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See Attachments for new applications of memristors

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The Memristor: A New Bond Graph Element

The "memristor," first defined by L. Chua for electrical circuits, is proposed as a new bond graph element, on an equal footing with R, L, & C, and having some unique modelling capabilities for nonlinear systems.

The Missing Constitutive Relation

In his original lecture notes introducing the bond graph technique, Paynter drew a "tetrahedron of state," (Fig. 1) which summarized the relationship between the state variables (e, f, p, q) [1, 2].[1] There are 6 binary relationships possible between these 4 state variables. Two of these are definitions: the displacement, $q(t) = q(0) + \int_0^t f(t)dt$ and the quantity $p(t) = p(0) + \int_0^t e(t)dt$, which is interpreted as momentum, magnetic flux, or "pressure-momentum" [2]. Of the remaining four possible relations, three are the elementary constitutive relations for the energy storage and dissipation elements:

$$F_C(e, q) = 0 \quad (1a)$$

$$F_I(p, f) = 0 \quad (1b)$$

$$F_R(e, f) = 0 \quad (1c)$$

What of the missing constitutive relation (which Paynter draws as a "hidden line" in Fig. 1)? From a purely logical viewpoint, this constitutive relation is as "fundamental" as the other three!

Recently, L. Chua pointed out that we may have been too hidebound in our physical interpretations of the dynamical variables [3]. After all, they are only mathematical definitions. He proposed that the missing constitutive relation,

$$F_m(q, p) = 0 \quad (2)$$

be called a "memristor," i.e., memory resistor, since it "remembers" both integrated flow and total applied effort.

What distinguishes a memristor from the other basic elements? What are its properties, and what effects, if any, does it model? Chua found few applications within the confines of electrical circuit theory; however, if we look beyond the electrical domain it is not hard to find systems whose characteristics are conveniently represented by a memristor model.

Properties

The constitutive relation for a 1-port memristor is a curve in the q-p plane, Fig. 2. In this context, we do not necessarily interpret the quantity $p(t) = p(0) + \int_0^t e(t)dt$ as momentum, flux, or pressure-momentum (2), but merely as the integrated effort ("impulse").

Depending on whether the memristor is charge- or impulse-controlled we may express the constitutive relation as

$$q = F(p) \quad \text{impulse-controlled} \quad (3a)$$

$$p = G(q) \quad \text{charge-controlled} \quad (3b)$$

Differentiating, we obtain

$$\dot{q} = F'(p)\ \dot{p} \text{ or } f = W(p)e \quad (4a)$$

$$\dot{p} = G'(q)\ \dot{q} \text{ or } e = M(q)f \quad (4b)$$

where $M(q)$ is called the incremental "memristance" and $W(p)$

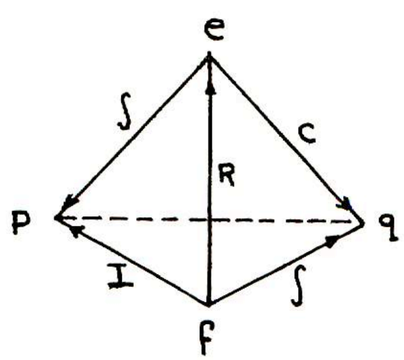

Fig. 1 Relation of state variables and constitutive relations ("Tetrahedron of State," Paynter, 1961)

[1] Numbers in brackets designate References at end of paper.

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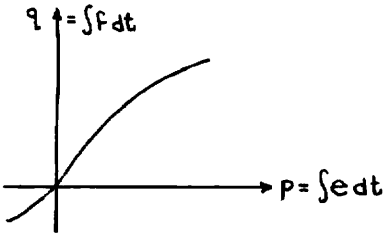
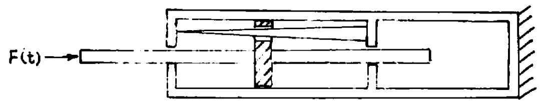
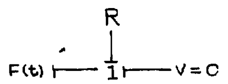


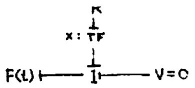
Fig. 2 Memristor constitutive relation



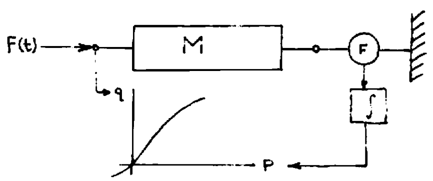
(a) A tapered dashpot (e.g. airplane strut)



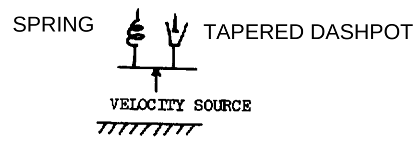
Bond graph for ordinary dashpot



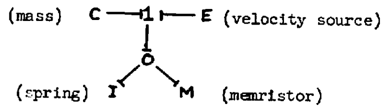
Bond graph for displacement modulated dashpot



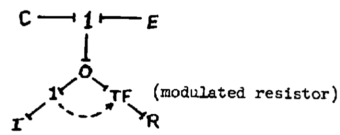
Measuring the memristor constittutive relation



(a) System Schematic



(b) Bond Graph, with memristor



(c) Bond Graph, with modulated resistor

Fig. 4 Schematic and bond graph of mechanical system with displacement-modulated dashpot

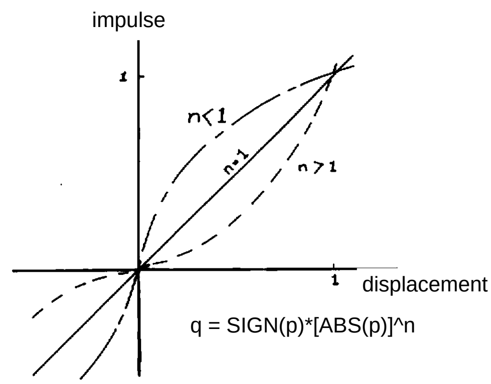


Fig. 5 Memductance curves

the incremental "memductance." We see that, dynamically, the memristor appears as either an impulse or a charge modulated resistor. Notice that for the special case of a linear constitutive relation, M = constant and W = constant, a memristor appears as an ordinary resistor. So memristors have meaning only for nonlinear systems (which may account in part for their neglect till now). Furthermore, a glance at the "tetrahedron of state" Fig. 1, shows that, since both an integration and a differentiation are involved in viewing the memristor as a "resistor" (i.e., on the e-f plane), the memristor, like the resistor, is causally neutral. That is, it may accept either an effort or a flow as input variable. However, there appear to be some restrictions insofar as device modeling is concerned which will be mentioned in the following.

Since the memristor is a 1-port device, it is trivially reciprocal in (q-p) coordinates. However, it is obvious that all definitions may be easily extended to the case of multiport, nonreciprocal resistors [3].

What then distinguishes a memristor from a resistor? Consider, for example, the tapered dashpot shown in Fig. 3. If we attempt to characterize this device on the e-f plane, mistaking it for a true resistor, we would not obtain a unique constitutive relation, $F(e, f) = 0$, but rather some peculiar hysteretic behavior, since the incremental resistance depends on the instantaneous piston displacement. On first glance one might attempt to model this device with a modulated R, so that the resistor constitutive relation could be parameterized by the state variable x, Fig. 3(b).[2] However, x is not a defined state variable for any element in the system. What is required is the displacement of the dashpot itself.[3] Modeling this device as a memristor eliminates the cumbersome modulation, and permits us to characterize the device as a *single* curve in the x-p plane. An important restriction, which is apparent from the form of the constitutive relation, is that the memristor can only be used to model linear, displacement modulated resistors. A real dashpot, for example, might have a characteristic like $e = F(q)f|f|$. The experimental setup to measure the constitutive relation for the tapered dashpot is shown in Fig. 3(c).

Examples

We have simulated both mechanical and electrochemical systems with memristors. The mechanical system, which includes a tapered dashpot of the type described in the foregoing, might

[2]Note: We are using the effort:velocity, flow:force analogy.

[3]An example of a true displacement modulated resistor is an electrolytic solution, where the number of charge carriers may vary with the electrolyte concentration [5]. Such concentration modulation is always implicitly present in electrical systems since the definition of the flow variable contains a concentration term which is included in the resistance: $f = qv_d$, where v_d = electron or ion drift velocity [4].

