Fronts and Frontogenesis

Definition

- The definition of a front varies:

 * from classical polar-front theory, and in popular usage, it is the boundary between two air masses. Media people often talk about “clash” between air masses; indeed, usage of term “front” was likely influenced by WWI being contemporary with its development (Bjerknes 1919). This idea suggests the front approaches a “discontinuity” in some atmospheric property

* Other treatments have tended to view the front as a broader zone of transition, or as a finite region of strong gradients. The implication here is that the front does NOT approach a discontinuity.

- In fact, it is observed that fronts may fit into either of these models. Some fronts have been observed as near-discontinuities while others have not. A front may evolve through a life cycle from a broad baroclinic zone to a near-discontinuity and then decay

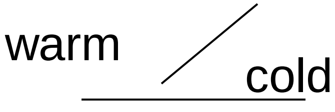
Frontogenesis

- Terminology: frontogenesis – creation or intensification of a front (front + genesis, birth, creation, formation, Genesis, gene, generate….)

frontolysis- destruction or weakening of a front (front + lysis, dissolution, destruction, paralysis, analysis….)

Structure of fronts

- We observe that fronts slope with height, and that they almost always slope toward the cold air. We can derive a simple formula that does a reasonable job of representing this slope



- First, we note that pressure must be continuous across the front. For the typical case of cold air to the north, and warm air to the south, the gradients are in the y-direction.

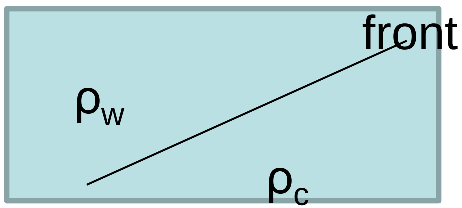
Structure (cont)

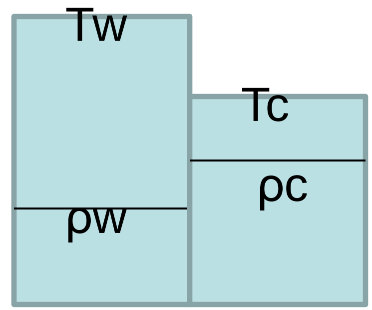
The reason the pressure must be continuous is that for a discontinuity, the pressure gradient would be infinite, that is…

$dp/dy \approx \Delta p/\ \Delta y$; for $\Delta y \rightarrow 0$ is $\Delta p = \infty$

If the pressure gradient were infinite, the corresponding accelerations in the eqns of motion would be infinite, leading to an infinitely strong wind, which of course is not observed.

- Since pressure is continuous, then both temperature and density must be discontinuous, or neither temperature and density are discontinuous….if the ideal gas law applies → $p=\rho RT$





Colder air has lower temperature and thus higher density

- For the continuous pressure field, we can express the pressure differential as

$$dp = \partial p/\partial y \, dy + \partial p/\partial z \, dz$$

(This is just the equation of a line). We recognize $\partial p/\partial z$ as physically meaningful and can apply the hydrostatic approximation…

$\partial p/\partial z = -\rho g$ to get

$dp = \partial p/\partial y \, dy - \rho g dz$

This equation must apply on both sides of the front. Then on the cold side we have

$dp=(\partial p/\partial y)_{cold} \, dy - \rho_{cold} \, g \, dz$

and on the warm side,

$dp=(\partial p/\partial y)_{warm} \, dy - \rho_{warm} \, g \, dz$

Equating the rhs of both eqns:

$(\partial p/\partial y)_c \, dy - \rho_c \, g \, dz = =(\partial p/\partial y)_w \, dy - \rho_w \, g \, dz$

Collecting terms in dy and dz gives

$[(\partial p/\partial y)_c - (\partial p/\rho y)_w]dy = (\rho_c - \rho_w)gdz$

Or $dz/dy = [(\partial p/\partial y)_c - (\partial p/\rho y)_w]/g\,(\rho_c - \rho_w)$

We thus see that non-zero dz/dy requires that $[(\partial p/\partial y)_c - (\partial p/\rho y)_w]$ also be non-zero.

