Game Theoretic Approach for Supply Chain Optimization under Demand Uncertainty

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Agenda

1. Introduction of our group
2. Why game theoretic approach is required ?
3. Nash Equilibrium
4. Stackelberg game
5. Supply chain coordination with quantity discount contract
6. Numerical example
7. Conclusion and future works

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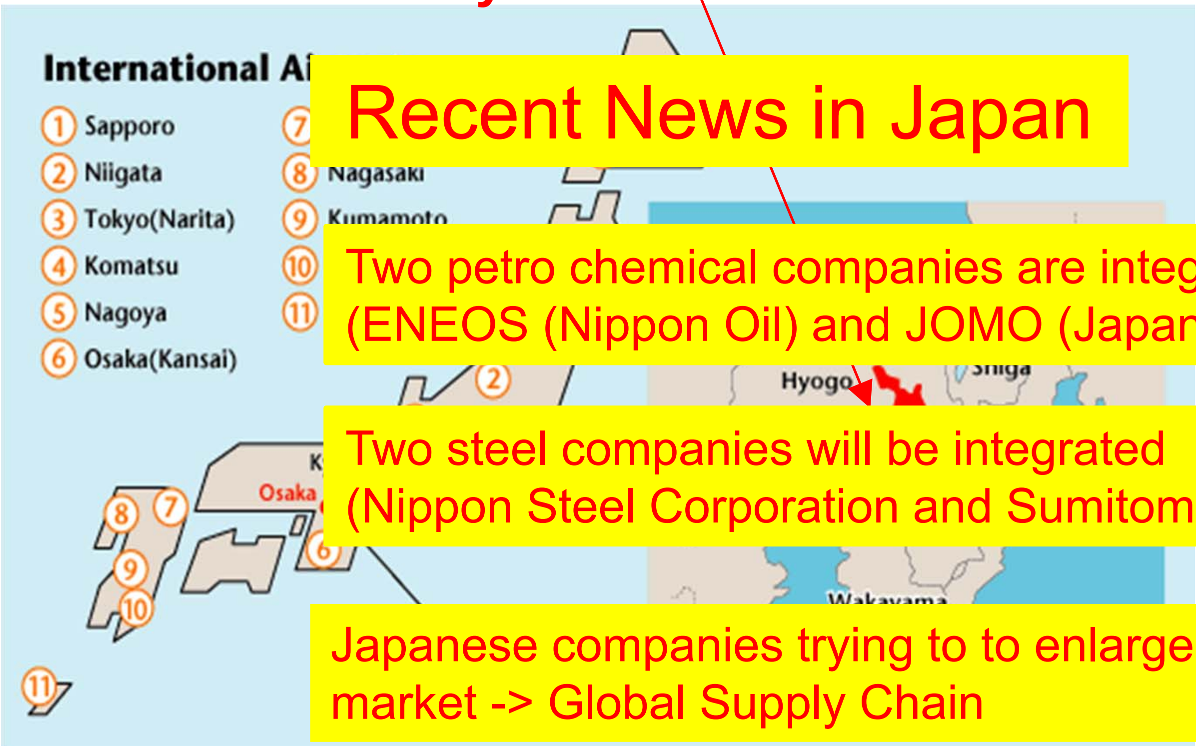


Recent News in Japan

Two petro chemical companies are integrated.
(ENEOS (Nippon Oil) and JOMO (Japan Energy))

Two steel companies will be integrated
(Nippon Steel Corporation and Sumitomo Steel Industries)

Japanese companies trying to to enlarge their global market -> Global Supply Chain

bachelor students: 15,865
master students : 4,352
doctor students : 2,053
Total students : 22,270
Foreign students: 1,608

