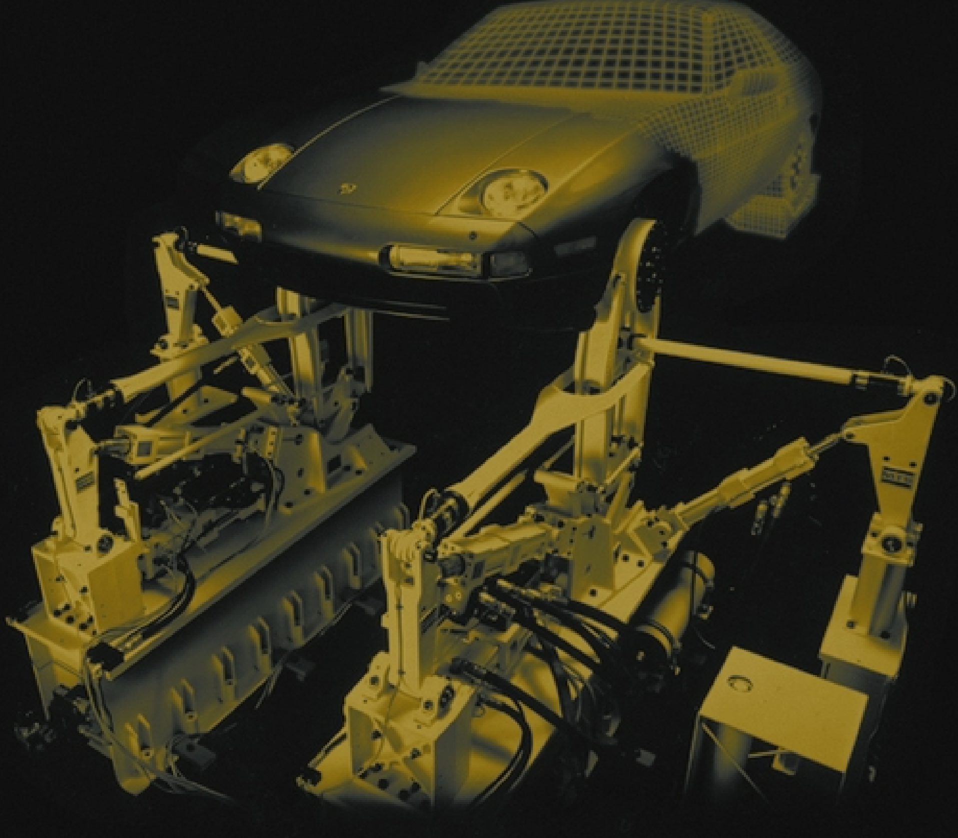
Second Edition



FUNDAMENTALS OF STRUCTURAL DYNAMICS

Roy R. Craig Jr. • Andrew J. Kurdila

Fundamentals of Structural Dynamics

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Second Edition

Roy R. Craig, Jr.
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The first author dedicates his work on this edition to his grandchildren: *Talia*, *Kyle*, and *Hart Barron*, and *Alex*, *Brandon*, and *Chase Lemens*. The second author dedicates his work to his wife, *Jeannie*, and to his children: *Patrick*, *Hannah*, and *Justin*.

Contents

Preface to Structural Dynamics—An Introduction to Computer Methods*

The topic of structural dynamics has undergone profound changes over the past two decades. The reason is the availability of digital computers to carry out numerical aspects of structural dynamics problem solving. Recently, the extensive use of the fast Fourier transform has brought about even more extensive changes in structural dynamics analysis, and has begun to make feasible the correlation of analysis with structural dynamics testing. Although this book contains much of the material that characterizes standard textbooks on mechanical vibrations, or structural dynamics, its goal is to present the background needed by an engineer who will be using structural dynamics computer programs or doing structural dynamics testing, or who will be taking advanced courses in finite element analysis or structural dynamics.

Although the applications of structural dynamics in aerospace engineering, civil engineering, engineering mechanics, and mechanical engineering are different, the principles and solution techniques are basically the same. Therefore, this book places emphasis on these principles and solution techniques, and illustrates them with numerous examples and homework exercises from the various engineering disciplines.

Special features of this book include: an emphasis on mathematical modeling of structures and experimental verification of mathematical models; an extensive introduction to numerical techniques for computing natural frequencies and mode shapes and for computing transient response; a systematic introduction to the use of finite elements in structural dynamics analysis; an application of complex frequency-response representations for the response of single- and multiple-degree-of-freedom systems; a thorough exposition of both the mode-displacement and mode-acceleration versions of mode superposition for computing dynamic response; an introduction to practical methods of component-mode synthesis for dynamic analysis; and the introduction of an instructional matrix algebra and finite element computer code, *ISMIS* (Interactive Structures and Matrix Interpretive System), for solving structural dynamics problems.

Although the emphasis of this book is on linear problems in structural dynamics, techniques for solving a limited class of nonlinear structural dynamics problems are also introduced. On the other hand, the topic of random vibrations is not discussed, since a thorough treatment of the subject is definitely beyond the scope of the book, and a cursory introduction would merely dilute the emphasis on numerical techniques for structural dynamics analysis. However, instructors wishing to supplement the text with material on random vibrations will find the information on complex frequency response to be valuable as background for the study of random vibrations.

A primary aim of the book is to give students a thorough introduction to the numerical techniques underlying finite element computer codes. This is done primarily through "hand" solutions and the coding of several subroutines in FORTRAN (or BASIC). Use of the *ISMIS* computer program extends the problem-solving capability of the student while avoiding the "black box" nature of production-type finite element codes. Although the *ISMIS* computer program is employed in Chapters 14 and 17, its use is by no means mandatory. The FORTRAN source code and a complete User's Manual for *ISMIS* are available for a very nominal fee and can be obtained by contacting the author directly at The University of Texas at Austin (Austin, TX 78712).

Computer graphics is beginning to play an important role in structural dynamics, for example, in computer simulations of vehicle collisions and animated displays of structural mode shapes. One feature of this book is that all figures that portray functional representations are direct computer-generated plots.