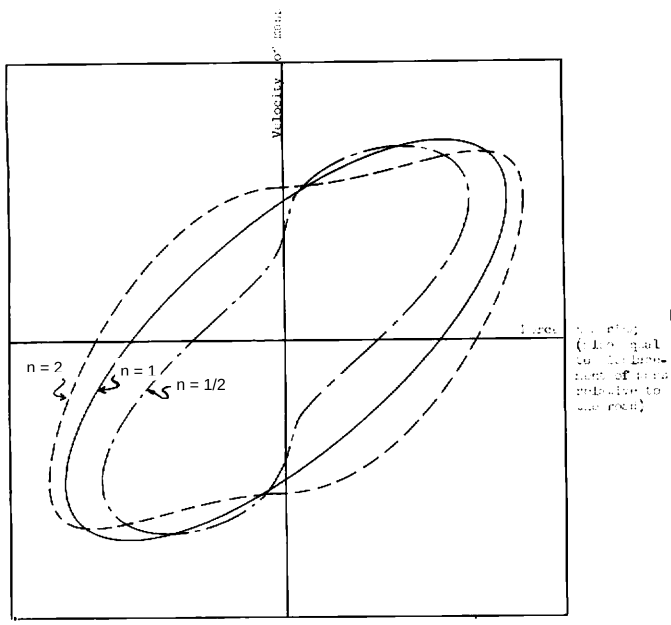
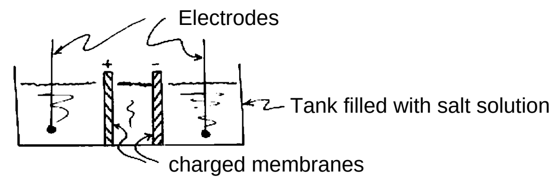
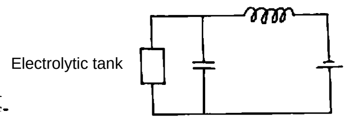


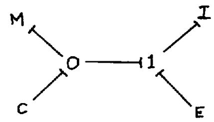
Fig. 6 Sinusoidal response on the state-plane



(a) Electrolytic tank with oppositely charged membranes



(b) Circuit diagram of system simulated



(c) Bond graph of system simulated

Fig. 7 Electro-chemical memristor system

be considered a crude model of an automobile suspension using a shock absorber whose characteristics depend on displacement. The electro-chemical system is a simple circuit containing a membrane rectifier [5]: an electrolytic cell whose electrical resistance depends on the electrolyte concentration between two charged membranes.

The schematic of the mechanical system is shown in Fig. 4(*a*). The mass could represent the mass of the car, the spring and dashpot its suspension system, and the velocity source the input due to undulations in the road. The bond graph, with the tapered dashpot as a memristor, is shown in Fig. 4(*b*) (using the e = velocity, f = force definitions). It is an accident of this particular system that it is also possible to represent the tapered dashpot as a modulated resistor since the displacement of the dashpot is the same as the displacement of the spring and thus proportional to the force in the spring. The bond graph for this system is shown in Fig. 4(*c*). Note that even in this case which has a standard bond graph representation, the bond graph with the memristor uses fewer elements and avoids the necessity of defining an entirely different kind of bond, the dashed bond for modulation.

Power-law relations used for the memductance are shown in Fig. 5 for the three cases that were simulated. Since, as was shown in the foregoing, the conductance depends on the slope of the memductance curves, the curve marked $n > 1$ corresponds to a dashpot that has a monotone increasing conductance. This is reversed for the curve marked $n < 1$, and for $n = 1$ the conductance is the same everywhere.

The response of the system to a sinusoidal forcing velocity is shown in the state plane in Fig. 6. The curves are coded in the same manner as used in Fig. 5; the solid line is for $n = 1$, the dashed line for $n = 2$, and the dash-dot line for $n = 1/2$. These results were consistent with those computed using the modulated resistor instead of the memristor. The case $n = 1$ corresponds to an ordinary linear dashpot, and, as expected, its state-plane trajectory is an ellipse. The other two trajectories are non-elliptical, indicating that the nonlinear p-q relation caused frequencies other than the forcing frequency to appear in the output. The presence of these harmonics is typical of nonlinear systems. The slopes of the trajectories near the velocity axis (that is, for spring-force close to zero) shows the characteristics of the displacement modulated dashpot; for $n = 2$ the dashpot is very soft around its center and the trajectory shows this by being nearly horizontal. On the other hand, for $n = 1/2$, the dashpot is very stiff near the center and its trajectory is very steep. (In theory, for $n = 1/2$ the slope is infinite at the origin. Use of a finite-difference solution, however, replaces the infinite slope with a large, but finite, slope near the origin, a much more realistic situation.)

In the system shown in Fig. 7(*a*) two oppositely charged membranes are introduced into a tank with two electrodes as shown. Because the oppositely charged membranes selectively prevent the passage of co-ions (i.e., ions with the same charge as the membranes), when an electric current flows through the electrodes the net electrolyte concentration in the inter-membrane space will increase or decrease, depending on the direction of current flow. Since the apparent electrical conductivity goes down as the concentration of ions decreases, the resistance of this device, as viewed from the external circuit, will depend on the total amount of current that has flowed through the cell. The concentration, and thus the resistance, will continue to change as long as there is any current flow.[4]

We have simulated the system shown in Fig. 7(*b*); its bond graph is shown in Fig. 7(*c*). In this case there are no variables anywhere in the system that could be used to provide the modulation for a modulated resistance. Thus, the memristor is the only possible element that can be used to model the electrolytic tank, short of a full-scale model of the ionic flows [5]. Note that, although the memristor appears as a dissipative element, it is a dynamic device requiring the independent specification of an initial condition, $q(0)$. The state space for the system of Fig. 7(*b*) is 3-dimensional, not 2-dimensional as would be expected on the basis of an RLC model.

Since concentrations can never be negative, we expect an asymmetric constitutive relation in the p-q plane; if we assume

[4]In the model simulated here it is assumed that the membranes are perfectly selective. In the actual case a steady-state can be established at very high (or very low) concentrations because the gradient for diffusion becomes high enough so that there will be some flow of the co-ions through the membranes, thus stabilizing the intermembrane concentration [5].

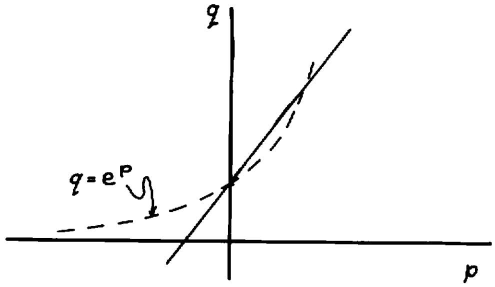


Fig. 8 Memristance curves

that the electrical resistance is inversely proportional to the concentration in the intermembrane space one obtains an exponential memristance curve, as shown by the dashed line in Fig. 8. The linear memristance shown by the solid line yields a completely linear system (i.e., constant resistance) for comparison purposes.

The results on the state plane projection are shown in Fig. 9 for sinusoidal excitation. As expected, the linear memristance (solid line) has an elliptical trajectory indicating the absence of any harmonics. The nonlinear case shows behavior indicative of an output containing more than just the forcing frequency and, because of the asymmetric constitutive relation of the memristor, its trajectory is also asymmetric.

There are two apparent restrictions on the use of the memristor as a modeling device. (i) According to equation (4), the memristor, viewed on the e-f plane, models a *linear* displacement-modulated resistor. That is, the device appears nonlinear by virtue of the nonlinear p-q relationship; but the $f = f(e, q)$ surface representing the resistor characteristic can only be a ruled surface, i.e., a surface swept out by a straight line. (ii) Memristors whose constitutive relation becomes horizontal (q = constant) are vertical (p = constant) are usually not admissible models, since the device continues to integrate the effort even though displacement is static. Therefore, when polarity is reversed across the device, a hysteretic behavior may occur in p-q as well as e-f coordinates.

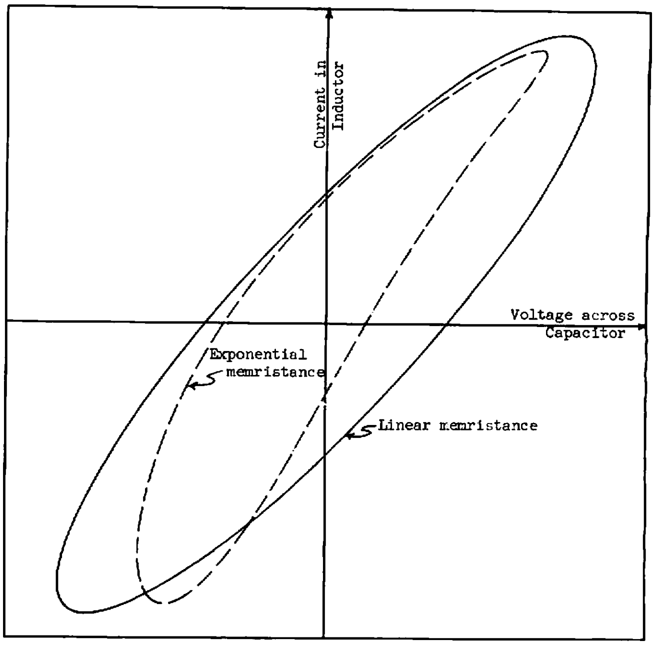


Fig. 9 State-plane trajectories for sinusoidal excitation

References

1 Paynter, H. M., *Analysis and Design of Engineering Systems,* The MIT Press, Cambridge, Mass., 1961.

2 Shearer, J. L., Murphy, A. T., and Richardson, H. H., *Introduction to Systems Dynamics,* Addison-Wesley, Reading, Mass., 1967.

