Graviton propagator

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Where we are in LQG

- General approach to background-independent QFT

- Kinematics well-defined:
  + Separable kinematical Hilbert space (spin networks, s-knots)
  + Geometrical interpretation (area and volume operators)
  – Euclidean/Lorentzian ? Immirzi parameter ?

- Dynamics:
  - Thiemann's operator (with its variants and ambiguities)
  - Spinfoam formalism (several models)
    - triangulation independence:
      Group Field Theory (+ good finiteness, - λ ?)
      Conrady-Bojowald-Perez triangulation invariance ?
    - Barrett-Crane vertex, or 10j symbol $A_{vertex} \sim e^{iS_{Regge}} + e^{-iS_{Regge}} + D$

good

bad

Barrett, Williams, Baez, Christensen, Egan, Freidel, Louapre

Developments

- Matter fields
- Mathematical formalization (uniqueness theorem)

Applications

- Spectra of Area and Volume
- Loop cosmology
- Black hole entropy
- Black hole singularity
- Astrophysical
- ...

Main open issues

- Which version is the good one?
- Low energy limit
- Newton's law
- Computing scattering amplitudes ← This talk

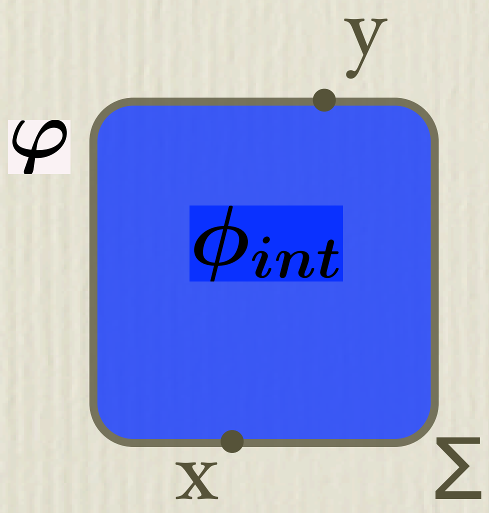
The problem

$$W(x,y) = \int D\phi\ \phi(x)\ \phi(y)\ e^{iS[\phi]}$$

if measure and action are diff invariant, then immediately

$$W(x,y) = W(f(x), f(y))$$

Idea for a solution: define the **boundary functional**



$$W[\varphi, \Sigma] = \int_{\phi_{int}|_\Sigma = \varphi} D\phi_{int}\ \ e^{iS[\phi_{int}]}$$

then

$$W(x,y;\Sigma,\Psi) = \int D\varphi\ \varphi(x)\ \varphi(y)\ W(\varphi,\Sigma)\ \Psi[\varphi]$$

cfr: R Oeckl (see also Conrady, Doplicher)

What happens in a diff invariant theory?

Remarkably $$W[\varphi, \Sigma] = W[\varphi]$$

Therefore $$W(x, y; \Psi) = \int D\varphi \; \varphi(x) \; \varphi(y) \; W(\varphi) \; \Psi[\varphi]$$

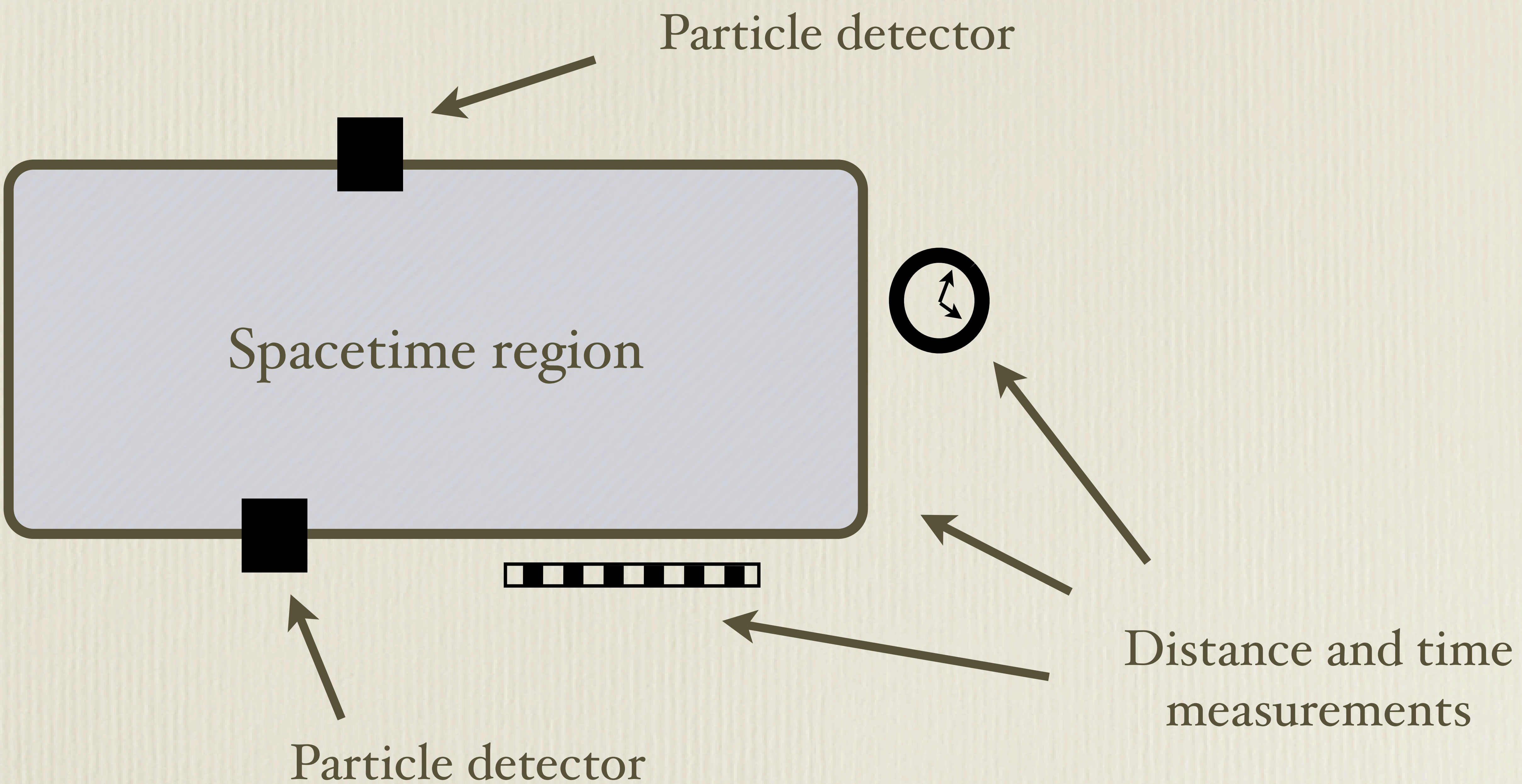
But in GR the information on the geometry of a surface is not in Σ
It is in the state of the (gravitational) field on the surface !

