Fundamentals of Risk Management

for Banks and Finance Companies

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Preface

The core purpose of banks and finance companies is to assume manageable levels of maturity risk and credit risk, and to generate a level of return consistent with it. This is hard to do, even in relatively small financial institutions because of the fact that each risk is being assumed by the institution on a near continuous basis and not just in head offices but also in distant branches. Despite these difficulties, management of these risks is of key strategic importance for banks and financial institutions. Decisions such as the choice of data generating processes for each source of risk; implementing appropriately chosen transfer pricing methodologies; and the setting of confidence bands to assess the requirements of capital, need more of the attention of top management than do issues such as locations of new branches or making individual credit decisions.

However, most top management teams are poorly equipped to make these decisions because the language is shrouded in arcana and there if often so much material to cover that it appears impossible to grasp without a life-time of study. While it is indeed true that there are many details that need to be well understood for the practicing risk manager, the concepts underlying these ideas are very few, and, with a little bit of effort, not at all hard even for somebody with only a basic understanding of mathematics or with very little time at their disposal. The material in this book has been especially developed with this in view. It needs ideally be studied or taught in small intimate groups of two or three people and intensively reviewed, on a line by line basis, rather than skimmed through rapidly. There is indeed some usage of mathematics and statistics since the essence of risk management is the quantification and management of uncertainty, but it is all carefully developed in the text itself, without assuming any prior knowledge of the associated tools and techniques. The focus is on ensuring that there is a good grasp of the concept rather than complete mastery of every aspect of the subject matter. Wherever possible the implications of the mathematical equations are made visible by using graphs, charts, and real-life examples.

The material in this book is broken into three parts. The first part delves into some of the fundamentals and explores basic ideas such as probability, random variables, and data generating processes. From there it goes on to build metrics such as Value at Risk and Return on Equity. The second part then takes these basic ideas and seeks to apply them to actual risks faced by banks and finance companies. Because the interest risk and credit risk are the most important it focusses on them. It also provides some ideas on how each of these risks is to be separately managed. The last part is focussed on the tools used by regulators and practitioners to manage these risks at a somewhat of an aggregate level and introduces concepts such as capital, securitisation, and options.

Fundamentals of Risk

Chapter 1: Data Generating Processes[1]

It is clear even to a casual observer that variables of interest to economists, bankers, and policy makers can be thought of as having a specific value at a particular point in time and also that this value can vary at other points in time. The key question of interest is: how does one better understand the behaviour of these variables? This section will set up some of the conceptual apparatus that is necessary to answer this question.

1. Probability[2]

While this concept is frequently used, it means different things to different people. For example the statement: "I'd say there's a thirty percent chance it will rain tomorrow" could mean slightly different things from different perspectives:

a. Neurological: When I think "it will rain tomorrow" the "truth-sensing" part of my brain exhibits 30 percent of its maximum electrical activity.
b. Frequentist: P(A) is the fraction of times A occurred during the previous (large number of) times we ran the experiment.
c. Market preference ("risk neutral probability"): P(A) is price of a contract paying one dollar if A occurs divided by price of contract paying one dollar regardless of whether A occurs or not.
d. Personal belief: P(A) is amount such that I'd be indifferent between contract paying 1 if A occurs and contract paying P(A) no matter what.

While each of these is potentially a plausible perspective it is consistent with the precise notion of probability only if it satisfies the Axioms of Probability:

a. $P(A) \in [0,1]$ for all $A \subset S$ where A the event in question [rain] and S is the set of the possibilities.
b. P(S) = 1.
c. Finite additivity: $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \phi$
d. Countable additivity: $P(\cup_{i=1}^{\infty} E_i) = \Sigma_{i=1}^{\infty} P(E_i)$ if $E_i \cap E_j = \phi$ for each pair i and j.

Applying this test to the four perspectives given above:

a. Neurological: When I think "it will rain tomorrow" the "truth-sensing" part of my brain exhibits 30 percent of its maximum electrical activity. Should satisfy $P(A) \in [0,1]$ for all $A \subset S$ and P(S) = 1 but not necessarily $P(A \cup B) = P(A) + P(B)$ if $A \cap B = \phi$
b. Frequentist: P(A) is the fraction of times A occurred during the previous (large number of) times we ran the experiment. Seems to satisfy all the axioms.
c. Market preference ("risk neutral probability"): P(A) is price of a contract paying one dollar if A occurs divided by price of contract paying one dollar regardless of whether A occurs or not. Seems to satisfy axioms, assuming no arbitrage, no bid-ask spread, complete market, etc.
d. Personal belief: P(A) is amount such that I'd be indifferent between contract paying 1 if A occurs and contract paying P(A) no matter what. Seems to satisfy axioms with some notion of utility units, strong assumption of "rationality", etc.

Random Variable

Each specific value of a variable (whether observed or unobserved[3]) may be thought of as being drawn from a function which describes all the possible set of values that the variable may take. This function is called a Random Variable. More precisely[4]:

a. A Random Variable X is a function from the state space[5] to the real numbers.

b. X can be interpreted as a quantity whose value depends on the outcome of an experiment.

c. Example:

   i. Toss n coins (so state space consists of the set of all 2^n possible coin sequences) and let X be the number of heads. The Random Variable X is a function (the "Number of heads that be obtained from tossing n coins" function) that can take a number of values k ∈ {0, 1, 2, ..., n} where k is the specific value of the Random Variable k. If n=1; i.e., one coin is being tossed then there are two ($=2^{[n=1]}$) possible coin sequences for the coin and only two possibilities exist for k ∈ {0, 1}. The Radom Variable X is therefore a function that has two possible values: 0 and 1. If n=2, i.e., two coins are being tossed then there are four ($=2^{[n=2]}$) possible coin sequences for the coin and three possibilities exist for k ∈ {0, 1, 2}. The Random Variable X is therefore a function that has three possible values: 0, 1, and 2. The specific value that the Random Variable X actually takes can only be determined once the experiment of tossing the coin / coins is actually conducted.

   ii. If, for example, the inter-bank interest rate for overnight money (call rate) observed at 9:00 am on May 22nd, 2014, for some reason, can only change by k ∈ {-1.0%, -0.5%, 0%, +0.5%, +1%} by 9:00 am on May 23rd, 2014 then, the Random Variable X which describes the change in the call rate within the 24 hour period between 9:00 am of May 22nd, 2014 and 9:00 am on May 23rd, 2014 is a function[6] that can take values k ∈ {-1.0%, -0.5%, 0%, +0.5%, +1%}. The specific value that the Random Variable X actually takes can only be observed at 9:00 am on May 23rd.

Probability Mass Function

Bringing the two concepts of probability and random variables together, a probability mass function for discrete random variables is defined as follows[7]:

a. Let X be a discrete random variable which take one of a countable set of values, with probability one.
b. For each "a" in this countable set, p(a) = P{X=a} is called the probability mass function of the Random Variable "X".
c. And, $F(a) = P\{X \le a\} = \Sigma_{X \le a} p(x)$ is called the Cumulative Mass Function (CMF) of the Random Variable "X".

For example, for a Random Variable which describes the probability of how often a particular value of the Random Variable X, say, the number of Heads, will obtain in a toss of n coins, the probability mass function and the cumulative distribution function can be specified as:

$P\{X = k\} = {}_nC_k/2^n$ where ${}_nC_k = \frac{n!}{k!(n-k)!}$
$P\{X \le k\} = \Sigma_0^k\, P\{X = k\} = \Sigma_0^k\, {}_nC_k/2^n$

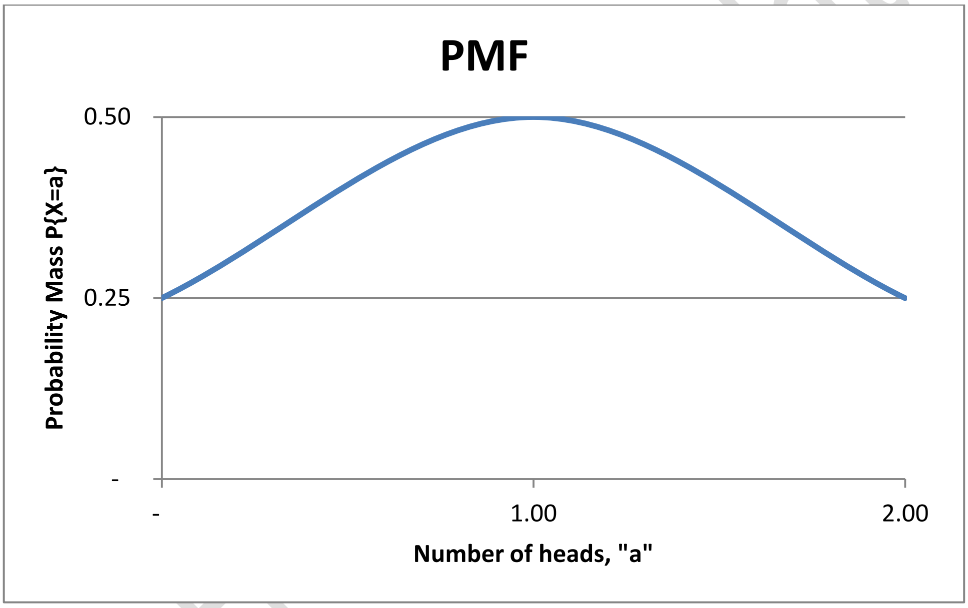
When n=2, i.e., there are two coins or a single coin is tossed twice, the probability distribution or the probability mass function can be easily derived by noting that the only four possibilities are:

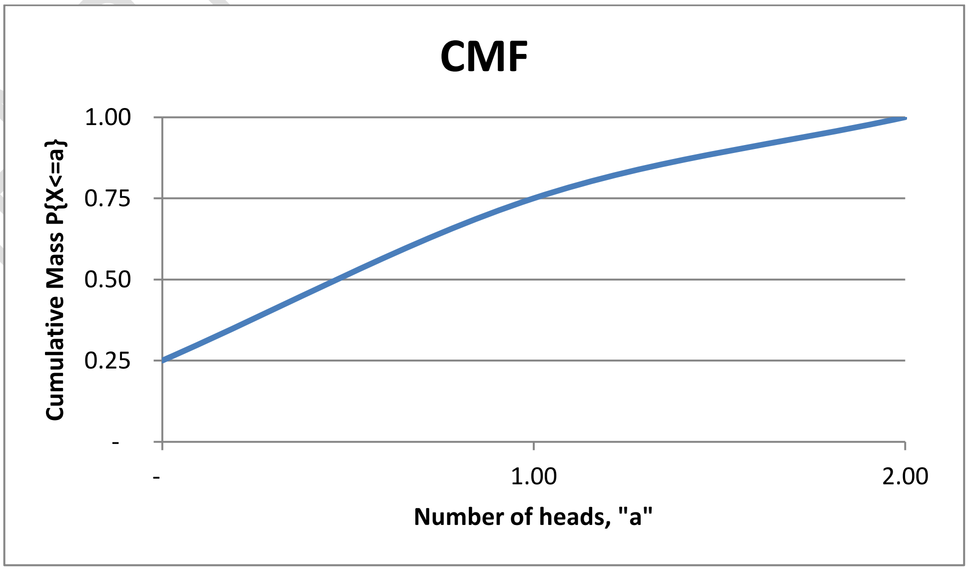
   i. Heads-Heads
   ii. Heads-Tails

iii. Tails-Heads
iv. Tails-Tails

The Probability Mass Function for the Random Variable X defined as the "Total Number of Heads Obtained after Flipping Two Coins" is therefore[8] is described by the table and the graph below. The Cumulative Mass Function is also described and graphed below.

Number of Heads "a"	**How Many Times X=a**	**Probability Mass P{X=a}**	**Cumulative Mass P{X<=a}**
0	1	$^1/_4 = 0.25$	0.25
1	2	$^2/_4 = 0.50$	0.75
2	1	$^1/_4 = 0.25$	1.00
Total	4	1.00	





2. Data Generating Process[9]

Now that the concepts of probability, random variables, probability mass functions (they are referred to as **probability density functions** for continuous variables), and cumulative distribution functions (of which cumulative mass function is an example specific to discrete variables) have been defined, the key challenge is to apply them to observed economic phenomena, and to attempt to describe them using these concepts.

For this to be done, the concept of a Data Generating Process (DGP) has to be understood first. The DGP is the hypothesised or "true but unobserved" function that guides the behaviour of the random variable in question. In the coin-toss example studied earlier the DGP can be described as a binomial process in which there are only two outcomes and each one is equally likely. However, for more complex random variables, which describe observed phenomena the answers may not be as straight forward. For example let Y be a random variable such that:

$$y_t = \beta_0 + \beta_1 y_{t-1} + u_t$$

where u_t is a specific realisation at time "t" of a random variable U which follows a Gaussian (or a Normal) Distribution[10], i.e.,

$$u_t \sim N(0, \sigma_u^2)$$

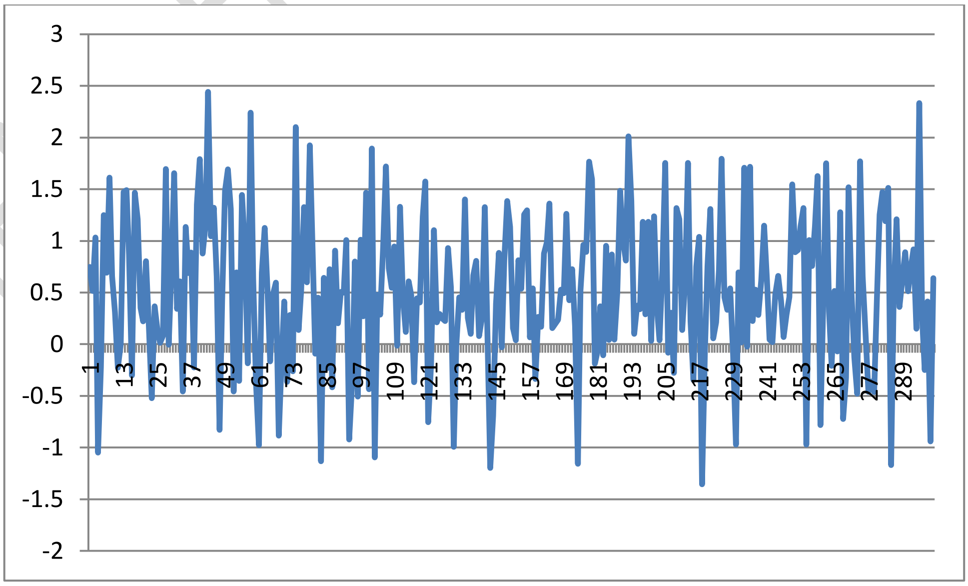
With the following values:

$$\beta_0 = 0.50, \beta_1 = 0.70, y_0 = 0, \sigma_u^2 = 0.50$$

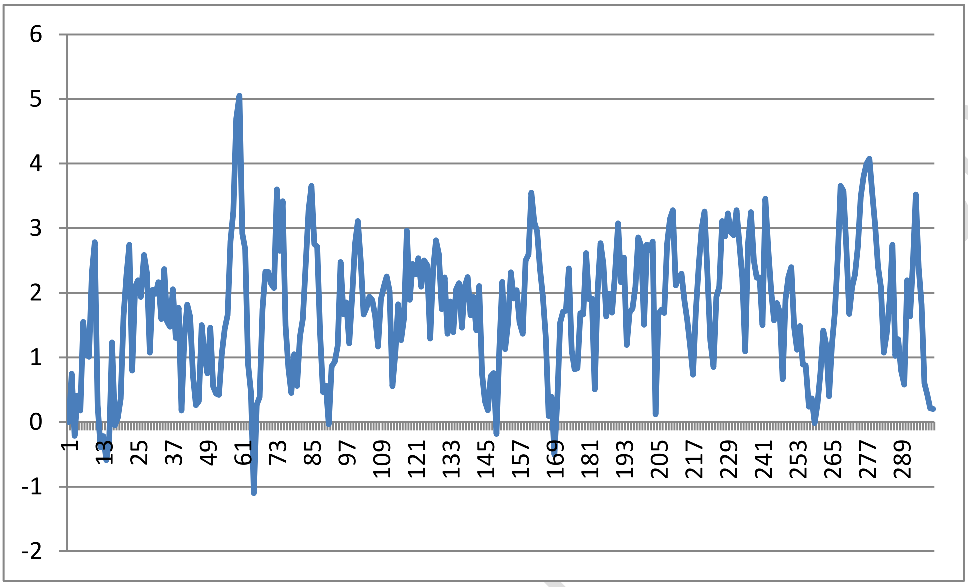
We get the value of y_1:

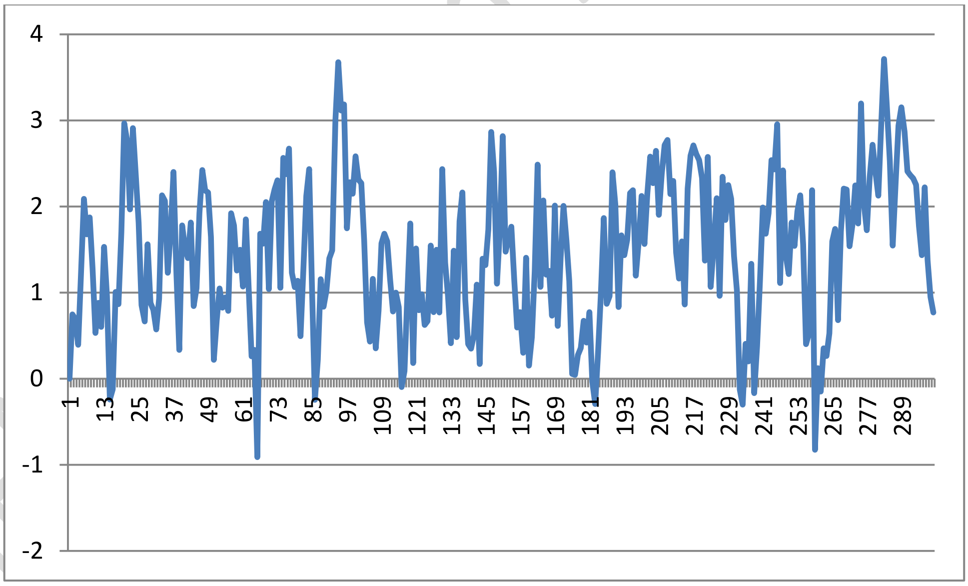
$$y_1 = 0.50 + 0.70 * 0 + \; u_1 \sim N(0, 0.50)$$

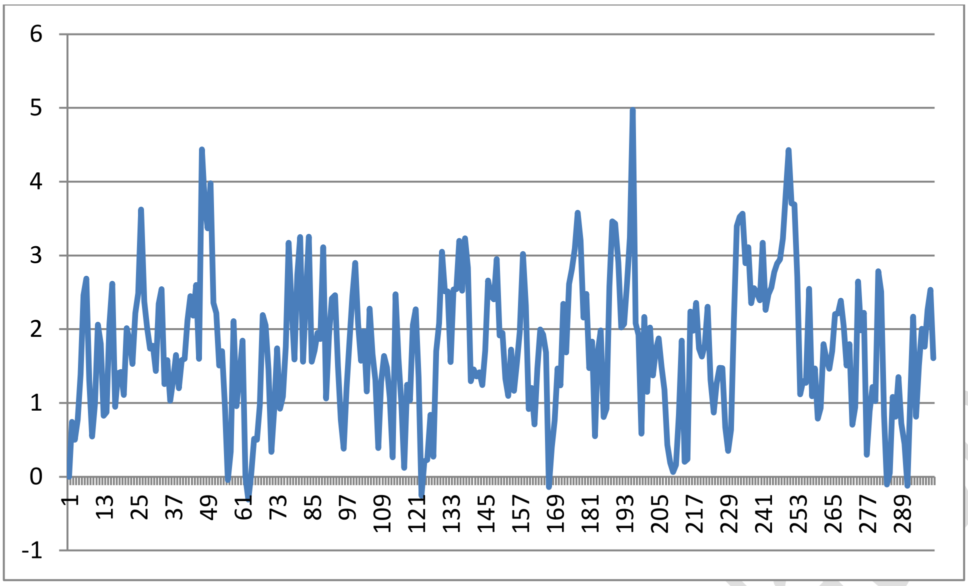
The variable y_1 can take on an infinite number of values. The 300 possible values of y_1 are given in the graph (below) and can range from a high of about 2.5 to a low of -1.5.



For each of the values of y_1 there will of course be an infinite number of values of y_2; for each value of y_2 there will in turn be infinite values of y_3; and so on. If it is assumed that the random variable Y describes a real-world economic variable such as the exchange rate or the interest rate observed at specific time on a particular day, then, if the behaviour of the random variable Y is indeed described by the above set of equations, then only one specific realisation of each of the infinite possible values of $y_1, y_2, y_3, \ldots$ will be observed. Three such series of specific realisations or pathways are given below which are associated with a value for $y_0 = 0$; $y_1 = 0.746659$.







Starting with a specific DGP, it was relatively easy, if laborious, to map out all the possible values of $y_0, y_1, \ldots, y_{300}$. However, the problem in the real world is that, given just one set of specific values of $y_0, y_1, \ldots, y_{300}$, it is necessary to make an "educated guess" what a possible DGP might be that is responsible for producing such a set of values. No meaningful risk management is possible without making this "educated guess" and the next few sections will help simplify the problem so that it is possible to start to make good "educated guesses".

3. A Short Primer on Differentiation and Integration[11]

Differentiation

It is clear that if $f(x)$ is a function then, the slope of this function is:

$$Slope = \frac{\Delta f(x)}{\Delta x}$$

However, even for a relatively simple function such as $f(x) = x^2$, the slope varies continuously at every point and using the above formula it becomes impossible to compute the precise slope a particular point x. However, as the value of Δx goes to zero it is possible to see how the slope in the above formula gets closer and closer to the exact value of the at point x.

$$f'(x) \equiv \frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f(x)}{\Delta x}$$

This exact value is the derivative of a function and is relatively easy to compute. In the earlier example where:

$$f(x) = x^2$$

$$f'(x) = 2x$$

While determining how the value of $f'(x) = 2x$ was derived at is beyond the scope of this primer, it is easy to see why this is indeed likely to be the correct answer.

If,

$$a = 2; b = 3$$

$$Slope = \frac{f(b) - f(a)}{(b-a)} = \frac{9-4}{1} = 5$$

while,

$$f'(x = a) = \ 2x = 4$$

with an error is as high as 25%.

However if,

$$a = 2; b = 2.01$$

$$Slope = \frac{f(b) - f(a)}{(b-a)} = \frac{4.0401 - 4}{0.01} = 4.01$$

while,

$$f'(x = a) = \ 2x = 4$$

with an error of only 0.25%.

<u>Integration</u>

If now the slope of any function is:

$$Slope = \frac{\Delta f(x)}{\Delta x}$$

then,

$$\Delta f(x) = \ Slope * \ \Delta x$$

and, if the desire was to compute the change in the numerical value of this function from point "a" to point "b", i.e.,

$$f(b) - f(a) = \sum_a^b \Delta f(x)$$

where each $\Delta f(x)$ is computed over a small range Δx.

However it is already known that each:

$$\Delta f(x) = \ Slope * \ \Delta x$$

Or, when the value of Δx tends to zero:

$$\mathrm{d}f(x) = \ f'(x)dx$$

Therefore,

$$f(b) - f(a) = \sum_a^b \Delta f(x) = \sum_a^b \mathrm{Slope} * \Delta \mathrm{x} = \lim_{\Delta x \to 0} \int_a^b f'(x)dx$$

or, more simply,

$$f(b) - f(a) = \int_a^b f'(x)dx$$

Now if there is an arbitrary function such that:

$$g(b) - g(a) = \int_a^b y(x)dx$$

then, g(x) is referred to as the integral of y(x) and equivalently y(x) is referred to as the differential of g(x).

And, from this relationship an important application of the integration process becomes apparent – that the integral:

$$\int_a^b y(x)dx$$

is nothing but the area under the curve y(x) between the points "a" and "b".

And, in the space of probability functions, it then becomes clear also that:

$$\int_x p(x)dx = 1$$

Is nothing but the area under the probability density function measured over the entire space over which x operates.

4. Moment Generating Function

Moments of a Distribution

In mathematics[12], a moment[13] is a quantitative measure of the shape of a set of points. The second moment, or more specifically the second central moment, for example, is widely used and measures the "width" (in a particular sense) of a set of points in one dimension, or in higher dimensions measures the shape of a cloud of points as it could be fit by an ellipsoid. Other moments describe other aspects of a distribution such as how the distribution is skewed from its mean (or skewness) or how much it bulges (or Kurtosis[14]).

For a given random variable X each of these moments can simply be computed as follows: n^{th} Moment = $E[X^n]$ or the n^{th} Centred Moment = $E[X - E(X)]^n$. The common names for each of the first four moments are[15]:

Parameter	Raw/Central	Moment	Definition	Measures
Mean (μ)	Raw	1	$\mu = \mu_1 = E(X)$	Central Location
Variance (σ^2)	Central	2	$\sigma^2 = E(X-\mu)^2 = E(X^2) - E(X)^2$ $= \mu_2 - (\mu_1)^2$	Dispersion
Skew (γ_2)	Central Standardized	3	$\gamma_2 = \dfrac{E(x-\mu)^3}{\sigma^3}$ $= \dfrac{\mu_3 - 3\mu_1\mu_2 + 2(\mu_1)^3}{\sigma^3}$	Asymmetry
Kurtosis	Central Standardized	4	$Kurtosis = \dfrac{E(x-\mu)^4}{\sigma^4}$ $= \dfrac{\mu_4 - 4\mu_1\mu_3 + 6(\mu_1)^2\mu_2 - 3(\mu_1)^4}{\sigma^4}$	Peakedness
Excess Kurtosis (γ_3)	Central Standardized	4	$\gamma_3 = Kurtosis - 3$	Peakedness relative to Gaussian

Given any data series these moments are easy to calculate and act as summary descriptors of the data for the sample. However, they do not directly indicate what the underlying Data Generating Process is likely to be. And, given underlying probability density/mass functions, these values are difficult to compute unless an easy process is available to compute the raw moments: $\mu_1 = E(X)$, $\mu_2 = E(X^2)$, … , $\mu_n = E(X^n)$, from which the Centred and the Standardised Moments may be computed relatively easily, as described above.

Deriving the Moment Generating Function

For a random variable X with a probability density function, P_X, if the expected value of e^{tX} exists[16] it is called the Moment Generating Function (MGF) of X:

$$M_X(t) = E(e^{tX})$$

For Discrete distributions this would be written as:

$$M_X(t) = \sum_x e^{tx} p_X(x)$$

While for continuous distributions it would be:

$$M_X(t) = \int_x e^{tx} p_X(x) dx$$

Whether the MGF is defined depends on the distribution and the choice of t. For example, $M_X(t)$ is defined for all "t" if X is Normal, defined for no "t" if X is Cauchy, and for t < λ if X ~ Exp (λ).

MGFs help in many ways:

1. Allow the calculation of raw moments easily by simple differentiation (instead of integration for each moment).
2. To determine distributions of functions of random variables.
3. To approximate distributions and sums of distributions – this feature is critical for the purposes of risk management.

It is already known that any function can be approximated around a point “a” by its Taylor Series expansion[17] as follows:

$$f(t) = f(a) + f'(a)(t-a) + \frac{f''(a)}{2!}(t-a)^2 + \cdots + \frac{f^n(a)}{n!}(t-a)^n + \cdots$$

While this series may be derived relatively easily[18] it is more important to get a visual sense of how it attempts to approximate the actual function through a series of ever more accurate approximations[19].