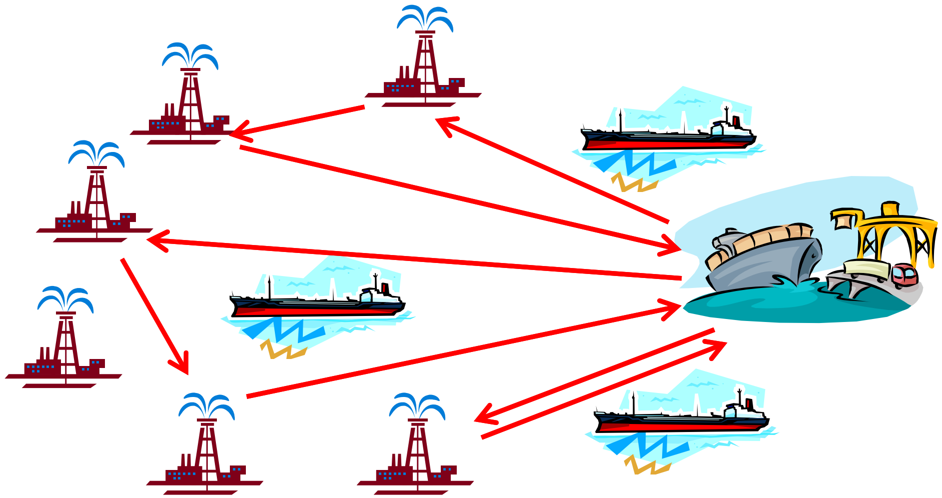
Professors : 2,978
: 2,606
: 5,584

us
• Toyonaka campus
Suita campus
pus

Collaboration with Japanese industries

- Semiconductor factory automation
  - Control of automated guided vehicles for transportation
  - Scheduling of cluster tool for silicon wafer production
- Railway scheduling automation
  - railway crew scheduling in Japan Railway
  - Train-set scheduling with maintenance constraints
  - Shift scheduling
- Petroleum chemical industry automation
  - International ship scheduling for crude oil transportation
  - Transportation network design

International ship scheduling problem



loading places

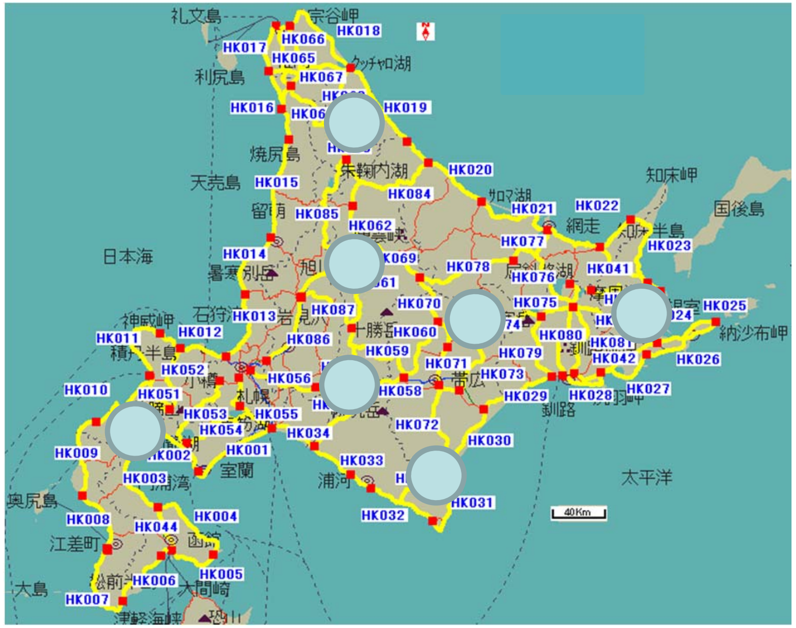
Split Delivery Vehicle Routing Problem (SDVRP)

Column generation is applied to solve the problem efficiently.

Is it OK for practitioners ? No, there will be uncertainty

Human operators are negotiating timing, due dates and costs in practical.

Transportation network design



: depot

: city

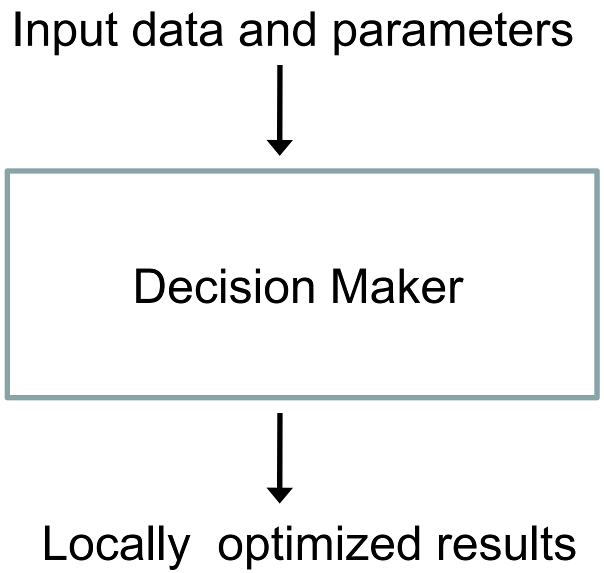
Energy demand will be change oil to electricity

How many number of depots will be required to satisfy demands from customers to minimize distribution costs ?

