will review it in Acta Eruditorum and respond with my own claim. We will allow our surrogates to dispute our claims for a short period of time. Then we will report in letters published simultaneously and over both of our signatures in Acta Eruditorum and Philosophical Transactions our scheme and our intention. I, at least, will not encourage my friends to criticize you after our

Continued on page 30

> **"As the smallest abundant number, the number twelve has held great mystical importance throughout the centuries."**

A Dozen Questions About A Dozen

James Tanton
St. Mark's Institute of Math, Southborough, Massachusetts

Heavens! After all several years of dozenal articles it has only just dawned upon your correspondent that the number 12 itself is worthy of its own set of a dozen questions. As the smallest abundant number (its proper factors sum to more than itself), the number twelve has held great mystical importance throughout the centuries. The 12-faced regular polygon, the dodecahedron, was the last of the five Platonic solids to be discovered. As the first four solids were already ascribed the properties of the four elements—earth, fire, water, and air—this fifth solid, the "quintessential" solid, came to represent the essence of all four elements combined, namely, the universe. (This was particularly fitting, for mystics at the time could match its twelve faces with the twelve signs of the zodiac.) The dozen became a standard unit of quantity (though bakers often threw in an extra loaf to make a "baker's dozen" of thirteen), and "12" appears in units of measure and time. There are precisely 12 ways to arrange five unit squares edge to edge, there are precisely 12 ways to arrange eight queens on a chessboard in mutual nonattack (not counting rotations and reflections of arrangements as distinct), and in January 2002, California high-school student Brittney Gallivan broke the world record and managed to fold a piece of paper in half 12 times. The number 12 is pentagonal, the equation "$12 = 3 \times 4$" is curious (as is "$56 = 7 \times 8$"), and as the expression T W ~~O~~ E L ~~E~~ V E ~~N~~ shows, two plus eleven minus one is indeed twelve!

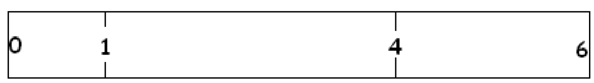
Here for your enjoyment are twelve curious questions about your correspondent's signature number.

Question 1: Quickies

1) Is 55,074,427 the sum of twelve consecutive integers?
2) Sally exclaims that twelve pages are missing from her textbook. Harold responds that the sum of the missing page numbers is even. Has Harold just incriminated himself?

Question 2: Cheap Rulers

In order to save ink, a manufacturer of six-inch rulers omitted the markings "2," "3," and "5" from their straightedges. Company managers noted correctly that it is still possible to measure all lengths one through six inches with their product.



Devise a design for a twelve-inch ruler that still allows its user to measure all lengths one through twelve inches using as few markings as possible. Prove that the number of markings you use is minimal.

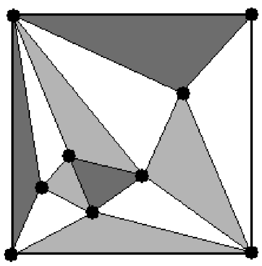
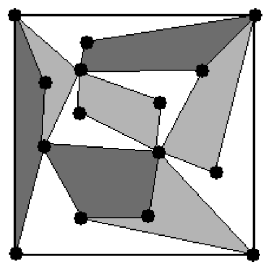
Question 3: Twisted Truths

In the following list, which statements are true and which are false?

- Precisely one of these statements is false.
- Precisely two of these statements are false.
- Precisely three of these statements are false.
- Precisely four of these statements are false.
- Precisely five of these statements are false.
- Precisely six of these statements are false.
- Precisely seven of these statements are false.
- Precisely eight of these statements are false.
- Precisely nine of these statements are false.
- Precisely ten of these statements are false.
- Precisely eleven of these statements are false.
- All twelve of these statements are false.

Question 4: Polygon Subdivision

It is possible to subdivide a square into a dozen triangles and into a dozen quadrangles in such a way that when two edges meet they do so along their entire edge-lengths.

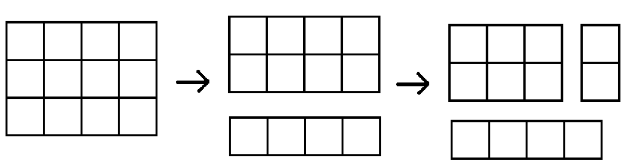
For which values of $k \geq 3$ is it possible to appropriately subdivide a square into 12 k-gons? For which values of k is it possible to subdivide a pentagon into a dozen k-gons?

Question 5: Random Clocks

Suppose the hour, minute, and second hands of a clock have come loose and one can spin them about the clock face to land at random positions. What is the probability that all three hands will land within the same twelfth of the clock face, say, between the 1 hour and 2 hour marks? How does this compare to the chances of choosing a time at random and finding all three hands of a (functional) clock lying within this same twelfth?

Question 6: Breaking Chocolate

It takes precisely eleven "breaks" to break a 1 × 12 bar of chocolate into its twelve constituent squares. What is the minimum number of breaks needed to divide a 3 × 4 bar of chocolate into twelve squares? (A "break" consists of picking up a piece of chocolate and snapping the piece along any one of its vertical or horizontal scored lines.)



Question 7: Divisibility Rules

A number is divisible by twelve if it is both divisible by three—its digits sum to a multiple of three—and divisible by four—its final two digits represent a multiple of four. Explain why the following bizarre rule for divisibility by a baker's dozen works:

> *To determine whether or not a number is divisible by 13, remove the last digit from the number and subtract nine times that digit from what remains. If the result is a multiple of 13, then so was the original number. (One can repeat this process many times to obtain a small number that is easily recognized as a multiple of thirteen or not.)*

To illustrate, test the divisibility of 88,582 by thirteen: delete "2" and subtract nine times this from 8,858 to obtain 8,840. Now delete "0" and subtract nine times this from what remains, giving 884. Now delete "4" and subtract nine times this from 88 to obtain 52. Now, if you like, delete "2" and subtract nine times this from 5 to obtain −13. This is a multiple of thirteen, thus so too was 88,582.

Question 8: A Classic Coin Puzzle

Suppose you are given twelve coins all identical in appearance. You are told that one coin is counterfeit and weighs a slightly different amount from the remaining eleven. Devise a method that will allow you to detect the false coin within just three uses of a simple two-armed balance. (Does your method also allow you to state whether the counterfeit coin is heavy or light?)

Question 9: Soccer Ball Designs

A traditional soccer ball is patterned with 20 regular hexagons and 12 regular pentagons arranged so that three edges meet at each vertex. Prove that any design on the surface of a sphere composed of hexagons and pentagons (not necessarily regular, or congruent, or even convex) with three edges meeting together at each vertex must contain precisely 12 pentagons.

Question 10: Another Clock Issue: Colored Numbers

Six of the numbers one through twelve on the face of a clock are selected at random and colored blue. The remaining numbers are painted red. Prove that one half of the clock face contains precisely three blue and three red numbers.

Question 11: Making Arrangements

Arrange the numbers one through the 12 in the boxes below in a way that respects the inequalities.

$$\square > \square < \square > \square > \square < \square > \square < \square < \square < \square > \square < \square$$

Can puzzles like these always be solved?

Question 12: A Candy Game

Twelve students sit in a circle and twelve pieces of candy are distributed randomly among them. (Some students might receive multiple pieces of candy, others none.) Lashana, who has candy, eats one piece and passes the rest to Andy on her left. Andy, who had no candy, eats a piece and passes what remains to A.J. on his left. A.J., who has candy, combines what he receives from Andy, eats a piece and passes what remains to Bea on his left. The students continue this way, hoping that the candy will “stretch out” sufficiently far so that everyone is able to eat a piece.

Prove that, no matter how the candy is distributed, it is always possible to select a student to start this process and succeed with the task of equal distribution.

Answers, Comments, and Further Questions:

1. 1) No. Any sequence of twelve consecutive numbers contains six even and six odd numbers and so sums to an even, not odd, quantity.

2) Harold has only incriminated himself as to cleverness. Each missing sheet from the text contains an odd page number on one side and an even page number on the other. The sum of the missing page numbers, whatever they may be, must be even.

Taking it Further: Is 55,074,426 the sum of twelve consecutive integers? How about 55,074,428?

Taking it Even Further: Completely classify those integers that can be expressed as a sum of two or more consecutive integers.

2. A ruler with just the six markings, “0,” “1,” “4,” “7,” “10,” and “12” does the trick. Notice that the spaces between these marks have lengths 1, 3, 3, 3, and 2, respectively, and that every number one through 12 can be represented as a sum of consecutive numbers from this list. Five markings, with spaces between them given as a, b, c, and d inches, can only produce (at most) ten distinct lengths:

a			
$a+b$	b		
$a+b+c$	$b+c$	c	
$a+b+c+d$	$b+c+d$	$c+d$	d

and so will never suffice for a twelve-inch ruler.

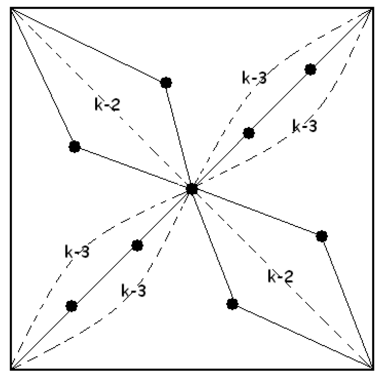
Taking it Further: What is the minimum number of markings needed to produce an N-inch ruler that successfully measures all lengths from one through N inches? Alternatively, if the number of marks permitted on a ruler is k, what is the longest ruler the manufacturer could produce? Is the “1 3 3 … 3 2” scheme always the most efficient?

3. Given their contradictory nature, no two statements can be true. If all twelve statements are false, then the final statement is a correct one, yielding a contradiction. It must be the case then that eleven statements are false and only one is true, namely, the eleventh one.

Taking it Further: Suppose the word “false” was replaced with the word “true” in each statement. Is it still possible to assign truth values to the statements in a meaningful way? What if each even numbered statement used the word “false,” and each odd numbered statement the word “true”?

Taking it Even Further: Which distributions of the words “true” and “false” among the twelve sentences yield a puzzle with a well-defined solution?

4. For a square, all values of k are permissible. For instance, if one places the indicated number of edges along the dotted lines shown, one obtains an appropriate subdivision of a square for values $k \geq 5$.