Consider:

$$f(t) = t^3$$

$$\Rightarrow f'(t) = \frac{df(t)}{dt} = 3t^2$$

$$\Rightarrow f''(t) = \frac{d^2f(t)}{dt^2} = 3*2*t$$

Therefore as a first approximation, if a=6 (arbitrarily chosen):

$$f(t) \approx f(a) = 6^3 = 216$$

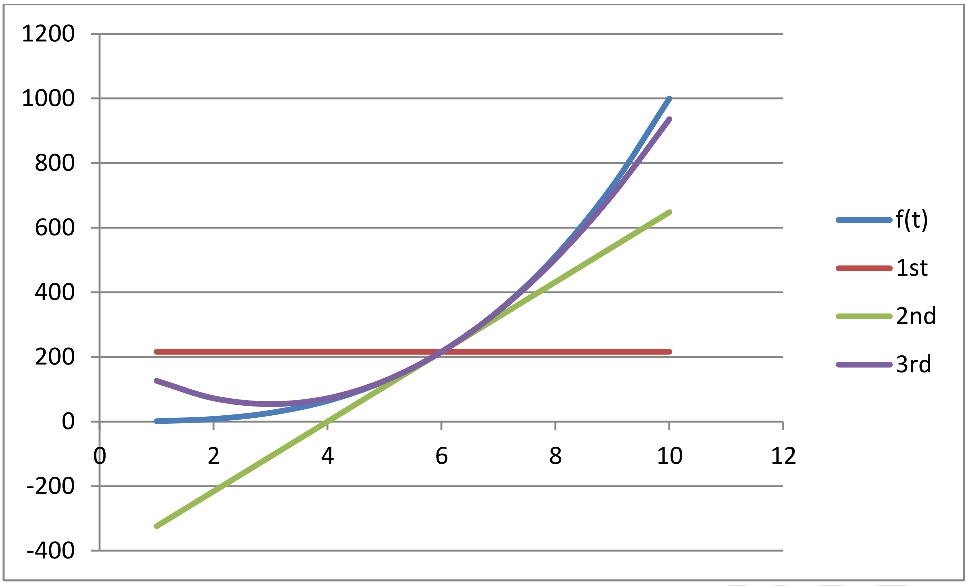
As a second approximation:

$$f(t) \approx f(a) + f'(a)(t-a) = a^3 + 3a^2(t-a) = 6^3 + 3*6^2(t-6)$$

As a third approximation:

$$\begin{aligned} f(t) &\approx f(a) + f'(a)(t-a) + \frac{f''(a)}{2!}(t-a)^2 \\ &= a^3 + 3a^2(t-a) + \frac{3*2*t}{2!}(t-a)^2 \\ &= 6^3 + 3*6^2(t-6) + \frac{3*2*t}{2!}(t-6)^2 \end{aligned}$$

The fourth approximation returns the full function once again. The quality of each of these successive approximations is visible from the graph below:



It is also the case that if $f(t) = e^{tX}$

$$f'(t) = \frac{df(t)}{dt} = \frac{de^{tX}}{dt} = Xe^{tX}$$

$$f''(t) = \frac{d^2e^{tX}}{dt^2} = X^2e^{tX}$$

and,

$$f^n(t) = \frac{d^ne^{tX}}{dt^n} = X^ne^{tX}$$

Therefore[20], using this expansion around $t = 0$,

$$M_X(t) = E(e^{tX}) = E[e^0 + Xe^0\,(t-0) + \frac{X^2e^0}{2!}(t-0)^2 + \cdots + \frac{X^ne^0}{n!}(t-0)^n + \cdots]$$

$$\Rightarrow M_X(t) = E(e^{tX}) = E[1 + Xt + \frac{X^2}{2!}t^2 + \cdots + \frac{X^n}{n!}t^n + \cdots]$$

$$\Rightarrow M_X(t) = E(e^{tX}) = E[1] + tE[X] + \frac{t^2}{2!}E[X^2] + \cdots + \frac{t^n}{n!}E[X^n] + \cdots$$

Now,

$$\frac{dM_X(t)}{dt} = 0 + E[X] + \frac{2t}{2!}E[X^2] + \frac{3t^2}{3!}E[X^3] \ldots + \frac{nt^{(n-1)}}{n!}E[X^n] + \cdots$$

$$\frac{d^2M_X(t)}{dt^2} = 0 + \ E[X^2] + \frac{2*3*t}{3!}E[X^3] + \cdots + \frac{n(n-1)t^{(n-2)}}{n!}E[X^n] + \cdots$$

Evaluated at $t = 0$, since the lower terms are constants and therefore have a differential of zero and all the higher terms contain “t”, therefore also have the value zero at t=0,

$$\frac{dM_X(t)}{dt} = M_X'(t = 0) = E[X] = \mu_1$$

$$\frac{d^2 M_X(t)}{dt^2} = M_X''(0) = E[X^2] = \mu_2$$

$$\frac{d^n M_X(t)}{dt^n} = M_X^{(n)}(0) = E[X^n] = \mu_n$$

From these values it is now possible to compute all of critical parameters of a distribution mentioned earlier. The table on the following page lists out the key moments of various distributions.

Examples of MGFs[21]:

Distribution	Probability Density Function	MGF	Mean	Variance	Skew	Excess Kurtosis
Exponential **E($\boldsymbol{\lambda}$)**	$\lambda e^{(-\lambda x)}$	$\lambda/(\lambda - t)$	$1/\lambda$	$2/\lambda^2 - 1/\lambda^2 = 1/\lambda^2$	2	6
Standard Normal **N(0,1)**	$\frac{1}{\sqrt{2\pi}} e^{-x^2/2}$	$e^{t^2/2}$	0	1	0	0
Normal **N($\boldsymbol{\mu}$, $\boldsymbol{\sigma}$)**	$\frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$	$e^{(\frac{\sigma^2 t^2}{2} + \mu t)}$	μ	σ^2	0	0
Poisson **P($\boldsymbol{\lambda}$)**	$\frac{e^{-\lambda}\lambda^x}{x!}$	$e^{\lambda(e^t - 1)}$	λ	λ	$\lambda^{-\frac{1}{2}}$	λ^{-1}
Uniform **U($\boldsymbol{\alpha}, \boldsymbol{\beta}$)**	$\frac{1}{\beta - \alpha}\ for\ \alpha < x < \beta$	$\frac{e^{\beta t} - e^{\alpha t}}{\beta - \alpha}$	$\frac{\beta + \alpha}{2}$	$\frac{(\beta - \alpha)^2}{12}$	0	$-\frac{6}{5}$
Gamma (Shape, Scale) $\boldsymbol{\Gamma(\kappa, \theta)}$	$\frac{1}{\Gamma(\kappa)\theta^\kappa} x^{\kappa - 1} e^{-\frac{x}{\theta}}$	$(1 - \theta t)^{-\kappa}$ $\forall\, t < \frac{1}{\theta}$	$\kappa\theta$	$\kappa\theta^2$	$\frac{2}{\sqrt{\kappa}}$	$\frac{6}{\kappa}$
$\boldsymbol{\Gamma(1, \theta)} \approx$ **Exponential**	$\frac{1}{\theta} e^{-\frac{x}{\theta}} \approx E(\frac{1}{\theta})$	$\frac{1}{(1 - \frac{1}{\theta} t)}$	$\frac{1}{\theta}$	$\frac{1}{\theta^2}$	2	6
$\boldsymbol{\Gamma(\frac{\upsilon}{2}, 2)} \approx$ **Chi-Squared ($\boldsymbol{\upsilon}$)**	$\frac{1}{\Gamma(\frac{\upsilon}{2}) 2^\kappa} x^{(\frac{\upsilon}{2} - 1)} e^{-\frac{x}{2}}$ $\approx \chi^2(\upsilon)$	$(1 - 2t)^{-\frac{\upsilon}{2}}$	υ	2υ	$2\sqrt{\frac{2}{\upsilon}}$	$\frac{12}{\upsilon}$
$\boldsymbol{\Gamma(\frac{1}{2}, 2)} \approx$ **Chi-Squared (1)**	$\frac{1}{\Gamma(\frac{1}{2}) 2^\kappa} x^{(\frac{1}{2} - 1)} e^{-\frac{x}{2}}$ $\approx \chi^2(1)$	$(1 - 2t)^{-\frac{1}{2}}$	1	2	$2\sqrt{2}$	12

Moment Generating Functions and the Equality of Distributions

If X and Y are two random variables with respectively distribution functions P_X and P_Y and MGFs M_X and M_Y, then X and Y have the same distribution function, i.e., $P_X(s) = P_Y(s)$ ∀ s, if and only if, $M_X(t) = M_Y(t) \forall\, t$.

This proposition is extremely important and relevant from a practical viewpoint: in many cases where we need to prove that two distributions are equal, it is much easier to prove equality of the moment generating functions than to prove equality of the distribution functions. Also note that equality of the distribution functions can be replaced in the proposition above by equality of the probability mass functions (if X and Y are discrete random variables) or by equality of the probability density functions (if X and Y are continuous random variables) [22].

Moment Generating Functions and the Sums of Random Variables[23]

If $Z = X + Y$, where X and Y are independent of each other then:

$$M_Z(t) = E[e^{tZ}] = E\left[e^{t(X+Y)}\right] = E[e^{tZ}e^{tZ}] = E[e^{tX}]E[e^{tY}] = M_X(t)M_Y(t)$$

This shows that adding independent random variables corresponds to multiplying Moment Generating Functions.

By extension, if $Z = X_1 + X_2 + X_3 + \cdots + X_n$ where each X_i is independent of all the others but identically distributed (i.i.d.) then:

$$M_Z(t) = E[e^{tZ}] = M_X^n(t)$$

This a big reason for studying moment generating functions. It helps reveal what happens when a lot of independent copies of the same random variable are summed up.

Similarly if, $Z = aX$, then:

$$M_Z(t) = E[e^{tZ}] = M_X(at);$$

and, if $Z = X + b$, then:

$$M_Z(t) = E[e^{tZ}] = M_X(t)M_b(t) = e^{bt}M_X(t)$$

5. Central Limit Theorem[24]

The central limit theorem states that if:

$$B_n = \frac{X_1 + X_2 + \cdots + X_n - n\mu}{\sigma\sqrt{n}}$$

where, $X_1 + X_2 + \cdots + X_n$ are i.i.d. each with mean μ and variance σ^2 then, B_n converges in law to a standard normal random variable as $n \to \infty$, i.e.,

$B_n \to N(0,1)$ as $n \to \infty$

The Proof of the CLT is relatively straight forward[25]:

If:

$$Y_i = (X_i - \mu)/\sigma$$

then each Y_i has mean 0 and variance 1.

i.e.,

$$M_{Y_i}(t) = e^{\frac{t^2}{2}}$$

and,

$$M_{\sum_{i=1}^{n} Y_i}(t) = \prod_{i=1}^{n} e^{\frac{t^2}{2}} = e^{n\left(\frac{t^2}{2}\right)}$$

Since,

$$B_n = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} Y_i$$

$$M_{B_n}(t) = e^{n\left(\frac{(\frac{t}{\sqrt{n}})^2}{2}\right)}$$

Using l'Hopital's rule[26], it is possible to prove that:

$$e^{n\left(\frac{(\frac{t}{\sqrt{n}})^2}{2}\right)} \rightarrow e^{\frac{t^2}{2}} as\, n \rightarrow \infty$$

i.e.,

$$B_n \rightarrow N(0,1) \text{ as } n \rightarrow \infty$$

Or, equivalently if:

$$Z_n = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

then,

$$Z_n \rightarrow N\left(\mu, \frac{\sigma}{\sqrt{n}}\right) \text{ as } n \rightarrow \infty$$

since,

$$Z_n = \mu + B_n \frac{\sigma}{\sqrt{n}}$$

The central limit theorem is fairly robust. Variants of the theorem still apply if the X_i are not identically distributed, or not completely independent. Roughly speaking, if there are a lot of little random terms that are "mostly independent" — and no single term contributes more than a "small fraction" of the total sum — then the total sum should be "approximately" normal.

6. Putting It All Back Together

Now that conceptual apparatus of Probability, Random Variables, Probability Density Functions, Data Generating Functions, Moments, Moment Generating Functions, and the Central Limit Theorem is available it is possible to now do the following:

1. Build an initial hypothesis about the behaviour (i.e., the DGP) of the variable of interest such as the movement of daily exchange rates and construct a model of its behaviour. For example, if it is possible to argue that a particular random variable (such as the daily exchange rate) is actually the sum of other random variables (such as hourly exchange rate) then it is possible to invoke the central limit theorem and argue that the daily exchange rate should be normally distributed – unless the independence assumption is strongly violated.

2. Collect data on the actual behaviour of the variable of interest.

3. Using the conceptual apparatus that has been built up in these pages, attempt to test the hypothesis against the data.

4. If the "goodness-of-fit" is within desired limits proceed ahead with the DGP but otherwise revise the hypothesis.

This process of determining the DGP of a variable of interest is the starting point of any exercise in risk management and is an extremely important step.

Chapter 2 : Risk, Return, and Value at Risk

1. Risk and Return

The world of finance is concerned, among other things, with understanding how to balance the likelihood of gain with the likelihood of loss. For two concrete reasons: (a) most financial variables can be hypothesised to be distributed Normally; and (b) most individuals appear to carry out the balancing of likelihood of gain with the likelihood of loss by seeking to maximise the return that they expect to make over a particular time horizon and minimise the uncertainty associated with that return. For both these reasons the words risk and expected return have come to acquire key importance and, generally speaking, if the return over a defined horizon is represented by the random variable R, then

expected return:

$$\mu = E[R]$$

and, risk or standard deviation:

$$\sigma = \sqrt{E[R^2] - \mu^2}$$

or,

$$\sigma^2 = E[R^2] - \mu^2$$

Therefore if there are two assets: A and B such that $A \sim [\mu, \sigma_1]; B \sim [\mu, \sigma_2]\ s.t. \sigma_1 > \sigma_2$; then the investment manager would clearly prefer the asset B over the Asset A. And, similarly if there are two assets: M and N such that $M \sim [\mu_1, \sigma]; B \sim [\mu_2, \sigma]\ s.t. \mu_1 > \mu_2$; then the investment manager would clearly prefer the asset M over the Asset N.

What about if the investment manager is told that she has two assets: $B \sim [\mu_1, \sigma_1]; M \sim [\mu_2, \sigma_2]$ and now must decide which asset she prefers? An important ratio that is used to rank investments is referred to as the Sharpe Ratio (SR) which is nothing but the ratio of return and risk or more accurately excess return over the risk-free return (μ_0) and the level of risk (or excess risk since a risk-free asset such as a government security with one day maturity is indeed available with $\sigma_0 = 0$):

$$SR_1 = \frac{\mu_1 - \mu_0}{\sigma_1}$$

$$SR_2 = \frac{\mu_2 - \mu_0}{\sigma_2}$$

In the context of financial institutions, if the entire institution is treated as an asset a similar ratio is used, which is referred to as return on equity or ROE where the return is the annual profit that institution has earned and the quantum of equity (or capital) is linked to the nature of the risks that it bears in a specific way. Much of the work in risk management is focussed on attempting to quantify the amount of equity an institution needs so that it is considered to be "adequately capitalised".

In order to ensure that they are able to offer their clients services at reasonable costs financial institutions need to operate with very thin margins (i.e., low returns) – a 2% Return on Assets (ROA) is considered more than adequate in most markets which means that the amount of equity that the financial institution holds needs also to be low so that the ROE may be adequate. For an ROE number to be 20% for example, the financial institution can afford to have no more than 10% capital against the assets that it holds:

$$ROE = \frac{Return}{Capital}$$

$$ROA = \frac{Return}{Assets}$$

Therefore,

$$\frac{Capital}{Assets} = \frac{ROA}{ROE} = \frac{2\%}{20\%} = \frac{1}{10}$$

Or, alternately,

$$ROE = \frac{ROA}{Capital/Assets} = \frac{2\%}{1/10} = 20\%$$

And, the lower that ratio is the higher is the ROE or the Sharpe Ratio associated with the institution.

However, before risk managers at financial institutions can figure out how to increase the ROE associated with their financial institutions they have the complex task of figuring out how to model the risk that their institution faces and arriving at a Data Generating Process (DGP) associated with each of the key risks that they face so that they can then proceed to analyse it and arrive at appropriate strategies. The five key risks that almost all financial institutions face are:

1. Interest Rate Risk
2. Liquidity Risk
3. Credit Risk
4. Operations Risk
5. Exchange Rate Risk

In all of these risk categories since it known that if $Z \sim [\mu, \sigma]$ then, $X \sim [0, \sigma]$, where $Z = X + \mu$, the return processes are typically modelled separately from the risk processes. And, in modelling risk processes, at least initially the returns are considered to be "0" so that there can be a sharp focus on the volatility process.

2. Data Generating Process for Volatility

In order to understand risk (or return for that matter), it becomes necessary to arrive at reasonable estimation of the "true but unobserved" Data Generating Process that guides the behaviour of the Random Variable.

During the period from April 2nd, 2013 the Indian Rupee fell from Rs. 54.3345 against the U. S. Dollar to Rs.59.2973 at the end of June 5, 2014. It was a fall of over 9% over the 429 day period. The rate of exchange at the end of every (business) business day is available and now that it is known what happened, the goal is to try and determine what the underlying DGP may have been with the belief that a similar DGP could then be used to estimate the future possible behaviour of the exchange rate.

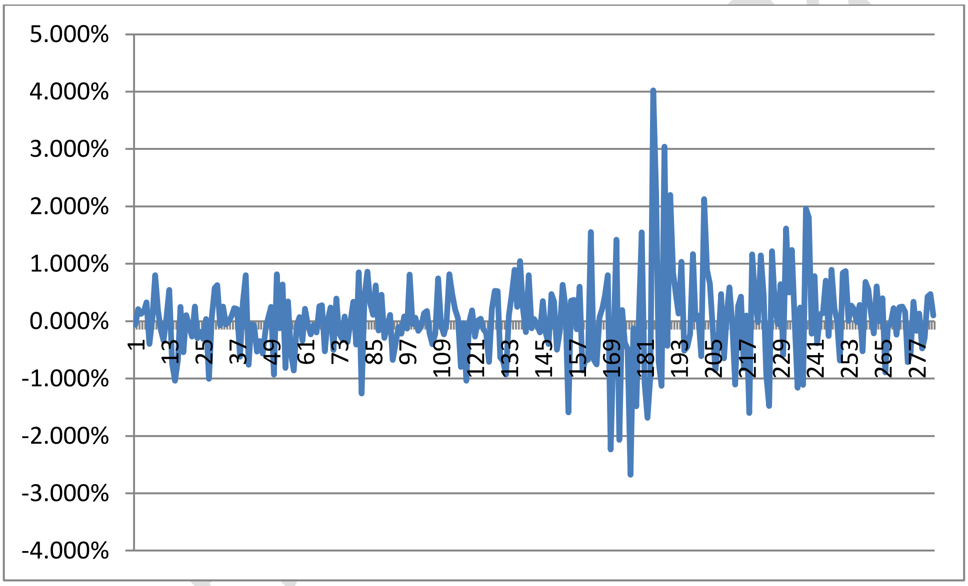
If, E_t is the exchange rate at the end of day t, and E_{t-1} is the exchange rate at the end of day t-1 then let, r_t be the change in the exchange rate such that:

$$E_t = E_{t-1}e^{r_t} \Rightarrow r_t = \ln(E_t) - \ln(E_{t-1}) \approx \frac{(E_t - E_{t-1})}{E_{t-1}}$$

Computed in this way, r_t is the continuously compounded rate at which the exchange rate changes from one day the next. Another way to understand this is to note that:

$$\frac{dlnx}{dx} = \frac{1}{x} \Rightarrow dlnx \approx \frac{dx}{x}$$

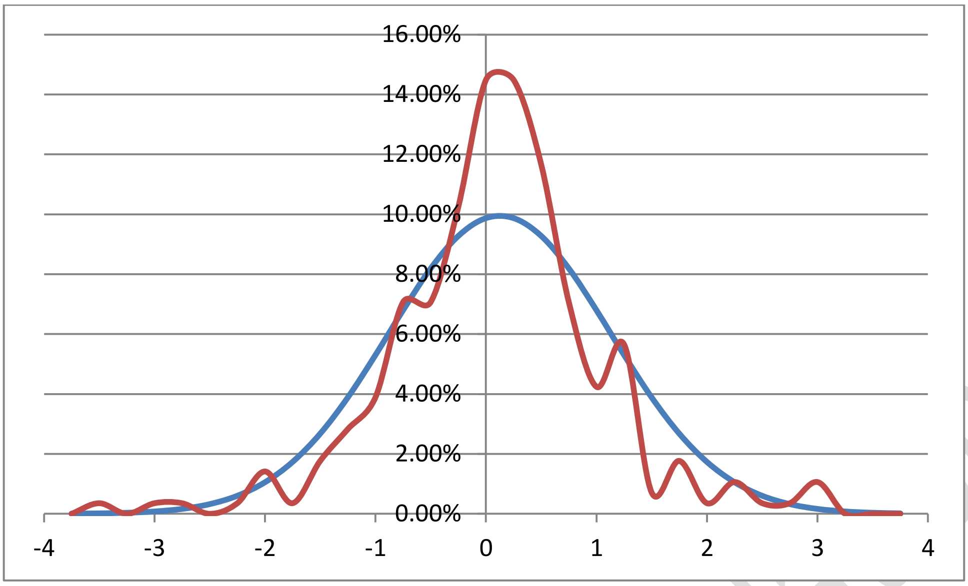
Given below is a graph of the daily values of r_t over this period:



It is obvious from the data that there are some episodes of high and low volatility but other than that it is difficult to draw any conclusions from the graph. The first step therefore is to compute the various moments of the return series and compare it with what would be expected if the series had been Normally distributed. It is clear from the table that the series is obviously not Normally Distributed and is strongly Leptokurtic.

Moment	**R**	**N(0,1)**	$S = (R - \mu_R)/\sigma_R$
μ	0.031%	0.0000	0.0000
σ^2	0.000054	1.0000	1.0000
σ	0.736%	1.0000	1.0000
Skewness	0.6887	0.0000	0.6886
Kurtosis	4.6527	0.0000	4.6527

If a graph of the probability density function of the adjusted series s_t is drawn against an empirically generated standard normal variable, the Leptokurtic (peaked and fat-tailed) nature is clearly visible.



There multiple ways of dealing with this issue. Two ways are commonly used:

1. Move to a higher level of aggregation (say, weekly) – as the level of aggregation grows, the effects of the Central Limit Theorem begin to be felt much more visibly.

2. Seek to model the "fat-tailed" behaviour of the distribution by exploring various models for the behaviour of the volatility.

A number of researchers have hypothesised that the random variable R_t for which only one specific observation r_t follows a "true but unobserved" distribution:

$$R_t \sim N(\mu, \sigma_t)$$

$$\mu \approx 0$$

$$\sigma_t^2 = \gamma\sigma_L^2 + \alpha r_{t-1}^2 + \beta\sigma_{t-1}^2$$

$$\gamma + \alpha + \beta = 1$$

$$\alpha + \beta < 1$$

where,

σ_L^2: is the long-run variance of the series, presumed to be constant
r_{t-1}^2: is the squared return of the previous days.
σ_{t-1}^2: is the variance associated with the previous day.

This process is referred to as the GARCH (1,1) process where GARCH stands for Generalised Autoregressive Conditional Heteroscedasticity[27]. This allows persistence of volatility to take place. When the volatility is high during the previous few days it takes time to die down and similarly periods of calm are disrupted only slowly as people adjust to the new data that they have received.

3. Value at Risk

Now that there is some clarity on how to estimate the volatility associated with a distribution the question is how is to be used in practice.

If the financial institution has invested Rs.1 million in US Dollars and if it expects the exchange rate to operate in accordance with the Random Variable R:

$$R \sim N(\mu, \sigma)$$

$$\mu = -0.031\%$$

$$\sigma = 0.736\%$$

then, it expects to have its financial return F behave as follows:

$$F \sim N(\mu, \sigma)$$

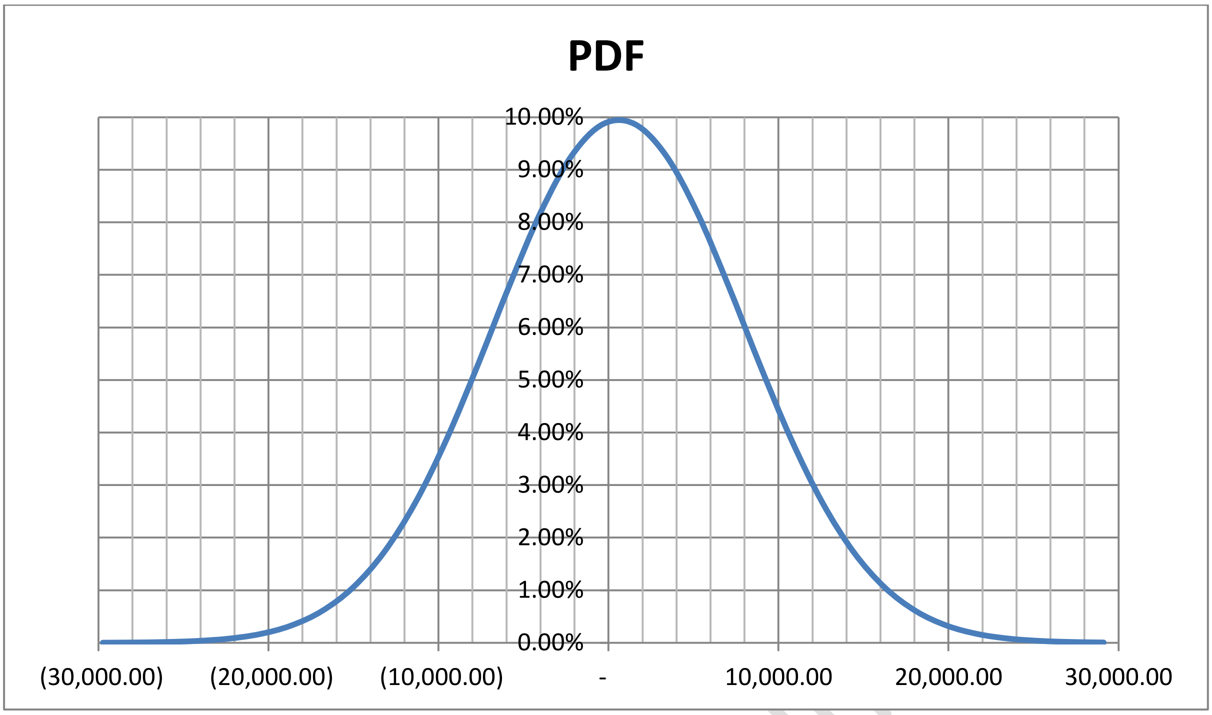
$$\mu = -0.031\% * 10^6 = -Rs. 308.85$$

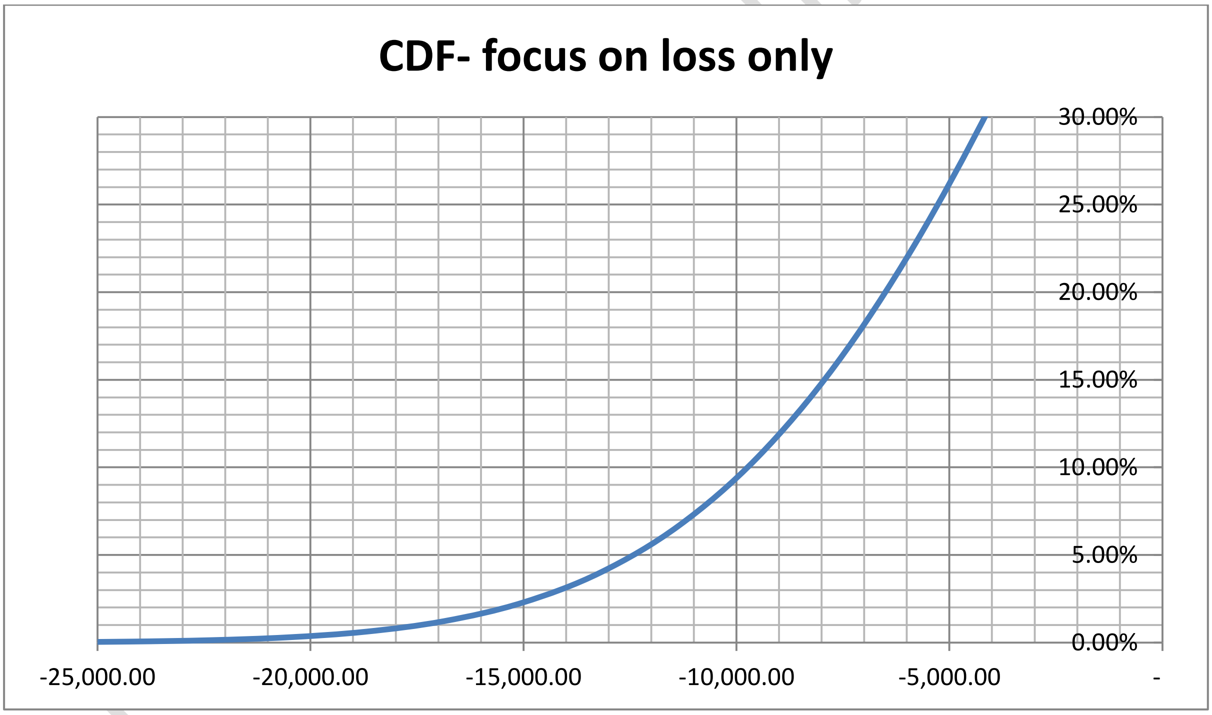
$$\sigma = 0.736\% * 10^6 = Rs. 7{,}359.46$$

And the probability density function of F looks as given below. From the properties of the Normal Distribution it is possible to see that while the average loss is expected to Rs. 308.85, by the end of the following day:

1. There is a 0.13% chance that the institution will lose more than Rs.22,387.24 (-3σ) (refer to graphed PDF on next page) or alternately the institution is 99.87% certain that it will not lose more than Rs.22,387.24 (see graphed CDF on next page) or alternately over the next 10,000 days there should be no more than 13 days during which losses exceed Rs.22,387.24.
2. There is a 2.28% chance that the institution will lose more than Rs.15,027.77 (-2σ) or alternately the institution is 97.72% certain that it will not lose more than Rs.15,027.77 or alternately over the next 100 days there are likely to be at most 3 days in which it loses more than Rs.15,027.77.
3. There is a 15.87% chance that the institution will lose more than Rs.7,668.31 (-1σ) or alternately the institution is 84.13% certain that it will not lose more than Rs.7,668.31 or alternately, over the next 100 days there are likely to be at most 16 days in which it loses more than Rs.7,668.31.

