FDOT TRANSPORTATION SYMPOSIUM 2019

FRP-RC Design - Part 3a

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Design of concrete structures internally reinforced with FRP bars

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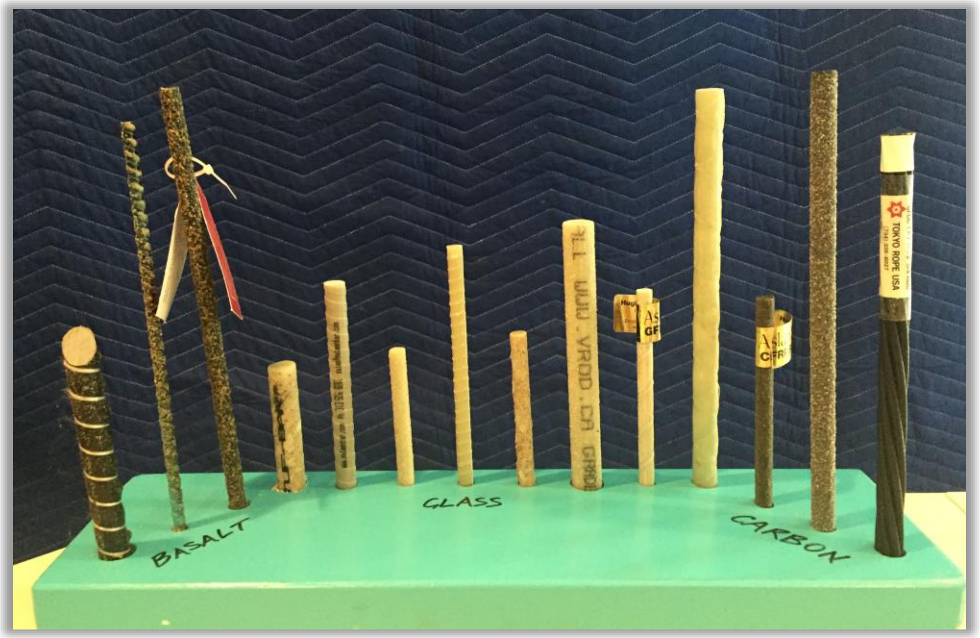
Course Description

Fiber-reinforced polymer (FRP) materials have emerged as an alternative for producing reinforcing bars for concrete structures. Due to other differences in the physical and mechanical behavior of FRP materials versus steel, unique guidance on the engineering and construction of concrete structures reinforced with FRP bars is necessary.



Learning Objectives

- Understand the mechanical properties of FRP bars
- Describe the behavior of FRP bars
- Describe the design assumptions
- Describe the flexural/shear/compression design procedures of concrete members internally reinforced with FRP bars
- Describe the use of internal FRP bars for serviceability & durability design including long-term deflection
- Review the procedure for determining the development and splice length of FRP bars.

Content of the Course

FRP-RC Design - Part 1, (50 min.)

This session will introduce concepts for reinforced concrete design with FRP rebar. Topics will address:

- Recent developments and applications
- Different bar and fiber types;
- Design and construction resources;
- Standards and policies;

FRP-RC Design - Part 2, (50 min.)

This session will introduce Basalt FRP rebar that is being standardized under FHWA funded project ***STIC-0004-00A*** with extended FDOT research under BE694, and provide training on the flexural design of beams, slabs, and columns for:

- Design Assumptions and Material Properties
- Ultimate capacity and rebar development length under strength limit states;
- Crack width, sustained load resistance, and deflection under service limit state;

Content of the Seminar

BFRP-RC Design - Part 3, (50 min.)

This session continues with Basalt FRP rebar from Part II, covering shear and axial design of columns at the strength limit states for:

- Ultimate capacity – Flexural behavior (Session 3)
- Fatigue resistance under the Fatigue limit state;
- Shear resistance of beams and slabs;
- Axial Resistance of columns;
- Combined axial and flexure loading.

FRP-RC Design - Part IV *(Not included at FTS - for future training)*:

This session continues with FRP rebar from Part III, covering detailing and plans preparation:

- Minimum Shrinkage and Temperature Reinforcing
- Bar Bends and Splicing
- Reinforcing Bar Lists
- General Notes & Specifications

Session 3: Flexural Behavior

Session 3:

Flexural behavior

- Balance failure
- Tension failure
- Compressive failure
- Design examples

Session 3: Flexural Behavior

Failure Modes:

- Under-reinforced sections would fail suddenly
  - FRP bars do not yield
- There will be warning in the form of cracking and large deflection
- Over-reinforced may be desirable to avoid sudden collapse of members
- Over or under-reinforced sections are acceptable provided that the strength and serviceability criteria are satisfied
- Flexural behavior is not ductile; therefore, safety factors are larger than in steel-RC

Session 3: Flexural Behavior

Assumptions:

- Maximum strain at the concrete compression fibre is 3500 x 10^{-6}
- Tensile strength of concrete is ignored for cracked sections
- The strain in concrete and FRP at any level is proportional to the distance from the neutral axi
- The stress-strain relationship for FRP is linear up to failure
- Perfect bond exists between the concrete and the FRP reinforcement

Session 3: Flexural Behavior

Ultimate Flexural Strength:

- M_n = nominal capacity
- M_u = factored moment
- ***f*** = strength reduction factor (CSA S806, CSA S6)

$$\phi M_n \geq M_u$$

As an example

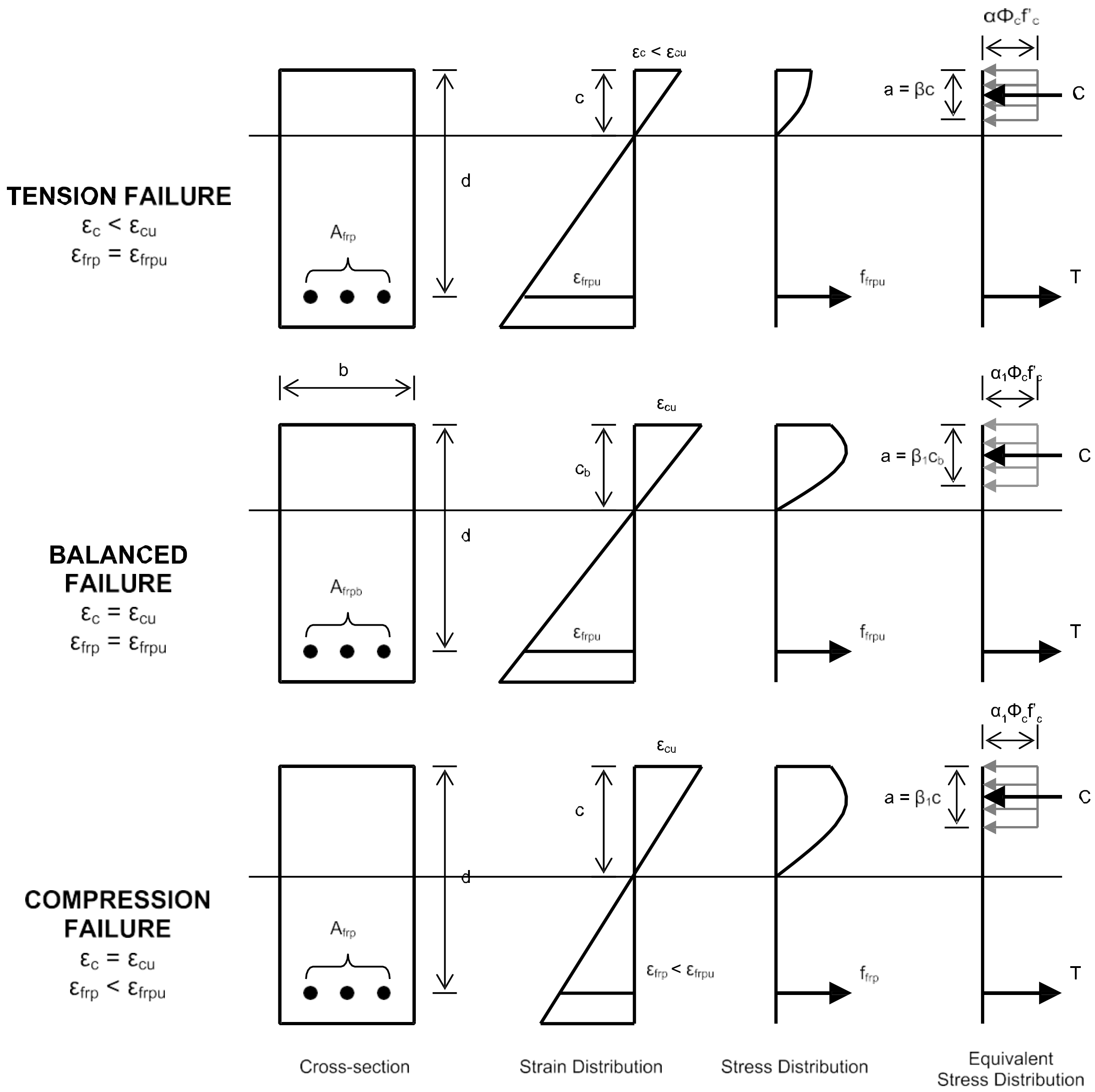
$$M_u = 1.2M_D + 1.6M_L$$

Session 3: Flexural Behavior

Modes of Failure:

- Balanced failure - simultaneous rupture of FRP and crushing of concrete;
- Compression failure - concrete crushing while FRP remains in the elastic range with a strain level smaller than the ultimate strain;
- Tension failure - rupture of FRP before crushing of concrete.