3 Chua, L., "Memristor—The Missing Circuit Element," *IEEE Transactions on Circuit Theory,* Sept. 1971.

4 Purcell, E. M., *Electricity and Magnetism, The Berkeley Physics Course,* Vol. II, McGraw-Hill, New York, 1966.

5 Oster, G., and Auslander, D., "Topological Representations of Thermodynamic Systems," *Journal of the Franklin Institute,* Parts I, II, 1971.

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songs worked like a charm. So it seems that the male forms of *fru* fine-tune these neurons in the male to perfect his song.

Even if it is not well tuned, a song circuit is present in females. So what makes them hide their singing talent? The selective activation of thoracic song circuits in males but not females is likely to be controlled by some subset of the *fru* neurons in the brain. Indeed, classic studies of gynandromorph flies (which have a mixture of male and female nervous tissues) indicated[4] that certain brain regions must be 'male' to trigger the song. In this context, it is interesting to note that several pairs of neurons descending from the brain to the thorax are *fru*-positive[1]. These neurons are prime candidates to convey sex-specific commands to the thoracic song circuits.

The picture that emerges from these studies is that the circuitry for song generation, like that for pheromone processing[9,10], is largely shared between the sexes. The crucial sex differences seem to lie somewhere in between these bisexual input and output circuits, in dimorphic 'decision-making' centres in the brain. A similar design has recently been proposed[11] for the circuits that regulate sexual behaviour in mice: in females unable to perceive certain olfactory cues, male-like sexual behaviour results, presumably reflecting the activation of otherwise dormant circuits for these male behaviours in females. This modular and bisexual design affords considerable flexibility, which may even be exploited within the animal's own lifetime. Some species of fish, for example, change their sexual behaviour in response to social cues[12]. They may do this by simply resetting a few critical switches in the decision-making centres of an otherwise bisexual nervous system.

There is great excitement in neuroscience these days, as genetic tools are used to anatomically and functionally dissect the neural circuits that mediate complex animal behaviours[13]. Clyne and Miesenböck's work[1] beautifully illustrates the essential role photoactivation methods will have in this endeavour. As biochemists and biophysicists have long appreciated, surprising insights come when one can address questions of causality as well as correlation, reducing a system to its essentials and pushing it beyond its normal operating range. The mating behaviours of the humble fruitfly seem to be particularly amenable to this type of reductionist approach. ■

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1. Clyne, J. D. & Miesenböck, G. *Cell* **133,** 354–363 (2008).
2. Baker, B. S., Taylor, B. J. & Hall, J. C. *Cell* **105,** 13–24 (2001).
3. Demir, E. & Dickson, B. J. *Cell* **121,** 785–794 (2005).
4. Manoli, D. S. *et al. Nature* **436,** 395–400 (2005).
5. Stockinger, P., Kvitsiani, D., Rotkopf, S., Tirián, L. & Dickson, B. J. *Cell* **121,** 795–807 (2005).
6. Lima, S. Q. & Miesenböck, G. *Cell* **121,** 141–152 (2005).
7. Zemelman, B. V., Lee, G. A., Ng, M. & Miesenböck, G. *Neuron* **33,** 15–22 (2002).
8. Popescu, I. R. & Frost, W. N. *J. Neurosci.* **22,** 1985–1993 (2002).
9. Kurtovic, A., Widmer, A. & Dickson, B. J. *Nature* **446,** 542–546 (2007).
10. Datta, S. R. *et al. Nature* **452,** 473–477 (2008).
11. Kimchi, T., Xu, J. & Dulac, C. *Nature* **448,** 1009–1014 (2007).
12. Shapiro, D. Y. *Science* **209,** 1136–1137 (1980).
13. Luo, L., Callaway, E. M. & Svoboda, K. *Neuron* **57,** 634–660 (2008).

ELECTRONICS

The fourth element

James M. Tour and Tao He

Almost four decades since its existence was first proposed, a fourth basic circuit element joins the canonical three. The 'memristor' might herald a step-change in the march towards ever more powerful circuitry.

We learn at school that there are three fundamental two-terminal elements used for circuit building: resistors, capacitors and inductors. These are 'passive' elements, capable of dissipating or storing energy — but not, as active elements are, of generating it. The behaviour of each of these elements is described by a simple linear relationship between two of the four basic variables describing a circuit: current, voltage, charge and magnetic flux.

As the electrical engineer Leon Chua pointed out[1] in 1971, for the sake of the logical completeness of circuit theory, a fourth passive element should in fact be added to the list. He named this hypothetical element, linking flux and charge, the 'memristor' (Fig. 1). Almost 40 years later, Strukov *et al.*[2] (page 80 of this issue) present both a simple model system in which memristance should arise and a first, approximate physical example.

So what? Beyond its fundamental interest, the excitement lies in the possibility that the memristor could markedly extend how we can make electronic circuits work. In doing so, it might provide us with a way to keep on exponentially increasing computing power over time — thus maintaining something approximating to Moore's law, the rule-of-thumb to that effect that has been valid over the past few decades.

But before we get ahead of ourselves, some basics. According to the theory, a memristor is essentially a device that works under alternating current (a.c.) conditions[1] in which the applied voltage varies sinusoidally with time. As the polarity of this voltage changes, the memristor can switch reversibly between a less conductive OFF state and a more conductive ON state. Crucially, the value of the current flow through the memristor (the measure of its resistance) does not in the second half of the cycle retrace the exact path it took in the first. Because of this 'hysteresis' effect, the memristor acts as a nonlinear resistor the resistance of which depends on the history of the voltage across it — its name, a contraction of 'memory resistor', reflects just that property.

The memristor is a special case of a more general class of nonlinear dynamical devices called memristive systems[3]. Whether physically realized or not, since memristance was first proposed the memristor has been successfully used as a conceptual tool for analysing signal processing and for modelling the workings of, for instance, electrochemical and nonlinear semiconductor devices.

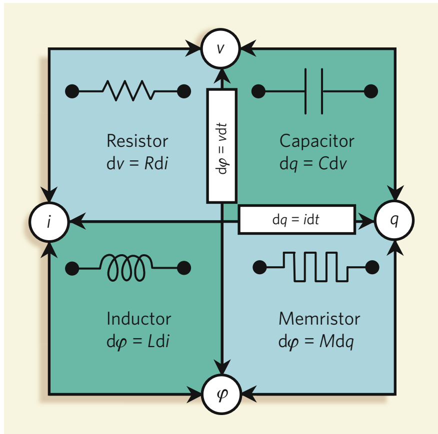

Figure 1 | Complete quartet. There are six independent permutations of two objects from a bank of four. Thus, six mathematical relations might be construed to connect pairs of the four fundamental circuit variables (current, i; voltage, v; charge, q; magnetic flux, φ)[1]. Of these, five are well known. Two arise from the definitions of two of the variables concerned: charge and magnetic flux are the time integrals of current and voltage ($dq = i\,dt$ and $d\varphi = v\,dt$), respectively. The other three lead to the axiomatic properties of three classic circuit elements: resistance, R, is the rate of change of voltage with current; capacitance, C, that of charge with voltage; and inductance, L, that of flux with current. The sixth relation leads to a fourth basic circuit element, which had been missing. Strukov *et al.*[2] have now found it: the memristor, with memristance, M, defined as the rate of change of flux with charge. (Figure adapted from refs 1 and 2.)

Even so, the concept has not been widely adopted, possibly because in normal microscale chips the memristance is minute. But everything changes on the nanoscale, because the size of memristance effects increases as the inverse square of device size. Strukov *et al.*[2] use a simple model to show how memristance arises naturally in a nanoscale system when electronic and atomic transport are coupled under an external voltage. The authors realize this memristive system by fabricating a layered platinum–titanium-oxide–platinum nanocell device. Here, the hysteretic current–voltage characteristics relate to the drift back and forth of oxygen vacancies in the titanium oxide layer driven by an applied voltage[4].

This observation provides a wonderfully simple explanation for several puzzling phenomena in nanoscale electronics: current–voltage anomalies in switching; hysteretic conductance; multiple-state conductances (as opposed to the normal instance of just two conductance states, ON and OFF); the often mischaracterized 'negative differential resistance', in which current decreases as voltage increases in certain nanoscale two-terminal devices; and metal–oxide–semiconductor memory structures, in which switching is caused by the formation and breakdown of metal filaments owing to the movement of metal atoms under applied bias.

But what of Moore's Law? Established by Intel co-founder Gordon Moore in 1965, this empirical rule states that the density of transistors on a silicon-based integrated circuit, and so the attainable computing power, doubles about every 18 months. It has held for more than 40 years, but there is a sobering consensus in the industry that the miniaturization process can continue for only another decade or so.

The memristor might provide a new path onwards and downwards to ever-greater processor density. By fabricating a cross-bar latch, consisting of one signal line crossed by two control lines[5], using (two-terminal) memristors, the function of a (three-terminal) transistor can be achieved with different physics. The two-terminal device is likely to be smaller and more easily addressable than the three-terminal one, and more amenable to three-dimensional circuit architectures. That could make memristors useful for ultra-dense, non-volatile memory devices.