Each of these quantities is referred to as the Value-at-Risk over the time horizon of one day with a confidence level of 99.87%, 97.72%, and 84.13%.





More precisely, VAR $V_{X,T}$ at a confidence level X, over a time horizon T is defined as[28]:

"There is X percent certainty that the financial institution will not lose more than $V_{X,T}$ rupees in time T"

VAR (or VaR) is used by regulators of financial institutions and by financial institutions themselves to determine the amount of capital they should keep. Regulators calculate the capital required for market risk as a multiple of the VaR calculated using a 10-day time horizon and a 99% confidence level. They calculate capital for credit risk and operational risk as the VaR using a one-year time horizon and a 99.9% confidence level. Suppose that the VaR of a portfolio for a confidence level of 99.9% and time horizon of one year is Rs. 50 million (Rs. 5 crore). This means that in extreme circumstances (theoretically, once very thousand years) the financial institution will lose more than Rs.50 million (Rs.5 crore) in a year. It therefore means that if it keeps Rs.50 million (Rs.5 crore) in capital, it will have a 99.9% probability of not running out of capital in the course of one year – or equivalently it is likely to fail only once every thousand years[29].

4. Maximising ROE

As discussed earlier, given the realities of the market place, while there is some room to increase returns, the principal focus of risk managers and top management of financial institutions is grow their balance sheets by seeking out revenue-accreting business opportunities and then to minimise the risks associated with them. There are only a few distinct ways in which risk can be minimised:

1. High quality origination
2. Optimal level of diversification
3. Hedging

If an investment manager has two assets: $X \sim [\mu_X, \sigma_X]; Y \sim [\mu_Y, \sigma_Y]$, as discussed earlier, she can choose the most attractive asset based on its Sharpe Ratio but it is possible that if she instead chooses a combination of these two assets she may be able to improve upon her Sharpe Ratio. Whether she is able to that or not depends on the manner in which the return distributions relate to each other. This relationship is measured by a statistic called Covariance which, when normalised, is also referred to as Correlation[30].

If is already known that if X and Y are independent random variables then:

$$E[XY] = E[X]E[Y]$$

and, more generally, if “g” and “h” are functions defined over X and Y:

$$E[g(X)h(Y)] = E[g(X)]E[h(Y)]$$

Now define Covariance of X and Y by:

$$Cov(X,Y) = E[(X - E[X])(Y - E[Y])]$$

If, X and Y are the same then, by definition:

$$Cov(X,Y) = E[(X - E[X])(X - E[X])] = E[(X - E[X])^2] = Var\ (X)$$

Just as in the case of Variance, it is also possible to show in the case of Covariance that,

$$Var(X) = E[(X - E[X])^2] = E[X^2] - E[X]^2$$

$$Cov(X,Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y]$$

If X and Y are independent, then:

$$Cov(X,Y) = E[XY] - E[X]E[Y] = E[X]E[Y] - E[X]E[Y] = 0$$

The Correlation of X and Y is defined as:

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var[X]Var[Y]}}$$

Notice that since both $Cov(X,Y)$ and $\sqrt{Var[X]Var[Y]}$ are in identical units, Correlation doesn't care what units are used for X and Y. And,

$$a > 0 \,\&\, c > 0 \;\Rightarrow\; \rho(aX+b, cY+d) = \rho(X,Y)\; s.t. -1 \le \rho(X,Y) \le 1$$

X and Y are said to be uncorrelated when $\rho(X,Y) = 0$

With these definitions at hand, it is now possible to construct as asset with a returns characterised by the Random Variable: $P(\omega_1, \omega_2)$, where ω_1 & ω_2 are the investment weights and the Random Variable:

$$P = \omega_1 X + \omega_2 Y$$

with,

$$\omega_1 + \omega_2 = 1$$

For this Random Variable:

$$\mu_P = \omega_1 \mu_X + \omega_2 \mu_Y$$

$$\sigma_P = \sqrt{\omega_1^2 \sigma_X^2 + \omega_2^2 \sigma_Y^2 + 2\rho(X,Y)\omega_1\omega_2\sigma_X\sigma_Y}$$

Let:

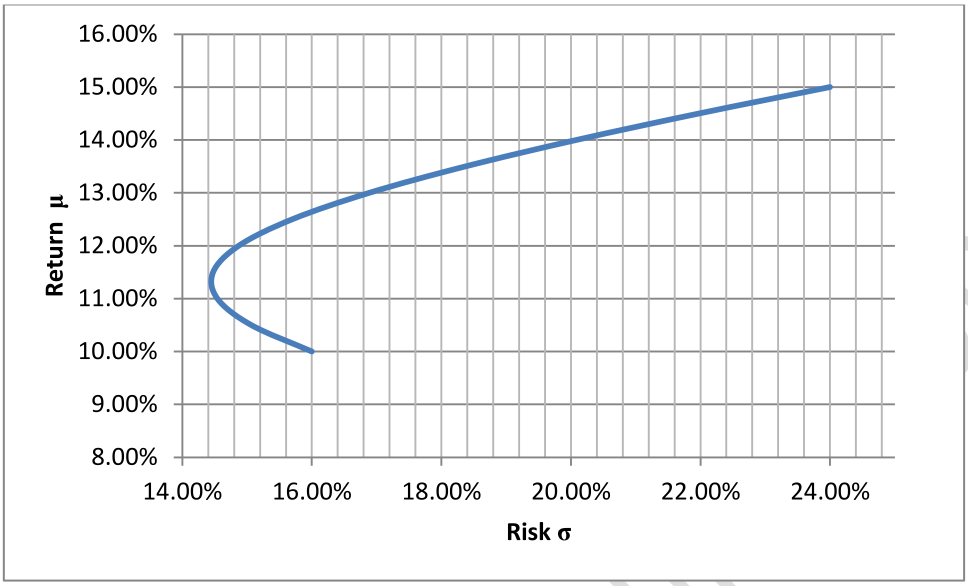
$$\mu_X = 10\%$$
$$\mu_Y = 15\%$$
$$\sigma_X = 16\%$$
$$\sigma_Y = 24\%$$
$$\rho = 0.20$$

The various values that μ_P and σ_P take are given below[31]:

$\boldsymbol{\omega_1}$	$\boldsymbol{\omega_2}$	$\boldsymbol{\sigma_P}$	$\boldsymbol{\mu_P}$	$\boldsymbol{SR_P}$
100%	0%	16.00%	10.00%	0.6250
90%	10%	15.06%	10.50%	0.6970
80%	20%	14.54%	11.00%	0.7565
70%	30%	14.48%	11.50%	0.7945
60%	**40%**	**14.87%**	**12.00%**	**0.8069**
50%	50%	15.70%	12.50%	0.7963
40%	60%	16.89%	13.00%	0.7698
30%	70%	18.37%	13.50%	0.7348
20%	80%	20.09%	14.00%	0.6970
10%	90%	21.98%	14.50%	0.6598

0%	100%	24.00%	15.00%	0.6250

These risk-return combinations are graphed below with risk on the x-axis and return on the y-axis:

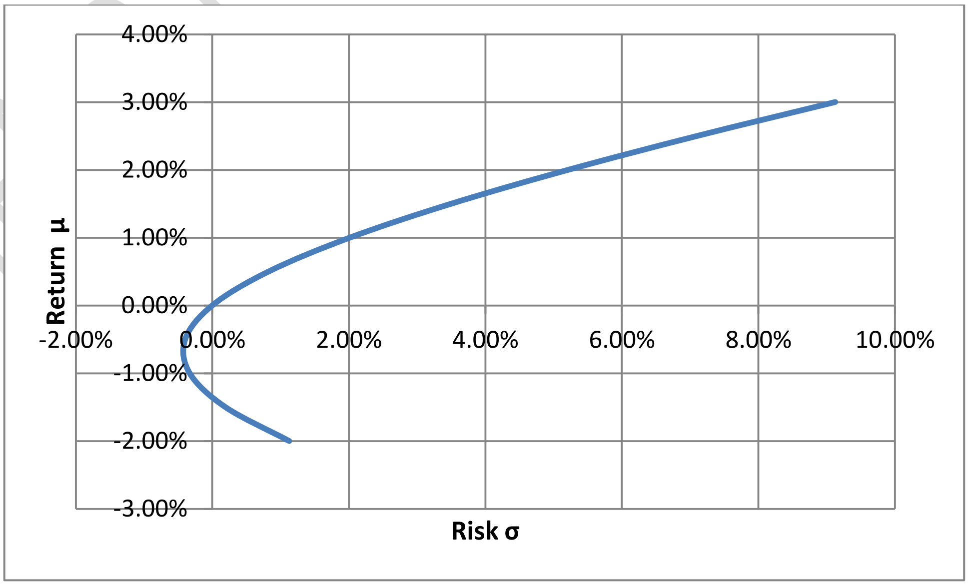


It is clear both from the table and the graph that:

1. Diversification does produce benefits in terms of reduced risks – in fact the lowest risk portfolio has a risk lower than either of the individual assets (14.48%).

2. If indeed the Sharpe Ratio is a good way to measure the risk-return trade-offs there is a portfolio which has the highest Sharpe Ratio of 0.8069 which involves a 60% investment in X and a 40% investment in Y.

If the above graph is re-scaled so that the highest Sharpe Ratio point becomes the 0,0 point the graph looks as follows, with "excess" risk being on the x-axis and "excess" return being on the y-axis:



The risk associated with the portfolio that has the highest Sharpe Ratio is referred to as Systematic Risk (14.87%) associated with this particular set of investments. The risk taken by the other portfolios / assets is considered to be a combination of Systematic Risk and Idiosyncratic Risk. Idiosyncratic risk can be diversified away through optimally choosing the combination of investment assets while Systematic risk cannot be.

Any financial institution seeking to maximise its ROE has therefore to ask the following questions:

1. Has it chosen investments that have the lowest possible risks given the level of return that they are able to give them? This is the origination function.

2. Is it holding these investments in amounts that ensure that it has the best risk-return ratio possible? This is the portfolio management function.

Systematic Risk is clearly not reducible through the process of additional diversification within asset classes available to the financial institution but may well be possible to eliminate / reduce through the use of instruments such as securitisation and credit derivatives. This is referred to as hedging. Here a larger financial institution or one with a different kind of exposure may willing to take on the Systematic Risk in whole or part but need to paid less than the full return associated with the portfolio in the hands of the smaller institution. This amounts to an investment in an asset with a mean return that is negative which has a risk level equal to the instrument with systematic risk and is perfectly correlated with it. This would leave the seller with essentially a risk free portfolio since the idiosyncratic risk has effectively been diversified away. Such a strategy would be strongly preferable to small financial institution that has a great deal of confidence in its ability to carry out strong origination and constructing well-diversified asset portfolios.

Sources of Risk

Chapter 3 : Transfer Pricing

1. Basic Principles

Banks and financial institutions, even small ones, are complex organisations. The core product has infinite flexibility as it relates to final maturity and repayment dates; types of bench-marks to be used and interest-reset dates; currency of denomination, and the nature of the credit risk, market risk, liquidity risk, operations risk to be assumed. This is compounded by the fact that several, if not all of these risks, are originated at multiple locations and by multiple individuals within the financial institution. This makes even the basic tasks of business such as management of liquidity (akin to inventory management for a manufacturing company) and the computation of profits associated with a transaction a significant challenge, leave alone the more complex task of risk management.

Different banks have gone about addressing these issues in different ways but they often apply one or more of the following principles in deciding the path to follow:

1. Banks need to possess specialised competencies and those that are best equipped to do so should be responsible for managing the specific risks that they originate.

2. Banks exist for a variety of core purposes and the risk management function should reveal what that core purpose is.

3. Banks often taken offsetting positions and it would be best to ensure that these positions are first “squared” internally before hedged in the market.

4. While banks may operate through multiple outlets, regulation tends to view the bank as a single central entity and applies all of its good conduct rules as well as capital management rules to that entity.

5. Risks can be broken up into systematic risks and idiosyncratic risks. While idiosyncratic risks are best managed close to the customer, systematic risks are best managed close to the markets.

The approach that is going to be discussed in this section will take a pathway that seeks to:

1. Isolate idiosyncratic credit risk close to the customer, particularly the component that has the maximum information asymmetry and will need to rely on “soft information” for the best possible decision making. Remove market risks such as interest rate risk and liquidity risk from the originating unit as well as systematic credit risk so that it has responsibility for managing only the pure idiosyncratic component.

2. Aggregate market risks such as interest rate risk and liquidity risk into a central treasury function where they can be best managed.

3. Aggregate systematic credit risk into a central portfolio management department so that it can manage and hedge that risk effectively using market instruments such as credit derivatives and securitisations.

4. Allocate capital with a pre-specified hurdle rate to each business / client unit, all the way down to the individual client so that RAROC / Economic Value Added can be computed at a highly granular level.

5. Aggregate operations risk into a central audit function which seeks to control the behaviour of operating units in this regard through a system of internal audit ratings which carry an associated capital charge.

A great deal of this isolation and aggregation is accomplished through the use of well specified transfer pricing systems.

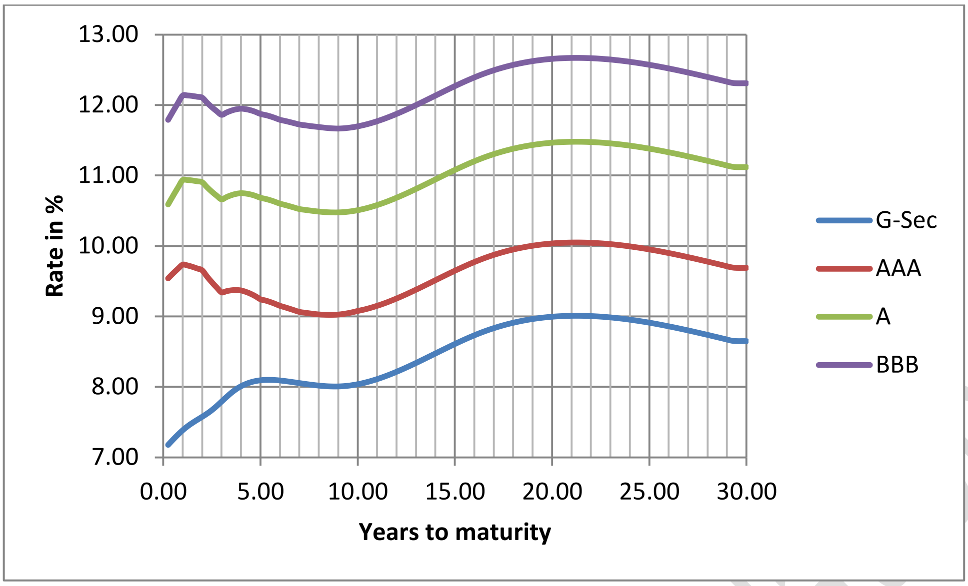
2. Matched Fund Transfer Pricing of Debt

This transfer pricing system seeks to isolate markets risk embedded in asset and liability transactions that go on at the branch level on a minute to minute basis and aggregate them upto a central treasury unit. This is accomplished very simply through the use of an internal market place for money that requires originating units to automatically borrow or lend "Matched Funds" (matched in every way – liquidity, maturity, reset-frequency, bench-mark applied) from an internal treasury which continuously quotes two way prices across the entire yield curve – from one day to 30 years.

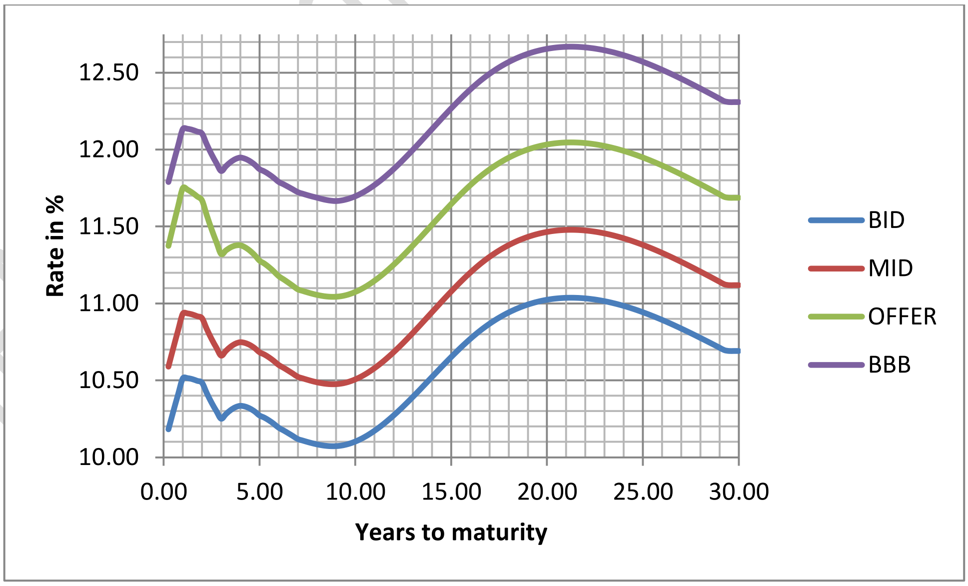
This process allows the central treasury unit to take the benefit of all the off-setting transactions across the entire bank, take a decision on how much capital it wishes to allocate to the residual mismatches that exist so that it can explicitly seek to benefit from anticipated movements in markets variables of interest, and to use internal pricing mechanisms to ensure that pockets of high demand and high supply and constantly balanced out.

In order to ensure that the central treasury does not have conflicts of interest, many banks require it to operate on a no-profit, no-loss basis, and to run a fully matched operation on an instantaneous basis. As a separate unit within the treasury there is often a trading unit which too faces the internal transfer pricing regime and acts as a bridge between it and the larger market. Such a unit is explicitly a profit centre and has capital allocated to it based on the extent of value that it puts at risk. While the Central Treasury unit regards the internal trading unit as a preferred trading partner it is normally expected to have the freedom to directly trade with the market as well so that an additional check is provided against deliberate mis-pricing by the internal trading unit.

Setting up a Matched Fund Transfer Pricing system is a complex endeavour but a good place to start would to map out the Yield Curve of interest rates being observed in the market. FIMMDA (http://www.fimmda.org/) is the institution that publishes such information on a regular basis. Given below is a sample curve from their files for February 28, 2011. It is clear from the graph that while the risk-free Government Securities Curve is markedly positively sloped, particularly from 0 to 5 year maturities, the curve for Corporate Bonds is positive sloped only from the 0 to 1 year maturities but negatively sloped thereafter.



Using this kind of data, an analysis of its own ability to access these markets, and the depth of these markets, a bank would set up its own bid-offer curve, which is intended to reflect, for each maturity, the point of indifference between what the central treasury could borrow on its own and lend on its own, without taking on any credit risk, as well as the relative demand and supply imbalances of the internal demand for funds from the various units within the financial institution. The curve below gives a possible bid-offer curve for an A rated Bank assuming that the A rated FIMMDA curve is sufficiently liquid and represents the rate at which an A rated bank/financial institution could borrow from the inter-bank market. The CRR adjusted rate is therefore the Bid Rate (since the inter-bank rate is CRR free) and the SLR adjusted rate is the offer rate (since the Central Treasury Unit can only invest in a matched-maturity SLR security and therefore incur the negative carry before making the money available to any other lending unit).



For reference the BBB curve is also provided. It is clear that given its own borrowing costs an A-rated financial institution / bank cannot hope to lend to clients who have a superior rating than it does but it can be seen that a BBB rated client can provide somewhat of a spread over the offer rates of a A-rated institution. However, the spread may not be sufficient to cover its operating costs or the amount of capital that would need to be allocated against the increased risk that a BBB rated client represents. This may leave the bank with no choice but to invest heavily in its origination machinery so that it can find clients who are of a high quality for a variety of reasons cannot access public markets and are therefore willing to pay a higher rate than would be strictly implied by their credit quality to obtain access to credit.

3. Transfer Pricing Capital

Just as funds are an important resource for a financial institution so is Capital. Within a bank or a financial institution, given the very high levels of leverage that are possible, it is most often viewed as a reserve against risk and therefore while modelling its use, even if indeed a whole or part of the capital raised is deployed into the business, the view taken is that of a reserve against risk and the presumption therefore is that the actual capital itself is invested in long-term risk free securities and the business goes onto use 100% debt to fund itself and seeks to provide capital with the added return required by shareholders to compensate them for the enhanced riskiness that the reserve-against-risk role implies.

While in the case of funding the only constraint that is provided to a business unit relates to the pricing at which they receive these funds and the supply of these funds is presumed to be unlimited as long as the business is prepared to pay the transfer price, in the case of capital there are three separate issues that need to be addressed:

1. An allocation of a fixed amount of capital resource (reserve) from the integrated balance sheet of the institution.
2. Capital consumption methodologies.
3. Pricing of the capital that has been allocated – also known as the hurdle rate.

<u>Allocation of Capital</u>

Whether or not such a phrase is used or such a process is formally carried out, in all businesses (and not just financial institutions) capital is being allocated to businesses and most Boards and Senior Managements are exercising some discipline over how much is being assigned to each business before of its importance. Within financial institutions given how scarce it is and how it is to be caught on the wrong-foot because of how easy it is to use up, there is often a formal process of ex-ante allocation of capital and the setting up of limits.

How an institution goes about doing this is a deep reflection of what the strategic intent of the business is and how it expects to both serve its customers, the core competency it expects to build (or already has), and the manner in which it expects to deliver superior returns to its shareholders. Typically, as a part of its budgeting exercises, the financial institution is expected to allocate the capital it has to all of its businesses. Given the natural correlations between returns expected by each of its businesses it may over-allocate capital and then monitor consumption and diversification carefully to ensure that total consumption of capital is within the actual amounts of capital it has to protect itself from bankruptcy. Capital requirements specified by regulators, of necessity, tend to be based on certain prototypical business profiles – the Basel Accords for example, somewhat arbitrarily, require all banks to have at least 8% of their risk-weighted assets in the form of capital. A specific bank may have lower credit ratings on its portfolios and therefore, on average, is likely to have regulatory capital levels lower than what it truly needs (Economic Capital levels) and therefore regulatory capital is likely to be lower than the capital the business has available to allocate. Financial Institutions that seek to operate at lower levels of risk will need to be very careful because while they may determine that specific businesses need only a specified amount of capital, regulation may require them to maintain a much higher level. One way to address this may be to maintain existing capital allocation methodologies based on Economic Capital but to require a much higher hurdle rate so that it sufficient to compensate the investors for a much higher level of capital than is strictly needed by the business.

Consumption of Capital

This depends up on the risk that the capital is being used as a reserve against and the precise Value at Risk framework being used. Broadly speaking the Central Treasury or the CFOs office will specify a set of rules that relate the risks being assumed by the business unit to the actual consumption of capital by the business unit (with the allocated capital setting up the upper limit beyond which businesses cannot take on additional risks without seeking an extra allocation).

In the case of credit risk there will often be another independent department which actually assigns an internal credit rating to the credit facility being offered to the client (loan / bond). This credit rating will often map into a specific amount of capital to be consumed by that facility. For other risks there is typically a middle-office that actually runs the algorithms and informs both the trading units as well as the central capital management unit how much capital has been consumed by the businesses.

The amount of capital that is consumed by any risk that is taken is also a function of the credit rating which the financial institution itself aspires to hold. The higher the credit rating aspiration, the higher is level of capital required for a particular asset.

Pricing of Capital

This, at least in theory, is dictated by the expectation of the shareholders for a desired rate of return on each unit of capital, given the underlying riskiness of the overall level of business and the market conditions in which the business operates. For listed financial institutions the Capital Asset Pricing Model[32] is used, which links the pricing of a particular share price to the riskiness of the stock or more precisely to the riskiness of the stock relative to a carefully chosen market index. One of the reasons that financial institutions seek to hedge out index risk, also known as Systematic Risk, is precisely this reasoning at work – since the investor can always invest directly in the index, the financial institution is best off focusing on managing idiosyncratic risk and using capital principally for that purpose.

Chapter 4 : Interest Rate Risk

1. Basic Principles

This is often a centrally managed function and processes such as Matched Fund Transfer Pricing (MFTP) tend to isolate business units from this risk and bring it back to the central treasury to manage.

Unlike the exchange rate or the stock price which are values of assets with infinite / perpetual lives, the interest rate tends to have a finite maturity associated with it and this makes it a much more complex risk class to manage adequately but after credit risk it is perhaps the most important risk that financial institutions seek to manage.

In order to manage these risks there are typically two steps taken:

1. Attempt to measure the level of exposure of the financial institution relative to a single interest rate benchmark so that it starts to look closer to a stock market or an exchange rate exposure.

2. Define a DGP for that benchmark so that a value at risk methodology can be deployed against it.

Some of the key concepts here are Duration of Equity of a Bank (D_E) and Convexity of Equity of a Bank (C_E)[33]:

2. Duration of Equity of a Bank

For a Bond with price "B" and yield to maturity "y" (continuously compounded), the Duration "D" of the bond that provides cash flow c_i at time t_i is:

$$D = \sum_{i=1}^{n} t_i \left(\frac{c_i e^{-y t_i}}{B} \right)$$

$$\Rightarrow \frac{\Delta B}{B} = -D \Delta y$$

i.e., the "D" is the sensitivity of the price of the Bond to a small change in its yield. Since $c_i e^{-y t_i}$ is nothing but the present value of each of the cash-flows of the Bond, and B its price is nothing but the sum of these present values (this is true by construction since the yield "y" is defined as the rate at which the sum of the present values of the individual cash-flows of the bond equal to its price B), the ratio:

$$\frac{c_i e^{-y t_i}}{B}$$

is nothing but a weight attached to each time point t_i each of which represents the relative contribution of that particular cash-flow to the price B of the Bond. Also, by construction, the sum total:

$$\sum_{i=1}^{n}\left(\frac{c_i e^{-yt_i}}{B}\right) = 1$$

$$\because \sum_{i=1}^{n} c_i e^{-yt_i} = B$$

It also follows from this discussion that the duration D of the Bond may be thought of as the weighted maturity of a Bond where each time point where there is a cash-flow is weighted by the relative contribution in the price of the Bond, i.e., bonds that have higher cash-flows early are likely to have lower durations and those that have them later are likely to have higher duration.