The text of this book has been used for a one-semester senior-level course in structural dynamics and a one-semester graduate-level course in computational methods in structural dynamics. The undergraduate course typically covers the following material: Chapters 1 through 6, Sections 9.1, 9.2, 10.1, 10.2, 11.1 through 11.4, and Chapter 12. The graduate course reviews the topics above (i.e., it assumes that students have had a prior course in mechanical vibrations or structural dynamics) and then covers the remaining topics in the book as time permits. Both undergraduate and graduate courses make use of the *ISMIS* computer program, while the graduate course also includes several FORTRAN coding exercises.

Portions of this text have been used in a self-paced undergraduate course in structural dynamics. This led to the statements of objectives at the beginning of each chapter and to the extensive use of example problems. Thus, the text should be especially valuable to engineers pursuing a study of structural dynamics on a self-study basis.

I express appreciation to my students who used the notes that led to the present text. Special thanks are due to Arne Berg, Mike Himes, and Rick McKenzie, who generated most of the computer plots, and to Butch Miller and Rodney Rocha, who served as proctors for the self-paced classes. Much of the content and "flavor" of the book is a result of my industrial experience at the Boeing Company's Commercial Airplane Division, at Lockheed Palo Alto Research Laboratory, and at NASA Johnson Space Center. I am indebted to the colleagues with whom I worked at these places.

I am grateful to Dr. Pol D. Spanos for reading Chapter 20 and making helpful comments. Dean Richard Gallagher reviewed the manuscript and offered many suggestions for changes, which have been incorporated into the text. This valuable service is greatly appreciated.

This book might never have been completed had it not been for the patience and accuracy of its typist, Mrs. Bettye Lofton, and to her I am most deeply indebted.

Finally, many of the hours spent in the writing of this book were hours that would otherwise have been spent with Jane, Carole, and Karen, my family. My gratitude for their sacrifices cannot be measured.

Roy R. Craig, Jr., Austin, TX

Preface to Fundamentals of Structural Dynamics

Although there has been a title change to *Fundamentals of Structural Dynamics*, this book is essentially the 2nd edition of *Structural Dynamics—An Introduction to Computer Methods*, published in 1981 by the senior author. As a textbook and as a resource book for practicing engineers, that edition had a phenomenal run of a quarter century. Although this edition retains the emphasis placed in the first edition on the topics of mathematical modeling, computer solution of structural dynamics problems, and the relationship of finite element analysis and experimental structural dynamics, it takes full advantage of the current state of the art in each of those topics. For example, whereas the first edition employed *ISMIS*, a FORTRAN-based introductory matrix algebra and finite element computer code, the present edition employs a MATLAB-based version of *ISMIS* and provides many additional structural dynamics solutions directly in MATLAB.

The new features of this edition are:

1. A coauthor, Dr. Andrew Kurdila, who has been responsible for Chapters 6, 15, 16, and 19 and Appendices D and E in this edition.
2. A greater emphasis on computer solutions, especially MATLAB-based plots; numerical algorithms in Chapters 6, 15, and 16; and digital signal-processing techniques in Chapter 18.
3. A new section (Section 5.6) on system response by the Laplace transform method, and a new appendix, Appendix C, on Laplace transforms.
4. An introduction, in Sections 10.4 and 10.5, to state-space solutions for complex modes of damped systems.
5. Greatly expanded chapters on eigensolvers (Chapter 15), numerical algorithms for calculating dynamic response (Chapters 6 and 16), and component-mode synthesis (Chapter 17).
6. New chapters on experimental modal analysis (Chapter 18) and on smart structures (Chapter 19).
7. A revised grouping of topics that places vibration of continuous systems after basic multiple-DOF topics, but before the major sections on computational methods and the advanced-topics chapters.
8. Many new or revised homework problems, including many to be solved on the computer.
9. A supplement that contains many sample MATLAB .m-files, the MATLAB-based *ISMIS* matrix structural analysis computer program, notes for an extensive short course on finite element analysis and experimental modal analysis, and other

study aids. This supplement, referred to throughout the book as the "book's website," is available online from the Wiley Web site www.wiley.com/college/craig.

The senior author would like to acknowledge the outstanding wealth of knowledge that has been shared with him by authors of papers presented at the many International Modal Analysis Conferences (IMACs) that he has attended over the past quarter century. Special appreciation is due to Prof. David L. Brown and his colleagues from the University of Cincinnati; to numerous engineers from Structural Dynamics Research Corporation, ATA Engineering, Inc., and Leuven Measurement Systems; and to the late Dominick J. (Dick) DeMichele, the founder of IMAC. Professor Eric Becker, a colleague of the senior author at The University of Texas at Austin, was responsible for many features of the original *ISMIS* (Interactive Structures and Matrix Interpretive System) FORTRAN code.

The authors would like to express appreciation to the following persons for their major contributions to this edition:

Prof. Peter Avitabile: for permission to include on the book's website his extensive short course notes on finite element analysis and experimental modal analysis.

Mr. Charlie Pickrel: for providing Boeing GVT photos (Fig. 1.9*a*,*b*) and his journal article on experimental modal analysis, the latter for inclusion on the book's website.

Dr. Matthew F. Kaplan: for permission to use substantial text and figures from his Ph.D. dissertation as the basis for the new Section 17.8 on multilevel substructuring.

Dr. Eric Blades: for conversion of the *ISMIS* FORTRAN code to form the MATLAB toolchest that is included on the book's website.

Mr. Sean Regisford: for assistance with the finite element case studies in Sections 15.6 and 16.5.

Mr. Garrett Moran: for assistance with solutions to new homework problems and for generating MATLAB plots duplicating the figures in the original *Structural Dynamics* book.

Their respective chapters of this edition were typeset in LaTeX by the authors.