For memristor memory devices to become reality, and to be readily scaled downwards, the efficient and reliable design and fabrication of electrode contacts, interconnects and the active region of the memristor must be assured. In addition, because (unlike with transistors) signal gain is not possible with a memristor, work needs to be put into obtaining high resistance ratios between the ON and OFF states. In all these instances, a deeper understanding of the memristor's dynamic nature is necessary.

It is often the simple ideas that stand the test of time. But even to consider an alternative to the transistor is anathema to many device engineers, and the memristor concept will have a steep slope to climb towards acceptance. Some will undoubtedly trivialize the realization of this ubiquitous nanoscale concept, whereas others will embrace it only after the demonstration of a well-functioning, large-scale array of these densely packed devices. When that happens, the race towards smaller devices will proceed at full steam. ■

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1. Chua, L. O. *IEEE Trans. Circuit Theory* **18,** 507–519 (1971).
2. Strukov, D. B., Snider, G. S., Stewart, D. R. & Williams, R. S. *Nature* **453,** 80–83 (2008).
3. Chua, L. O. & Kang, S. M. *Proc. IEEE* **64,** 209–223 (1976).
4. Yang, J. J. *et al. Nature Nanotech.* (in the press).
5. Kuekes, P. J., Stewart, D. R. & Williams, R. S. *J. Appl. Phys.* **97,** 034301 (2005).

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LETTERS

The missing memristor found

Dmitri B. Strukov[1], Gregory S. Snider[1], Duncan R. Stewart[1] & R. Stanley Williams[1]

Anyone who ever took an electronics laboratory class will be familiar with the fundamental passive circuit elements: the resistor, the capacitor and the inductor. However, in 1971 Leon Chua reasoned from symmetry arguments that there should be a fourth fundamental element, which he called a memristor (short for memory resistor)[1]. Although he showed that such an element has many interesting and valuable circuit properties, until now no one has presented either a useful physical model or an example of a memristor. Here we show, using a simple analytical example, that memristance arises naturally in nanoscale systems in which solid-state electronic and ionic transport are coupled under an external bias voltage. These results serve as the foundation for understanding a wide range of hysteretic current–voltage behaviour observed in many nanoscale electronic devices[2–19] that involve the motion of charged atomic or molecular species, in particular certain titanium dioxide cross-point switches[20–22].

More specifically, Chua noted that there are six different mathematical relations connecting pairs of the four fundamental circuit variables: electric current i, voltage v, charge q and magnetic flux φ. One of these relations (the charge is the time integral of the current) is determined from the definitions of two of the variables, and another (the flux is the time integral of the electromotive force, or voltage) is determined from Faraday's law of induction. Thus, there should be four basic circuit elements described by the remaining relations between the variables (Fig. 1). The 'missing' element—the memristor, with memristance M—provides a functional relation between charge and flux, $d\varphi = M dq$.

In the case of linear elements, in which M is a constant, memristance is identical to resistance and, thus, is of no special interest. However, if M is itself a function of q, yielding a nonlinear circuit element, then the situation is more interesting. The i–v characteristic of such a nonlinear relation between q and φ for a sinusoidal input is generally a frequency-dependent Lissajous figure[1], and no combination of nonlinear resistive, capacitive and inductive components can duplicate the circuit properties of a nonlinear memristor (although including active circuit elements such as amplifiers can do so)[1]. Because most valuable circuit functions are attributable to nonlinear device characteristics, memristors compatible with integrated circuits could provide new circuit functions such as electronic resistance switching at extremely high two-terminal device densities. However, until now there has not been a material realization of a memristor.

The most basic mathematical definition of a current-controlled memristor for circuit analysis is the differential form

$$v = \mathcal{R}(w)i \quad (1)$$

$$\frac{dw}{dt} = i \quad (2)$$

where w is the state variable of the device and $\mathcal{R}$ is a generalized resistance that depends upon the internal state of the device. In this case the state variable is just the charge, but no one has been able to propose a physical model that satisfies these simple equations. In 1976 Chua and Kang generalized the memristor concept to a much broader class of nonlinear dynamical systems they called memristive systems[23], described by the equations

$$v = \mathcal{R}(w,i)i \quad (3)$$

$$\frac{dw}{dt} = f(w,i) \quad (4)$$

where w can be a set of state variables and $\mathcal{R}$ and f can in general be explicit functions of time. Here, for simplicity, we restrict the discussion to current-controlled, time-invariant, one-port devices. Note that, unlike in a memristor, the flux in memristive systems is no longer uniquely defined by the charge. However, equation (3) does serve to distinguish a memristive system from an arbitrary dynamical device; no current flows through the memristive system when the voltage drop across it is zero. Chua and Kang showed that the i–v characteristics of some devices and systems, notably thermistors, Josephson junctions, neon bulbs and even the Hodgkin–Huxley model of the neuron, can be modelled using memristive equations[23]. Nevertheless, there was no direct connection between the mathematics and the physical properties of any practical system, and hence, almost forty years later, the concepts have not been widely adopted.

Here we present a physical model of a two-terminal electrical device that behaves like a perfect memristor for a certain restricted

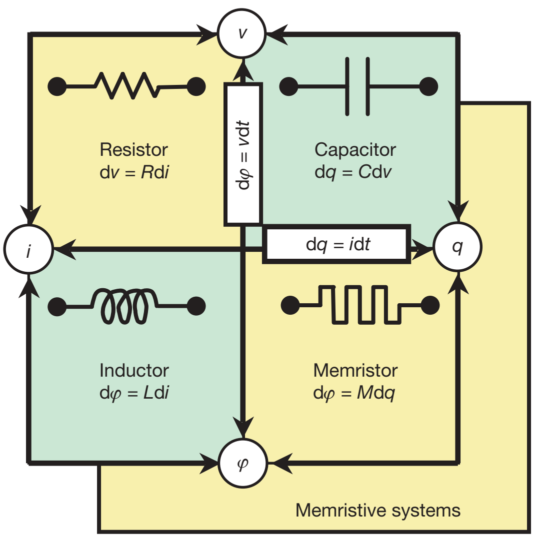

Figure 1 | The four fundamental two-terminal circuit elements: resistor, capacitor, inductor and memristor. Resistors and memristors are subsets of a more general class of dynamical devices, memristive systems. Note that R, C, L and M can be functions of the independent variable in their defining equations, yielding nonlinear elements. For example, a charge-controlled memristor is defined by a single-valued function $M(q)$.

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range of the state variable w and as a memristive system for another, wider (but still bounded), range of w. This intuitive model produces rich hysteretic behaviour controlled by the intrinsic nonlinearity of M and the boundary conditions on the state variable w. The results provide a simplified explanation for reports of current–voltage anomalies, including switching and hysteretic conductance, multiple conductance states and apparent negative differential resistance, especially in thin-film, two-terminal nanoscale devices, that have been appearing in the literature for nearly 50 years[2–4].

Electrical switching in thin-film devices has recently attracted renewed attention, because such a technology may enable functional scaling of logic and memory circuits well beyond the limits of complementary metal–oxide–semiconductors[24,25]. The microscopic nature of resistance switching and charge transport in such devices is still under debate, but one proposal is that the hysteresis requires some sort of atomic rearrangement that modulates the electronic current. On the basis of this proposition, we consider a thin semiconductor film of thickness D sandwiched between two metal contacts, as shown in Fig. 2a. The total resistance of the device is determined by two variable resistors connected in series (Fig. 2a), where the resistances are given for the full length D of the device. Specifically, the semiconductor film has a region with a high concentration of dopants (in this example assumed to be positive ions) having low resistance $\mathcal{R}_{ON}$, and the remainder has a low (essentially zero) dopant concentration and much higher resistance $\mathcal{R}_{OFF}$.

The application of an external bias $v(t)$ across the device will move the boundary between the two regions by causing the charged dopants to drift[26]. For the simplest case of ohmic electronic conduction and linear ionic drift in a uniform field with average ion mobility μ_V, we obtain

$$v(t)=\left(\mathcal{R}_{ON}\frac{w(t)}{D}+\mathcal{R}_{OFF}\left(1-\frac{w(t)}{D}\right)\right)i(t) \quad (5)$$

$$\frac{\mathrm{d}w(t)}{\mathrm{d}t}=\mu_V\frac{\mathcal{R}_{ON}}{D}i(t) \quad (6)$$

which yields the following formula for $w(t)$:

$$w(t)=\mu_V\frac{\mathcal{R}_{ON}}{D}q(t) \quad (7)$$

By inserting equation (7) into equation (5) we obtain the memristance of this system, which for $\mathcal{R}_{ON}\ll\mathcal{R}_{OFF}$ simplifies to:

$$M(q)=\mathcal{R}_{OFF}\left(1-\frac{\mu_V\mathcal{R}_{ON}}{D^2}q(t)\right)$$

The q-dependent term in parentheses on the right-hand side of this equation is the crucial contribution to the memristance, and it becomes larger in absolute value for higher dopant mobilities μ_V and smaller semiconductor film thicknesses D. For any material, this term is 1,000,000 times larger in absolute value at the nanometre scale than it is at the micrometre scale, because of the factor of $1/D^2$, and the memristance is correspondingly more significant. Thus, memristance becomes more important for understanding the electronic characteristics of any device as the critical dimensions shrink to the nanometre scale.

