

# Fundamentals of Heat Transfer



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# Determination of heat transfer coefficient

Following methods may be employed for the determination of heat transfer coefficient:

- ✚ Experimental determination (Experimental correlations)
- ✚ Analysis of boundary layer
- ✚ Using analogies

# Factors affecting heat transfer coefficient

The heat transfer coefficient from hot horizontal surface to the surrounding fluid has been found to be influenced by:

- ✚ Specific heat capacity
- ✚ Thermal conductivity
- ✚ Density
- ✚ Viscosity
- ✚ Linear dimension of the surface (pipe diameter)
- ✚ Flow velocity
- ✚ Temperature difference
- ✚ Coefficient of thermal expansion of the fluid, and
- ✚ Acceleration due to gravity.

# Dimensional Analysis

By arranging the variables affecting the heat transfer coefficient in certain dimensionless groups it is convenient to observe the effect of variables as number of groups are less than the individual parameters.

See [class notes](#) for the derivation.

# Variables along with their dimensions

Variable	Symbol	Dimension
Heat transfer coefficient	$h = \frac{q}{A \cdot \Delta T}$	$\frac{H}{\theta \cdot L^2 \cdot T}$
Velocity of fluid	$u$	$\frac{L}{\theta}$
Linear dimension (Characteristic length)	$l$	L
Viscosity of fluid	$\mu$	$\frac{M}{L \cdot \theta}$
Specific heat capacity of fluid	$c_p$	$\frac{H}{M \cdot T}$
Density of fluid	$\rho$	$\frac{M}{L^3}$
Thermal conductivity of fluid	$k$	$\frac{H}{\theta \cdot L \cdot T}$
Temperature difference between fluid and wall of the conduit	$\Delta T$	T
Coefficient of thermal expansion of fluid	$\beta$	$\frac{1}{T}$
Acceleration due to gravity	$g$	$\frac{L}{\theta^2}$
Dimensionless constant used when mechanical energy is converted into heat or vice-versa	$J$	$\frac{M \cdot L^2}{H \cdot \theta^2}$

# Nusselt Equation

$$Nu = f(Re, Pr, Gr)$$

$$\frac{h \cdot l}{k} = \text{Nusselt number} = Nu$$

$$\frac{c_p \cdot \mu}{k} = \text{Prandtl number} = Pr$$

$$\frac{l \cdot u \cdot \rho}{\mu} = \text{Reynolds number} = Re$$

$$\frac{\beta \cdot g \cdot \Delta T \cdot l^3 \cdot \rho^2}{\mu^2} = \text{Grashof number for heat transfer} = Gr$$

# Cases for Nusselt equation

Free Convection:  $Nu = f(Pr, Gr)$

Forced Convection:  $Nu = f(Re, Pr)$

# Physical interpretation of $Nu$ , $Re$ , $Pr$ , and $Gr$

Group	Symbol	Proportional to the ratio	Formula	Applications
Nusselt number	$Nu$	$\frac{\text{heat transfer by convection}}{\text{heat transfer by conduction}}$ or $\frac{\text{conductive thermal resistance}}{\text{convective thermal resistance}}$ <p>Note: Convective heat transfer means total heat transfer by both molecular and bulk flow mechanisms</p>	$\frac{h \cdot l}{k}$	Heat transfer in fluids in motion
Reynolds number	$Re$	$\frac{\text{inertial forces}}{\text{viscous forces}}$	$\frac{l \cdot u \cdot \rho}{\mu}$	Criteria for type of flow behavior  Forced convection heat and mass transfer
Prandtl number	$Pr$	$\frac{\text{molecular diffusivity of momentum}}{\text{molecular diffusivity of heat}}$ <p>Note: Molecular diffusivity of momentum is also called kinematic viscosity</p>	$\frac{\nu}{\alpha} = \frac{\mu / \rho}{k / (\rho \cdot c_p)}$ $= \frac{c_p \cdot \mu}{k}$	Simultaneous heat and momentum transfer
Grashof number	$Gr$	$\frac{\text{bouyant forces}}{\text{viscous forces}}$	$\frac{\beta \cdot g \cdot \Delta T \cdot l^3 \cdot \rho^2}{\mu^2}$	Free convection heat and mass transfer



# Schmidt number and Lewis number

**Schmidt number** is a ratio of molecular diffusivity of momentum (kinematic viscosity) to molecular diffusivity of mass.

**Lewis number** is a ratio of molecular diffusivity of heat (thermal diffusivity) to molecular diffusivity of mass.

Write expression for each of the above.

# Heat transfer correlations

✚ Increasing velocity of a fluid decreases the corresponding film thickness and therefore heat transfer coefficient is increased by increasing the velocity of the fluid. Increasing heat transfer coefficient means increasing heat transfer rate. A higher rate of heat transfer means generally a small size of heat transfer equipment and less consumption of heating (or cooling) utility and less time of operation in a batch process. All these lead to more profit.

✚ Viscosity is one of the major factor that influences the heat transfer coefficient negatively. Higher viscosity of fluid means thicker film and higher heat transfer resistance and therefore lower heat transfer coefficient and lower rate of heat transfer.

✚ Think the ways by which one can increase the heat transfer coefficient in various day to day heat transfer applications.

# Heat transfer correlations

A heat transfer coefficient depends upon many factors (geometric, hydrodynamic, and fluid properties) and there is not a single value of heat transfer coefficient for a given system. Therefore, for the given situation, one needs to do experiments. However, in the literature, concerned people have developed (based on experiments) correlations that can be used, in usual cases, with sufficient accuracy for a problem in hand. Many such correlations have been developed over the years and cited in heat transfer texts and related journals. A few of the important correlations are discussed in the coming slides.

# Heat transfer correlations [1]

**For laminar flow inside a horizontal pipe:**

[Seider and Tate correlation](#)

$$Nu = \frac{h \cdot D}{k} = 1.86 \cdot \left( Re \cdot Pr \cdot \frac{D}{L} \right)^{1/3} \cdot \left( \frac{\mu}{\mu_w} \right)^{0.14}$$

**For turbulent flow inside a pipe:**

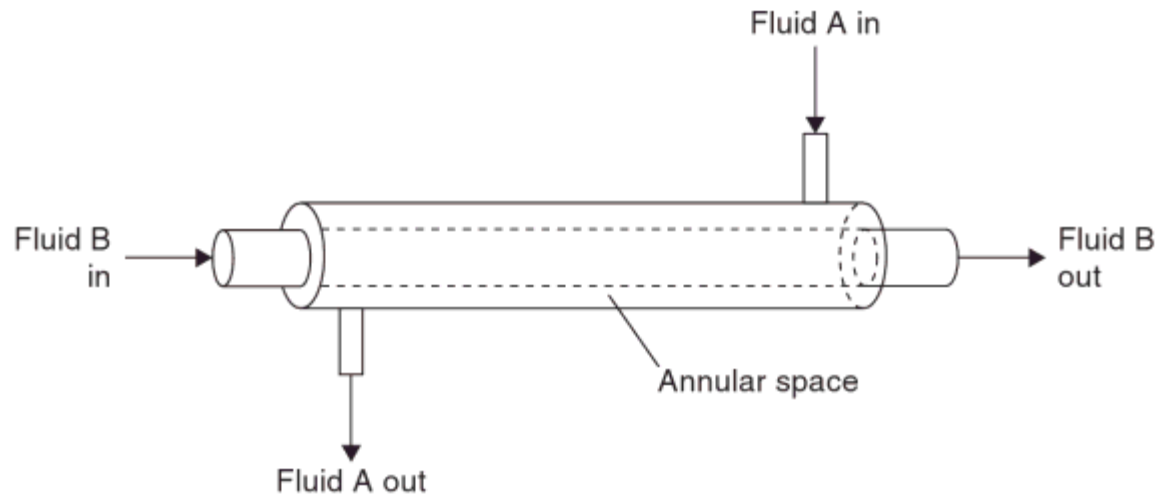
$$Nu = \frac{h \cdot D}{k} = 0.027 \cdot Re^{0.8} \cdot Pr^{\frac{1}{3}} \cdot \left( \frac{\mu}{\mu_w} \right)^{0.14}$$

For detail see Reference 1.