ROY R. CRAIG, JR., AUSTIN, TX

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About the Authors

Roy R. Craig, Jr. is the John J. McKetta Energy Professor Emeritus in Engineering in the Department of Aerospace Engineering and Engineering Mechanics at The University of Texas at Austin. He received his B.S. degree in civil engineering from the University of Oklahoma, and M.S. and Ph.D. degrees in theoretical and applied mechanics from the University of Illinois at Urbana–Champaign. Dr. Craig's research and publications have been principally in the areas of structural dynamics analysis and testing, structural optimization, control of flexible structures, and the use of computers in engineering education. He is the developer of the *Craig–Bampton Method* of component-mode synthesis, which has been used extensively throughout the world for analyzing the dynamic response of complex structures, and he is the author of many technical papers and reports and of one other textbook, *Mechanics of Materials*. His industrial experience has been with the U.S. Naval Civil Engineering Laboratory, the Boeing Company, Lockheed Palo Alto Research Laboratory, Exxon Production Research Corporation, NASA, and IBM.

Dr. Craig has received numerous teaching awards and faculty leadership awards, including the General Dynamics Teaching Excellence Award in the College of Engineering, the John Leland Atwood Award presented jointly by the Aerospace Division of the American Society for Engineering Education and by the American Institute of Aeronautics and Astronautics "for sustained outstanding leadership and contributions in structural dynamics and experimental methods," and the D. J. DeMichele Award of the Society for Experimental Mechanics "for exemplary service and support in promoting the science and educational aspects of modal analysis technology." He is a member of the Society for Experimental Mechanics and a Fellow of the American Institute of Aeronautics and Astronautics.

Andrew J. Kurdila is the W. Martin Johnson Professor of Mechanical Engineering at the Virginia Polytechnic Institute and State University. He received his B.S. degree in applied mechanics in 1983 from the University of Cincinnati in the Department of Aerospace Engineering and Applied Mechanics. He subsequently entered The University of Texas at Austin and was awarded the M.S. degree in engineering mechanics the following year. He entered the Department of Engineering Science and Mechanics at the Georgia Institute of Technology as a Presidential Fellow and earned his Ph.D. in 1989.

Dr. Kurdila joined the faculty of the Aerospace Engineering Department at Texas A&M University in 1990 as an assistant professor. He was tenured and promoted to associate professor in 1993. He joined the faculty of the University of Florida in 1997 and was promoted to full professor in 1998. In 2005 he joined the faculty of the Virginia Polytechnic Institute and State University. He was recognized as a Select Faculty Fellow at Texas A&M University in 1994 and as a Faculty Fellow in 1996 and was awarded the Raymond L. Bisplinghoff Award at the University of Florida in 1999 for Excellence in Teaching.

Dr. Kurdila is the author of over 50 archival journal publications, 100 conference presentations and publications, four book chapters, two edited volumes, and two books. He has served as an associate editor of the *Journal of Vibration and Control* and of the *Journal of Guidance, Control and Dynamics*. He was named an Associate Fellow of the AIAA in 2001. His current research is in the areas of dynamical systems theory, control theory, and computational mechanics. His research has been funded by the Army Research Office, the Office of Naval Research, the Air Force Office of Scientific Research, the Air Force Research Laboratory, the National Science Foundation, the Department of Energy, the Army Research and Development Command, and the State of Texas.

1

The Science and Art of Structural Dynamics

What do a sport-utility vehicle traveling off-road, an airplane flying near a thunderstorm, an offshore oil platform in rough seas, and an office tower during an earthquake all have in common? One answer is that all of these are structures that are subjected to *dynamic loading*, that is, to time-varying loading. The emphasis placed on the safety, performance, and reliability of mechanical and civil structures such as these has led to the need for extensive analysis and testing to determine their response to dynamic loading. The structural dynamics techniques that are discussed in this book have even been employed to study the dynamics of snow skis and violins.

Although the topic of this book, as indicated by its title, is *structural dynamics*, some books with the word *vibrations* in their title discuss essentially the same subject matter. Powerful computer programs are invariably used to implement the modeling, analysis, and testing tasks that are discussed in this book, whether the application is one in aerospace engineering, civil engineering, mechanical engineering, electrical engineering, or even in sports or music.

1.1 INTRODUCTION TO STRUCTURAL DYNAMICS

This introductory chapter is entitled "The Science and Art of Structural Dynamics" to emphasize at the outset that by studying the principles and mathematical formulas discussed in this book you will begin to understand the *science* of structural dynamics analysis. However, structural dynamicists must also master the *art* of creating mathematical models of structures, and in many cases they must also perform dynamic tests. The cover photo depicts an automobile that is undergoing such dynamic testing. *Modeling*, *analysis*, and *testing* tasks all demand that skill and judgment be exercised in order that useful results will be obtained; and all three of these tasks are discussed in this book.

A *dynamic load* is one whose magnitude, direction, or point of application varies with time. The resulting time-varying displacements and stresses constitute the *dynamic response*. If the loading is a known function of time, the loading is said to be *prescribed loading*, and the analysis of a given structural system to a prescribed loading is called a *deterministic analysis*. If the time history of the loading is not known completely but only in a statistical sense, the loading is said to be *random*. In this book we treat only prescribed dynamic loading.

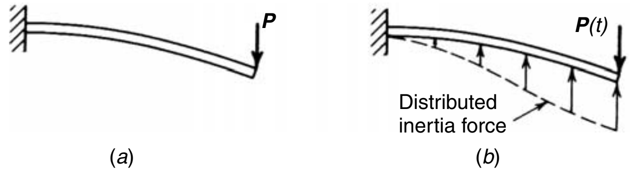

Figure 1.1 Cantilever beam under (*a*) static loading and (*b*) dynamic loading.

A structural dynamics problem differs from the corresponding static problem in two important respects. The first has been noted above: namely, the time-varying nature of the excitation. Of equal importance in a structural dynamics problem, however, is the role played by *acceleration*. Figure 1.1*a* shows a cantilever beam under static loading. The deflection and internal stresses depend directly on the static load P. On the other hand, Fig. 1.1*b* shows a similar cantilever beam subjected to a time-varying load $P(t)$. The acceleration of the beam gives rise to a distributed *inertia force*, as indicated in the figure. If the inertia force contributes significantly to the deflection of the structure and the internal stresses in the structure, a dynamical investigation is required.

