Funding, Margin and Capital Valuation Adjustments for Bilateral Trade Portfolios

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Abstract

We apply to the concrete setup of a bank engaged into bilateral trade portfolios the XVA conceptual framework of Albanese and Crépey (2016), whereby so-called contra-liabilities and cost of capital need to be charged by a bank to its clients at trade inceptions, on top of the fair valuation of counterparty credit risk, in order to account for the incompleteness of this risk. Our funding cost for variation margin (FVA) is defined asymmetrically since there is no benefit in holding excess capital in the future. Capital is fungible as a source of funding for variation margin (but not for initial margin), causing a material FVA reduction. We introduce specialist initial margin lending schemes that drastically reduce the funding cost for initial margin (MVA). By contrast with the other approaches in the literature, our capital valuation adjustment (KVA) is not defined as the risk-neutral valuation of some cash flows, but as a risk premium, i.e. the cost of remunerating shareholder capital at risk at some hurdle rate.

Keywords: Counterparty credit risk, cost of funding variation margin (FVA), cost of funding initial margin (MVA), cost of capital (KVA).

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1 Introduction

Albanese and Crépey (2016) develop an XVA conceptual framework based on a capital structure model acknowledging the impossibility for a bank to replicate jump-to-default related cash flows. This approach results in a two-step XVA methodology.

First, the so-called contra-assets (CA) are valued as the expected counterparty default losses and funding expenditures. These expected costs can be decomposed as the sum between the fair valuation, dubbed CCR, of counterparty credit risk to the bank as a whole, plus an add-on compensating bank shareholders for a wealth transfer, corresponding to the so-called contra-liabilities (CL), triggered by the impossibility for the bank to hedge its own jump-to-default risk.

Second, a KVA risk premium is computed as the cost of a sustainable remuneration of the shareholder capital at risk earmarked to absorb the exceptional (beyond expected) losses due to the impossibility for the bank to replicate counterparty jump-to-default risk.

The all-inclusive XVA charge appears as

$$\mathrm{CA} + \mathrm{KVA} = \mathrm{CCR} + \mathrm{CL} + \mathrm{KVA}, \tag{1}$$

which is charged to clients on an incremental run-off basis at every new trade. CA payments flow into a reserve capital (RC) account of the bank used for coping with expected counterparty default losses and funding expenditures. KVA payments flow into a risk margin (RM) account from which they are gradually released to shareholders as a remuneration for their capital at risk.

In the present paper we apply this framework to the concrete setup of a bank engaged in bilateral trade portfolios. The ***main contributions*** are concrete equations for the corresponding CVA, FVA, MVA (see below) and KVA, rooted in the balance sheet analysis of Albanese and Crépey (2016), the XVA algorithm and numerical results, including on a real life banking derivative portfolio.

1.1 Overview of the Paper

In the context of bilateral trading, the discounted expectation of losses due to the default of counterparties or of the bank itself are respectively known as CVA (credit valuation adjustment) and DVA (debt valuation adjustment). Counterparty credit risk mitigants include variation margin (VM) tracking the mark-to-market of client portfolios and initial margin (IM) set as a cushion against gap risk, i.e. the risk of slippage between the portfolio and its variation margin during the liquidation period. The cost of funding cash collateral for variation margin is known as funding valuation adjustment (FVA), while the cost of funding segregated collateral posted as initial margin is the margin valuation adjustment (MVA). Contra-liability counterparts of the FVA and the MVA arise as the FDA (funding debt adjustment) and the MDA (margin debt adjustment). A contra-liability component of the CVA, dubbed $\mathrm{CVA}^{\mathrm{CL}}$, appears as the cost of an insurance subscribed by the bank in order to comply with a regulatory unilateral CVA requirement.

1.2 Outline of the Paper

Section 2 provides an executive summary of the XVA methodology of Albanese and Crépey (2016). Section 3 specifies all the cash flows involved in the case of bilateral trade portfolios. The ensuing XVA formulas, as well as the corresponding loss process L required as input data in the KVA computations, are derived in Section 4. Section 5 deals with the FVA reduction provided by the possibility for a bank to post economic capital as variation margin. Section 6 introduces specialist initial margin lending schemes that drastically reduce the funding cost for initial margin (MVA). Section 7 contrasts the risk premium KVA approach of this paper with alternative approaches in the literature. Section 8 shows how our XVA approach can be implemented by means of nested Monte Carlo simulations. In Section 9 this is illustrated numerically by two case studies. A list of the main acronyms used in the paper is provided in Section A.

2 Conceptual XVA Framework

This section provides a brief recap of the XVA methodology that arises from Albanese and Crépey (2016).

We consider a pricing stochastic basis $(\Omega, \mathbb{G}, \mathbb{Q})$, with model filtration $\mathbb{G} = (\mathcal{G}_t)_{t\in\mathbb{R}_+}$ and risk-neutral pricing measure $\mathbb{Q}$, such that all the processes of interest are $\mathbb{G}$ adapted and all the random times of interest are $\mathbb{G}$ stopping times. The corresponding expectation and conditional expectation are denoted by $\mathbb{E}$ and $\mathbb{E}_t$. We denote by r a $\mathbb{G}$ progressive OIS rate process, where OIS rate stands for overnight indexed swap rate, which is together the best market proxy for a risk-free rate and the reference rate for the remuneration of cash collateral. We write $\beta_t = e^{-\int_0^t r_s ds}$ for the corresponding risk-neutral discount factor. All cash flows are valued by their risk-free discounted $(\mathbb{G}, \mathbb{Q})$ conditional expectation, assumed to exist. This ensures the internal consistency of the valuation setup. We assume that the historical probability measure $\mathbb{P}$ required for capital calculations coincides with the pricing measure $\mathbb{Q}$, the discrepancy between $\mathbb{P}$ and $\mathbb{Q}$ being left to model risk.

In order to focus on counterparty credit risk and XVA analysis, we assume throughout the paper that the market risk of the bank is perfectly hedged by means of perfectly collateralized back-to-back trades. Hence only the counterparty credit risk related cash flows remain.

We assume that the reserve capital (RC) account of the bank is continuously reset to its theoretical target CA level so that, much like with futures, the position of the bank is reset to zero at all times, but it generates a trading loss-and-profit process L, or loss process for brevity.

At least this holds until the default of the bank, modeled as a totally unpredictable event calibrated to the bank CDS spread, which we view as the most reliable and informative credit data regarding anticipations of markets participants about future recapitalization, government intervention, etc.

Accounting for the bank default time τ, the time horizon of the model is $\bar{\tau} = \tau \wedge T$, where T is the final maturity of the portfolio. Technically, the contra-assets (CA) process corresponds to the value of the counterparty credit risk related cash flows posted by the bank prior its default time τ, whereas the contra-liabilities (CL) process corresponds to the value of the counterparty credit risk related cash flows received by the bank during its default resolution period starting at τ.

We denote by $J = \mathbb{1}_{[0,\tau)}$ the survival indicator process of the bank. For any left-limited process Y, we denote by $\Delta_\tau Y = Y_\tau - Y_{\tau-}$ the jump of Y at τ and by

$Y^{\tau-} = JY + (1-J)Y_{\tau-}$ the process Y stopped before time τ, so that

$$dY_t = dY_t^{\tau-} + (-\Delta_\tau Y)\, dJ_t,\ 0 \le t \le \bar{\tau}. \quad (2)$$

We denote by $\mathcal{C}$ and $\mathcal{F}$ the cumulative streams of counterparty credit and funding cash flows related to the derivative portfolio of the bank. Hence counterparty default losses and funding expenditures contributing to CA correspond to $\mathcal{C}^{\tau-}$ and $\mathcal{F}^{\tau-}$, whereas $(-\Delta_\tau \mathcal{C})$ and $(-\Delta_\tau \mathcal{F})$ contribute to CL. Accordingly:

Lemma 5.1 and Theorem 5.1 in Albanese and Crépey (2016) (i) *We have*

$$\begin{aligned}
\mathrm{CA}_t &= \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s d\mathcal{C}_s^{\tau-} + \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s d\mathcal{F}_s^{\tau-},\\
\mathrm{CL}_t &= \mathbb{E}_t\big[\beta_t^{-1}\beta_\tau \mathbb{1}_{\{\tau<T\}}(-\Delta_\tau \mathcal{C})\big] + \mathbb{E}_t\big[\beta_t^{-1}\beta_\tau \mathbb{1}_{\{\tau<T\}}(-\Delta_\tau \mathcal{F})\big],\\
\mathrm{CCR}_t &= \mathrm{CA}_t - \mathrm{CL}_t = \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s d\mathcal{C}_s.
\end{aligned} \quad (3)$$

(ii) *The loss process L is a risk-neutral local martingale such that*

$$\beta_t dL_t = d(\beta_t \mathrm{CA}_t) + \beta_t(d\mathcal{C}_t^{\tau-} + d\mathcal{F}_t^{\tau-}),\ 0 \le t \le \bar{\tau}, \quad (4)$$

starting from some initial value $L_0 = z$ unknown but immaterial, as only the fluctuations of L matter in the computations.