The coupled equations of motion for the charged dopants and the electrons in this system take the normal form for a current-controlled (or charge-controlled) memristor (equations (1) and (2)). The fact that the magnetic field does not play an explicit role in the

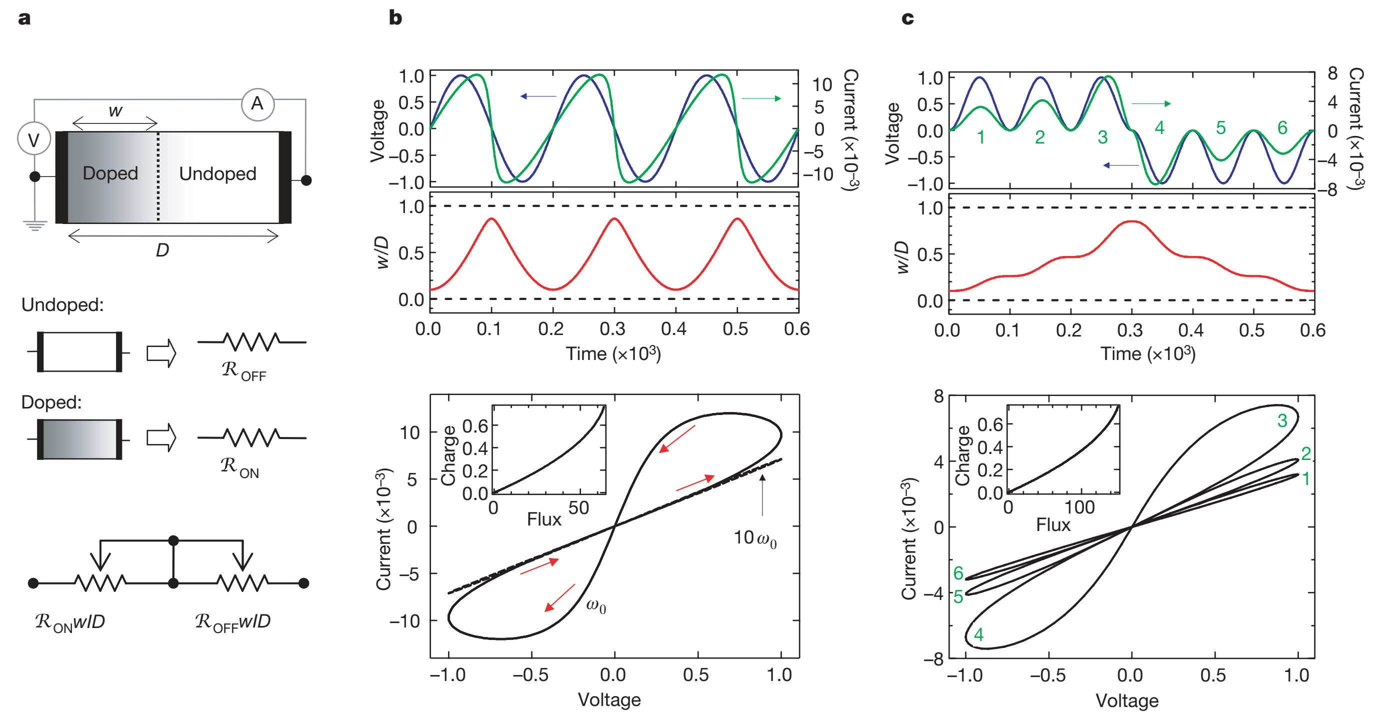

Figure 2 | The coupled variable-resistor model for a memristor. **a**, Diagram with a simplified equivalent circuit. V, voltmeter; A, ammeter. **b**, **c**, The applied voltage (blue) and resulting current (green) as a function of time t for a typical memristor. In **b** the applied voltage is $v_0\sin(\omega_0 t)$ and the resistance ratio is $\mathcal{R}_{OFF}/\mathcal{R}_{ON}=160$, and in **c** the applied voltage is $\pm v_0\sin^2(\omega_0 t)$ and $\mathcal{R}_{OFF}/\mathcal{R}_{ON}=380$, where v_0 is the magnitude of the applied voltage and ω_0 is the frequency. The numbers 1–6 label successive waves in the applied voltage and the corresponding loops in the i–v curves. In each plot the axes are dimensionless, with voltage, current, time, flux and charge expressed in units of $v_0=1$ V, $i_0\equiv v_0/\mathcal{R}_{ON}=10$ mA, $t_0\equiv 2\pi/\omega_0\equiv D^2/\mu_V v_0=10$ ms, v_0t_0 and i_0t_0, respectively. Here i_0 denotes the maximum possible current through the device, and t_0 is the shortest time required for linear drift of dopants across the full device length in a uniform field v_0/D, for example with $D=10$ nm and $\mu_V=10^{-10}$ cm^2 s^{-1} V^{-1}. We note that, for the parameters chosen, the applied bias never forces either of the two resistive regions to collapse; for example, w/D does not approach zero or one (shown with dashed lines in the middle plots in **b** and **c**). Also, the dashed i–v plot in **b** demonstrates the hysteresis collapse observed with a tenfold increase in sweep frequency. The insets in the i–v plots in **b** and **c** show that for these examples the charge is a single-valued function of the flux, as it must be in a memristor.

mechanism of memristance is one possible reason why the phenomenon has been hidden for so long; those interested in memristive devices were searching in the wrong places. The mathematics simply require there to be a nonlinear relationship between the integrals of the current and voltage, which is realized in equations (5) and (6). Another significant issue that was not anticipated by Chua is that the state variable w, which in this case specifies the distribution of dopants in the device, is bounded between zero and D. The state variable is proportional to the charge q that passes through the device until its value approaches D; this is the condition of 'hard' switching (large voltage excursions or long times under bias). As long as the system remains in the memristor regime, any symmetrical alternating-current voltage bias results in double-loop i–v hysteresis that collapses to a straight line for high frequencies (Fig. 2b). Multiple continuous states will also be obtained if there is any sort of asymmetry in the applied bias (Fig. 2c).

Obviously, equation (7) is only valid for values of w in the interval $[0, D]$. Different hard-switching cases are defined by imposing a variety of boundary conditions, such as assuming that once the value of w reaches either of the boundaries, it remains constant until the voltage reverses polarity. In such a case, the device satisfies the normal equations for a current-controlled memristive system (equations (3) and (4)). Figure 3a, b shows two qualitatively different i–v curves that are possible for such a memristive device. In Fig. 3a, the upper boundary is reached while the derivative of the voltage is negative, producing an apparent or 'dynamical' negative differential resistance. Unlike a true 'static' negative differential resistance, which would be insensitive to time and device history, such a dynamical effect is simply a result of the charge-dependent change in the device resistance, and can be identified by a strong dependence on the frequency of a sinusoidal driving voltage. In another case, for example when the boundary is reached much faster by doubling the magnitude of the applied voltage (Fig. 3b), the switching event is a monotonic function of current. Even though in the hard-switching case there appears to be a clearly defined threshold voltage for switching from the 'off' (high resistance) state to the 'on' (low resistance) state, the effect is actually dynamical. This means that any positive voltage v_+ applied to the device in the off state will eventually switch it to the on state after time $\sim D^2\mathcal{R}_{OFF}/(2\mu_V v_+ \mathcal{R}_{ON})$. The device will remain in the on state as long as a positive voltage is applied, but even a small negative bias will switch it back to the off state; this is why a current-hysteresis loop is only observed for the positive voltage sweep in Fig. 3a, b.

In nanoscale devices, small voltages can yield enormous electric fields, which in turn can produce significant nonlinearities in ionic transport. Figure 3c illustrates such a case in which the right-hand side of equation (6) is multiplied by a window function $w(1-w)/D^2$, which corresponds to nonlinear drift when w is close to zero or D. In this case, the switching event requires a significantly larger amount of charge (or even a threshold voltage) in order for w to approach either boundary. Therefore, the switching is essentially binary because the on and off states can be held much longer if the voltage does not exceed a specific threshold. Nonlinearity can also be expected in the electronic transport, which can be due to, for example, tunnelling at the interfaces or high-field electron hopping. In this case, the hysteresis behaviour discussed above remains essentially the same but the i–v characteristic becomes nonlinear.