# Problem [p. 262, 1]

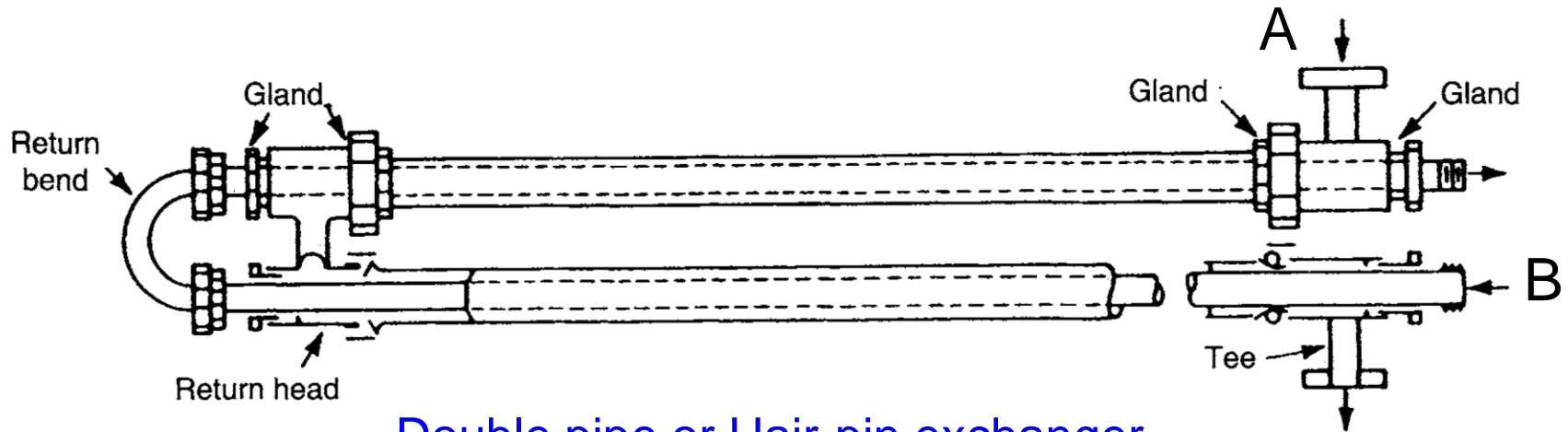
Air at 206.8 kPa and an average of 477.6 K is being heated as it flows through a tube of 25.4 mm inside diameter at a velocity of 7.62 m/s. The heating medium is 488.7 K steam condensing on the outside of the tube. Since the heat transfer coefficient of condensing steam is several thousand  $\text{W}/\text{m}^2\cdot\text{K}$  and the resistance of the metal wall is very small, it will be assumed that the surface wall temperature of the metal in contact with the air is 488.7 K. Calculate the heat transfer coefficient for an  $L/D > 60$ .

# Solution to the problem



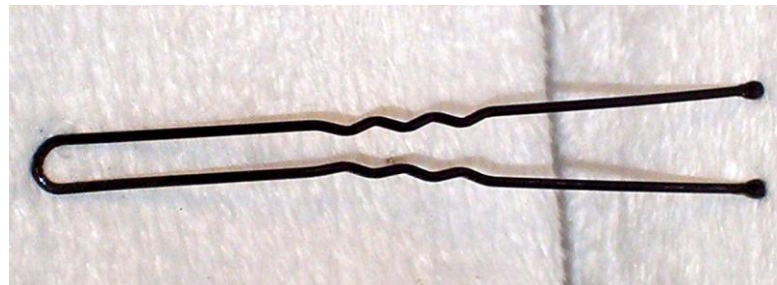
- Identify air and steam (A or B)
- Locate heat transfer resistances

# Solution to the problem



Double pipe or Hair-pin exchanger

- Identify air and steam (A or B)
- Locate heat transfer resistances



Hair-pin

# Solution to the problem [p. 262, 1]

For application of the equation:

$$Re > 10^4$$

$$Pr \text{ between } 0.7 \text{ and } 16,000, \text{ and } \frac{L}{D} > 60$$

$h$  is based on the log-mean driving force

Fluid properties except for  $\mu_w$  are evaluated at the mean bulk temperature.

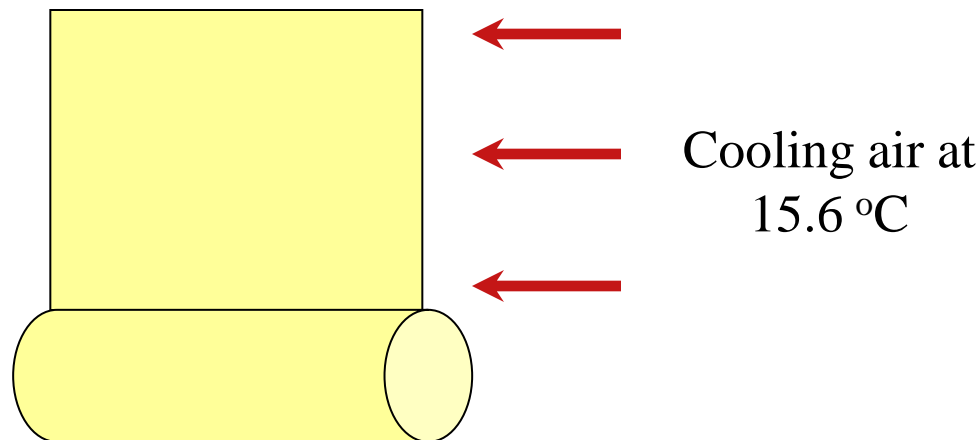
Solution: See [classwork](#).



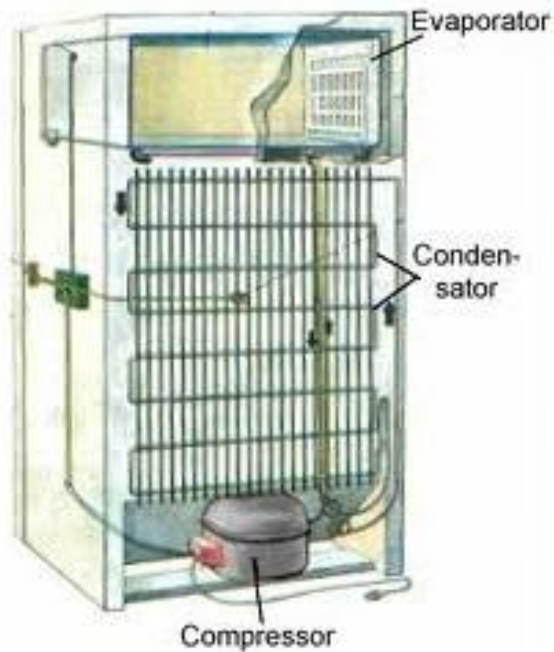
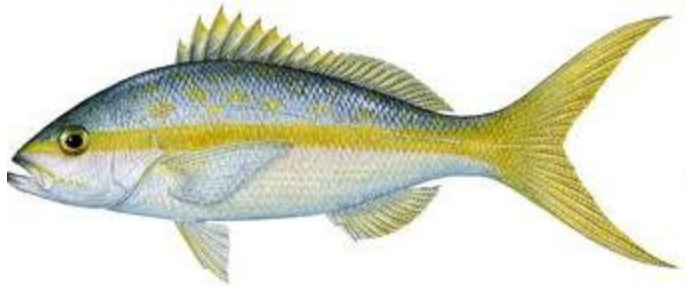
# Problem [p. 272, 1]

A smooth, flat, thin fin of copper extending out from a tube is 51 mm by 51 mm square. Its temperature is approximately uniform at 82.2 °C. Cooling air at 15.6 °C and 1 atm abs flows parallel to the fin at a velocity of 12.2 m/s.

- For laminar flow, calculate the heat transfer coefficient,  $h$ .
- If the leading edge of the fin is rough so that all of the boundary layer or film next to the fin is completely turbulent, calculate  $h$ .



# Fins or extended surfaces



Longitudinal fins



Traverse fins



# Problem-2 [p. 272, 1]

Laminar:  $Nu = 0.664 \cdot Re_L^{0.5} \cdot Pr^{1/3}$

For application of the equation:

$$Re < 3 \times 10^5$$

$$Pr > 0.7$$

Turbulent:  $Nu = 0.0366 \cdot Re_L^{0.8} \cdot Pr^{1/3}$

For application of the equation:

$$Re > 3 \times 10^5$$

$$Pr > 0.7$$

Solution: See [classwork](#).