Since L fluctuates over time as of (4), economic capital $\mathrm{EC} = \mathrm{EC}_t(L)$ needs to be earmarked by the bank in order to absorb exceptional losses (beyond the expected level of the losses accounted for by reserve capital). Under the cost of capital pricing approach of Albanese and Crépey (2016), derivative entry prices include, on top of the valuation of the corresponding cash-flows, a KVA risk premium, devised as the cost of a remuneration of shareholder capital at risk (SCR) at some hurdle rate h, i.e.

$$\mathrm{KVA}_t = h\mathbb{E}_t \int_t^{\bar{\tau}} e^{-\int_t^s (r_u+h)du} \mathrm{EC}_s(L) ds,\ t \in [0, \bar{\tau}], \quad (5)$$

where $\mathrm{EC}_s(L)$ is computed based on a 97.5% expected shortfall (ES) of $\int_s^{s+1} \beta_s^{-1}\beta_u dL_u$ conditional on $\mathcal{G}_s \vee \{\tau > (s+1)\}$, which we denote by $\mathrm{ES}_t(L)$. The "$+h$" in the discount factor in (5) reflects the fact that the KVA is loss-absorbing, hence part of EC, so that shareholder capital at risk that needs to be remunerated reduces to SCR=EC−KVA. As a further consequence, an increase of economic capital above ES may be required in order to ensure the consistency condition KVA $\le$ EC, i.e. EC − KVA = SCR ≥ 0. This ends up in a fixed-point problem (5), where

$$\mathrm{EC}_t(L) = \max(\mathrm{ES}_t(L), \mathrm{KVA}_t(L)), \quad (6)$$

so that (5)-(6) result in a Lipschitz BSDE for the KVA process, shown to be well posed in Albanese and Crépey (2016, Section 6).

All value and price processes are modeled as semimartingales in a càdlàg version. We write $x^{\pm} = \max(\pm x, 0)$ and $\int_a^b = \int_{(a,b]}$.

3 Bilateral Trading Cash Flows

In this paper we assume that the bank is engaged in bilateral trading of a derivative portfolio split into several netting sets corresponding to counterparties indexed by $i = 1, \dots, n$, with default times τ_i and survival indicators $J^i = \mathbb{1}_{[0,\tau_i)}$. The bank is also default prone, with default time τ and survival indicator $J = \mathbb{1}_{[0,\tau)}$.

We suppose that all these default times are positive and admit a finite intensity. In particular, defaults occur at any given $\mathbb{G}$ predictable stopping time with zero probability, so that such events can be ignored in all the computations.

3.1 Exposures at Defaults

Let MtM^i_t be the mark-to-market of the i-th netting set, i.e. the trade additive risk-neutral conditional expectation of future discounted promised cash flows, ignoring counterparty credit risk and assuming risk-free funding. Let VM^i_t denote the corresponding variation margin, counted positively when received by the bank. Hence

$$P^i_t = \mathrm{MtM}^i_t - \mathrm{VM}^i_t \tag{7}$$

is the net spot exposure of the bank to the i-th netting set. In addition to the variation margin VM^i_t that flows between them, the bank and the counterparty i post respective initial margins PIM^i and RIM^i_t, respectively posted and received by the bank in some segregated accounts. Finally, we denote by R and R_i the unsecured borrowing recovery rate of the bank and of counterparty i.

In practice, there is a positive liquidation period, usually a few days, between the default of a counterparty or the bank and the liquidation of their portfolio. The gap risk of slippage of MtM^i_t and of unpaid contractual cash flows during the liquidation period is the motivation for the initial margins.

A positive liquidation period is explicitly introduced in Armenti and Crépey (2016) and Crépey and Song (2016) (see also Brigo and Pallavicini (2014)) and involves introducing the random variables

$$\mathrm{MtM}^i_{\tau_i+\delta t} + \delta\mathrm{MtM}^i_{\tau_i+\delta t} - \mathrm{VM}^i_{\tau_i}, \tag{8}$$

where δt is the length of the liquidation period and $\delta\mathrm{MtM}^i_{\tau_i+\delta t}$ is the accrued value of all the cash flows owed by the counterparty of the bank during the liquidation period.

To alleviate the notation in this paper, we take the limit as $\delta t \to 0$ and approximate $\mathrm{MtM}^i_{\tau_i+\delta t} + \delta\mathrm{MtM}^i_{\tau_i+\delta t}$ by $\widehat{\mathrm{MtM}}^i_{\tau_i}$, and therefore (8) by $Q^i_{\tau_i} = \widehat{\mathrm{MtM}}^i_{\tau_i} - \mathrm{VM}^i_{\tau_i}$, for a suitable $\mathbb{G}$-optional process $\widehat{\mathrm{MtM}}^i$. A related issue is wrong-way risk, i.e. the risk of adverse dependence between the default exposures and the credit risk of the bank and its counterparties. As illustrated in Crépey and Song (2016), this impact can also be captured in the modelling of the $Q^i_{\tau_i}$.

Lemma 3.1 *The exposure of the bank to the default of each counterparty $i = 1, \dots, n$ is*

$$J_{\tau_i-}(1 - R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+. \tag{9}$$

The exposure of each counterparty $i = 1, \dots, n$ to the default of the bank is

$$J^i_{\tau-}(1 - R)(Q^i_\tau - \mathrm{PIM}^i_\tau)^-. \tag{10}$$

Proof. By symmetry, it is enough to prove (9). Let $C^i = \mathrm{VM}^i + \mathrm{RIM}^i$ and

$$\epsilon_i = (Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+ = (\widehat{\mathrm{MtM}}^i_{\tau_i} - C^i_{\tau_i})^+.$$

When the counterparty i defaults:

- If $\epsilon_i = 0$, meaning that $\widehat{\mathrm{MtM}}^i_{\tau_i} \le C^i_{\tau_i}$, then:
  - Either $\widehat{\mathrm{MtM}}^i_{\tau_i} \ge 0$ and the ownership of an amount $\widehat{\mathrm{MtM}}^i_{\tau_i}$ of collateral is transferred to the bank,
  - Or $\widehat{\mathrm{MtM}}^i_{\tau_i} \le 0$ and an amount $(-\widehat{\mathrm{MtM}}^i_{\tau_i})$ is paid by the bank to the liquidator of the counterparty i, who keeps ownership of all its collateral.

  In both cases, the bank gets $\widehat{\mathrm{MtM}}^i_{\tau_i}$;
- Otherwise, i.e. if $\epsilon_i > 0$, meaning that the overall collateral C^i of the counterparty i does not cover the totality of its debt to the bank, then, at time τ_i, the ownership of C^i is transferred in totality to the bank, which also recovers a fraction R_i of ϵ_i.

Also accounting for the unwinding of the back-to-back hedge of the netting set i at the time of liquidation of the counterparty i, the loss of the bank in case of default of the counterparty i appears as (assuming $\tau \ge \tau_i$)

$$\begin{aligned}&\widehat{\mathrm{MtM}}^i_{\tau_i} - \mathbb{1}_{\epsilon_i=0}\widehat{\mathrm{MtM}}^i_{\tau_i} - \mathbb{1}_{\epsilon_i>0}(C^i_{\tau_i} + R_i\epsilon_i)\\ &\qquad = \mathbb{1}_{\epsilon_i>0}(\widehat{\mathrm{MtM}}^i_{\tau_i} - C^i_{\tau_i} - R_i\epsilon_i) = (1-R_i)\epsilon_i. \blacksquare\end{aligned}$$

As an immediate corollary to Lemma 3.1, denoting by $\boldsymbol{\delta}_t$ a Dirac measure at time t:

Lemma 3.2 *The cumulative stream of counterparty credit cash flows $\mathcal{C}$ satisfies, for $0 \le t \le \bar{\tau}$,*

$$\begin{aligned}d\mathcal{C}_t &= \sum_i (1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+ \boldsymbol{\delta}_{\tau_i}(dt) - \sum_i J^i_{\tau-}(1-R)(Q^i_\tau - \mathrm{PIM}^i_\tau)^- \boldsymbol{\delta}_\tau(dt)\\ d\mathcal{C}^{\tau-}_t &= \sum_i J_{\tau_i}(1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+ \boldsymbol{\delta}_{\tau_i}(dt)\\ \Delta_\tau \mathcal{C} &= \sum_{i;\tau_i=\tau} (1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+ \boldsymbol{\delta}_{\tau_i}(dt) - \sum_i J^i_{\tau-}(1-R)(Q^i_\tau - \mathrm{PIM}^i_\tau)^- \boldsymbol{\delta}_\tau(dt).\end{aligned} \tag{11}$$

3.2 Margining and Funding Schemes

Variation margin typically consists of cash that is re-hypothecable, meaning that received variation margin can be reused for funding purposes, and is remunerated at OIS by the receiving party. Initial margin typically consists of liquid assets deposited in a segregated account, such as government bonds, which naturally pay coupons or otherwise accrue in value. The poster of the collateral receives no compensation, except for the natural accrual or coupons of its collateral.