The model of equations (5) and (6) exhibits many features that have been described as bipolar switching, that is, when voltages of opposite polarity are required for switching a device to the on state and the off state. This type of behaviour has been experimentally observed in various material systems: organic films[5–9] that contain charged dopants or molecules with mobile charged components; chalcogenides[4,10–12], where switching is attributed to ion migration rather than a phase transition; and metal oxides[2–4,20], notably TiO_2 (refs 4, 13, 14, 21) and various perovskites[4,15–19]. For example, multistate[8–14,16–18,20,21] and binary[3,4,7,15,16] switching that are similar to those modelled in Figs 2c and 3c, respectively, have been observed, with some showing dynamical negative differential resistance. Typically, hysteresis such as in Fig. 3c is observed for both voltage polarities[7,9–12,14–17,21], but observations of i–v characteristics resembling Fig. 3a, b have also been reported[8,17–20]. In our own studies of TiO_x devices, i–v behaviours very similar to those in Figs 2b, 2c and 3c are regularly observed. Figure 3d illustrates an experimental i–v characteristic from a metal/oxide/metal cross-point device within which the critical 5-nm-thick oxide film initially contained one layer of insulating TiO_2 and one layer of oxygen-poor TiO_{2-x} (refs 21, 22). In this system, oxygen vacancies act as mobile +2-charged dopants, which drift in the applied electric field, shifting the dividing line between the TiO_2 and TiO_{2-x} layers. The switching characteristic observed for a particular memristive system helps classify the nature of the boundary conditions on the state variable of the device.

The rich hysteretic i–v characteristics detected in many thin-film, two-terminal devices can now be understood as memristive behaviour defined by coupled equations of motion: some for (ionized) atomic degrees of freedom that define the internal state of the device, and others for the electronic transport. This behaviour is increasingly relevant as the active region in many electronic devices continues to shrink to a width of only a few nanometres, so even a low applied voltage corresponds to a large electric field that can cause charged species to move. Such dopant or impurity motion through the active region can produce dramatic changes in the device resistance. Including memristors and memristive systems in integrated circuits has the potential to significantly extend circuit functionality as long as the dynamical nature of such devices is understood and properly

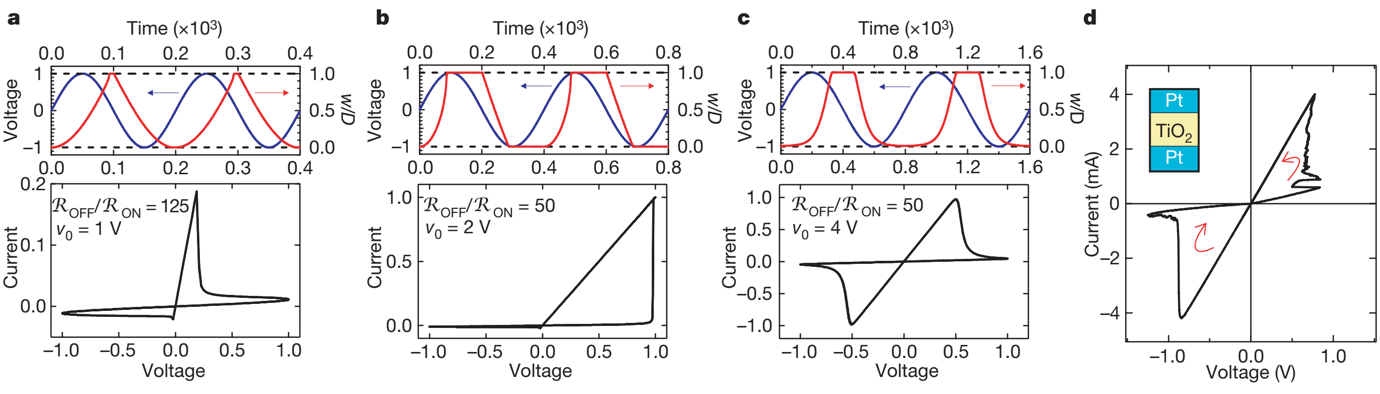

Figure 3 | Simulations of a voltage-driven memristive device. **a**, Simulation with dynamic negative differential resistance; **b**, simulation with no dynamic negative differential resistance; **c**, simulation governed by nonlinear ionic drift. In the upper plots of **a**, **b** and **c** we plot the voltage stimulus (blue) and the corresponding change in the normalized state variable w/D (red), versus time. In all cases, hard switching occurs when w/D closely approaches the boundaries at zero and one (dashed), and the qualitatively different i–v hysteresis shapes are due to the specific dependence of w/D on the electric field near the boundaries. **d**, For comparison, we present an experimental i–v plot of a Pt–TiO_{2-x}–Pt device[21].

used. Important applications include ultradense, semi-non-volatile memories and learning networks that require a synapse-like function.

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1. Chua, L. O. Memristor - the missing circuit element. *IEEE Trans. Circuit Theory* **18,** 507–519 (1971).
2. Hickmott, M. T. Low-frequency negative resistance in thin anodic oxide films. *J. Appl. Phys.* **33,** 2669–2682 (1962).
3. Dearnaley, G., Stoneham, A. M. & Morgan, D. V. Electrical phenomena in amorphous oxide films. *Rep. Prog. Phys.* **33,** 1129–1192 (1970).
4. Waser, R. & Aono, M. Nanoionics-based resistive switching memories. *Nature Mater.* **6,** 833–840 (2007).
5. Scott, J. C. & Bozano, L. D. Nonvolatile memory elements based on organic materials. *Adv. Mater.* **19,** 1452–1463 (2007).
6. Collier, C. P. *et al.* A [2]catenane-based solid state electronically reconfigurable switch. *Science* **289,** 1172–1175 (2000).
7. Zhitenev, N. B., Sidorenko, A., Tennant, D. M. & Cirelli, R. A. Chemical modification of the electronic conducting states in polymer nanodevices. *Nature Nanotechnol.* **2,** 237–242 (2007).
8. Smits, J. H. A., Meskers, S. C. J., Janssen, R. A. J., Marsman, A. W. & de Leeuw, D. M. Electrically rewritable memory cells from poly(3-hexylthiophene) Schottky diodes. *Adv. Mater.* **17,** 1169–1173 (2005).
9. Lai, Q. X., Zhu, Z. H., Chen, Y., Patil, S. & Wudl, F. Organic nonvolatile memory by dopant-configurable polymer. *Appl. Phys. Lett.* **88,** 133515 (2006).
10. Terabe, K., Hasegawa, T., Nakayama, T. & Aono, M. Quantized conductance atomic switch. *Nature* **433,** 47–50 (2005).
11. Kozicki, M. N., Park, M. & Mitkova, M. Nanoscale memory elements based on solid-state electrolytes. *IEEE Trans. Nanotechnol.* **4,** 331–338 (2005).
12. Dietrich, S. *et al.* A nonvolatile 2-Mbit CBRAM memory core featuring advanced read and program control. *IEEE J. Solid State Circuits* **42,** 839–845 (2007).
13. Jameson, J. R. *et al.* Field-programmable rectification in rutile TiO_2 crystals. *Appl. Phys. Lett.* **91,** 112101 (2007).
14. Jeong, D. S., Schroeder, H. & Waser, R. Coexistence of bipolar and unipolar resistive switching behaviors in a Pt/TiO_2/Pt stack. *Electrochem. Solid State Lett.* **10,** G51–G53 (2007).
15. Beck, A., Bednorz, J. G., Gerber, C., Rossel, C. & Widmer, D. Reproducible switching effect in thin oxide films for memory applications. *Appl. Phys. Lett.* **77,** 139–141 (2000).
16. Szot, K., Speier, W., Bihlmayer, G. & Waser, R. Switching the electrical resistance of individual dislocations in single-crystalline $SrTiO_3$. *Nature Mater.* **5,** 312–320 (2006).
17. Sawa, A., Fujii, T., Kawasaki, M. & Tokura, Y. Interface resistance switching at a few nanometer thick perovskite manganite active layers. *Appl. Phys. Lett.* **88,** 232112 (2006).
18. Hamaguchi, M., Aoyama, K., Asanuma, S., Uesu, Y. & Katsufuji, T. Electric-field-induced resistance switching universally observed in transition-metal-oxide thin films. *Appl. Phys. Lett.* **88,** 142508 (2006).
19. Oligschlaeger, R., Waser, R., Meyer, R., Karthauser, S. & Dittmann, R. Resistive switching and data reliability of epitaxial (Ba,Sr)TiO_3 thin films. *Appl. Phys. Lett.* **88,** 042901 (2006).
20. Richter, C. A., Stewart, D. R., Ohlberg, D. A. A. & Williams, R. S. Electrical characterization of Al/AlO_x/molecule/Ti/Al devices. *Appl. Phys. Mater. Sci. Process.* **80,** 1355–1362 (2005).
21. Stewart, D. R. *et al.* Molecule-independent electrical switching in Pt/organic monolayer/Ti devices. *Nano Lett.* **4,** 133–136 (2004).
22. Blackstock, J. J., Stickle, W. F., Donley, C. L., Stewart, D. R. & Williams, R. S. Internal structure of a molecular junction device: chemical reduction of PtO_2 by Ti evaporation onto an interceding organic monolayer. *J. Phys. Chem. C* **111,** 16–20 (2007).
23. Chua, L. O. & Kang, S. M. Memristive devices and systems. *Proc. IEEE* **64,** 209–223 (1976).
24. Kuekes, P. J., Snider, G. S. & Williams, R. S. Crossbar nanocomputers. *Sci. Am.* **293,** 72–78 (2005).
25. Strukov, D. B. & Likharev, K. K. Defect-tolerant architectures for nanoelectronic crossbar memories. *J. Nanosci. Nanotechnol.* **7,** 151–167 (2007).
26. Blanc, J. & Staebler, D. L. Electrocoloration in SrTiO - vacancy drift and oxidation-reduction of transition metals. *Phys. Rev. B* **4,** 3548–3557 (1971).