# Natural convection from various geometries [p. 278, 1]

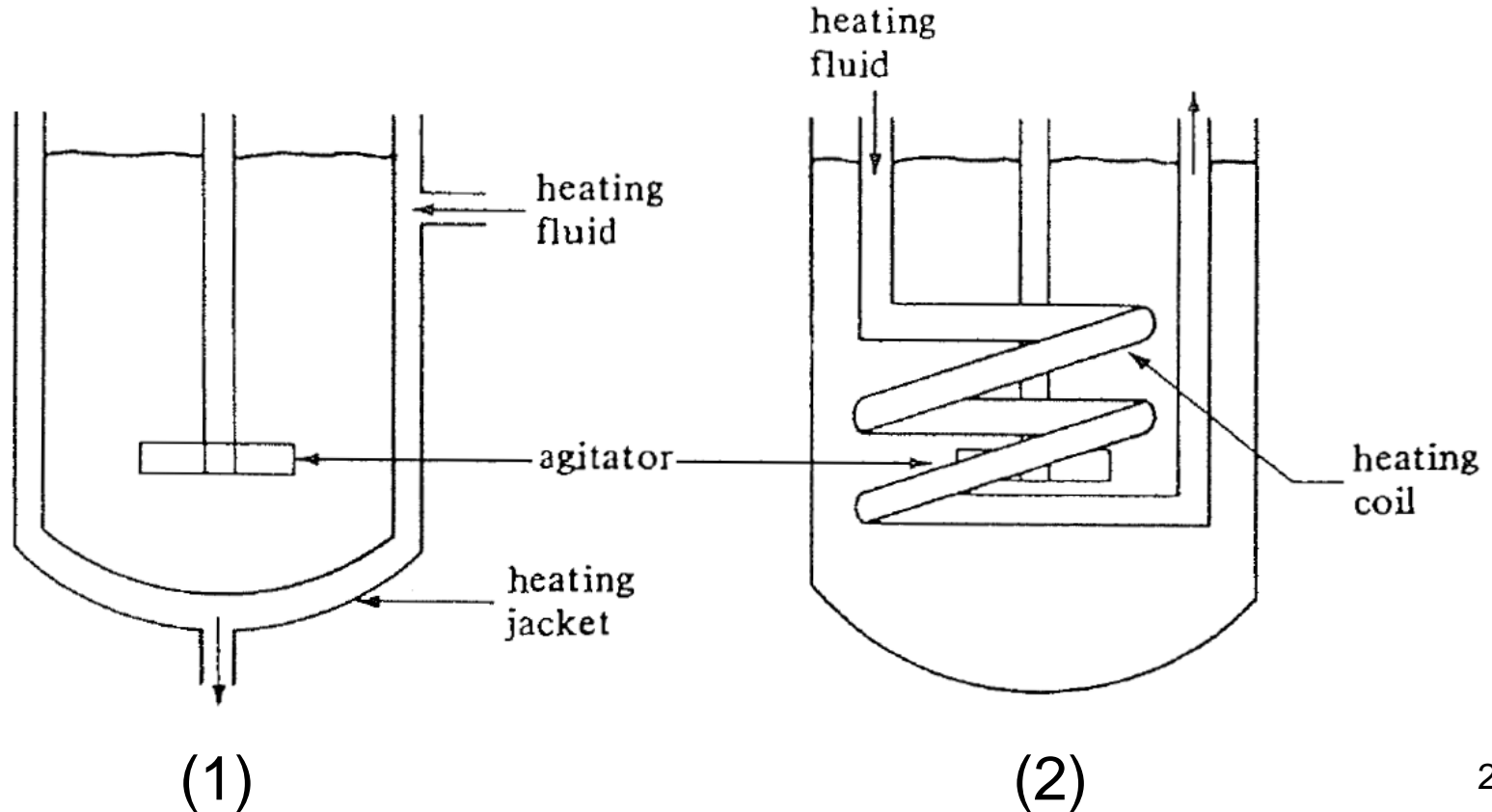
$$Nu = \frac{h \cdot L}{k} = a \cdot \left( \frac{L^3 \cdot \rho^2 \cdot g \cdot \beta \cdot \Delta T}{\mu^2} \cdot \frac{c_p \cdot \mu}{k} \right)^m = a \cdot (Gr \cdot Pr)^m$$

Physical Geometry	$N_{Gr} N_{Pr}$	$a$	$m$	Ref.
Vertical planes and cylinders [vertical height $L < 1$ m (3 ft)]				
	$< 10^4$	1.36	$\frac{1}{3}$	(P3)
	$10^4 - 10^9$	0.59	$\frac{1}{4}$	(M1)
	$> 10^9$	0.13	$\frac{1}{3}$	(M1)
Horizontal cylinders [diameter $D$ used for $L$ and $D < 0.20$ m (0.66 ft)]				
	$< 10^{-5}$	0.49	0	(P3)
	$10^{-5} - 10^{-3}$	0.71	$\frac{1}{25}$	(P3)
	$10^{-3} - 1$	1.09	$\frac{1}{16}$	(P3)
	$1 - 10^4$	1.09	$\frac{1}{5}$	(P3)
	$10^4 - 10^9$	0.53	$\frac{1}{4}$	(M1)
	$> 10^9$	0.13	$\frac{1}{3}$	(P3)
Horizontal plates				
Upper surface of heated plates or lower surface of cooled plates	$10^5 - 2 \times 10^7$	0.54	$\frac{1}{4}$	(M1)
	$2 \times 10^7 - 3 \times 10^{10}$	0.14	$\frac{1}{3}$	(M1)
Lower surface of heated plates or upper surface of cooled plates	$10^5 - 10^{11}$	0.58	$\frac{1}{3}$	(F1)

# Heat transfer in agitated vessels

## [p. 327, 1]

1. Vessels with heating jackets
2. Vessels with heating coils



# Heat transfer coefficient in agitated jacketed vessel [p. 327, 1]

$$\frac{h \cdot D_t}{k} = a \cdot \left( \frac{D_a^2 \cdot N \cdot \rho}{\mu} \right)^b \cdot \left( \frac{c_p \cdot \mu}{k} \right)^{\frac{1}{3}} \cdot \left( \frac{\mu}{\mu_w} \right)^m$$

# Heat transfer coefficient in agitated jacketed vessel [p. 327, 1]

1. Paddle agitator with no baffles (C5, U1)

$$a = 0.36, \quad b = \frac{2}{3}, \quad m = 0.21, \quad N'_{Re} = 300 \text{ to } 3 \times 10^5$$

2. Flat-blade turbine agitator with no baffles (B4)

$$a = 0.54, \quad b = \frac{2}{3}, \quad m = 0.14, \quad N'_{Re} = 30 \text{ to } 3 \times 10^5$$

3. Flat-blade turbine agitator with baffles (B4, B5)

$$a = 0.74, \quad b = \frac{2}{3}, \quad m = 0.14, \quad N'_{Re} = 500 \text{ to } 3 \times 10^5$$

4. Anchor agitator with no baffles (U1)

$$a = 1.0, \quad b = \frac{1}{2}, \quad m = 0.18, \quad N'_{Re} = 10 \text{ to } 300$$

$$a = 0.36, \quad b = \frac{2}{3}, \quad m = 0.18, \quad N'_{Re} = 300 \text{ to } 4 \times 10^4$$

5. Helical ribbon agitator with no baffles (G4)

$$a = 0.633, \quad b = \frac{1}{2}, \quad m = 0.18, \quad N'_{Re} = 8 \text{ to } 10^5$$

# Types of impellers



Paddle impeller



Paddle impeller



Flat-blade  
turbine impeller



3-blade marine  
propeller impeller



Ribbon impeller



Achor impeller



# Typical overall heat transfer coefficients in jacketed vessels [p. 328, 1]

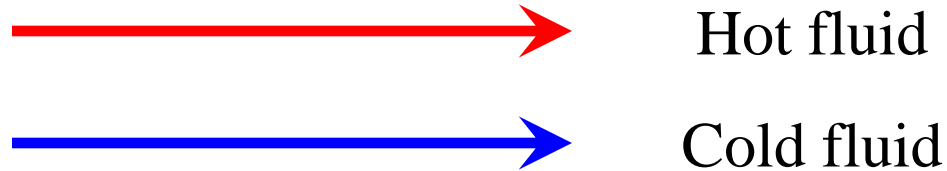
<i>Fluid in Jacket</i>	<i>Fluid in Vessel</i>	<i>Wall Material</i>	<i>Agitation</i>	<i>U</i>		<i>Ref.</i>
				$\frac{btu}{h \cdot ft^2 \cdot ^\circ F}$	$\frac{W}{m^2 \cdot K}$	
Steam	Water	Copper	None	150	852	(P1)
			Simple stirring	250	1420	
Steam	Paste	Cast iron	Double scrapers	125	710	(P1)
Steam	Boiling water	Copper	None	250	1420	(P1)
Steam	Milk	Enameled cast iron	None	200	1135	(P1)
			Stirring	300	1700	
Hot water	Cold water	Enameled cast iron	None	70	398	(P1)
Steam	Tomato purée	Metal	Agitation	30	170	(C1)

# Homework Problems

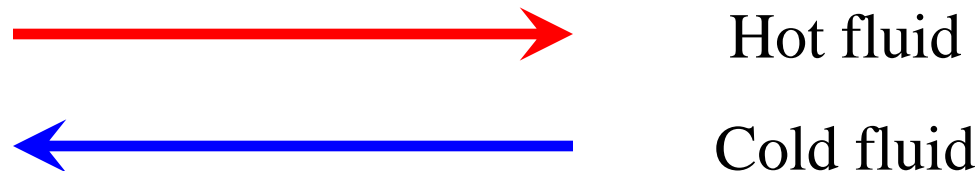
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✚ Ref. 1: 4.5-1, 4.6-1, 4.6-3