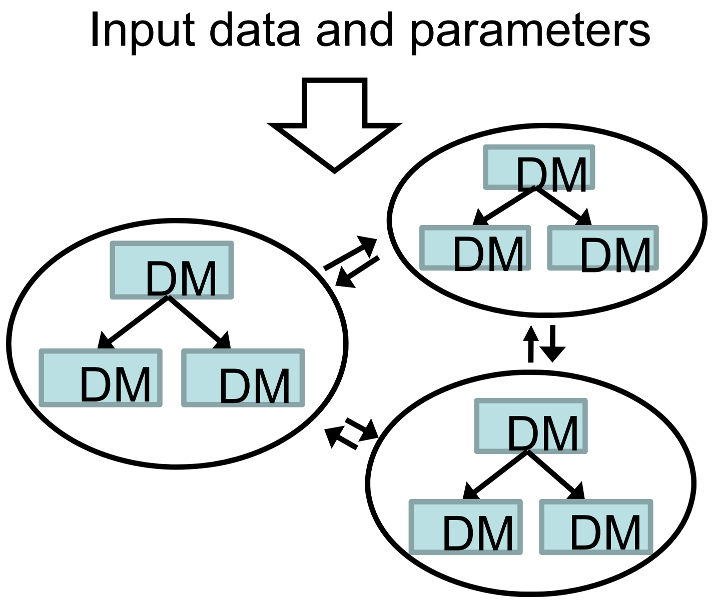
Classical facility location problem

Is it OK for practitioners ? The answer is no.
There will be competing companies.
We have to consider future marketing and competition.

Why game theoretic approach is required ?



Conventional problem setting



Current realistic problem

What is the final results of the negotiation and optimization between decision makers ?

Game theoretic approach

simultaneous optimization > game theoretic approach > separate optimization

Game theoretical analysis

- Stackelberg game on channel coordination (Cai et al. 2009)
  – leader and follower

Nash equilibrium analysis

- Equilibrium behavior of decentralized supply chain with competing retailers（Bernstein et al. 2005）
  ―analysis on supermodularity of profit function

- Price competing model with risky supplier（Serel, 2008）
  ―analysis on quasi-concavity of profit function

Nash equilibrium

For n player game,

x_i is the strategy for player i ,

x_{-i} is the strategy for players except player i .

The strategy $(x_1^*, x_2^*, \ldots, x_n^*)$ is Nash equilibrium
when the optimal strategy for player i is x_i^*
on the condition that all the players except player i is x_{-i}^*

$$x_i^* = \arg\max_{x_i} \pi_i(x_i, x_{-i}^*)$$

Supermodularity

The real-valued function $f(x)$ is supermodular if

$$f(x') + f(x'') \leq f(\min\{x', x''\}) + f(\max\{x', x''\})$$

The real-valued function $g(x,t)$ is increasing differences if

$$g(x', t'') - g(x', t') \leq g(x'', t'') - g(x'', t')$$

The 2nd differential function $f(x)$ is increasing differences if

$$\frac{\partial f(x)}{\partial x_i \partial x_j} \geq 0$$

Supermodularity and Nash equilibrium (Topkis 1998)

Theorem 1

$f(x)$ is supermodular when $f(x)$ is increasing differences and $f(x)$ is supermodular on the fixed x_i

Theorem 2

If the profit function π_i is supermodular,

Nash equilibrium exists when player i determines the strategy by

$$x_i^{k+1} = \arg\max_{x_i} \pi_i(x_i, x_{-i})$$

Existence of Nash equilibrium

Supermodular function

$$\phi(\max\{x, y\}) + \phi(\min\{x, y\}) \geq \phi(x) + \phi(y)$$

x, y : pricing vector

The profit function for each company is supermodular

↓

Increase or decrease of pricing is identical

↓

Existence of Nash equilibrium

Game theoretic approach for supply chain coordination under demand uncertainty with quantity discount contract