Flexural Failure Modes for FRP Reinforced Beams



Concrete does not crush

Undesirable for design with GFRP

FRP ruptures

Concrete crushes

FRP ruptures

Concrete crushes

FRP does not rupture

Determine Flexural Failure Mode

- Calculate the reinforcement ratio for balanced strain condition:

  $$\rho_{FRPb} = \alpha_1\ \beta_1\ \varphi_c\ f'_c/f_{FRPu}\ \varepsilon_{cu}\ /(\varepsilon_{cu} + \varphi_{FRP}\ \varepsilon_{FRPu})$$

- Calculate reinforcement ratio for FRP-reinforced beam:

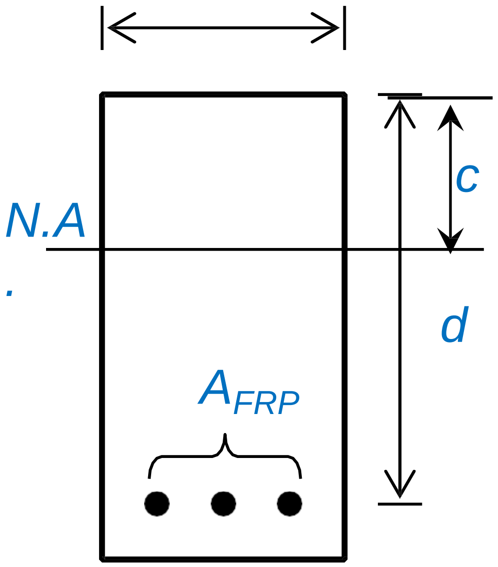
  $$\rho_{FRP} = A_{FRP}/(d\ \ b)$$

- $\rho_{FRP} < \rho_{FRPb} \rightarrow$ Tension Failure
- $\rho_{FRP} > \rho_{FRPb} \rightarrow$ Compression Failure

- Calculate the depth to neutral axis c_b for the balanced strain condition:

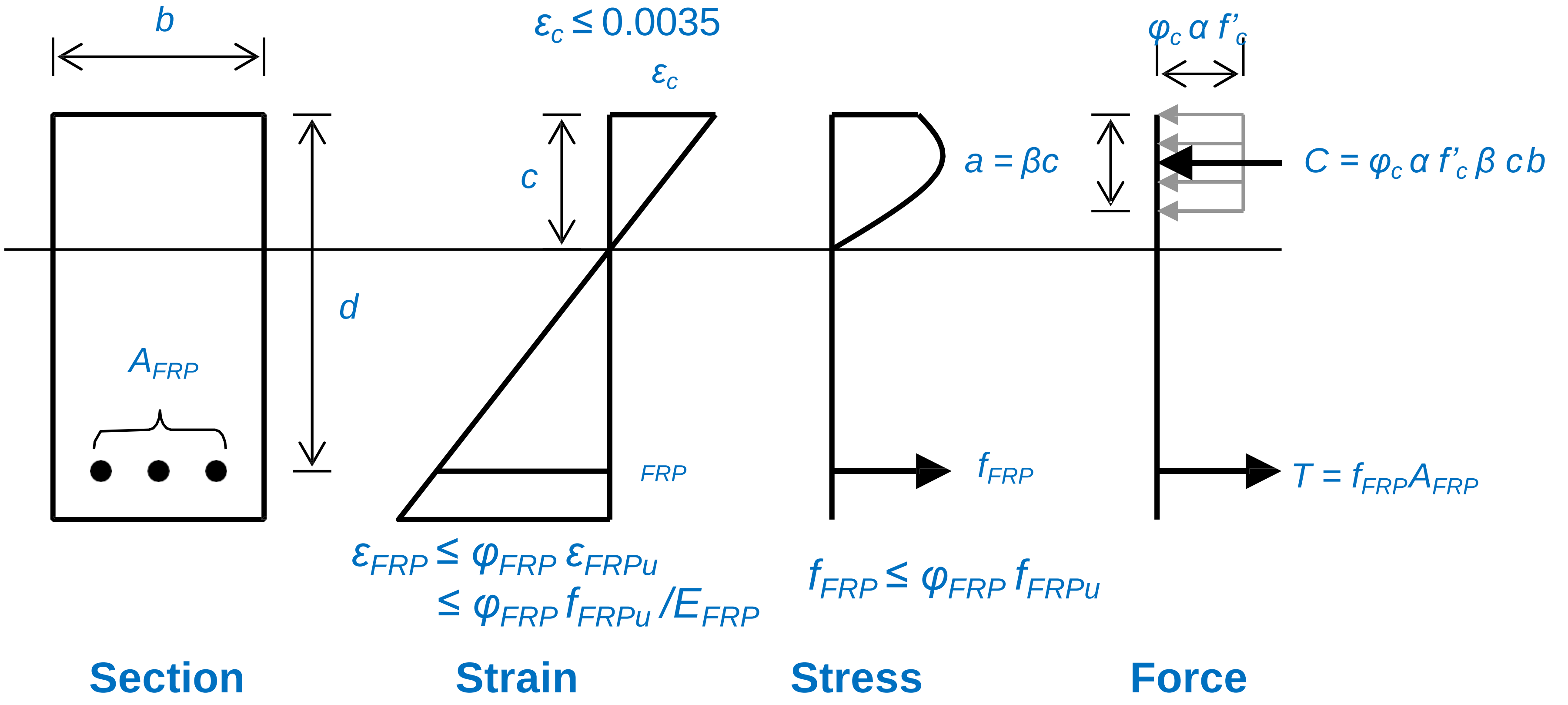
  $$c_b = d\ \varepsilon_{cu}/(\varepsilon_{cu} + \varphi_{FRP}\ \varepsilon_{FRPu})$$

- Tension Failure $\rightarrow$ c < c_b
- Compression Failure $\rightarrow$ c > c_b



Basic Flexural Theory Applied to FRP RC Beams

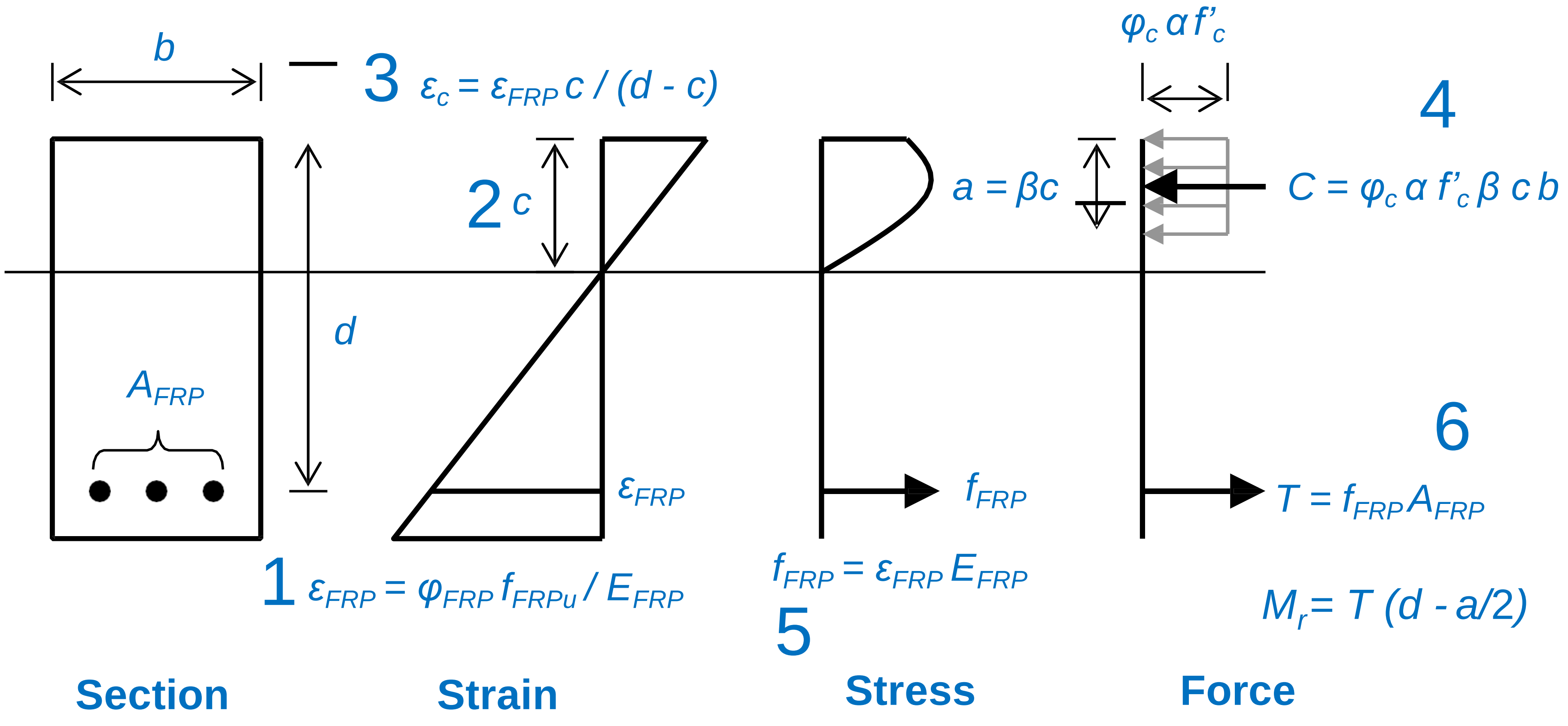
- Plane section remains plane with linear strain variation
- $\varepsilon c \leq 0.0035$ and $\varepsilon_{FRP} \leq \varphi_{FRP}\, \varepsilon_{FRPu}$ *at ULS*