# Co-current and Counter-Current flow patterns

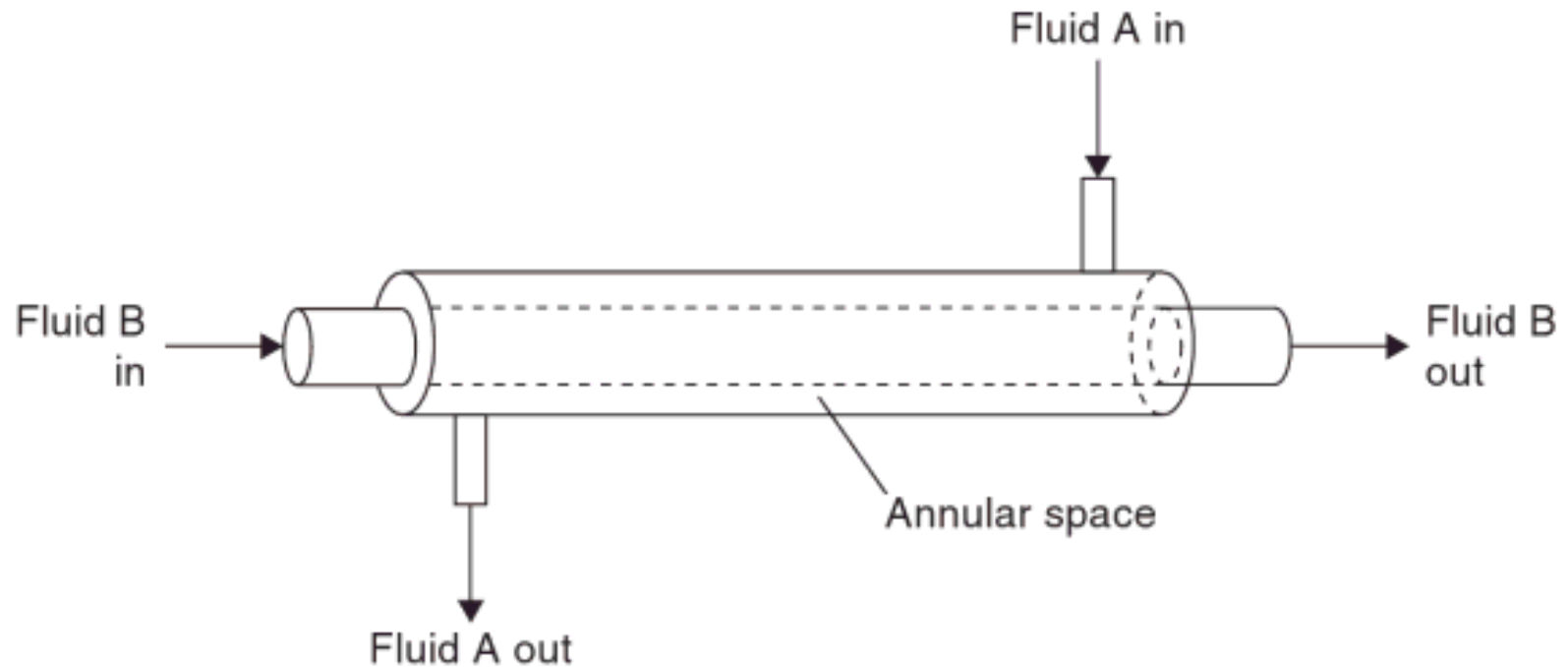


Co-current flow or  
parallel flow



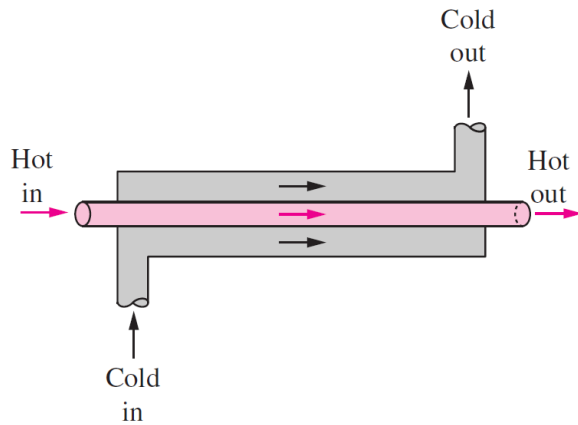
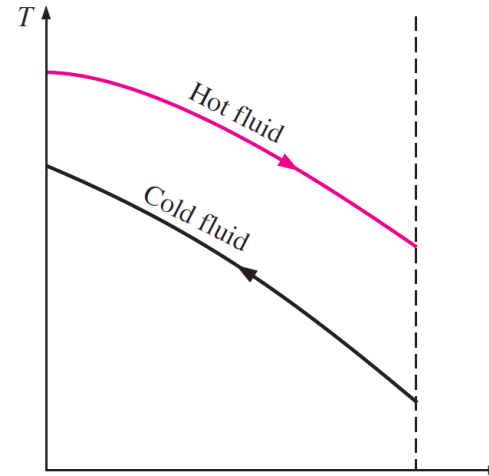
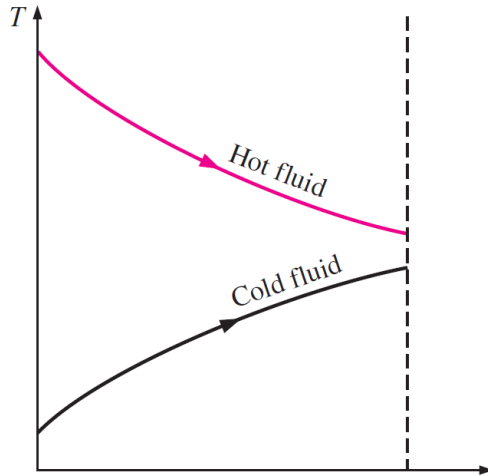
Counter-current flow  
or anti-parallel flow

# Co-current and Counter-Current flow patterns

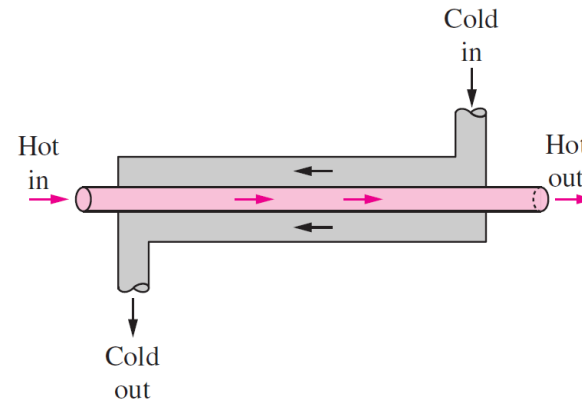


Identify the type of flow?

# Identify the type of flow?



?



?

# Co-current and Counter-Current flow patterns (Class input)

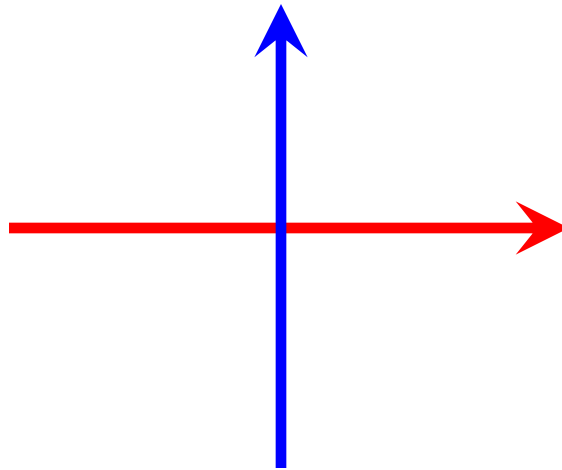
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What information you can get by comparing co-current and counter-current flows?

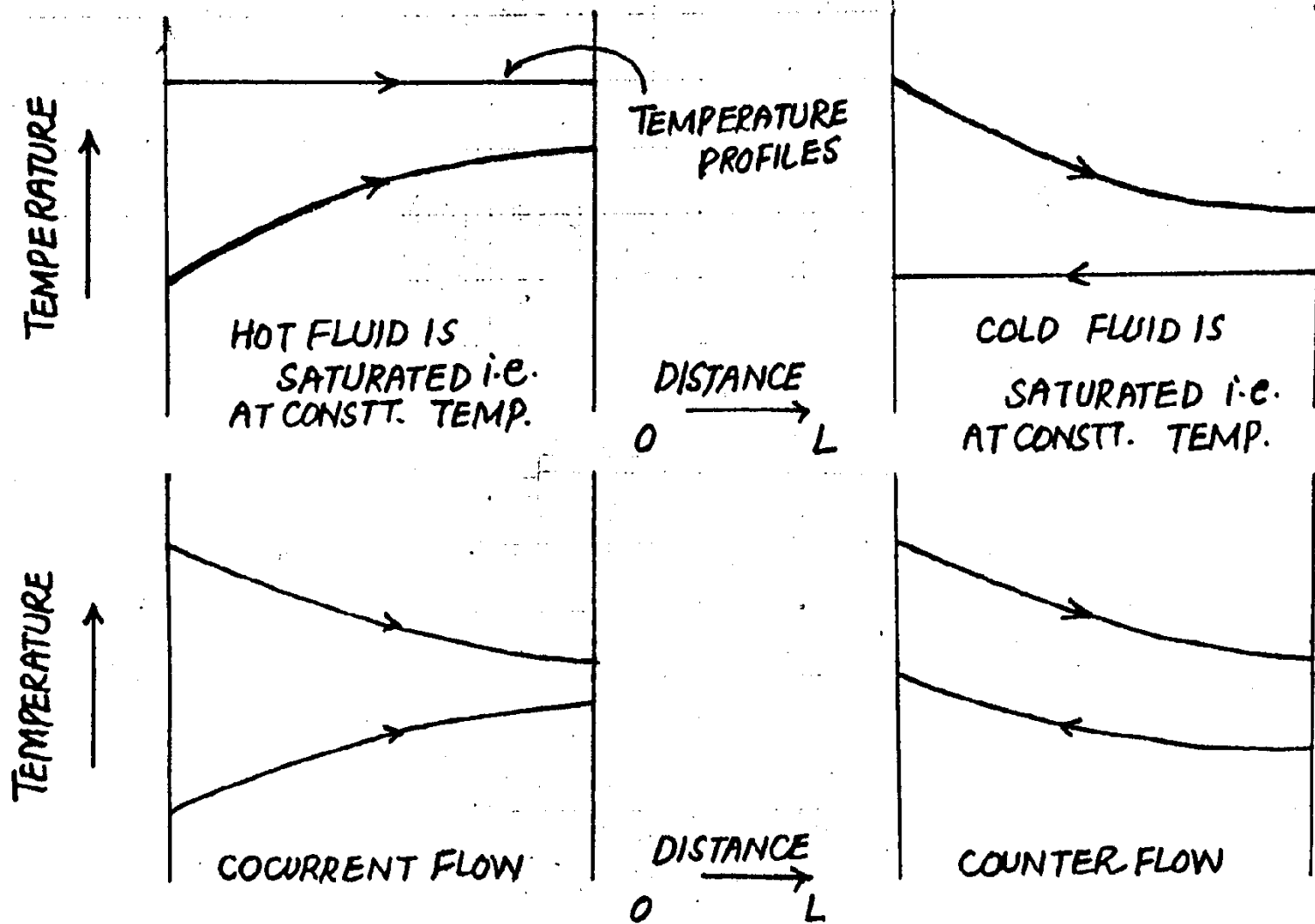
- 1.
- 2.
- 3.