Another interesting feature of Duration that follows from this discussion is that if the Bond has no intermediate cash-flows (i.e., it is a Zero Coupon or a Deep Discount Bond) because it has no intermediate interest payments, then it follows that:

$$D = \sum_{i=1}^{n} t_i\left(\frac{c_i e^{-yt_i}}{B}\right) = t_n$$

This gives rise to another interpretation of Duration D of a bond as the maturity of an equivalent Deep-Discount Bond since both are expected to have an identical sensitivity to small changes in their yield.

For a Bond with a yield of 12%, maturity of 3 years, and a semi-annual coupon of 10% the Duration D is 2.653 (years):

Time	Cashflow	PV of Cashflow	Proportional Cashflow PV	Time x Proportional Cashflow PV
0.5	5	4.709	0.050	0.025
1	5	4.435	0.047	0.047
1.5	5	4.176	0.044	0.066
2	5	3.933	0.042	0.083
2.5	5	3.704	0.039	0.098
3	105	73.256	0.778	2.333
Total	**130**	**94.213**	**1.000**	**2.653**

For a Bond with an identical price (Rs.94.213) and final maturity, if the coupon is zero, then while the final cashflow would need to be larger (since there are no intermediate cashflows), irrespective of the quantum of the cashflows or the price of the Bond, the Duration of the Bond will always be exactly equal to its final maturity which, in this case, is 3 years.

Time	Cashflow	PV of Cashflow	Proportional Cashflow PV	Time x Proportional Cashflow PV
0.5	0	0.000	0.000	0.000
1	0	0.000	0.000	0.000
1.5	0	0.000	0.000	0.000
2	0	0.000	0.000	0.000
2.5	0	0.000	0.000	0.000
3	135.0383	94.213	1.000	3.000
Total	**135.0383**	**94.213**	**1.000**	**3.000**

While the concept has been applied here to the cashflows of a single Bond, nothing prevents its application to the integrated cashflows of multiple bonds and loans as may be found in the assets and liabilities of a Bank. The steps to be followed involve bucketing each asset or liability side cashflow into a time bucket and then using a process identical to the one described above to respectively calculate the asset and the liability durations of the Bank's balance sheet. Going one step further if the assets and liability side cashflows are bucketed together the duration thus obtained is referred to as the "Duration of Equity" of a Bank's Balance sheet, since that is all that would be left on the liabilities side of a Bank's balance sheet, even though it is strictly speaking the Duration of the Net Assets of the Bank.

$$E = A - L$$

$$D_A = \sum_{i=1}^{n} t_i \left(\frac{c_{Ai} e^{-y_A t_i}}{B_A} \right)$$

$$D_L = \sum_{i=1}^{n} t_i \left(\frac{c_{Li} e^{-y_L t_i}}{B_L} \right)$$

$$\Rightarrow \; D_E = \sum_{i=1}^{n} t_i \left(\frac{c_{Ei} e^{-y_E t_i}}{B_E} \right)$$

Where,

$$c_{Ei} = c_{Ai} - c_{Li}$$

Now, if:

$$y_A = y_L = y_E = y$$

Then,

$$c_{Ei} e^{-y t_i} = c_{Ai} e^{-y t_i} - c_{Li} e^{-y t_i}$$

$$\Rightarrow \sum_{i=1}^{n} t_i \, c_{Ei} e^{-y t_i} = \sum_{i=1}^{n} t_i \left(c_{Ai} e^{-y t_i} - c_{Li} e^{-y t_i} \right)$$

$$\Rightarrow D_E B_E = D_A B_A - D_L B_L$$

$$\Rightarrow D_E = \frac{D_A B_A - D_L B_L}{B_E}$$

And, since,

$$B_E = B_A - B_L$$

$$\Rightarrow D_E = \frac{D_A B_A - D_L B_L}{B_A - B_L}$$

If the market values of each component is replaced by its book value then,

$$D_E = \frac{D_A A - D_L L}{A - L} = \frac{D_A A - D_L L}{E}$$

$$\frac{\Delta E}{E} = -D_E \Delta y$$

This implies that if "Duration of Equity" of a Bank is D_E, then it is equal to the percentage reduction in the book value of the bank for every per cent rise in interest rates. This is a powerful result because if the change in interest rates Δy can be modelled in some manner and DGP obtained it becomes possible to assign a Value at Risk number. If the Value at Risk is too high then all that the risk manager has to do is to further reduce the value of D_E by either reducing D_A or increasing D_L.

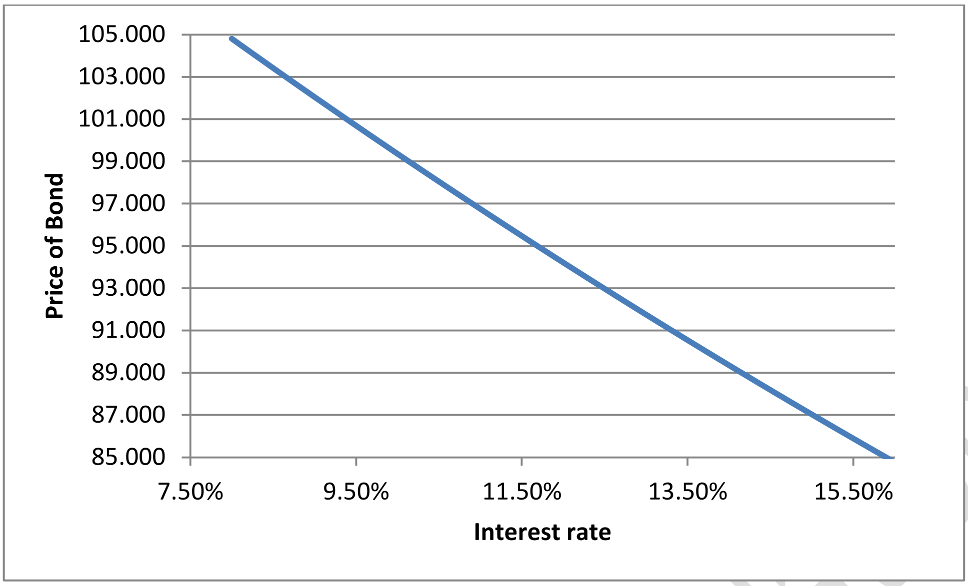
Often large banks like to keep this risk exposure very low, both against increases or decreases in interest rates and will require risk manager to adhere to:

$$-0.25 \leq D_E \leq 0.25$$

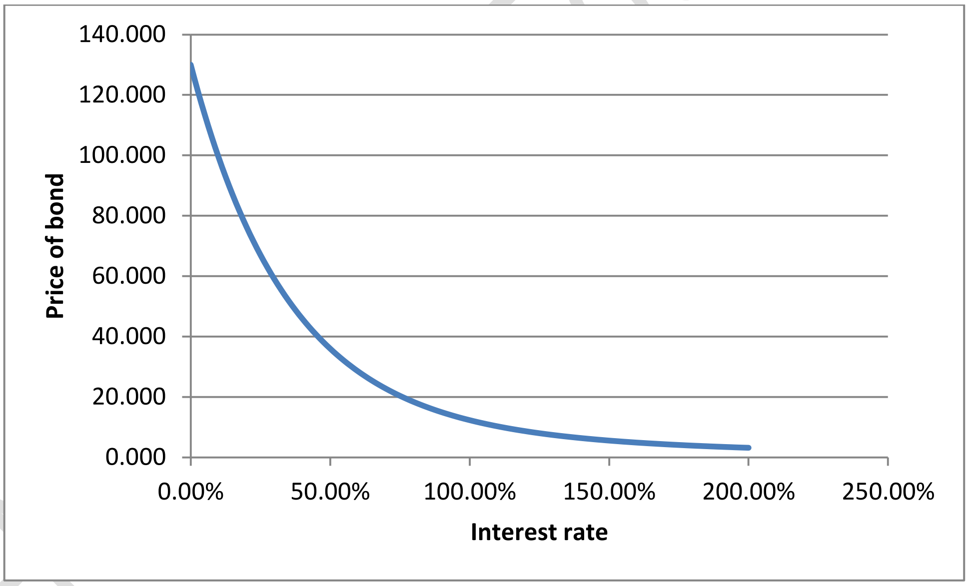
which will ensure that no matter what the underlying DGP of interest rates may be the Value At Risk exposed to interest rate changes is nearly zero.

3. Convexity of Equity of a Bank

When the value of a Bond is examined over small horizons the changes in value of the Bond appear to operate in a linear manner. For the above Bond the graph given below plots the changes in value when the interest rate changes from 8% to 16% in increments of 0.10%.



However, when for the same bond a much larger jump is envisaged, and the change in value of the bond is plotted when the interest rate changes from 0% to 200% in increments of 2.5%, the curvature in the shape of the curve become apparent.



Using techniques similar to those developed when understanding the shapes of probability distributions, it is relatively easy to show that:

$$\frac{\Delta B}{B} = -D\Delta y$$

is only a first order approximation and a more accurate equation (to the second order) is:

$$\frac{\Delta B}{B} = -D\Delta y + \frac{1}{2}C(\Delta y)^2$$

where,

$$C = \sum_{i=1}^{n} t_i^2 \left(\frac{c_i e^{-yt_i}}{B}\right)$$

It is clear from the equation that when the changes in interest rate are very small, i.e.,

$$\Delta y \ \leq 0.01 \ \Rightarrow \ (\Delta y)^2 \leq \ 0.0001$$

$$\frac{\Delta B}{B} = \ -D\Delta y + \frac{1}{2}C(\Delta y)^2 \ \approx -D\Delta y$$

but when interest rates movements are larger this approximation is no longer a good one to make the concept of convexity becomes an important one to use. While assessing Value at Risk of necessity a wider range of interest rate movements have to be examined and therefore it becomes important to make use of this concept as well.

It is interesting to note that Convexity works in the opposite direction from Duration and that for a given level of Duration if the Convexity of a Bond portfolio is increased then the interest rate sensitivity of a the portfolio behaves in an interesting way:

1. When interest rates rise a great deal, for a portfolio with positive duration there is a large loss but if the same portfolio had a large convexity then the loss is significantly lowered on account of the beneficial impact of the convexity.

2. However, when interest rates fall steeply, for a portfolio with positive duration there is a large gain and this gain is increased further if the same portfolio had a large convexity.

And, since the Convexity term takes the square of the interest change, it has a beneficial impact even on portfolios with a negative Duration. These are some of the ways in which risk managers seek to lower the risk exposure of their portfolios (i.e., reduce value at risk) while not giving up the return potential from beneficial interest rate moves. It can be shown that portfolios of identical durations, other things being equal, tend to higher convexity when the cashflows have a bar-bell character so that the mass towards both the extremes is increased, instead of having a smooth character, i.e., a deep discount bond of 5 year duration would have a lower convexity than a portfolio with an identical market price and duration but one which combined two deep discount bonds – one of 1 year duration and the other of a 10 year duration.

As in the case of Duration the entire analysis carries over unchanged when the net asset duration of a bank or a financial institution is being considered and:

$$\frac{\Delta E}{E} = \ -D_E\Delta y + \frac{1}{2}C_E(\Delta y)^2$$

4. Data Generating Process for Interest Rates

The term structure of interest rates (or the yield curve) can behave in multiple ways[34] and before a Value at Risk Methodology can be applied it becomes important to model its behaviour. Given the complexity of the behaviour of bond prices and interest rates, Factor models are used to study the behaviour of interest rates. Factors models assume that the terms structure of interest rates is driven by a set of variables or factors. Most empirical studies using a principal component analysis have decomposed the motion of the interest rate term structure (or yield curve) into three independent and non-correlated factors:

1. The first one is a shift of the term structure, i.e., a parallel movement of all the rates. It usually accounts for up to 80 to 90% of the total variance.
2. The second one is a twist, i.e., a situation in which long rates and short-term rates move in opposite directions. It usually accounts for an additional 5% to 10% of the total variance.
3. The third one is called a butterfly (the intermediate rate moves in the opposite direction of the short and the long term rate). Its influence is generally small (1% to 2% of the total variance).

As the first component generally explains a large fraction of the yield curve movements the modelling of interest rate behaviour is often carried out using a single factor model. This does not necessarily imply that the whole term structure is forced to move in parallel, but simply that one single source of uncertainty is sufficient to explain the movements of the term structure.

A popular single factor for modelling interest rates is the famous Cox-Ingersoll-Ross or the CIR model. It can be written as follows:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t$$

$$\Rightarrow r_{t+\Delta t} - r_t = \kappa(\theta - r_t)\Delta t + \sigma\sqrt{r_t}\sqrt{\Delta t}\varepsilon_t$$

Here, W_t models the random market risk factor. In the discrete form of the equation ε_t is assumed to be unit normal and κ,θ, σ are the estimated parameters. The CIR model posits the existence of a long run value θ of the interest rate r such that if the interest rate moves away from θ it will revert towards it at a speed factor determined by κ. Since W_t is unit normal, the factor $\sigma\sqrt{r_t}$ gives the volatility associated with the changes in interest rates. The standard deviation factor $\sigma\sqrt{r_t}$ also avoids the possibility of negative interest rates for all the positive values of κ and θ. An interest rate of zero is also precluded if the following condition is met:

$$2\kappa\theta \geq \sigma^2$$

The interest rate behaviour implied by this structure thus has the following empirically relevant properties[35]:

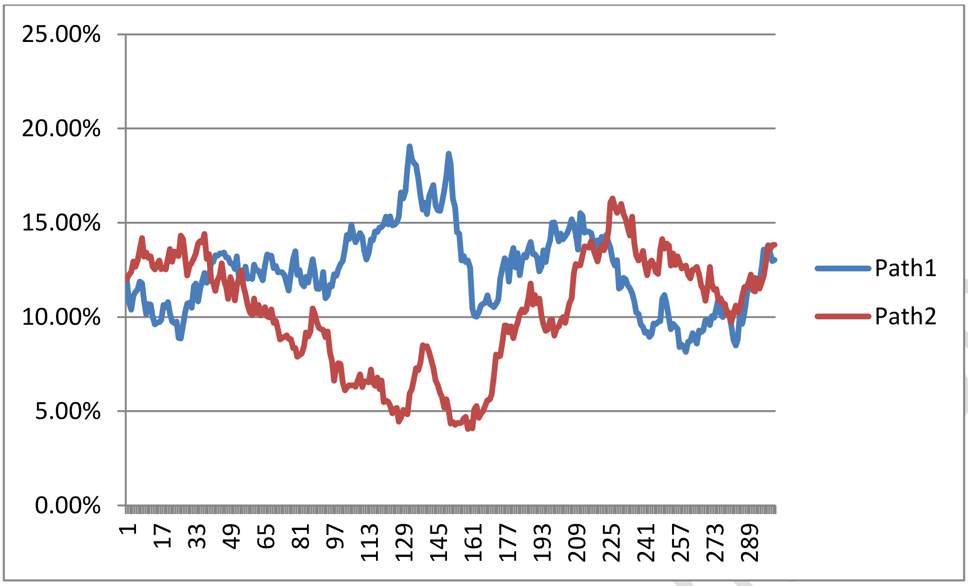
1. Negative interest rates are precluded.
2. If the interest rate reaches zero, it can subsequently become positive.
3. The absolute variance of the interest rate increases when the interest rate itself increases.
4. There is a steady state distribution for the interest rate.

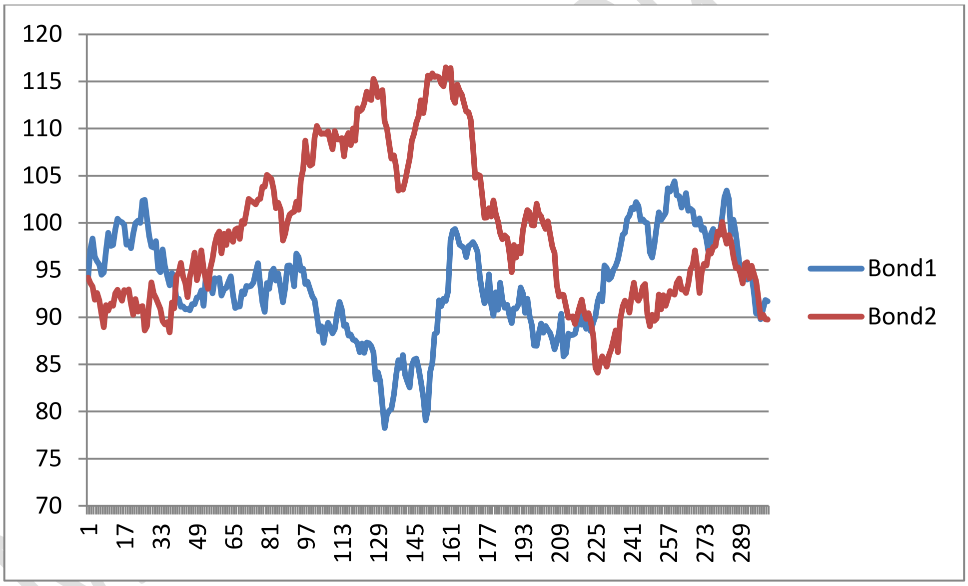
Using the following assumed values:

$$\kappa = 3$$
$$\theta = 10\%$$
$$\sigma^2 = 10\%$$
$$r_0 = 12\%$$

It is possible to simulate the behaviour of the movement of interest rates over a one year horizon.

The graph below gives the possible paths that interest rates could follow with each path starting off at 12% but with a long-term interest rate of 10%. The one below that gives the associated movements in Bond prices. The parameters that have been taken to construct these pathways would, in practice, need to be estimated from actual historical interest rates, assuming that the CIR model provided an accurate representation of the behaviour of the interest rates.





Having obtained the parameters it is now possible to simulate the behaviour of the changes in interest rates and actually plot the PDFs and CDFs. Starting from:

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t$$

it is possible to show that, for a given κ and $\theta > 0$, the long-run distribution of the interest rate converges to a Gamma Distribution $\Gamma(shape, scale)$, where[36,37]:

$$shape = {}^{2\kappa\theta}/_{\sigma^2}$$

$$scale = {}^{\sigma^2}/_{2\kappa}$$

i.e., if R is the long-run interest rate then:

$$R \sim \Gamma({}^{2\kappa\theta}/_{\sigma^2}, {}^{\sigma^2}/_{2\kappa})$$

with,

$$\mu_R = shape * scale = \theta$$

$$\sigma_R^2 = shape * scale^2 = {}^{\sigma^2\theta}/_{2\kappa}$$

This implies that,

$$\theta = \mu_R$$

and,

$$\frac{\sigma^2}{2\kappa} = \frac{\sigma_R^2}{\theta}$$

Therefore,

$$shape = \frac{2\kappa\theta}{\sigma^2} = \frac{{\mu_R}^2}{\sigma_R^2}$$

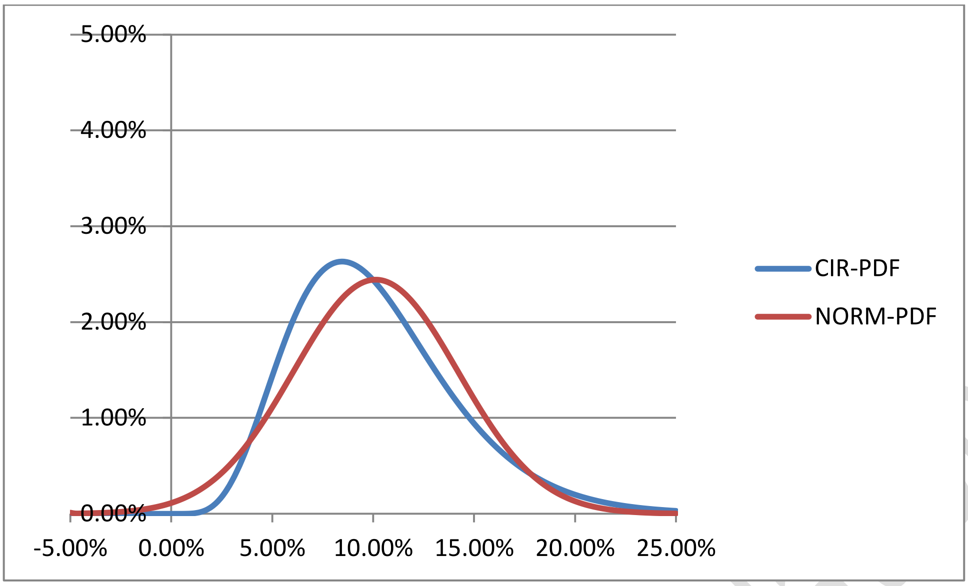
and,

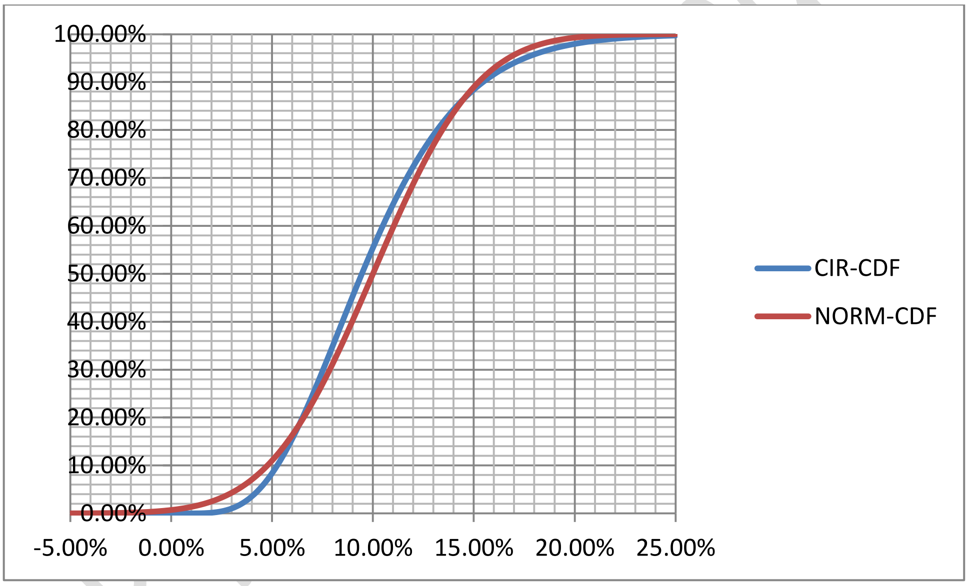
$$scale = \frac{\sigma^2}{2\kappa} = \frac{\sigma_R^2}{\mu_R}$$

Therefore, given an actual interest rate series, which is sufficiently long, it should be possible to calculate the shape and the scale parameters very easily, which are needed to power the Gamma distribution.

For the moment, continuing to use the following assumed values, the PDF and CDF of the Gamma Distribution are generated and compared with the PDF and CDF of a Normal Distribution: $N(\mu_R, \sigma_R)$

$$\kappa = 3$$
$$\theta = 10\%$$
$$\sigma^2 = 10\%$$





It is obvious from the distributions that the behaviour of the interest rate under the CIR model is not Normal, in part because the interest rate remains anchored around $\theta = 10\%$ and the variance is directly proportional to θ and inversely to the speed of mean-reversion$\kappa = 3$. It therefore has fatter tails than does the normal distribution. From these distributions it is possible to infer that if a Bank with Rs.100 crore equity wished to allocate no more than Rs.2.5 crore to interest rate risk at a 95% level of certainty, it would need to be prepared for an interest rate of as high as 17.50% (would have been only 16.75% under the Normal Distribution). Since the current interest rate is 12%, this implies that it would need to work with a 5.5% interest rate shock and ensure that, with that level of shock:

$$\frac{\Delta E}{E} = -0.055D_E + \frac{1}{2}C_E(0.055)^2 \geq -\frac{2.5\ crore}{100\ crore} = -2.5\%$$

If the Bank's equity had a duration and convexity equal to the Bond examined earlier:

$$\frac{\Delta E}{E} = -0.055(D_E = 2.653) + \frac{1}{2}(C_E = 7.570)(0.055)^2 = -13.45\% \ll -2.5\%$$

it would clearly have a loss level far higher than the comfort level of the Board and the risk manager would need to sharply reduce the duration of its equity as well as increase its convexity.

Using actual date on call rates for the Indian market from March 2nd, 2009 to June 5th, 2014 the following parameter values are obtained:

$$\mu_R = shape * scale = \theta = 6.645\%$$

$$\sigma_R^2 = shape * scale^2 = {}^{\sigma^2\theta}/_{2\kappa} = 2.231\%^2$$

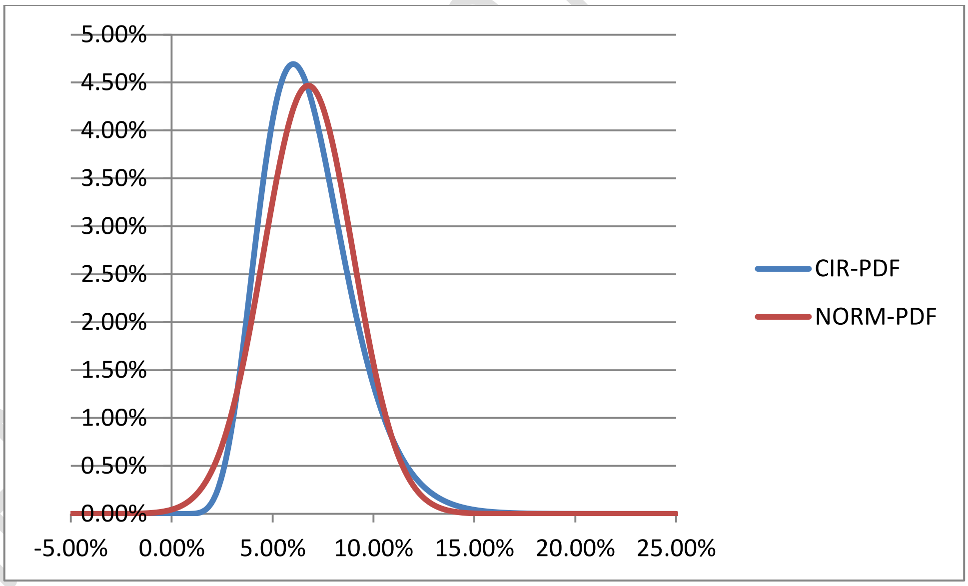
This implies:

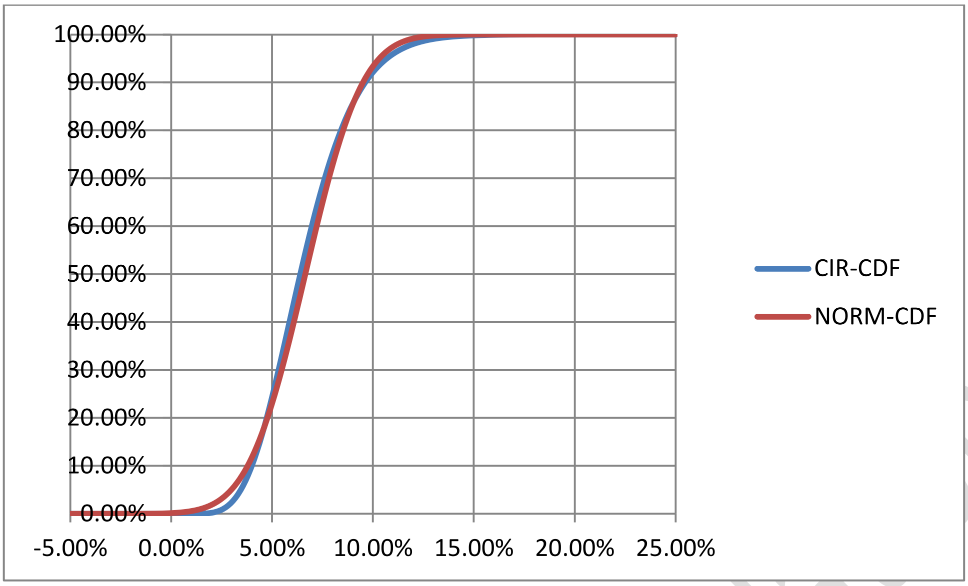
$$shape = \frac{2\kappa\theta}{\sigma^2} = \frac{{\mu_R}^2}{\sigma_R^2} = (\frac{6.645}{2.231})^2$$

and,

$$scale = \frac{\sigma^2}{2\kappa} = \frac{\sigma_R^2}{\mu_R} = \frac{(2.231\%)^2}{6.645\%}$$

Using these shape and scale parameters the PDF and CDF of the Gamma Distribution are generated and compared with the PDF and CDF of a Normal Distribution: $N(\mu_R, \sigma_R)$.





