Fundamentals of Reliability Engineering and Applications

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IIE

February 21, 2012

Outline
Part 1. Reliability Definitions

- Reliability Definition…Time dependent characteristics
- Failure Rate
- Availability
- MTTF and MTBF
- Time to First Failure
- Mean Residual Life
- Conclusions

Outline
Part 2. Reliability Calculations

1. Use of failure data
2. Density functions
3. Reliability function
4. Hazard and failure rates

Outline
Part 3. Failure Time Distributions

1. Constant failure rate distributions
2. Increasing failure rate distributions
3. Decreasing failure rate distributions
4. Weibull Analysis – Why use Weibull?

Outline
Part 2. Reliability Calculations

1. Use of failure data
   a) Interval data (no censoring)
   b) Exact failure times are known
2. Density functions
3. Reliability function
4. Hazard and failure rates

Basic Calculations

Suppose n_0 identical units are subjected to a test. During the interval $(t, t+\Delta t)$, we observed $n_f(t)$ failed components. Let $n_s(t)$ be the surviving components at time t, then we define:

Failure density function $$\hat{f}(t) = \frac{n_f(t)}{n_0 \Delta t}$$

Failure rate function $$\hat{h}(t) = \frac{n_f(t)}{n_s(t)\Delta t},$$

Reliability function $$\hat{R}(t) = P_r(T > t) = \frac{n_s(t)}{n_0}$$

Basic Definitions Cont'd

The unreliability $F(t)$ is

$$F(t) = 1 - R(t)$$

Example: 200 light bulbs were tested and the failures in 1000-hour intervals are

Time Interval (Hours)	Failures in the interval
0-1000	100
1001-2000	40
2001-3000	20
3001-4000	15
4001-5000	10
5001-6000	8
6001-7000	7
Total	200

Calculations

Time Interval (Hours)	Failures in the interval
0-1000	100
1001-2000	40
2001-3000	20
3001-4000	15
4001-5000	10
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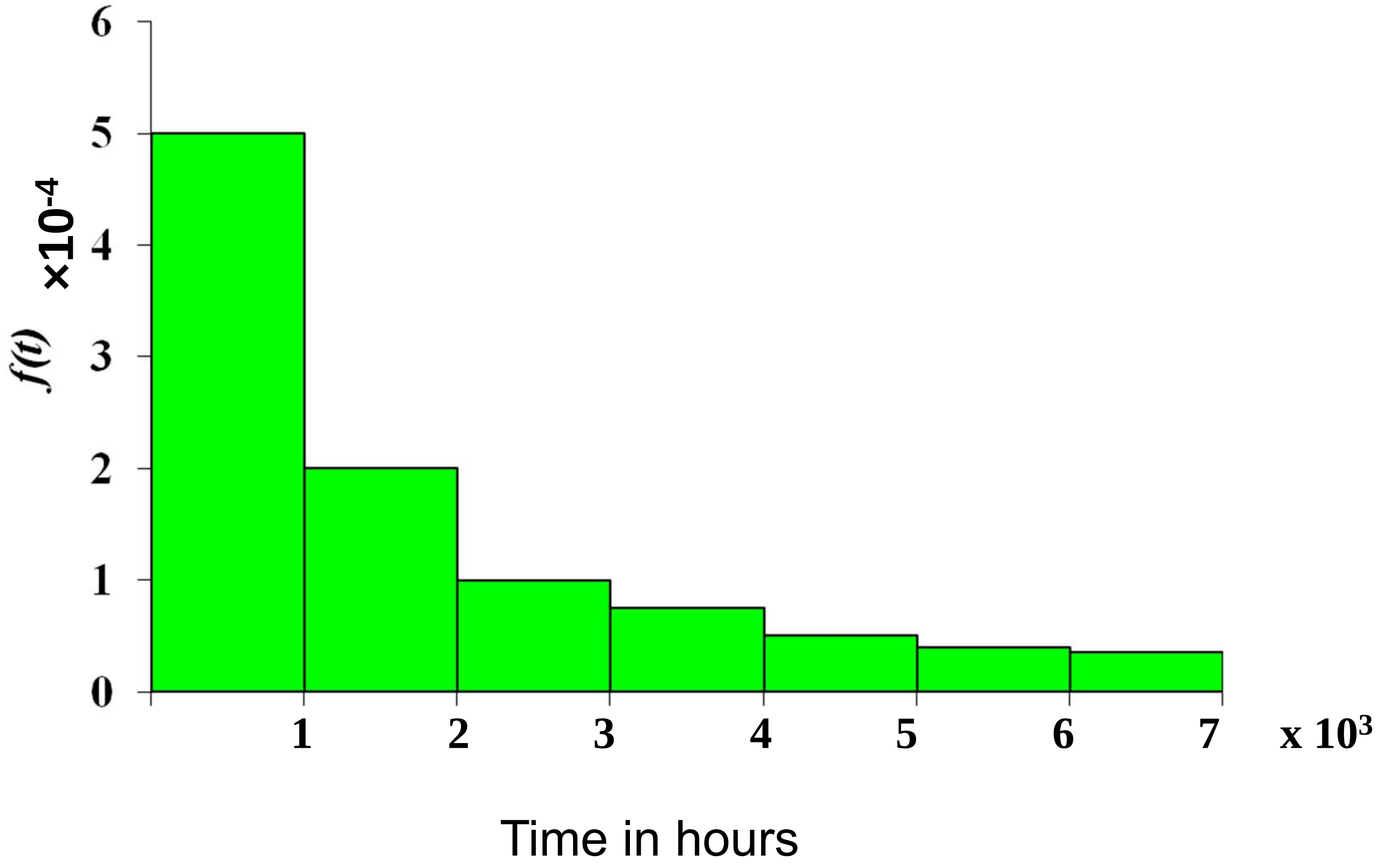
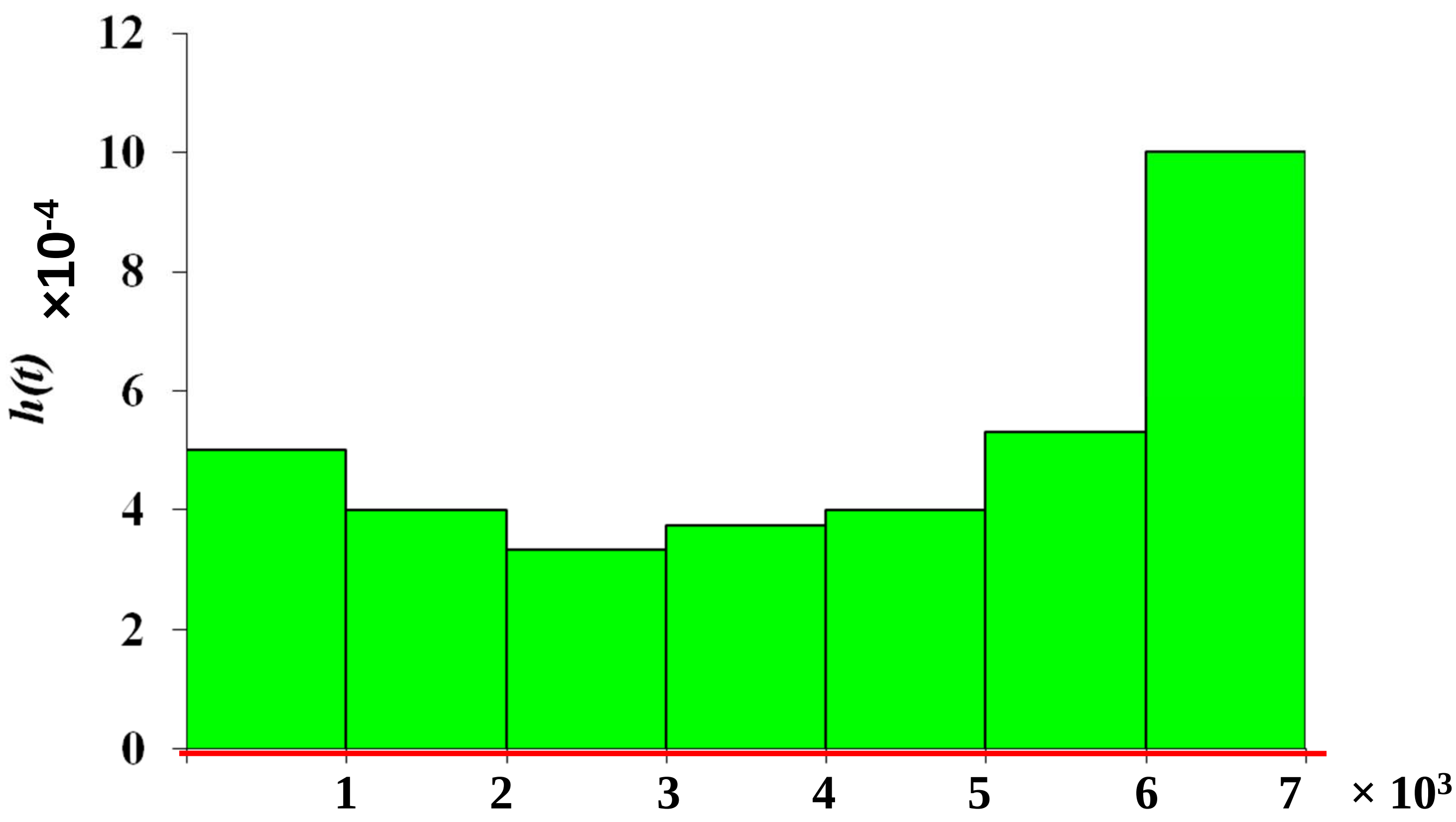
6001-7000	7
Total	200