Hence: choose Ψ to be a state picked on a given geometry q of Σ !

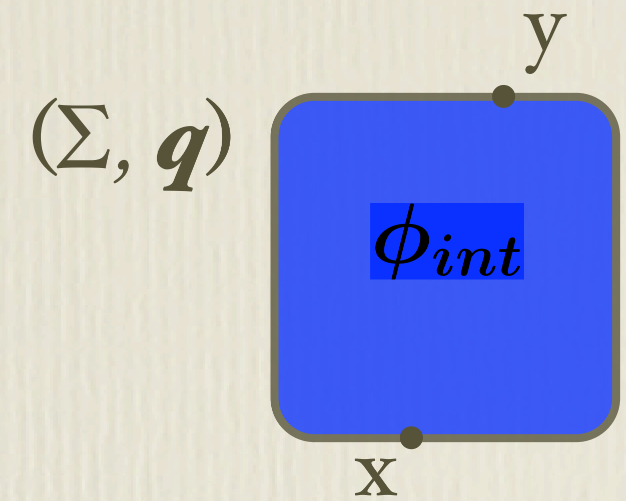
$$W(x, y; q) = \int D\varphi \; \varphi(x) \; \varphi(y) \; W(\varphi) \; \Psi_q[\varphi]$$

Distance and time separations between x and y are now well defined with respect to the mean boundary geometry q.

Conrady ,Doplicher, Oeckl, Testa, CR



In GR distance and time measurements are field measurements like the other ones: they determine the **boundary data** of the problem.



Give meaning to the expression

$$W(x,y;q) = \int D\varphi\ \varphi(x)\ \varphi(y)\ W[\varphi]\ \Psi_q[\varphi]$$

- $\int D\varphi$ ➞ $\sum_{\text{s-knots}}$
- $W[\varphi]$ ➞ $W[s]$ defined by GFT spinfoam model
- Ψ_q ➞ a suitable coherent state on the geometry q
- $\varphi(x)$ ➞ graviton field operator from LQG.

$$W^{abcd}(x,y;q) = N \sum_{ss'} W[s']\ \langle s'|h^{ab}(x)h^{cd}(y)|s\rangle\ \Psi_q[s]$$

Modesto, CR, PRL 05

$\boldsymbol{W}$[s] : Group field theory (here GFT/B):

$$\boldsymbol{W}[\boldsymbol{s}] = \int \mathrm{D}\boldsymbol{\phi}\ \ \boldsymbol{f_s}(\boldsymbol{\phi})\ \ \mathrm{e}^{-\int \phi^2 - \frac{\lambda}{5!}\int \phi^5}$$

The Feynman expansion in λ gives a sum over spinfoams

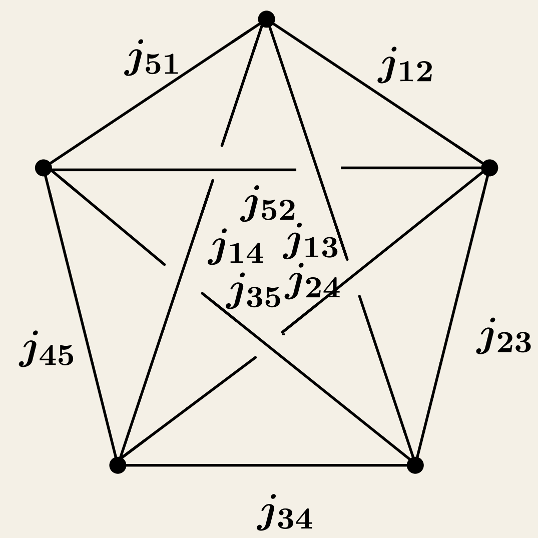
$$\boldsymbol{W}[\boldsymbol{s}] = \sum_{\partial\sigma = s} \prod_{faces} A_{faces} \prod_{vertices} A_{vertex}$$

which has a nice interpretation as a discretization of the Misner-Hawking sum over geometries

$$\boldsymbol{W}({}^3\boldsymbol{g}) = \int_{\partial g = {}^3 g} Dg\ e^{iS_{Einstein-Hilbert}[g]}$$

with background triangulations summed over as well.