$M_r = T\,(d - a/2)$

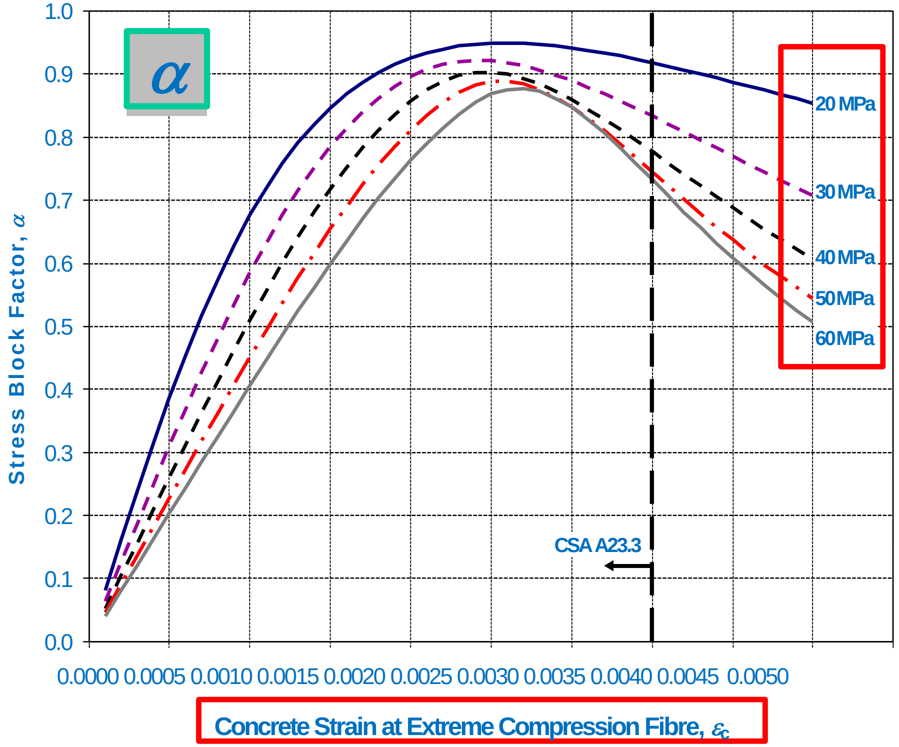
Basic Flexural Theory – Tension Failure

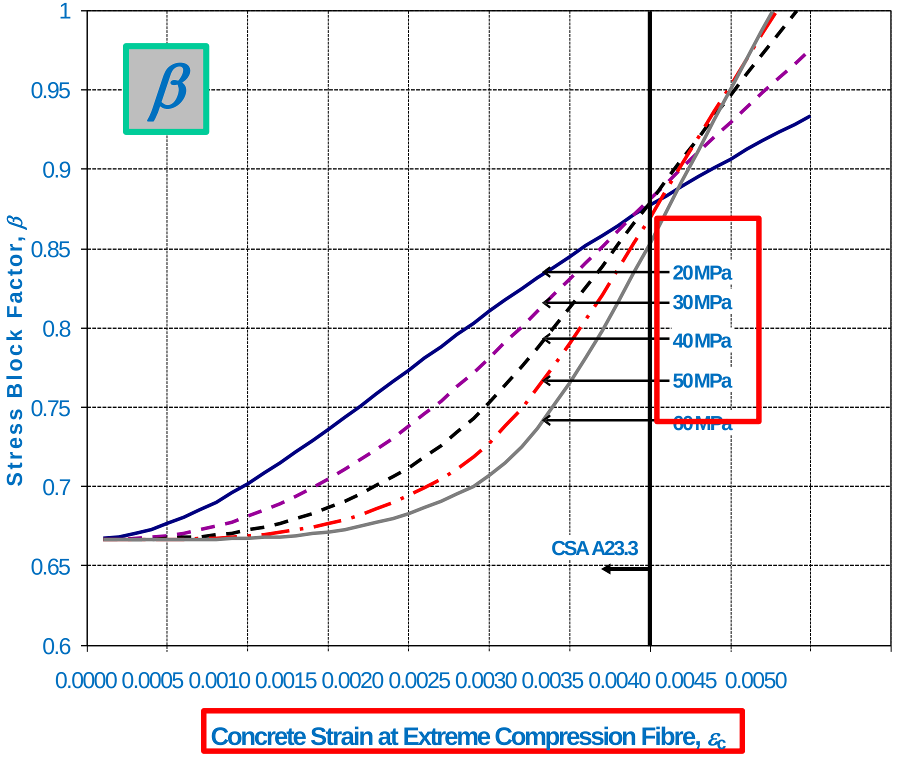
- Assume ***c*, calculate $\boldsymbol{\varepsilon_c}$** and C, T, revise ***c*** until C = T.
- For ***α***, ***β***, you may use tables or detailed formulas.



Flexural Analysis

Tension Failure





α & β

- Concrete strain, ε_c
- Concrete strength, f'_c

Stress Block Factors for $\varepsilon_c < 0.0035$

- For a constant width section, we may assume that the stress-strain curve of the concrete is parabolic and the following equations can be used (more convenient than tables for spreadsheet calculations).
- For strengths higher than 60 MPa, consult tables in Collins and Mitchell (1997).

$$\beta = \frac{4 - (\varepsilon_c / \varepsilon'_c)}{6 - 2(\varepsilon_c / \varepsilon'_c)} \qquad \alpha = \frac{1}{\beta_1}\left[\left(\frac{\varepsilon_c}{\varepsilon'_c}\right) - \frac{1}{3}\left(\frac{\varepsilon_c}{\varepsilon'_c}\right)^2\right]$$

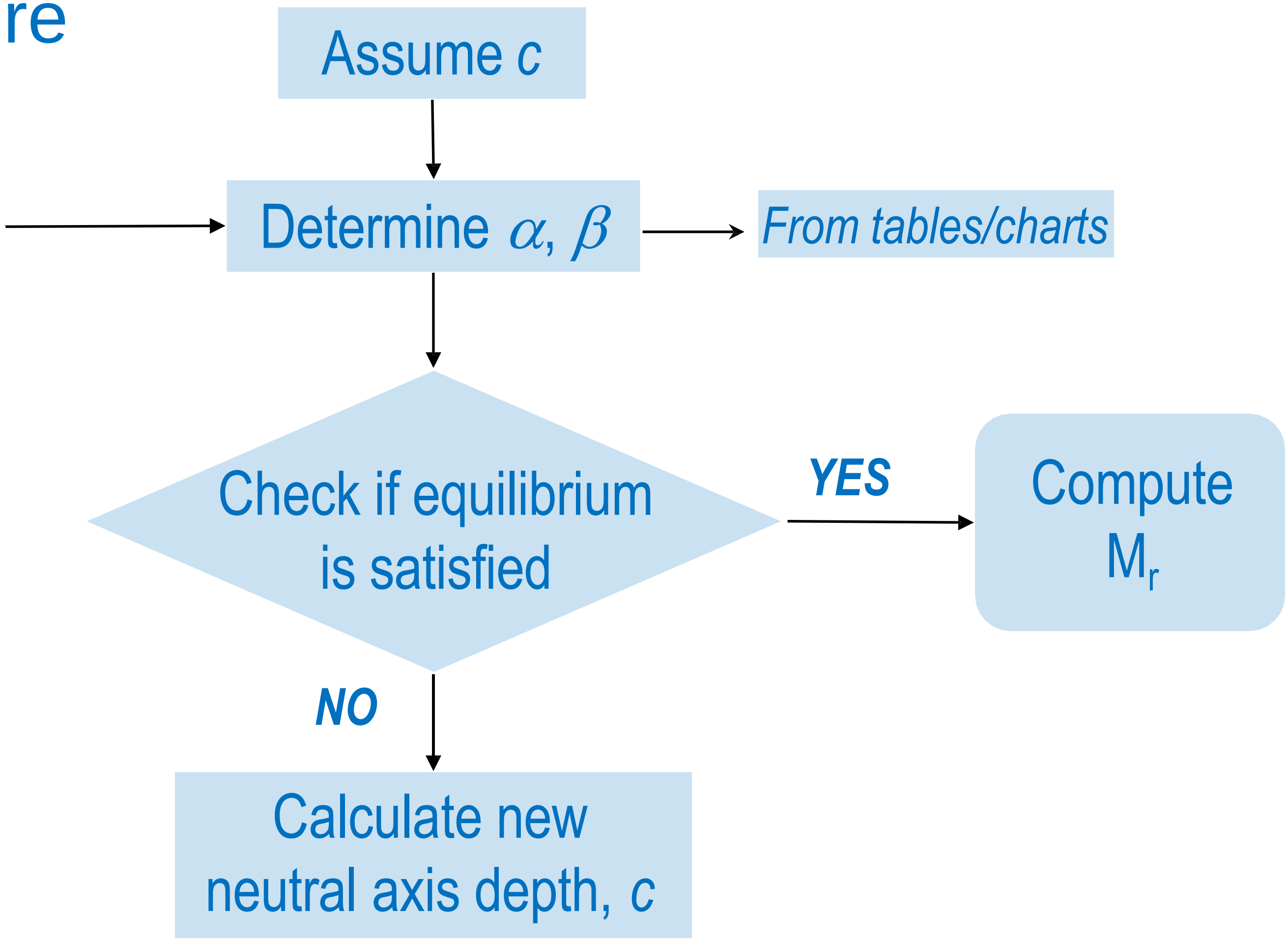
where the concrete compressive strain is ε_c