Time Interval	Failure Density $f(t)$ x 10^{-4}	Hazard rate $h(t)$ x 10^{-4}
0–1000	$\frac{100}{200 \times 10^3} = 5.0$	$\frac{100}{200 \times 10^3} = 5.0$
1001–2000	$\frac{40}{200 \times 10^3} = 2.0$	$\frac{40}{100 \times 10^3} = 4.0$
2001–3000	$\frac{20}{200 \times 10^3} = 1.0$	$\frac{20}{60 \times 10^3} = 3.33$
……	……..	……
6001–7000	$\frac{7}{200 \times 10^3} = 0.35$	$\frac{7}{7 \times 10^3} = 10$

Failure Density vs. Time



Hazard Rate vs. Time



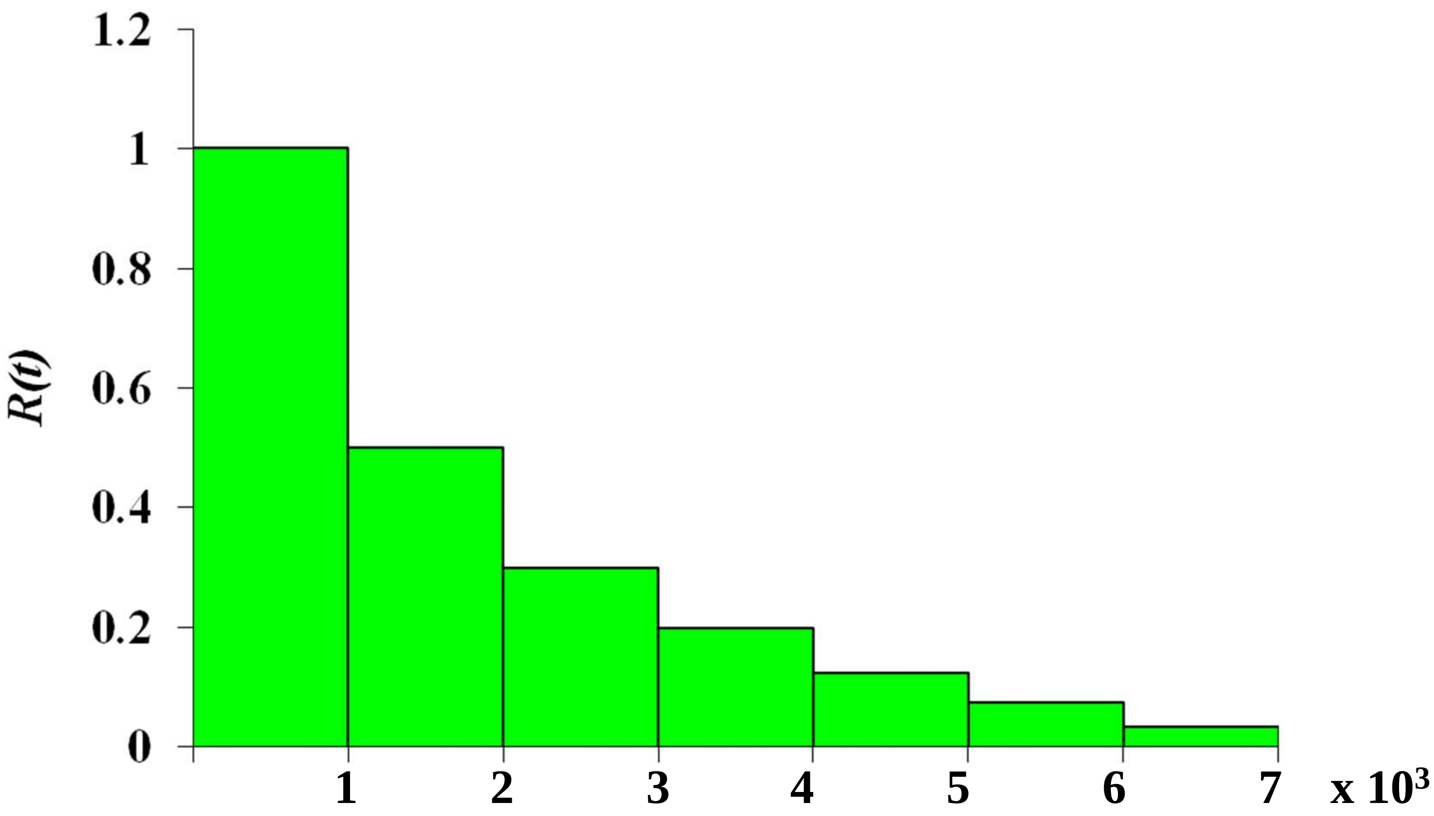
Time in Hours

Reliability Calculations

Time Interval (Hours)	Failures in the interval
0-1000	100
1001-2000	40
2001-3000	20
3001-4000	15
4001-5000	10
5001-6000	8
6001-7000	7
Total	200

Time Interval	Reliability $R(t)$
0–1000	5/5=1.0
1001–2000	2.0/4.0=0.5
2001–3000	1/3.33=0.33
……	……
6001–7000	0.35/10=.035

Reliability vs. Time



Time in hours

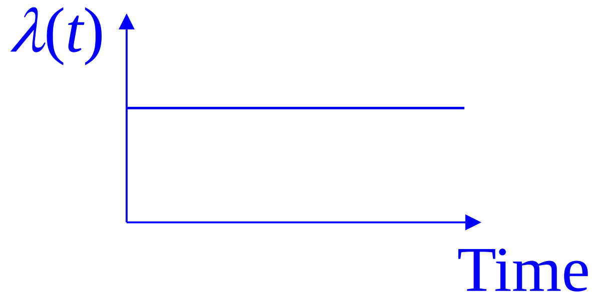
Exponential Distribution

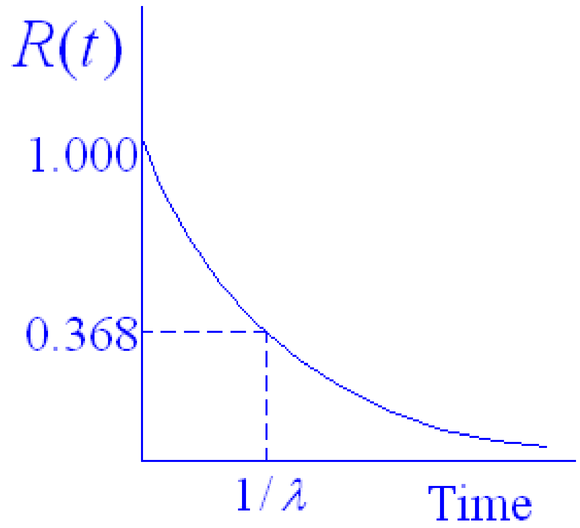
Definition

$$h(t) = \lambda \qquad \lambda > 0,\ \ t \geq 0$$

$$f(t) = \lambda \exp(-\lambda t)$$

$$R(t) = \exp(-\lambda t) = 1 - F(t)$$





Exponential Model Cont'd

Statistical Properties

$$MTTF = \frac{1}{\lambda}$$

$\lambda = 5 \times 10^{-6}$ Failures/hr

MTTF=200,000 hrs or 20 years

$$Variance = \frac{1}{\lambda^2}$$

$\lambda = 5 \times 10^{-6}$ Failures/hr

Standard deviation of MTTF is 200,000 hrs or 20 years

$$\text{Median life} = (\ln 2)\frac{1}{\lambda}$$

Median life =138,626 hrs or 14 years

Exponential Model Cont'd

Statistical Properties

$$MTTF = \frac{1}{\lambda}$$

$\lambda = 5 \times 10^{-6}$ Failures/hr

MTTF=200,000 hrs or 20 years

It is important to note that the MTTF= (1/failure rate) is only applicable for the constant failure rate case (failure time follow exponential distribution.