The derivative portfolio strategy of the bank needs funding for raising variation margin and initial margin that need to be posted as collateral. As happens in practice in the current regulatory environment, the back-to-back market hedge of the derivative portfolio of the bank is assumed to be with other financial institutions and attracts variation margin at zero threshold (i.e. is fully collateralized), so that the variation margin posted by the bank on its back-to-back hedge is constantly equal to $\sum_i J^i\mathrm{MtM}^i$. Hence, the bank posts $\sum_i J^i\mathrm{MtM}^i$ as VM on the back-to-back hedge and receives $\sum_i J^i\mathrm{VM}^i$ as VM on client trades.

Moreover the bank can use reserve capital (and also economic capital, which will be the topic of Sect. 5) as variation margin. Note that the marginal cost of capital for using capital as a funding source for variation margin is nil, because when one posts cash against variation margin, the valuation of the collateralized hedge is reset to zero and the total capital amount does not change. If instead the bank were to post capital as initial margin, the bank would record a "margin receivable" entry on its balance sheet, which however cannot contribute to capital since this asset is too illiquid and impossible to unwind without unwinding all underlying derivatives. Hence capital can be used as VM, while IM must be borrowed entirely.

Under the continuous reset assumption of this paper, the amount RC of reserve capital that can be used as VM coincides at all times with the theoretical CA value. The cash held by the bank, whether borrowed or received as variation margin, is deemed fungible across netting sets in a unique funding set. In conclusion,

$$\begin{aligned}(\text{VM funding needs})_t &= \Big(\sum_i J^i_t\mathrm{MtM}^i - \sum_i J^i_t\mathrm{VM}^i - \mathrm{CA}_t\Big)^+ = \Big(\sum_i J^i_t P^i_t - \mathrm{CA}_t\Big)^+ \\ (\text{IM funding needs})_t &= \sum_i J^i_t\mathrm{PIM}^i_t.\end{aligned} \tag{12}$$

We assume that the bank can invest at the OIS rate r_t and obtain unsecured funding at rate $(r_t+\lambda_t)$ for funding VM and $(r_t+\bar\lambda_t)$ for funding IM, via two bonds of different seniorities issued by the bank, with respective recoveries R and $\bar R$. Given our standing valuation setup, it must hold that

$$\lambda = (1-R)\gamma,\ \bar\lambda = (1-\bar R)\gamma, \tag{13}$$

where γ is the risk-neutral default intensity process of the bank. The reason why a blended spread $\bar\lambda < \lambda$ may arise for the funding of IM is the topic of Sect. 6.

The regulator says quite explicitly that the bank capital (reserve capital in particular) cannot be seen increasing as a consequence of the sole deterioration of the bank credit, all else being equal (see Albanese and Andersen (2014, Section 3.1)). In particular regulators decided that the CVA should be computed unilaterally as UCVA, given in the present setup as (cf. (9))

$$\mathrm{UCVA}_t = \mathbb{E}_t \sum_{t<\tau_i<T} \beta_t^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+,\ 0\le t\le\bar\tau, \tag{14}$$

as opposed to a first-to-default CVA with summation over $t<\tau_i\le\bar\tau$. We assume that the bank, in order to comply with this regulatory requirement, buys insurance from some risk-free third-party yielding an amount $\mathrm{UCVA}_{\tau-}$ at time τ in exchange of a continuously paid insurance fee $J_t\gamma_t\mathrm{UCVA}_t dt$. As we will see below, this implies that an amount UCVA_t sits in its RC account at any point in time (under our continuous reset assumption RC=CA), in agreement with the regulatory prescription.

We denote by $d\mu_t = \gamma_t dt + dJ_t$ the compensated jump-to-default martingale of the bank.

Lemma 3.3 *The cumulative stream of funding cash flows $\mathcal{F}$ satisfies, for $0 \le t \le \bar{\tau}$,*

$$
\begin{aligned}
&d\mathcal{F}_t = \big((1-R)(\sum_i J^i_{t-}P^i_{t-} - \mathrm{CA}_{t-})^+ + (1-\bar{R})(\sum_i J^i_{t-}\mathrm{PIM}^i_{t-}) + \mathrm{UCVA}_{t-}\big)d\mu_t \\
&d\mathcal{F}^{\tau-}_t = \big(\sum_i J^i_t P^i_t - \mathrm{CA}_t\big)^+\lambda_t dt + (\sum_i J^i_t \mathrm{PIM}^i_t)\bar{\lambda}_t dt + \mathrm{UCVA}_t\gamma_t dt \\
&-\Delta_\tau\mathcal{F} = (1-R)\big(\sum_i J^i_{\tau-}P^i_{\tau-} - \mathrm{CA}_{\tau-}\big)^+ + (1-\bar{R})(\sum_i J^i_{\tau-}\mathrm{PIM}^i_{\tau-}) + \mathrm{UCVA}_{\tau-}.
\end{aligned}
\tag{15}
$$

Proof. In view of the above description, we have

$$
\begin{aligned}
&d\mathcal{F}^{\tau-}_t = (\text{VM funding needs})_t\lambda_t dt + (\text{IM funding needs})_t\bar{\lambda}_t dt + \mathrm{UCVA}_t\gamma_t dt \\
&-\Delta_\tau\mathcal{F} = (1-R)(\text{VM funding needs})_{\tau-} + (1-\bar{R})(\text{IM funding needs})_{\tau-} + \mathrm{UCVA}_{\tau-},
\end{aligned}
$$

i.e.

$$
\begin{aligned}
d\mathcal{F}_t = &(\text{VM funding needs})_{t-}(\lambda_t dt + (1-R)dJ_t) \\
&+ (\text{IM funding needs})_{t-}(\bar{\lambda}_t dt + (1-\bar{R})dJ_t) + \mathrm{UCVA}_{t-}(\gamma_t dt + dJ_t).
\end{aligned}
$$

Hence, given (13),

$$
\begin{aligned}
d\mathcal{F}_t = \big(&(1-R)(\text{VM funding needs})_{t-} \\
&+ (1-\bar{R})(\text{IM funding needs})_{t-} + \mathrm{UCVA}_{t-}\big)d\mu_t.
\end{aligned}
$$

In view of (12), this yields (15). ∎

4 Bilateral Trading XVA Formulas and Loss Process

We work under the technical assumption that the martingales

$$
\mathbb{E}_t\big[\beta_{\tau_i}\mathbb{1}_{\{\tau_i<T\}}(1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+\big],\ i = 1,\ldots,n
$$

do not jump at time τ. This is a mild regularity assumption intended to ensure that:

Lemma 4.1 *For $t \le \bar{\tau}$:*

$$
\mathrm{UCVA}_t = \mathbb{E}_t\Big[\sum_{\{i;\, t<\tau_i<\bar{\tau}\}} \beta_t^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+ + \beta_t^{-1}\beta_\tau\mathbb{1}_{\{t<\tau<T\}}\mathrm{UCVA}_{\tau-}\Big]. \tag{16}
$$

Proof. Recalling the definition (14) of UCVA, we have, on $\{t \le \tau\}$:

$$
\begin{aligned}
\mathbb{E}_t[\beta_\tau\mathbb{1}_{\{t<\tau<T\}}\mathrm{UCVA}_{\tau-}] &= \mathbb{E}_t[\beta_\tau\mathbb{1}_{\{t<\tau<T\}}\lim_{s\uparrow\uparrow\tau}\mathrm{UCVA}_s] \\
&= \mathbb{E}_t\big[\beta_\tau\mathbb{1}_{\{t<\tau<T\}}\lim_{s\uparrow\uparrow\tau}\mathbb{E}_s\sum_{\{i;\, s<\tau_i<T\}}\beta_s^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+\big] \\
&= \mathbb{E}_t\Big[\beta_\tau\mathbb{1}_{\{t<\tau<T\}}\sum_i\lim_{s\uparrow\uparrow\tau}\Big(J^i_s\beta_s^{-1}\mathbb{E}_s\big[\beta_{\tau_i}\mathbb{1}_{\{\tau_i<T\}}(1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+\big]\Big)\Big] \\
&= \mathbb{E}_t\Big[\beta_\tau\mathbb{1}_{\{t<\tau<T\}}\sum_i J^i_{\tau-}\beta_\tau^{-1}\mathbb{E}_{\tau-}\big[\beta_{\tau_i}\mathbb{1}_{\{\tau_i<T\}}(1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+\big]\Big] \\
&= \mathbb{E}_t\Big[\mathbb{1}_{\{t<\tau<T\}}\sum_i J^i_{\tau-}\mathbb{E}_\tau\big[\beta_{\tau_i}\mathbb{1}_{\{\tau_i<T\}}(1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+\big]\Big] \\
&= \mathbb{E}_t\sum_{\{i;\, t<\tau\le\tau_i<T\}}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+,
\end{aligned}
$$

where our technical assumption was used in the next-to-last equality. As a consequence,

$$\begin{aligned}&\mathbb{E}_t \sum_{\{i;\, t<\tau_i<\bar{\tau}\}} \beta_t^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i}-\mathrm{RIM}^i_{\tau_i})^+ + \mathbb{E}_t\Big[\beta_t^{-1}\beta_{\tau}\mathbb{1}_{\{t<\tau<T\}}\mathrm{UCVA}_{\tau-}\Big]\\&\qquad = \mathbb{E}_t \sum_{\{i;\, t<\tau_i<T\}} \beta_t^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i}-\mathrm{RIM}^i_{\tau_i})^+ = \mathrm{UCVA}_t,\end{aligned}$$

by definition (14) of UCVA. ∎

Let

$$\mathrm{MVA}_t = \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s \bar{\lambda}_s \sum_i J^i_s \mathrm{PIM}^i_s ds,\ 0 \le t \le \bar{\tau}. \tag{17}$$

We denote by $\mathcal{L}^p$ the space of $\cdot^p$-integrable processes over $[0,\bar{\tau}]$, for any $p \ge 1$.