Figure 1.2 shows the typical steps in a complete dynamical investigation. The three major steps, which are outlined by dashed-line boxes, are: *design*, *analysis*, and *testing*. The engineer is generally required to perform only one, or possibly two, of these steps. For example, a civil engineer might be asked to perform a dynamical analysis of an existing building and to confirm the analysis by performing specific dynamic testing of the building. The results of the analysis and testing might lead to criteria for retrofitting the building with additional bracing or damping to ensure safety against failure due to specified earthquake excitation.[1.1,1.2] Automotive engineers perform extensive analysis and vibration testing to determine the dynamical behavior of new car designs.[1.3,1.4] Results of this analysis and testing frequently lead to design changes that will improve the ride quality, economy, or safety of the vehicle.

In Section 1.2 we introduce the topic of mathematical models. In Section 1.3 we introduce the *prototype single-degree-of-freedom model* and indicate how to analyze the dynamic response of this model when it is subjected to certain simple inputs. Finally, in Section 1.4 we indicate some of the types of vibration tests that are performed on structures.

1.2 MODELING OF STRUCTURAL COMPONENTS AND SYSTEMS

Perhaps the most demanding step in any dynamical analysis is the creation of a *mathematical model* of the structure. This process is illustrated by steps 2a and 2b of Fig. 1.2. In step 2a you must contrive an idealized model of the structural system to be studied, a model essentially like the real system (which may already exist or may merely be in the design stages) but easier to analyze mathematically. This *analytical model* consists of:

1. A list of the simplifying assumptions made in reducing the real system to the analytical model
2. Drawings that depict the analytical model (e.g., see Fig. 1.3)
3. A list of the design parameters (i.e., sizes, materials, etc.)

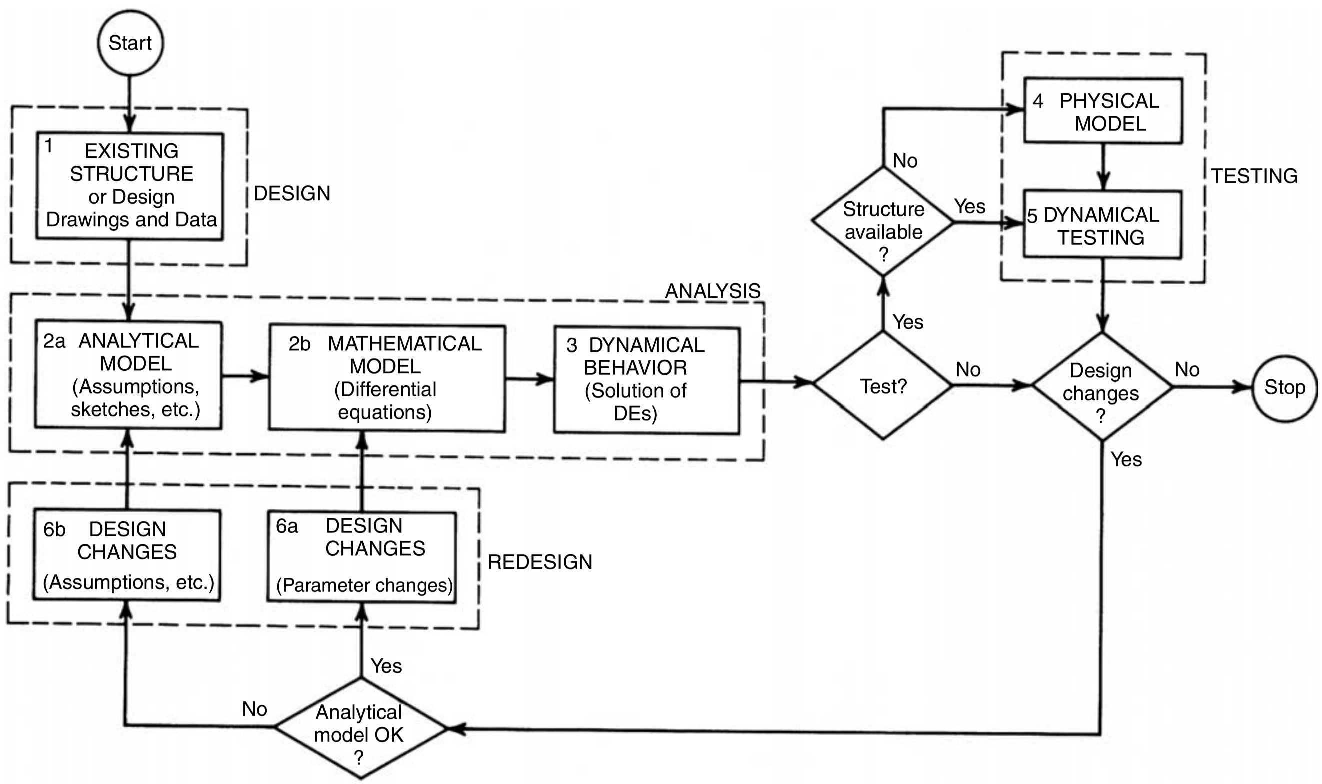

Figure 1.2 Steps in a dynamical investigation.

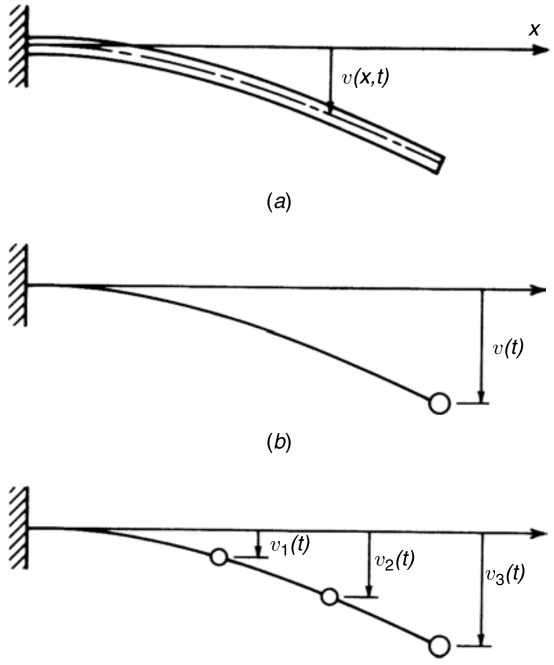

Figure 1.3 Analytical models of a cantilever beam: (*a*) distributed-mass cantilever beam, a continuous model (or distributed-parameter model); (*b*) one-degree-of-freedom model, a discrete-parameter model; (*c*) three-degree-of-freedom model, a more refined discrete-parameter model.