That is, if the front slopes, then there must be a discontinuity of the pressure gradient. This is the reason why fronts should be analyzed with a kink in the isobars!

- Let us now assume that the component of the wind parallel to the front is in geostrophic balance:
- $u=u_g = -1/\rho f\ \partial p/\partial y$
- Solving for the pressure gradient, $\partial p/\partial y=-\rho f u_g$
- Substitute into our eqn for frontal slope:
- $dz/dy = [(\rho f u_g)_w - (\rho f u_g)_c]/g\ (\rho_c-\rho_w)$
- For a narrow frontal zone, we observe that the proportional difference in the Coriolis parameter is very small; e.g., for a frontal zone 10 km wide at 40 N, we have $(f_w-f_c)/f \approx 0.002$ (0.2%)
- Similarly, density differences are small (≈ 1%)

- But the differences in the geostrophic wind can be of similar magnitude to the wind itself, i.e.,

$(u_{gc} - u_{gw})/ 0.5*(u_{gc}+u_{gw}) \approx 0.1 - 1$

Then the number in the numerator in the frontal slope eqn is dominated by the change in geostrophic wind, so we can rewrite the eqn as:

$dz/dy \approx \rho f(u_{gw} - u_{gc})/g(\rho_c-\rho_w)$

- By the ideal gas law, $p=\rho RT$. Then for $p \approx$ constant (continuous across front), the discontinuous step increase (or decrease) of ρ must be balanced by a corresponding step decrease (or increase) of T; that is,

$\Delta\rho/\rho \approx \Delta T/T$ or in terms of our problem,

$(\rho_c - \rho_w)/\rho \approx (T_w - T_c)/T$ Then we can rewrite our frontal slope eqn as:

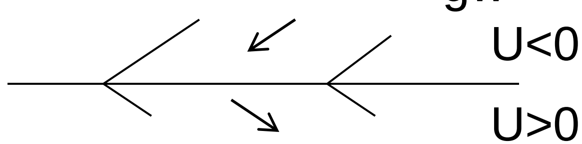
$$dz/dy \approx fT/g\,(u_{gw} - u_{gc})/(T_w - T_c)$$

Insights from the equation

- Velocity difference ($u_{gw} – u_{gc}$) across a frontal zone of width Δy can be expressed as $(u_{gw} – u_{gc})/\Delta y$
- Recall the vertical component of vorticity is

$\zeta = \partial v/\partial x - \partial u/\partial y$

Then, if u decreases northward (i.e., u decreases as y increases), the front is a zone of positive geostrophic vorticity, $u_{gw} – u_{gc} > 0$ so



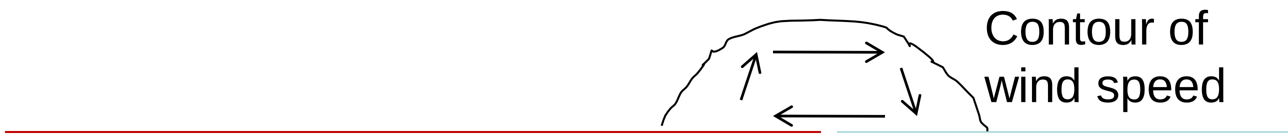
This is consistent with the observed kink in the isobars.

- Strong fronts (large temperature contrast) do not necessarily slope more than weak ones, since $(u_{gw} - u_{gc})$ also is likely to increase for a strong front
- If the shear is cyclonic $(u_{gw} - u_{gc}) > 0$, then $dz/dy > 0$. So for cold air to the north, the front slopes toward the cold air.