Theorem 4.1 *Assuming that* r *is bounded from below and that the processes* r, λ, UCVA, MVA *and* $\lambda(\sum_i J^i P^i - \mathrm{UCVA} - \mathrm{MVA})^+$ *are in* $\mathcal{L}^2$*:*
(i) *Contra-assets are given as*

$$\mathrm{CA} = \mathrm{UCVA} + \mathrm{FVA} + \mathrm{MVA}, \tag{18}$$

meant in the sense of the following backward stochastic differential equation (BSDE) for the CA *process:*

$$\begin{aligned}\mathrm{CA}_t = &\underbrace{\mathbb{E}_t \sum_{\{i;\, t<\tau_i<T\}} \beta_t^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i}-\mathrm{RIM}^i_{\tau_i})^+}_{\mathrm{UCVA}_t} + \underbrace{\mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s\bar{\lambda}_s \sum_i J^i_s \mathrm{PIM}^i_s ds}_{\mathrm{MVA}_t}\\ &+ \underbrace{\mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s\lambda_s\Big(\sum_i J^i_s P^i_s - \mathrm{CA}_s\Big)^+ ds}_{\mathrm{FVA}_t},\ 0 \le t \le \bar{\tau}.\end{aligned} \tag{19}$$

This BSDE is well-posed in $\mathcal{L}^2$ *and it therefore uniquely defines a square integrable contra-assets value process* CA*.*
(ii) *Contra-liabilities are given as*

$$\mathrm{CL} = \mathrm{FTDDVA} + \mathrm{CVA}^{\mathrm{CL}} + \mathrm{FDA} + \mathrm{MDA}, \tag{20}$$

namely

$$\begin{aligned}\mathrm{CL}_t = &\underbrace{\sum_i \mathbb{E}_t\Big[\beta_t^{-1}\beta_{\tau_i}\mathbb{1}_{\{t<\tau\le\tau_i\wedge T\}}(1-R)(Q^i_{\tau}-\mathrm{PIM}^i_{\tau})^-\Big]}_{\mathrm{FTDDVA}_t=\sum_i \mathrm{FTDDVA}^i_t}\\ &+\underbrace{\mathbb{E}_t \sum_{\{i;\, \tau\le\tau_i<T\}} \beta_t^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i}-\mathrm{RIM}^i_{\tau_i})^+}_{\mathrm{CVA}^{\mathrm{CL}}_t}\\ &+\underbrace{\mathbb{E}_t\Big[\beta_t^{-1}\beta_{\tau}\mathbb{1}_{\{\tau<T\}}(1-R)\Big(\sum_i J^i_{\tau-}P^i_{\tau-} - \mathrm{CA}_{\tau-}\Big)^+\Big]}_{\mathrm{FDA}_t=\mathrm{FVA}_t}\\ &+\underbrace{\mathbb{E}_t\Big[\beta_t^{-1}\beta_{\tau}\mathbb{1}_{\{\tau<T\}}(1-\bar{R})\Big(\sum_i J^i_{\tau-}\mathrm{PIM}^i_{\tau-}\Big)^+\Big]}_{\mathrm{MDA}_t=\mathrm{MVA}_t},\ 0 \le t \le \bar{\tau}.\end{aligned} \tag{21}$$

(iii) *The value of counterparty credit risk to the bank as a whole is given by*

$$\begin{aligned}\mathrm{CCR}_t = &\underbrace{\sum_i \mathbb{E}_t\Big[\beta_t^{-1}\beta_{\tau_i}\mathbb{1}_{\{t<\tau_i\leq\bar{\tau}\}}(1-R_i)(Q^i_{\tau_i}-\mathrm{RIM}^i_{\tau_i})^+\Big]}_{\mathrm{FTDCVA}_t=\sum_i \mathrm{FTDCVA}^i_t} \\ &-\underbrace{\sum_i \mathbb{E}_t\Big[\beta_t^{-1}\beta_{\tau_i}\mathbb{1}_{\{t<\tau\leq\tau_i\wedge T\}}(1-R)(Q^i_{\tau}-\mathrm{PIM}^i_{\tau})^-\Big]}_{\mathrm{FTDDVA}_t=\sum_i \mathrm{FTDDVA}^i_t},\ 0\leq t\leq\bar{\tau},\end{aligned} \tag{22}$$

i.e. we have

$$\mathrm{CA} = \underbrace{\mathrm{FTDCVA}-\mathrm{FTDDVA}}_{\mathrm{CCR}} + \underbrace{\mathrm{FTDDVA}+\mathrm{CVA}^{\mathrm{CL}}+\mathrm{FDA}+\mathrm{MDA}}_{\mathrm{CL}}, \tag{23}$$

where the different terms are detailed in (19), (21) *and* (22).

(iv) *The loss process L satisfies the following forward SDE on* $[0,\bar{\tau}]$*:*

$$\begin{aligned}&L_0 = z \text{ (the accrued trading loss of the bank at time 0) and, for } t\in(0,\bar{\tau}],\\ &dL_t = d\mathrm{CA}_t + \sum_i J_{\tau_i}(1-R_i)(Q^i_{\tau_i}-\mathrm{RIM}^i_{\tau_i})^+\boldsymbol{\delta}_{\tau_i}(dt)\\ &+\Big(\lambda_t\big(\sum_i J^i_tP^i_t-\mathrm{CA}_t\big)^+ + \bar{\lambda}_t\sum_i J^i_t\mathrm{PIM}^i_t + \gamma_t\mathrm{UCVA}_t - r_t\mathrm{CA}_t\Big)dt.\end{aligned} \tag{24}$$

(v) *The all-inclusive XVA add-on to the entry price for a new deal, which we call funds transfer price (FTP), appears as*

$$\begin{aligned}\mathrm{FTP} &= \underbrace{\Delta\mathrm{UCVA}+\Delta\mathrm{FVA}+\Delta\mathrm{MVA}}_{\Delta\mathrm{CA}} + \underbrace{\Delta\mathrm{KVA}}_{\textit{Risk premium}}\\ &= \underbrace{\Delta\mathrm{FTDCVA}-\Delta\mathrm{FTDDVA}}_{\Delta\mathrm{CCR}}\\ &\quad+\underbrace{\Delta\mathrm{FTDDVA}+\Delta\mathrm{CVA}^{\mathrm{CL}}+\Delta\mathrm{FDA}+\Delta\mathrm{MDA}}_{\Delta\mathrm{CL}} + \underbrace{\Delta\mathrm{KVA}}_{\textit{Risk premium}},\end{aligned} \tag{25}$$

computed on an incremental run-off basis, where all the underlying XVA metrics as well as the processes L to be used as input data in the economic capital and KVA computations are defined as in parts (i) through (iv) relative to the portfolios with and without the new deal.

Proof. **(i)** By the first line in (3) and Lemmas 3.2–3.3, the CA process satisfies, for $t\in[0,\bar{\tau}]$,

$$\begin{aligned}\mathrm{CA}_t = &\mathbb{E}_t\sum_{\{i;\,t<\tau_i<\bar{\tau}\}}\beta_t^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i}-\mathrm{RIM}^i_{\tau_i})^+\\ &+\mathbb{E}_t\int_t^{\bar{\tau}}\beta_t^{-1}\beta_s\Big(\lambda_s\big(\sum_i J^i_sP^i_s-\mathrm{CA}_s\big)^+ + \bar{\lambda}_s\big(\sum_i J^i_s\mathrm{PIM}^i_s\big)+\gamma_s\mathrm{UCVA}_s\Big)ds.\end{aligned} \tag{26}$$

By the martingale property of $d\mu_t = \gamma dt + dJ_t$, this is equivalent to

$$\begin{aligned}
\mathrm{CA}_t = \mathbb{E}_t \sum_{\{i;\, t<\tau_i<\bar{\tau}\}} \beta_t^{-1}\beta_{\tau_i}(1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+ + \mathbb{E}_t\Big[\beta_t^{-1}\beta_\tau \mathbb{1}_{\{t<\tau<T\}}\mathrm{UCVA}_{\tau-}\Big] \\
+ \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s\bar{\lambda}_s\big(\sum_i J^i_s \mathrm{PIM}^i_s\big)ds + \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s\lambda_s\Big(\sum_i J^i_s P^i_s - \mathrm{CA}_s\Big)^+ ds,\ 0 \le t \le \bar{\tau}.
\end{aligned} \tag{27}$$