To first order in λ, the only nonvanishing connected term in W[s] is for

$s =$ 

And the dominant contribution for large j is given by the spinfoam σ dual to a *single* four-simplex. This is

$$W[s] = \frac{\lambda}{5!} \left(\prod_{n<m} dim(j_{nm}) \right) A_{vertex}(j_{nm})$$

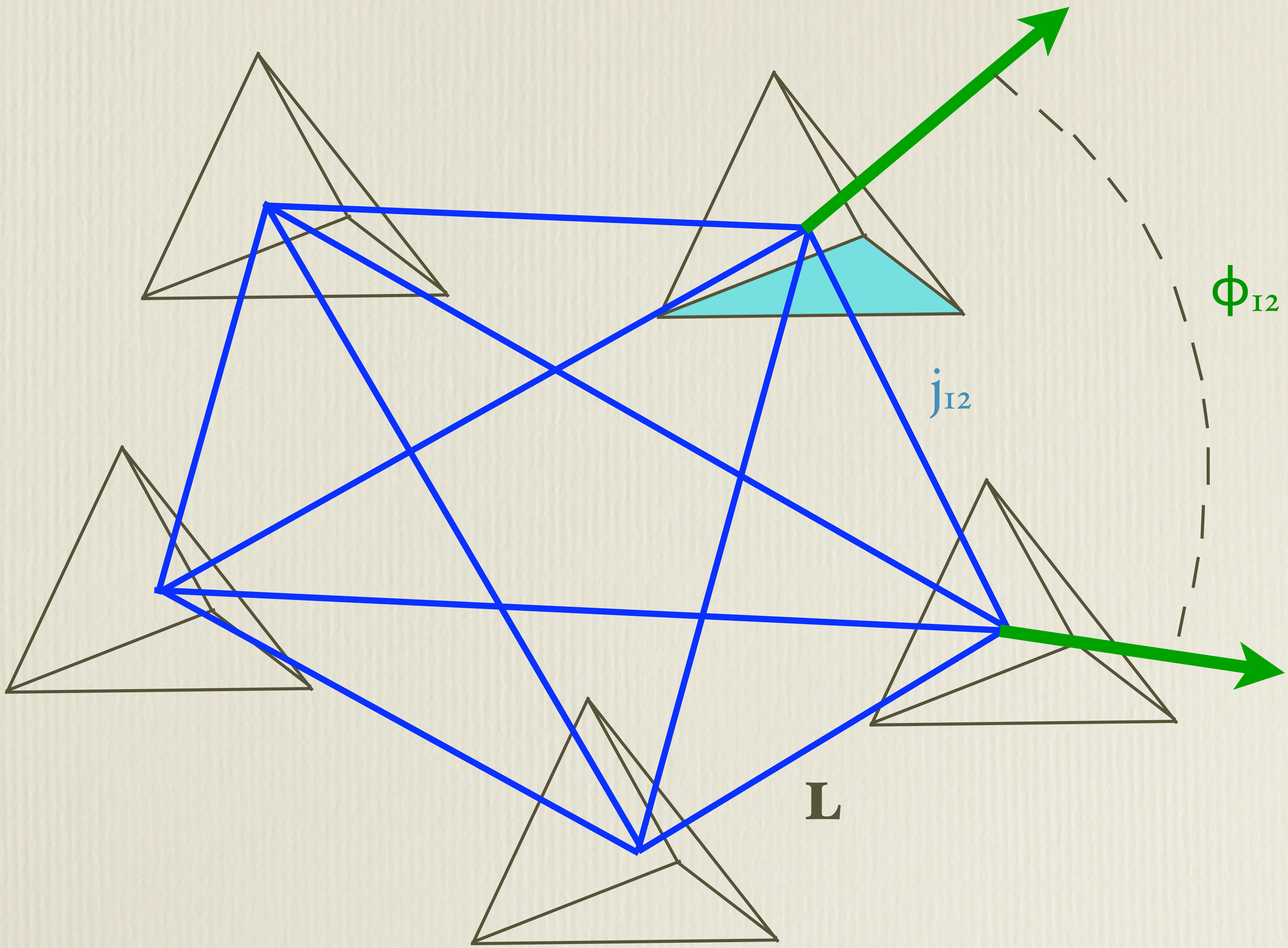
The boundary state $\Psi_q(s)$

- Choose a boundary geometry *q*: let *q* be the geometry of the 3d boundary (Σ,q) of a spherical 4d ball, with linear size $L >> \sqrt{\hbar G}$.
- Interpret *s* as the (dual) of a triangulation of this geometry. Choose a *regular* triangulation of (Σ, q); interpret the spins as the areas of the corresponding triangles, using the standard LQG interpretation of spin networks.
- This determine the "background" spins $j^{(o)}_{nm}=j_L$. $\Psi_q(s)$ must be picked on these values. Choose a Gaussian state around these values with with α, to be determined.
- A Gaussian can have an arbitrary *phase:*

$$\Psi_q[s] = \exp\left\{-\frac{\alpha}{2}\sum_{n<m}(j_{nm}-j^{(0)}_{nm})^2 + i\sum_{n<m}\Phi^{(0)}_{nm}j_{nm}\right\}$$

➡ $\Psi_q(s)$ must be a coherent state, determined by coordinate *and momentum*, namely by intrinsic 3-geometry *and extrinsic* 3-geometry *q* !!

➡ The $\Phi^{(o)}_{nm} = \Phi$ are the background **dihedral angles**.