Empirical Estimate of *F(t)* and *R(t)*

When the exact failure times of units is known, we use an empirical approach to estimate the reliability metrics. The most common approach is the Rank Estimator. Order the failure time observations (failure times) in an ascending order:

$$t_1 \leq t_2 \leq \ldots \leq t_{i-1} \leq t_i \leq t_{i+1} \leq \ldots \leq t_{n-1} \leq t_n$$

Empirical Estimate of *F(t)* and *R(t)*

$F(t_i)$ is obtained by several methods

1. Uniform “naive” estimator $\frac{i}{n}$
2. Mean rank estimator $\frac{i}{n+1}$
3. Median rank estimator (Bernard) $\frac{i-0.3}{n+0.4}$
4. Median rank estimator (Blom) $\frac{i-3/8}{n+1/4}$

Empirical Estimate of *F(t)* and *R(t)*

Assume that we use the mean rank estimator

$$\hat{F}(t_i) = \frac{i}{n+1}$$

$$\hat{R}(t_i) = \frac{n+1-i}{n+1} \qquad t_i \le t \le t_{i+1} \qquad i = 0, 1, 2, \ldots, n$$

Since *f* (*t*) is the derivative of *F*(*t*), then

$$\hat{f}(t_i) = \frac{\hat{F}(t_{i+1}) - \hat{F}(t_i)}{\Delta t_i .(n+1)} \qquad \Delta t_i = t_{i+1} - t_i$$

$$\hat{f}(t_i) = \frac{1}{\Delta t_i .(n+1)}$$

Empirical Estimate of *F(t)* and *R(t)*

$$\hat{\lambda}(t_i) = \frac{1}{\Delta t_i \,.\, (n+1-i)}$$

$$\hat{H}(t_i) = -\ln\left(\hat{R}(t_i)\right)$$

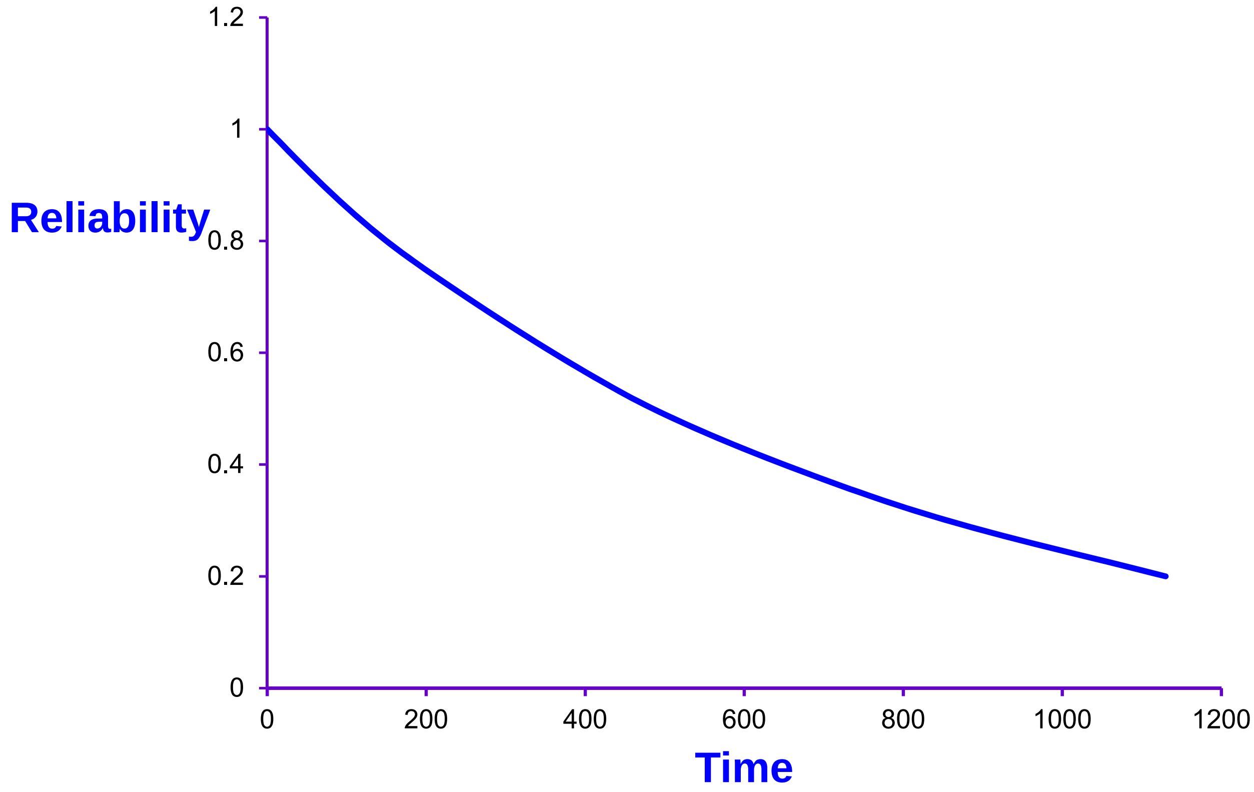
Example:

Recorded failure times for a sample of 9 units are observed at *t*=70, 150, 250, 360, 485, 650, 855, 1130, 1540. Determine F(t), R(t), f(t), $\lambda(t)$,H(t)