But Lemma 4.1 implies that the first line in (27) is UCVA_t, so that (27) is equivalent to the CA BSDE (19). Moreover, since (UCVA + MVA) is an exogenous process in $\mathcal{L}^2$, the CA BSDE (19) in $\mathcal{L}^2$ is equivalent to defining a CA process through (18), for an FVA process in $\mathcal{L}^2$ defined in the first place through the following BSDE:

$$\beta_t \mathrm{FVA}_t = \mathbb{E}_t \int_t^{\bar{\tau}} \beta_s\lambda_s\Big(\sum_i J^i_s P^i_s - \mathrm{UCVA}_s - \mathrm{MVA}_s - \mathrm{FVA}_s\Big)^+ ds,\ 0 \le t \le \bar{\tau}. \tag{28}$$

As a result, in order to prove (i), we need only to show that the FVA BSDE (28) is well-posed in $\mathcal{L}^2$ under the assumptions of the theorem. Let $X_t = \sum_i J^i_t P^i_t - \mathrm{UCVA}_t - \mathrm{MVA}_t$. In terms of the coefficient

$$g_t(y) = \lambda_t\Big(X_t - y\Big)^+ - r_t y,\ y \in \mathbb{R}, \tag{29}$$

the FVA BSDE (28) is rewritten as

$$\mathrm{FVA}_t = \mathbb{E}_t \int_t^{\bar{\tau}} g_s(\mathrm{FVA}_s)ds,\ 0 \le t \le \bar{\tau}. \tag{30}$$

For any real $y, y' \in \mathbb{R}$ and $t \in [0, \bar{\tau}]$, we have

$$\begin{aligned}
\big(g_t(y) - g_t(y')\big)(y - y') &= -r_t(y-y')^2 + \lambda_t(y-y')\Big(\big(X_t - y\big)^+ - \big(X_t - y'\big)^+\Big) \\
&\le -r_t(y-y')^2 \le C(y-y')^2,
\end{aligned}$$

for some constant C (having assumed r bounded from below and recalling $\lambda \ge 0$), so that the coefficient g satisfies the so-called monotonicity condition. Moreover, for $|y| \le \bar{y}$, we have:

$$|g_\cdot(y) - g_\cdot(0)| = \lambda\big(X - y\big)^+ - ry - \lambda X^+ \le (\lambda + |r|)\bar{y}.$$

Hence, assuming that r, λ and $\lambda\big(\sum_i J^i P^i - \mathrm{UCVA} - \mathrm{MVA}\big)^+ = \lambda X^+$ are in $\mathcal{L}^2$, the following integrability conditions hold:

$$\sup_{|y|\le\bar{y}} |g_\cdot(y) - g_\cdot(0)| \in \mathcal{L}^1,\ \text{for any } \bar{y} > 0, \qquad \text{and} \qquad g_\cdot(0) \in \mathcal{L}^2.$$

Therefore, by application of the general filtration BSDE results of Kruse and Popier (2016, Sect. 5), the FVA BSDE (30) is well-posed in $\mathcal{L}^2$, where well-posedness includes existence, uniqueness and comparison.
(ii) follows from the second line in (3) and Lemmas 3.2–3.3.
(iii) The third line in (3) and Lemmas 3.2–3.3 yield (22), detailed as (23).
(iv) follows from (4) and Lemmas 3.2–3.3.
(v) immediately follows from (i) through (iv) by application of the generic formula (1). ∎

Note that our back-to-back hedge setup results in a direct derivation of the CA BSDE, as opposed to, in most of the previous XVA BSDE literature, a CA BSDE derived in two steps, as the difference between a linear equation for the mark-to-market of the portfolio ignoring counterparty risk and funding costs, minus the BSDE for the "risky value" of the portfolio.

We emphasize that the formulas (3) and in turn Theorem 4.1(i)–(iv) are derived from a pure valuation perspective on CA. In most other former XVA references in the literature, XVA equations are based on hedging arguments. The reason is that previous XVA works were not considering KVA yet. Under our approach, the KVA is the risk premium for the market incompleteness related to the impossibility for the bank of replicating counterparty default losses. Hence, for consistency, our KVA treatment requires a pure valuation (as opposed to hedging) view on CA.

In Theorem 4.1, the CA process (similar comments apply to the FVA process) is viewed as the solution to a BSDE through which it depends on all the other processes in the equation, including itself. This might seem in contradiction with our standing linear valuation rule or with the additive appearance of the "formula' (3). The two points of view are in fact equally valid. The reconciliation between the two comes from the fact that CA is together the value process of contra-assets, by definition, and the amount RC in the reserve capital account, under our continuous reset assumption RC=CA. As RC is a deduction to the VM funding needs, which appear in the CA equation, it follows that CA depends on itself. Hence we can see CA either from a linear perspective, as conditional expectation of the future cash flows that it is valuing, RC (i.e. CA itself) included, or as a solution to a BSDE. Of course technically (mathematically and numerically) one needs to solve a BSDE.

The CA BSDE (19) is independent of the initial condition $L_0 = z$, which therefore does not affect CA, nor a KVA as of (5). Hence the value of the unknown constant z is immaterial in all XVA computations.

Without the regularity assumption made at the beginning of Sect. 4, we would end up with, instead of UCVA_t in CA_t in (19), the expression given by the right-hand side in (16) .

4.1 Connection with Duffie and Huang (1996)'s Formula

The formula (22) for the valuation of counterparty credit risk is derived in Duffie and Huang (1996) in the limit case of a perfect market (complete counterparty credit risk market without trading restrictions). Theorem 4.1(iii) extends the validity of this formula for the valuation (CCR) of counterparty credit risk from the point of view of the bank of the whole in our incomplete market setup.

Formula (22) is symmetrical, i.e. consistent with the law of one price, in the sense that each term ($\mathrm{FTDCVA}^i - \mathrm{FTDDVA}^i$) in (22) corresponds to the negative of the analogous quantity considered from the point of view of the counterparty i. It only involves the first-to-default CVAs and DVAs, where the counterparty default losses are only considered until the first occurrence of a default of the bank or its counterparty in the deal. This is consistent with the fact that later cash flows will, as first emphasised in Duffie and Huang (1996), Bielecki and Rutkowski (2002) and Brigo and Capponi (2008), not be paid in principle.

Since the presence of collateral has a direct reducing impact on FTDCVA/DVA, this formula may give the impression that collateralization achieves a reduction in counterparty credit risk at no cost to either the bank or the clients. However, in the present incomplete market setup, the value CCR from the point of view of the bank as a whole ignores the misalignement of interest between the shareholders and the creditors of a bank. Theorems 4.1(i) and (ii) give explicit decompositions of the respective cost of counterparty credit risk to shareholders (CA) and of the wealth transfer (CL) triggered from the shareholders to the creditors by the impossibility for the bank to hedge its own jump-to-default exposure. Due to the latter and to the impossibility for the bank to replicate counterparty default losses, these contra-liabilities (CL) as well as the cost of capital (KVA) are material to shareholders and need to be reflected in entry prices on top of the fair valuation (CCR) of counterparty credit risk (cf. (1)).

5 Using Economic Capital as Variation Margin

In this section we account for the FVA reduction provided by the possibility for a bank to post economic capital, on top of reserve capital already included in the above, as variation margin. Note that, in principle, uninvested capital of the bank could be used for VM as well. But, since the amount of uninvested capital is not known and could as well be zero in the future, capital is conservatively taken in FVA computations as (RC+EC).

The quantity $\mathrm{EC} = \mathrm{EC}_t(L)$ corresponds to the amount of economic capital required by the loss process L (cf. (5)-(6) and Albanese and Crépey (2016, Theorem 6.1)). Accounting for the use of EC as VM, the VM funding needs are reduced from $(\sum_i J^i P^i - \mathrm{CA})^+$to $(\sum_i J^i P^i - \mathrm{EC}(L) - \mathrm{CA})^+$ in (12). Lemma 3.3 is still valid provided one replaces $(\sum_i J^i P^i - \mathrm{CA})^+$ by $(\sum_i J^i P^i - \mathrm{EC}(L) - \mathrm{CA})^+$ in (15).

As a consequence, instead of an exogenous CA value process as of (19) feeding the dynamics (24) for L, we obtain the following FBSDE system, made of a forward SDE for L coupled with a backward SDE for the CA value process:

$$\begin{aligned}
&L_0 = z \text{ and, for } t \in (0, \bar{\tau}],\\
&dL_t = d\mathrm{CA}_t + \sum_i J_{\tau_i}(1 - R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+ \boldsymbol{\delta}_{\tau_i}(dt)\\
&+\Big(\lambda_t\big(\sum_i J^i_t P^i_t - \mathrm{EC}_t(L) - \mathrm{CA}_t\big)^+ + \bar{\lambda}_t \sum_i J^i_t \mathrm{PIM}^i_t + \gamma_t \mathrm{UCVA}_t - r_t \mathrm{CA}_t\Big)dt,
\end{aligned} \tag{31}$$

where

$$\begin{aligned}
\mathrm{CA}_t = &\underbrace{\mathbb{E}_t \sum_{t<\tau_i<T} \beta_t^{-1}\beta_{\tau_i}(1 - R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+}_{\mathrm{UCVA}_t} + \underbrace{\mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s \bar{\lambda}_s \sum_i J^i_s \mathrm{PIM}^i_s ds}_{\mathrm{MVA}_t}\\
&+ \underbrace{\mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s \lambda_s \Big(\sum_i J^i_s P^i_s - \mathrm{EC}_s(L) - \mathrm{CA}_s\Big)^+ ds}_{\mathrm{FVA}_t},\ 0 \le t \le \bar{\tau}.
\end{aligned} \tag{32}$$

However, the results of Crépey, Élie, and Sabbagh (2016) show that, accounting for the use of economic capital as variation margin, Theorem 4.1 is still valid (at least for λ bounded) provided one replaces $(\sum_i J^i P^i - \mathrm{CA})^+$ by $(\sum_i J^i P^i - \mathrm{EC}(L) - \mathrm{CA})^+$ in all the formulas.