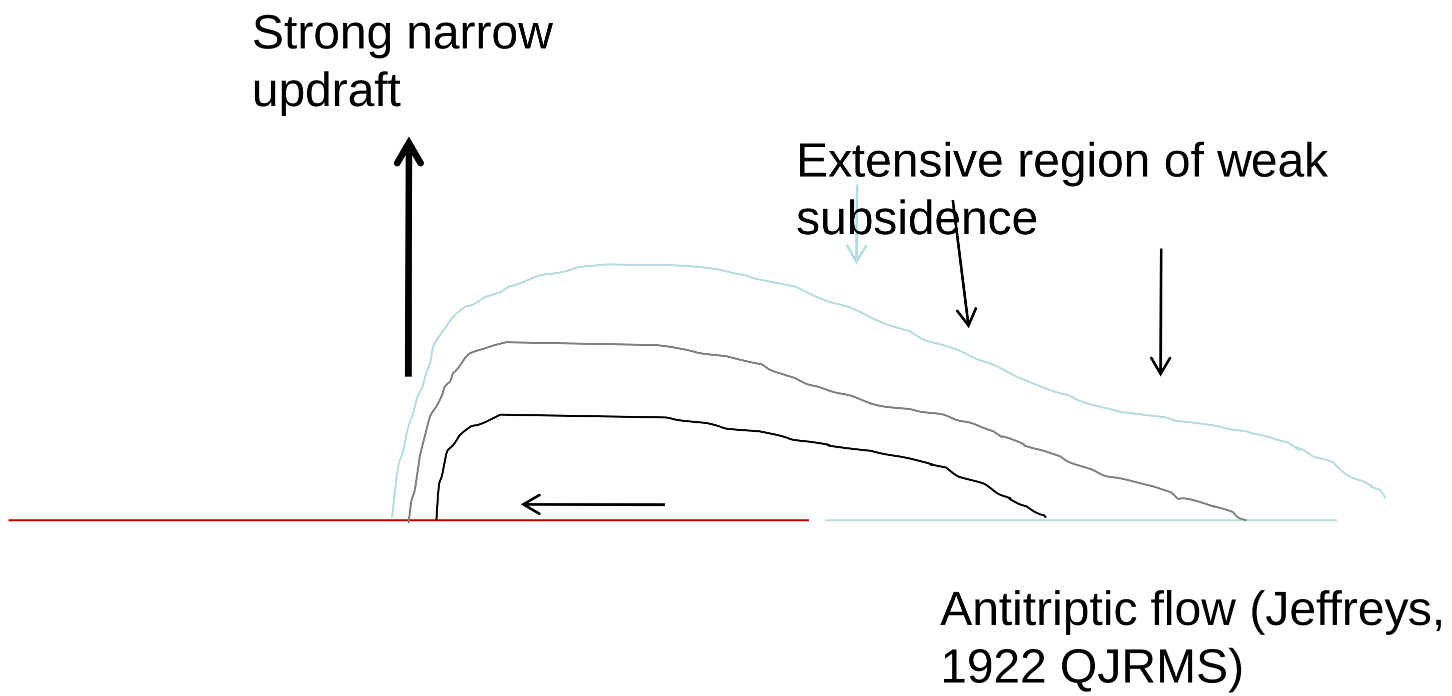
Typical evolution of sea breeze

- Assume atmosphere at rest, early in the morning

Just after sunrise, land heats up. Initially the perturbations are small so the response is linear.



Once perturbations become large, the nonlinear effects cause a front to form (we will study this in detail) on the inland side.



- Sea breeze is deeper on the inland side because stable stratification over the water suppresses the vertical extent.

Frontogenesis in the sea breeze

- We will begin with a 2D framework, and try to create an equation for $\partial\theta/\partial t$ =

*If we define the front as $\partial\theta/\partial x$, we can get

$\partial/\partial t(-\partial\theta/\partial x) = -\partial/\partial x(\partial\theta/\partial t) = -(\partial u/\partial x)(\partial\theta/\partial x) - (\partial\omega/\partial x)(\partial\theta/\partial p) - 1/c_p(p_0/p)^k\, \partial/\partial x(dQ/dt)$

- We could also define the front in other ways such as with the convergence of wind. In that case, we start with u-momentum equation

$\partial u/\partial t = -1/\rho\ \partial p/\partial x - u\ \partial u/\partial x - w\ \partial u/\partial z + fv - \partial/\partial z(\overline{u'w'})$

If we put a minus sign in so that positive values give us a stronger front, then..