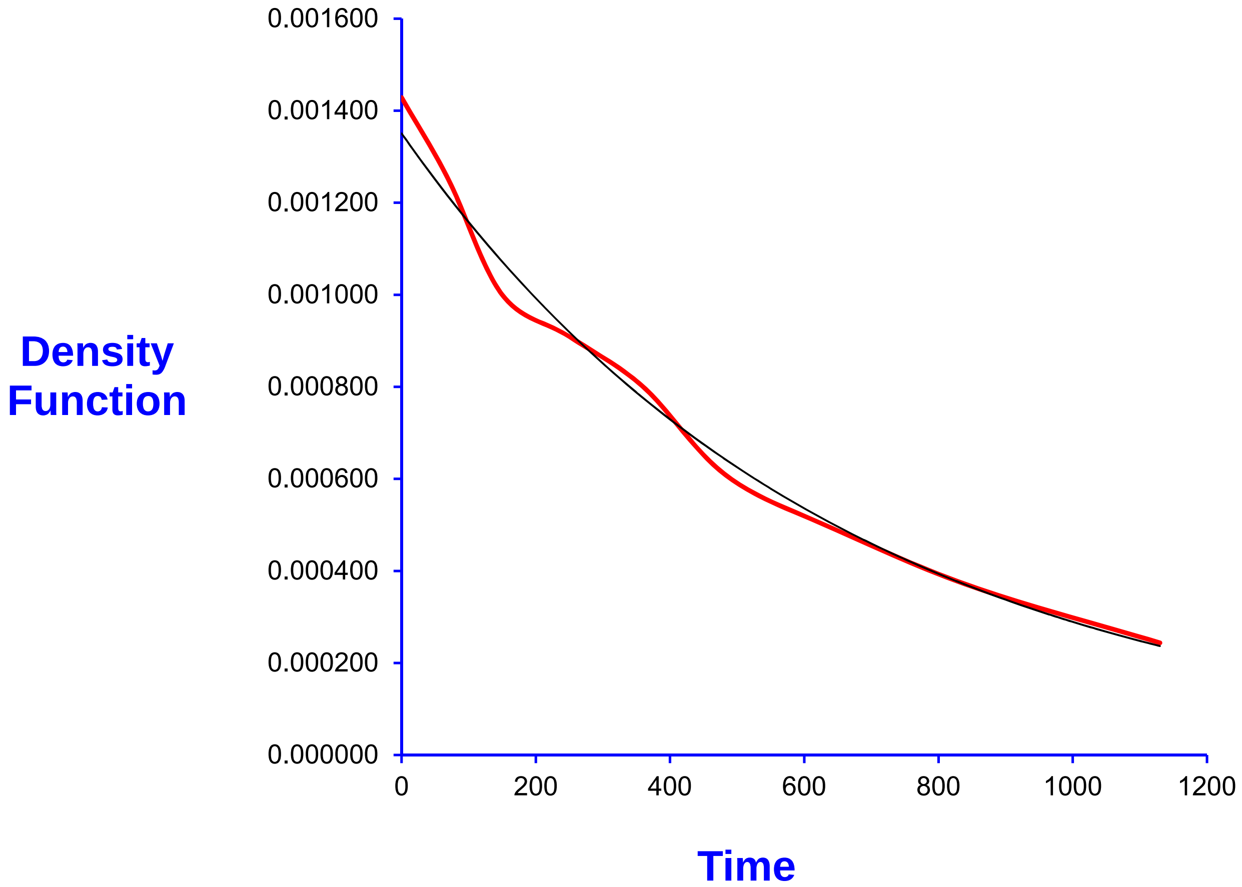
Calculations

i	t (i)	t(i+1)	F=i/10	R=(10-i)/10	f=0.1/Δt	λ =1/(Δt.(10-i))	H(t)
0	0	70	0	1	0.001429	0.001429	0
1	70	150	0.1	0.9	0.001250	0.001389	0.10536052
2	150	250	0.2	0.8	0.001000	0.001250	0.22314355
3	250	360	0.3	0.7	0.000909	0.001299	0.35667494
4	360	485	0.4	0.6	0.000800	0.001333	0.51082562
5	485	650	0.5	0.5	0.000606	0.001212	0.69314718
6	650	855	0.6	0.4	0.000488	0.001220	0.91629073
7	855	1130	0.7	0.3	0.000364	0.001212	1.2039728
8	1130	1540	0.8	0.2	0.000244	0.001220	1.60943791
9	1540	-	0.9	0.1			2.30258509