6 Specialist Lending of Initial Margin

If IM is unsecurely funded by the bank, then $\bar{\lambda} = \lambda$. However, instead of an unsecured IM funding scheme, one can consider a more efficient scheme where initial margin is funded through a specialist lender that lends only IM and, in case of default of the bank, receives back the portion of IM unused to cover losses.

Hence the exposure of the specialist lender to the default of the bank is $(1 - R)\sum_i J^i_\tau\big((Q^i_\tau)^- \wedge \mathrm{PIM}^i_\tau\big)$. Recalling the risk-neutral valuation condition $\lambda = (1-R)\gamma$ in (13), where γ is the risk-neutral default intensity of the bank, the ensuing instantaneous IM funding charge for the bank is, for $t \in [0, \bar{\tau}]$,

$$\gamma_t(1 - R)\sum_i J^i_t\big((Q^i_t)^- \wedge \mathrm{PIM}^i_t\big) = \lambda_t \sum_i J^i_t\big((Q^i_t)^- \wedge \mathrm{PIM}^i_t\big)$$

(assuming here for simplicity $\mathbb{G}$ predictable processes Q^i and PIM^i).

By identification with the general form $\bar{\lambda}_t \sum_i J^i_t \mathrm{PIM}^i_t$ of IM costs in this paper, this specialist lending scheme corresponds to

$$\bar{\lambda}_t = \frac{\sum_i J^i_t\big((Q^i_t)^- \wedge \mathrm{PIM}^i_t\big)}{\sum_i J^i_t \mathrm{PIM}^i_t}\lambda_t \leq \lambda_t.$$

In fact, given the very conservative (such as 99% or more) levels of IM prescribed by the regulation for bilateral transactions in the coming years, such a blended spread $\bar{\lambda}_t$ is typically much smaller than the unsecured funding spread λ. Equivalently, the blended recovery rate $\bar{R}$ in (13), i.e.

$$\bar{R}_t = (1 - \frac{\bar{\lambda}_t}{\lambda_t}) + \frac{\bar{\lambda}_t}{\lambda_t}R$$

(noting that everything in the paper can be readily extended to a $\mathbb{G}$ predictable recovery rate process $\bar{R}$), is much larger then the unsecured borrowing recovery rate R.

Note that, for the argument to be valid, the IM lender does not need to anticipate the nature of future trades, which in the case of a market maker, such as a bank, would be impossible. The argument is robust and independent of future dealings. The IM lender simply needs to know (which is public regulatory information) that the collateral posted by the bank is supposed to be sufficient to cover losses in the 99% of the cases, no matter what trades are entered in the future.

Such an IM funding policy is not a violation of pari passu rules, just as repo or mortgages are not. It is just a form of collateralised lending, which does not transfer wealth from senior creditors in the baseline case.

For similar ideas regarding VM, see Albanese, Brigo, and Oertel (2013). However, such funding schemes are much more difficult to implement for VM because VM is far larger and more volatile than IM.

7 KVA Is Not a CET1 Deduction

Despite what the "valuation adjustment" terminology fallaciously induces one to believe, our KVA (better named risk margin) as of (5) is not part of the value of the derivative portfolio, but only part of entry prices as a risk premium. As risk margin is retained earnings meant to be released to bank shareholders, the KVA in our sense does not belong to the balance-sheet as a liability, at least not statically as part of contra-assets. But, in some sense, our KVA is a measure of the fluctuations of the balance sheet (i.e. of the process $(-L)$, see Albanese and Crépey (2016, Section 4) for more details).

By contrast, in Green et al. (2014) and Green and Kenyon (2016), and discussed in some theoretical actuarial literature (see Salzmann and Wüthrich (2010, Sect. 4.4)), the KVA is instead treated as part of the value of the derivative portfolio, so that there is a single account for reserve capital and risk margin altogether and a further KVA contra-asset.

In our notation, the KVA in their sense would correspond to the value of an additional $h\mathrm{EC}_t dt$ cash-flow in the loss process (24). But the nature and meaning of this cash-flow are not clear. As the KVA is loss-absorbing, hence part of EC, shareholder capital at risk reduces to SCR=EC−KVA, so that putting $h(\mathrm{EC}_t - \mathrm{KVA}_t)dt$ instead of $h\mathrm{EC}_t dt$ would look a bit better. But then the KVA itself would appear in the modified loss process (24), making the resulting CA BSDE intractable.

Moreover, based on such premises, one is forced to focus on regulatory instead of economic capital in KVA computations. Otherwise, forward starting one-year-ahead fluctuations of the KVA should be simulated for capital and in turn KVA calculation, which would both involve a conceptual circularity and be intractable numerically. Using regulatory instead of economic capital is motivated by practical considerations but is less self-consistent. It loses the connection, established from structural balance-sheet considerations in Albanese and Crépey (2016, Section 4), whereby the right KVA input should be the contra-asset mis-hedge loss process L (cf. (4)-(5)). This connection is what makes the KVA and CA equations, hence the XVA problem as a whole, a self-contained and well-posed problem.

Remark 7.1 For KVA computations entailing capital projections over decades, an equilibrium view based on Pillar II economic capital (EC) is more attractive than the ever-changing Pillar I regulatory charges supposed to approximate it (see Pykhtin (2012)). However, Pillar I regulatory capital requirements could be incorporated into our approach, if desired, by replacing ES by its maximum with the regulatory capital pertaining to the portfolio. ∎

In addition, Green et al. (2014) and Green and Kenyon (2016) derive their KVA-inclusive CA equation based on a replication argument, whereas the main motivation for capital requirements is that credit markets are very incomplete and hedging is not possible.

Example 7.1 The fact that regulators require CVA reserve capital to be held reflects the fact that there are no CDS contracts for most names and as a consequence there is credit risk which cannot be hedged. ∎

The core function of banks in the economy is to take credit risk notwithstanding this fundamental incompleteness and, for this purpose, regulators require capital. From this point of view, deriving a KVA equation from replication arguments seems self-contradictory. A KVA-inclusive CA equation similar to the ones in Green et al. (2014) or Green and Kenyon (2016) is derived in an expectation setup in Elouerkhaoui (2016).

8 The XVA Algorithm

Under our analysis in this paper, prices of individual trades are no longer computable in isolation. Instead, they can only be computed incrementally with respect to existing endowment. This is a major innovation in mathematical finance, which so far has mainly revolved on option pricing theory for individual payoffs in isolation. There are antecedents in this direction in the XVA literature, but the new KVA dimension pushes this logic to an unprecedented level. By current industry practice, XVA desks are the first consulted desks in all major trades, whose pros and cons are assessed in terms of incremental XVAs (incremental KVA in particular).

This portfolio view raises computational and modeling challenges. Our XVA approach can be implemented by means of nested Monte Carlo simulations for approximating the loss process L required as input data in the KVA computations. Contra-assets (and contra-liabilities if wished) are computed at the same time.

Since one of our goals in the numerics is to emphasize the impact on the FVA of the funding sources provided by reserve capital and economic capital, we consider the FBSDE (31)–(32) which accounts for the use of EC (on top of RC) as VM. Let

$$\mathrm{FVA}_t^{(0)} = \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s \lambda_s \Big(\sum_i J_s^i P_s^i \Big)^+ ds, \tag{33}$$

which corresponds to an FVA that accounts only for the re-hypothecation of the variation margin received on hedges, but ignores the FVA deductions reflecting the possible use of reserve and economical capital as VM. Based on nested simulated paths, we compute UCVA, MVA and $\mathrm{FVA}^{(0)}$ at all nodes of the primary simulation grid. We consider the following Picard iteration in the search for the solution to (31)–(32): $L^{(0)} = z$, $\mathrm{CA}^{(0)} = \mathrm{UCVA} + \mathrm{FVA}^{(0)} + \mathrm{MVA}$ with $\mathrm{FVA}^{(0)}$ as of (33), $\mathrm{KVA}^{(0)} = 0$ and, for $k \geq 1$,

$$\begin{aligned}
&L_0^{(k)} = z \text{ and, for } t \in (0, \bar{\tau}],\\
&dL_t^{(k)} = d\mathrm{CA}_t^{(k-1)} dt - r_t \mathrm{CA}_t^{(k-1)} dt + \sum_i J_{\tau_i}(1-R_i)(Q^i_{\tau_i} - \mathrm{RIM}^i_{\tau_i})^+ \boldsymbol{\delta}_{\tau_i}(dt)\\
&+ \lambda_t \Big(\sum_i J_t^i P_t^i - \max\big(\mathrm{ES}_t(L^{(k-1)}), \mathrm{KVA}_t^{(k-1)}\big) - \mathrm{CA}_t^{(k-1)} \Big)^+ dt\\
&+ \bar{\lambda}_t \sum_i J_t^i \mathrm{PIM}_t^i dt + \gamma_t \mathrm{UCVA}_t dt\\
&\mathrm{CA}_t^{(k)} = \mathrm{UCVA}_t + \mathrm{FVA}_t^{(k)} + \mathrm{MVA}_t, \text{ where } \mathrm{FVA}_t^{(k)} =\\
&\qquad \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s \lambda_s \big(\sum_i J_s^i P_s^i - \max\big(\mathrm{ES}_s(L^{(k)}), \mathrm{KVA}_s^{(k-1)}\big) - \mathrm{CA}_s^{(k-1)} \big)^+ ds\\
&\mathrm{KVA}_t^{(k)} = h \mathbb{E}_t \int_t^{\bar{\tau}} e^{-\int_t^s (r_u + h) du} \max\big(\mathrm{ES}_s(L^{(k)}), \mathrm{KVA}_s^{(k-1)}\big) ds.
\end{aligned} \tag{34}$$