Analytical models fall into two basic categories: *continuous models* and *discrete-parameter models*. Figure 1.3*a* shows a continuous model of a cantilever beam. The number of displacement quantities that must be considered to represent the effects of all significant inertia forces is called the *number of degrees of freedom* (DOF) of the system. Thus, a continuous model represents an infinite-DOF system. Techniques for creating continuous models are discussed in Chapter 12. However, Fig. 1.3*b* and *c* depict finite-DOF systems. The discrete-parameter models shown here are called *lumped-mass models* because the mass of the system is assumed to be represented by a small number of point masses, or particles. Techniques for creating discrete-parameter models are discussed in Chapters 2, 8, and 14.

To create a useful analytical model, you must have clearly in mind the intended use of the analytical model, that is, the types of behavior of the real system that the model is supposed to represent faithfully. The complexity of the analytical model is determined (1) by the types and detail of behavior that it must represent, (2) by the computational analysis capability available (hardware and software), and (3) by the time and expense allowable. For example, Fig. 1.4 shows four different analytical models used in the 1960s to study the dynamical behavior of the *Apollo Saturn V* space vehicle, the vehicle that was used in landing astronauts on the surface of the moon. The 30-DOF beam-rod model was used for preliminary studies and to determine full-scale testing requirements. The 300-DOF model on the right, on the other hand, was required to give

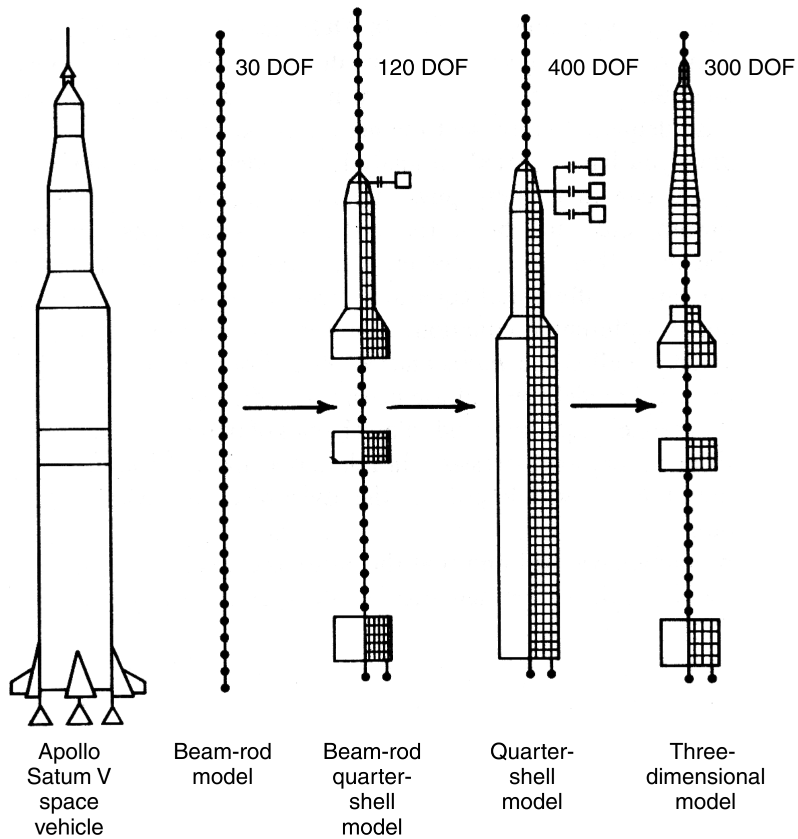

Figure 1.4 Analytical models of varying complexity used in studying the space vehicle dynamics of the *Apollo Saturn V*. (From C. E. Green et al., *Dynamic Testing for Shuttle Design Verification*, NASA, Washington, DC, 1972.)

a more accurate description of motion at the flight sensor locations. All of these *Saturn V* analytical models are extremely small compared with the multimillion-DOF models that can be analyzed now (see Section 17.8). However, supported by extensive dynamical testing, these analytical models were sufficient to ensure successful accomplishment of *Apollo V*'s moon-landing mission. Simplicity of the analytical model is very desirable as long as the model is adequate to represent the necessary behavior.

Once you have created an analytical model of the structure you wish to study, you can apply physical laws (e.g., Newton's Laws, stress–strain relationships) to obtain the differential equation(s) of motion that describe, in mathematical language, the analytical model. A continuous model leads to partial differential equations, whereas a discrete-parameter model leads to ordinary differential equations. The set of differential equations of motion so derived is called a *mathematical model* of the structure. To obtain a mathematical model, you will use methods studied in *dynamics* (e.g., Newton's Laws, Lagrange's Equations) and in *mechanics of deformable solids* (e.g., strain–displacement relations, stress–strain relations) and will combine these to obtain differential equations describing the dynamical behavior of a deformable structure.

In practice you will find that the entire process of creating first an analytical model and then a mathematical model may be referred to simply as *mathematical modeling*. In using a finite element computer program such as ABAQUS[1.5], ANSYS[1.6], MSC-Nastran[1.7], OpenFEM[1.8], SAP2000[1.9], or another computer program to carry out a structural dynamics analysis, your major modeling task will be to simplify the system and provide input data on dimensions, material properties, loads, and so on. This is