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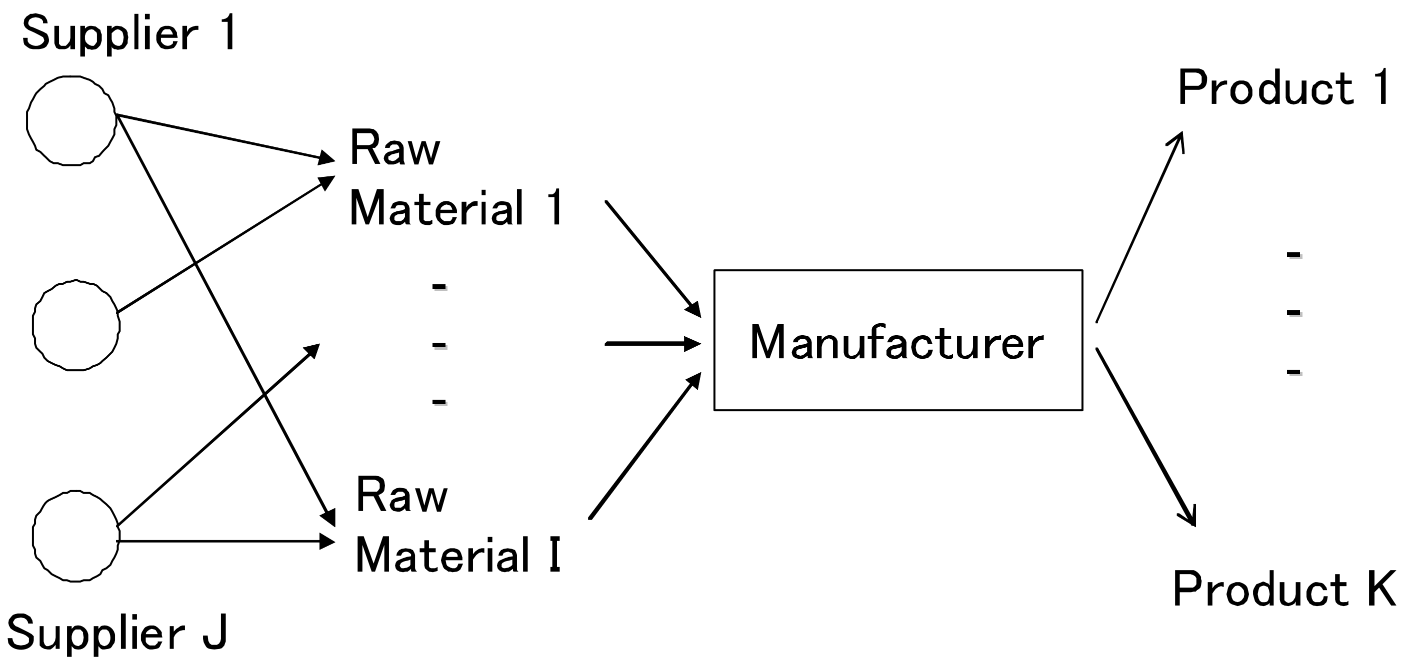
Outline

1. Game theoretic approach
2. Problem definition
3. suppliers and manufacturers model
4. Stackelberg game
5. Quantity discount as a coordination mechanism
6. Optimization algorithm
7. Numerical Example

Problem definition



A supply network consists of a manufacturer and its suppliers

- Multiple products
- Uncertain demand
- Multiple suppliers
- Capacity limits

Problem

How to select suppliers for each raw material/part and determine purchasing allocation of each RM among the suppliers to maximize the total profit of the manufacturer and suppliers

Simultaneously determine

Manufacturer (leader)

- optimal annual production levels
- supplier selection
- demand uncertainty

suppliers (followers)

- quantity of raw material
- price of raw material
- setup and ordering cost

Literature review

Quantity discount model:

Kim et al.(2002) Configuring a Manufacturing Firm's Supply Network with Multiple Suppliers

Tsai (2007) An optimization Approach for Supply Chain Management Models with Quantity Discount Policy

Zhang and Ma (2009) Optimal Acquisition Policy with Quantity Discounts and Uncertain Demands

Coordination Model:

Qin et al. (2007) Channel Coordination and Volume Discounts with Price-sensitive Demand

Yu et al. (2009) Stackelberg Game-theoretic Model for Optimizing Advertising ,Pricing and Inventory Policies in Vendor Managed Inventory Production Supply Chains

Leng et al. (2010) Game-theoretic Analyses of Decentralized Assembly Supply Chains

Modeling: assumptions

- A manufacturer and suppliers two-tier supply chain (leader-follower relationship)

- One cycle of the manufacturer's long term production period

- PDF f(z) of demand z is known

- Unit understocking cost a_k , and
  unit overstocking cost b_k are known

Decision variables and parameters

Decision variables

D_{ij} quantity of raw materials i purchased from supplier j.

y_k production quantity of product k for manufacturer.