Controlling this iteration for establishing the convergence of $(L^{(k)}, \mathrm{CA}^{(k)}, \mathrm{KVA}^{(k)})$ to the solution $(L, \mathrm{CA}, \mathrm{KVA})$ of (31)-(32) and (5)-(6) is challenging due to the terms of the form $\mathrm{ES}_t(L)$. This convergence is studied in Crépey et al. (2016).

Numerically, one iterates (34) as many times as is required to reach a fixed point within a preset accuracy. In the case studies we considered, one iteration ($k = 1$) was found sufficient. In other words, the KVA is computed based on the linear formula

(5) with $\mathrm{ES}_s(L^{(1)})$ instead of $\mathrm{EC}_s(L)$ there and a refined FVA is obtained as a value $\mathrm{FVA}_0 \approx \mathrm{FVA}_0^{(1)}$ accounting for the use of reserve capital and economic capital as VM. A second iteration did not bring significant change, as:

- In (31)-(32), the FVA feeds into EC only through FVA volatility and EC feeds into FVA through a capital term which is typically not FVA dominated.
- In (5)-(6), in most cases as in Figure 2, we have that EC = ES. The inequality only stops holding when the hurdle rate h is very high and the term structure of EC starts very low and has a sharp peak in a few years, which is quite unusual for a portfolio held on a run-off basis, as considered in XVA computations, which tends to amortize in time.

It could be that particular portfolios and parameter choices would necessitate two or more iterations. We did not encounter such situations and did not try to build artificial ones.

However, going even once through (34) necessitates the conditional risk measure simulation of $\mathrm{ES}_t(L^{(1)})$. On realistically large portfolios, some approximation is required for the sake of tractability. Namely, the simulated paths of $L^{(1)}$ are used for inferring a deterministic term structure

$$\mathrm{ES}_{(1)}(t) \approx \mathrm{ES}_t(L^{(1)}) \tag{35}$$

of economic capital, obtained by projecting in time instead of conditioning with respect to $\mathcal{G}_t$ in ES, i.e. taking the 97.5% expected shortfall of $\int_s^{s+1} \beta_s^{-1}\beta_u dL_u$ conditional on $\{\tau > (s+1)\}$ instead of, in principle, $\mathcal{G}_s \vee \{\tau > (s+1)\}$ (see the explanation following (5)). Simulating the full-flesh conditional expected shortfall process would involve not only nested, but doubly-nested Monte Carlo simulation, because of the conditional one-year-ahead $\mathrm{CA}^{(0)}$ fluctuations that are part of the conditional one-year-ahead fluctuations of the loss process $L^{(1)}$.

Note that, if a corporate holds a bank payable, it typically has an appetite to close it, receive cash, and restructure the hedge otherwise with a par contract (the bank would agree to close the deal as a market maker, charging fees for the new trade). Because of this natural selection, a bank is mostly in the receivables in its derivative business with corporates. Hence, the tail-fluctuations of its loss process L are mostly driven by the counterparty default events rather than by the volatility of the underlying market exposure. As a consequence, working with a deterministic term structure approximation $\mathrm{ES}_{(1)}(t)$ of economic capital should be acceptable. If, by exception, the derivative portfolio of a bank is mostly in the payables, then all the XVA numbers are small and matter much less anyway.

Remark 8.1 A similar argument is sometimes used to defend a symmetric FVA (or SFVA) approach, such as, instead of FVA_t in (19):

$$\mathrm{SFVA}_t = \mathbb{E}_t \int_t^{\bar{\tau}} \beta_t^{-1}\beta_s \widetilde{\lambda}_s \Big(\sum_i J_s^i P_s^i\Big) ds,\ 0 \le t \le \bar{\tau}, \tag{36}$$

for some VM blended funding spread $\widetilde{\lambda}_t$ (cf. Piterbarg (2010), Burgard and Kjaer (2013) and the discussion in Andersen et al. (2016)). This explicit, linear SFVA formula can be implemented by standard (non-nested) Monte Carlo simulations. For a suitably chosen blended spread $\widetilde{\lambda}_t$, the equation yields reasonable results in case of a typical bank portfolio dominated by unsecured receivables. However, in the case of a portfolio dominated by unsecured payables, this equation could yield a negative FVA, i.e. an FVA benefit, proportional to the own credit spread of the bank, which is a not acceptable from a regulatory point of view. ∎

9 Numerical Results

To illustrate our XVA methodology, we present two XVA case studies on fixed-income and foreign-exchange portfolios. Toward this end we use the GARCH-type market and credit portfolio models of Albanese, Bellaj, Gimonet, and Pietronero (2011) calibrated to the relevant market data.

We use nested simulation with primary scenarios and secondary scenarios generated under the risk neutral measure $\mathbb{Q}$ calibrated to derivative data using broker datasets for derivative market data.

All the computations are run using a 4-socket server for Monte Carlo simulations, Nvidia GPUs for algebraic calculations and Global Valuation Esther as simulation software. Using this super-computer and GPU technology the whole calculation takes a few minutes for building the models, followed by a nested simulation time in the order of about an hour for processing a billion scenarios on a real-life bank portfolio.

We assume a hurdle rate $h = 10.5\%$ and no variation or initial margins on the portfolio (but perfect variation-margining on the portfolio back-to-back hedge). In particular, the MVA numbers are all equal to zero and hence not reported in the tables below. We take $Q^i = P^i$ in all the counterparty credit exposures (9)-(10).

9.1 Toy Portfolio Results

We first consider a portfolio of ten USD currency fixed-income swaps depicted in Table 1, on the date of 11 January 2016. The nominal of each swap is 10^4. The swaps are traded with four counterparties $i = 1, \ldots, 4$, with 40% recovery rate and credit curves as of Figure 1.

We use 20,000 primary scenarios up to 30 years in the future run on 54 underlying time point with 1,000 secondary scenarios starting from each primary simulation node, which amounts to a total of $20,000 \times 54 \times 1,000 =$ 1,080 million scenarios. In this toy portfolio case the whole calculation takes roughly ten minutes to run, including two to three minutes for building the pre-calibrated market and credit models.

The corresponding XVA results are displayed in the left panel of Table 2. Since the portfolio is not collateralized, its UCVA is quite high as compared with the nominal (10^4) of each swap. But its KVA is even higher. Note that given our deterministic term structure approximation (35) for expected shortfalls, the computation of the KVA reduces to a deterministic time-integration, which is why there is no related standard error in Table 2. The number $\mathrm{FVA}_0^{(0)}$, accounting only for re-hypothecation of variation margin received on hedges, amounts to \$73.87. However, if we consider the additional funding sources due to economic capital and reserve capital, we arrive at an FVA figure of $\mathrm{FVA}_0^{(1)}$=\$3.87 only. The FTDCVA and FTDDVA metrics, whose difference corresponds to the fair valuation CCR of counterparty credit risk (cf. Theorem 4.1(iii)), are also shown for comparison.

The right panel of Table 2 shows the incremental XVA results when the fifth (resp. ninth) swap in Table 1 is last added in the portfolio. Note that, under an

Mat.	Receiver Rate	Payer Rate	i
10y	Par 6M	LIBOR 3M	3
10y	LIBOR 3M	Par 6M	2
5y	Par 6M	LIBOR 3M	2
5y	LIBOR 3M	Par 6M	3
30y	Par 6M	LIBOR 3M	2
30y	LIBOR 3M	Par 6M	1
2y	Par 6M	LIBOR 3M	1
2y	LIBOR 3M	Par 6M	4
15y	Par 6M	LIBOR 3M	1
15y	LIBOR 3M	Par 6M	4

Table 1: Toy portfolio of swaps (the nominal of each swap is $\$10^4$).