# Cross-flow pattern

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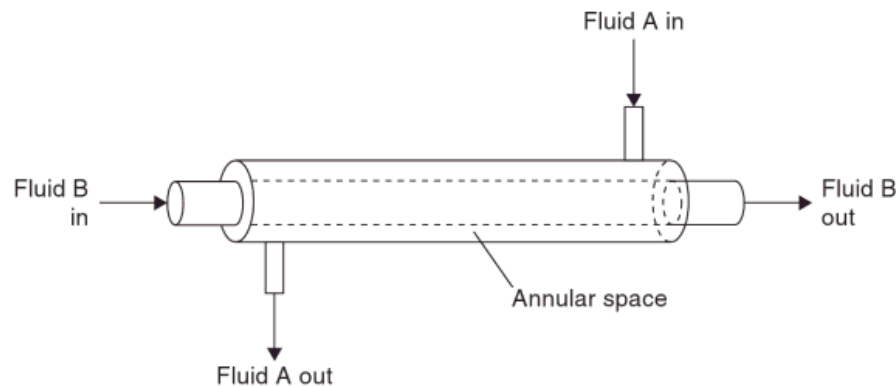
# Temperature distributions



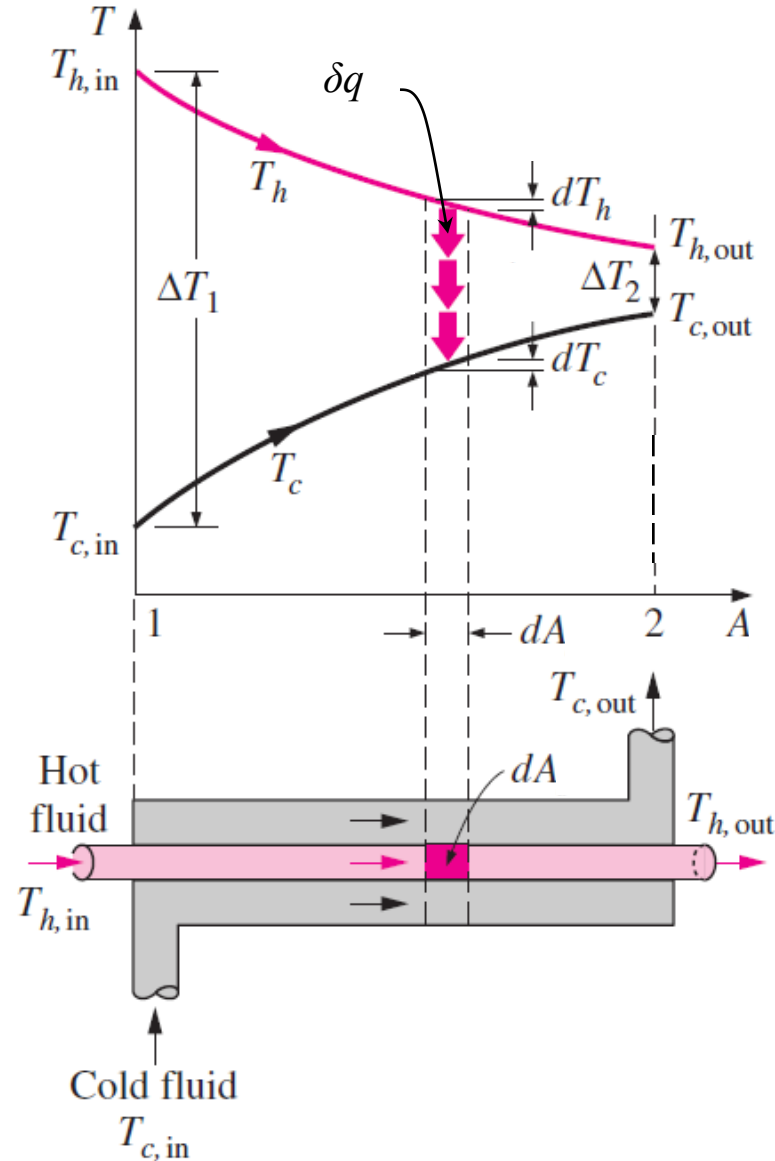


# The Logarithmic (Log) Mean Temperature Difference Method

For the heat exchanger system shown in the figure below, the temperatures of the heating and cooling fluids are not constant but vary along the length of the heat exchanger. Generally, both these temperature variations are not straight lines and an arithmetic mean temperature difference is not the appropriate temperature difference. Under such conditions, we define **logarithmic mean temperature difference** (LMTD) which is a better representation of mean temperature difference.



# The logarithmic (Log) mean temperature difference method



# The logarithmic (log) mean temperature difference method

## Assumptions:

- The overall heat transfer coefficient is constant over the entire length of the heating surface (of the heat exchanger).
- The flowrates of cold and hot fluids are not varying with time (constant). The steady-state conditions are therefore established.
- The system is considered adiabatic and the heat losses are negligibly small.
- Variations of the properties of the fluids with temperature are small enough to be neglected. Thus specific heat capacities are taken as constants.
- No phase change occurs in either of the fluids. The derivation is applicable for the cases when there are sensible heat changes or when vaporization and condensation is isothermal for the whole length of the effective heat transfer surface.

# The logarithmic (log) mean temperature difference method

$$LMTD = \Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

$$\Delta T_1 = T_{h1} - T_{c1}$$

$$\Delta T_2 = T_{h2} - T_{c2}$$

# The logarithmic (log) mean temperature difference method

If the temperature difference between  $T_{h1}$  and  $T_{c1}$  is not more than 50% greater than  $T_{h2}-T_{c2}$ , the arithmetic mean temperature difference will be within 1% of the LMTD. This can simplify the calculations.

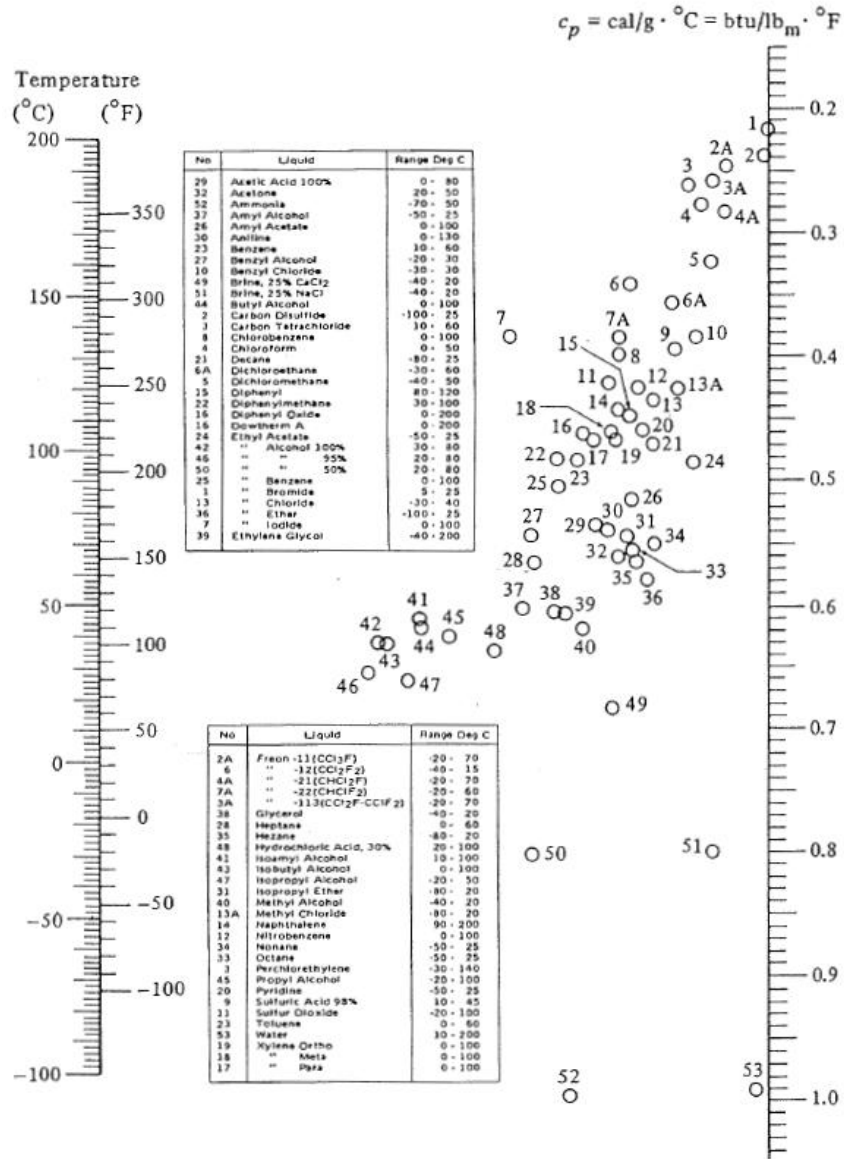
# Tube dimensions [p. 997, 1]