$\partial/\partial t(-\partial u/\partial x) = 1/\rho\ \partial^2 p/\partial x^2 + \partial u/\partial x\ \partial u/\partial x + u\ \partial^2 u/\partial x^2 + \partial w/\partial x\ \partial u/\partial z + w\ \partial^2 u/\partial z \partial x - f\ \partial v/\partial x + \partial/\partial x(\ \partial/\partial z \overline{u'w'}) =$

$[-u\ \partial/\partial x\ (-\partial u/\partial x) - w\ \partial/\partial z(-\partial u/\partial x)]$ adv. of frontal character

$+ 1/\rho\ \partial^2 p/\partial x^2$ requires non-constant PGF (2nd derivative → curvature)

$+ \partial u/\partial x\ \partial u/\partial x$ convergence (nonlinear)

$+ \partial w/\partial x\ \partial u/\partial z$ tilting of vertical shear into horizontal

$-f\ \partial v/\partial x$ differential coriolis force – this becomes frontolytic later in day

$+ \partial/\partial x\ (\partial/\partial z\ \overline{u'w'})$ differential friction (usually frontolytic)

Frontogenesis

- The classical definition of the frontogenetical function is

$F = D/Dt\ |\nabla\theta|$

This is just a generalization of our earlier expression used in discussion of sea breeze frontogenesis. Here we consider gradients in any direction (i.e., $\nabla\theta = \partial\theta/\partial x + \partial\theta/\partial y$) and of any sign (as per the absolute value).

Isentropes along front

- Consider a case with frontal zone along x axis and isentropes parallel to front, with no wind variations along front. Also, temperature decreases toward north (increasing y).
- Then $F = D/Dt\,(-\partial\theta/\partial y) = (\partial v/\partial y)(\partial\theta/\partial y) + (\partial\omega/\partial y)(\partial\theta/\partial p) - 1/c_p (p_0/p)^k\, \partial/\partial y(dQ/dt)$
- Here – the gradient is in y-direction (not x) and vertical coordinate is pressure

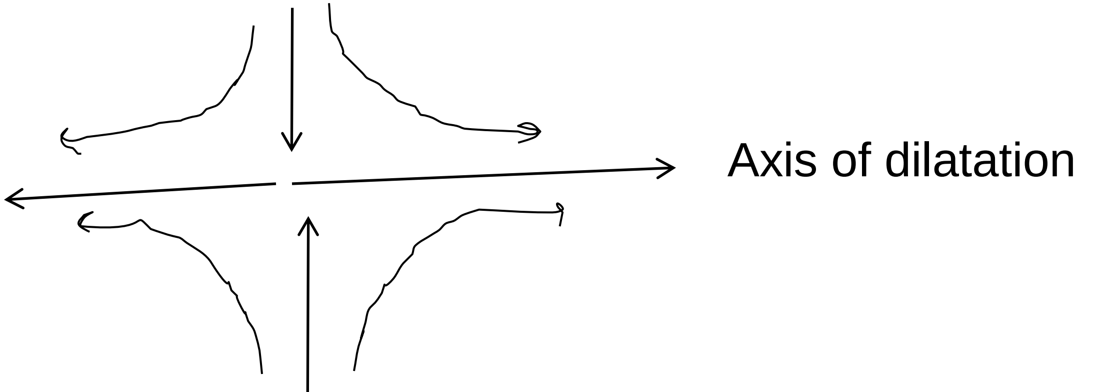
Role of deformation

- Pure deformation flow is defined as:

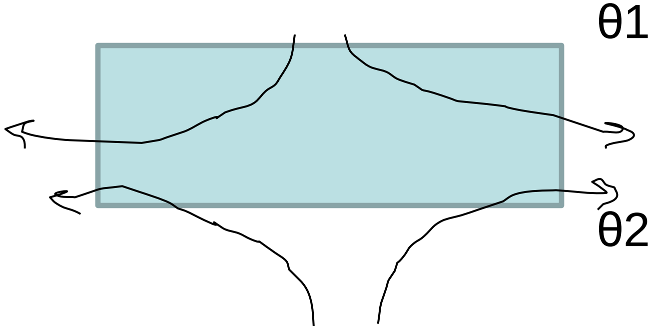
$\partial u/\partial x + \partial v/\partial y = 0$ (non-divergent)

Or equivalently, $\partial u/\partial x = -\partial v/\partial y$

- Therefore, if we have divergence in x-direction, it has to be exactly balanced by convergence in y-direction, and vice-versa



- In this case, the x-axis would be called the axis of dilatation and the y-axis the axis of contraction.
- Qualitatively, we can diagram the effect of deformation on the gradient as follows:
- Consider a control area defined as a rectangle. If the long edge of the rectangle is aligned with the axis of dilatation, the rectangle gets stretched out longer : since there is no divergence, area is unchanged



- If we assume the long sides of the rectangle correspond to isotherms (or adiabats), then the effect of deformation in this case is frontogenetic.
- Conversely if the long edge of the rectangle is along the axis of contraction, the rectangle becomes more of a square, and if the long sides are isotherms, the deformation is frontolytic.
- For other orientations, the effect of deformation will depend on the relative angle of the axis of dilatation and the isotherms

- The effect of horizontal convergence, as we have seen, is frontogenetic; conversely divergence is frontolytic. Combining the effects of deformation and divergence in the along wind direction,
- $F=|\nabla\theta|/2\,(D\cos 2b - \delta)$ where b= angle between axis of dilatation and isotherms.

b=0, cos2b=1 (frontogenetic)

b = 90, cos 2b=-1 (frontolytic)

b=45, cost2b = 0

Frontogenesis in 3D

We can extend our frontogenesis eq. to 3D by taking $\frac{\partial}{\partial x}$, $\frac{\partial}{\partial y}$, and $\frac{\partial}{\partial z}$ of the thermodynamic eq. and adding together: $\frac{d\theta}{dt} = \left(\frac{p_0}{p}\right)^{\kappa} \frac{1}{c_p} \frac{dQ}{dt}$

function is therefore defined as

diabatic — horiz Deformation — tilt — vert def. — vert div.

$$
\begin{aligned}
F &= \frac{D}{Dt}|\nabla\theta| \\
&= \frac{1}{|\nabla\theta|}\Bigg\{\frac{\partial\theta}{\partial x}\Bigg[\underbrace{\frac{1}{C_p}\left(\frac{p_o}{p}\right)^{\kappa}\frac{\partial}{\partial x}\left(\frac{dQ}{dt}\right)}_{1} - \underbrace{\frac{\partial u}{\partial x}\frac{\partial\theta}{\partial x}}_{2} - \underbrace{\frac{\partial v}{\partial x}\frac{\partial\theta}{\partial y}}_{3} - \underbrace{\frac{\partial w}{\partial x}\frac{\partial\theta}{\partial z}}_{4}\Bigg] \\
&\quad + \frac{\partial\theta}{\partial y}\Bigg[\underbrace{\frac{1}{C_p}\left(\frac{p_0}{p}\right)^{k}\frac{\partial}{\partial y}\left(\frac{dQ}{dt}\right)}_{5} - \underbrace{\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial x}}_{6} - \underbrace{\frac{\partial v}{\partial y}\frac{\partial\theta}{\partial y}}_{7} - \underbrace{\frac{\partial w}{\partial y}\frac{\partial\theta}{\partial z}}_{8}\Bigg] \\
&\quad + \frac{\partial\theta}{\partial z}\Bigg[\underbrace{\frac{p_0{}^{\kappa}}{C_p}\frac{\partial}{\partial z}\left(p^{-\kappa}\frac{dQ}{dt}\right)}_{9} - \underbrace{\frac{\partial u}{\partial z}\frac{\partial\theta}{\partial x}}_{10} - \underbrace{\frac{\partial v}{\partial z}\frac{\partial\theta}{\partial y}}_{11} - \underbrace{\frac{\partial w}{\partial z}\frac{\partial\theta}{\partial z}}_{12}\Bigg]\Bigg\} \qquad (9.11)
\end{aligned}
$$