$w_j \in \{0,1\}$ binary variable indicating whether manufacturer buys from supplier j.

Parameters

z_k random demand for product k on normal distribution

Manufacturer's planning model under demand uncertainty

Maximize J= $\sum_{k=1}^{K}\{\int_0^{y_k}[r_k z_k - b_k(y_k - z_k)f(z_k)dz_k + \int_{y_k}^{\infty} r_k y_k - a_k(z_k - y_z)f(z_k)dz_k\} - \sum_{k=1}^{K} e_k y_k - \sum_{i=1}^{I}\sum_{j=1}^{J} q_{ij} D_{ij} - \sum_{j=1}^{J} m_j w_j$

Total Profit=Expected Revenue (while it is overstocking and understocking)-Production cost-Procurement cost-Management cost

Subject to

$\sum_{k=1}^{K} y_k g_k \le \sum_{j=1}^{J} D_{ij}, \forall i = 1, \ldots I$ (1) **Bills of Materials**

$\sum_{k=1}^{K} t_k y_k \le C$ (2) **Production Capacity**

$\sum_{i=1}^{I} n_{ij} D_{ij} \le c_j w_j, \forall j = 1, \ldots J$ (3) **Procurement Capacity**

$D_{ij}, y_k \ge 0, w_j \in \{0,1\}, \forall i, \forall j, \forall k$ (4) **Variables**

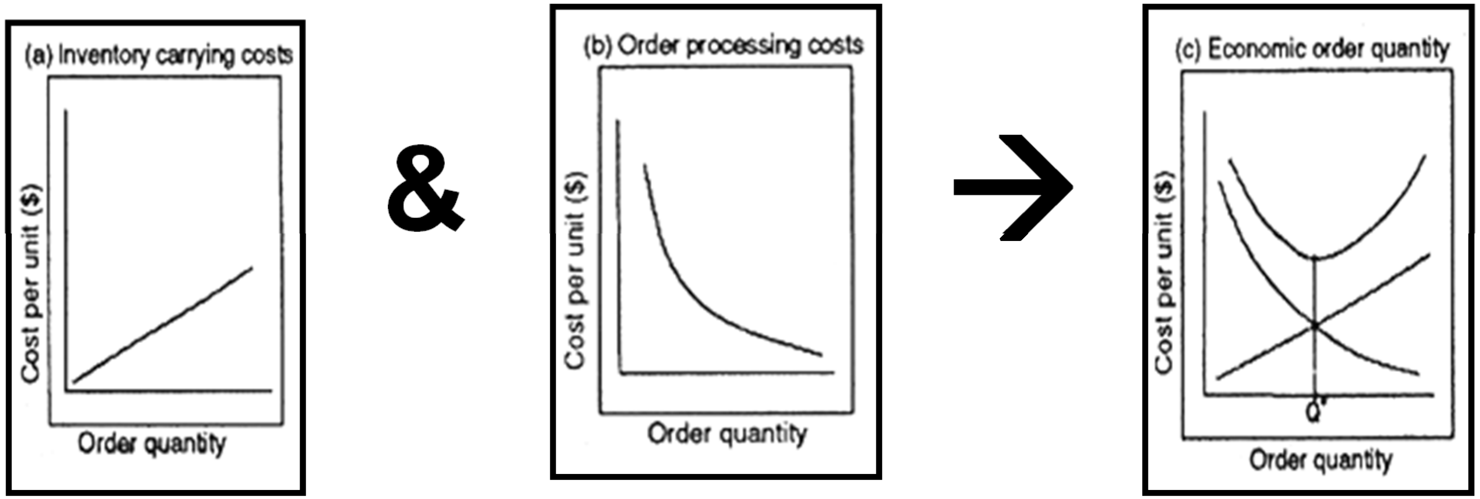
Supplier's EOQ model

Total profit= Management cost
+Gross revenue- Inventory holding cost –Order processing cost

$$\text{Maximize } \Pi_j = m_j w_j + \sum_{i=1}^{I}\left[(q_{ij} - h_{ij})D_{ij} - \frac{A_{ij}Q_{ij}}{2} - \frac{S_{ij}D_{ij}}{Q_{ij}}\right] \quad (5)$$