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Memristor minds

What connects our own human intelligence to the unsung cunning of slime moulds? An electronic component that no one thought existed, as **Justin Mullins** explains

EVER had the feeling something is missing? If so, you're in good company. Dmitri Mendeleev did in 1869 when he noticed four gaps in his periodic table. They turned out to be the undiscovered elements scandium, gallium, technetium and germanium. Paul Dirac did in 1929 when he looked deep into the quantum-mechanical equation he had formulated to describe the electron. Besides the electron, he saw something else that looked rather like it, but different. It was only in 1932, when the electron's antimatter sibling, the positron, was sighted in cosmic rays that such a thing was found to exist.

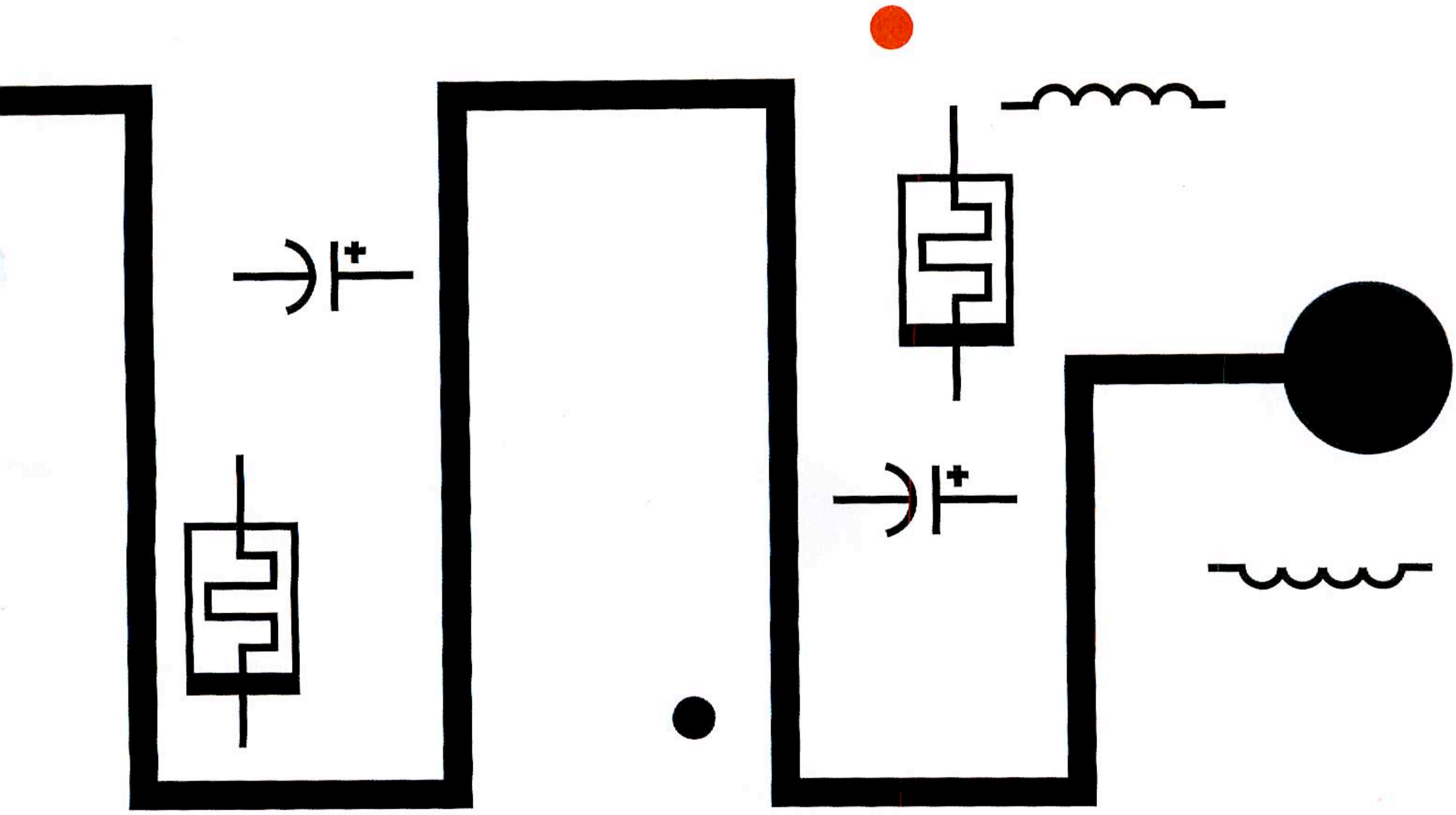
In 1971, Leon Chua had that feeling. A young electronics engineer with a penchant for mathematics at the University of California, Berkeley, he was fascinated by the fact that electronics had no rigorous mathematical foundation. So like any diligent scientist, he set about trying to derive one.

And he found something missing: a fourth basic circuit element besides the standard trio of resistor, capacitor and inductor. Chua dubbed it the "memristor". The only problem was that as far as Chua or anyone else could see, memristors did not actually exist.

Except that they do. Within the past couple of years, memristors have morphed from obscure jargon into one of the hottest properties in physics. They've not only been made, but their unique capabilities might revolutionise consumer electronics. More than that, though, along with completing the jigsaw of electronics, they might solve the puzzle of how nature makes that most delicate and powerful of computers – the brain.

That would be a fitting pay-off for a story which, in its beginnings, is a triumph of pure logic. Back in 1971, Chua was examining the four basic quantities that define an electronic circuit. First, there is electric charge. Then there is the change in that charge over time, better known as current. Currents create magnetic fields, leading to a third variable, magnetic flux, which characterises the field's strength. Finally, magnetic flux varies with time, leading to the quantity we call voltage.

Four interconnected things, mathematics says, can be related in six ways. Charge and current, and magnetic flux and voltage, are connected through their definitions. That's two. Three more associations correspond to the three traditional circuit elements. A resistor is any device that, when you pass current through it, creates a voltage. For a given voltage a capacitor will store a certain



amount of charge. Pass a current through an inductor, and you create a magnetic flux. That makes five. Something missing?

Indeed. Where was the device that connected charge and magnetic flux? The short answer was there wasn't one. But there should have been.

Chua set about exploring what this device would do. It was something that no combination of resistors, capacitors and inductors would do. Because moving charges make currents, and changing magnetic fluxes breed voltages, the new device would generate a voltage from a current rather like a resistor, but in a complex, dynamic way. In fact, Chua calculated, it would behave like a resistor that could "remember" what current had flowed through it before (see diagram, page 44). Thus the memristor was born.

"Though memristors were welcome in theory, no physical device seemed capable of the effect"

And promptly abandoned. Though it was welcome in theory, no physical device or material seemed capable of the resistance-with-memory effect. The fundamentals of electronics have kept Chua busy ever since, but even he had low expectations for his baby. "I never thought I'd see one of these devices in my lifetime," he says.

He had reckoned without Stan Williams, senior fellow at the Hewlett-Packard Laboratories in Palo Alto, California. In the early 2000s, Williams and his team were wondering whether you could create a fast, low-power switch by placing two tiny resistors made of titanium dioxide over one another, using the current in one to somehow toggle the resistance in the other on and off.

Nanoscale novelty

They found that they could, but the resistance in different switches behaved in a way that was impossible to predict using any conventional model. Williams was stumped. It took three years and a chance tip-off from a colleague about Chua's work before the revelation came. "I realised suddenly that the equations I was writing down to describe our device were very similar to Chua's," says Williams. "Then everything fell into place."

What was happening was this: in its pure state of repeating units of one titanium and two oxygen atoms, titanium dioxide is a semiconductor. Heat the material, though, and some of the oxygen is driven out of the structure, leaving electrically charged bubbles that make the material behave like a metal.

In Williams's switches, the upper resistor was made of pure semiconductor, and the › lower of the oxygen-deficient metal. Applying a voltage to the device pushes charged bubbles up from the metal, radically reducing the semiconductor's resistance and making it into a full-blown conductor. A voltage applied in the other direction starts the merry-go-round revolving the other way: the bubbles drain back down into the lower layer, and the upper layer reverts to a high-resistance, semiconducting state.