Note also that if an N-gon is divided into 12 k-gons, then the quantity “$12k$” counts each interior edge present in the diagram twice (let’s say there are I of them) and each exterior edge once. We have $12k = 2I + N$ showing that N must be even. It is therefore impossible to subdivide a pentagon, or any odd-sided shape, into 12 polygons of the same type.

Taking it Further: Can every even-sided polygon be subdivided into 12 k-gons for any given value of k?

Taking it Even Further: For which values of N, k, and c is it possible to demonstrate an N-gon subdivided into c k-gons?

5. The probabilities are the same! Consider one full rotation of the hour hand of a functional clock. It lies within the given twelfth of the clock face for one twelfth of this time (one hour). During this hour, the minute hand makes a single rotation of the face, landing in the same twelfth for a fraction of one twelfth of this time (five minutes). During these five minutes the second hand makes five full rotations, landing in the same twelfth of the face one twelfth of the time. The chances of choosing a time of day for which all three hands lie in any specified twelfth of the clock face is thus (1/12)(1/12)(1/12), the same probability as if the hands of the clocks were positioned at random.

Comment: This argument does not rely on knowing which (contiguous) twelfth of the clock face was selected for consideration. Also, the fraction one-twelfth (alas) is only mildly special here: the same argument shows that for any multiple of one-sixtieth, the chances of finding all three hands within this same segment of the clock face is the same for functional and dysfunctional clocks.

Taking it Further: Let θ be an angle between 0 and 360 degrees. Write a general formula for the probability of finding the three hands of a functional clock within a given sector of angle-width θ.

Taking it Even Further: Select a time of day at random. What is the probability of finding all three hands of a functional clock lying in some *unspecified* sector of angle-width θ? How does this compare to the random placement of hands?

6: Eleven! As each "break" increases the count of pieces by one, one must perform precisely this number of breaks to obtain twelve pieces.

Taking it Further: Suppose one were allowed to stack pieces of chocolate on top of one another to perform a simultaneous break along a common vertical or horizontal score mark. Call this a "stacked break." What is the minimum number of ordinary or stacked breaks needed to subdivide a 3 × 4 bar of chocolate into its twelve squares? Is this minimal number the same as for a 1 × 12 or a 2 × 6 bar of chocolate?

7. Any number N with final digit b can be written in the form $N = 10a + b$ for some integer a and note too that "91" is a multiple of 13 with final digit one. Now $10a + b$ is divisible by 13 if, and only if, $10a + b - 91b = 10(a - 9b)$ is. Also, since 13 is coprime to ten, this is divisible by 13 if, and only if, $a - 9b$ is. This final quantity is the number N with its final digit removed and nine times that digit subtracted from what remains.

Taking it Further: Establish divisibility rules for the numbers 7, 19, and 463. Also, use this method to describe a new divisibility rule for the number three.

8. Notice that every number from –12 to 12 has a unique three-digit ternary expression using the digits "1," "0," and "–1." For example, the number 10 (= 9+ 1) in base three is 1 0 1, the number 11 (= 9 + 3 – 1) is 1 1 – 1, and –2 (= –3 + 1) is 0 –1 1. Our plan is to number the coins one through 12 and arrange a method of weighing so that the three digits of the counterfeit coin are revealed during the process of three weighings.

Conveniently, there are three possible states of a two-pan balance: the left pan is down (call this state "1"), the pan is balanced (call this state "0"), or the left pan is raised (call this "–1"). If, for example, the twelfth coin is counterfeit and heavy, we would like to have a scheme that places coin number 12 in the left pan for the first two weighings. Then the code "1 1 0" will be revealed to us. Similarly, if the eighth coin were light, then we'd like a scheme that places the coin number 8 in the left pan for the first weighing, does not use the eighth coin in the second weighing, but places this coin in the right pan for the third weighing. This reveals the code "–1 0 1" for negative eight, the light eighth coin. This strategy suggests the following weighing scheme:

	Left Pan	Right Pan
First Weighing	5 6 7 8 9 10 11 12	
Second Weighing	2 3 4 11 12	5 6 7
Third Weighing	1 4 7 10	2 5 8 11

Unfortunately, this schema is meaningless: one cannot weigh eight coins in the left pan against no coins in the right pan, for example. However, we have the advantage that the effect of a heavy coin on the left balance, for instance, is the same as that of a light coin on the right balance. Thus we are permitted to shift the coin numbers in the above table between the columns and, with luck, obtain a schema that balances four coins against four at each step. Beginning with shifting the number "12" in each instance that it occurs, here's one possible arrangement:

	Left Pan	Right Pan
First Weighing	5 7 8 10	6 9 11 12
Second Weighing	2 3 6 11	4 5 7 12
Third Weighing	1 7 10 11	2 5 8 4

To see how this revised plan works suppose again coin 12 is counterfeit and heavy. The scheme in the table reveals the code "–1 –1 0" for "–12," identifying coin twelve as counterfeit. Since we have shifted the place of the numbers in our second table, we can no longer rely on the sign of the numbers we read to indicate "heavy" or "light," but this information is not lost to us! Knowing now that coin 12 is counterfeit and that on the first weighing the scale tipped to the right we can ascertain that coin 12 is heavy! As another example, suppose we conducted the three weighings as shown and obtained the code "1 –1 0." This shows we are dealing with coin 9 – 3 = 6 as counterfeit. That the scale tipped to the left on the first weighing also tells us that coin six is light. This method will always correctly identify and classify the counterfeit coin.

Taking it Further: Show that with n uses of a simple two-armed balance one can detect a counterfeit coin among a maximum of $(3n - 3)/2$ coins.

9. We make use of Euler's formula for a sphere which states that if v is the number of vertices, e the number of edges, and r the number of regions in any polygonal design drawn on a sphere, then $v - e + r = 2$. (There is a technical issue behind this. See "Taking it Further" below.) Consider a soccer ball design with h hexagons and p pentagons. Since three edges meet at each vertex, we have: $3v = 2e$. (Edges are double counted.) Also, each hexagon possesses six edges and each pentagon five. We have: $6h + 5p = 2e$. (Again each edge is double counted.) Substitute $v = 2e/3$ and $h = e/3 - 5p/6$ into $v–e + h + p = 2$ to obtain $p/6 = 2$, or $p = 12$.

Taking it Further: Euler's formula is valid only if each region considered is "simply connected," that is, possesses no hole like that of an annulus. Is it possible to create a soccer ball design with non-simply connected hexagons and pentagons, again with three edges meeting at each vertex, with a count different than a dozen pentagons?

10. For each number x from one to twelve, let $b(x)$ be the count of blue numbers among the set x to $x + 6$, working modulo twelve. Notice that the number of blue numbers from $x + 1$ to $x + 7$ differs from this count, if at all, by at most one, depending on whether we "lose" x of one color to "gain" $x + 7$ of a different color. Thus the sequence $b(1)$, $b(2)$, …, $b(12)$ represents a list of numbers that change value up or down, if at all, by at most one (and $b(12)$ differs from $b(1)$ by at most one). If the value $b(1)$ equals three, then, great, we're done. If, on the other hand, $b(1)$ is greater/less than three, then since $b(1) + b(7) = 6$ (there are six blue numbers in all), $b(7)$ is less/greater than three. There must be some intermediate value x then with $b(x) = 3$.

Comment: This argument shows that if any even count of numbers on a clock face are colored blue, then there always exists one half that contains half the blue numbers.

Taking it Further: Suppose that, in some random fashion, four of the numbers on a clock face are colored blue, another four red, and the remaining four plum. Prove that there are two periods of a day, each starting and ending on a half hour and summing to a total of six hours for which the hour hand passes through precisely two blue, two red, and two plum numbers.

11. Notice that there is nothing special about the numbers 1 through 12 here: one could just as well be asked to arrange the numbers 34 to 45, or for that matter, any twelve distinct real numbers in these boxes. The nature of the puzzle does not change.

Here's an approach that generalizes to a method of solution for any puzzle of this type. That the first inequality sign on the left is ">" suggests that we begin by inserting the largest number available to us, namely "12," in the first box. This leaves us with the challenge of arranging the numbers 1 through 11 in eleven boxes. The next inequality sign is "<," suggesting we next use the smallest number currently available, namely "1." We now have the challenge of arranging the numbers 2 through 11 in ten boxes. That the next inequality listed is ">" suggests we place the number "11" in the third box, and so on. It is clear that this method will never falter. It yields the solution: $12 > 1 < 11 > 10 > 2 < 9 > 3 < 4 < 5 < 8 > 6 < 7$.

Open Question: How many different solutions are there to this puzzle? Is there a general method for counting solutions?

Taking it Further: Twelve inequality signs are randomly selected for the "half hour" spaces between the numbers on the face of a clock. What is the probability of it being possible to rearrange the clock numbers on the face to respect those inequalities?

12. Suppose Lashana is given 100 pieces of candy in addition to the supply she receives from the initial distribution of 12 pieces and consider what happens to this pile of 100 pieces as it moves about the group. (Clearly there is now enough candy to "stretch around" the full circle once to make it back to Lashana.) The pile is depleted by one piece as it reaches each student, but is "boosted" up by 12 pieces as piles of candy from the initial distribution are picked up along the way. Thus precisely 100 pieces of candy return to Lashana. Also, there is one student, Ralph say, for which the pile of candy was reduced to its minimum count after eating his piece. This identifies Ralph as the student "most in danger" in playing this game: if 100 pieces of candy were not added to the game, then the initial distribution of candy is not likely to make it Ralph's way. But suppose we started the game not with Lashana, but with the person to Ralph's left, giving her the 100 pieces instead. We know that no person is in a worse position than Ralph, yet Ralph is able to pass 100 pieces of candy along, even after eating his piece. This means that if the 100 pieces of candy were not in play, Ralph is still able to eat the twelfth piece of candy (and pass zero candy along), and the game, without additional candy, is successfully completed.

Taking it Further: Does a "continuous" version of this problem hold? For example, suppose the hour hand of an unusual clock is powered by gasoline and that it takes precisely one ounce of gas for the hand to make one full revolution. Suppose that this amount of gas is randomly distributed among a finite (or infinite?) number of small reservoirs hidden behind the rim of the clock face. (Assume that there is a mechanism to pick up gas as the hand passes each reservoir). Is there a starting position for the hour hand for which we can be sure that the clock will run for 12 hours? If 14 ounces of gas is distributed among the reservoirs can we be sure that the clock could run for one full week?

Miracle tot solves PDEs using ESP

Strange attractors stole my girlfriend

Was Ramanujan an ET?