With these parameters, if a Bank with Rs.100 crore equity wished to allocate no more than Rs.2.5 crore to interest rate risk at a 95% level of certainty, it would need to be prepared for an interest rate of as high as 10.75% (would have been only 10.25% under the Normal Distribution) and ensure that with that level of shock the loss is no more than 2.5%. Since the current interest rate is 12%, this implies that it would need to work with a reduction of 1.25% in the interest rate and therefore:

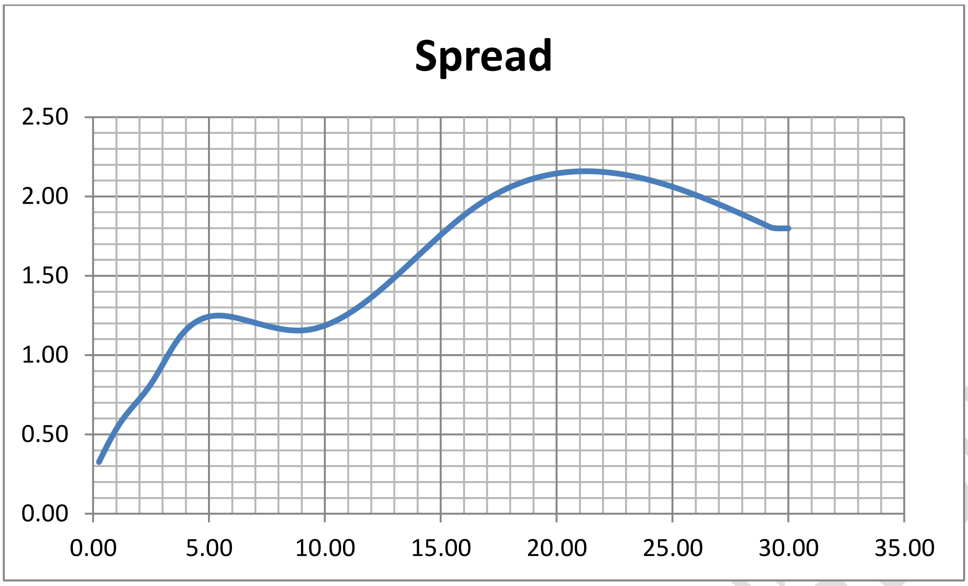
$$\frac{\Delta E}{E} = -0.0125(D_E = 2.653) + \frac{1}{2}(C_E = 7.570)(0.0125)^2 = 3.375\% > -2.5\%$$

Unlike in the previous case, with an equilibrium rate of 6.645%, the current interest rate is so far above that there is a less than 2% probability that the rate will go above 12%. So in this case, with actual data, the bank is in a much better position.

5. Modelling the Entire Yield Curve

While in a single factor model any particular rate could be the subject of modelling, it would be best to use the most liquid and traded benchmark so that all the potential market factors have an opportunity to work themselves through. In the data-based example discussed earlier the rate being used was the over-night rate (or the Call rate). However from this rate it would now be important develop the shape of the full yield curve that would be used to actually calculate parameters of interest. In a single factor model this is relatively easy since the curve is expected to shift in a parallel manner with each shock and to maintain its shape and no twists or butterflies are allowed for.

If for example $\theta_{Overnight} = 6.645\%$, then the entire zero-coupon curve for could be assumed to be 6.645% plus the spread given in the chart below[38]:


Spread
2.50
2.00
1.50
1.00
0.50
0.00
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00

Chapter 5 : Credit Risk

1. Basic Principles

Credit Risk is defined as the likelihood of default of a client of a bank that has a credit facility. This likelihood is expressed the distribution of a Random Variable D which typically associated with a Mean Default Rate D_μ and Standard Deviation of Default Rate or Default Volatility D_σ. Credit risk has the following characteristics:

1. Maximum Default can only be 100%
2. Minimum Default can only be 0%
3. The likelihood of zero to small losses is high and that of large losses is very low.

Given these characteristics there are two fundamental problems associated with credit risk modelling and management:

1. Determination of D_μ and D_σ for any given asset or credit facility.
2. Developing a Data Generating Function for the Random Variable D.

2. Data Generating Process for Default Rates

One DGP that appears to broadly meet with the requirements is the Chi-Squared Function. If:

$$X \sim N(\mu, \sigma)$$

then,

$$D = X^2 \sim \chi^2(1)$$

with,

$$D_\mu = \sigma^2 + \mu^2$$

And, since it is already known that for a normal distribution:

$$\frac{E(x-\mu)^3}{\sigma^3} = \frac{\mu_3 - 3\mu_1\mu_2 + 2\mu_1^3}{\sigma^3} = 0$$

and,

$$\frac{E(x-\mu)^4}{\sigma^4} = \frac{\mu_4 - 4\mu_1\mu_3 + 6\mu_1^2\mu_2 - 3\mu_1^4}{\sigma^4} = 3$$

It is possible to show that:

$$D_{\sigma^2} = [3\sigma^4 + 6\mu^2\sigma^2 + \mu^4] - D_\mu^2$$

This implies that:

$$D_\mu = \sigma^2 + \mu^2$$

and,

$$D_{\sigma^2} = 2\sigma^4 + 4\mu^2\sigma^2$$

It is therefore possible to state that:

$$D = X^2 \sim \chi_1^2(\sigma^2 + \mu^2, \sqrt{2\sigma^4 + 4\mu^2\sigma^2})$$

And, if

$$D = X^2 \sim \chi_1^2(a, b)$$

then,

$$X \sim N[\mu, \sigma]$$

where,

$$\mu = \sqrt[4]{a^2 - 0.5b^2}$$

$$\sigma = \sqrt{a - \mu^2}$$

or, alternately,

$$X \sim N[\sqrt[4]{a^2 - 0.5b^2}, \sqrt{a - \sqrt{a^2 - 0.5b^2}}]$$

In the special case where $\mu = \sigma$,

$$D = X^2 \sim \chi_1^2(2\mu^2, \sqrt{6\mu^2}) \cong \chi_1^2(2\mu^2, 2.45\mu^2)$$

i.e.,

$$a = 2\mu^2$$

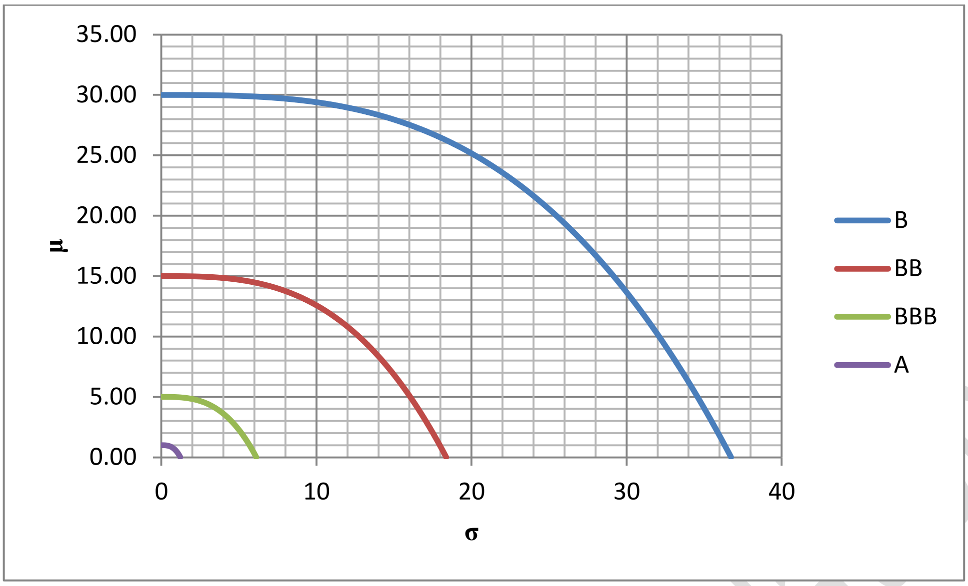
$$b = \sqrt{6\mu^2} \cong 2.45\mu^2$$

3. Credit Ratings

Having arrived at a workable hypothesis for the data generating process for the default rate it becomes necessary to now arrive at as estimate of D_μ and D_σ that can be used to eventually estimate the capital that is needed to support such an asset and to price it correctly.

Unlike interest rates and exchange rates which are economy-wide market variables, loans and bonds are specific to an individual company and are more akin to stock prices associated with the company. For stock prices, taking the lead from Capital Asset Pricing Model[39], an approach would be to start with a stock index such as the NIFTY-50[40] and to arrive at an estimate of the "beta" of every stock. It then becomes possible to focus on the behaviour only of the index for all risk management purposes and to use the "beta" of the stock as a fixed scaling factor.

For loans and bonds a very different approach is followed and the concept of credit ratings is used to group diverse sets of assets in what are hypothesized to be broad bands of D_μ and D_σ, as follows, with D_σ on the x-axis and D_μ on the y-axis.



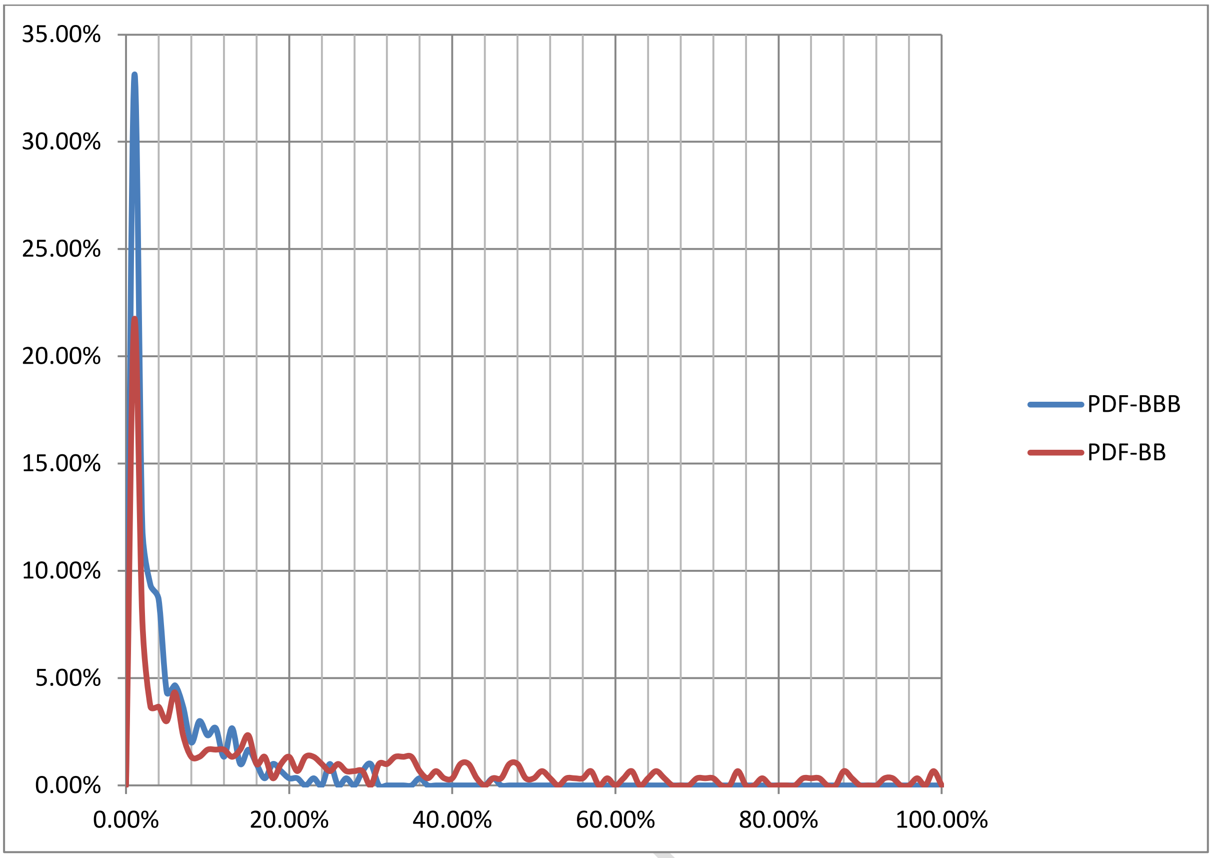
Using these very broad bands as reference points, and highly subjective but broadly stable criteria, Rating agencies separately go ahead and assign each loan or bond of a company a credit rating. Over a period of time, these bonds experience default. Using these observed default rates rating agencies then regularly publish tables of the following kind, where D_μ and D_σ are estimated directly from these observed defaults. It is also possible to arrive at these estimates working backwards from observed pricing of bonds that are traded in the market or using more complex models (such as the KMV model[41]) that attempt to infer these parameters from the movements of stock prices of specific companies[42].

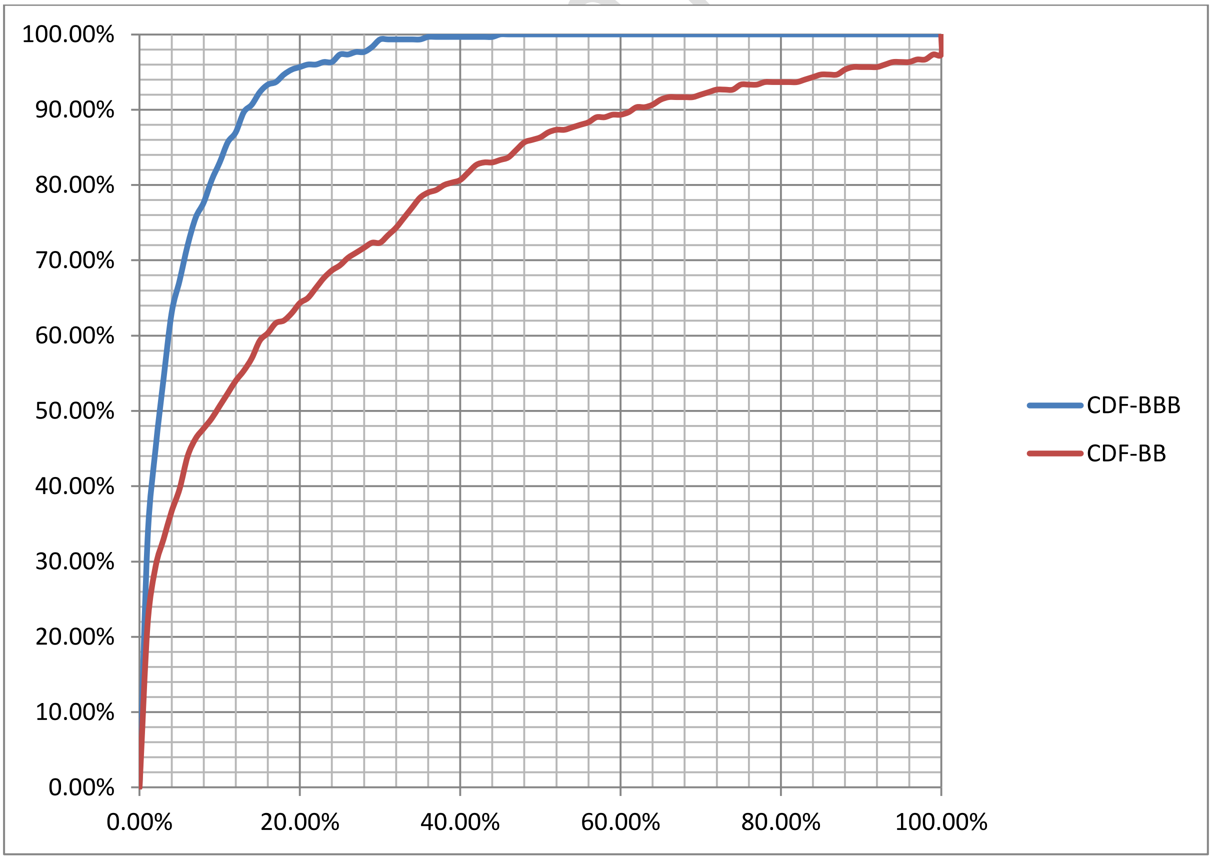
CRISIL Rating	Moody's Rating	Expected Default (D_μ)	Default Volatility (D_σ)
AA	Baa	0.12%	0.18%
A	Ba	1.01%	1.34%
BBB	B	5.46%	6.75%
BB	Caa	18.66%	24.36%

Using these tables and the rating assigned by a rating agencies (or internally by the bank), it then becomes possible to simulate the χ_1^2 distributions for each rating. This loss distribution is presumed to apply to all bonds and loan facilities that have the same rating. The following graphs plot the loss curve of a BBB asset and a BB asset with:

$$D_{\mu,BBB} = 5.46\%;\ D_{\sigma,BBB} = 6.75\%$$

$$D_{\mu,BB} = 18.66\%;\ D_{\sigma,BBB} = 24.36\%$$





From the graphs the difference in default behaviour can easily be seen. It can also be seen that the 95% probability cut-off points are very different for the two assets:

$$D_{\mu,BBB} = 5.46\%;\ D_{95\%,BBB} = 18.50\%;\ \Rightarrow\ D_{95\%,BBB} - D_{\mu,BBB} = 13.04\%$$

$$D_{\mu,BB} = 18.66\%;\ D_{95\%,BB} = 75\%;\ \Rightarrow\ D_{95\%,BB} - D_{\mu,BB} = 56.34\%$$

As will be discussed later these numbers are critical to the pricing of credit assets are referred to as Expected Loss (E_L) and Unexpected Loss (U_L) numbers.

4. Pricing of Credit Risk[43]

Unlike in the case of interest rate risk where the only real possibility is to operate at the portfolio level and attempt maximize ROE, in the case of credit risk, it is possible to do this asset by asset attempt to both measure the consumption of equity for each asset as well as seek to price it in such a way that it becomes possible to earn a reasonable return on the equity capital assigned to each asset.

The underlying thought is that an asset rated BBB has an expected loss rate specified by $D_{\mu,BBB}$ and therefore needs to charge each BBB rated client that much money and make a provision (an expected loss provision or general provision) in its books for such losses. There is another factor though that needs to accounted for, this is the recovery rate once default has happened or the Loss Given Default rate (= 1 – recovery rate) which can be written as LGD. Therefore:

$$E_{L,BBB} = D_{\mu,BBB} LGD_{BBB}$$

In the next step the Unexpected Loss Charge needs to be computed and provided for. As in the case of other risks the thought here is that Capital (as a reserve) needs to be set aside (typically assumed to be invested in long-term risk-free government securities) with a level of certainty determined by the credit rating expectations of the bank itself. Assuming for the moment that this level of certainty is 95% the Unexpected Loss Charge (or Unexpected Loss) is computed as follows:

$$U_{L,BBB} = (D_{95\%,BBB} - D_{\mu,BBB}) LGD_{BBB} r_{Hurdle}$$

where,

$$r_{Hurdle} = \frac{TargetROE_{posttax}}{(1 - taxrate)} - r_{riskfree}$$

The interest rate r_{client} charged to the client would therefore be:

$$r_{client} = r_{transferprice} + c_{operations} + E_{L,BBB+} U_{L,BBB}$$

LGD is dependent on the nature of collateral and the specific loan contract between the bank and its client and can vary a great deal even for a specific rating. However the values of expected default and default volatility can be grouped by rating categories as follows:

CRISIL Rating	Moody's Rating	Expected Default (D_μ)	Default Volatility (D_σ)	Default Rate at 95%
AA	Baa	0.12%	0.18%	0.50%
A	Ba	1.01%	1.34%	5.00%
BBB	B	5.46%	6.75%	18.50%
BB	Caa	18.66%	24.36%	75.00%

and the pricing for each rating level can then be developed at each level of LGD. If:

$$LGD = 70\%$$

$$r_{riskfree} = 7\%$$

$$taxrate = 35.7\%$$

$$TargetROE_{posttax} = 20\%$$

then,

$$r_{Hurdle} = \frac{TargetROE_{posttax}}{(1 - taxrate)} - r_{riskfree} = \frac{0.20}{(1 - 0.357)} - 0.07 = 24.10\%$$

and,

$$E_L = D_\mu LGD = 0.70D_\mu$$

$$U_L = (D_{95\%} - D_\mu)LGDr_{Hurdle} = 0.70 * 0.2410 * (D_{95\%} - D_\mu) = 0.1687(D_{95\%} - D_\mu)$$

The E_L, U_L, and the Credit Spread = $E_L + U_L$ numbers for each rating are as follows:

CRISIL Rating	**Expected Default (D_μ)**	**Default Rate at 95%**	**LGD=70% $E_L = 0.70D_\mu$**	**LGD=70% $U_L = 0.17(D_{95\%} - D_\mu)$**	**Credit Spread $E_L + U_L$**
AA	0.12%	0.50%	0.084%	0.06%	0.148%
A	1.01%	4.00%	0.707%	0.50%	1.212%
BBB	5.46%	18.00%	3.822%	2.12%	5.938%
BB	18.66%	60.00%	13.062%	6.98%	20.037%

For lower rated categories the credit spreads are clearly too high but with better collateral strategies the recovery rates improve to say 70%, i.e., LGD falls to 30%, then the spreads improve quite dramatically suggesting that for lower quality credit risks collateral dependent strategies may be far more effective than those that have poor security cover.

CRISIL Rating	**Expected Default (D_μ)**	**Default Rate at 95%**	**LGD=30% $E_L = 0.30D_\mu$**	**LGD=30% $U_L = 0.07(D_{95\%} - D_\mu)$**	**Credit Spread $E_L + U_L$**
AA	0.12%	0.50%	0.036%	0.03%	0.063%
A	1.01%	4.00%	0.303%	0.22%	0.519%
BBB	5.46%	18.00%	1.638%	0.91%	2.545%
BB	18.66%	60.00%	5.598%	2.99%	8.587%

Using a variety of internal models, a number of rating agencies publish credit ratings that are associated with Bond and credit facilities. It is then possible to track, over a period of time which of these facilities defaulted and after how many years and from such data compute the relevant values of D_μ and D_σ[44].

Management of Risk

Chapter 6 : Capital and Balance Sheet Management

1. Basic Principles

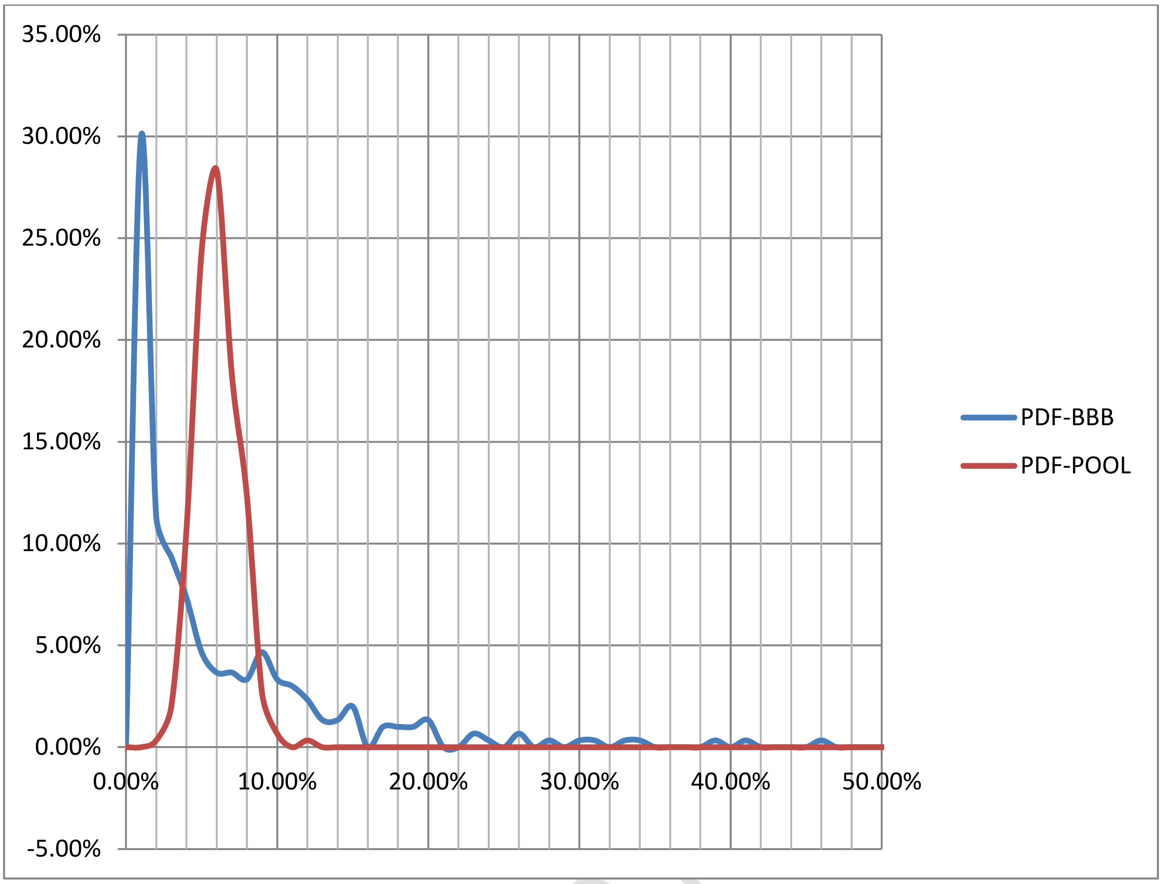
There are three core types of financial institutions:

1. Those that seek to atomize credit and markets risks and distribute it – these are the mutual funds.
2. Those that seek to aggregate risks such as life and non-line, thus eliminating idiosyncratic risk and holding on to systematic risks or hedging them out – these are the insurance companies
3. Those that seek to transform risks by assuming on its assets side a variety of interest rate, liquidity, operations, and credit risks; and using equity capital obtained from its investors in a judicious manner as a reserve to ensure that its depositors and lenders face only a defined and very low level of risk. These are banks and finance companies.

In order to compute the amount of capital that a bank or a finance company requires to achieve this balance it is first necessary to decide the precise level of risk it is prepared to assume. One way to do this is to for the institution to set for itself a rating aspiration. This is often set based on assessment of clients it wishes to serve – a single A rated bank is unlikely to be able to serve AAA rated clients for example, while a AAA rated bank is likely to find itself having to restrict itself to lending to the highest quality names such as sovereigns and having to raise enormous amounts of capital to maintain its high credit ratings. Markets such as India do not have sufficient depth in lower rated segments and while a bank is almost by definition required to maintain very high credit ratings, even finance companies seeking to operate with low credit ratings may not be able to find an adequate number of debt investors who are prepared to lend to them. If a bank decides to maintain a AA credit rating it implies that it wishes to offer to its depositors and lenders an expected default rate[45] of not more than $D_{\mu,AA} = 0.021\%$. This implies that for any asset it seeks to finance or any risk it seeks to take it needs to ensure that the confidence level is set higher than or equal[46] to 99.979%.