Subject to

$$q_{ij} \geq h_{ij}, Q_{ij} > 0, \forall i = 1, \ldots, I, \forall j = 1, \ldots, J \quad (6)$$



Stackerberg game model

Stackerberg Game model

Max : manufacturer's total profit function ← **Leader**

Subject to :

- Max : Supplier's function ← **Follower**
- constraints

Bilevel Programming Problem

Supplier's optimal response function

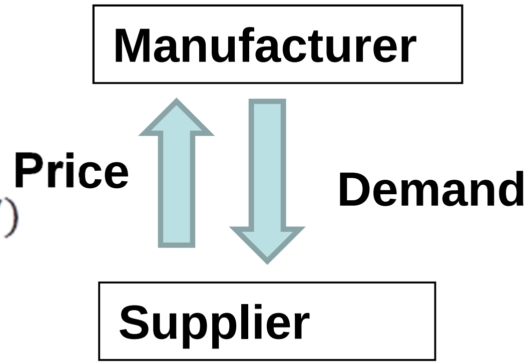
The supplier's optimal response function

First derivative of supplier's profit function (5) with respect to Q_i:

$$\frac{\partial \Pi_j}{\partial Q_i^j} = -\frac{S^j D_i^j}{{Q_i^j}^2} - \frac{A^j}{2}, \forall i = 1, \dots, I, \forall j = 1, \dots . J$$

By solving the equation $\frac{\partial \Pi_j}{\partial Q_i^j} = 0$:

$$Q_i^{*j}\left(q_i^j, D_i^j\right) = \sqrt{\frac{2S^j D_i^j}{A^j}}, \forall i = 1, \dots, I, \forall j = 1, \dots . J \quad \dots\dots (7)$$



Price of Raw Materials

First derivative of $\frac{\partial \Pi_j}{\partial D_i^j} = 0$:

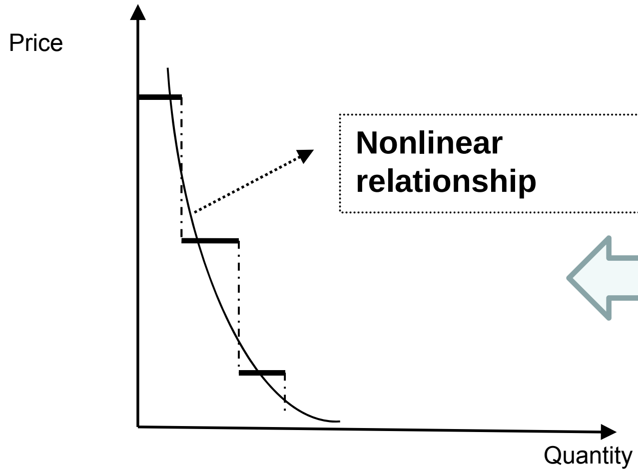
$$\frac{\partial \Pi_j}{\partial D_i^j} = q_i^j - c_i^j - \sqrt{\frac{A^j S^j}{2D_i^j}} = 0 \Leftrightarrow q_{ij}^* = c_{ij} + \sqrt{\frac{A_{ij} S_{ij}}{2D_{ij}}} \quad \dots\dots (8)$$

Quantity of purchased materials

Coordination between manufacturer and suppliers

Analysis of the Stackelberg game equilibrium

Relation function of quantity and price



$$q^*_{ij} = c_{ij} + \sqrt{\frac{A_{ij}S_{ij}}{2D_{ij}}}$$

Apply quantity discount policy
in the manufacturing decision model

Manufacturing planning model with quantity discount model

The quantity discount model can be applied to solve manufacturer's decision problem

The total profit function:

Mixed Integer Nonlinear Programming Problem

Max

$$\sum_{k=1}^{K}\left\{\int_0^{y_k}[r_k z_k - b_k(y_k - z_k)]f(z_k)dz_k + \int_{y_k}^{\infty}[r_k y_k - a_k(z_k - y_k)]f(z_k)dz_k\right\} - \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L_j} q_{ijl} D_{ijl} - \sum_{k=1}^{K} e_k y_k - \sum_{j=1}^{J} m_j w_j \quad (9)$$