Mathematical ENQUIRER

Volume 1 Issue 0

April 1, 2007

New Platonic Solid Discovered

A new Platonic solid has been discovered, according to mathematicians at Metropolitan University. The solid, tentatively named the *docentahedron*, has 2000 identical faces, each of which is a regular byegon. According to Metropolitan officials, "we found the solid by accident. We had models of the five Platonic solids in a classroom when Prof. Siddhartha "Sidd" Finch, one of our older faculty members, tripped over a cube. He landed on the other four solids, destroying them. When he looked up, he noticed the new solid somehow had been created from the various broken pieces of the old models." He modestly rejected the initial name of the solid, the "siddfinchahedron."

The solid has some special, 'magical' properties, the spokesman continued. "Our chemists were able to sustain a cold fusion reaction in the interior of the docentahedron. This reaction cannot take place in any of the five, previously known Platonic solids. Somehow, all laws of physics are suspended inside this solid. We think this significant achievement will have far-reaching consequences for our world's energy needs for the future." ■

Student Missing After Math Mishap

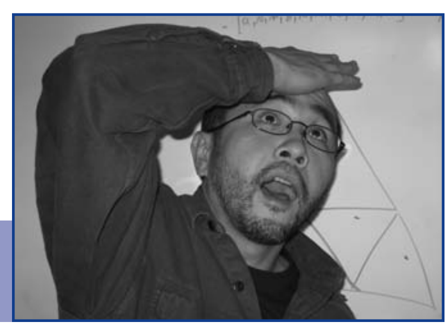
Moving with the dispatch for which faculty committees are known, the Marcenia University Special Committee on Disturbing Events (MUSCODE) has issued its official report in the matter of Stan Slapernarski. A popular math student, Slapernarski disappeared last fall, and has not been heard from since.

Although details are sketchy, the report concludes that Slapernarski (known as Slap throughout the university) fell victim to a Halloween stunt gone awry. According to the testimony of campus police officer Julius Orange, Slapernarski was getting ready for the Math/Stat Department's Halloween party, and suffered a catastrophic wardrobe malfunction. "He intended to go as a Klein Bottle and was ignorant of, or unconcerned about, that figure's inherent instability," Orange said. "Apparently, while using some stage props to alter his homotopy type, he became disoriented, fell into a pole, and diverged to infinity." Orange did not know whether alcohol was a factor.

Media coverage at the time depicted a campus in a state of shock as news of the calamity circulated. Famous mathemagician Arthur Benjamin, on hand to perform before the Halloween party, was quoted in the Daily Marcenian as saying, "It is just a terrible, senseless, tragedy, and one that I'm afraid will become all too common unless we wake up to the dangers of mathematics." He explained that the subject's rising popularity is attracting more and more students: "Unfortunately, they get caught up in the thrill of advanced experimental techniques, without adequate training or supervision. Slapernarski should never have attempted that transformation without the guidance of an experienced topologist."

Department Chairman Otto Morfeck was saddened by the release of the MUSCODE report. "The night it happened, we were in a state of chaos," Morfeck said. "Even now, months later, it is hard to accept." He explained that

Putnam Coach Forgets Pigeonhole Principle



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Slapernarski was a key member of the math club, serving as ringleader for the group. "Losing Slap has really thrown us a curve," he added elliptically.

The chair's mental state is apparently still upset, judging from the disarray of his attire. When interviewed for this story, Morfeck had his pants on inside out. Other members of the faculty tolerantly overlooked this eccentricity. "It's nothing out of the ordinary for Otto," one remarked.

Slapernarski was a celebrated campus figure and a popular student leader. His exploits as captain of the campus mathematics squad were legendary. In addition, he was the front man for a successful garage band called "Moobius." In a review in the trade weekly Vocalville, the band's signature smooth covers were described as "complex and irrational, though somewhat derivative."

Strangely, Slapernarski's disappearance is not without precedent in his family. By a macabre twist of fate, he is the name-sake of a revered grand uncle who suffered a similar misfortune. The uncle's story was chronicled by popular mathematics writer Martin Gardner, whose account can still be found in "The No-Sided Professor." Attempts to reach Gardner for comment were unsuccessful.

Some math and physics faculty still hold out hope for Slapernarski's eventual return. It is speculated that he ascended into a higher dimensional space when he assumed the Klein bottle configuration. "If he managed to untangle himself, he might have emerged again at any point of space-time. It could be somewhere far away, or nearby but still in the future," said cosmologist Ellen Hurley-Braun. "He could pop up again at any time. We can only hope that when he rematerializes, he will resume his original orientation. It would be very inconvenient if he comes back inside-out, for example." ■

Math Societies Announce Corporate Sponsorships

The Mathematical Association of America (MAA) and The American Mathematical Society (AMS) have decided to follow the lead of the NCAA and major sporting arenas by renaming several important theorems and axioms. (See bottom of the page.)

In addition, the AMS announced that the Fundamental Theorems of Arithmetic, Calculus, Algebra and Galois Theory will now be known as the Capital One, AFLAC, Halliburton and Nazareth National Bank Fundamental Theorems, respectively. According to the AMS website, "this may make it difficult to remember what individual theorems say, but most students don't remember those theorems fifteen minutes after the final exam, anyway!" The MAA proudly revealed that these corporate sponsorship deals have already raised over $100 for the society. The MAA asks authors and teachers to henceforth use the new, improved names. They also encourage all mathematicians to attend next year's Annual Joint AMS-MAA-Enron Meetings. ■

Electronic Journal of Computational Mathematics Launched

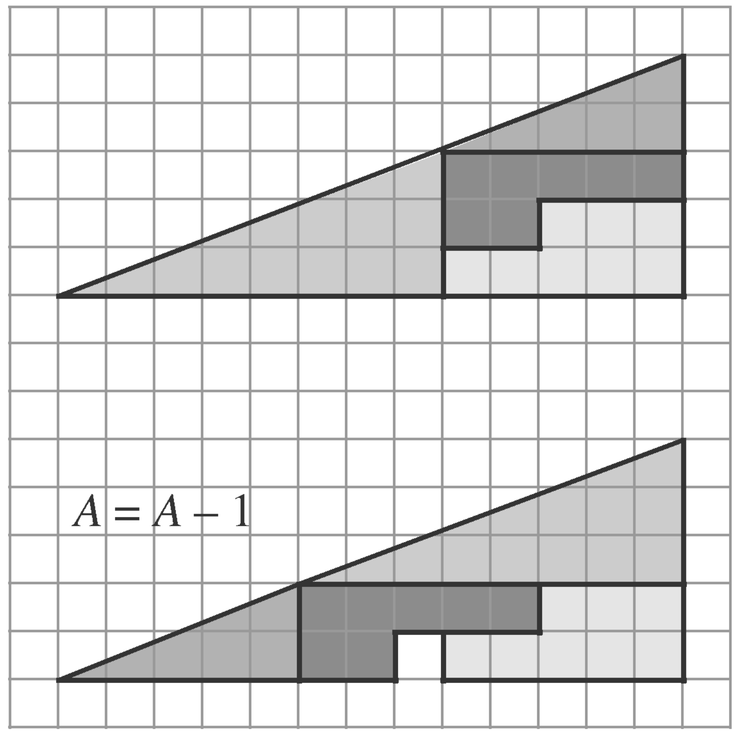
The Atari Corporation announced a new research journal, exclusively devoted to articles written by computers, for computers. This journal fills a much-needed gap in what current research journals offer. According to Shalosh B. Ekhad, spokescomputer for the new journal, "Computers have made tremendous advances in all fields of mathematics in the past forty years. Humans are no longer necessary for the most important new research." The spokescomputer offered a spectacular example: "We were able to prove the twin-primes conjecture simply by checking all positive integers. No human could possibly do that," it boasted.

The journal, which will be edited by computers which have been discarded, is referred to affectionately by its nick-

Old Name	New Name
Mean Value Theorem	Costco Value Theorem
Pythagorean Theorem	Dr. Pepper's Triangle Rule
Axiom of Choice	People's Bank Choice Axiom
Zorn's Lemma	Just Born's Hot Tamales Lemma
Banach-Tarskii Paradox	Sun Microsystems Oxymoron Paradox
Hilbert's Nullstellensatz	Mrs. Filbert's Zero-calorie Satz
Riemann-Roch Theorem	Rolling Rock Theorem

name: 11235813. The editorial offices will be located beneath the Hackensack River bridge on the New Jersey Turnpike. The first volume, which occupies some 20,000 yottabytes, gives a 'new' proof of the four-color theorem, eliminating any step that could possibly be checked by human beings. In a subsequent issue, the computers plan to give a 'one-line solution' to the P vs. NP problem (although the line will include an infinite loop). ■

Proof without words:
The area A of a triangle equals A minus one.



Dr. Cadaver—Mathemortician

Meet Dr. Cadaver, Mathemortician. That is the pseudonym of Professor Sweeney Todman. But to Professor Todman the Dr. Cadaver character is more than just a dramatic role—it is a matter of life and death.

Todman got the idea for his alter ego by combining his two great passions: mathematics and mortuary science. As a child, Todman was fascinated by the allure of numbers and the austere beauty of mathematical truths. However, while growing up as part of his family's respected mortuary business, he developed a great affection and respect for this important profession. Today he is both a successful mathematician and professor, and a certified funeral director. He finds these two affinities surprisingly complementary.

"As a math teacher, I understand how important it is for students to see that mathematics can connect with life. Mortuary science gives me a novel and unique way to do that. After all, what could be more universal in life than death?" he asked in a recent interview. "And when students see how, even in the mortuary arts, mathematics has a role to play, well, they cannot help but be impressed. Plus, the combination of these topics provides a terrific contrast. Once my students learn about rates of decay and embalming theory, they seem eager to return to the study of calculus with a renewed rigor."