2. Capital for Credit Risk

If a Bank ends up building an asset portfolio with the average credit rating of BBB which has an expected default rate of $D_{\mu,BBB} = 5.46\%$, then in order to ensure that despite its asset portfolio quality of BBB, it is able to maintain a AA quality, it will need to protect against a loss rate of as high as 30%. Since $D_{\mu,BBB} = 5.46\%$, after adjust for the Loss Given Default (LGD) of, say, 70%, the Bank would need to provide capital of as much as 24.54% * 0.70 = 17.18% against the loans that it makes[47]. However, if the Bank is able to pool a variety of BBB rated assets and using the combined power of low-correlations between different asset classes and the central limit theorem, is able to build a portfolio with has an expected default rate of $D_{\mu,BBB} = 5.46\%$ but $D_{\sigma,A} = 1.34\%$ then the PDF and CDF distributions will look as follows:



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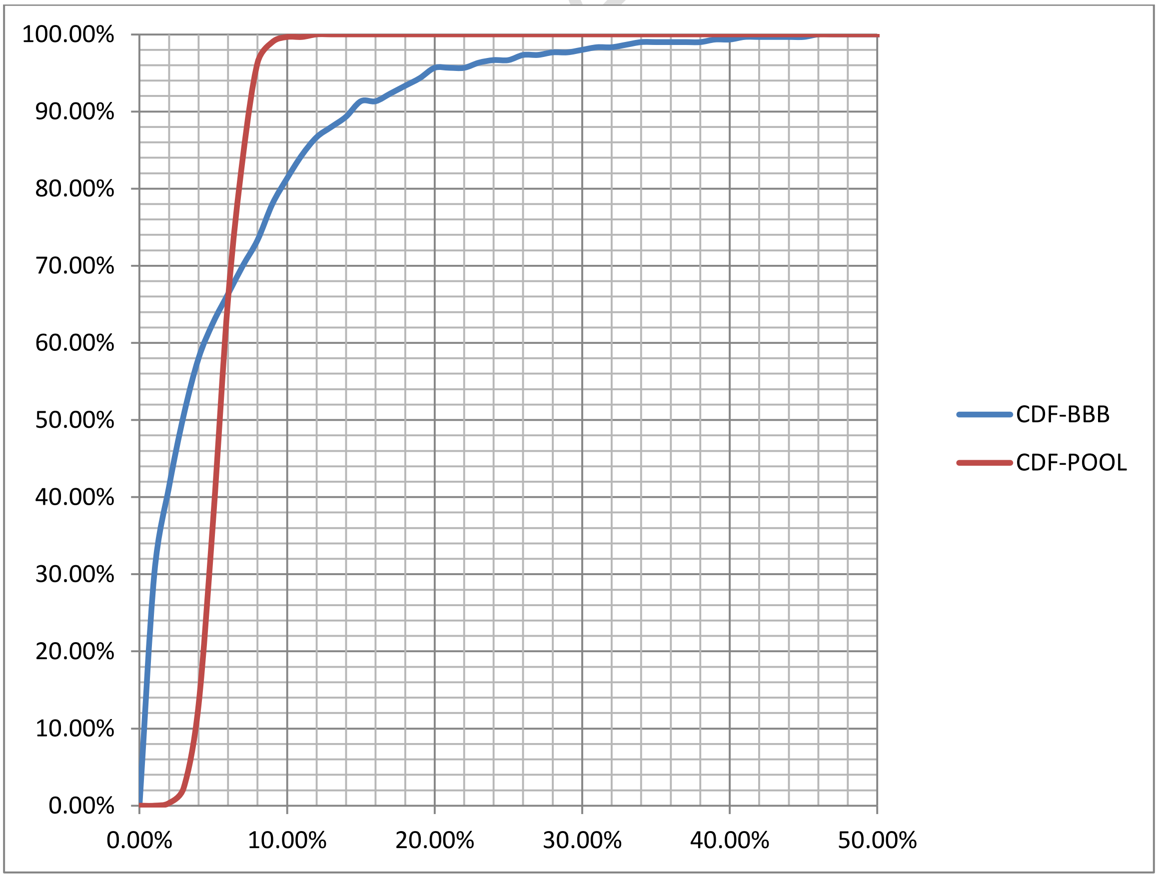
PDF-BBB
PDF-POOL



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CDF-BBB
CDF-POOL

Under these circumstances the Bank now needs to provide capital only to cover total losses of 10%, which implies that the capital requirement has fallen sharply to 4.54% * 0.70 = 3.18%

Some of the ways in which banks are able to reduce the capital consumption of their entire portfolio are as follows:

1. Incentivise lending units to lend to un-correlated / low-correlated assets by lowering the incremental capital requirements for them and therefore effectively lowering the capital charge so that they are able to benefit at least to some degree by the diversification benefit they are contributing to the entire bank.

2. Conversely, penalise lending units that continue to build high levels of concentration into highly correlated sectors by adding an a concentration capital penalty.

3. Liquidate part of the balance sheet by selling assets in which the bank is over-exposed and use the cash thus generated to originate assets of lower-correlations.

4. Improve loss-given default to much higher levels by improving the monitoring of asset quality and initiating prompt corrective action in case the client starts to face difficulties.

3. Capital for Interest Rate Risk

It is already known that, if "NA" is the value of Net Assets of the Bank (or the value of its equity capital) then "E", the amount of capital required for market risk equals:

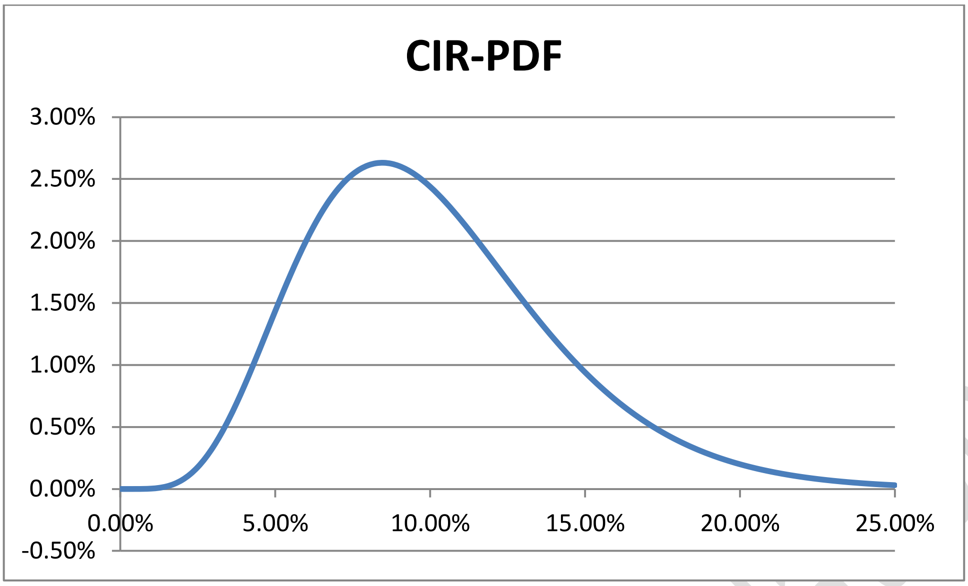
$$E = \left(-D_{NA}\Delta y + \frac{1}{2}C_{NA}(\Delta y)^2\right) * NA$$

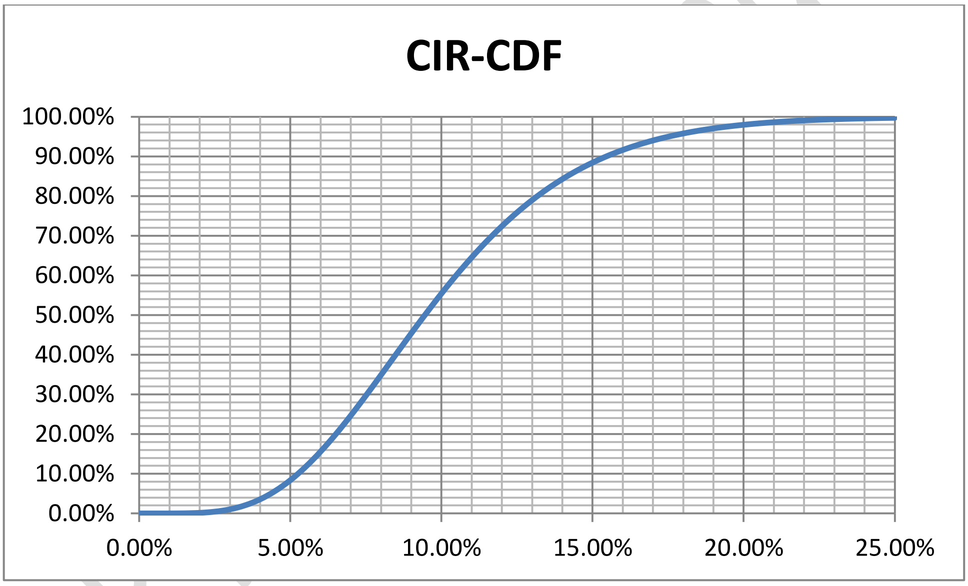
To arrive at the appropriate value of Δy, if the interest rates are governed by the CIR model then, it is already known that, if R is the long-run interest rate then:

$$R \sim \Gamma({}^{2\kappa\theta}/_{\sigma^2}, {}^{\sigma^2}/_{2\kappa})$$

Therefore, using the following parameter values, the PDF and CDF of the Gamma Distribution appear as follows:

$$\kappa = 3$$
$$\theta = 10\%$$
$$\sigma^2 = 10\%$$
$$r_0 = 12\%$$





it is now possible to infer that if a Bank with Rs.100 crore of Net Assets (NA), $D_E = 2.653$, and $C_E = 7.570$, wishes to maintain a credit rating of AA, it would need to have adequate capital to protect itself against a move of over 9% from the currently prevailing rate, which implies that it would need:

$$\frac{E}{NA} = -0.09(D_{NA} = 2.653) + \frac{1}{2}(C_{NA} = 7.570)(0.09)^2 = -20.81\%$$

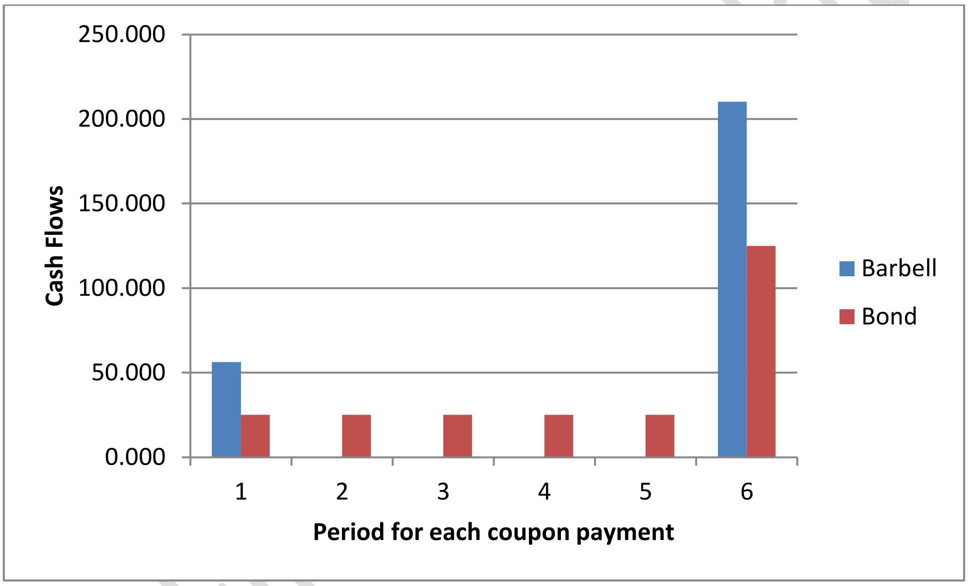
This implies that a bank with Rs.100 crore of net assets with $D_\mu = 5.46\%$; $D_\sigma = 1.34\%$; $D_E = 2.653$; and $C_E = 7.570$ would need to provide credit risk capital of 6.54% and interest rate risk capital of 20.81%, i.e., a total capital of 27.35% or Rs. 27.35 crore. If it felt that this level of capital was excessive, it would need to restructure its books and appropriately change the value of the above parameters.

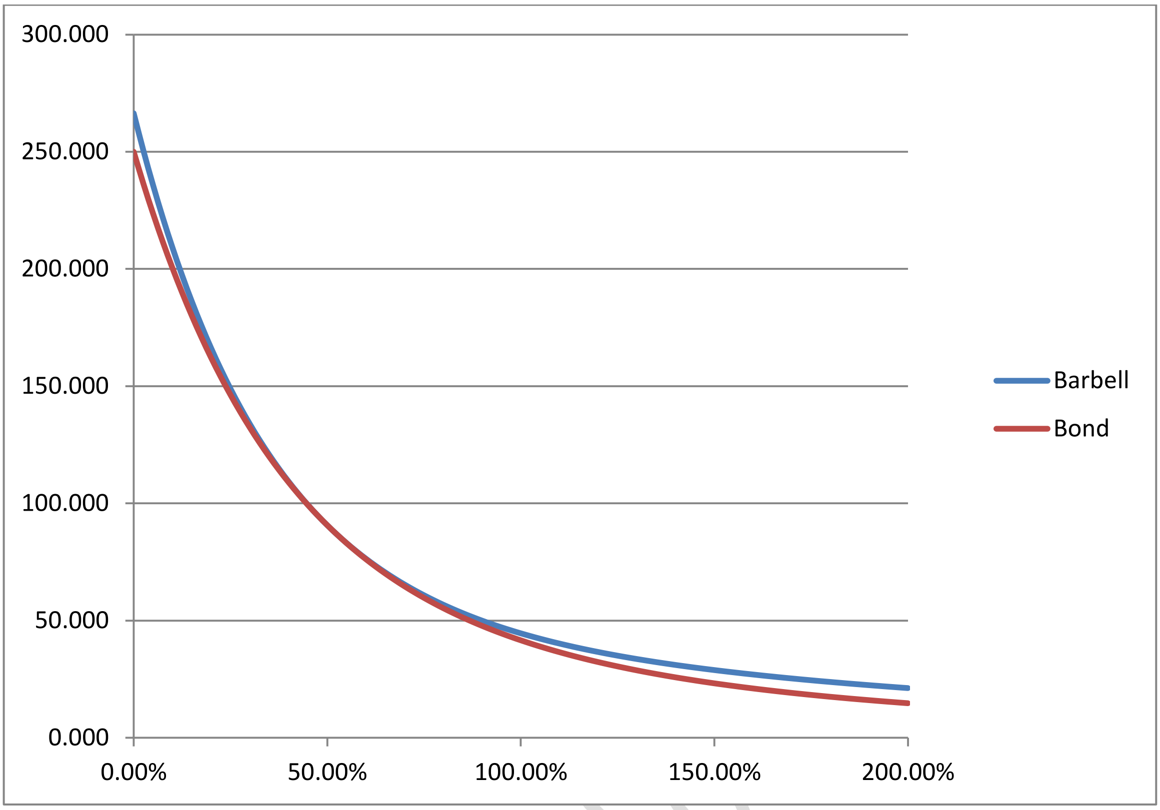
Some of the mechanisms that banks and finance companies use to tweak their balance sheet so that they can lower their exposure to interest rate risk are as follows:

1. Change transfer prices so that the various operating units of the bank face higher prices at those maturities where the bank needs to raise more money and lower prices where it has excess liquidity so that it is able to successfully address both liquidity and duration management challenges.

2. Often banks are required to hold Statutory Reserves, the profile of these could be changed quite dramatically so that the overall duration and convexity of the balance sheet is closer to the desired level. Similarly loan / bond assets which are required to adjust duration and convexity up and down could be sold down as well.

3. In carrying out these adjustments (often referred to as immunization strategies) there are a number of ways risk managers are able to maximize the gains from the trades. One example is one in which risk manager seeks to have a bar-bell structure to the cash-flows. This ensures that for any given level of duration the Barbell has a higher degree of convexity, as is visible from the graphs below:





4. National and Global Capital Regulations[48]

Given all the earlier discussions, it would appear that if banks were to be sensibly managed by their managements, they would hold an adequate amount of capital, there would be no need for national regulations on capital leave along global regulations. However, for a number of reasons, including:

1. Concern that for financial institutions to be referred to as banks they could not be permitted to have a rating lower than "A" and therefore would need to have an adequate amount of capital (this is where the 8% number comes from and the phrase Capital Adequacy).

2. Lack of agreement on the underlying DGPs governing the behaviour of each source of risk and therefore a desire for some standardised approaches for each risk category.

The Basel 1988 Accord (Basel I) required banks to keep capital of least 8% of their risk-weighted assets where the risk-weight was nothing but an estimate of the riskiness (credit risk only) of each asset on the bank's balance sheet relative to the risk of a benchmark "A" rated asset. If the asset was less risky than an "A" rated asset (such as cash, gold, and government securities) it would have a lower risk weight (zero in this case) and if it was more risky it would have a higher weight. That way instead of applying a different capital ratio to each asset class it would be possible to apply a single capital ratio of 8% on the entire risk-weighted asset base.

The 1996 Amendment (implemented in 1998 therefore also called BIS 98), for the first time recognized the existence of market risk and required capital to be allocated to it using a 10-day time horizon and a 99% confidence interval (instead of a one year, 99.98% in the case of credit risk).

In June 1999, the Basel Committee proposed new rules that have become known as Basel I. Prior to Basel II all corporate assets were treated as having a 100% risk weight, the new accord allowed different risk weights with BBB+ o BB- rated assets now being seen as having a 100% risk weight and others having higher or lower risk weights depending upon their credit rating. Capital was also required for the first time against operational risks.

Basel III proposals were first published in December 2009, with the final version being published in December 2010. These proposals (not yet implemented in India) imposed, for the first time, minimum liquidity requirements to be maintained by a Bank, and a leverage ratio – harking back to the Basel I unweighted capital adequacy rule.

5. Deutsche Bank's Economic Capital and Overall Risk Assessment Report[49]

Key risk categories for us include credit risk, market risk, operational risk, business risk(including tax and strategic risk), reputational risk and liquidity risk. We manage the identification, assessment and mitigation of top and emerging risks through an internal governance process and the use of risk management tools and processes. Our approach to identification and impact assessment aims to ensure that we mitigate the impact of these risks on our financial results, long term strategic goals and reputation.

As part of our regular risk and cross-risk analysis, sensitivities of the key portfolio risks are reviewed using a bottom-up risk assessment and through a top-down macro-economic and political scenario analysis. This two-pronged approach allows us to capture not only risks that have an impact across our risk inventories and business divisions but also those that are relevant only to specific portfolios. Current portfolio-wide risks on which we continue to focus include: the potential re-escalation of the European sovereign debt crisis, a potential slowdown in Asian growth, disruptive US monetary tightening and its impact in particular on Emerging Markets and the potential risk of a geopolitical shock. These risks have been a consistent focus throughout recent quarters. The assessment of the potential impacts of these risks is made through integration into our group-wide stress tests which assess our ability to absorb these events should they occur. The results of these tests showed that we currently have adequate capital and liquidity reserves to absorb the impact of these risks if they were to materialize in line with the tests' parameters.

The year 2013 saw a continuation of the global trend for increased regulation in the financial services industry which is likely to persist through the coming years. We are focused on identifying potential political and regulatory changes and assessing the possible impact on our business model and processes.

The overall focus of Risk and Capital Management throughout 2013 was on maintaining our risk profile in line with our risk strategy, increasing our capital base and supporting our strategic management initiatives with a focus on balance sheet optimization. This approach is reflected across the different risk metrics summarized below.

For purposes of Article 431 CRR, we have adopted a formal risk disclosure policy aiming to support a conclusion that our risk disclosures are in compliance with applicable legal, regulatory and accounting risk disclosure standards. A Risk Reporting Committee comprising senior representatives and subject matter experts from Finance and Risk governs our respective risk disclosure processes. Based upon our assessment and verification we believe that our risk disclosures presented throughout this risk report appropriately and comprehensively convey our overall risk profile.

Our mix of various business activities results in diverse risk taking by our business divisions. We measure the key risks inherent to their respective business models through the undiversified Total Economic Capital metric, which mirrors each business division's risk profile before taking into account cross-risk effects at the Group level. The changes from year-end 2012 mainly reflect offsetting effects of our de-risking strategy and methodology updates across risk types. Corporate Banking & Securities' (CB&S) risk profile is dominated by its trading in support of origination, structuring and market making activities, which gives rise to market risk and credit risk. Further credit risks originate from exposures to corporates and financial institutions. Under CB&S' current business model, the remainder is derived from operational risks and business risk, primarily from potential legal and earnings volatility risks, respectively.

Global Transaction Banking's (GTB) focus on trade finance implies that the vast majority of its risk originates from credit risk with a small portion from market risk mainly in relation to derivative positions.

The main risk driver of Deutsche Asset & Wealth Management's (DeAWM) business are guarantees on investment funds, which we report as non-trading market risk. Otherwise DeAWM's advisory and commission focused business attracts primarily operational risk. In contrast to this, Private & Business Clients' (PBC) risk profile is comprised of credit risk from retail and small and medium-sized enterprises (SMEs) lending and non-trading market risk from Postbank's investment portfolio.

The Non-Core Operations Unit (NCOU) portfolio includes activities that are non-core to the Bank's future strategy; assets materially affected by business, environment, legal or regulatory changes; assets earmarked for de-risking; assets suitable for separation; assets with significant capital absorption but low returns; and assets exposed to legal risks. NCOU's risk profile covers risks across the entire range of our operations comprising credit risks and also market and operational risks (including legal risks) targeted where possible for accelerated de-risking.

The execution of our divestment strategy in NCOU has resulted in a reduced balance sheet, which triggered a review of our operational risk allocation framework. In line with the NCOU business wind down, we reallocated economic capital for operational risk amounting to € 892 million to our Core Bank in the third quarter of 2013.

Deutsche Bank Economic Capital Table 2013

in % (unless stated otherwise)	Corporate Banking & Securities	Global Transaction Banking	Deutsche Asset & Wealth Management	Private & Business Clients	Non-Core Operations Unit	Consoli-dation & Adjustments	Total in € m.	Total
Credit Risk	17	7	1	14	5	0	12,013	44
Market Risk	18	1	6	11	5	7	12,738	47
Operational Risk	9	0	2	3	5	0	5,253	19
Diversification Benefit	(7)	(1)	(2)	(3)	(3)	0	(4,515)	(17)
Business Risk	5	0	0	0	1	0	1,682	6
Total EC in € m.	11,398	2,033	2,010	6,671	3,349	1,710	27,171	100
in %	42	7	7	25	12	6	100	0

Deutsche Bank Economic Capital Table 2012

in % (unless stated otherwise)	Corporate Banking & Securities	Global Transaction Banking	Deutsche Asset & Wealth Management	Private & Business Clients	Non-Core Operations Unit	Consoli-dation & Adjustments	Total in € m.	Total
Credit Risk	16	6	1	13	8	0	12,574	44
Market Risk	14	1	5	11	10	5	13,185	46
Operational Risk	7	0	2	1	7	0	5,018	17
Diversification Benefit	(5)	0	(2)	(2)	(6)	0	(4,435)	(15)
Business Risk	7	0	0	0	1	0	2,399	8
Total EC in € m.	11,118	1,781	2,009	6,720	5,782	1,331	28,741	100
in %	39	6	7	23	20	5	100	0

Chapter 7: Off-Balance Sheet Management

1. Securitization

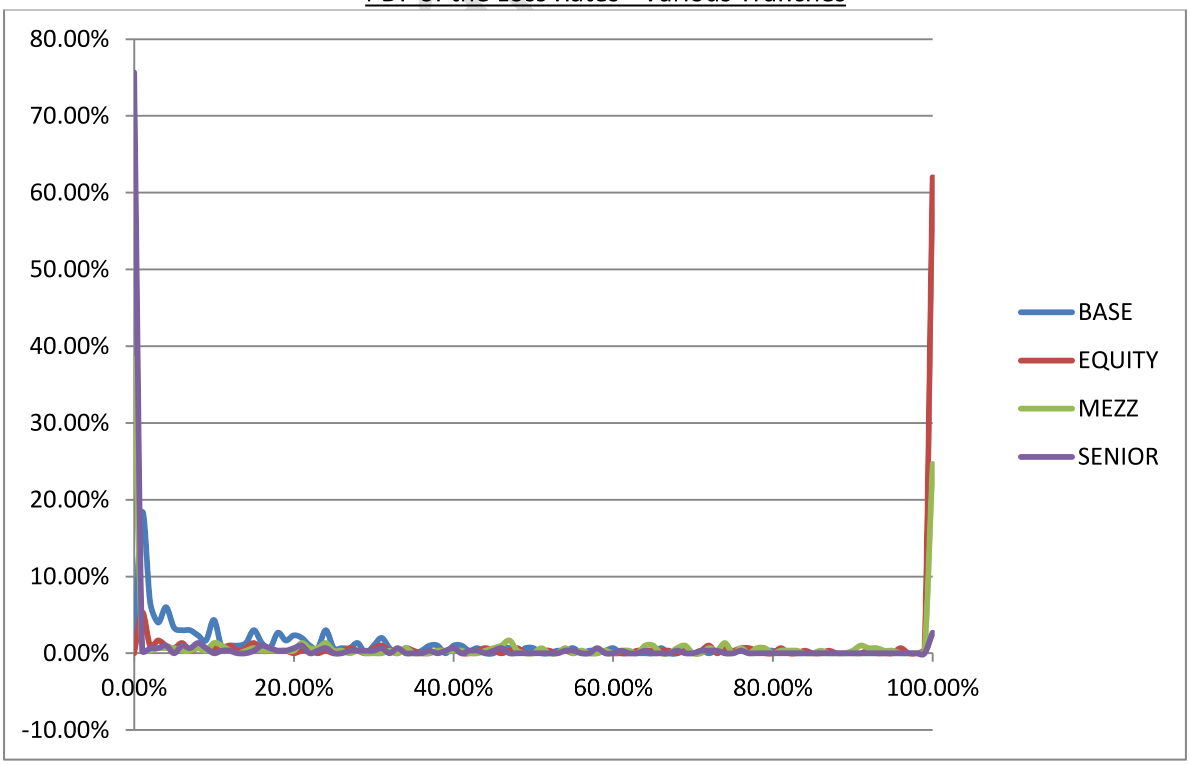
This is a popular tool for pools of assets to be transacted in a standardised way. In this instead of using capital to protect against risk a process called tranching is used create multiple risk categories and then offered up for sale to investors.

CRISIL Rating	Moody's Rating	Expected Default (D_μ)	Default Volatility (D_σ)
AA	Baa	0.12%	0.18%
A	Ba	1.01%	1.34%
BBB	B	5.46%	6.75%
BB	Caa	18.66%	24.36%

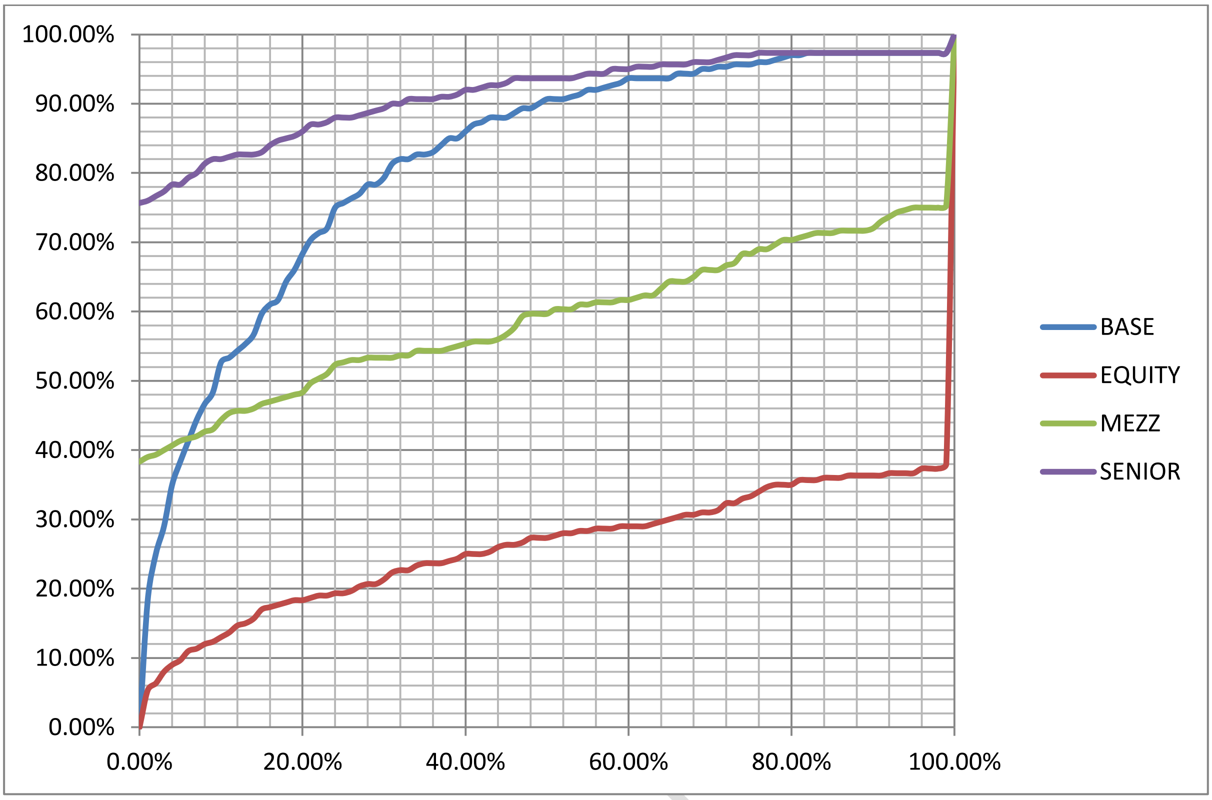
The two simulations given below provide an interesting insight into the effects of tranching. In the first simulation the starting point is a BB rated pool with $D_\mu = 18.66\%$ and $D_\sigma = 24.36\%$ and it is tranched into three buckets:

1. A 5% Equity bucket – any actual losses less than or equal to 5% will be absorbed by this bucket with none of the more senior buckets facing any losses.
2. A 25% Mezzanine Bucket – any actual losses exceeding 5% but less than 25% will be absorbed by this bucket. At this point the Equity Bucket will be empty since losses upto 5% already have been absorbed by that bucket.
3. A Senior Bucket – only losses above 25% would be absorbed by this bucket. At this point both the Equity Bucket and Mezzanine Bucket will be empty since the actual losses upto 25% would have been absorbed by those two buckets.