the peak strain at peak stress f'_c is $\varepsilon'_c = 1.71 f'_c / E_c$

Flexural Analysis

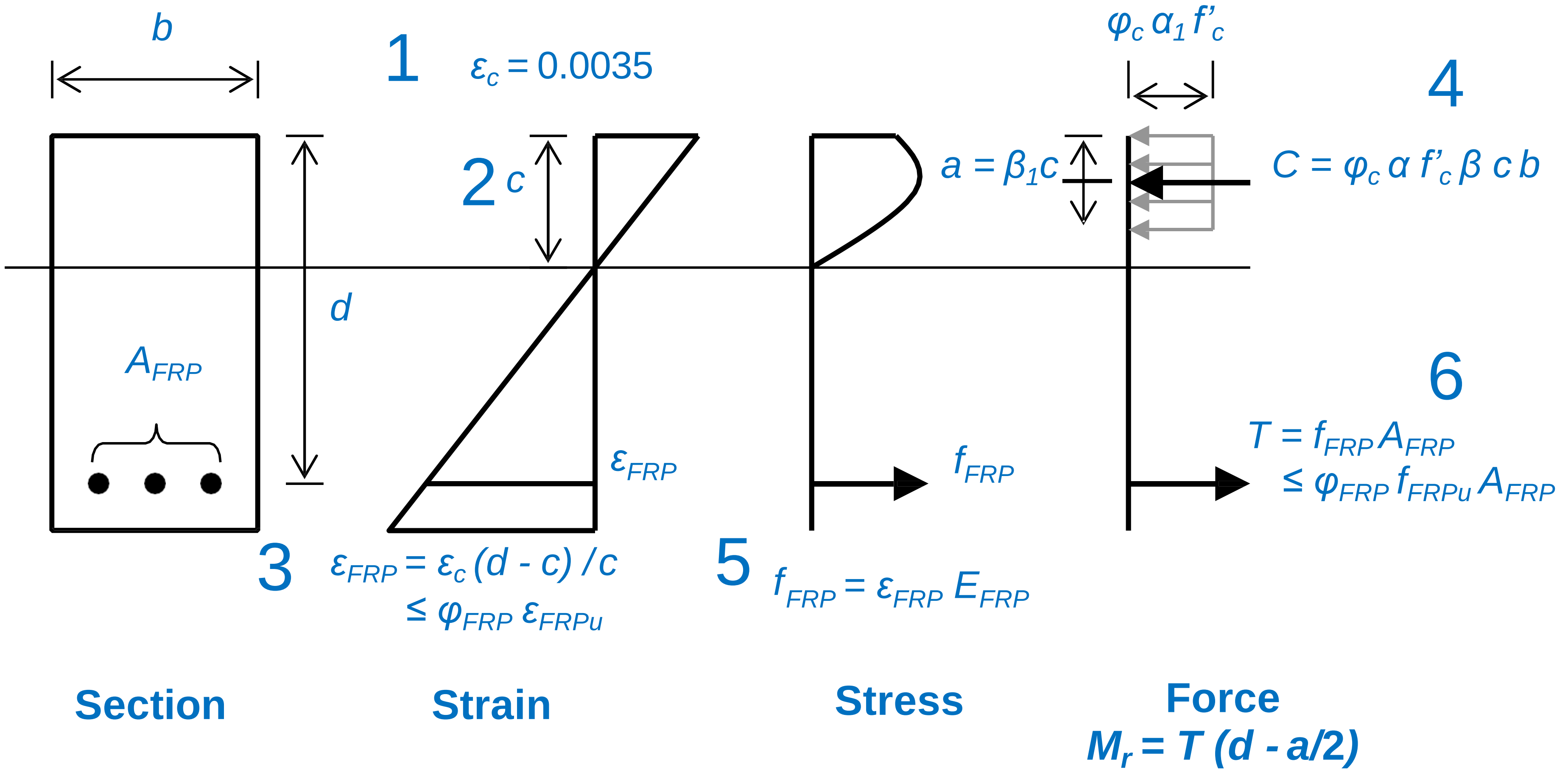
Tension Failure

To summarize:



Basic Flexural Theory – Compression Failure

- Assume c, calculate ε_{FRP}, f_{FRP} and C, T, revise c until C=T.
- Use $\alpha_1 = 0.85\text{-}0.0015f'_c$, $\beta_1 = 0.97\text{-}0.0025f'_c$ for $\varepsilon_c = 0.0035$.



Flexural Failure

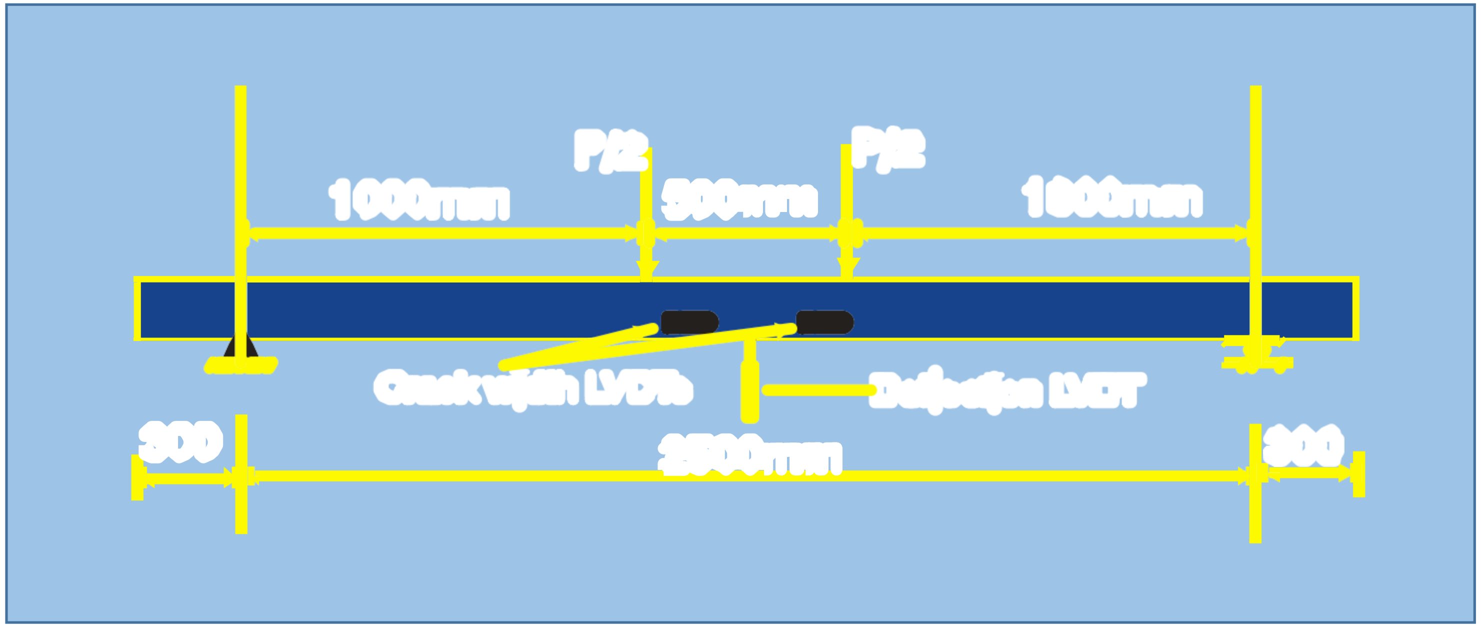
	Tension	Balanced	Compression
Behavior	FRP Rupture	FRP Rupture and Concrete crushing	Concrete crushing
Desirability	Least desirable : rupture is sudden and violent		Most desirable : sufficient warning
Reinf. Ratio	$\rho_{frp} < \rho_{bal}$	$\rho_{frp} = \rho_{bal}$	$\rho_{frp} > \rho_{bal}$
Strains	$\varepsilon_{frp} = \varepsilon_{frpu}$ $\varepsilon_c < \varepsilon_{cu}$	$\varepsilon_{frp} = \varepsilon_{frpu}$ $\varepsilon_c = \varepsilon_{cu}$	$\varepsilon_{frp} < \varepsilon_{frpu}$ $\varepsilon_c = \varepsilon_{cu}$

Session 3: Flexural Behaviour

FRP-Reinforced Concrete Beams & One-Way Slabs under Flexure Load

Test Set-up

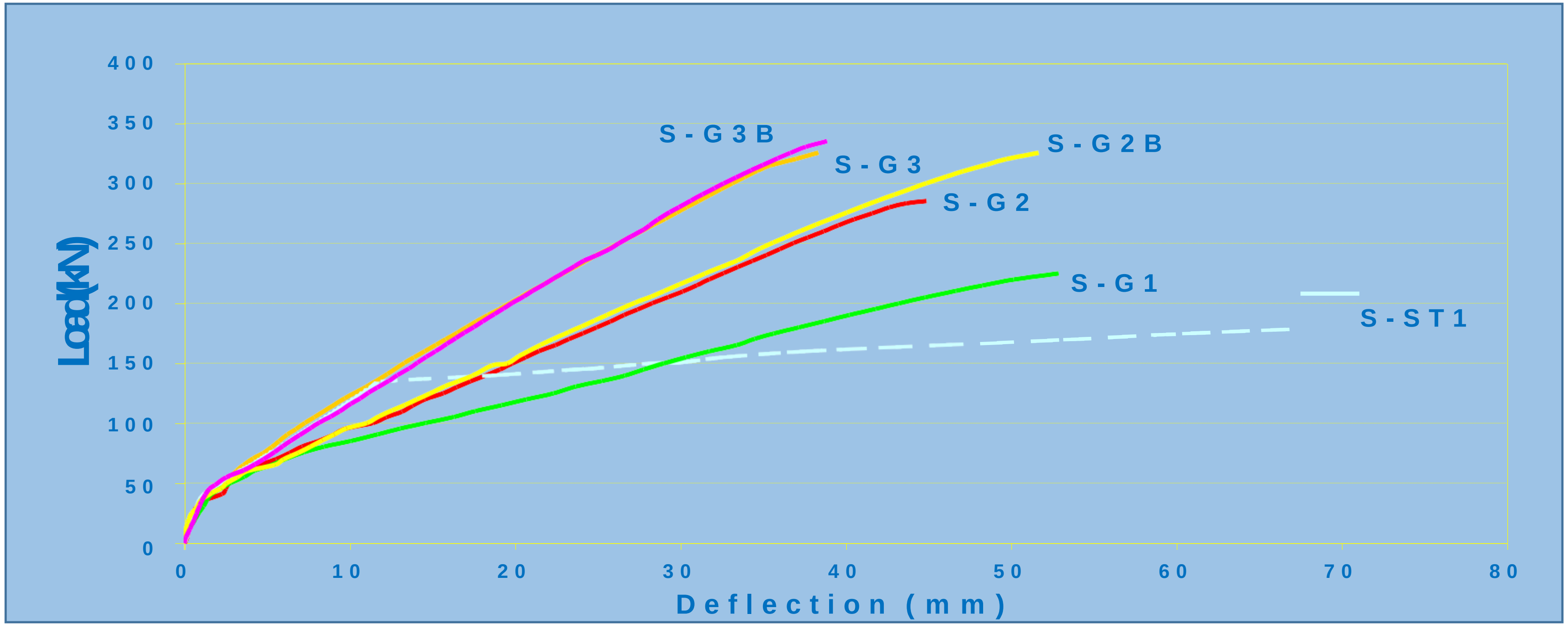
4-point bending over a clear span of 2.5 m.