Dr. Cadaver arrives to teach Calculus

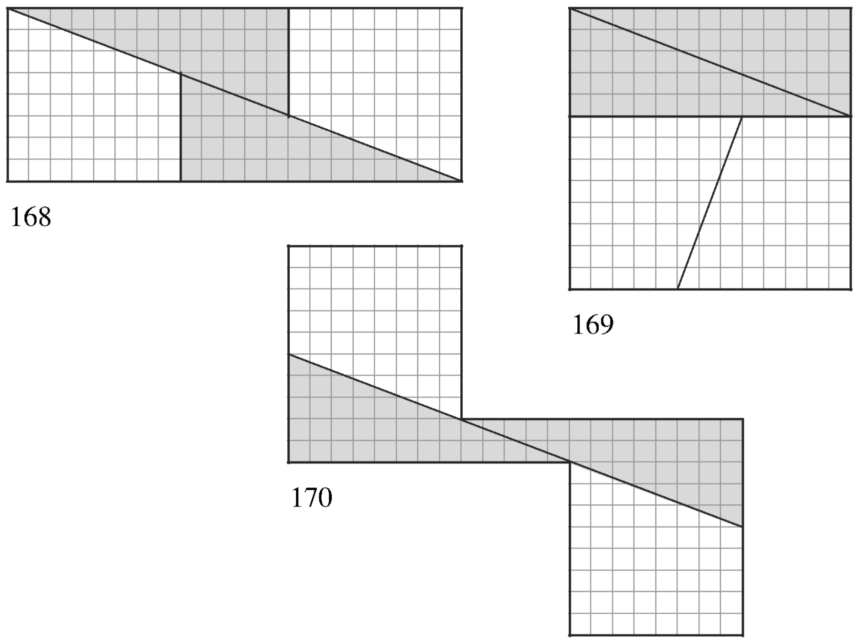
As Dr. Cadaver, Professor Todman can bring the abstract mathematical topics from his classes into the real world. He explained it this way: "Death is something everyone knows about and everyone can relate to. My experience and accomplishments in the field give me a terrific credibility. Some students walk into my class convinced that math is a dead subject. But believe me; they don't walk out that way. I know what's dead and what isn't. And my students know I know."

Professor Todman says his dual professions serve the university in multiple ways: "It enriches my classes, for sure. Plus, I can offer my colleagues and members of the university community great discounts during times of bereavement." Anyone who wishes to take advantage of this opportunity is encouraged to visit the website `www.mathemortician.com`. Tell them Dr. Cadaver sent you. ■

NBC announces new TV show: *ƎQUATIONS*

NBC has announced a new mathematics themed show, described as "a highly original, boldy creative programming move." The show will center on the criminal exploits of Quinn Tick, a mathematical genius who uses her abilities to perpetrate a series of crimes as a professor at fictional 'New York Univer-

Proof Without Words: 168 = 169 = 170



Corollary: All natural numbers are equal.

sity.' According to a press release, Quinn will use "wavelets, chaos theory, representation theory, game theory, genetic algorithms and other buzzwords to defraud the National Science Foundation. She does this by writing enormous grants claiming to solve famous open problems in mathematics, when she actually intends to settle more obscure conjectures. As a public service, forty minutes of the pilot episode are devoted to the intricacies of NSF regulations governing grant-writing." Surprisingly, eighty percent of the test market fell asleep during the pilot.

The director's first choice for the lead was the actress Danica McKellar (*The Wonder Years, The West Wing*), but unfortunately, she was busy doing mathematics. Instead, in an effort to appeal to older viewers, the network decided to cast Judy Garland in the lead. Although Garland died in 1969, the network has pieced together scenes from *The Wizard of Oz, Meet Me In St. Louis* and several other movies in which she appeared to create a seamless production. In future shows, look for Quinn to find a proof of the Riemann hypothesis by visiting a wizard, a counterexample to Goldbach's Conjecture by following a yellow brick road and an entirely new result to be called "The Trolley Theorem". ■

Math Psychic Predicts the Following Headlines!

Anguish of an aging epsilon: "I can't approach zero anymore!"

Your favorite power series reveals your past lives.

37% (really!) of all parenthesis are unnecessary.

Farmer builds three-sided rectilinear pen along river and maximizes its area!

Long suppressed government documents reveal Galois was annihilated in his dual space.

"I am not a liar," claims noted Cretan.

73% of all statistics are made up on the spot!

If the IRS had discovered the quadratic formula…

Daniel J. Velleman
Amherst College

Who Can Use Form QF?

You can use Form QF if all of the following apply.

- You need to solve an equation of the form $Ax^2 + Bx + C = 0$.
- A is not equal to zero.

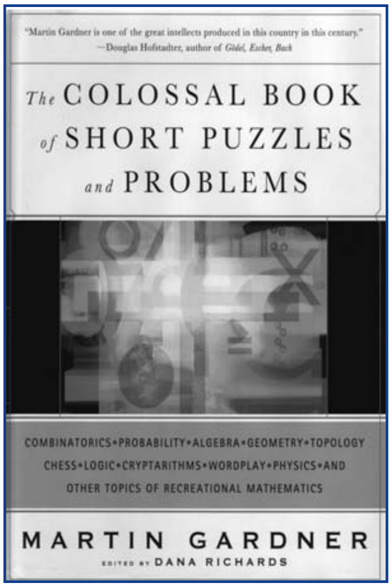
Form QF

1	Enter A here. If line 1 is zero, stop. You cannot use Form QF...	1	
2	Enter B here	2	
3	Enter C here..	3	
4a	Do you have evidence to support your values of A, B, and C?.........	☐ Yes	☐ No
b	If "Yes," is the evidence written?..................................	☐ Yes	☐ No
5	Multiply line 1 by 2....................................	5	
6	Divide line 2 by line 5..................................	6	
7	Multiply line 6 by -1	7	
8	Multiply line 3 by line 5	8	
9	Amount from line 2.....................................	9	
10	Multiply line 2 by line 9	10	
11	Multiply line 8 by 2....................................	11	
12	Subtract line 11 from line 10. If line 11 is more than line 10, leave blank and fill out Negative Discriminant Worksheet	12	
13	If amount on line 12 is zero, enter amount from line 7 on line 15, write "Dbl Rt" in space to left of line 15, and leave line 16 blank. Otherwise, take square root of amount on line 12. Check if square root is from: **a** ☐ Square root tables **b** ☐ Calculator	13	
14	Divide line 13 by line 5	14	
15	Root 1: Add lines 7 and 14	15	
16	Root 2: Subtract line 14 from line 7...........................	16	

Negative Discriminant Worksheet

1	Amount from Form QF line 5................................	1	
2	Amount from Form QF line 7................................	2	
3	Amount from Form QF line 11	3	
4	Amount from Form QF line 10...............................	4	
5	Subtract line 4 from line 3	5	
6	Take square root of line 5. Check if square root is from: **a** ☐ Square root tables **b** ☐ Calculator.........................	6	
7	Divide line 6 by line 1....................................	7	
8	Write amount from line 2, a plus sign, amount from line 7, and the letter "i." Enter here and on Form QF line 15...............	8	
9	Write amount from line 2, a minus sign, amount from line 7, and the letter "i." Enter here and on Form QF line 16...............	9	

Colossal Conundrums and Fiendish Foolers



The Colossal Book of Short Puzzles and Problems, by Martin Gardner. Published by W.W Norton. ISBN 978-0-39306-114-7. List: $35.00

The Colossal Book of Short Puzzles and Problems

Jacob McMillen, Emory University

Martin Gardner is undoubtedly one of the most prolific and beloved intellectuals in recent history. Gardner is largely responsible for popularizing and sustaining interest in recreational mathematics all over the world. He is well known for his *Mathematical Games* column that ran in the *Scientific American* for 25 years. In 2001, W. W. Norton & Company released *The Colossal Book of Mathematics*. It is a survey of many of the longer, more involved problems that Gardner addressed in his columns. The recently published *Colossal Book of Short Puzzles and Problems* is designed to complement and serve as a companion book to it. As suggested by the title, it is a collection of many of the shorter problems that appeared in Gardner's columns.

Dana Richards, a computer scientist and friend of Gardner's, compiled and organized this collection. He did a superb job with both of these tasks. The problems included have all the hallmarks of Gardner's style. The majority of problems do not require sophisticated techniques of any sort. Rather, one usually needs only basic mathematical knowledge, along with some patience and creativity to solve them. Furthermore, as can be expected from Gardner, most problems have elegant and often surprising solutions.

The layout and organization of the problems is one of the nicest features of this book. The problems are first divided by type into four main categories: combinatorial and numerical problems, geometric puzzles, algorithmic puzzles and games, and other miscellaneous puzzles. These four groups are then subdivided into a total of 17 different topics. Many different fields of mathematics are represented including combinatorics, probability, algebra, geometry, topology, and logic. Finally, for each topic, problems are arranged in ascending order of difficulty. This is highly convenient as it prevents amateurs from unwittingly running into intimidating problems, and it allows puzzle veterans to skip directly to problems that are appropriate for them.

Not only does the organization of the book make life easy for puzzle solvers, but it makes the book an extremely useful tool for educators who are looking for entertaining ways to introduce new topics to students. One can simply find the section corresponding to the desired topic and then easily choose problems of the appropriate difficulty level.

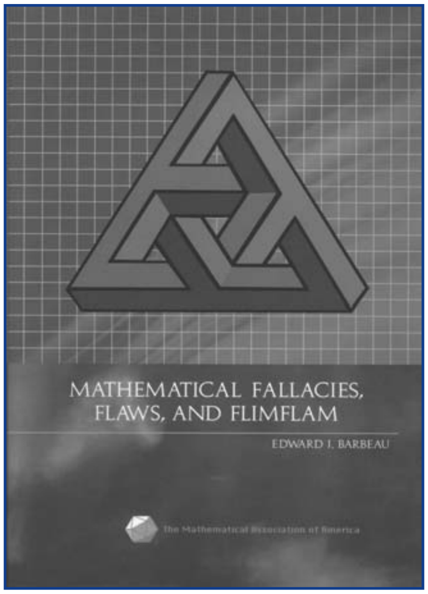
There are a total of 340 puzzles from the *Scientific American* columns in this collection. In addition to these, Gardner has included twelve of his favorite puzzles that he encountered after he stopped writing the *Mathematical Games* column in 1976. Of these twelve, nine of them have not been published previously. There is also new content in the form of additional comments that Gardner has added to various solutions.