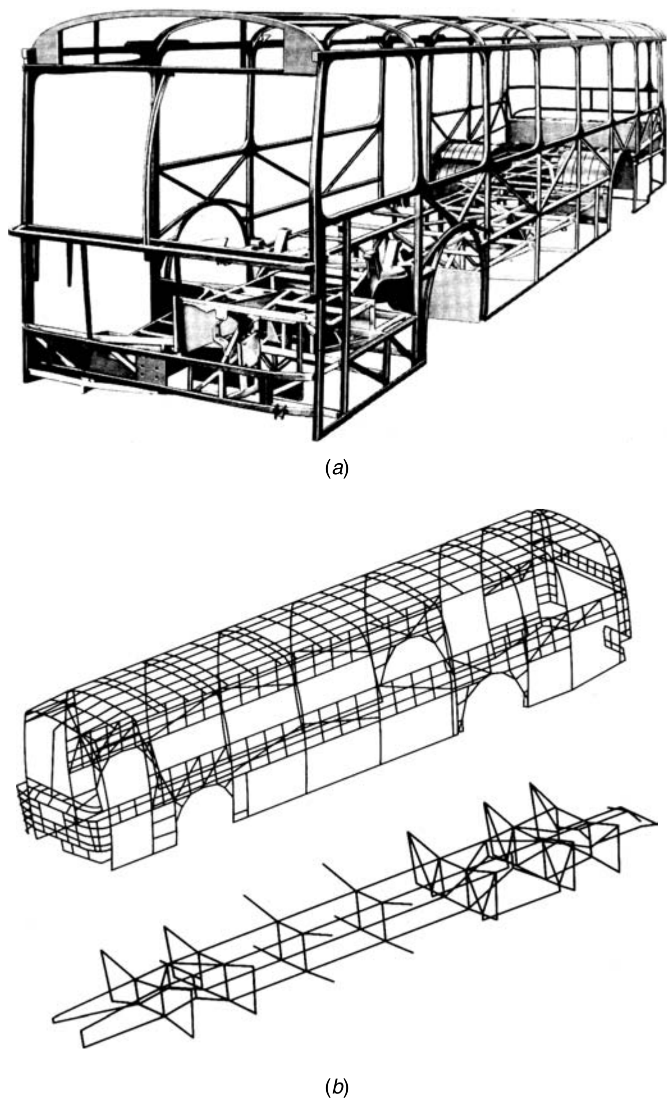

Figure 1.5 (*a*) Actual bus body and frame structure; (*b*) finite element models of the body and frame. (From D. Radaj et al., *Finite Element Analysis: An Automobile Engineer's Tool*, Society of Automotive Engineers, 1974. Used with permission of the Society of Automotive Engineers, Inc. Copyright © 1974 SAE.)

where the "art" of structural dynamics comes into play. On the other hand, actual creation and solution of the differential equations is done by the computer program. Figure 1.5 shows a picture of an actual bus body and a computer-generated plot of the idealized structure, that is, analytical model, which was input to a computer. Computer graphics software (e.g., MSC-Patran[1.7]) has become an invaluable tool for use in creating mathematical models of structures and in displaying the results of the analyses that are performed by computers.

Once a mathematical model has been formulated, the next step in a dynamical analysis is to solve the differential equation(s) to obtain the dynamical response that is predicted. (*Note:* The terms *dynamical response* and *vibration* are used interchangeably.) The two types of dynamical behavior that are of primary importance in structural applications are *free vibration* and *forced vibration* (or *forced response*), the former being the motion resulting from specified initial conditions, and the latter being the motion resulting directly from specified inputs to the system from external sources. Thus, you solve the differential equations of motion subject to specified initial conditions and to specified inputs from external sources, and you obtain the resulting time histories of the motion of the structure and stresses within the structure. This constitutes the behavior predicted for the (real) structure, or the *response*.

The analysis phase of a dynamical investigation consists of the three steps just described: defining the *analytical model*, deriving the corresponding *mathematical model*, and solving for the *dynamical response*. This book deals primarily with the second and third steps in the analysis phase of a structural dynamics investigation. Section 1.3 illustrates these steps for the simplest analytical model, a lumped-mass single-DOF model. Section 1.4 provides a brief discussion of dynamical testing.

1.3 PROTOTYPE SPRING–MASS MODEL

Before proceeding with the details of how to model complex structures and analyze their dynamical behavior, let us consider the simplest structure undergoing the simplest forms of vibration. The structure must have an *elastic component*, which can store and release potential energy; and it must have *mass*, which can store and release kinetic energy. The simplest model, therefore, is the *spring–mass oscillator*, shown in Fig. 1.6*a*.

1.3.1 Simplifying Assumptions: Analytical Model

The simplifying assumptions that define this *prototype analytical model* are:

1. The mass is a point mass that is confined to move along one horizontal direction on a frictionless plane. The displacement of the mass in the x direction from the position where the spring is undeformed is designated by the displacement variable $u(t)$.
2. The mass is connected to a fixed base by an idealized massless, linear spring. The fixed base serves as an inertial reference frame. Figure 1.6*b* shows the linear relationship between the *elongation* (u positive) and *contraction* (u negative) of the spring and the force $f_S(t)$ that the spring exerts on the mass. When the spring is in tension, f_S is positive; when the spring is in compression, f_S is negative.
3. A specified external force $p(t)$ acts on the mass, as shown in Fig. 1.6*a*.

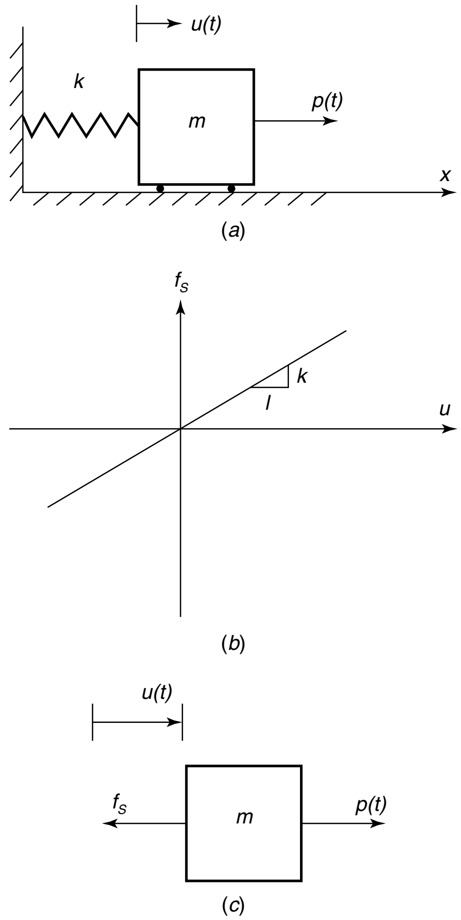

Figure 1.6 (*a*) Spring–mass oscillator; (*b*) force–elongation behavior of a linear spring; (*c*) free-body diagram of the spring–mass oscillator.