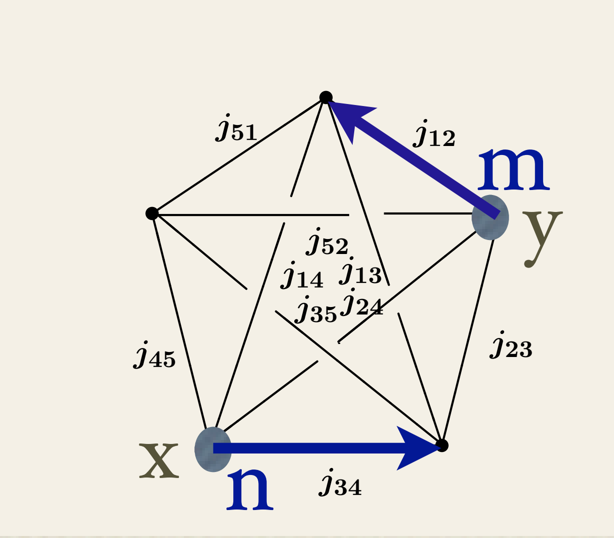
ϕ_{12}
j_{12}
L

The field operator

$$h^{ab}(\vec{x}) = g^{ab}(\vec{x}) - \delta^{ab} = E^{ai}(\vec{x})E^{bi}(\vec{x}) - \delta^{ab}$$

Choose x to be on the nodes and contract the indices with two parallel vectors along the links.
Then we have the standard action on boundary spin networks, well known from LQG

$$E^{Ii}(n)E^{I}_{i}(n)|s\rangle = (8\pi\hbar G)^2\, j_I(j_I+1)|s\rangle$$

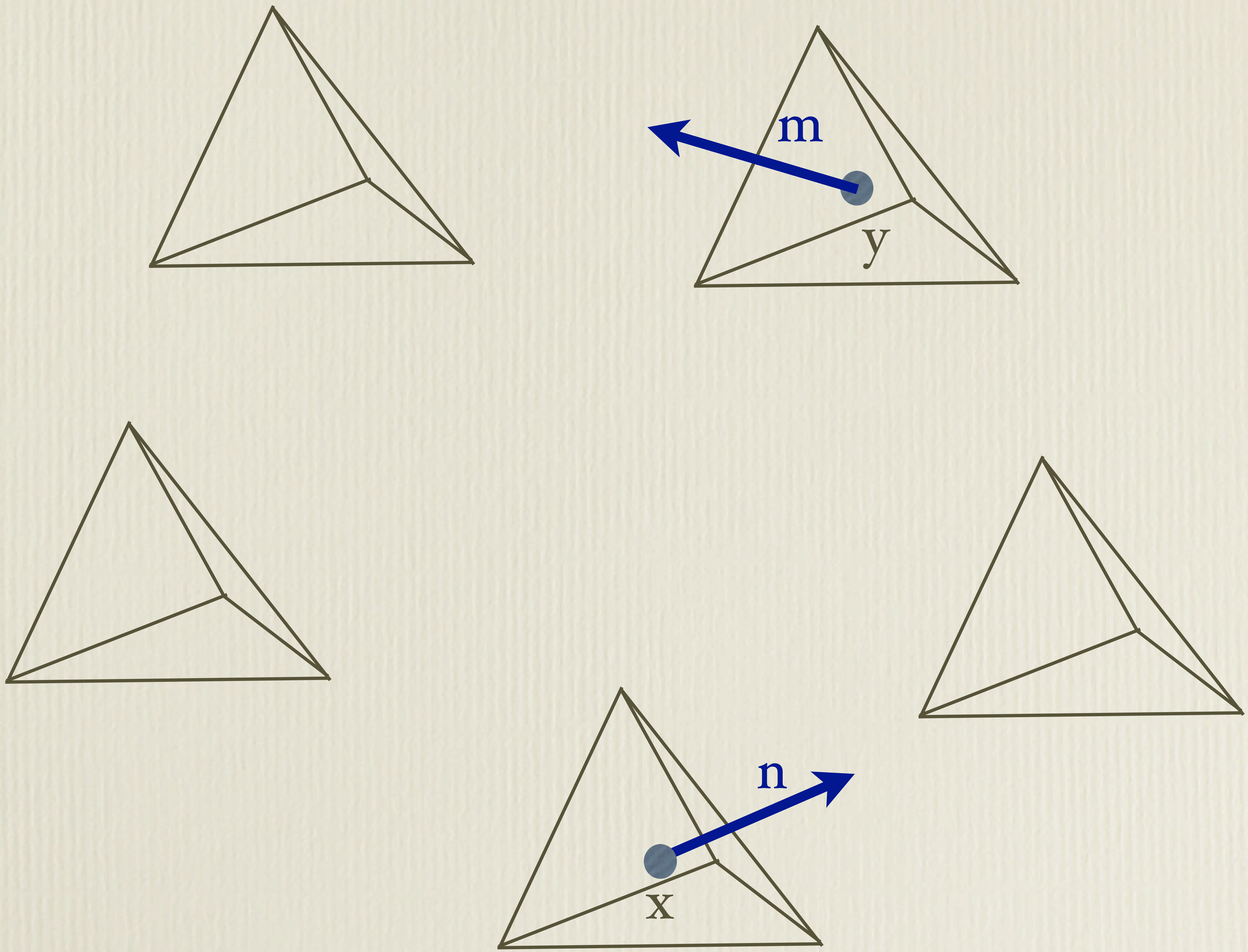


Define

$$W(L) = W^{abcd}(x,y;q)\; n_a n_b m_c m_d$$

Standard perturbative theory gives

$$W(L) = i\frac{8\pi}{4\pi^2}\frac{1}{|x-y|^2_q} = i\frac{8\pi\hbar G}{4\pi^2}\frac{1}{L^2}$$


m
y
n
x

The expression for the propagator is then well defined:

$$W(L) = W^{abcd}(x,y;q)n_a n_b m_c m_d =$$

$$N\frac{\lambda(\hbar G)^4}{5!}\sum_{j_{nm}} (j_{12}(j_{12}+1)-j_L^2)\,(j_{34}(j_{34}+1)-j_L^2)\; A_{vertex}(j_{nm}) \;\; e^{-\frac{\alpha}{2}\sum_{n,m}(j_{nm}-j_L)^2 - i\Phi\sum_{n,m} j_{nm}}$$

$$A_{vertex} \sim e^{iS_{Regge}} + e^{-iS_{Regge}} + D$$

rapidly oscillating phase

But since

$$S_{Regge}(j_{nm}) = \sum_{n<m}\Phi_{nm}(j)\; j_{nm}$$

and

$$S_{Regge}(j_{nm}) \sim \Phi\sum_{nm} j_{nm} + \frac{1}{2}G_{(mn)(kl)}\delta j_{mn}\delta j_{kl}$$

only the “good” component of A_{vertex} survives !