PDF of the Loss Rates - Various Tranches



CDF of the Loss Rates -Various Tranches

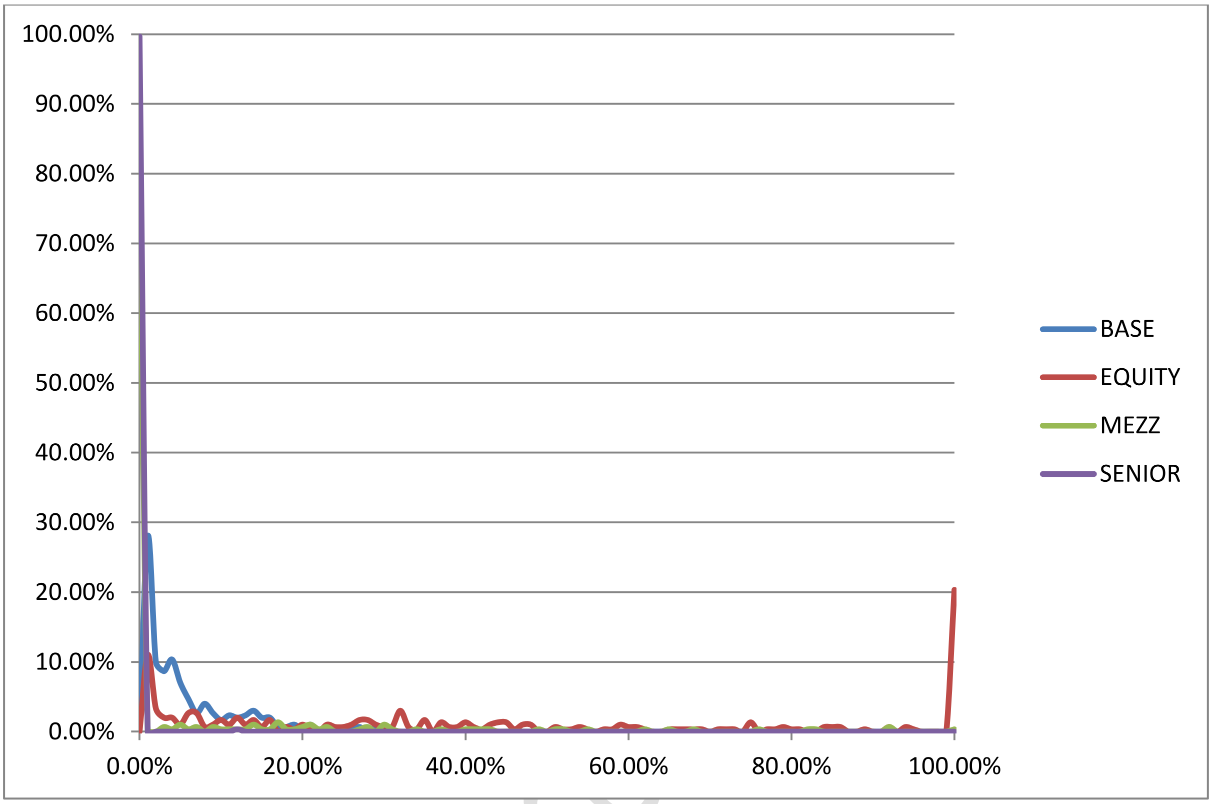


Summary Statistics	BASE (BB)	EQUITY (5%)	MEZZ (25%)	SENIOR
Mean	18.23%	75.84%	41.22%	8.26%
Standard Deviation	21.67%	37.67%	42.41%	19.46%
Skewness	1.73	-1.10	0.39	2.95
Kurtosis	2.93	-0.58	-1.60	9.16

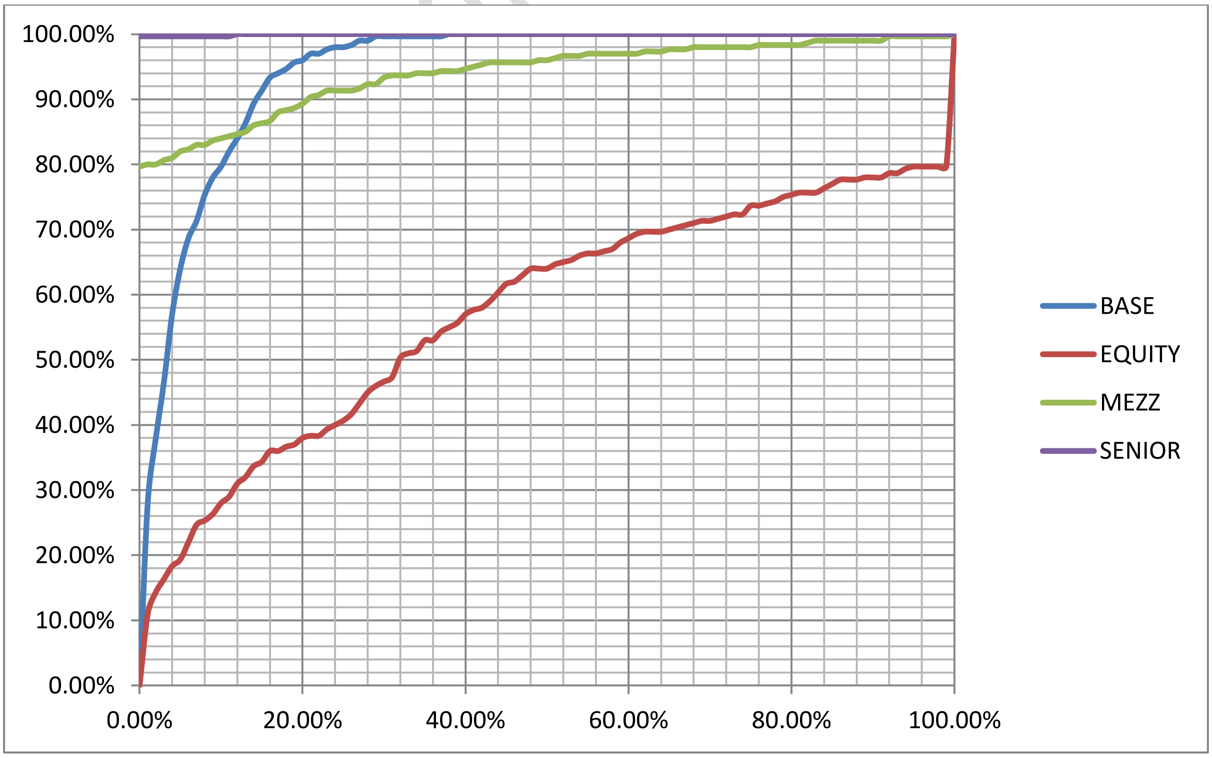
In the second simulation the starting point is a BBB rated pool with $D_\mu = 5.46\%$ and $D_\sigma = 6.75\%$ and it is tranched into three buckets:

1. A 10% Equity bucket – any actual losses less than or equal to 10% will be absorbed by this bucket with none of the more senior buckets facing any losses.
2. A 30% Mezzanine Bucket – any actual losses exceeding 10% but less than 30% will be absorbed by this bucket. At this point the Equity Bucket will be empty since losses upto 5% already have been absorbed by that bucket.
3. A Senior Bucket – only losses above 30% would be absorbed by this bucket. At this point both the Equity Bucket and Mezzanine Bucket will be empty since the actual losses upto 25% would have been absorbed by those two buckets.

PDF of the Loss Rates Various Tranches



CDF of the Loss Rates Various Tranches



Summary Statistics	BASE (BBB)	EQUITY (10%)	MEZZ (30%)	SENIOR
Mean	5.58%	40.65%	6.54%	0.30%
Standard Deviation	7.75%	36.36%	19.74%	3.26%
Skewness	3.158152	0.508858	3.542503	12.40339
Kurtosis	14.76776	-1.23527	12.31842	160.5442

It can be seen from the tables that the Senior Tranches have considerably lower levels of expected default than the original portfolio. The Senior Tranche of the BB portfolio now looks comparable to a BBB asset while the Senior Tranche of the BBB portfolio looks comparable to a AA rated asset. This is the fundamental appeal of the Securitisation process – it allows the construction of fundamentally lower-risk assets which have deeper markets are priced in such a way that the total cost of borrowing to the financial institution goes down considerably.

However, one can also see the unusual behaviour of the Mezzanine Tranche that will not be easily picked-up just by looking at the expected default rates. In the case of the BBB (original) pool, the Mezzanine Tranche has retained the expected loss rates of the original BBB pool and could be priced and sold as a BBB asset but suffers from what is referred to as the "cliff effect" which makes it very different from the original BBB rated asset. This is the sudden drop in value of the asset to 0 when the observed loss rates on the pool exceed the tranching cut-off. This effect is much more visible in the CDF for the BB pool and exists even for the BBB pool. This is partially captured in the higher level of standard deviation of the mezzanine tranche but more accurately captured by the Credit Value at Risk (CVAR) number for each asset:

$$\mu_{BBB,Base} = 5.46\%;\ CVAR_{BBB,Base,98\%} = 24\%$$

$$\mu_{BBB,30\%\,Tranche} = 6.54\%;\ CVAR_{BBB,30\%\,Tranche,98\%} = 68\%$$

2. Swaps

A. Basic Principles

Swaps are transactions in which one form of exposure is exchanged for another, often with a common underlying amount involved. so that the entire transaction may be carried out on an off-balance-sheet basis. Swaps have a deterministic character in the sense that unlike options there are no contingencies involved.

B. Interest Rate Swaps

If the risk management unit wishes to increase its duration of equity it needs to go long some long-dated exposures and would carry out the following trades:

1. Take a 5 year loan of a $100 million with a quarterly interest pay out at say, LIBOR+6%, where the LIBOR is set every three months (in arrears), i.e., PAY FLOATING.

2. Use the money thus generated to make a five year maturity fixed interest rate fixed interest rate loan for $100 million to a client with quarterly payouts at say a 12% interest rate per year or 3% per quarter, i.e., RECEIVE FIXED.

If these two on-balance sheet transactions are now combined into a single transaction (i.e., the lender to the bank and the borrower from the bank is the same institution) then:

1. Since the two principal amounts are the same there is no need to actually lend or borrow the $100 million, it merely becomes a notional amount for the computation of interest amounts.

2. And, since there is no exchange of principal the entire set of transactions are not recorded on the balance sheet of a bank but are treated as a single off-balance sheet transaction.

3. On the exchange of interest payments, each counterparty either pays that actually interest amounts involved, or they simply agree to pay / receive the net amount.

This, in its simplest form, is an interest rate swap. In order to value it, it is possible to assume that the LIBOR linked loan has near zero duration and therefore no matter what happens to interest rates will always be present valued at Par. However the fixed rate loan has a specific present value. The differences between the two present values is the price of the swap – usually charged as an extra spread over the interest rate in the fixed leg of the swap and the swap is therefore quoted as spread to some agreed fixed interest rate benchmark such as a 5 year Government Bond Yield.

C. Credit Default Swaps

A Credit Default Swap (CDS) extends the idea of a Swap to credit risk. Here the seller of the swap (or protection) agrees to compensate the buyer if a reference instrument (typically a bond) goes into default. In return the buyer pays the seller a fixed fee. On the face of it this appears similar to an options contract but is actually closer to a loan agreement that the seller of the CDS has entered into with the entity that is the issuer of the reference instrument and is reflective of the credit risk associated with the reference instrument. In fact just as the interest rate swap is priced off two independent balance sheet based loans (one fixed and one floating) the CDS is priced off a loan that is made to the issuer of the reference instrument. The fixed fee that the seller receives is directly comparable to the credit spread that a lender to the reference entity would have received.

This instrument is often used by banks to rebalance their books by:

1. Buying a CDS on a specific loan instead of attempting to sell it.

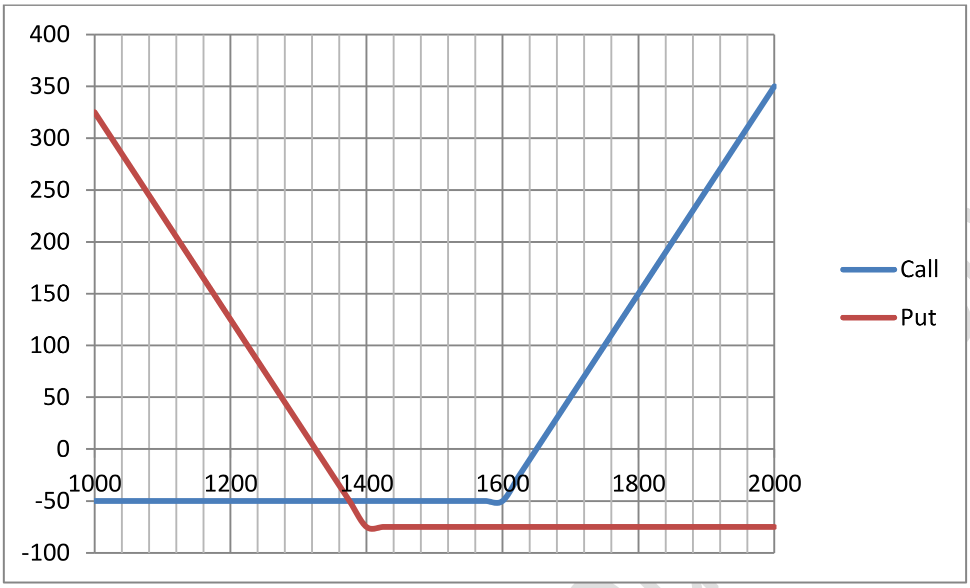
2. Buying a CDS on a reference instrument that is in some way correlated with the asset book of the bank so that the resultant portfolio has the characteristics that the bank wants.

3. Selling a CDS on a reference instrument instead of lending money to a particular client so that it is able to operate in much more liquid and an off-balance sheet instrument while continuing to stay with its core business of lending.

3. Options

A. Basic Principles

Options are an important hedging tool for a number of risks. They differ from other instruments in that they confer on the buyer a right to benefit from movements of variables of interest only in one direction and not have to worry about movements in the other direction. The most basic of these instruments are the Call Option and the Put Option. The Call Option confers a right on the buyer to buy when a trigger price is reached (referred to as the Strike) while the Put Option confers a right on the buyer to sell at a pre-agreed strike price. The pay-off profiles (profit and loss) for these generic options on the expiration date look as follows:



B. Options Pricing

The key idea behind pricing options is that, given a predefined and stable DGP for the evolution of, say, the stock price, there exists a ratio "δ" such that, in a one period model, the investment I_0:

$$I_0 = S\delta - C$$

is risk free, where "C" is the price paid for a Call option at particular strike price, and "S" is the stock price.

For example[50], if the stock price evolution process is such that, if the price today is: \$100, in the next period (say one month), the price can either be \$110 or \$90 (how likely a particular price is not significant because the argument is based on the logic that no matter what the prices the payoff will be the same). Assume we have a call option available for this stock.

If "C" is the price of the Call Option, then:

$$I_0 = S\delta - C = 100\delta - C$$

At the end of the period, if the call option is assumed to have a strike price also of \$100 and priced fairly, say at \$10, then, a person with such a portfolio will get the same payoff no matter what happens – if the stock price goes up to \$110, the person gets 110δ but has to pay off \$10 (the price of the call option); if the stock price goes down to \$90, then the option is worthless, but since the stock price has gone down to \$90, the payoff is now 90δ. Since the payoff must be equal in each state, if there is no arbitrage possible, and the option has been priced fairly then it must be the case that:

$$110\delta - 10 = 90\delta$$

$$\Rightarrow \ \delta = \frac{10}{110-90} = 0.50$$

Our payoff in both cases (up-state and down-state) is $45. If the stock price is $110 and we apply a δ of 0.50 (as calculated above), our stock position is worth $55, but we have to pay the call option price of $10 and our payoff is $45. On the other hand, if the stock price falls to $90, the value of our stock position would be $45 (and the value of our option position would be zero we obviously do not have to pay the $10 for exercising the call option). In order to find the current price of the call option, we take the present value of the payoffs.

and, it follows that:

$$\mathrm{I}_0 = S\delta - C = 0.5 * 100 - C = 50 - C = PV(45) = 45e^{-rt}$$

where, r is the annual risk free discount rate and t is the associated time period for the option. If the risk free rate is 6% per annum and the time period is one month

$$\Rightarrow C = 50 - 45e^{-rt} = 50 - 45e^{-0.06\left(\frac{1}{12}\right)} = \$5.22$$

If the option strike rate were to change to $85 instead of $100, then:

$$110\delta - 25 = \ 90\delta - 5 \Rightarrow \ \delta = \frac{20}{110-90} = 1$$

and,

$$C = 100 - 85e^{-rt} = 100 - 85e^{-0.06\left(\frac{1}{12}\right)} = \$15.42$$

Or alternately if the prices process were to change such that in the up-state the price goes to $120 instead of $110 but in the down state remains $90, then, for a call option with Strike 100,

$$120\delta - 20 = \ 90\delta \Rightarrow \ \delta = \frac{20}{120-90} = \frac{2}{3}$$

and,

$$C = 100\left(\frac{2}{3}\right) - 80e^{-rt} = 100\left(\frac{2}{3}\right) - 80e^{-0.06\left(\frac{1}{12}\right)} = -\$12.93$$

From all these examples it can be seen that:

1. The probabilities associated with each price level is not required.
2. The sizes of moves themselves are important though as is the strike price of the Option.
3. The only condition being imposed is no-arbitrage.

If the Stock Price is distributed normally with volatility σ (μ does not matter), then the up and down moves can be shown to be:

$$u = \ \sigma^{\sqrt{\Delta t}}$$
$$d = \sigma^{-\sqrt{\Delta t}}$$

and, the stock price at the end of any time interval Δt can be shown to be:

$$S_u = S_0\sigma^{\sqrt{\Delta t}}$$
$$S_d = S_0\sigma^{-\sqrt{\Delta t}}$$

And, once these prices are known, then, for any option with a particular strike price, a price may be computed easily.   It is also possible to show that[51] if stock price indeed varies continuously then, in the limit, the formula for a Call option price reduces to the famous Black-Scholes Option pricing formula:

$$C = S_0N(d_1) - Ke^{-rt}N(d_2)$$

where,

$$d_1 = \frac{\ln\left({S_0}/{K}\right) + \left(r + {\sigma^2}/{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left({S_0}/{K}\right) + \left(r - {\sigma^2}/{2}\right)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

The function N() is the cumulative probability distribution function for a standard normal distribution and the variable K is the strike price. From the formula it can also be seen that:

$$\delta = N(d_1)$$

C. Usage of Options

Options have become a very powerful instrument and are used to manage stock price risk, interest rate risk, credit risk, and exchange rate risk. And because they are unidirectional in nature they can be used quite effectively by manufacturing companies as well as banks and financial institutions. As can be seen from the above discussion, since the core idea behind pricing of options is the creation of replicating on-balance sheet portfolios, there is no fundamental reason why such positions cannot be crafted by risk managers using entirely balance sheet based strategies. However there is one essential difference between purchasing an option in the market and in attempting to replicate it on the balance sheet – the risk of sudden changes in the values of underlying variables (or the so called "jump" risk) is assumed by the seller of the option and taken away from the purchaser of the option. Attempts to replicate options positions however leave the risk of these jumps in the hands of the bank or the finance company. The value of the transfer of jump risk cannot be under-estimated but has to be weighed against the costs (or premium) to be paid for purchasing the option versus replicating it internally.

Case Studies

Simulated Finance Company Case Study
Achankovil Financial Services Private Limited ("Achankovil")
Pallathuruthy Bridge South, Alappuzha[52], Kerala 688 001

A Bad Start

When B. Arunkumar ("Kumar") sauntered into the office on April 1st, a full half an hour late, he could sense that there was trouble brewing in the office and that he was at the centre of it. He was never late but he had gone out with his colleagues last night to celebrate the end of a good year in which both he and his company had done well. He had gone to bed late and had gotten delayed getting ready to get to office. None of his colleagues were prepared to make eye contact with him and when he got to his desk he saw why. His boss Ms. Akkamma Cherian[53] had left a note on his table which read:

> "Kumar: where the devil are you? Come to my office the minute you see this note."

People had seen the note and while nobody seemed to know what the issue was, the tone of the note was clear.

When he went into see Ms. Cherian she seemed pretty upset. She had received a call from the head of the AccuRate Rating Agency ("Accurate") first thing in the morning, giving her an informal feedback on the rating application that Achankovil had submitted to them based on provisional figures for the previous year. Based on a review of the figures, Accurate had informed Ms. Cherian that not only was Achankovil not going to get the "A" rating that they had hoped for but that the best that they could expect to get was a BB+ rating, not even a BBB.

She had been informed that to get a "A" rating for the bonds to be issued by Achankvovil, they needed to convince Accurate that the one year default expectations would be no more than 1%, i.e., that Achankovil would not default on its obligations more than once in 100 years. Accurate had determined that Achankovil was nowhere near that expected default rate and instead had numbers closer to 20%, i.e., it was likely to experience failure over the next 5 years.

This was indeed devastating news and even Kumar was pretty upset when he came out of the office. He could not figure out what had gone so wrong.

Assets and Liabilities

Kumar was a simple uncomplicated man and he liked to keep things simple and clear. Achankovil had closed the year with a Rs.100 crore balance sheet, with the following profile

1. Kumar had borrowed long-term money at 16% rate with a bullet maturity but quarterly interest payments. He had paid a little bit extra for it but he knew how hard it was to persuade lenders to give money to an unrated financial services company, that too with a rural focus, so when an offer of Rs.90 crore had come along, he had jumped at it. At the end of March 31st all his loans had a balance maturity of 5 years.

2. Achankovil had a capital base of Rs.10 crore and while he was not sure why he felt that way, in his view this was a good amount of capital and his investors expected to give them an annual return of 20% after all taxes – this was thus his most expensive source of money and he tried to rely on it to the minimum.

3. He had asked his branches to lend all this money out for a five to six year term with quarterly amortization schedules because Kumar knew liquidity was important and wanted to get money back from his clients as soon as possible so that he would not have to struggle for it when the time came to repay his lenders. The money he received he kept as a deposit with another much larger highly rated finance company who agreed to pay him the market rate for it, which at the moment was 16%, the same rate as the rate he borrowed money so he figured he was in good shape there.

4. At the end of March 31st all his loans, it turned out, had a balance maturity of exactly five years. They had been made to rural retailers who served the local backwaters tourist trade. This was a business Achankovil staff knew well so he, from his perch at the head-office, encouraged them to stick to only these borrowers. It is true there was steady attrition amongst these retailers as the tourist traded waxed and waned from year to year but he figured that on average only about 5 to 6% of the retailers went bust every year and could not pay anything back. The collateral that Achankovil held was largely inventories and by the time the staff got to it almost 70% of the inventory had disappeared.

5. To take care of all of these issues Kumar had required all his staff to charge a spread of 8% over the cost of funds and over and above any additional staff and other costs that they incurred – producing a lending rate of as high as 28%. This was high by urban market standards but Kumar felt that with an "A" rating at hand he could lower his cost of funds quite dramatically and, over time, would also have benefits of scale so that he could further reduce his operational costs.

6. He and Ms. Cherian both felt very strongly that serving clients like these, whom the larger banks were not prepared to touch, was the very purpose of the existence of Achankovil and that there was no question of starting to serve clients who had a lower risk of failure and moving away from the current remote rural focus.

What Had Gone Wrong?

While this whole area of ratings was very new to him, he knew he was the inheritor of the great traditions of the famous Kerala School of Mathematics which had produced giants like Madhava of Sangamagrama (whose work on "Taylor" series expansions for example, pre-dated the work of Brook Taylor by over 200 years), and he set about understanding these issues in a very systematic manner. The questions he needed to answer were very clear:

1. Why did the Accurate think that Achankovil was worth only a BB+ rating?

2. Was Achankovil's lending strategy or his own borrowing strategy somehow responsible?

3. What could he do about it so that the all-important "A" rating was possible?

Simulated Commercial Bank Case Study

CP and Berar Bank
CPB Bank Towers
Sitabuldi, Nagpur 440012

Mudhoji Bhonsale was a career banker and when he was appointed as the Managing Director and Chief Executive Officer (MD & CEO) of CP & Berar Bank (CPB) at the age of 55 nobody was surprised. Mudhoji was also happy to return to the city of his birth in such a prominent position. He came from a distinguished Nagpur family and it was rumoured that he was a direct descendent of the last Maratha King of Nagpur[54]. He had studied Engineering at the Visvesvaraya Regional College of Engineering (VREC), and joined one of the larger national banks as a probationary officer immediately after his graduation and gradually, over time, he had advanced in his career, had uniformly received good ratings, and was generally known to be a strong banker. Banking had changed from the days when Mudhoji had started his career. When he was a young banker the good name of the client and his social standing was all that really mattered and he had prided himself that he was a good judge of this.

These days however, things had become a lot more uncertain, and before his own eyes he had seen several prominent families of his own city driven to bankruptcy unable to service their debts. Interest rates, which for a long time had remained unchanged, now varied continuously and issues such as interest rate risk had to be taken into account as well. Fortunately, CPB, because of its impeccable pedigree dating right back to the very formation of the Central Provinces in 1861[55], had a strong reputation amongst generations of its depositors and had a vast branch network in every nook and corner of Madhya Pradesh, Vidarbha region of Maharashtra, Chattisgarh, Northern Karnataka, and Telangana. But Mudhoji did not feel as if he was as well prepared for his new responsibilities as he would like to be.

On his second day after taking charge when he arrived into office, his secretary told him that his Executive Director Nirmala Deshpande[56] had sought an urgent meeting. From the tone of the secretary's voice he knew that the news was not likely to be good. From what he had heard about Nirmala and from his first interactions she had struck him as a calm person with a strong character. Unlike himself and most of his other colleagues at CPB, she had had a much wider exposure to modern banking from her stints as a General Manager at a larger Mumbai-based Bank, and the years she had spent as the head of the New York branch of that Bank. He knew he was lucky to have somebody like her as a part of his team. At CPB as its Chief Financial Officer (CFO), she was in-charge of profitability and risk-management of the Bank and he was sure that this urgent meeting had something to do with these aspects of the Bank.