Session 3: Flexural Behaviour

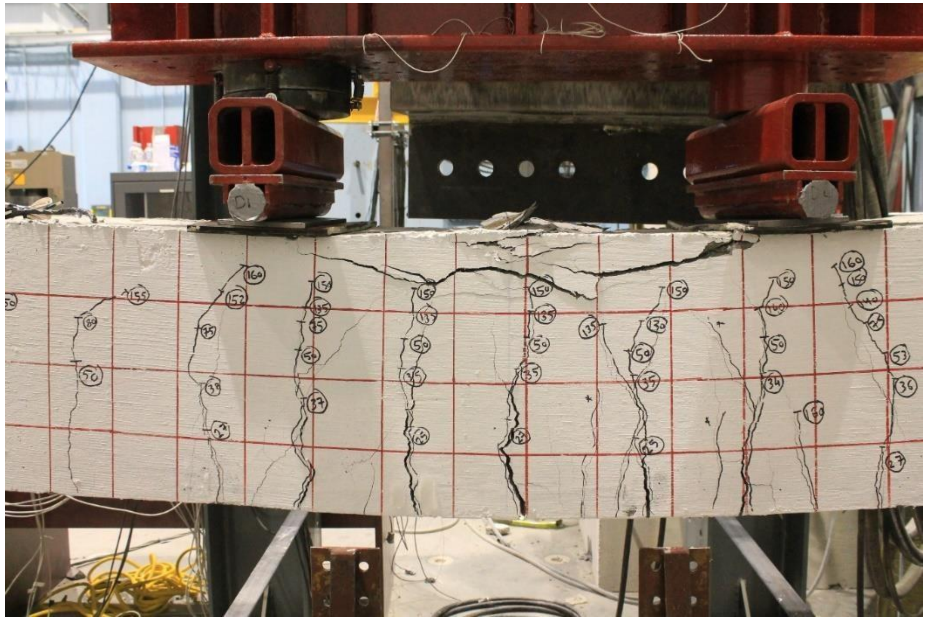
Load-Deflection of FRP-Reinforced Concrete Beams & One-Way Slabs under Flexure Load

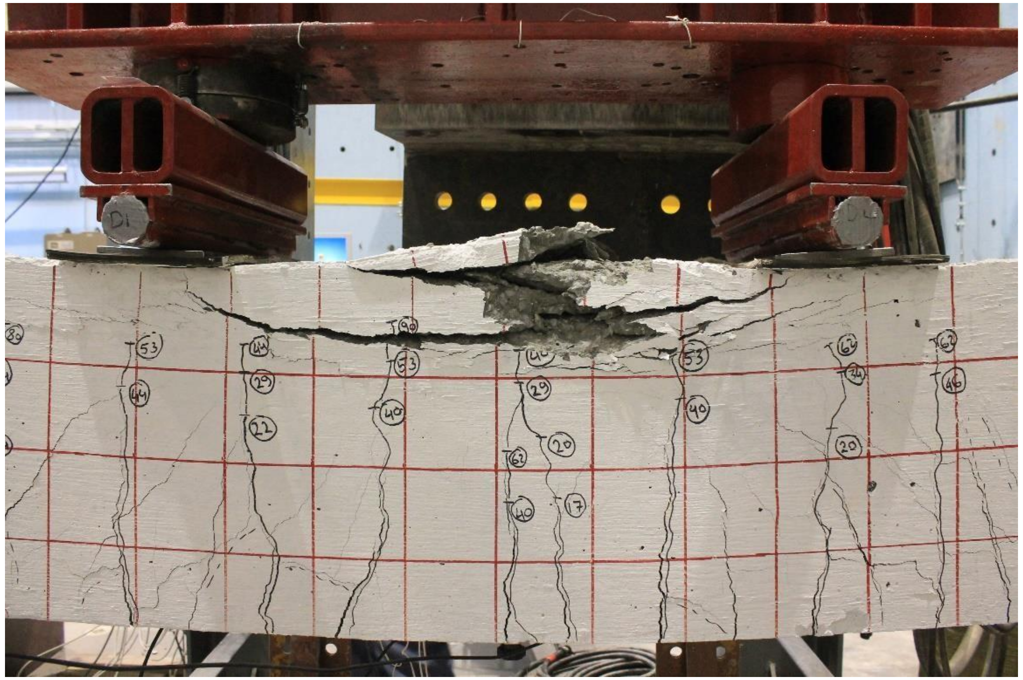
Effect of GFRP reinforcement ratio



Session 3: Flexural Behaviour

Mode of failure: Compression failure (gradual concrete crushing)





Session 3: Flexural Behaviour

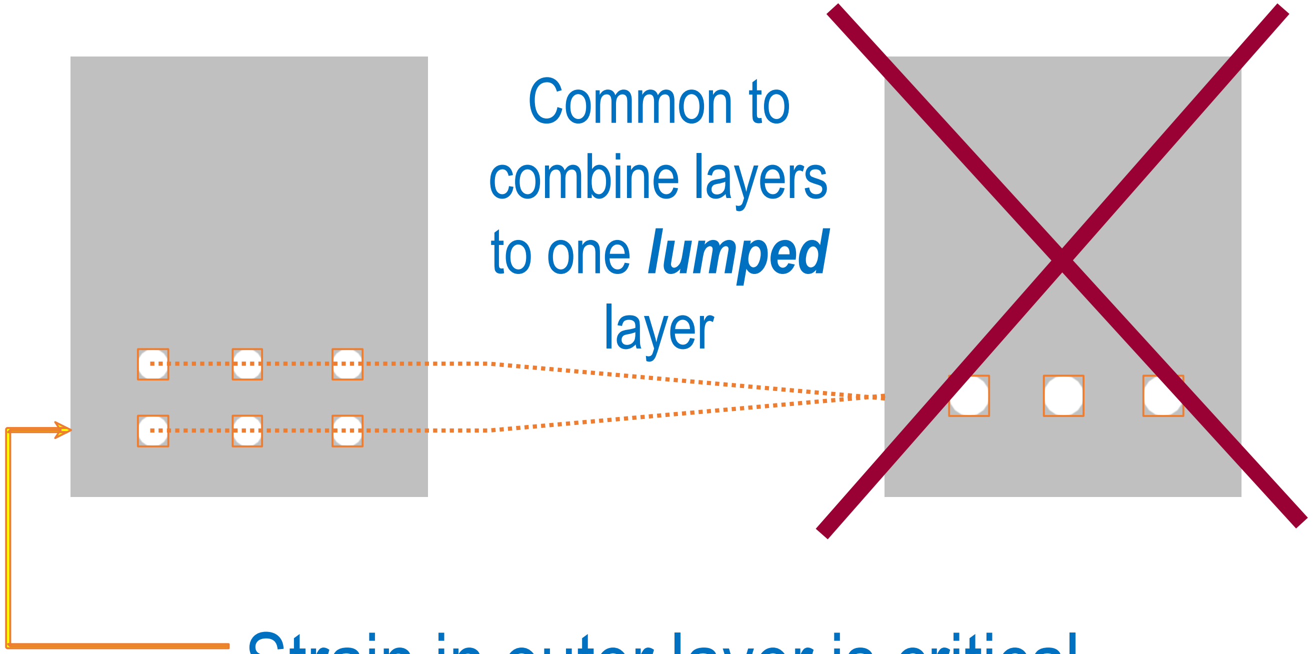
Compression failure – GFRP RC Beam

When the applied load was released, the FRP RC beam recovered most of their deflection during the unloading process, because the FRP bars on the tension side did not reach rupture strain; In contrast, the steel specimen retained deflection after unloading

(Elastic behavior of FRP: Resilient structural element which maintains its functionality)

Flexural Analysis - Additional Considerations

Beams with FRP reinforcement in multiple layers



Strain in outer layer is critical

Lumping of reinforcement not allowed, strain compatibility is used to design on the basis of tensile failure of the outermost FRP layer

Flexural Analysis - Additional Considerations

Minimum Flexural Resistance ***(CSA-S6)***

- Minimum reinforcement required to prevent brittle failure when concrete cracks on tensile face:

  $$\boldsymbol{M_r} \geq 1.5\, \boldsymbol{M_{cr}}$$

- If the ULS resistance of the section is governed by FRP rupture (*tension failure*):

  $$\boldsymbol{M_r} \geq 1.5\, \boldsymbol{M_f}$$

- For tension failure, the code requires a purposely conservative design to ensure that ample deformation and cracks will develop before failure of the beam.
- Neglect compression FRP.