Constrains:

(1), (2), (3), (4), (5), (6)

$$D_{ijl} \le d_{ijl}^{H} u_{ijl}, \forall i, j, l \quad (10)$$

$$D_{ijl} \ge d_{ijl}^{S} u_{ijl}, \forall i, j, l \quad (11)$$

$$\sum_{l=1}^{L_j} u_{ijl} = v_{ij}, \forall j \quad (12)$$

$$w_j \ge v_{ij}, \forall \quad (13)$$

$$u_{ijl}, v_{ijl}, w_j \in \{0,1\}, \forall i, j, l \quad (14)$$

assignment

Management cost

Optimization algorithm

Fix 0-1 binary variables u_{ijl}, v_{ij}, w_j to a 0-1 binary configuration

Primal Problem:

$$\max \sum_{k=1}^{K} \left\{ \int_0^{y_k} [r_k z_k - b_k (y_k - z_k)] f(z_k) dz_k + \int_{y_k}^{\infty} [r_k y_k - a_k (z_k - y_k)] f(z_k) dz_k \right\}$$

$$- \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{l=1}^{L_j} c_{ijl} \, x_{ijl} - \sum_{k=1}^{K} e_k \, y_k$$

solved by SQP algorithm

Master Problem:

$$\min(\mu OA + \sum_{j=1}^{J} (\sum_{i=1}^{I} c_{ij} x_{ij}) \sum_{l=1}^{L_j} (1 - d_{jl}) u_{jl} + \sum_{j=1}^{J} m_j w_j)$$

s.t $\quad \mu OA \geq -P(y_k^h) - \left.\frac{\delta P}{\delta y_k}\right|_{y_k = y_k^h} (y_k - y_k^h)$

$$\sum_{j \in B^h} (w_j + \sum_{i \in B^h} v_{ij} + \sum_{l \in B^h} u_{jl}) - \sum_{j \in NB^h} (w_j + \sum_{i \in NB^h} v_{ij} + \sum_{l \in NB^h} u_{jl}) \leq |B^h| - 1$$

where $P(y) = - \sum_{k=1}^{K} \left\{ \int_0^{y_k} [r_k z_k - b_k (y_k - z_k)] f(z_k) dz_k + \int_{y_k}^{\infty} [r_k y_k - a_k (z_k - y_k)] f(z_k) dz_k \right\} - \sum_{k=1}^{K} e_k \, y_k$

Improvement of OA algorithm

The deviation between realized demand and production.

min (z_k, y_k)

Set $X_k = \frac{z_k - \check{z}}{\sigma_k}$ $Y_k = \frac{y_k - \check{z}}{\sigma_k}$

Therefore, the expected revenue from products is

$$RE_k(Z_k) = r_k \min(z_k, y_k) = \begin{cases} r_k z_k & if\ z_k < y_k \\ r_k y_k & if\ z_k > y_k \end{cases}$$

According to the definition of normal distribution:

$$F(x) = \frac{1}{\sqrt{2\pi}} e^{(-\frac{1}{2}(x)^2)}$$

$$=> \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t^2} dt$$

$$E(RE_k) = \Phi(Y_k) E[r_k z_k \mid \frac{z_k - \check{z}}{\sigma_k} \le \frac{y_k - \check{z}}{\sigma_k}]$$

$$+ (1 - \Phi(Y_k))\ E[r_k y_k \mid \frac{z_k - \check{z}}{\sigma_k} \ge \frac{y_k - \check{z}}{\sigma_k}]$$

$$= r_k \check{z} + r_k \sigma_k \{\Phi(Y_k) E[X_k | X_k \le Y_k]$$

$$+ (1 - \Phi(Y_k))\ E[Y_k | X_k \ge Y_k]\}$$

$$E[Y_k | Y_k \le X_k] = Y_k$$

$$E[X_k | X_k \le Y_k] = \frac{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Y_k} X_k e^{-\frac{1}{2}X_k^2} dX_k}{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Y_k} e^{-\frac{1}{2}X_k^2} dX_k} = \frac{-1}{\Phi(Y_k)} \frac{e^{-\frac{1}{2}X_k^2}}{\sqrt{2\pi}} = -\frac{F(Y_k)}{\Phi(Y_k)}$$