The Colossal Book of Short Puzzles and Problems should appeal to a very wide audience. Mathematicians, students, and laymen alike can enjoy the problems that are presented in it. Gardner fans who already own the complete collection of the *Mathematical Games* columns published by the MAA will still want to consider picking it up due to its convenient layout, new content not included in the original columns, and the twelve new problems. Together, this book and *The Colossal Book of Mathematics* form an excellent survey of much of Gardner's work and are a worthy addition to anyone's personal library. ■

Mathematical Fallacies, Flaws, and Flimflam

Michael Flake, Davidson College

"Mathematics is a dangerous enterprise" warns Edward Barbeau in the introduction to his book *Mathematical Fallacies, Flaws, and Flimflam*. He is



Mathematical Fallacies, Flaws, and Flimflam, by Edward J. Barbeau. Published by The Mathematical Association of America. ISBN 978-0-88385-529-4. List: $29.00.

right! Any struggling calculus student would agree with those words as would a professor looking for an elusive research result. However, for Barbeau, the danger in mathematics comes from all the mistakes that people can make. Some mistakes are little; some mistakes are big; some mistakes appear on student's quizzes; and some mistakes are published. Analyzing and recognizing these mistakes can incite critical thought for both students and teachers. This critical thought is Barbeau's aim.

This book comes from eleven years of Barbeau's work published in *The College Mathematics Journal*. He presents some tidbits: a faulty textbook solution, a student's specious result, statistics used in journalism, or even flimflam carefully crafted by a mathematician. Then, with a mixture of mathematical prowess and wit, Barbeau tries to develop more critical thought within his readers.

Barbeau, for example, "proves" that $0 = 1$ two different ways—my personal favorite uses the function $\exp(\exp(z))$ and Picard's Little Theorem. Both proofs seem sensible, and thus Barbeau must walk his reader through the mathematical folly that allows people to arrive at the result that $0 = 1$. Readers can learn a lot from these carefully constructed examples as they must ask themselves (as I did constantly), "Why did the flimflam work?"

Not everything in the book requires knowledge of advanced topics like Picard's Little Theorem. Barbeau includes many humorous stories, such as the magazine publisher who remarked that anyone who could solve a problem like $6 - 50 = x + 20$ would "probably qualify as a nuclear scientist." In another instance Barbeau shows that people can arrive at the right answer for the derivative of x^x by using two incorrect methods to differentiate that function.

In *Mathematical Fallacies, Flaws, and Flimflam*, Barbeau covers a range of mathematical fields in great depth. This aspect of the book is perhaps its most noteworthy feature. The book includes over 170 examples from topics such as numbers, probability, geometry, finite mathematics, trigonometry, calculus, linear algebra, and modern algebra. Few people will appreciate every example in the book, but most people should understand and enjoy a large amount of the mathematical gobbledygook that Barbeau has amassed.

Personally, I enjoyed the wry, witty way in which Barbeau spoke about certain solutions, newspaper statistics, and "proofs." The sheer variety of mathematics with which Barbeau makes his readers wrestle is impressive. Barbeau's eclectic mass of examples ranges from the strange to the unexpected. Some results are puzzling and some are humorous. But his aim is clear and attained: his readers must think more critically about mathematics and must enjoy doing so. ■

Michael Flake and Jacob McMillen serve on the Student Advisory Group for Math Horizons.

"Things are seldom what they seem: Skim milk masquerades as cream." —Sir Arthur Sullivan

"Clio, the muse of history, often is fickle in the matter of attaching names to theorems!" —Carl B. Boyer

Whodunit?

Ezra Brown
Virginia Tech

The Quiz

1. L'Hôpital's Rule is as follows. If f and g are differentiable on some interval I about a, except possibly at a, with $g' \neq 0$ on $I - \{a\}$, and if $\lim_{x\to a} f(x) = \lim_{x\to a} g(x) = 0$, or if these limits are both infinite, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f'(x)}{\lim_{x\to a} g'(x)}$$

provided the latter limit exists or is infinite. L'Hôpital's Rule is due to which of these?

(a) Johann Bernoulli (b) Jacob Bernoulli (c) Big Julie Bernoulli (d) Guillaume François Marquis de l'Hôpital's (e) Gottfried Wilhelm Leibniz.

2. Laplace's Equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$$

is due to

(a) Alexis Clairaut (b) Leonhard Euler (c) Jean le Rond D'Alembert (d) Joseph Louis Lagrange (e) Pierre Simon de Laplace.

3. The Gaussian or normal distribution is the probability density function defined by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2},$$

where μ and σ denote the mean and variance, respectively. The Gaussian distribution is due to

(a) Karl Friedrich Gauss (b) Abraham deMoivre (c) Brooke Taylor (d) Pierre Simon de Laplace (e) Adrien-Marie Legendre.

4. Pascal's Triangle is the triangular arrangement of positive integers in which the k^{th} entry in the n^{th} row is equal to $C(n, k)$, the number of k-combinations of n objects. Pascal's Triangle is due to

(a) Blaise Pascal (b) Michel Stifel (c) Niccolo Tartaglia (d) Yang Hui (e) Zhu Shijie.

5. Simpson's Rule for numerical integration states that if f is continuous on the interval $[a,b]$, if n is an even integer and if $\{x_0, x_1, \ldots, x_n\}$ is the regular partition of $[a,b]$ into n equal subintervals, then

$$\int_a^b f(x)\,dx \approx \frac{b-a}{3n}(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)).$$

Simpson's Rule is due to

(a) Homer Simpson (b) the Moscow Papyrus (c) James Gregory (d) Isaac Newton (e) Thomas Simpson.

6. Pell's Equation $x^2 - dy^2 = 1$ is due to

(a) Bhaskara (b) Brahmagupta (c) Lord Brouncker (d) Pierre Fermat (e) John Pell.

7. A Vandermonde matrix is the following $n \times n$ matrix V_n formed from a set $\{x_1, \ldots, x_n\}$:

$$V_n = \begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{pmatrix}$$

The Vandermonde Determinant Theorem states that

$$\det(V_n) = \prod_{1\le i<j\le n}(x_j - x_i),$$

and is due to

(a) Augustin-Louis Cauchy (b) Arthur Cayley (c) Joseph Louis Lagrange (d) Gottfried Wilhelm Leibniz (e) Alexandre-Theophile Vandermonde.

8. The Archimedean Principle states that for any real number a, there exists a positive integer n such that $n > a$. The Archimedean Principle is due to

(a) Archimedes (b) Euclid (c) Eudoxus (d) Richard Dedekind (e) Archie Bunker.

9. The Sylow Theorems are three major results about the structure of finite groups, and are as follows: let G be a finite group of order $p^k m$, where p is a prime and p and m are relatively prime. Then (1) G has a subgroup of order p^k, called a p-Sylow subgroup, (2) all p-Sylow subgroups of G are conjugate, and (3) the number of p-Sylow subgroups divides the order of G and is of the form $np + 1$ for some integer $n \ge 0$. The Sylow Theorems are due to

(a) Peter Ludwig Mejdell Sylow (b) Georg Frobenius (c) Arthur Cayley (d) Augustin-Louis Cauchy (e) Sophus Lie.

10. The equation $e^{i\theta} = \cos\theta + i\sin\theta$, known as Euler's Formula, is due to

(a) Leonhard Euler (b) Roger Cotes (c) Abraham de Moivre (d) James Bernoulli (e) Colin Maclaurin.

11. The Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

have many combinatorial interpretations, such as the number of distinct ways to parenthesize a string of n symbols. The first few are 1, 2, 5, 14, 42, …. The Catalan numbers are due to

(a) Eugène Charles Catalan (b) Euler (c) Thomas Kirkman (d) Antu Ming (e) Johann Andreas von Segner.

12. The sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, … is commonly known as the Fibonacci sequence, and was first used by

(a) Fibonacci (b) Leonardo of Pisa (c) Acarya Hemachandra (d) Bugs Bunny or some other wascawwy wabbit (e) Édouard Lucas.

13. Stirling's Approximation $n! \approx (n/e)^n\sqrt{2n\pi}$ for $n!$ is due to

(a) Gabriel Cramer (b) Abraham de Moivre (c) Gottfried Wilhelm Leibniz (d) Colin Maclaurin (e) James Stirling (f) Brook Taylor.

14. The Maclaurin expansion of a function f having derivatives of all orders at $x = 0$ is given by

$$f(x) = f(0) + \sum_{n=1}^{\infty} f^{(n)}(0)\frac{x^n}{n!}$$

and is due to

(a) Gabriel Cramer (b) Abraham de Moivre (c) Gottfried Wilhelm Leibniz (d) Colin Maclaurin (e) James Stirling (f) Brook Taylor.

15. Cramer's Rule for linear systems is as follows. If the $n \times n$ matrix $M = [a_{ij}]$ has a nonzero determinant, then the system $\sum_{j=1}^{n} a_{ij}x_j = b_i$ $(i = 1, \ldots, n)$ of n linear equations in n unknowns has a unique solution given by

$$x_j = \frac{\det M_j}{\det M},$$

where M_j is the matrix obtained from M by replacing the j^{th} column by the column vector $(b_1,\ldots, b_n)^T$. Cramer's Rule is due to

(a) Gabriel Cramer (b) Abraham de Moivre (c) Gottfried Wilhelm Leibniz (d) Colin Maclaurin (e) James Stirling (f) Brook Taylor.

Bonus Question. Boyer's Law states that mathematical formulas and theorems are usually not named after their original discoverers. Boyer's Law is due to

(a) Carl B. Boyer (b) someone else.

See page 28 for answers and explanations

Statistician's BLUEs—

Larry Lesser
University of Texas at El Paso

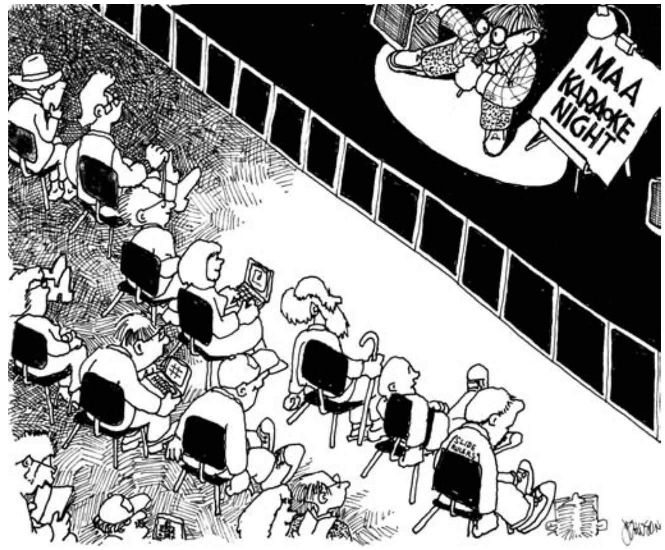
(May be sung as a standard 12-bar blues, with the words in parentheses more spoken than sung during the final 2 bars of each 12. It helps if you play a *mean* guitar!