in which the thermodynamics equation was used to eliminate $D\theta/Dt$, and height is used as the vertical coordinate so that the vertical component of the gradient of θ can be neatly combined with $\nabla_z\theta$. (The reader should verify that (9.9) and (9.10) are special cases of (9.11).) Terms 1, 5, and 9 are the diabatic terms; 2, 3, 6, and 7 are the horizontal-deformation terms; 10 and 11 are the vertical-deformation terms; 4 and 8 are the tilting terms; and 12 is the vertical divergence term. The physical interpretation of each term is just a generalization of the physical interpretation given to each term in (9.9). Reed and Sanders (1953), Newton (1954), and Sanders (1955) were the first to use observations to evaluate some of the terms in the frontogenetical function.

QG frontogenesis

n.o. D^{st} Heating
level sfc → no w there

$$F = \frac{D}{Dt}\left(-\frac{\partial\theta}{\partial y}\right) = \left(\frac{\partial v}{\partial y}\right)\left(\frac{\partial \theta}{\partial y}\right)$$

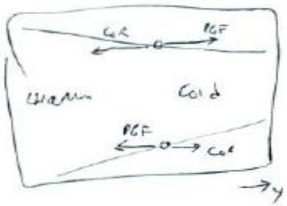
if $v = v_g$ and $\frac{\partial v_g}{\partial y}$ is held fixed

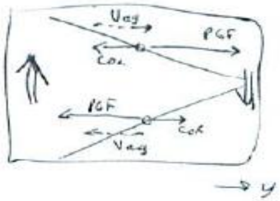
$$\left(-\frac{\partial\theta}{\partial y}\right)_t = -\frac{\partial\theta}{\partial y}\Big|_{t=0} \exp\left(-\frac{\partial v_g}{\partial y} t\right)$$

$\frac{10}{10^5}$ ≤ sec

e folding time: $\left(-\frac{\partial v_g}{\partial y}\right)^{-1} = 1$ day.

it would take 2.5 days to double temp gradient
— much slower than observed





intensifying ∆T gradient means we have
Vag eastwardly aloft + westerly below
which is turned by Coriolis into ≤ aloft + N below

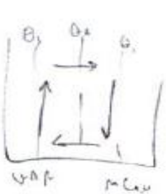
Cold air increases at low levels
rising warm air / inverse
diminishes temp gradient

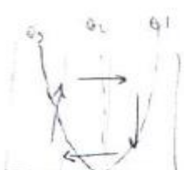
COR turns $\underset{\sim}{v}_{ag}$ into more w aloft & below.

Orogen Thermal wind — increased shear result

QG weaknesses →

1.) slow frontogenesis at sfc, intense gradients don't form
2.) No tilt with height to frontal zone
3.) Field of rel vorticity has large anticyclonic and cyclonic vorticity
4) static instability may be produced





0 0 1
0 0 1
i j k

or equivalently,

$$\boxed{\vec{V}_a = \frac{1}{f}\,\hat{k} \times \frac{d\vec{V}}{dt}}$$ [leave up]

Substituting for $\vec{V}_a$ in the eq. for the total wind,

$$\vec{V} = \vec{V}_g + \vec{V}_a$$

$$= \vec{V}_g + \frac{1}{f}\left[\hat{k} \times \frac{d\vec{V}}{dt}\right]$$

and using our definition $\vec{V} = \vec{V}_g + \vec{V}_a$ on the rhs,

$$\vec{V} = \vec{V}_g + \frac{1}{f}\,\hat{k} \times \left[\frac{d}{dt}\left(\vec{V}_g + \vec{V}_a\right)\right]$$