Flexural Analysis - Additional Considerations

Flexure Design ***(CSA S806)***

- **Assumptions**
  - Compressive strength of FRP shall be ignored when calculating the resistance of a member
  - Strain compatibility method shall be used to calculate the factored resistance of a member
  - Flexural members shall be designed such that failure at ultimate is initiated by the failure of concrete at the extreme compression fiber. This condition is satisfied by the c/d requirement shown below:

$$\frac{c}{d} \geq \frac{7}{7 + 2000\,\varepsilon_{Fu}}$$

Flexural Analysis - Additional Considerations

*Flexure Design **(CSA S806)***

- **Assumptions**
  - Minimum Flexural Reinforcement Requirement

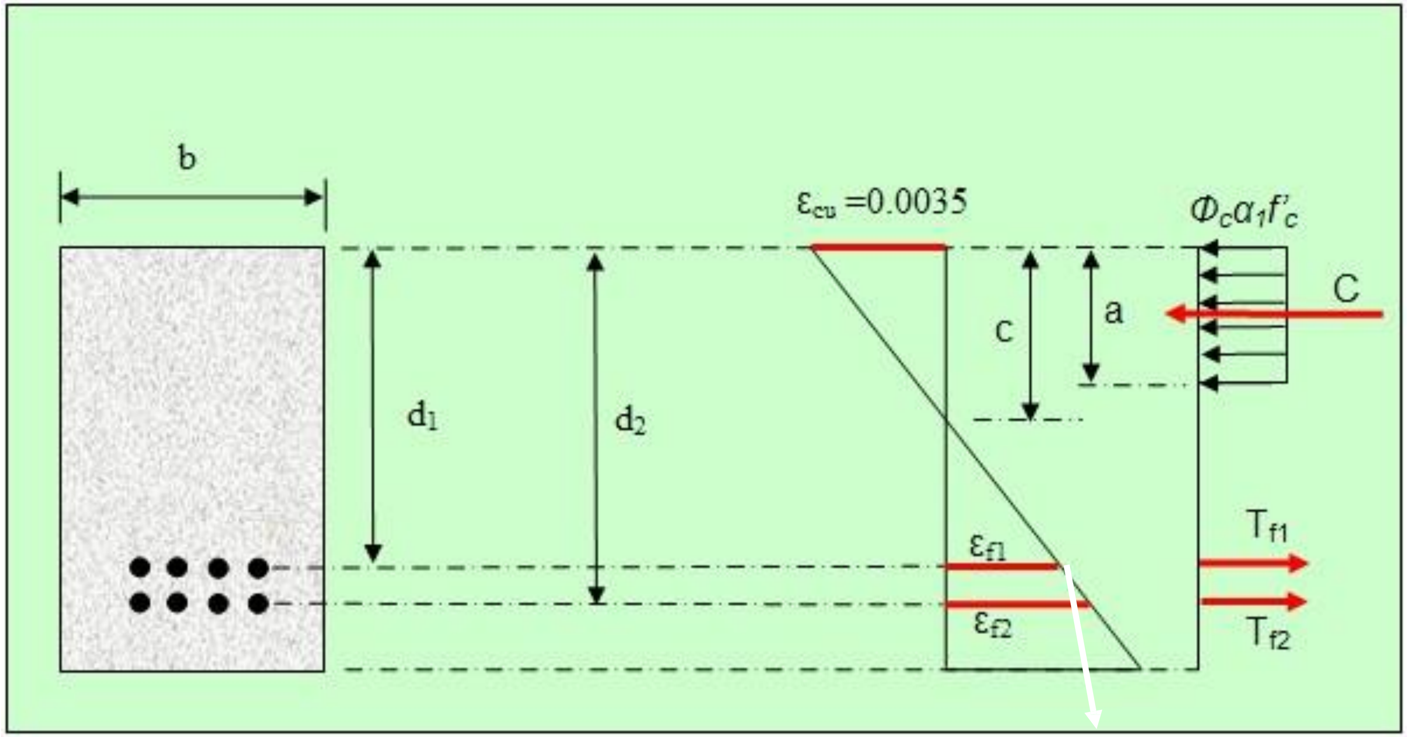
$$M_r \geq 1.5 M_{cr} = 1.5 f_r \frac{I_g}{y_t}$$

  - For Slabs:
    - $\boldsymbol{A_{F.min}} = 400\boldsymbol{E_F}/\boldsymbol{A_g} \geq 0.0025\boldsymbol{A_g}$
    - Spacing of $\boldsymbol{A_{F.min}} \leq$ 300mm or 3 times slab thickness

Flexural Analysis - Additional Considerations

*Flexure Design **(CSA S806)***

Strain Compatibility Analysis:



$$\varphi_c = 0.65$$
$$\varphi_F = 0.75$$
$$\alpha_1 = 0.85 - 0.0015 f'c \geq 0.67$$
$$\beta_1 = 0.97 - 0.0025 f'c \geq 0.67$$
$$a = \beta_1 c$$

$$T_{f1} = \alpha_F \varepsilon_{f1} E_f A_{f1}$$
$$T_{f2} = \alpha_F \varepsilon_{f2} E_f A_{f2}$$

$$\varepsilon_{fi} = \frac{d_i - c}{} \varepsilon_{cu} = \frac{d_i - c}{} 0.0035$$

Flexural Analysis - Additional Considerations

Flexure Design ***(CSA S806)***

Resisting Moment:

$$M_r = C\left(c - \frac{a}{2}\right) + T_{f1}\left(d_1 - c\right) + T_{f2}\left(d_2 - c\right) \geq M_f$$

Flexural Analysis

Analysis at Service Limit State

- Serviceability considerations (stresses, crack widths, and deflections) may govern the design of FRP-reinforced concrete members

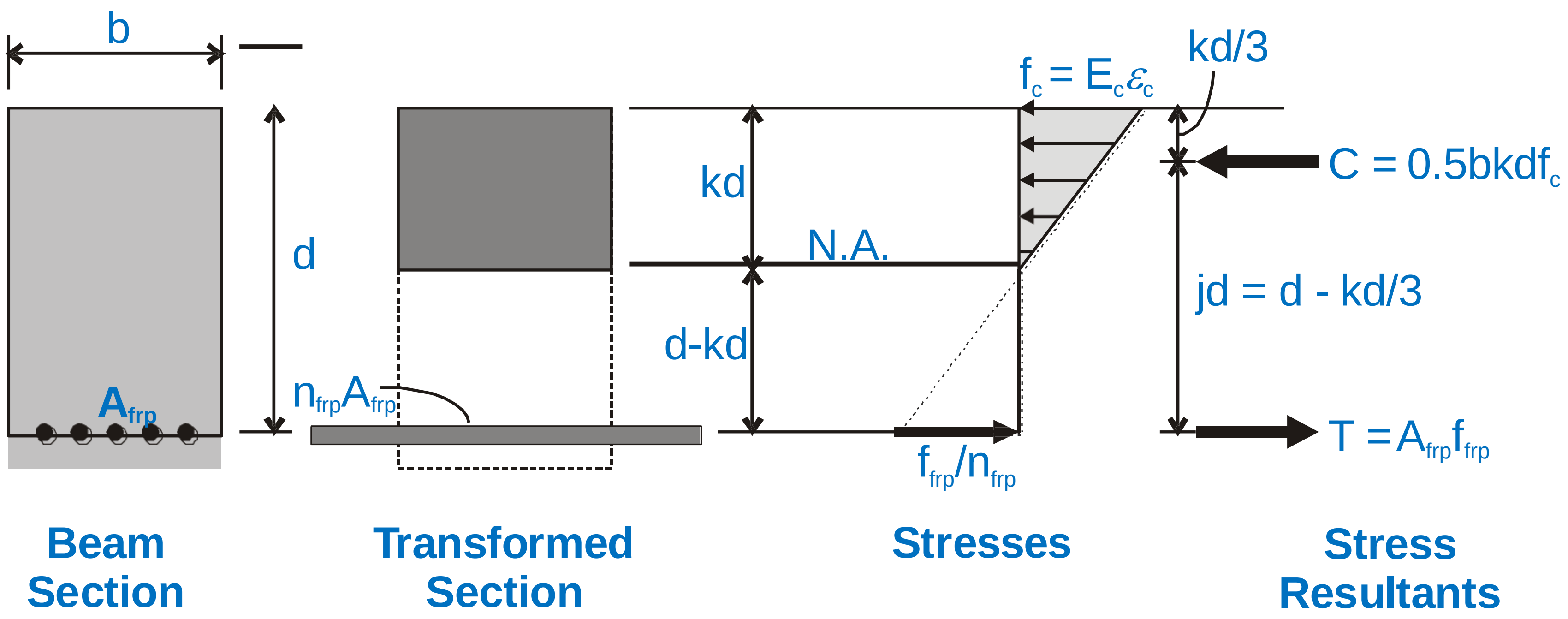
- Analysis at Service Limit State can be performed assuming linear-elastic behavior (straight-line theory)
  - FRP materials are linear-elastic to failure
  - Concrete stress-strain relationship is linear for compression stresses less than 60% of f'_c

Flexural Analysis

Analysis at Service Limit State

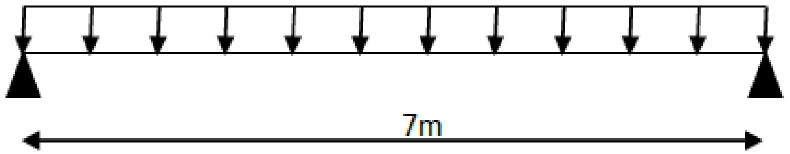
- Linear-elastic cracked transformed section analysis:

A_{FRP} transformed to equivalent area of concrete



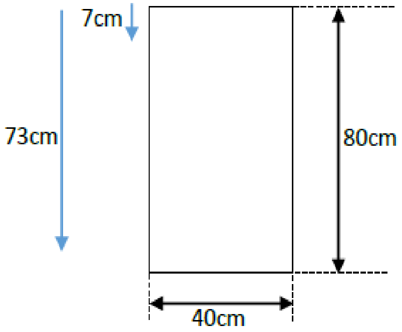
Design of Concrete Beam Reinforced with GFRP Bars According to ***CSA S806-12***

Loads



Dead load (D.L) = 85 kN/m

Live load (L.L) = 40 kN/m



Service limit state ($W_{s.l.s}$) = 85 + 40 = 125 kN/m

Ultimate limit state ($W_{u.l.s}$) = 1.25 * 85 + 1.5 * 40 = 166.25 kN/m

	S.L.S	U.L.S
V = W L / 2 (kN)	*437.50*	*581.88*
M = W L^2 / 8 (kN.m)	*765.63*	*1018.28*

Mechanical Properties

- **Concrete**

  $f_c' = 30$ MPa

- **GFRP (Grade III)**

  GFRP straight bar #8 (25 mm-diameter)

  $A_f = 506.7\ mm^2$; $E_f = 66.4$ GPa; Guaranteed tensile strength (f_{fu}) = 1000 MPa.

  GFRP bent bar #3 (10 mm-diameter)

  $A_f = 71.3\ mm^2$; $E_f = 50$ GPa; Guaranteed tensile strength (f_{fu}) = 460 MPa.

Notes and Assumptions

- Concrete cover = 30 mm or $2d_b$ (Clause 8.2.3).
- Assume exterior exposure of the beam for crack control.
- Minimum clear spacing between longitudinal bars = 20 mm (for vertical and horizontal spacing).
- Concrete resistance factor (φ_c) = 0.65 (Clause 6.5.3.2).
- GFRP resistance factor (φ_{FRP}) = 0.75 (Clause 7.1.6.3).
- Bond dependent coefficient (k_b) = 0.8 (sand-coated bars)

Design Steps

Assume flexure reinforcement

- Calculate internal forces.
- Check section flexural ultimate capacity and cracking moment.
- Check maximum stress under service load.
- Check crack width parameter.

Design for shear force

- Calculate concrete contribution.
- Calculate GFRP stirrups contribution

Flexural Design

- **Assuming 16 #8 GFRP straight bars, using stirrups #3**

d = distance from top chord to reinforcement C.G

d = 800 – 2 * 25 – 32.4 = 716.6 mm

$A_f \geq A_{f,min} = \text{Max}\left(0.0025 * (t * b), \frac{400*(t*b)}{E_f}\right)$ **Clause 8.4.2.3**

$A_f = 16 * 506.7 = 8107.2\ mm^2 \geq A_{f,min} = 1972.7\ mm^2$

Concrete stress block factors

$\alpha_1 = 0.85 - 0.0015f'_c = 0.805 \geq 0.67$

$\beta_1 = 0.97 - 0.0025f'_c = 0.895 \geq 0.67$

Flexural Design

- **Internal Forces**

$$C_c = \alpha_1 \varphi_c f'_c ba = \alpha_1 \varphi_c f'_c b(\beta c)$$

$$C_c = 0.805 * 0.65 * 30 * 400 * 0.895 * c$$

C_c **= 5619.7 c**

$$T_F = \varphi_F \varepsilon_f E_F A_F = \varphi_F \left[\frac{\varepsilon_c}{c}(d-c)\right] E_F A_F$$

$$T_F = 0.75\left[\frac{0.0035}{\ }(716.6-c)\right] * 66400 * 8107.2$$

Flexural Design

- **Internal Forces**

$$C_c = T_F \rightarrow \textbf{5619.7 c} = \frac{1012669673}{c} - 1413085$$

c = 317.0 mm $\rightarrow C_c = T_F$ **= 1781.5 kN**

$$\frac{c}{d} > \frac{7}{7 + 2000\varepsilon_f}$$

$$\frac{c}{d} = \frac{317.0}{716.6} = 0.442 > \frac{7}{7 + 2000 * 0.0151} = 0.188$$

$$C_c = 5619.7\ \text{c}$$

$$T_F = \frac{1012669673}{c} - 1413085$$

Flexural Design

- **Check section flexural ultimate capacity**

$$M_r = C_c\left(c - \frac{a}{2}\right) + T_s(d - c)$$

$$M_r = 1781.5 * \left(317.0 - \frac{0.895 * 317.0}{2}\right)$$

$$+ 1781.5 * (716.6 - 317.0) = 1023.9\ kN.m > M_f$$

- **Check cracking moment**

$$M_{cr} = f_r \frac{I_g}{y_t} = \boldsymbol{f}_r * \frac{b * h^3/12}{h/2} = 0.6\sqrt{30} * \frac{400 * 800^3/12}{800/2}$$
$$= 140.2\ kN.m$$

$$1.5M_{cr} = 210.3\ kN.m < M_r$$

Clause 8.4.2.1

Flexural Design

- **Check maximum stress under service load**

$$f_f = \frac{M_s}{A_f\, d\,(1 - k/3)} \leq 0.25\, f_{fu}$$

Clause 7.1.2.2

$$E_c = \left(3300\sqrt{f_c'} + 6900\right)\left(\frac{\gamma_c}{2300}\right)^{1.5}$$

CSA A23.3 Equation (8-1)

$$E_c = \left(3300\sqrt{30} + 6900\right)\left(\frac{2300}{2300}\right)^{1.5} = 24975 MPa$$

$$n_f = \frac{E_f}{E_c} = \frac{66400}{24975} = 2.659$$

$$k = \sqrt{2\,\rho_f\, n_f + \left(\rho_f\, n_f\right)^2} - \rho_f\, n_f = \sqrt{2 * 0.0283 * 2.659 + (0.0283 * 2.659)^2} - 0.0283 * 2.659 = 0.320$$

Flexural Design

- **Check maximum stress under service load**

$$f_f = \frac{765.63 * 10^6}{8107.2 * 716.6 * (1 - 0.320/3)} = 147.5MPa \leq 250.0\,MPa$$

$$\varepsilon_f = \frac{f_f}{E_f} = \frac{147.5}{66400} = 0.0022 > 0.0015$$ ***Clause 8.3.1.1***

D.L ratio = 85 / 125 = 0.68

$$\varepsilon_{f-Sustain} = 0.68 * \boldsymbol{0.0022} = 0.001\boldsymbol{5} < 0.002$$ ***Clause 7.1.2.3***

Flexural Design

- **Check crack width parameter**

$$z = k_b \frac{E_s}{E_F} f_f \sqrt[3]{d_c A}$$ *Equation (8-1)*

$E_s = 200\ GPa$

d_c = *Distance from extreme tension fiber of concrete to centerline of flexural reinforcement.*

$$d_c = h - d = 800 - 716.6 = 83.4 \leq 50mm$$ *Clause 8.3.1.1*

$$A = 2 * \frac{d_c * b}{bars\#} = 2 * \frac{50 * 400}{16} = 2500.0mm^2$$

$$z = 0.8\, x \frac{200000}{66400} x\, 147.5 * \sqrt[3]{2500x50} = 17771.6 < 38000\ N/mm$$