This is the “forward propagating” (Oriti, Livine) component of A_{vertex}
cfr. Colosi, CR

$$S_{Regge}(j_{nm}) \sim \Phi \sum_{nm} j_{nm} + \frac{1}{2} G_{(mn)(kl)} \delta j_{mn} \delta j_{kl}$$

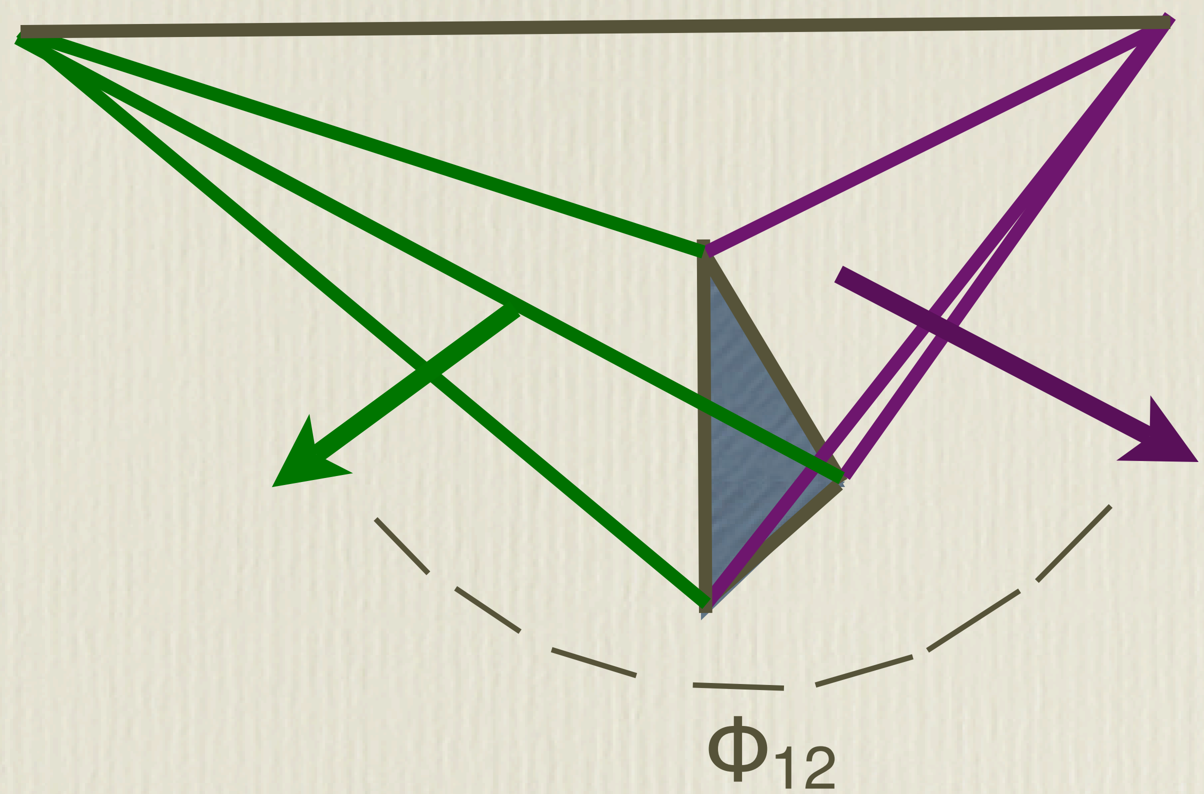
The gaussian "integration" gives finally

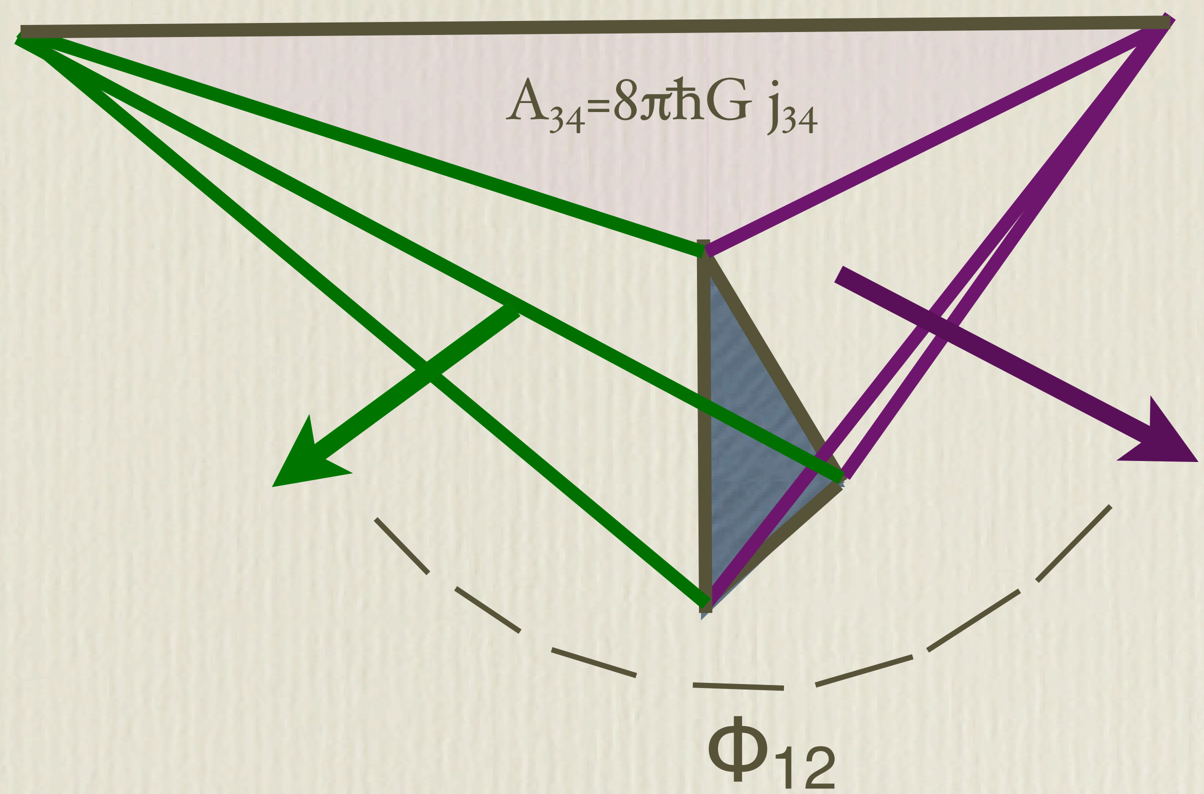
$$W(L) = \frac{4i}{\alpha^2}\, G_{(12)(34)}$$