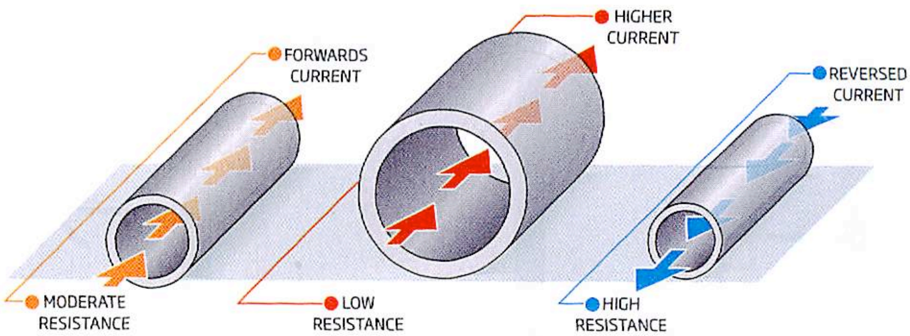
The crucial thing is that, every time the voltage is switched off, the merry-go-round stops and the resistance is frozen. When the voltage is switched on again, the system "remembers" where it was, waking up in the same resistance state (*Nature*, vol 453, p 80). Williams had accidentally made a memristor just as Chua had described it.

Williams could also show why a memristor had never been seen before. Because the effect depends on atomic-scale movements, it only popped up on the nanoscale of Williams's devices. "On the millimetre scale, it is essentially unobservable," he says.

Nanoscale or no, it rapidly became clear just how useful memristors might be. Information can be written into the material as the resistance state of the memristor in a few nanoseconds using just a few picojoules of energy – "as good as anything needs to be", according to Williams. And once written, memristive memory stays written even when the power is switched off.

A memristor never forgets

The "resistor with memory" that Leon Chua described behaves like a pipe whose diameter varies according to the amount and direction of the current passing through it



IF THE CURRENT IS TURNED OFF, THE PIPE'S DIAMETER STAYS THE SAME UNTIL IT IS SWITCHED ON AGAIN – IT "REMEMBERS" WHAT CURRENT HAS FLOWED THROUGH IT

Memory mould

This was a revelation. For 50 years, electronics engineers had been building networks of dozens of transistors – the building blocks of memory chips – to store single bits of information without knowing it was memristance they were attempting to simulate. Now Williams, standing on the shoulders of Chua, had showed that a single tiny component was all they needed.

The most immediate potential use is as a powerful replacement for flash memory – the kind used in applications that require quick writing and rewriting capabilities, such as in cameras and USB memory sticks. Like flash memory, memristive memory can only be written 10,000 times or so before the constant atomic movements within the device cause it to break down. That makes it unsuitable for computer memories. Still, Williams believes it will be possible to improve the durability of memristors. Then, he says, they could be just the thing for a superfast random access memory (RAM), the working memory that computers use to store data on the fly, and ultimately even for hard drives.

Were this an article about a conventional breakthrough in electronics, that would be the end of the story. Better memory materials alone do not set the pulse racing. We have come to regard ever zippier consumer electronics as a basic right, and are notoriously insouciant about the improvements in basic physics that make them possible. What's different about memristors?

Explaining that requires a dramatic change of scene – to the world of the slime mould *Physarum polycephalum*. In an understated way, this large, gloopy, single-celled organism is a beast of surprising intelligence. It can sense and react to its environment, and can even solve simple puzzles. Perhaps its most remarkable skill, though, was reported last year by Tetsu Saisuga and his colleagues at Hokkaido University in Sapporo, Japan: it can anticipate periodic events.

"Somehow, an organism without a neuron to call its own had memorised a pattern of events"

Here's how we know. *P. polycephalum* can move around by passing a watery substance known as sol through its viscous, gelatinous interior, allowing it to extend itself in a particular direction. At room temperature, the slime mould moves at a slothful rate of about a centimetre per hour, but you can speed this movement up by giving the mould a blast of warm, moist air.

You can also slow it down with a cool, dry breeze, which is what the Japanese researchers did. They exposed the gloop to 10 minutes of cold air, allowed it to warm up again for a set period of time, and repeated the sequence three times. Sure enough, the mould slowed down and sped up in time with the temperature changes.

But then they changed the rules. Instead of giving *P. polycephalum* a fourth blast of cold air, they did nothing. The slime mould's reaction was remarkable: it slowed down again, in anticipation of a blast that never came (*Physical Review Letters*, vol 100, p 018101).

It's worth taking a moment to think about what this means. Somehow, this single-celled organism had memorised the pattern of events it was faced with and changed its behaviour to anticipate a future event. That's something we humans have trouble enough with, let alone a single-celled organism without a neuron to call its own.

The Japanese paper rang a bell with Max Di Ventra, a physicist at the University of California, San Diego. He was one of the few who had followed Chua's work, and recognised that the slime mould was behaving like a memristive circuit. To prove his contention, he and his colleagues set about building a circuit that would, like the slime mould, learn and predict future signals.

The analogous circuit proved simple to derive. Changes in an external voltage applied to the circuit simulated changes in the temperature and humidity of the slime mould's environment, and the voltage across a memristive element represented the slime mould's speed. Wired up the right way, the memristor's voltage would vary in tempo with an arbitrary series of external voltage pulses. When "trained" through a series of three equally spaced voltage pulses, the memristor voltage repeated the response even when subsequent pulses did not appear (www.arxiv.org/abs/0810.4179).

Di Ventra speculates that the viscosities of the sol and gel components of the slime mould make for a mechanical analogue of memristance. When the external temperature rises, the gel component starts to break down and become less viscous, creating new pathways through which the sol can flow and speeding up the cell's movement. A lowered temperature reverses that process, but how the initial state is regained depends on where the pathways were formed, and therefore on the cell's internal history.

In true memristive fashion, Chua had anticipated the idea that memristors might have something to say about how biological organisms learn. While completing his first paper on memristors, he became fascinated by synapses – the gaps between nerve cells in higher organisms across which nerve impulses must pass. In particular, he noticed their complex electrical response to the ebb and flow of potassium and sodium ions across the membranes of each cell, which allow the synapses to alter their response according to the frequency and strength of signals. It looked maddeningly similar to the response a memristor would produce. "I realised then that synapses were memristors," he says. "The ion channel was the missing circuit element I was looking for, and it already existed in nature."

They might not look much, but slime moulds can be surprisingly quick-witted beasts

EYE OF SCIENCE/SPL

To Chua, this all points to a home truth. Despite years of effort, attempts to build an electronic intelligence that can mimic the awesome power of a brain have seen little success. And that might be simply because we were lacking the crucial electronic components – memristors.

So now we've found them, might a new era in artificial intelligence be at hand? The Defense Advanced Research Projects Agency certainly thinks so. DARPA is a US Department of Defense outfit with a strong record in backing high-risk, high-pay-off projects – things like the internet. In April last year, it announced the Systems of Neuromorphic Adaptive Plastic Scalable Electronics Program, SyNAPSE for short, to create "electronic neuromorphic machine technology that is scalable to biological levels".

"The behaviour of synapses looked maddeningly similar to a memristor's response"

I, memristor

Williams's team from Hewlett-Packard is heavily involved. Late last year, in an obscure US Department of Energy publication called *SciDAC Review*, his colleague Greg Snider set out how a memristor-based chip might be wired up to test more complex models of synapses. He points out that in the human cortex synapses are packed at a density of about 10^{10} per square centimetre, whereas today's microprocessors only manage densities 10 times less. "That is one important reason intelligent machines are not yet walking around on the street," he says.

Snider's dream is of a field he calls "cortical computing" that harnesses the possibilities of memristors to mimic how the brain's neurons interact. It's an entirely new idea. "People confuse these kinds of networks with neural networks," says Williams. But neural networks – the previous best hope for creating an artificial brain – are software working on standard computing hardware. "What we're aiming for is actually a change in architecture," he says.

The first steps are already being taken. Williams and Snider have teamed up with Gail Carpenter and Stephen Grossberg at Boston University, who are pioneers in reducing neural behaviours to systems of differential equations, to create hybrid transitor-memristor chips designed to reproduce some of the brain's thought processes. Di Ventra and his colleague Yuriy Pershin have gone further and built a memristive synapse that they claim behaves like the real thing(www.arxiv.org/abs/0905.2935).

The electronic brain will be a time coming. "We're still getting to grips with this chip," says Williams. Part of the problem is that the chip is just too intelligent – rather than a standard digital pulse it produces an analogue output that flummoxes the standard software used to test chips. So Williams and his colleagues have had to develop their own test software. "All that takes time," he says.

Chua, meanwhile, is not resting on his laurels. He has been busy extending his theory of fundamental circuit elements, asking what happens if you combine the properties of memristors with those of capacitors and inductors to produce compound devices called memcapacitors and meminductors, and then what happens if you combine those devices, and so on.

"Memcapacitors may be even more useful than memristors," says Chua, "because they don't have any resistance." In theory at least, a memcapacitor could store data without dissipating any energy at all. Mighty handy – whatever you want to do with them. Williams agrees. In fact, his team is already on the case, producing a first prototype memcapacitor earlier this year, a result that he aims to publish soon. "We haven't characterised it yet," he says. With so many fundamental breakthroughs to work on, he says, it's hard to decide what to do next. Maybe a memristor could help. ■

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