Design of Concrete Bridge Deck Slab

Reinforced with GFRP Bars

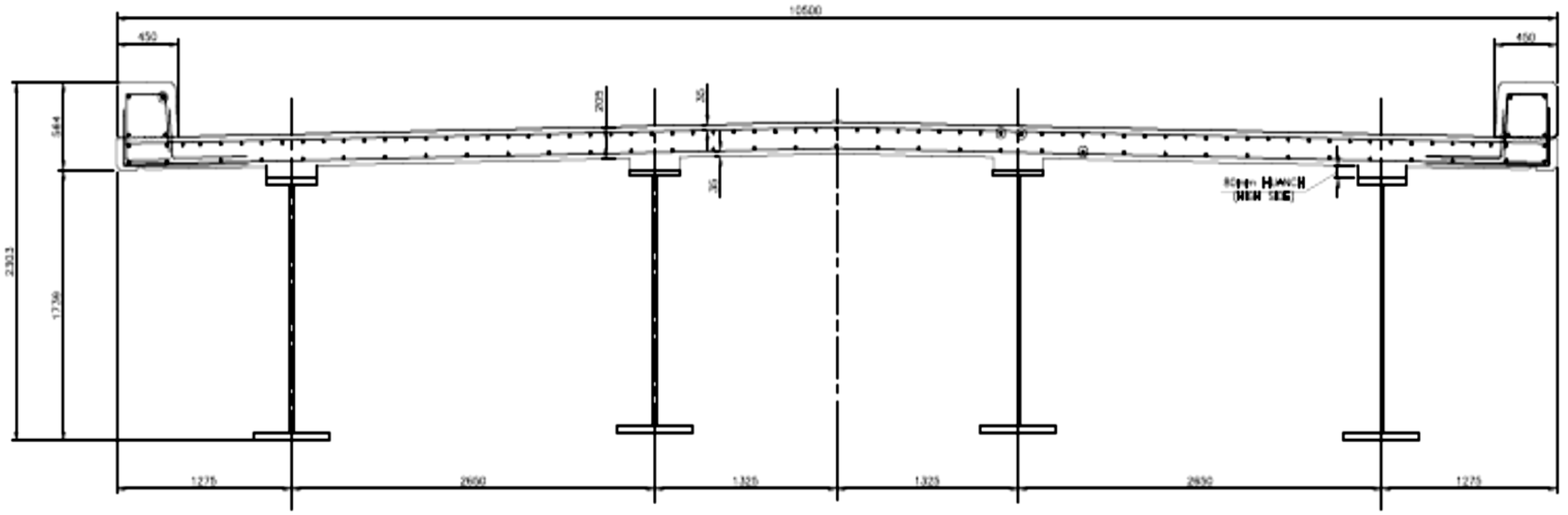
According to **CHBDC (CSA S6-14)**

Design Criteria

Thickness of deck slab = 225 mm

Spacing between girder beams = 3750 mm

Thickness of asphalt and waterproofing = 90 mm

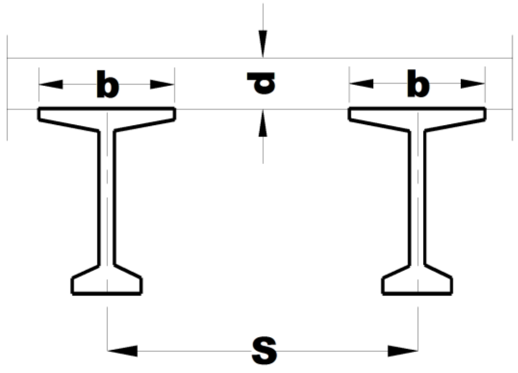


Dead Loads

Own weight of the slab = 0.225 x 23.5 = 5.29 kN/m²

Own weight of the pavement = 0.09 x 24 = 2.16 kN/m²

- Service dead load (w_{ds}) = 5.29 + 2.16 = 7.45 kN/m²
- Factored dead load (w_{du}) = 1.2 x 5.29 + 1.5 x 2.16 = 9.59 kN/m²
- Design Span = S or S - (b) + d
- S_e = 3.750 – 0.220 + 0.225 = 3.755 m > 3.75 m



Service dead load moment (M_{ds}) = 0.071 w_{ds} S_e^2 = 7.44 kN.m/m

Factored dead load moment (M_{du}) = 0.071 w_{ds} S_e^2 = 9.58 kN.m/m

Wheel Load (Live Load)

$$M_y = \frac{(S_e + 0.6)P}{10} = \frac{(3.75 + 0.6)\,87.5}{10} = 37.19\ kN.m$$

For deck slabs continuous over three or more supports, the maximum bending moment, shall be assumed to be 80% of that determined for a simple span. These moments shall be increased by the dynamic load allowance for a single axle (Clause 3.8.4.5.3).

$$M_{ym} = 0.8 * 37.19 * 1.4 = 41.65\ kN.m/m$$

Service wheel load moment (M_{ws}) = 0.9 M_{ym}= 37.5 kN.m/m

$M_{s\ -total}$ = 44.9 kN.m/m

Factored wheel load moment (M_{wu}) = 1.7 M_{ym} = 70.8 kN.m/m

$M_{f\ -total}$ = 80.4 kN.m/m

Mechanical Properties (Concrete & GFRP Bar)

- **Concrete**

  f'_c= 35 MPa

- **GFRP (Grade III)**

  GFRP straight bar #5 (15.875 mm-diameter)

  A_f = 197.9 mm^2; E_f = 62.6 GPa; Guaranteed tensile strength (f_{fu}) = 1184 MPa.

Notes and Assumptions

- Concrete cover = 35 *mm.*
- Minimum clear spacing between longitudinal bars = 20 mm (for vertical and horizontal spacing).
- Concrete resistance factor (φ_c) = 0.75 *(Clause 8.4.6).*
- GFRP resistance factor *(φ_{FRP})* = 0.55 *(Clause*

Concrete stress block factors

$$\alpha_1 = 0.85 - 0.0015f'_c = 0.798 \geq 0.67$$

$$\beta_1 = 0.97 - 0.0025f'_c = 0.883 \geq 0.67$$

Design Steps

Design for flexural moment for main direction

- **Calculate cracking moment.**
- **Assume main reinforcement.**
- **Calculate internal forces**
- **Check section flexural ultimate capacity.**
- **Check maximum stress under factored and service loads.**
- **Check crack control.**

Flexural Design

- **Check cracking moment**

$$M_{cr} = f_r \frac{I_g}{y_t} = \boldsymbol{f}_r * \frac{b * h^3/12}{h/2} = 0.4\sqrt{45} * \frac{1000 * 225^3/12}{225/2} = 20.0\ kN.m$$

- **Assuming #5 GFRP straight bars each 135 mm**

d = distance from top chord to reinforcement C.G

d = 225 – 35 – 15.875 / 2 = 182.06 mm

$$\rho_{fb} = 0.85\,\beta_1 \frac{f_c'}{f_{fu}} \frac{E_f\,\varepsilon_{cu}}{E_f\,\varepsilon_{cu} + f_{fu}} = 0.35\%$$

$$\rho_f = \frac{no\ x\ A_f}{b\,d} = \frac{197.9}{135\,x 182.06} = 0.81\% > \rho_{fb} \quad \text{Over reinforced Section}$$

Flexural Design

- **Internal forces**

$$C_c = \alpha_1 \varphi_c f'_c ba = \alpha_1 \varphi_c f'_c b(\beta c)$$

$$C_c = 0.798 * 0.75 * 35 * 1000 * 0.86 * c$$

C_c **= 18474.6 c**

$$T_F = \varphi_F \varepsilon_f E_F A_F = \varphi_F \left[\frac{\varepsilon_c}{c}(d - c)\right] E_F A_F$$

$$T_F = 0.55 \left[\frac{0.0035}{c}(182.06 - c)\right] * 62600 * 1000/135 * 197.9$$

$$T_F = \frac{32161596.2}{c} - 176651.4$$

Flexural Design

- **Internal forces**

$C_c = T_F \rightarrow$ 18474.6 c =32161596.2/c−176651.4

c = 37.22 mm $\rightarrow C_c = T_F$ = 687.5 kN

$$\frac{c}{d} = \frac{37.22}{182.06} = 0.204$$

$$\frac{c_b}{d} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_f} = \frac{0.0035}{0.0035 + \frac{1184}{62600}} = 0.156$$

$\frac{c}{d} > \frac{c_b}{d}$ **Concrete Crushing**

Flexural Design

- **Check section flexural ultimate capacity**

$$M_r = C_c\left(c - \frac{a}{2}\right) + T_s(d - c)$$

$$M_r = 687.5 * \left(37.22 - \frac{0.883 * 37.22}{2}\right)$$

$$+ 687.5 * (182.06 - 37.22) = 113.9\ kN.m > M_f\ \ , Mr > 1.5\ Mcr$$

(Clause 16.8.2.2)

- **Check maximum stress at ultimate load**

$$f_f = \frac{T_f}{A_f} = \frac{687.5}{197.9 * 1000/135} = 496\ MPa < \varphi_F f_{fu} = 651\ MPa$$

Flexural Design

- **Check maximum stress under service load**

$$f_f = \frac{M_s}{A_f\, d\,(1 - k/3)} \leq 0.25\, f_{fu}$$

Clause 16.8.3

$$E_c = \left(3000\sqrt{f_c'} + 6900\right)\left(\frac{\gamma_c}{2300}\right)^{1.5}$$

Clause 8.4.1.7

$$E_c = \left(3000\sqrt{35} + 6900\right)\left(\frac{2300}{2300}\right)^{1.5} = 24648\, MPa$$

$$n_f = \frac{E_f}{E_c} = \frac{62600}{24648} = 2.54\,,\ k = \sqrt{2\,\rho_f\, n_f + \left(\rho_f\, n_f\right)^2} - \rho_f\, n_f$$

$$= \sqrt{2 * 0.0081 * 2.54 + (0.0081 * 2.54)^2} - 0.0081 * 2.54$$

$$= 0.183$$

Flexural Design

- **Check maximum stress under service load**

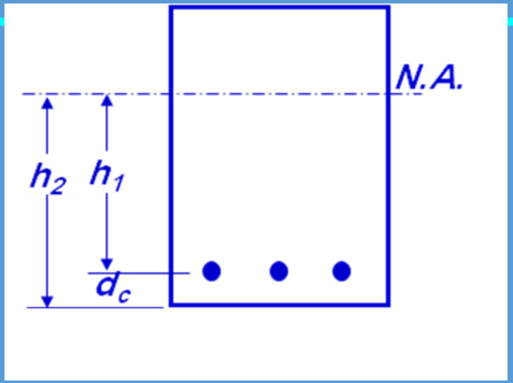
$$f_f = \frac{44.93 * 10^6}{\frac{1000}{135} * 197.9 * 182.06 * (1 - 0.183/3)} = 179.3\ MPa \leq 296.0\ MPa$$

$$\varepsilon_f = \frac{179.27}{62600} = 0.0029 > 0.0015 \Rightarrow \textbf{check crack control}$$

Flexural Design

- **Check crack width parameter**

$$w = 2\,\frac{f_f}{E_f}\,\frac{h_2}{h_1}\,k_b\,\sqrt{d_c^2 + (0.5\,s)^2}$$ ***Clause 16.8.2.***



$$h_1 = d - kd = 182.06 - 0.183\,x\,182.06 = 148.78\,mm$$

$$h_2 = h - kd = 225 - 0.183\,x182.06 = 191.72\,mm$$

$$d_c = h - d = 225 - 182.06 = 42.94 \leq 50mm$$

$$w = 2\,x\,\frac{179.27}{62600}\,x\,\frac{191.72}{148.78}\,x\,0.8\,x\,\sqrt{42.94^2 + (0.5\,x135)^2}$$

$$\underline{w = 0.472\,mm \quad < \ 0.5\,mm}$$

Courtesy of:

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Session 1: Introduction

End of Session

Questions

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