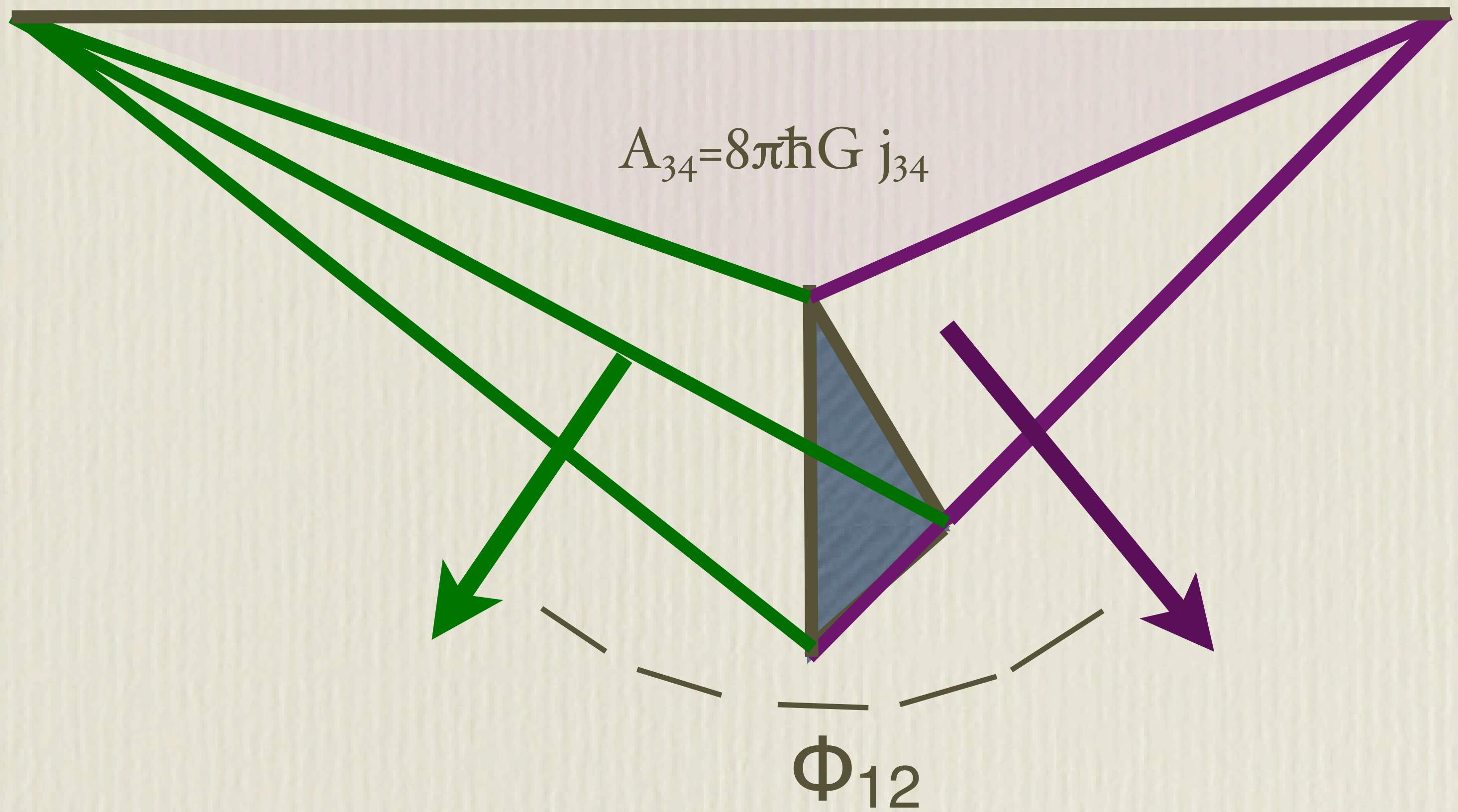
Where $G_{(mn)(kl)}$ is the ("discrete") derivative of the dihedral angle, with respect to the area (the spin).

$$G_{(mn)(kl)} = \left.\frac{\partial \Phi_{mn}(j_{ij})}{\partial j_{kl}}\right|_{j_{ij}=j_L}$$