Substituting again for $\vec{V}_a = \frac{1}{f}\,\hat{k} \times \frac{d\vec{V}}{dt}$, we get

$$\vec{V} = \vec{V}_g + \vec{V}_a = \vec{V}_g + \frac{1}{f}\,\hat{k} \times \left[\frac{d\vec{V}_g}{dt} + \frac{d}{dt}\left(\frac{1}{f}\,\hat{k} \times \frac{d\vec{V}}{dt}\right)\right]$$

and cancelling $\vec{V}_g$ on both sides,

$$\vec{V}_a = \frac{1}{f}\,\hat{k} \times \left[\frac{d\vec{V}_g}{dt} + \frac{d}{dt}\left(\frac{1}{f}\,\hat{k} \times \frac{d\vec{V}}{dt}\right)\right]$$

or equivalently,

$$\vec{V}_a = \frac{1}{f}\,\hat{k} \times \frac{d\vec{V}_g}{dt} - \frac{1}{f^2}\frac{d^2\vec{V}}{dt^2}$$ → truncate (later)

Now we could continue this process of substituting for $\vec{V}_a$ indefinitely. This would give a definition of $\vec{V}_a$ as an <u>infinite</u> <u>series</u> of higher-order time derivatives of $\vec{V}$. If we <u>truncate</u> the second-order and higher terms, we end up with the <u>approximation</u>:

$$\vec{V}_a = \frac{1}{f}\hat{k} \times \frac{d\vec{V}_g}{dt}$$

or

$$\boxed{\frac{d\vec{V}_g}{dt} = -f\,\hat{k} \times \vec{V}_a}$$

Physically, the truncation implies that our time scale is larger than $\frac{1}{f}$.

This is the eq. of motion adopting the <u>geostrophic</u> <u>momentum</u> approximation. It is important to recognize that the total derivative is defined in the usual way as:

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{V} \cdot \nabla + \omega \frac{\partial}{\partial p}$$

$$= \frac{\partial}{\partial t} + (\vec{V}_g + \vec{V}_a) \cdot \nabla + \omega \frac{\partial}{\partial p}$$

that is, the advection includes transport by both the geostrophic and ageostrophic components.

Compare to the quasi-geostrophic momentum eq:

$$\frac{d_g \vec{V}_g}{dt} = -f \hat{k} \times \vec{V}_a$$

The difference is in the definition of the total derivative; i.e.,

$$\frac{d_g \vec{V}_g}{dt} = \frac{\partial}{\partial t} + \vec{V}_g \cdot \nabla$$

Here the transport is done only by the geostrophic part of the horizontal wind.

We see that by adopting the geostrophic momentum approx. we extend the QG eq. by including

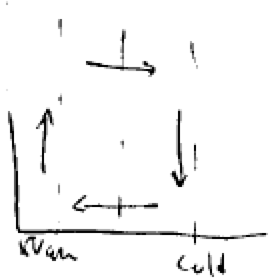
- advection by the ageostrophic wind;
- vertical advection

These are processes that are likely to be important in the vicinity of fronts, where there are pronounced vertical motions (e.g., lifting at the front and subsidence behind) as well as thermally-direct ageostrophic circulation.

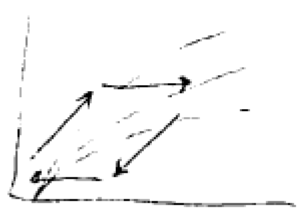
Looking back to the "full" momentum eq.,

$$\frac{d\vec{V}}{dt} = -f\,\hat{k} \times \vec{V}_a$$

the only difference is that we substitute $\frac{d\vec{V}_g}{dt}$ for $\frac{d\vec{V}}{dt}$; i.e., we neglect $\frac{d\vec{V}_a}{dt}$.



QG Frontogenes.



Geos mom. Approx

ageos. circ. affects things more.