I've been mean-in' to tell ya 'bout my last co-relation,
I've been median to tell ya 'bout my last co-relation:
She wasn't from Asia, but she was vari-ation!
(unexplained and uncontrolled!)

I saw her with ANOVA man, and they were not discrete,
I saw her with ANOVA man, and they were not discrete—
I went proba-ballistic and let out a Pearson scream!
(those deviates! I was confounded!)

Told her,
"If you're gamma data me, mu beta change your mode.
If you gamma data me, mu beta change your mode.
Chi-square you'll be inference, if you random that road!
(you'll be skewed!)"

Called up my dad: "Hi Pa! This is testing my heart!"
Yeah I told my dad: "Hypothesis testing my heart!"
He said, "What's your expectation? Ya met her at an X-bar."
("You're right, Dad! Sim-u-lator!")



Cartoon by John Johnson

She was my significant other—significant at point-oh-three,
She was my significant other—significant at point-oh-three,
But alpha get her soon—as sample as can be!
(That'll Fisher! Time serious!)

Continued from page 25

The Answers

1. (a) 2. (b) or (c) 3. (b) 4. (d) 5. (b) 6. (c) 7. (a) 8. (c) 9. (b) 10. (b) 11. (d) 12. (c) 13. (b) 14. (e) or (f) 15. (a) Bonus question. (b)

The Explanations

1. (a) Johann Bernoulli. The rule appeared in l'Hôpital's 1696 book *Analyse des infiniment petits pour l'intelligence des lignes courbes*. It is now known that the result was due to Johann Bernoulli in the early 1690s. Much ink has been spilled about both the origin and the (mis)application of l'Hôpital's Rule.

2. (b) Euler or (c) D'Alembert. Their 1761 papers on hydrodynamics introduced the equation to the mathematical world, but Laplace's work on potential (1785) and his monumental *Méchanique Céleste* (1799) made it widely known.

3. (b) Abraham de Moivre. The normal distribution was first introduced by Abraham de Moivre in an article in 1734 (reprinted in the second edition of his *The Doctrine of Chances*, 1738) in the context of approximating certain binomial distributions for large n. His result was extended by Laplace in his book *Analytical Theory of Probabilities* (1812), and is now called the theorem of de Moivre-Laplace. Laplace used the normal distribution in the analysis of errors of experiments. The important method of least squares was introduced by Legendre in 1805. Gauss, who claimed to have used the method since 1794, justified it rigorously in 1809 by assuming a normal distribution of the errors.

4. (d) Yang Hui. Yang (ca. 1275) displayed the triangle in one of his works, according to Zhu Shijie in his own 1303 treatise *The Precious Mirror of the Four Elements*. Nicolo Tartaglia first published the generalization of the figurate numbers in 1523 and a tabular form of the triangle appears thirty years later in his *General Treatise*. Stifel displayed the left half of the triangle, through the sixteenth row, in a 1544 treatise. Pascal's principal work on the triangle dates from 1665 and was used in connection with his work on probability.

5. (c) James Gregory. *The Moscow Papyrus* (ca. 1890 BCE) contains the formula $V = h/3\ (a^2 + ab + b^2)$ for the volume of a truncated pyramid of height h and base widths a and b. Let $A(x)$ be the area of a cross-section parallel to the bases at the height x. Then $A(0) = b^2$, $A(h) = a^2$, and in general

$$A(x) = \left(b + \frac{a-b}{h}x\right)^2.$$

Thus, $V = \int_0^h A(x)\,dx$. Simpson's Rule for $n = 2$ is exact for quadratic polynomials and gives the volume as $V = h/6\ (A(0) + 4A(h/2) + A(h))$, and a little algebra shows that this agrees with the result from *The Moscow Papyrus*. This degree of mathematical sophistication is perhaps unique in ancient Egyptian mathematics. The earliest recognizable forms of Simpson's Rule, however, appear in works by Gregory (ca. 1668) and Newton (1711); these precede Simpson's 1743 publication of the formula that bears his name. As for Homer Simpson, well, "D'oh!" Ironically, Thomas Simpson attributed the rule to Newton!

6. (b) Brahmagupta. In his *Brahmasphutasiddhanta* from 628, Brahmagupta described some methods for solving the equation $x^2 - dy^2 = 1$ for several values of d. He gives $x = 1{,}766{,}319{,}049$, $y = 226{,}153{,}980$ as the smallest solution for $d = 61$, an impressive piece of hand calculation. Bhaskara considerably extended Brahmagupta's work in 1150. In 1658, William Lord Brouncker (1620–1684) discovered a general method of solution equivalent to the continued fraction algorithm, in response to one of Fermat's challenge problems. John Pell's name was attached to the equation by Euler in a classic case of misattribution: Pell had nothing to do with it. The website `www.gap-system.org/history/HistTopics/Pell.html` tells the story very well.

7. (a) Augustin-Louis Cauchy. Vandermonde studied this matrix in the case in the 1770s, but the theorem for general $n \times n$ matrices is due to Cauchy in an 1815 paper.

8. (c) Eudoxus. Archimedes (late 3rd century BCE) used this principle in several treatises, including *On the Sphere and Cylinder* and *On the Quadrature of the Parabola*, and he attributes the principle to Eudoxus (early 4th century BCE). A form of the principle appears in Book V of Euclid's *Elements* (early 3rd century BCE). Dedekind's Completeness Axiom is late 19th century. Although Archie Bunker frequently invoked the "Archie-Meathead-ean Principle," his only principle is "Keep It All In The Family."

9. (b) Georg Frobenius. Frobenius proved Sylow's theorems for arbitrary groups in 1884. Sylow's original proofs from 1872 are for permutation groups. Cauchy's theorem on the existence of an element of order p in a finite group of order divisible by p was motivation for Sylow's work. Cayley and Lie had nothing to do with it—they are just two more giant groupers.

10. (b) Roger Cotes. The equivalent equation $\ln(\cos\theta + i\sin\theta) = i\theta$ was discovered by Cotes about 1713. De

Moivre's related formula $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$ appeared in 1722, and Euler used it to derive the result we now call Euler's formula. The celebrated equation $e^{i\pi} + 1 = 0$ is due to Euler and first appeared in 1748 in *Introductio in analysin infinitorum*.

11. (d) Antu Ming. The Mongolian mathematician and astronomer Antu Ming (ca. 1692–1763) wrote about these numbers around 1730. Segner's 1758 recurrence formula gives the solution to Euler's polygon division problem $E_n = E_2E_{n-1} + E_3E_{n-2} + \ldots + E_{n-1}E_2$ (J. A. von Segner, Enumeratio modorum, quibus figurae planae rectilineae per diagonales dividuntur in triangula, *Novi Comm. Acad. Scient. Imper. Petropolitanae*, 7 (1758/1759), 203-209). In 1838, Catalan showed that the number of ways of parenthesizing a product of $n + 1$ symbols using n pairs of parentheses is equal to the n^{th} Catalan number C_n.

12. (c) Acarya Hemachandra. Hemachandra presented what is now called the Fibonacci sequence around 1150, about fifty years before Fibonacci (1202). He was counting the number of rhythmic patterns $p(n)$ made up of single-beat and double-beat notes of length n and showed that these could be formed by adding a single-beat note to a pattern of length n–1 or a double-beat note to one of length n–2. The resulting recurrence $p(n) = p(n - 1) + p(n - 2)$ with the initial conditions $p(0) = p(1) = 1$ define the Fibonacci sequence. The famous Rabbit Problem of Leonardo of Pisa, also known as Fibonacci, appeared as Chapter 12, Part 7, Problem 18 in the *Liber Abaci* from 1202 and introduced the sequence to the European world. Lucas gave the sequence its name in the 19th century. The Fibonacci sequence is the only bit of mathematics that Bugs Bunny knows.

13. (b) Abraham de Moivre. The approximation

$$n! \approx cn^{n+1/2} e^{-n}$$

appears in de Moivre's 1730 work *Miscellanea Analytica*. Stirling's own subsequent contribution was the exact value of $\sqrt{2\pi}$ for the constant c.

14. (e) James Stirling or (f) Brook Taylor. Taylor described the series expansions

$$f(x) = f(a) + \sum_{n=1}^{\infty} f^{(n)}(a)\frac{(x-a)^n}{n!}$$

bearing his name in 1715, and Stirling gave the Maclaurin expansion (the case $a = 0$) in the 1730s. Maclaurin's 1742 work *Treatise of Fluxions* contains those expansions. Incidentally, Taylor series appeared in earlier works of James Gregory and James Bernoulli—but that's another story.

15. (a) Gabriel Cramer. Credit could go to Leibniz (1683) or Maclaurin (ca. 1730), but we'll go with Cramer, who treated the $n \times n$ case in 1750. Kosinski (*Math. Magazine*, October 2001, 310-312) tells the whole story.

Bonus Question. (b) Someone else. It's only logical! ■

Suggested Reading

Carl B. Boyer and Uta C. Merzbach, *A History of Mathematics* (2nd edition), John Wiley & Sons, New York, NY, 1989.

David M. Burton, *The History of Mathematics: An Introduction* (6th edition), McGraw-Hill, New York, NY, 2006.

Howard M. Eves, *An Introduction to the History of Mathematics* (6th edition), Saunders/HBJ, New York, NY, 1990.

The website

`lahabra.seniorhigh.net/pages/teachers/pages/math/timeline/MpreAndAncient.html`

has a wealth of information. Begin here and follow the time line.

Continued from page 11

public claims, I tire of making them our fools and pawns. I confess to you that I feel no great pride in my actions in this conspiracy, I wish that we had never concocted this plot. I very much doubt that we have accomplished any part of our goal of diminishing priority disputes, I fear we have made things worse. I am

Your most humble Servant.
Leibniz

In public, events proceeded as described in this letter, at least up to the point of confession, which never came. Newton asserted in his 1704 *On Quadratures* that he had invented calculus in "1665 and 1666" and Leibniz in an anonymous review of that work published in *Acta Eruditorum* (the journal he had founded) described Newton's work as essentially the same as the differential calculus "discussed by its inventor G.W. Leibniz in these *Acta*." In private, Leibniz regretted this review even before it was published and the next several letters to Newton renounce it and urge Newton to join him in an immediate public confession and apology. It is not clear from Leibniz's letters what arguments Newton is making against this course, but Leibniz's letters exhibit growing shame and increasing frustration over the next few years and eventually, in 1707, he gives up.