Since it takes only one variable [e.g., $u(t)$] to specify the instantaneous position of the mass, this is called a *single-degree-of-freedom* (SDOF) *system*.

1.3.2 Mathematical Model: Equation of Motion

Newton's Second Law To obtain a mathematical model describing the behavior of the spring–mass oscillator, we start by drawing a *free-body diagram* of the mass (Fig. 1.6*c*) and applying *Newton's Second Law*,

$$\sum F_x = ma_x \tag{1.1}$$

where m is the mass and a_x is the acceleration of the mass, taken as positive in the $+x$ direction. Acceleration a_x is given by the second derivative of the displacement, that is, $a_x = \ddot{u}(t)$; similarly, the velocity is given by $\dot{u}(t)$. By assuming that the mass is displaced u to the right of the position where the spring force is zero, we can say that the spring will be in tension, so the spring force will act to the left on the mass, as shown on the free-body diagram. Thus, Eq. 1.1 becomes

$$-f_S + p(t) = m\ddot{u} \tag{1.2}$$

Force–Displacement Relationship As indicated in Fig. 1.6*b*, there is assumed to be a linear relationship between the force in the spring and its elongation u, so

$$f_S = ku \tag{1.3}$$

where k is the *stiffness* of the spring.

Equation of Motion Finally, by combining Eqs. 1.2 and 1.3 and rearranging to place all u-terms on the left, we obtain the *equation of motion* for the prototype undamped SDOF model:

$$\boxed{m\ddot{u} + ku = p(t)} \tag{1.4}$$

This equation of motion is a linear second-order ordinary differential equation. It is the *mathematical model* of this simple SDOF system.

Having Eq. 1.4, the equation of motion that governs the motion of the SDOF spring–mass oscillator in Fig. 1.6*a*, we now examine the dynamic response of this prototype system. The response of the system is determined by its *initial conditions*, that is, by the values of its displacement and velocity at time $t = 0$:

$$u(0) = u_0 = \text{initial displacement}, \qquad \dot{u}(0) = v_0 = \text{initial velocity} \tag{1.5}$$

and by $p(t)$, the *external force* acting on the system. Here we consider two simple examples of vibration of the spring–mass oscillator; a more general discussion of SDOF systems follows in Chapters 3 through 7.

1.3.3 Free Vibration Example

The spring–mass oscillator is said to undergo *free vibration* if $p(t) \equiv 0$, but the mass has nonzero initial displacement u_0 and/or nonzero initial velocity v_0. Therefore, the equation of motion for free vibration is the homogeneous second-order differential equation

$$m\ddot{u} + ku = 0 \tag{1.6}$$

The general solution of this well-known simple differential equation is

$$u = A_1 \cos \omega_n t + A_2 \sin \omega_n t \tag{1.7}$$

where ω_n is the *undamped circular natural frequency*, defined by

$$\boxed{\omega_n = \sqrt{\frac{k}{m}}} \tag{1.8}$$

The units of ω_n are radians per second (rad/s).

The constants A_1 and A_2 in Eq. 1.7 are chosen so that the two initial conditions, Eqs. 1.5, will be satisfied. Thus, *free vibration of an undamped spring–mass oscillator* is characterized by the time-dependent displacement

$$\boxed{u(t) = u_0 \cos \omega_n t + \frac{v_0}{\omega_n} \sin \omega_n t} \tag{1.9}$$

It is easy to show that this solution satisfies the differential equation, Eq. 1.6, and the two initial conditions, Eqs. 1.5.

Figure 1.7 depicts the response of a spring–mass oscillator released from rest from an initial displacement of u_0. Thus, the motion depicted in Fig. 1.7 is given by

$$u(t) = u_0 \cos \omega_n t = u_0 \cos \frac{2\pi t}{T_n} \tag{1.10}$$

From Eq. 1.10 and Fig. 1.7, free vibration of an undamped SDOF system consists of harmonic (sinusoidal) motion that repeats itself with a *period* (in seconds) given by

$$\boxed{T_n = \frac{2\pi}{\omega_n}} \tag{1.11}$$

as illustrated in Fig. 1.7. The *amplitude* of the vibration is defined as the maximum displacement that is experienced by the mass. For the free vibration depicted in Fig. 1.7, the amplitude is equal to the initial displacement u_0.

Free vibration is discussed further in Chapter 3.

1.3.4 Forced Response Example

The spring–mass oscillator is said to undergo *forced vibration* if $p(t) \neq 0$ in Eq. 1.4. Solution of the differential equation of motion for this case, Eq. 1.4, requires both a *complementary solution* u_c and a *particular solution* u_p. Thus,

$$u(t) = u_c(t) + u_p(t) \tag{1.12}$$

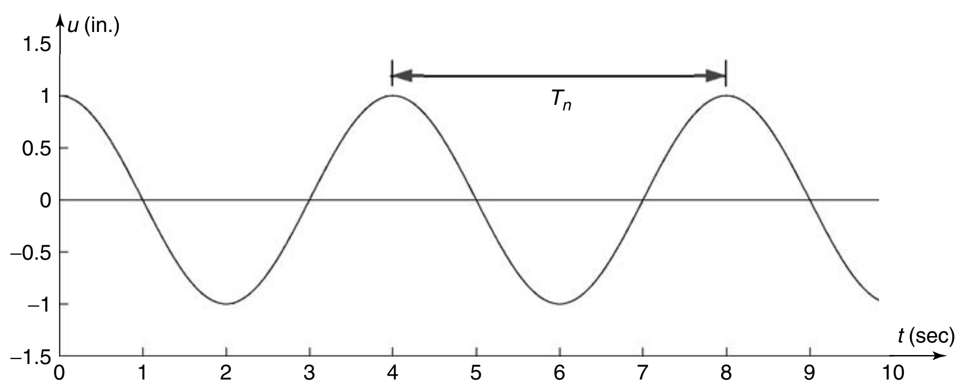

Figure 1.7 Free vibration of a spring-mass oscillator with $u_0 = 1.0$ in., $v_0 = 0$, and $T_n = 4.0$ sec.