<i>Outside Diameter</i>		<i>BWG Number</i>	<i>Wall Thickness</i>		<i>Inside Diameter</i>		<i>Inside Cross-Sectional Area</i>	
<i>in.</i>	<i>mm</i>		<i>in.</i>	<i>mm</i>	<i>in.</i>	<i>mm</i>	<i>ft<sup>2</sup></i>	<i>m<sup>2</sup> × 10<sup>4</sup></i>
$\frac{5}{8}$	15.88	12	0.109	2.77	0.407	10.33	0.000903	0.8381
		14	0.083	2.11	0.459	11.66	0.00115	1.068
		16	0.065	1.65	0.495	12.57	0.00134	1.241
		18	0.049	1.25	0.527	13.39	0.00151	1.408
$\frac{3}{4}$	19.05	12	0.109	2.77	0.532	13.51	0.00154	1.434
		14	0.083	2.11	0.584	14.83	0.00186	1.727
		16	0.065	1.65	0.620	15.75	0.00210	1.948
		18	0.049	1.25	0.652	16.56	0.00232	2.154
$\frac{7}{8}$	22.23	12	0.109	2.77	0.657	16.69	0.00235	2.188
		14	0.083	2.11	0.709	18.01	0.00274	2.548
		16	0.065	1.65	0.745	18.92	0.00303	2.811
		18	0.049	1.25	0.777	19.74	0.00329	3.060
1	25.40	10	0.134	3.40	0.732	18.59	0.00292	2.714
		12	0.109	2.77	0.782	19.86	0.00334	3.098
		14	0.083	2.11	0.834	21.18	0.00379	3.523
		16	0.065	1.65	0.870	22.10	0.00413	3.836
$1\frac{1}{4}$	31.75	10	0.134	3.40	0.982	24.94	0.00526	4.885
		12	0.109	2.77	1.032	26.21	0.00581	5.395
		14	0.083	2.11	1.084	27.53	0.00641	5.953
		16	0.065	1.65	1.120	28.45	0.00684	6.357

# Steel pipe dimensions [p. 996, 1]

Nominal Pipe Size (in.)	Outside Diameter		Sched- ule Number	Wall Thickness		Inside Diameter		Inside Cross- Sectional Area	
	in.	mm		in.	mm	in.	mm	ft <sup>2</sup>	m <sup>2</sup> × 10 <sup>4</sup>
1/8	0.405	10.29	40	0.068	1.73	0.269	6.83	0.00040	0.3664
			80	0.095	2.41	0.215	5.46	0.00025	0.2341
1/4	0.540	13.72	40	0.088	2.24	0.364	9.25	0.00072	0.6720
			80	0.119	3.02	0.302	7.67	0.00050	0.4620
3/8	0.675	17.15	40	0.091	2.31	0.493	12.52	0.00133	1.231
			80	0.126	3.20	0.423	10.74	0.00098	0.9059
1/2	0.840	21.34	40	0.109	2.77	0.622	15.80	0.00211	1.961
			80	0.147	3.73	0.546	13.87	0.00163	1.511
3/4	1.050	26.67	40	0.113	2.87	0.824	20.93	0.00371	3.441
			80	0.154	3.91	0.742	18.85	0.00300	2.791
1	1.315	33.40	40	0.133	3.38	1.049	26.64	0.00600	5.574
			80	0.179	4.45	0.957	24.31	0.00499	4.641
1 1/4	1.660	42.16	40	0.140	3.56	1.380	35.05	0.01040	9.648
			80	0.191	4.85	1.278	32.46	0.00891	8.275
1 1/2	1.900	48.26	40	0.145	3.68	1.610	40.89	0.01414	13.13
			80	0.200	5.08	1.500	38.10	0.01225	11.40
2	2.375	60.33	40	0.154	3.91	2.067	52.50	0.02330	21.65
			80	0.218	5.54	1.939	49.25	0.02050	19.05