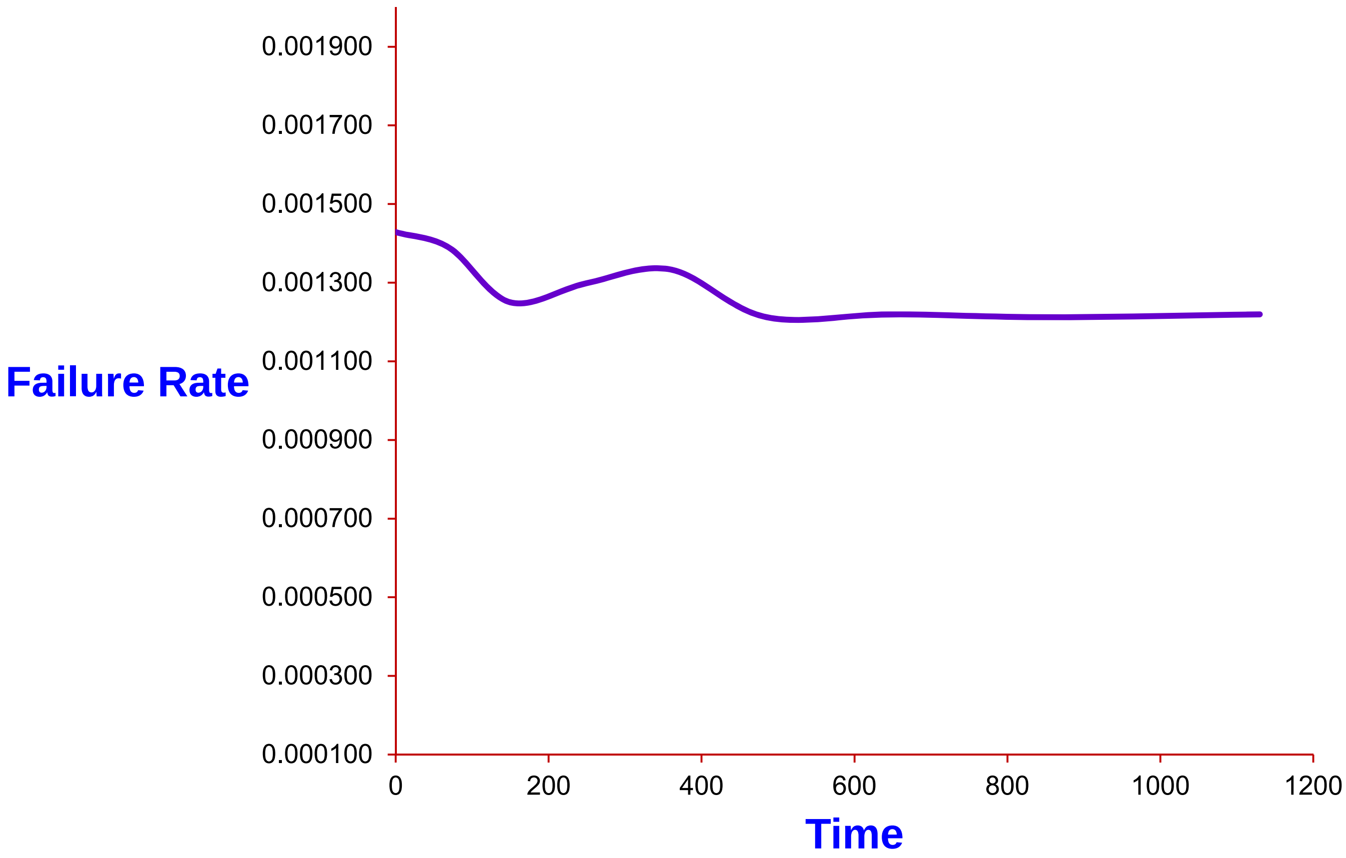
Reliability Function



Probability Density Function



Constant Failure Rate



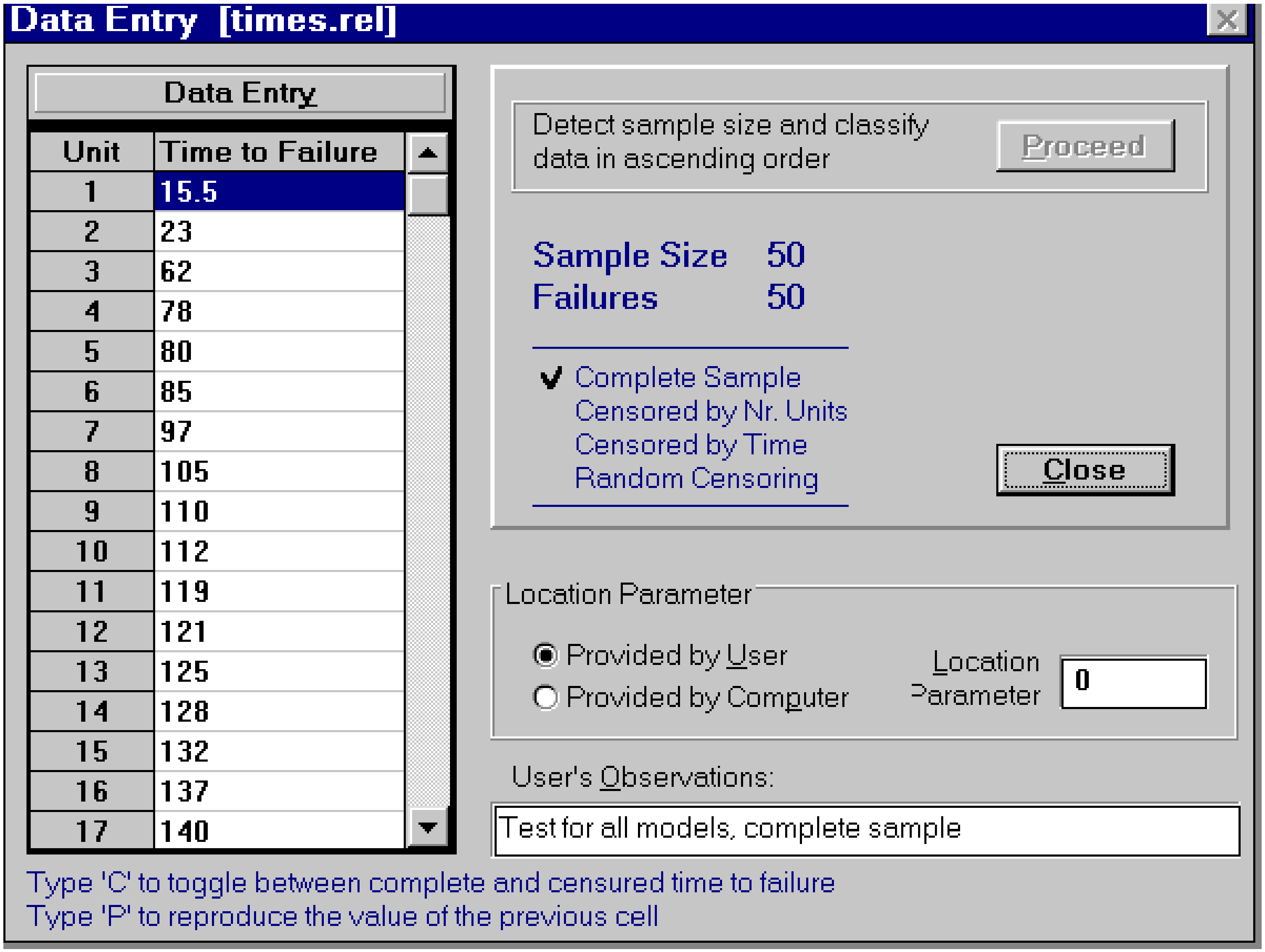
Exponential Distribution: Another Example

Given failure data:

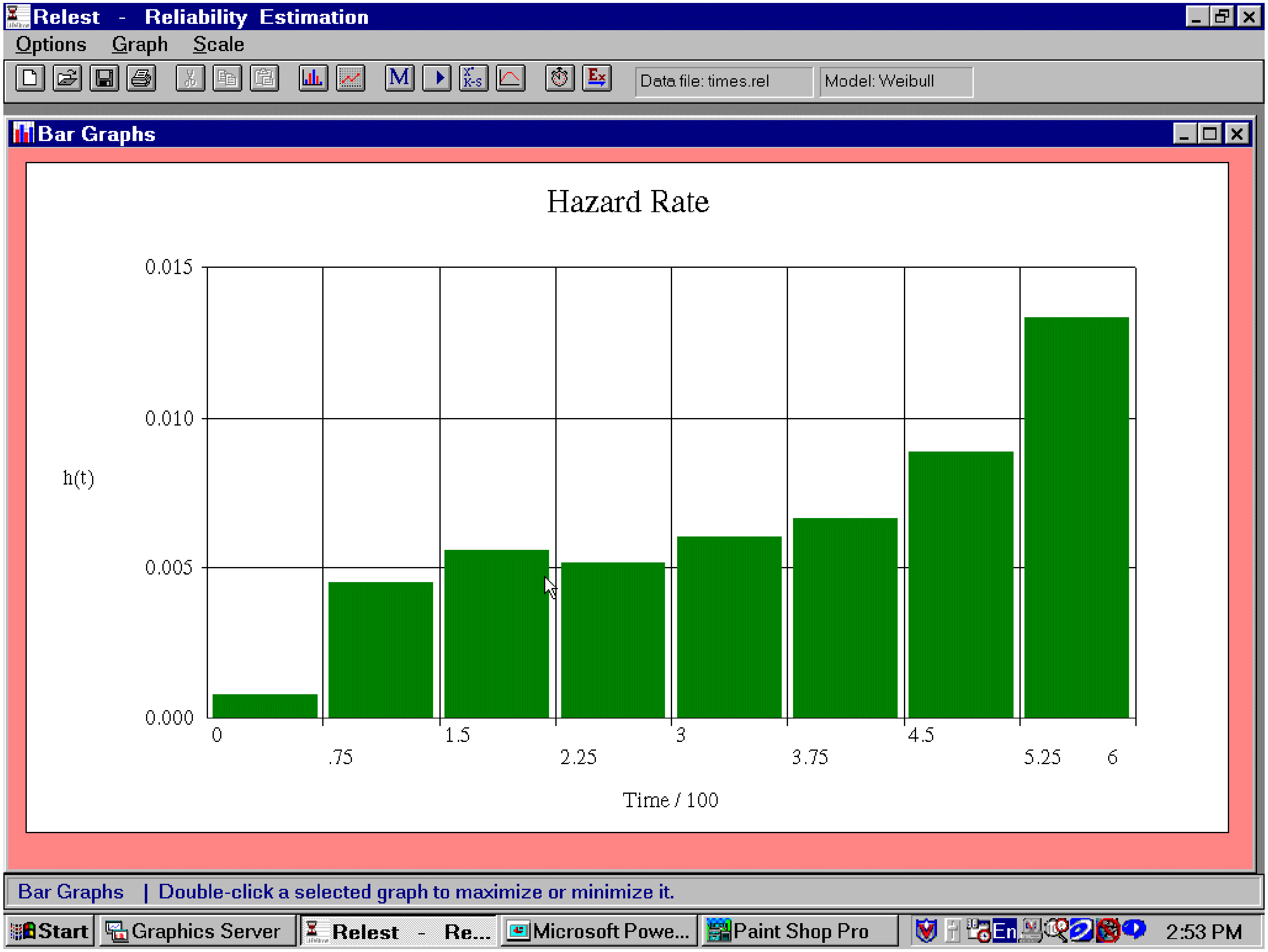
Plot the hazard rate, if constant then use the exponential distribution with $f(t)$, $R(t)$ and $h(t)$ as defined before.

We use a software to demonstrate these steps.