As a simple illustration of forced vibration we consider *ramp response*, the response of the spring–mass oscillator to the linearly varying ramp excitation force given by

$$p(t) = p_0 \frac{t}{t_0}, \qquad t > 0 \tag{1.13}$$

and illustrated in Fig. 1.8*a*. (The time t_0 is the time at which the force reaches the value p_0.) The particular solution, like the excitation, varies linearly with time. The complementary solution has the same form as given in Eq. 1.7, so the total response has the form

$$u(t) = A_1 \cos \omega_n t + A_2 \sin \omega_n t + \frac{p_0}{k} \frac{t}{t_0} \tag{1.14}$$

where the constants A_1 and A_2 must be selected so that the initial conditions $u(0)$ and $\dot{u}(0)$ will be satisfied.

Figure 1.8*b* depicts the response of a spring–mass oscillator that is initially at rest at the origin, so the initial conditions are $u(0) = \dot{u}(0) = 0$. The corresponding *ramp response* is thus given by

$$\boxed{u(t) = \frac{p_0}{k}\left(\frac{t}{t_0} - \frac{1}{\omega_n t_0} \sin \omega_n t\right)} \tag{1.15}$$

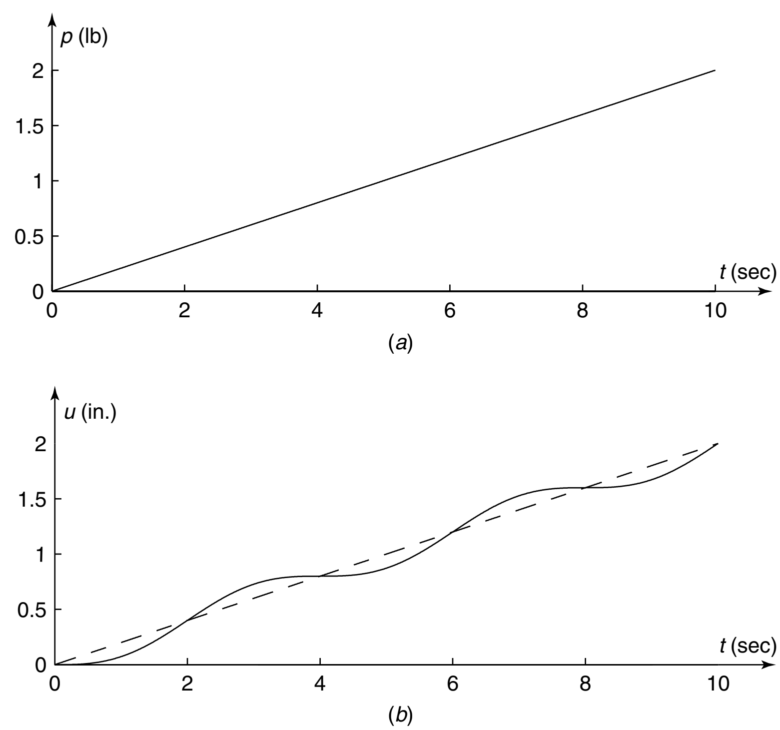

Figure 1.8 (*a*) Ramp excitation $p(t) = p_0(t/t_0)$ for $t > 0$, with $p_0 = 2$ lb, $t_0 = 10$ sec; (*b*) response of a spring–mass oscillator to ramp excitation. For (*b*), $k = 1$ lb/in. and $T_n = 4$ sec.

Clearly evident in this example of forced response are two components: (1) a linearly time-varying displacement (dashed curve), which is due directly to the linearly time-varying ramp excitation, and (2) an induced oscillatory motion at the undamped natural frequency ω_n. Of course, this ramp response is only valid as long as the spring remains within its linearly elastic range.

1.3.5 Conclusions

In this section we have taken a preliminary look at several characteristics that are typical of the response of structures to nonzero initial conditions and/or to time-varying excitation. We have especially noted the oscillatory nature of the response. In Chapters 3 through 7, we consider many additional examples of free and forced vibration of SDOF systems, including systems with damping.

1.4 VIBRATION TESTING OF STRUCTURES

A primary purpose of dynamical testing is to confirm a mathematical model and, in many instances, to obtain important information on loads, on damping, and on other quantities that may be required in the dynamical analysis. In some instances these tests are conducted on reduced-scale *physical models*: for example, wind tunnel tests of airplane models. In other cases, when a full-scale structure (e.g., an automobile) is available, the tests may be conducted on it.

Aerospace vehicles (i.e., airplanes, spacecraft, etc.) must be subjected to extensive static and dynamic testing on the ground prior to actual flight of the vehicle. Figure 1.9*a* shows a *ground vibration test* in progress on a Boeing 767 airplane. Note the electrodynamic shaker in place under each wingtip and the special soft support under the nose landing gear.

Dynamical testing of physical models may be employed for determining qualitatively and quantitatively the dynamical behavior characteristics of a particular class of structures. For example, Fig. 1.9*b* shows an aeroelastic model of a Boeing 777 airplane undergoing ground vibration testing in preparation for testing in a wind tunnel to aid in predicting the dynamics of the full-scale airplane in flight. Note the soft bungee-cord distributed support of the model and the two electrodynamic shakers that are attached by stingers to the engine nacelles. Figure 1.10 shows a fluid-filled cylindrical tank structure in place on a shake table in a university laboratory. The shake table is used to simulate earthquake excitation at the base of the tank structure.

Chapter 18 provides an introduction to *Experimental Modal Analysis*, a very important structural dynamics test procedure that is used extensively in the automotive and aerospace industries and is also used to test buildings, bridges, and other civil structures.

1.5 SCOPE OF THE BOOK

Part I, encompassing Chapters 2 through 7, treats single-degree-of-freedom (SDOF) systems. In Chapter 2 procedures are described for developing SDOF mathematical models; both Newton's Laws and the Principle of Virtual Displacements are employed. The free vibration of undamped and damped systems is the topic of Chapter 3, and