He asked his secretary to clear his calendar for the morning and politely request the number of people who had asked to see him to welcome him to his new position, to come later in the day and to apologise to them for the inconvenience. He wanted to have enough time to get to grips with whatever issues that she wanted to discuss with him. Nirmala arrived as soon as his office gave her the word and in a no-nonsense manner got down to business with the minimal of pleasantries. She informed him that she had received a call from the newly created Stress Testing Advisory Unit of Accurate (a prominent Mumbai based rating agency) that at 10% Capital Adequacy (all Tier 1), while the bank had the requisite amount of capital to meet the Basel III guidelines that were to become applicable shortly, it would fail the Stress Test Capital requirements specified in the draft guidelines that had been circulated last year[57]. While the Bank had a rating of "AAA" because of the expectation of support in case it failed, its current stand-alone rating was "BBB" and the stress tests were being carried out with an aspirational rating of "AA". The bank's stock was trading at a price to book ratio of 0.50, with several analysts having issued a sell recommendation even at this price.

The largest shareholders of the bank had also privately indicated to the Bank that, given all their existing commitments, they were not in a positon to provide any additional capital nor were they willing to dilute their shareholding below current levels, which meant that they were not likely to approve any additional capital raising plans. CPB had a Rs. 300,000 crore balance sheet with a CASA of Rs. 100,000 crore on which the Bank paid an average interest rate of 4%. Nirmala had also brought a summary sheet of numbers for him to mull over during the day. She wanted to resume this meeting the next morning and to arrive at a broad operating strategy on how to deal with the set of issues that CPB faced.

The sheets that Nirmala had left with him gave the following information:

1. The Bank had an estimated duration of assets equal to 1.40 years (on account of the Base Rate-linked pricing for the entire loan portfolio) while the duration of liabilities was 4.26 years.

2. Of the FDs that CPB has, Rs.50,000 crore had come due last year and had been renewed for one more year with an interest rate that had gone down from 9% to 7%. The rest of the FDs were of a 5 year maturity and had an interest rate unchanged at 10% -- since CPB was not planning to grow its portfolio this year, no attempt had been made to add to the FD balances and the one-year FDs had renewed almost instantaneously at the new rate.

3. All of the loans of the Bank were at 11% last year, with the exception of the capital base which was already invested entirely in 5 year government securities.

4. There was also a large table (Annexure 1) which gave the credit exposure of the Bank to various sectors as well as indication of how default rates of each sector were related to each other.

This was a lot of information. And, as Mudhoji sat in cabin which was full of beautiful bouquets of flowers, long after all the well-wishers had gone, mulling over this he realised he need to connect all the information to the following questions that he needed to answer:

1. Why had CPB failed the stress tests?

2. What corrective action would he need to take since he was not going to be able to raise new capital at least in the immediate future?

3. How could he make a convincing argument for a movement towards Matched-Fund-Transfer-Pricing (MFTP) and away from the current Base Rate system?

Annexure 1

CPB Credit Portfolio Distribution

	Portfolio Proportions	Portfolio Rating	Mean Default	Default Volatility
CNX Auto	30.00%	A	1.01%	1.34%
CNX Energy	20.00%	BB	18.66%	24.36%
CNX Finance	0.00%	AA	0.12%	0.18%
CNX FMCG	0.00%	AA	0.12%	0.18%
CNX Infra	20.00%	BB	18.66%	24.36%
CNX IT	0.00%	AA	0.12%	0.18%
CNX Media	0.00%	A	1.01%	1.34%
CNX Metal	20.00%	BBB	5.46%	6.75%
CNX PSU Bank	0.00%	AA	0.12%	0.18%
CNX Realty	10.00%	BB	18.66%	24.36%
CNX MNC	0.00%	AA	0.12%	0.18%
CNX Pharma	0.00%	A	1.01%	1.34%

Sector Default Rate Correlation Matrix[58]

1.000	0.704	0.793	0.520	0.753	0.202	0.659	0.645	0.681	0.708	0.743	0.502
0.704	1.000	0.796	0.381	0.836	0.170	0.714	0.735	0.756	0.829	0.722	0.216
0.793	0.796	1.000	0.524	0.874	0.084	0.761	0.764	0.872	0.836	0.760	0.298
0.520	0.381	0.524	1.000	0.443	0.033	0.461	0.232	0.315	0.361	0.575	0.420
0.753	0.836	0.874	0.443	1.000	0.097	0.821	0.795	0.850	0.876	0.786	0.268
0.202	0.170	0.084	0.033	0.097	1.000	0.204	0.179	-0.043	0.135	0.077	0.311
0.659	0.714	0.761	0.461	0.821	0.204	1.000	0.642	0.726	0.806	0.688	0.337
0.645	0.735	0.764	0.232	0.795	0.179	0.642	1.000	0.734	0.777	0.748	0.162
0.681	0.756	0.872	0.315	0.850	-0.043	0.726	0.734	1.000	0.827	0.639	0.092
0.708	0.829	0.836	0.361	0.876	0.135	0.806	0.777	0.827	1.000	0.723	0.136
0.743	0.722	0.760	0.575	0.786	0.077	0.688	0.748	0.639	0.723	1.000	0.452
0.502	0.216	0.298	0.420	0.268	0.311	0.337	0.162	0.092	0.136	0.452	1.000

Exporter Hedging Case Study
Knowledge Matters KPO Services Limited
Purchasing Insurance against Foreign Exchange Losses

Background

Knowledge Matters KPO Services Limited (KMKS) is a well-established Knowledge Process Outsourcing (KPO) company with clients situated around the globe. It enters into contract negations with its clients starting in the month of January and reaches agreement with them on the rate per FTE (Full Time Equivalent Employee) in U.S. Dollars. The company start to bill at new rates on the first of every month for the work completed during the previous month. The client pays two months after the receipt of the bill.

It is February 1, 2010 and Mr. B. Arunkumar the Chief Financial Officer of KMKS has just completed his negotiations with his overseas clients for $5 million a month for the twelve month period starting April 1, 2010. At the exchange rate prevailing today (US$ 1 = Rs. 46.3750) he expects to make a profit margin of 30.00%. Table 1 (below) gives the calculations that Mr. B. Arunkumar made while finalising the contract.

Table 1
Contract Negotiations (2010-2011)

	Monthly	**Annual**
	Million	Million
Business in US$	$5	$60
Business in Rupees	Rs. 231.875	Rs. 2782.50
Costs in Rupees	Rs. 162.3125	Rs. 1947.75
Profits in Rupees	Rs. 69.5625	Rs. 834.75
Profit Margin	30.00	30.00%

Mr. Arunkumar had every reason to be satisfied with this outcome. In a very competitive market scenario he had been able to preserve the profit margins of the company and if all went as planned Mr. Arunkumar expected KMKS to show a very nice profit of Rs. 834.75 million at the end of the year from this contract.

Currency Concerns

While Mr. Arunkumar was indeed quite happy with the outcome of the negotiation, he had a serious concern the movement in foreign currency prices could have an adverse impact on these profit projections. In the previous year around the same time Mr. Arunkumar had achieved an identical outcome from the negotiations (the exchange rate prevailing on February 1, 2009 was: US$ 1 = Rs. 48.8750) and had predicted that KMKS profits from this business would be Rs. 879.7500 million. Not only were the actual profits that he was going to report on March 31, 2010 much lower, he had the acute embarrassment to facing the Board and KMKS shareholder with an accounting loss on account of exchange rate movements.

Table 2
Contract Negotiations (2009-2010)

	Monthly	**Annual**
	million	Million
Business in US$	$ 5.0000	$60.0000
Business in Rupees	Rs. 244.3750	Rs. 2932.5000
Costs in Rupees	Rs. 171.0625	Rs. 2052.7500
Profits in Rupees	Rs. 73.3125	Rs. 879.7500
Profit Margin (%)	30.00%	30.00%

Table 3
Actual Profits (2009-2010)

	Billed Amount	Exchange Rate	Received Amount	Accounting Loss	Unbilled Loss	Hedge All Receipts	Insurance @46.7624
Date	$ Million	US$ 1	Rs. Million	Rs. Million	Rs. Million	Rs. Million	Rs. Million
01-02-2009		**48.8750**					
01-03-2009	5	51.1612					
01-04-2009	5	50.7300					
01-05-2009	5	50.0925	250.4625	-5.3435	11.4310	1.2000	11.4310
01-06-2009	5	46.9475	234.7375	-18.9125	9.2750	1.2000	9.2750
01-07-2009	5	47.8925	239.4625	-11.0000	6.0875	1.2000	6.0875
01-08-2009	5	47.9350	239.6750	4.9375	-9.6375	1.2000	-9.6375
01-09-2009	5	49.0250	245.1250	5.6625	-4.9125	1.2000	-4.9125
01-10-2009	5	47.7550	238.7750	-0.9000	-4.7000	1.2000	-4.7000
01-11-2009	5	46.9750	234.8750	-10.2500	0.7500	1.2000	0.7500
01-12-2009	5	46.3175	231.5875	-7.1875	-5.6000	1.2000	-5.6000
01-01-2010	5	46.6200	233.1000	-1.7750	-9.5000	1.2000	-9.5000
01-02-2010	5	46.3750	231.8750	0.2875	-12.7875	1.2000	-10.5628
01-03-2010		46.0850	230.4250	-2.6750	-11.2750	1.2000	-10.5628
01-04-2010		44.9175	224.5875	-7.2875	-12.5000	1.2000	-10.5628
Total			2834.6875	-54.4435	-43.3690	14.4000	-38.4949

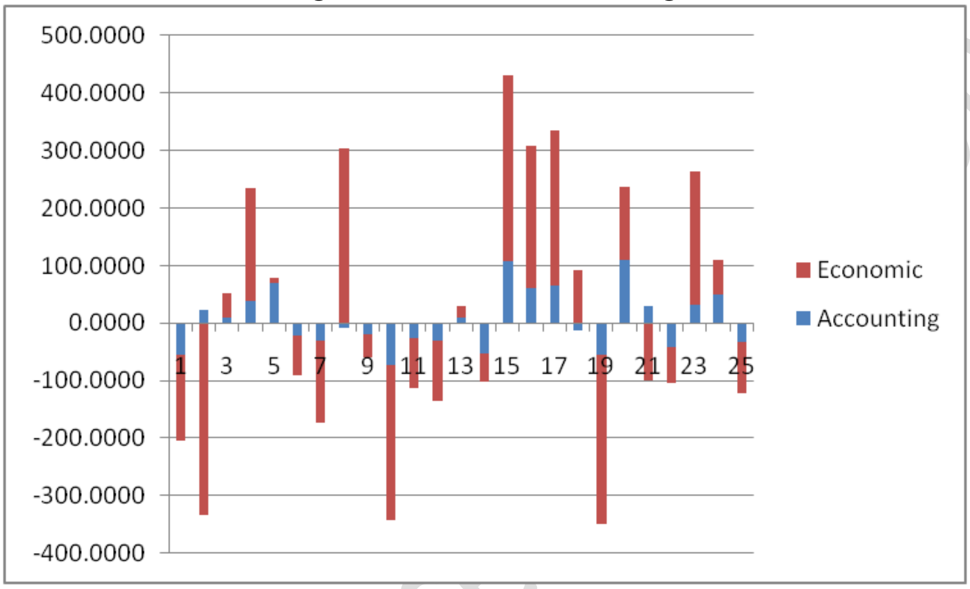
From Table 3 it can be seen that against expected revenue of Rs. 2932.50 KMKS received an income of only Rs. 2834.6875, i.e., a reduction in profit of Rs. 97.8125 million which could further be broken up into an accounting loss of Rs. 54.4435 million (the difference between the exchange rate at which the work was billed and the rate at which it was received) and an additional economic loss Rs. 43.3690 million (the difference between the exchange rate at which the work was contracted for and the rate at which the work was billed).

Mr. Arunkumar also knew something his Board and his shareholders did not – he had gotten extremely lucky that the day he signed the contract the exchange rate was US$ 1 = Rs. 48.8750 and fell shortly thereafter to Rs. 51.1612 giving him a substantial gain in the initial period and reducing his losses when it fell again almost to Rs. 44.9175. He could not bear to think about what would have happened if the negotiations had stretched a little bit more and the contract had been signed when the exchange rate was US$ 51.1612.

An Uncertain Future

An analysis of the last four years of currency movements suggests that the monthly volatility of the exchange rate is about 2.5%. Assuming that on average the currency does not appreciate or depreciate over the next one year period, it is possible to simulate the possible movement of the exchange rate over the next twelve month period and to compute the amount of accounting loss and economic loss / gain implied by these simulated currency movements. Figure 1 gives the result of one such simulation in which 25 possible currency paths are simulated and the net consequence in terms of account loss and economic loss is computed.

Figure 1
Unhedged Economic and Accounting Loss



It can be seen that relatively high level of volatility assumed for the currency movement translates into a very high volatility in both the economic loss numbers as well as accounting loss numbers for the firm. The average accounting loss based on this simulation is Rs. 5.7625 million (standard deviation of Rs. 51.3191) and the average economic loss based on this simulation is Rs. 5.3220 million (standard deviation of Rs. 184.1285), generating a total (average) loss of Rs. 11.0846 and a very high standard deviation of Rs. 221.5258 million. The company could earn an additional profit of Rs. 430.1354 million but if things did not go well the profits of KMKS would fall by as much as Rs. 350.3023 million, which would be a disaster for KMKS.

Hedging Strategy

From the above analysis it appears that KMKS needs to observe the following rules while managing its foreign exchange exposures:

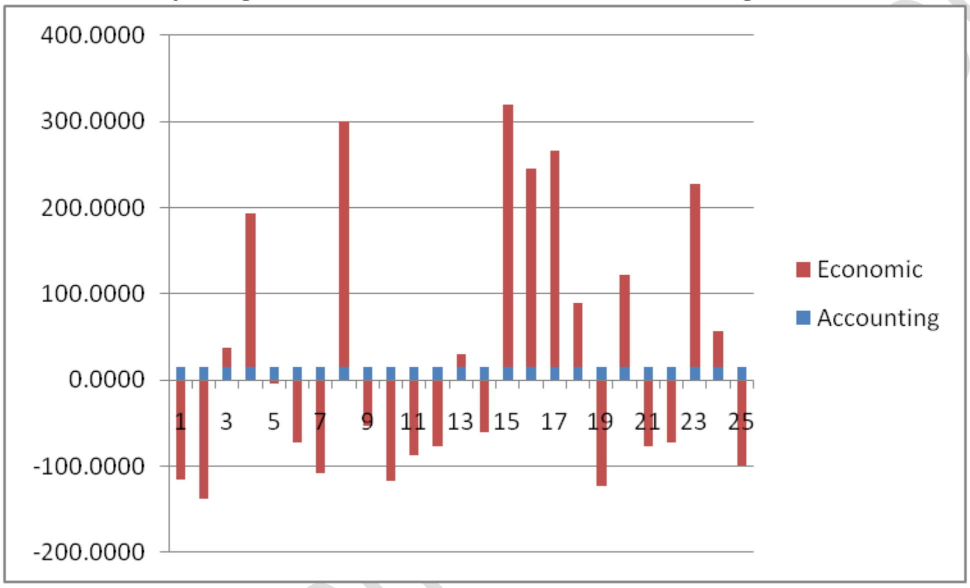
1. Sell all the US$ on the date of billing itself using a forward contract route and deliver the currency upon its receipt from the international client. This hedging strategy would completely eliminate the accounting loss and replace it with a modest gain. It would also eliminate the possibility of any significant accounting gain but since the company is continuously billing the client this would not amount to a loss of economic value should there a secular depreciation in the value of the rupee.

2. Purchase currency insurance on the entire contracted amount using a series of deep-out of the money put (catastrophic insurance) which would reduce the cost of the insurance but allow KMKS to preserve a minimal profit margin even if the rupee should appreciate dramatically over the next twelve months.

It may be seen from Table 3 that if the company had hedged[1] all the billed amounts on the date of billing itself and bought insurance[2] on the entire $60 million on the date the contract was signed (February 1, 2009), the net amount received would have been higher by Rs. 55.7176 after deducting the cost of purchasing the currency insurance and there would have been no accounting loss that would need to be reported.

Similarly if the company does decide to follow this policy going forward, using the simulated values mentioned earlier, the hedged and insured incremental numbers are captured in Figure 2.

Figure 2
Fully Hedged and Insured Economic and Accounting Losses



From Figure 2 it can be seen that on account of the twin hedging and insurance strategy, while the maximum gains that are possible have been reduced somewhat from Rs. 430.1354 million to Rs. 319.4805 million, the low has sharply fallen to Rs. 123.8701 million.

The hedged and insured average accounting gains based on this simulation are Rs. 14.4 million (standard deviation of nil) and the average economic loss based on this simulation is Rs. 20.7858 million (standard deviation of Rs. 143.9572 million), generating a total (average) gain of Rs. 35.1858 million and a lower standard deviation of Rs. 143.9572 million relative to the un-hedged position. Here the put option purchased is once again at a much lower exchange rate of Rs. 44.3705 (spot is 46.3750) and at a price of Rs. 18 million for the full year (the price of the option has already been deducted from gains / losses shown in Figure 2.

[1]By booking a forward contract which has the effect of selling the billed US$ amount on the date of billing itself by borrowing the US$ needed and then repaying the US$ loan when the billed amount is actually received. In Table 3 it is assumed that 24 points is the gain on account of hedging because of the fact that rupee interest rates are higher than US$ interest rates.

[2] Through the purchase of a Put Option on the US$ at US$ 1 = Rs. 46.7624 at a price of Rs. 0.30 million per US$ million, i.e., a total payment of Rs. 18.00 million.

Endnotes

[1] This section is principally based on: Sheffield, Scott. *18.440 Probability and Random Variables, Spring 2011*. (MIT OpenCourseWare: Massachusetts Institute of Technology),http://ocw.mit.edu/courses/mathematics/18-440-probability-and-random-variables-spring-2011 (Accessed 25 May, 2014). License: Creative Commons BY-NC-SA

[2] Drawn from Lecture 3 (page 4) and Lecture 4 (pages 4,5).

[3] In economics and finance there are a number of unobserved variables that are known (or hypothesized) to exist but cannot be directly observed. An example of this could be the volatility of the interest rate – while the actual level of the interest rate can be directly observed (or computed) from market transactions the volatility of the interest rate itself cannot be. However this does not mean volatility of the interest rate is not important and therefore the "true but unobserved" volatility will feature in a number of equations and discussions about the evolution of interest rates. In fact it is possible to argue that even the level of the interest rate prevailing at a particular point of time is unobservable and the number derived from a particular transaction is only an estimate. It is interesting that even in the physical world where it would be expected that very precise measurements can be made, Werner Heisenberg in his 1927 paper titled: "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik" or "The Actual Content of Quantum Theoretical Kinematics and Mechanics" (see this link for the original manuscript of this defining document: http://scarc.library.oregonstate.edu/coll/pauling/bond/papers/corr155.1.html) formulated the famous Heisenberg Uncertainty Principle ($\Delta X \Delta P \geq h/2\pi$, where h is the Planck's Constant = 6.634×10^{-34}, itself one of the key fundamental constants of the known universe) which says that that it is not possible to measure the position (x) and the momentum (p) of a particle with absolute precision because the process of observation requires the object to be bombarded with photons which themselves moves the object (see this article in the Guardian of November 10, 2013 for an easily accessible explanation of this fascinating insight: http://www.theguardian.com/science/2013/nov/10/what-is-heisenbergs-uncertainty-principle which Einstein himself took some time to be persuaded of: http://www.aip.org/history/heisenberg/p07c.htm).

[4] Drawn from Lecture 8 (page 4).

[5] The phrase "State Space" comes from Systems Theory and is used in the context of Dynamical Systems. Any Dynamical System is a System whose State evolves over time. The set of variables that are used to describe the State of a System are known as State Variables. The set of all possible values of the State Variables is the State Space (drawn from: Nykamp DQ, "The idea of a dynamical system." From Math Insight. http://mathinsight.org/dynamical_system_idea).

[6] The most well-known equation that seek to model the behaviour of the short-term interest rate is the one specified by Cox, Ingersoll, and Ross in their 1985 paper: Cox, John C., Jonathan E. Ingersoll Jr, and Stephen A. Ross. "A theory of the term structure of interest rates." Econometrica: Journal of the Econometric Society (1985): 385-407. Here the Random Variable of interest is the change in the instantaneous interest rate (equation 17 on page 391):$dr = \kappa(\theta - r)dt + \sigma\sqrt{r}dz_1$. For $\kappa, \theta > 0$, this is a continuous time process where the randomly moving interest rate is elastically pulled toward a central location or long-term value, θ. The parameter κ determines the speed of adjustment. See the original paper for a more detailed discussion: http://efinance.org.cn/cn/FEshuo/A%20Theory%20of%20the%20Term%20Structure%20of%20Interest%20Rates1985.pdf

[7] Drawn from Lecture 8 (pages 9, 10)

[8] Drawn from FELIH Page 28

[9] This section is based on: Fanelli, Luca (2014), "The concept of Data Generating Process (DGP) and its relationships with the analysis of specification", pages 1 to 9, University of Bologna, http://www.rimini.unibo.it/fanelli/econometric_models2_2014.pdf (Accessed 1 June, 2014).

[10] This is arguably the most important distribution in economics and will be discussed in much greater detail later. However, this is but one of many important well-known distributions. This link gives the gallery of common distributions: http://www.itl.nist.gov/div898/handbook/eda/section3/eda366.htm and this one gives the
binomial calculator: http://stattrek.com/online-calculator/binomial.aspx.

[11] Taken from: http://www.understandingcalculus.com/chapters/09/9-1.php

[12] Taken from http://en.wikipedia.org/wiki/Moment_(mathematics) (accessed June 1, 2014).

[13] The origin of why this word came to be used in this manner is unclear. It is very likely that it is related to the word "moment" as it is used in Physics.
[14] http://en.wiktionary.org/wiki/kurtosis
[15] Definitions drawn from: http://www.youtube.com/watch?v=kXIdNfvOWak
[16] This portion is drawn from: Irwin, Mark (2006), "Moment Generating Functions", Pages 6,7, Statistics 110, Harvard University, 2006 (accessed June 1, 2014).
[17] The insight here is that curve maybe approximated around a point by its level at that point and a series of closely fitting straight lines such as its slope, the slope of its slope, and so on. Also see Appendix G of Hull.
[18] See this:
http://web.hep.uiuc.edu/home/serrede/P435/Lecture_Notes/Derivation_of_Taylor_Series_Expansion.pdf
[19] Interestingly while the Taylor expansion is named after Brook Taylor who provided the most general method for arriving at these approximations, it was learnt in 1830 that the Kerala School of Mathematics had come upon these ideas many centuries earlier. See: Whish, C. M. (1834). XXXIII. On the Hindú Quadrature of the Circle, and the infinite Series of the proportion of the circumference to the diameter exhibited in the four S'ástras, the Tantra Sangraham, Yucti Bháshá, Carana Padhati, and Sadratnamála. *Transactions of the Royal Asiatic Society of Great Britain and Ireland*, *3*(03), 509-523
[20] This discussion is drawn from: http://www.youtube.com/watch?v=uEpXVvl7YWY.
[21] Drawn from http://mathworld.wolfram.com/topics/ContinuousDistributions.html
[22] Drawn from: http://www.statlect.com/subon2/momgen1.htm
[23] Drawn from Lecture 27 (pages 6,7,8)
[24] Drawn from Lecture 31 (pages 10,15)
[25] Drawn from the class notes of Professor Susan Holmes (http://statweb.stanford.edu/~susan/) of Stanford University (seen on June 6, 2014): http://statweb.stanford.edu/~susan/courses/s116/node120.html
[26] See this for a more detailed explanation of the rule: http://en.wikipedia.org/wiki/L'H%C3%B4pital's_rule
[27] Also see Hull Section 10.7, Pages 218-220.
[28] From Hull, Section 9.1, Page 183
[29] Entire paragraph has been taken from Hull Section 9.4, page 188.
[30] Drawn from Lecture 25, Pages 4 to 8.
[31] This example has been taken from Hull Section 1.1, Page 4. Also see FELIH Chapter 4 for another example – pages 52 to 55.
[32] The Capital Asset Pricing Model (CAPM) posits a linear relationship between a security's systematic risk exposure and its expected rate of return. The formula is as follows $r_a = r_f + \beta(r_m - r_f)$, where r_a is the return on the security, r_f is the risk-free rate of return (or the return gained from investing in the safest, least-risk or zero-risk security such as T-bills), β is the risk attached to the security, and r_m is the expected return of the market
[33] From John Hull Chapter 8
[34] Drawn from Gibson, R., Lhabitant, F. S., & Talay, D. (2010). Modeling the term structure of interest rates: A review of the literature. *Foundations and Trends in Finance*,*5*(1-2).
[35] Drawn from Cox, J. C., Ingersoll Jr, J. E., & Ross, S. A. (1985). A theory of the term structure of interest rates. *Econometrica: Journal of the Econometric Society*, 385-407.
[36] See Cox, J. C., Ingersoll Jr, J. E., & Ross, S. A. (1985). A theory of the term structure of interest rates. *Econometrica: Journal of the Econometric Society*, Page 392:
[37] See: Lepage, T., Lawi, S., Tupper, P., & Bryant, D. (2006). Continuous and tractable models for the variation of evolutionary rates. *Mathematical biosciences*, *199*(2),Page 224.
http://www.maths.otago.ac.nz/~dbryant/Papers/05Rates.pdf
[38] Based on the actual shape of the Yield Curve on February 28, 2011
[39] See: Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk*. *The journal of finance*, *19*(3), 425-442.
[40] See: http://www.nseindia.com/products/content/equities/indices/cnx_nifty.htm
[41] See: https://www.math.ust.hk/~maykwok/Web_ppt/KMV/KMV.pdf
[42] The Risk Management Institute at the National University of Singapore publishes real-time probabilities of defaults for listed companies using a variant of the KMV model (http://www.rmi.nus.edu.sg/).
[43] Discussion drawn from: Chakrabarti, Sanmoy; Sarfaraz Ahmed; and Sandipan Mullick (2002), "An Approach to risk-based pricing of loans', January 2002, ICICIResearchCentre.org.

[44] For two such studies see: CRISIL Annual Default and Ratings Transition Study 2013: http://www.crisil.com/pdf/ratings/crisil-rating-default-study-2013.pdf; and Historical Default Rates of Corporate Bond Issuers, 1920-1999 by Moody's Investors Services, January 2000: http://cours2.fsa.ulaval.ca/cours/gsf-60808/Moodys_Historical_Default_Rates.pdf

[45] From Table 16.1 in Hull. Page 350.
[46] From Hull, Page 491-492.
[47] See example 23.1 on page 492 of Hull.
[4848] Drawn from Hull Chapter 12
[49] Refer Deutsche Bank Annual Report 2013, Pages 58 to 60
[50] Drawn from a teaching note prepared by Professor Robert Conroy for the University of Virginia Darden School of Business in 2003.
[51] Rendleman, R. J., & Bartter, B. J. (1979). Two-State Option Pricing. *The Journal of Finance*, *34*(5), 1093-1110.
[52] Alappuzha is one of the most important tourist centres in the State of Kerala, with a large network of inland canals and is often referred to as the "Venice of the East". The name Alappuzha is derived from 'Aal (Sea) + Puzhai (River-mouth). The correct pronunciation of this name can be heard here: http://upload.wikimedia.org/wikipedia/commons/2/26/Alappuzha.ogg
[53] Her parents had named her after the famous Kerala freedom fighter who Gandhiji referred to as the Jhansi Rani of Travancore.
[54] A historic battle for the control of Nagpur was fought at Sitabuldi between the Martha forces led by Mudhoji II Bhonsale and the British army led by Lieutenant Colonel Scott on November 26, 1817. The battle lasted all of one morning and by afternoon the Martha troops had been routed and the control of Sitabuldi Fort passed into British hands: http://en.wikipedia.org/wiki/Sitabuldi_Fort
[55] http://en.wikipedia.org/wiki/Central_Provinces_and_Berar
[56] While Nirmala was in no way related to the noted Gandhian and Padma Bhushan awardee who was her namesake, Nirmala held the same convictions towards upright behaviour and honest dealings and had a picture of Nirmala Deshpande on her wall to act as a reminder: http://en.wikipedia.org/wiki/Nirmala_Deshpande
[57] Stress tests amounted to calculating the economic capital requirements of a bank based on an unsupported credit rating.
[58] This correlation matrix has been computed using monthly log returns on various stock indices to provide a proxy for how defaults rates of assets in each sector are likely to be correlated to each other.