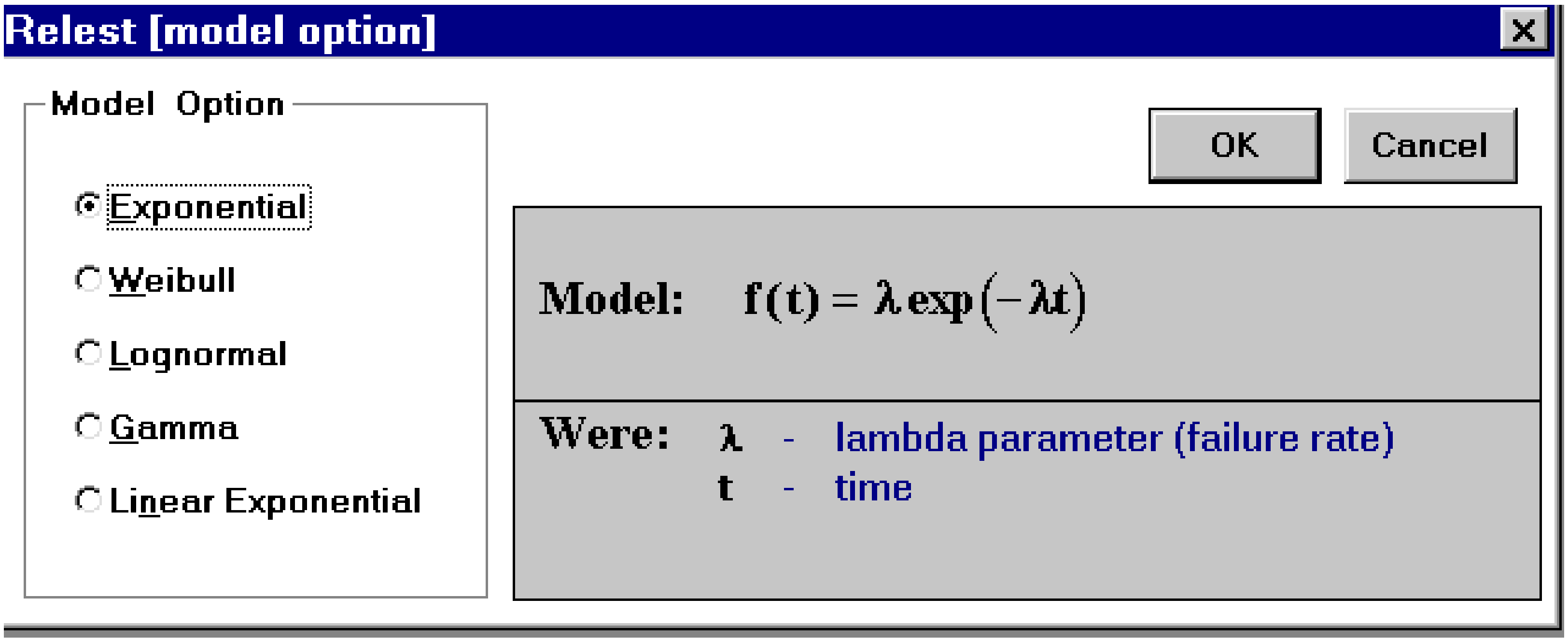
Input Data


Data Entry [times.rel]
Data Entry
Unit
Time to Failure
1 15.5
2 23
3 62
4 78
5 80
6 85
7 97
8 105
9 110
10 112
11 119
12 121
13 125
14 128
15 132
16 137
17 140
Detect sample size and classify data in ascending order
Proceed
Sample Size 50
Failures 50
Complete Sample
Censored by Nr. Units
Censored by Time
Random Censoring
Close
Location Parameter
Provided by User
Provided by Computer
Location Parameter
0
User's Observations:
Test for all models, complete sample
Type 'C' to toggle between complete and censured time to failure
Type 'P' to reproduce the value of the previous cell

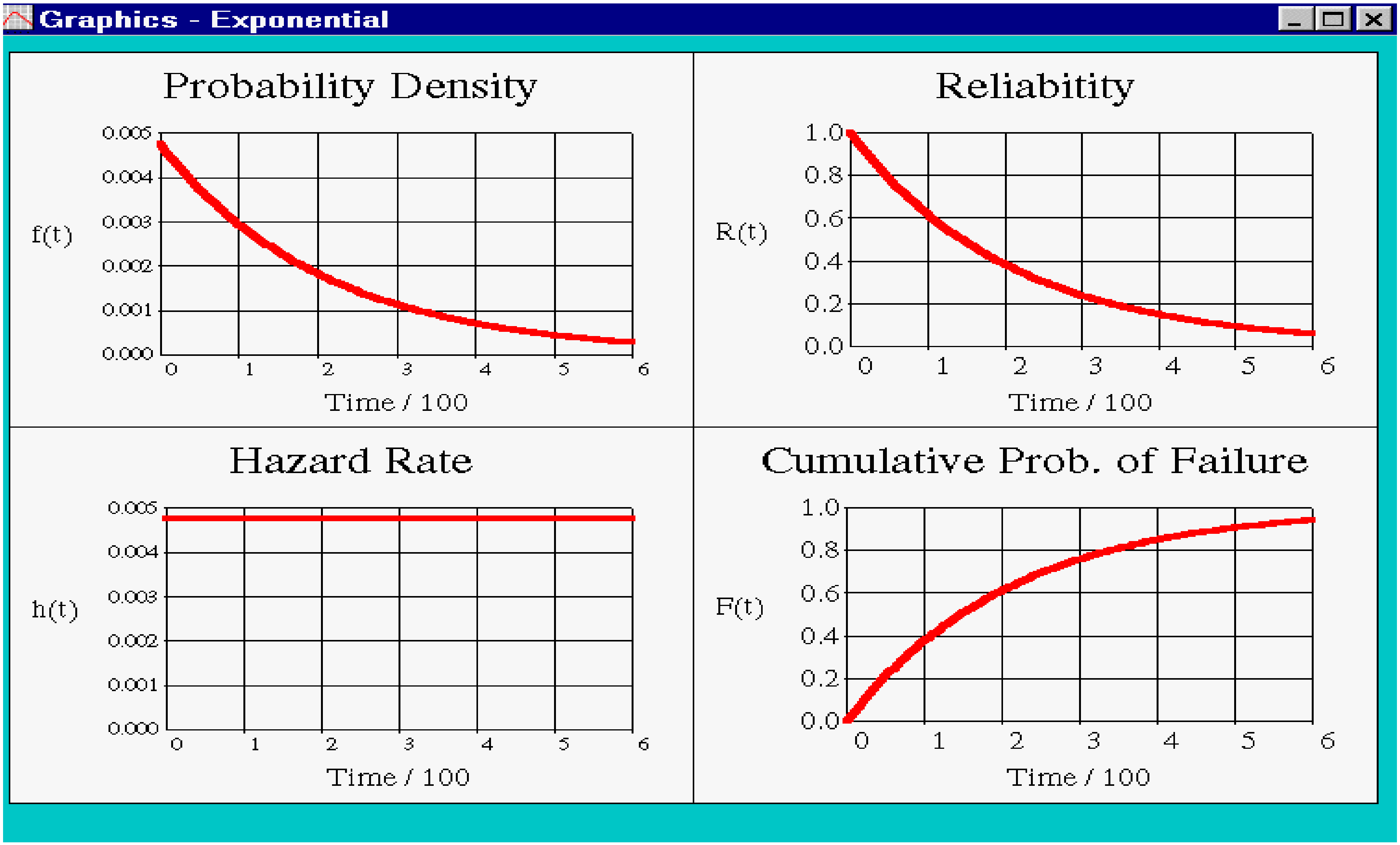
Plot of the Data



Exponential Fit



Exponential Analysis



Summary

In this part, we presented the three most important relationships in reliability engineering.

We estimated obtained estimate functions for failure rate, reliability and failure time. We obtained these function for interval time and exact failure times.