# Nomogram for sp. heat capacity [p. 984, 1]





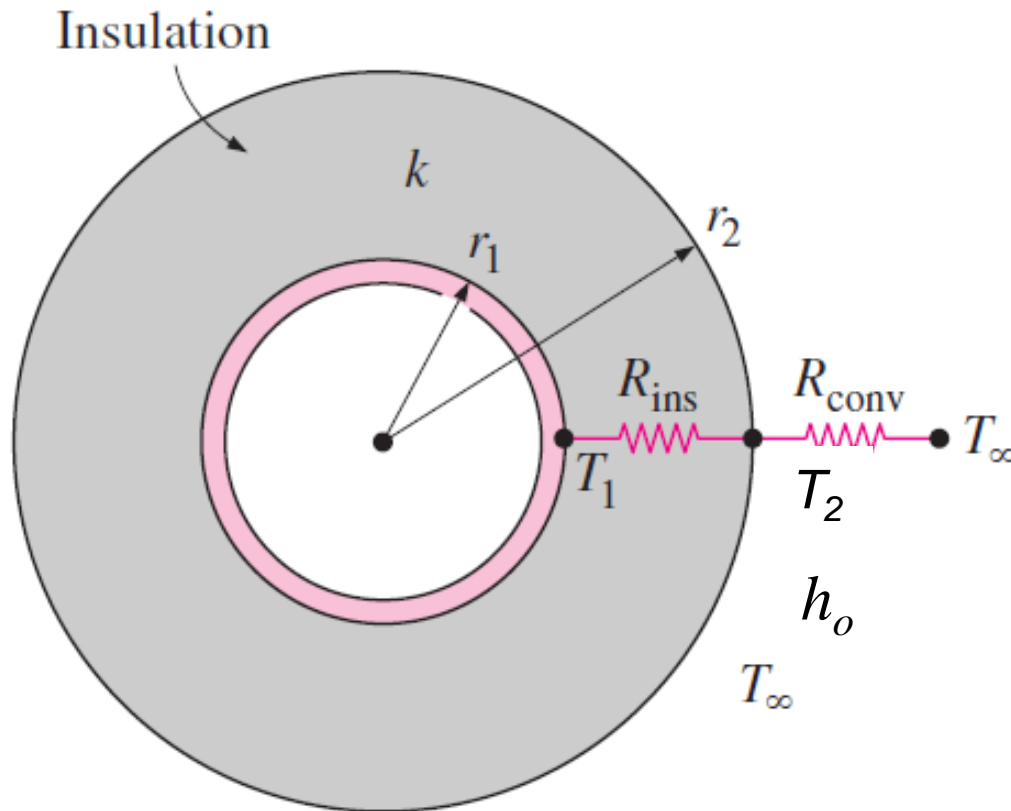
# Homework problems

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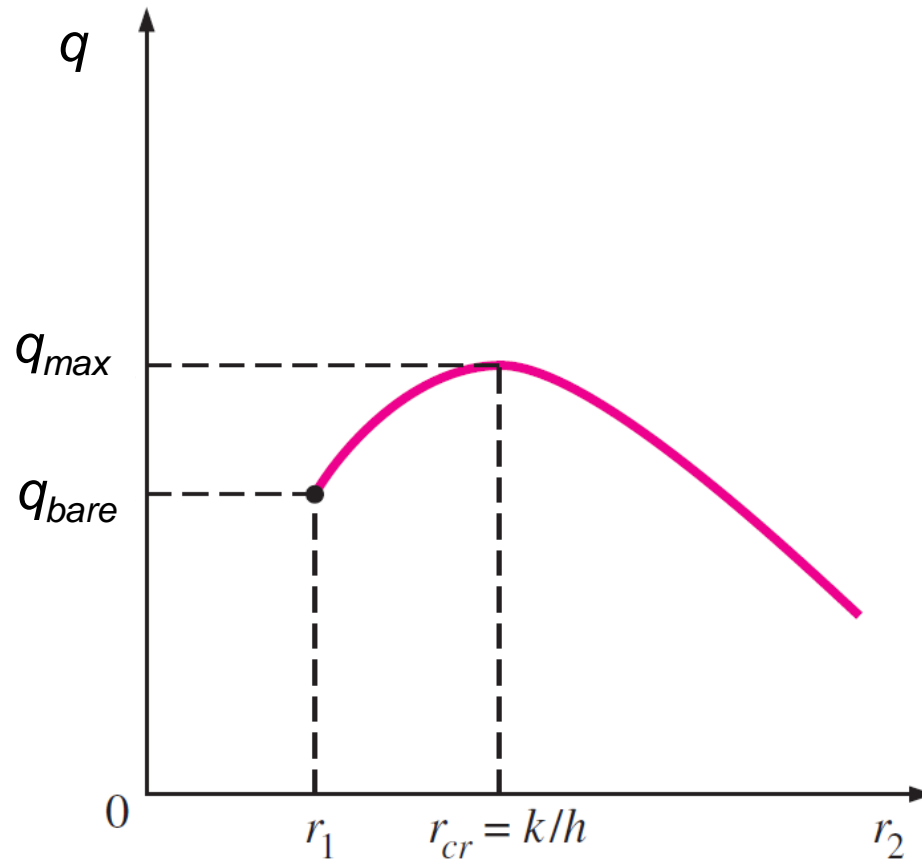
✚ Ref 1: 14.5-2, 4.5-3, 4.5-4, 4.5-4, 4.5-4.

See also separate sheet of the problems, i.e., Part-12.

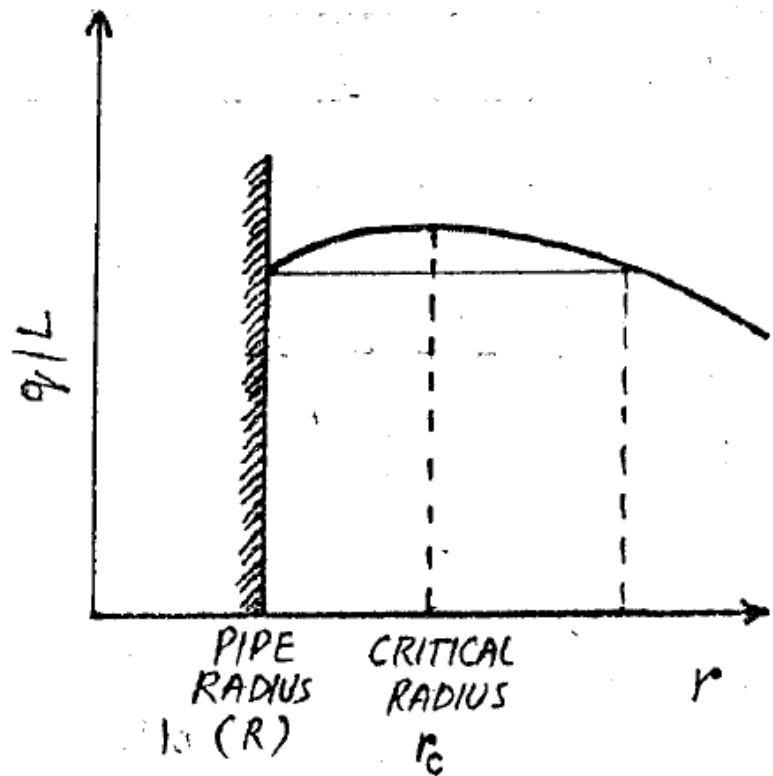
# Critical thickness (radius) of insulation for a cylinder



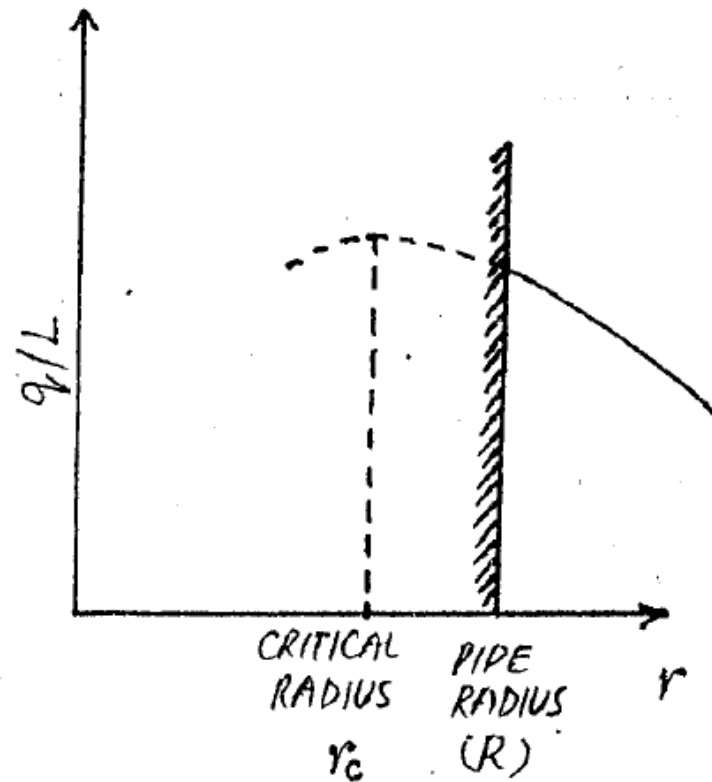
# Critical thickness (radius) of insulation for a cylinder



# Critical thickness (radius) of insulation for a cylinder



$$R \leq r_c$$

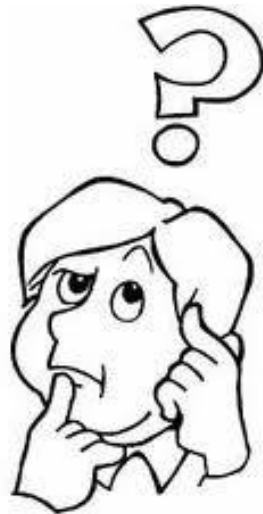


$$R > r_c$$

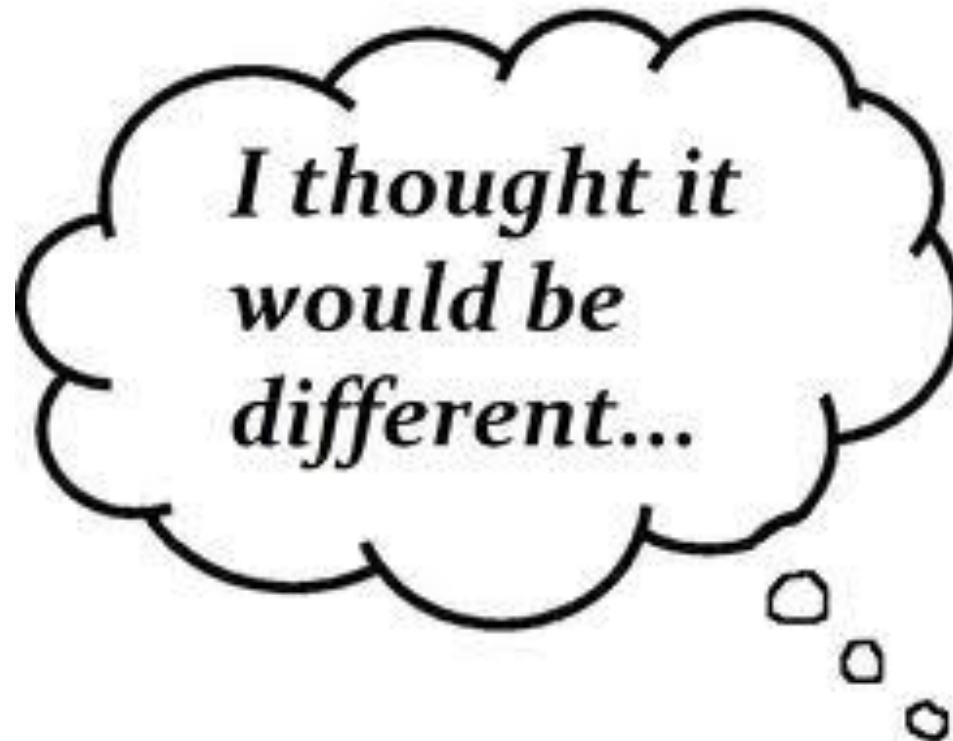
For larger pipes.

# Critical thickness (radius) of insulation for a flat wall

What would be the critical thickness for a flat wall?