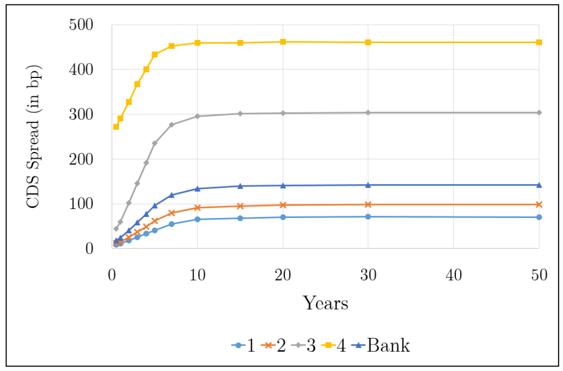

Figure 1: Credit curves of the bank and its four conterparties.

incremental run-off XVA methodology, introducing financial contracts one after the other in one or the reverse order in the portfolio at time 0 would end-up in the same aggregated incremental FTP amounts for the bank, equal to its "portfolio FTP" (1), but in different FTPs for each given contract and counterparty.

Interestingly enough, all the incremental XVAs of Swap 9 (and also the incremental FVA of Swap 5) are negative. Hence, Swap 9, when added last, is XVA profitable to the portfolio, meaning that a price maker should be ready to enter the swap for less than its mark-to-market value, assuming it is already trading the rest of the portfolio: the corresponding FTP amounts to \$-85.82, versus \$202.87 in the case of Swap 5.

	\$Value	SE
UCVA_0	471.23	0.46%
$\mathrm{FVA}_0^{(0)}$	73.87	1.06%
FVA_0	3.87	4.3%
KVA_0	668.83	N/A
FTDCVA_0	372.22	0.46%
FTDDVA_0	335.94	0.51%

	Swap 5	Swap 9
$\Delta\mathrm{UCVA}_0$	155.46	-27.17
$\Delta\mathrm{FVA}_0^{(0)}$	-85.28	-8.81
$\Delta\mathrm{FVA}_0$	-80.13	-5.80
$\Delta\mathrm{KVA}_0$	127.54	-52.85
$\Delta\mathrm{FTDCVA}_0$	98.49	-23.83
$\Delta\mathrm{FTDDVA}_0$	122.91	-80.13

Table 2: Toy portfolio. *Left*: XVA values and standard relative errors (SE). *Right*: Respective impacts when Swaps 5 and 9 are added last in the portfolio.

9.2 Large Portfolio Results

We now consider a representative portfolio with about 2,000 counterparties, 100,000 fixed income trades including swaps, swaptions, FX options, inflation swaps and CDS trades.

We use 20,000 primary scenarios up to 50 years in the future run on 100 underlying time points, with 1,000 secondary scenarios starting from each primary simulation node, which amounts to a total of two billion scenarios. Using super-computer and GPU technologies, the whole calculation takes about 2 hours.

Table 3 shows the XVA results for the large portfolio. The FVA is much smaller than the KVA, especially after accounting for the economic and reserve capital funding

XVA	$Value
UCVA_0	242 M
$\mathrm{FVA}_0^{(0)}$	126 M
FVA_0	62 M
KVA_0	275 M
FTDCVA	194 M
FTDDVA	166 M

Table 3: XVA values for the large portfolio.

sources. The KVA amounts to $275 M, which makes it the greatest of the XVA numbers. Figure 2 shows the term structure of economic capital along with the term structure of the KVA obtained by a deterministic term structure approximation $\mathrm{ES}_{(1)}$ as of (35) for economic capital and by the linear KVA formula (5) with $\mathrm{ES}_{(1)}$ instead of EC there. Such a term structure of economic capital, with a starting hump followed by a slow decay after 2 or 3 years, is rather typical of an investment bank derivative portfolio assumed held on a run-off basis until its final maturity, where the bulk of the portfolio consists of trades with 3y to 5y maturity. In relation with the second point made after (34), note that the KVA computed by the linear formula (5) based on this term structure of economic capital is below the latter at all times.

The funding needs reduction achieved by EC and RC $=$ CA $=$ UCVA$+$FVA is also shown in Figure 3 by the FVA blended curve. This is the FVA funding curve which, whenever applied to the FVA computed neglecting the impact of economic and reserve capital, gives rise to the same term structure for the forward FVA as the calculation carried out with the CDS curve $\lambda(t)$ of the bank as the funding curve but accounting for the economic and reserve capital funding sources. This blended curve is often inferred by consensus estimates based on the Markit XVA service. However, here it is computed from the ground up based on full-fledged capital projections.

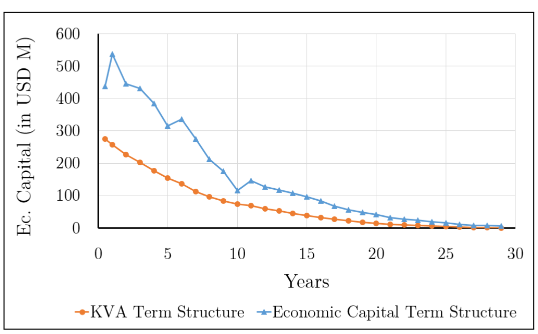

Figure 2: Term structure of economic capital compared with the term structure of KVA.

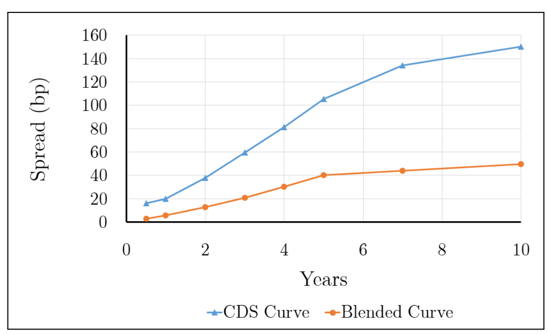

Figure 3: FVA blended funding curve computed from the ground up based on capital projections.

10 Conclusions

To conclude this paper we put its main technical insights in perspective with the existing XVA literature.

Theorem 4.1 yields a complete specification of all the XVA metrics and of the loss process L required as input data in the KVA computations, in the case of a bank engaged in bilateral trade portfolios. It identifies the FTP (all-inclusive XVA add-on to the entry price) of a new trade as its incremental (UCVA + FVA + MVA + KVA), the difference with the complete market formula (FTDCVA − FTDDVA) being explained by the contra-liabilities (CL) wealth transfer and the KVA risk premium triggered by the impossibility for the bank to replicate jump-to-default exposures.

Our FVA is defined asymmetrically since in no way can we recognise, even approximately, a positive funding benefit to excess capital at hand in the future. Symmetric variants of the FVA have been advocated in Piterbarg (2010), Burgard and Kjaer (2013) and Andersen, Duffie, and Song (2016), on the premise that most unsecured bank derivative books are net receivables. Asymmetric FVA is more rigorous and has been considered in Albanese and Andersen (2014), Albanese, Andersen, and Iabichino (2015), Crépey (2015), Crépey and Song (2016), Brigo and Pallavicini (2014), Bielecki and Rutkowski (2015) and Bichuch, Capponi, and Sturm (2016). In Section 5 we improve upon these asymmetric FVA models by accounting for the funding source provided by economic capital: The refined FVA in (32) captures the intertwining between the FVA and economic capital, which leads to a significantly lower FVA as a result of the fungibility of economic capital (on top of reserve capital) as a source of funding for variation margin.

Section 6 shows that specialist initial margin lending schemes may drastically reduce the funding cost for initial margin (MVA).

In contrast to the other XVAs, our KVA is not the valuation of some cash flows, but a risk premium. As risk margin is retained earnings meant to be released to the bank shareholders, the KVA in our sense does not belong to the balance-sheet as a liability, at least not statically as part of contra-assets. But, in some sense, our KVA is a measure of the fluctuations of the balance sheet. In Section 7 this is contrasted with alternative KVA approaches in the literature where all the pricing adjustments are viewed as part of fair valuation and where, in particular, the KVA is treated as a contra-asset for reserve capital such as CVA, FVA and MVA, as if the KVA was a capital deduction.

An interesting direction of research would be to see how the XVA analysis of this paper can be adapted to the case of a bank trading through a CCP, which is becoming the standard for vanilla products. The magnifying impact of model risk on the different XVA metrics is another important topic of future investigation.

A Acronyms

BSDE Backward stochastic differential equation.

CA Contra-assets (or their valuation).

CCR Counterparty credit risk (or its valuation).

CL Contra-liabilities (or their valuation).

CDS Credit default swap.

CVA Credit valuation adjustment (can be either UCVA or FTDCVA).

$\mathrm{CVA}^{\mathrm{CL}}$ Difference (UCVA − FTDCVA).

DVA Debt valuation adjustment (can be either unilateral or FTDDVA).

EC Economic capital.

ES Expected shortfall at the confidence level 97.5%.

FDA Funding debt adjustment (the contra-liability counterpart of the FVA).

FTDCVA First-to-default CVA.

FTDDVA First-to-default DVA.

FTP Funds transfer price (all-inclusive XVA add-on to the entry price of a deal).

FVA Funding valuation adjustment.

IM Initial margin (with **PIM** and **RIM** for IM posted and received by the bank).

KVA Capital valuation adjustment.

MDA Margin debt adjustment (the contra-liability counterpart of the MVA).

MtM Mark-to-market of a portfolio when all XVAs are ignored.

MVA Margin valuation adjustment.

OIS rate Overnight index swap rate.

RC Reserve capital (or CA account).

RM Risk margin (or KVA) account.

SCR Shareholder capital at risk.

UCVA Unilateral CVA.

VM Variation margin.

XVA Generic "X" valuation adjustment

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