September 21, 1707
Dear Sir,

I have burnt your letters. Please burn mine.

Leibniz

Warfare

One might think Newton would have been satisfied with this level of torment but there was more to come. Learning that his letters had been destroyed, Newton took advantage of the fact that his tracks were now untraceable and unleashed his harshest assault to date. In 1710, Englishman John Keill in his "On the Laws of Centripetal Force" (published in the Royal Society's *Philosophical Transactions*) included a blunt accusation that Leibniz had stolen his ideas from Newton and changed the notation to cover up his crime. Leibniz, a Fellow of the Royal Society, immediately demanded an apology. Newton, at this time, was President of the Royal Society and essentially allowed Keill to reiterate his claim in the form of a public document that was sent directly to Leibniz by the Society. Leibniz wrote again the Secretary of the Society demanding an apology and, meanwhile, sent a private letter to Newton.

December 14, 1711
Sir,

I am, as you well know, not a plagiarist. I have acted dishonorably in our schemes and I am guilty of deceptions of which I did not think I was capable. I should add that I am guilty of transgressions of which you yourself may not be aware. I am sorry, I am ashamed, and I am prepared to confess all. But I am not guilty of the crimes for which I have been charged by the Royal Society, and I ask your assistance in repairing my reputation.

Leibniz

Was this what Newton wanted? Was Leibniz finally owning up to a moment of weakness from 18 years earlier in which, as Newton saw it, Leibniz tried to win the calculus priority debate by incapacitating his rival? Uninterested in reconciliation at this point, Newton had Leibniz right where he wanted him and was not about to let up. The Royal Society formed a commission to study the dispute and its report, largely written by Newton himself, concluded that Newton had invented the calculus in the 1660s and communicated the essential ideas to Leibniz in the two letters of 1676. Leibniz had digested the ideas, made some modest improvements and published the ideas with a new notation as his own creation in the 1680s. A vicious and ugly public war of words ensued as scientists all over Europe joined in on one side or the other.

At this point the correspondence breaks off, and there is not another letter for nearly five years. In fact only two letters remain, both written by Leibniz in the year of his eventual death. The first echoes with the sound of contrition as the great German mathematician feels compelled to finally lay everything bare to perhaps the only intellectual equal he has ever known.

August 12, 1716
Dear Newton,

I do not need doctors to tell me that I am coming into the twilight of my days and it is time to make peace with the moments in my life when I let my less noble self take charge of decisions to be made. It is thus with a sad, but honest, heart that I write to tell you of my real motives that dictated my dealings with you. Back in 1691 when you first proposed we stage our faux dispute, your admirable goal was to use it as a means to further proliferate the ideas and methods of our newly discovered geometry as well as provide a moral lesson to our colleagues about priority disputes. However, when you proposed your "scheme," I was at that time in a bitter personal dispute of my own with the young Johann Bernoulli. I shall not give you the details, but let it be said that he was *only an average intellectual talent, conspiring to make a name for himself, and constantly pestering me for solutions to problems that he was not able to solve with his own wit. On several earlier occasions I tried to orchestrate his mathematical demise, once stepping so low as to enlist his brother to instruct Johann that the harmonic series converged. But Johann persisted. (To this day, I am still baffled by how this mediocre mind was able to determine that Jakob's catenary was in fact a logarithmic semi-sum.)*

Needless to say, in your plan I saw a way to pursue my own devilish goal of thwarting the career of the young Bernoulli. I regret this deceit most deeply, but what I most regret is the loss of our friendship. Although my motives were not pure, I am still confused as to why you forever postponed the time at which we were to announce the end of our charade. As punishment for my sins, I accept that I may never know the whole truth, but before departing this world I felt it best to let you know the truths of my own heart in this matter.

Yours most sincerely,
Leibniz

Leibniz's candor must have affected Newton deeply because the Englishmen responded in a timely fashion with a letter of his own. Reading Leibniz's next, and final, letter it is clear that in his reply Newton finally saw fit to confront his now dying rival with the accusations that Leibniz had tried to poison him years earlier.

November 12, 1716
My Dearest Newton,

You must accept that a man perched so precipitously on death's doorstep would have no cause to utter anything but what is true and pure. In this spirit, I must tell you that on this particular charge I am as innocent as I am shocked. As to how this confusion has come about, I have no proposition. I can only assert that, not only do I have no memory of sending you any sort of "gift" in 1693, I have never concerned myself with alchemy and would not have access to any sort of offering of this kind to send. In fact, I believe it was around this time that I swore off any possible researches in this area altogether when I learned that the ever-bothersome Bernoulli, while working on his medical degree, accidentally poisoned the family hound with some derived metal that rendered the dog raving mad for the remainder of its life. My decision then was to embrace the advice I gave Johann which was, as I recall, to 'deliver these wretched ingredients to some far away place before you kill someone.'

Toward the end of the letter, Leibniz makes one final plea for reconciliation:

…I am still shamed by our scheme, and my particular role in it, but I am now ready to wholeheartedly denounce it. My shame has until now prevented confession, but no longer. I have begun work on a manuscript describing our plot and our motivation and most abjectly apologizing for the foul deception we have perpetuated upon men who counted themselves our friends and admirers. I entreat you to accept my testimony above and join me in its publication. I will send it to you imminently. It will be published in my Acta as soon as possible. I remain, Sir,

Your humble servant
Leibniz

Forensic analysis of the August 5, 1693 letter containing the poisonous vial has already provided incontrovertible evidence that the letter most certainly did *not* come from Leibniz. Indeed, when we consider both motive and means, all leads now point squarely to Bernoulli as the guilty party. (As of this writing, the handwriting and the chemical composition of the ink are being compared to known samples of Bernoulli correspondence from 1693.) The implications here are, of course, stunning. Leibniz died two days after writing the letter quoted above. No trace of the manuscript he describes in the closing paragraph has ever been found and we assume he did not have the strength to pursue this final confession. Bernoulli went on to a celebrated career, teaching Liebniz's calculus to whoever would listen. As deplorable as we now see this snake in the garden to be, we must also give Bernoulli his due. Although he was the intended target of retribution of the two greatest minds of the 17th century, he somehow managed to slither his way out of the crossfire and ended up dancing on the heads and shoulders of both of the giants.

And what about Isaac Newton? Did he accept Leibniz's denials and, by necessity, then recognize the colossal error in his own calculus? We may not ever know the answer. If, however, we are to insist on a moral to the story then perhaps it should be this: the history of mathematics, like all human endeavors, is rich with miscalculations and misconceptions, and things are not always what they seem. Let us just affirm that skepticism should ever be our byword. ■

References

[1] S.D. Abbott and S.F. Kennedy, *When Lions Battle*, Videri Nunquam Press, forthcoming.

[2] S.D. Abbott and S.F. Kennedy, "The Leibniz Letters to Newton 1677-1716", *Histeria Mathematica* [23], June 2007.

[3] A. Rupert Hall, *Philosophers at War*, Cambridge University Press, 1980.

[4] Richard S. Westfall, *Never at Rest*, Cambridge University Press, 1980.

Problem Section

Editor
Andy Liu
University of Alberta

This section features problems for students at the undergraduate and (challenging) high school levels. Problems designated by "S" are especially well-suited for students. All problems and/or solutions should be submitted to Andy Liu, Mathematics Department, University of Alberta, Edmonton, Alberta T6G 2G1, Canada. Electronic submissions may also be sent to aliu@math.ualberta.ca. Please include your name, email address, school affiliation, and indicate if you are a student.

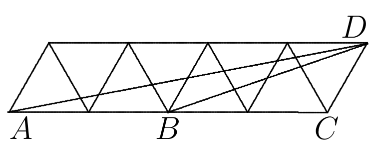
Starting from the first issue of the next volume (September 2007), the column will feature four problems per issue. The high school problems will appear in November and April, and I will continue to look after that. The regular problems will appear in September and February, and will be handled by Derek Smith. His contact information is: Department of Mathematics, Lafayette College, Easton, PA 18042; phone: 610-330-5283; FAX: 610-330-5721; email: smithder@lafayette.edu. The deadlines are as before.

In this issue, we offer two problems so as to bring each set to the next multiple of four. Continue to send solutions up to this issue to me.

Proposals

To be considered for publication, solutions to the following problems should be received by August 10, 2007.

S119. *Proposed by Cheng-Chiang Tsai, student, Taiwan University*. Compute $\angle BAD + \angle CBD$ in the following parallelogram that is formed from eight congruent equilateral triangles.



S120. *Proposed by Yakub Aliyev, Baku State University*.
(a) Does there exist a function f from the set of positive real numbers to itself such that $f(x+y) = xf(y) + y^2 f(x)$ holds for all positive real numbers x and y?

(b) Does there exist a function f from the set of positive real numbers to itself such that $f(xy) = f(y)^x f(x)^{y^2}$ holds for all positive real numbers x and y?

Solutions:

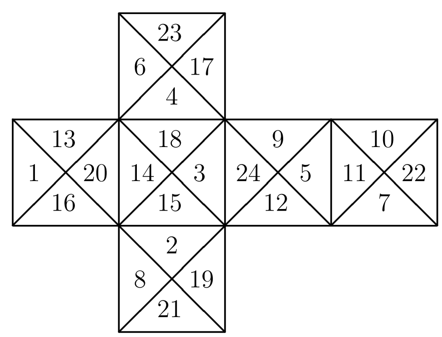
S113. A Balanced Die. *Proposed by Lou Zocchi*. Divide each face of a cube into four isosceles right triangles by its diagonals. Label the 24 triangles so obtained with the numbers from 1 to 24, using each of them exactly once, such that the following conditions are satisfied:

1. The sum of the numbers on the four triangles of each face of the cube is exactly 50.

2. The sum of the numbers on two opposite triangles is exactly 25. (Two triangles are opposite if they lie on opposite faces of the cube and their hypotenuse are opposite edges of the cube.)

3. The sum of the numbers on the six triangles surrounding each vertex of the cube is as close to 75 as possible.

Solution by Alex Briasco-Brin, Freeport Middle School, Maine and Josh Purinton, student, University of Massachusetts, independently.



Also solved by Solomon Adera (student), Alma Caballero with Tony Tam and the proposer. All these solutions, however, do not satisfy condition 3, in view of the featured solution where the sum of the numbers around each vertex of the original cube is exactly 75.