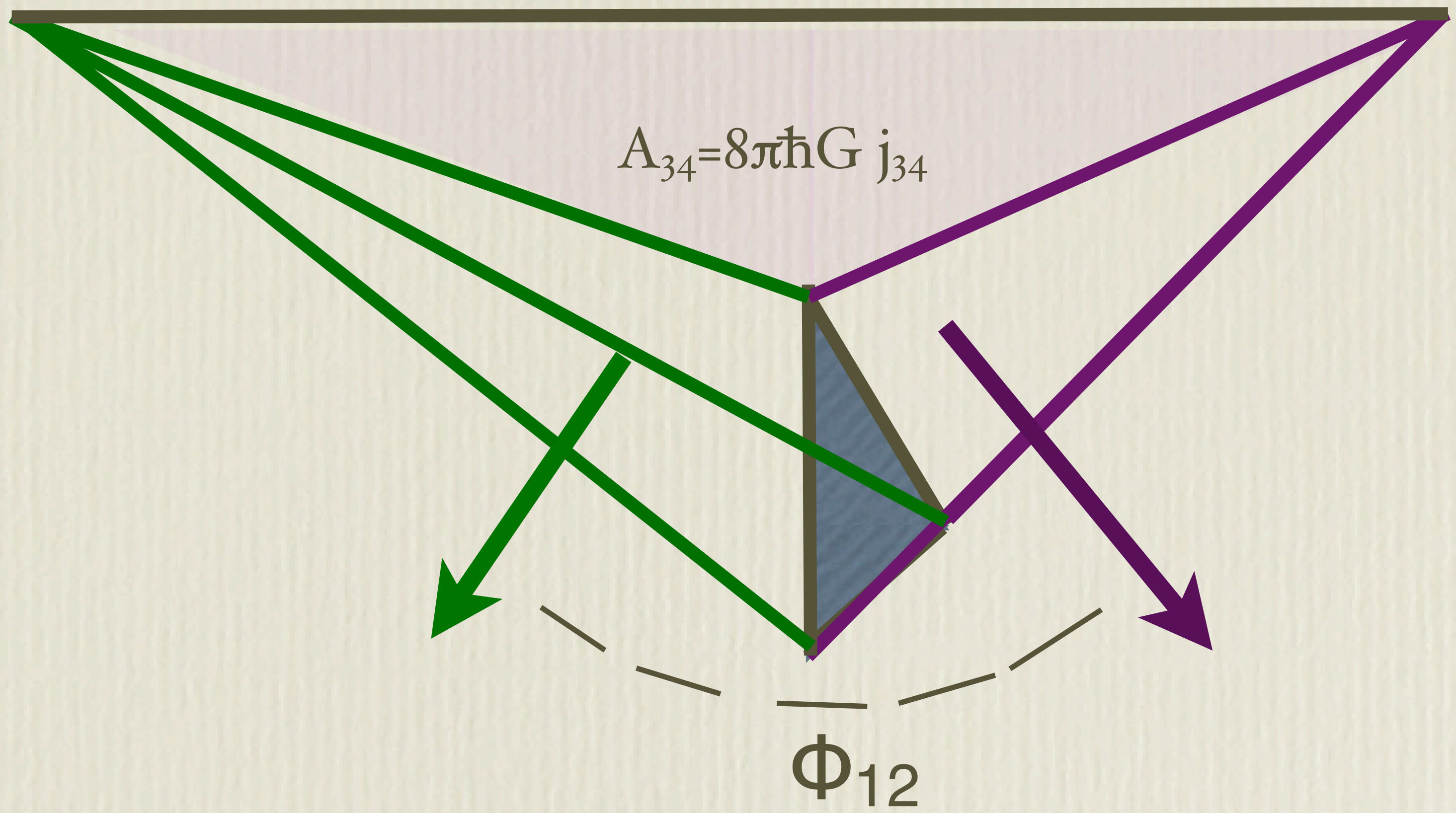
It can be computed from geometry, giving $G_{(12)(34)} = \frac{8\pi\hbar G k}{L^2}$, where k is a numerical factor ~ 1