# Critical thickness (radius) of insulation for a flat wall



# Critical thickness (radius) of insulation for a sphere

Derive an expression for critical radius of insulation for a spherical geometry.

$$r_{cr} = \frac{2k}{h_o}$$

# Boiling heat transfer

## Types of boiling:

1. Pool boiling (just as water heating in a kettle)
2. Convective or flow boiling (just as flowing water boiling inside a tube)



# Pool boiling

Pool boiling regimes:

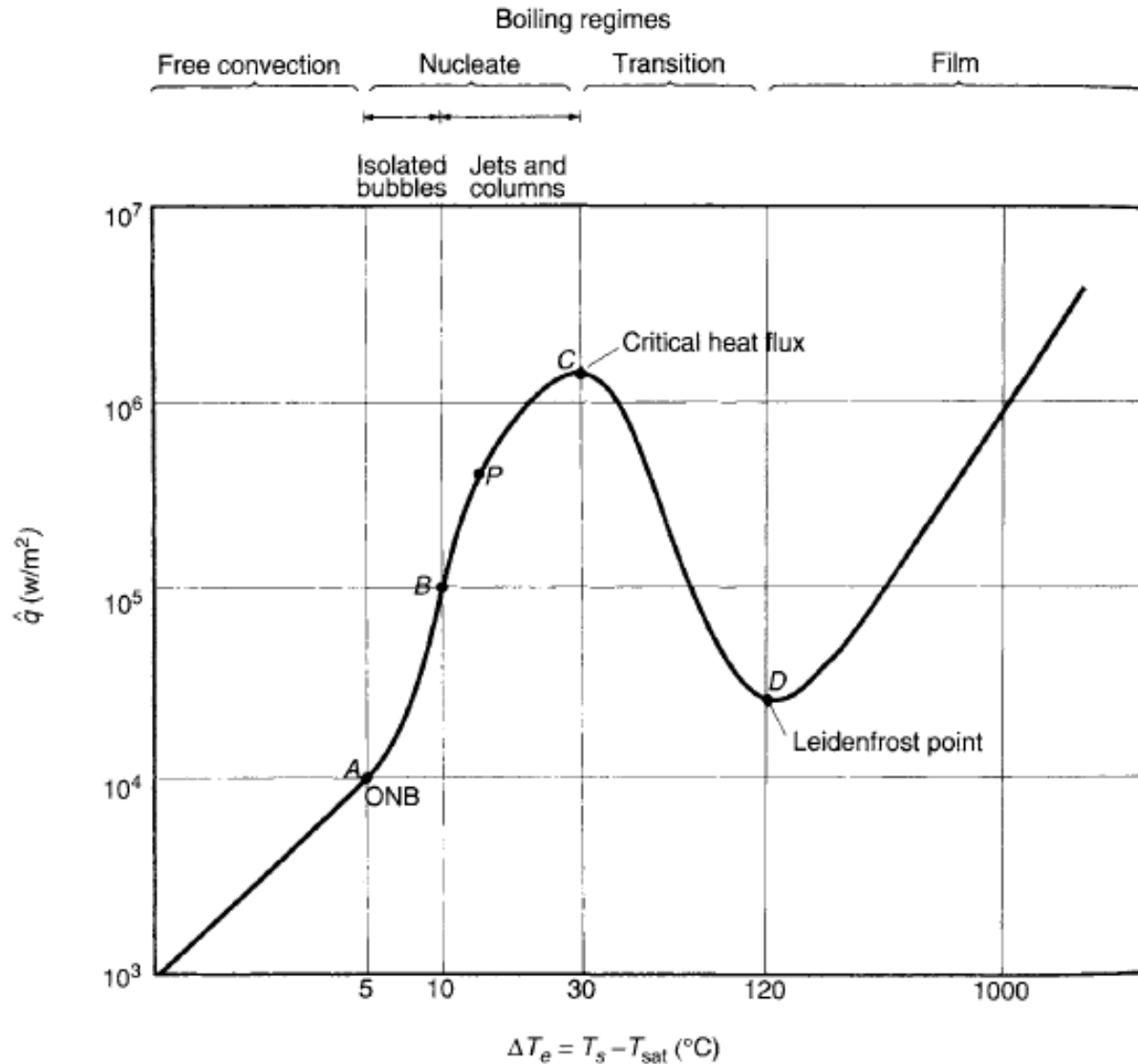
A: Natural or free convection boiling

B: Nucleate boiling

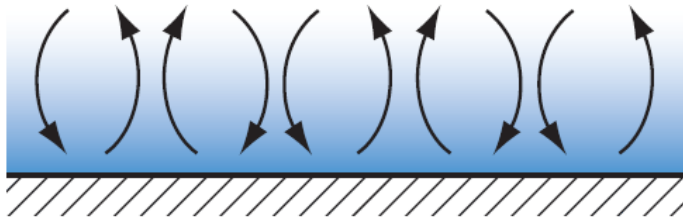
C: Transition boiling

D: Film boiling

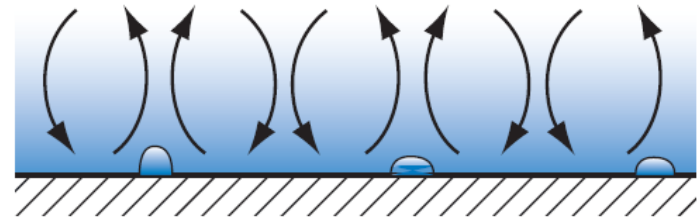
# Pool boiling curve for water at 1.013 bar [p. 387, 9]



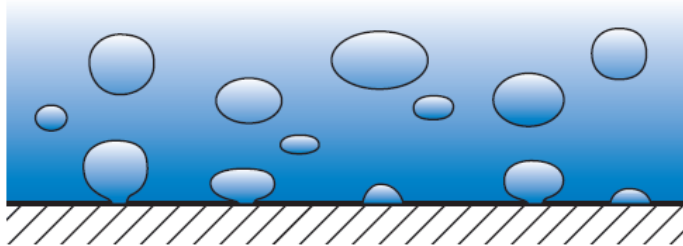
# Pictorial representation of various pool boiling regimes [p. 627, 10]



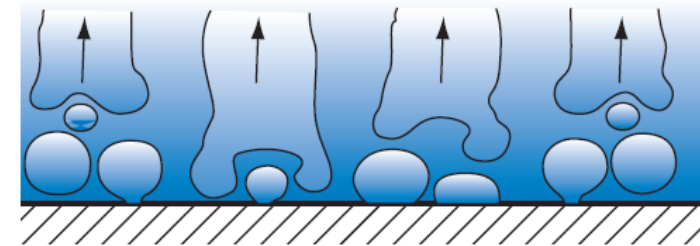
1. Natural convection



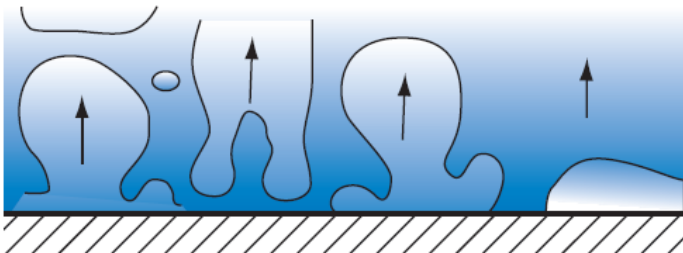
Onset of boiling



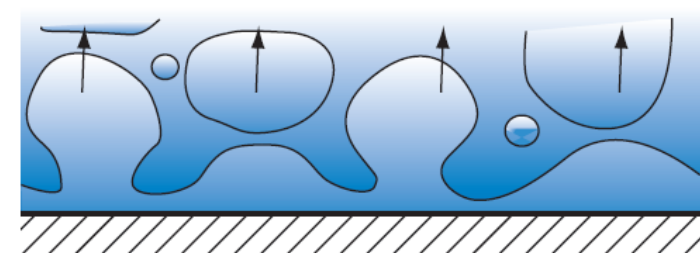
2. Individual bubble regime



3. Regime of slugs and bubbles



4. Transition film boiling



5. Stable film boiling

# Rate of heat transfer in pool boiling

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How can we calculate the rate of heat transfer for a pool boiling problem?

What do we need?

# References

1. Geankoplis, C.J. (2003). Transport processes and separation process principles: includes unit operations. 4<sup>th</sup> ed. Prentice-Hall International, Inc.
2. Holman, J.P. (2010). Heat transfer. 10<sup>th</sup> ed. McGraw-Hill Higher Education, Singapore.
3. Cengel, Y.A. (2003). Heat transfer: A practical approach. 2<sup>nd</sup> ed. McGraw-Hill.
4. Incropera, F.P.; DeWitt, D.P.; Bergman, T.L.; Lavine. A.S. (2007) Fundamentals of heat and mass transfer. 6<sup>th</sup> ed. John Wiley & Sons, Inc.
5. Kern, D.Q. (1965). Process heat transfer. McGraw-Hill International Book Co., Singapore.
6. McCabe, W.L.; Smith, J.C.; Harriott, P. (1993). Unit operations of chemical engineering. 5<sup>th</sup> ed. McGraw-Hill, Inc., Singapore.
7. Coulson, J.M.; Richardson, J.F.; Backhurst, J.R.; Harker, J.H. 1999. Coulson and Richardson's Chemical engineering: Fluid flow, heat transfer and mass transfer. vol. 1. 6<sup>th</sup> ed. Butterworth-Heinemann, Oxford.
8. Staff of Research and Education Association. (1984). The heat transfer problem solver. Research and Education Association, New Jersey.
9. Serth, R.W. (2007). Process Heat Transfer: Principles and Applications. Academic Press.
10. Kreith, F.; Manglik, R.J.; Bohn, M.S. 2011. Principles of heat transfer, 7<sup>th</sup> ed. Cengage Learning.