S114. Upsets in a Tournament. *Proposed by Linda Yu*. In a chess tournament, each of the 89 participants plays a game against each of the other 88. A win is worth 1 point, a draw 1/2 point and a loss 0 points. A participant's score is the sum of the points gained from all 88 games. After the tournament, each of the (89)(88)/2 = 3916 games is reviewed. A game is called an upset if it is not a draw, and the score of the winner is lower than that of the loser. Is it possible for the number of upsets to exceed 2000?

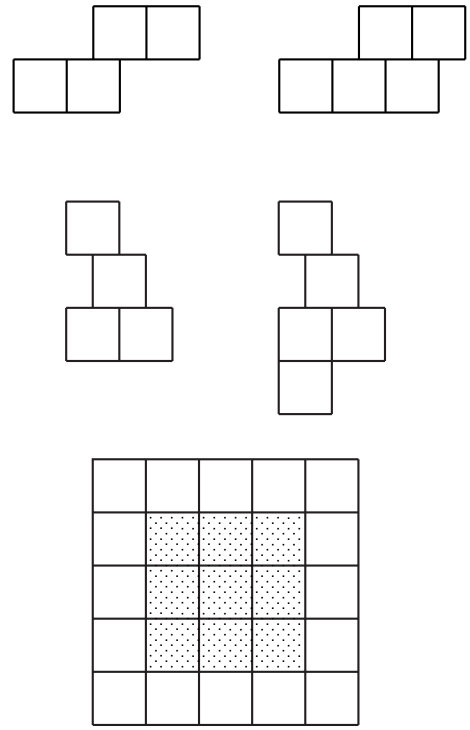
Solution by the proposer. It is possible, and we give a construction. Divide the 89 players into ten groups, $A_0, A_1, A_2, \ldots, A_8$ and B, each with 9 players except that B has 8. All games within each group are draws, except that each of four players in B has exactly one win and each the other four has exactly one loss. For $0 \le k \le 8$, each player in A_k wins $8 - k$ games and loses k games against the players in B. The players in B collectively win 324 games and lose 324 games. Each of the four players with a win within B wins 40 of these games and loses 41 of them, while each of the other four wins 41 of these games and loses 40 of them. It follows that the score of each player in B is 44. For $0 \le k \le 8$, let each player in A_k win against every player in A_{k+5}, A_{k+6}, A_{k+7}, and A_{k+8}, and lose against every player in A_{k+1}, A_{k+2}, A_{k+3}, and A_{k+4}, with the subscripts adjusted modulo 9. It follows that the score of each player in A_k is $48 - k$. We now count the number of upsets.

1.
Each player in A_0 is upset by the players in A_1, A_2, A_3, and A_4. Each player in A_1 is upset by the players in A_2, A_3, A_4, and A_5. Each player in A_2 is upset by the players in A_3, A_4, A_5, and A_6. Each player in A_3 is upset by the players in A_4, A_5, A_6, and A_7. Each player in A_4 is upset by the players in A_5, A_6, A_7, and A_8. Each player in A_5 is upset by the players in A_6, A_7, and A_8. Each player in A_6 is upset by the players in A_7 and A_8. Each player in A_7 is upset by the players in A_8. We have a total of $9\times9\times26 = 2106$ upsets.

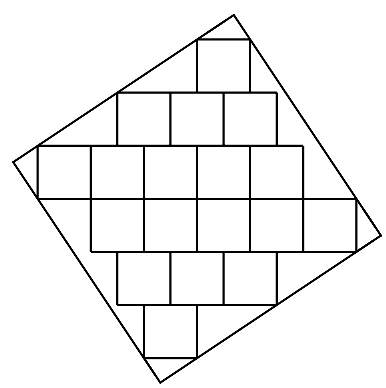
2. Each player in A_1 is upset by one player in B. Each player in A_2 is upset by two players in B. Each player in A_3 is upset by three players in B. We have a total of $9\times6 = 54$ upsets.

3. The players in B are upset by each player in A_5 three times, each of those in A_6 twice and each of those in A_7 once. We have a total of $9\times6 = 54$ upsets. The grand total is $2106+54+54=2214$. *While it may be surprising that the fraction of games that are upsets can be higher than a half, the maximum value actually approaches three-quarters.*

S115. Packing and Covering. *Proposed by Rikishi Yamada.* Two of the following four pieces have area 4 and the other two have area 5. Place all of them into the square board of area 25, without overlap, so that the central 3×3 subboard is completely covered up. The pieces may be rotated or reflected.



Solution by Sheng-Ho Chiang, student, National Experimental High School, Hsinchu, with Chen-Yu Yang, student, Chien-Kuo High School, Taipei.

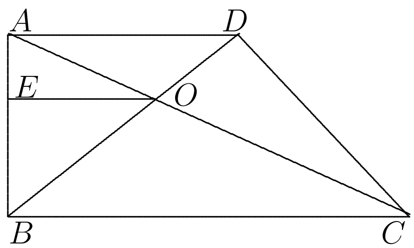


Also solved by Matthew Vicksell (student) and the proposer. It is tempting to conclude that the problem has no solutions, but this is only if we restrict ourselves to orthogonal placements of the pieces.

Problem 205. Root of a Quartic. *Proposed by Árpád Bényi.* In the quadrilateral $ABCD$, both AD and BC are perpendicular to

AB. The diagonals *AC* and *BD* have respective lengths 3 and 2, and they intersect at the point *O*. Let *E* be the foot of perpendicular from *O* to *AB*. If $OE = 1$ and $f(x) = \frac{x^2+x+1}{\sqrt{4x^2+2x+1}}$. Determine $f(BE/AE)$.

Solution by Megan Zigarovich, student, Washington and Jefferson College.



Let $AE = a$ and $BE = b$. Since triangles *AEO* and *ABC* are similar, $BC = (a+b)/a$. Since triangles *BEO* and *BAD* are similar, $AD = (a+b)/b$. By Pythagoras' Theorem, $9 = (a+b)^2 + (a+b)^2/a^2$ and $4 = (a+b)^2 + (a+b)^2/b^2$. Subtraction yields $5 = (a+b)^2\,(b^2-a^2)\,/a^2b^2$. Now let $x = b/a$. Then we have $x^4+2x^3-5x^2-2x-1=0$. This may be rewritten as $(x^2+x+1)^2 = 2(4x^2+2x+1)$. It follows that $f(x) = \frac{x^2+x+1}{\sqrt{4x^2+2x+1}} = \sqrt{2}$.

Also solved by Tyler Raspat (student), Ben Small (student) and the proposer. Two other solutions by numerical methods yield only approximate answers.

Problem 206. Limit of a Sum-Product. *Proposed by Paolo Perfetti*. Let $\{a_i\}$ be an infinite sequence of nonnegative real numbers with a finite sum. Determine

$$\lim_{n\to\infty} \sum_{k=n+1}^{2n} \frac{k}{n}\left(\prod_{j=1}^{n} a_{k+j}\right)^{\frac{1}{n}}.$$

Solution by Northwestern University Math Problem Solving Group.

By the Arithmetic-Geometric Means Inequality and using $k \le 2n$, we have

$$0 \le \sum_{k=n+1}^{2n} \frac{k}{n}\left(\prod_{j=1}^{n} a_{k+j}\right)^{\frac{1}{n}} \le \frac{2}{n}\sum_{k=n+1}^{2n}\sum_{j=1}^{n} a_{k+j}$$
$$\le 2\sum_{k=n+2}^{3n} a_k \le 2\sum_{k=n+2}^{\infty} a_k.$$

Since

$$\sum_{k=1}^{\infty} a_k$$

is convergent, the last expression tends to 0 as *n* tends to infinity. The desired result follows immediately.

Also solved by Robert Agnew and the proposer.

Loose-ends:

Robert Agnew was the first reader to draw my attention that in the February 2007 issue, problems S113, S114, S115, 205, and 206 should have numbered S116, S117, S118, 207, and 208, respectively. Also, in the solution to Problem 204, the restated problem should be evaluate (a) $\int \sqrt{\tan\theta}\, d\theta$; (b) $\int_0^{\pi} \frac{\theta \sin\theta}{1+\cos^4\theta}\, d\theta$.

New Mathematical Definitions and Contest

Ken Suman
Winona State University

Math Horizons has scoured primary sources across the globe to bring you these new, exciting mathematical definitions.

affine variety: what a good store carries

asymptote: how the farmer got his lame donkey home

bias: the two words parents hear most in a toy store

catenoid: what you get if you grab a tiger by the tail

cochain: what Joe Paterno is known for

demand curve: what students often do

Euler: Jiffy Lube employee

field axioms: ground rules

floor function: to "give me a place to stand"

integration by substitution: forced busing

into mapping: what a cartographer is

lim sup: cannibal favorite on cold damp days

partition into odd parts: gerrymander

polygon: bird flu

recursive relations: Aunt Marge casting invectives again

semigroup: convoy

subcover: lettuce and tomatoes

subtracted: nonplussed

topologically distinct: holier than thou

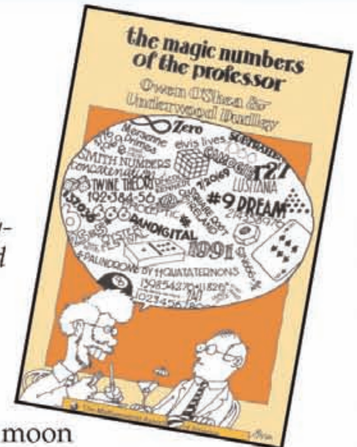
More definitions like these, called "Mathematical Equivoques," can be found at `http://course1.winona.edu/ksuman/Dictionary/Dictionary.htm`.

Contest:

Seek, search, or sift through your own mathematical "definitions" and send your best examples to us at jquinn@awm-math.org. The *Math Horizons* Student Advisory Group will judge all entries received by June 30, 2007. Winners to be announced and prizes to be awarded at a future time.

Check out these new titles from The Mathematical Association of America

The Magic Numbers of the Professor



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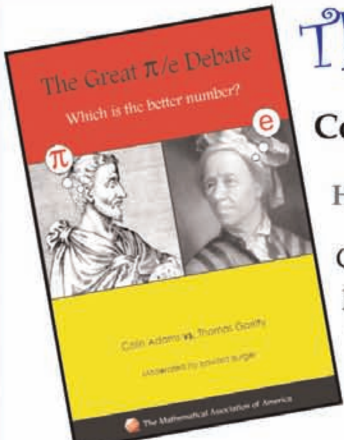

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