Φ_{12}


A_{34}=8πħG j_{34}
Φ_{12}

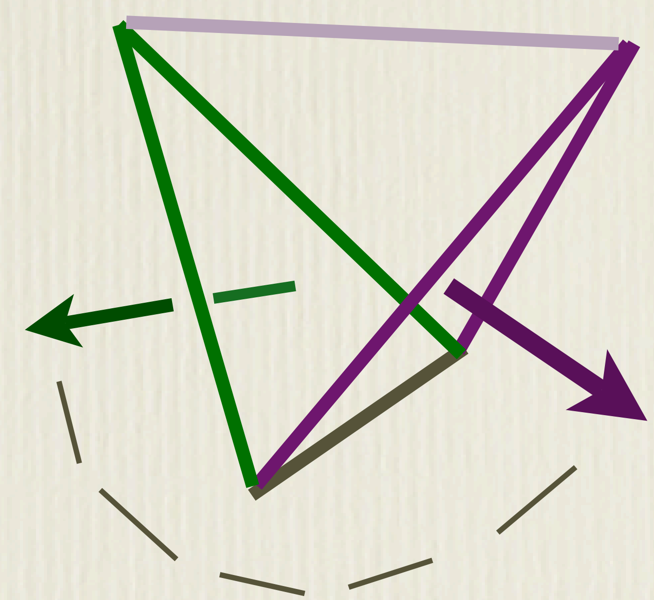


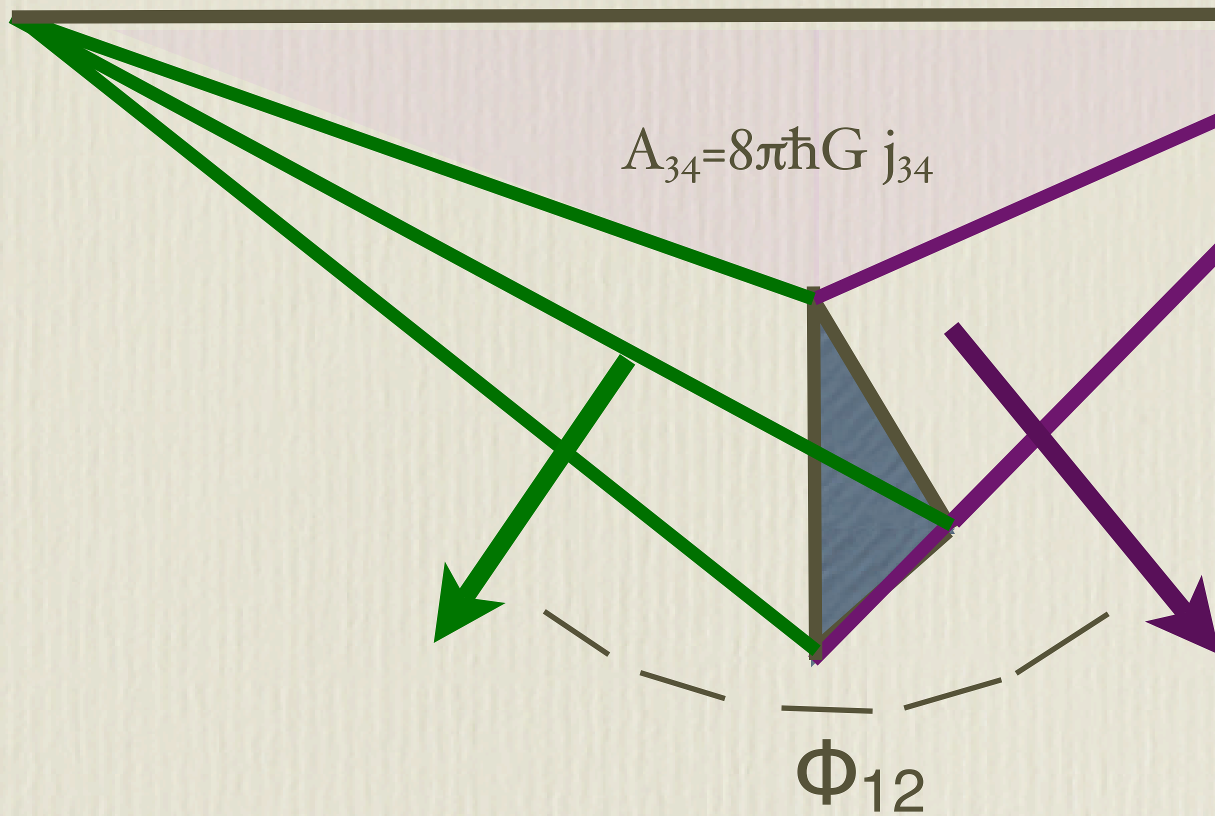
$A_{34}=8\pi\hbar G\ j_{34}$
Φ_{12}



Cfr the “nutshel’ dynamics in 3d gravity

Colosi, Doplicher, Fairbairn, Modesto, Noui, CR





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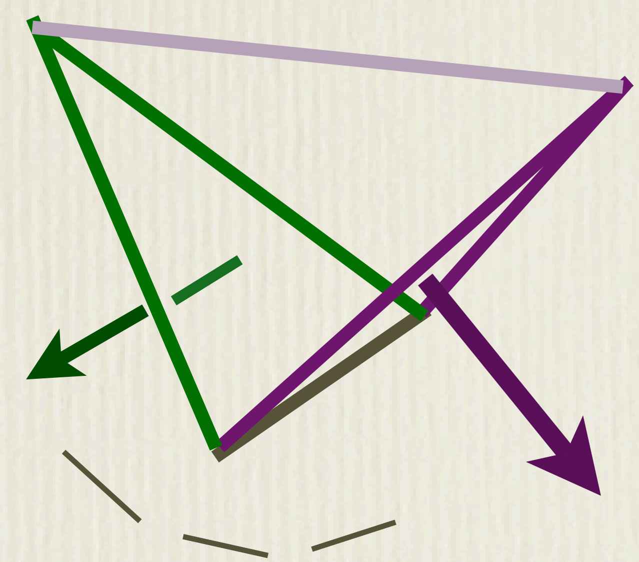

Adjusting numerical factors $\alpha^2 = 16\pi^2 k$, this gives

$$W(L) = i\frac{8\pi\hbar G}{4\pi^2}\frac{1}{L^2} = i\frac{8\pi}{4\pi^2}\frac{1}{|x-y|_q^2}$$

which is the correct graviton-propagator (component).

➞ This is only valid for $L^2 >> \hbar G$.
For small L, the propagator is affected by quantum gravity effects, and is given by the 10j symbol combinatorics.

➞ **This is equivalent to the Newton law**

Open issues

- Is it just by chance?
- Other components? Full tensorial structure (Modesto, Speziale) (intertwiners)
- Do higher order terms in λ change the result ? (Modesto)
- Other models (GFT/C seems to give the same result (Modesto))
- n - point functions ? Computing the undetermined constants of the non-renormalizable perturbative QFT ?
- ...

Conclusion

i. Low energy limit. (One component of) the graviton propagator (or the Newton law) appears to be correct, to first order in λ.

ii. Barret-Crane vertex. Only the "good" component of the 10j symbol survives, because of the phase of the state, given by the extrinsic geometry of the boundary state. The BC vertex works.

iii. Scattering amplitudes. A technique to compute n-point functions within a background-independent formalism exists.