Final Equation:

$$E(\text{overproduction}) = b_k \{\check{z} + \sigma_k[-F(Y_k) + (1 - \Phi(Y_k))\ Y_k]\}$$

$$E(\text{shortfall}) = a_k \{\ E\{\min(z_k, y_k)\} - \check{z}\ \}$$

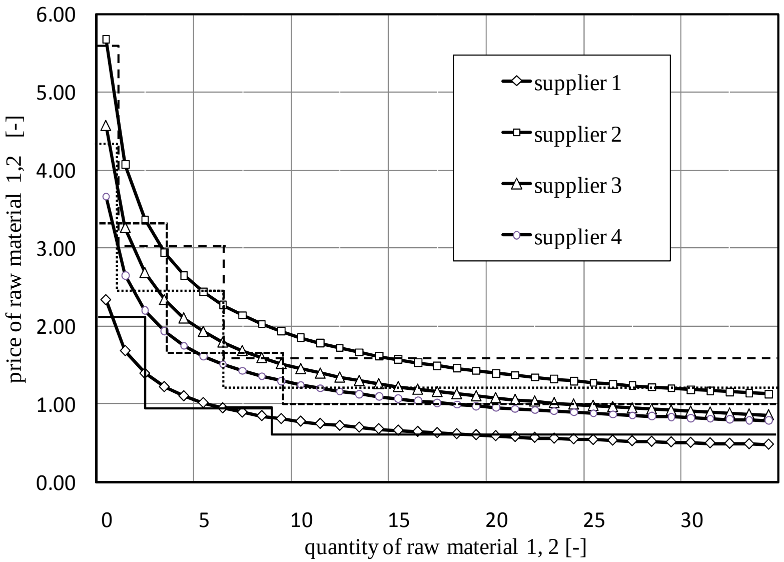
$$= a_k\ \sigma_k[-F(Y_k) + (1 - \Phi(Y_k))\ Y_k]$$

$$E(RE_k) = \check{z} + \sigma_k[-F(Y_k) + (1 - \Phi(Y_k))\ Y_k]$$

Numerical example

4 suppliers, 1 manufacturer, 2 finished products,5 raw material,3 quantity discount interval

An Intel(R) Core2Duo E8400 3 GHz with 3GB memory (Matlab R2009a and CPLEX 9.0 with CPLEXINT library)



Incremental Discount Schedule

Numerical example

Optimal Solutions

Revenue	28,169
Overstock cost	5.2
Shortage cost	1,378
Production cost	5,090
Material cost	98.2
Management cost	259
Total profit	21,338.6

Manufacturer's Optimal Solution

Results:
Total computation time: 28.9 second
The derived upper bound is 21365.6, and the lower bound is 21338.6
Duality gap : 0.127%

suppl.1	D_{i11}	D_{i12}	D_{i13}	q^*_{i1}	Q^*_{i1}	EOQ_{i1}
mater.1	0	0	29.48	0.51	12.1	-0.412
mater.2	0	0	27.38	0.53	11.7	-0.427
mater.3	0	0	13.00	0.59	5.10	-0.392
mater.4	0	0	21.50	0.50	6.56	-0.305
mater.5	0	0	33.64	0.44	8.20	-0.244
suppl.2	D_{i21}	D_{i22}	D_{i23}	q^*_{i2}	Q^*_{i2}	EOQ_{i2}
mater.1-5	0	0	0	-	-	0
suppl.3	D_{i31}	D_{i32}	D_{i23}	q^*_{i3}	Q^*_{i3}	EOQ_{i3}
mater.1-5	0	0	0	-	-	0
suppl.4	D_{i41}	D_{i42}	D_{i43}	q^*_{i4}	Q^*_{i4}	EOQ_{i4}
mater.1	0	0	0	-	-	0
mater.2	0	3.50	0	2.05	4.32	-1.852
mater.3	0	0	7.00	1.51	5.29	0.094
mater.4	0	0	7.00	1.51	5.29	0.094
mater.5	0	3.00	0	2.20	3.46	-0.928

Supplier's Optimal Solution

Conclusion and future works

Contract decision models including quantity discount model and the Stackelberg game theoretic model are studied.

The outer approximation method is applied to solve the mixed integer nonlinear programming problem.

The computational experiments demonstrate the effectiveness of the proposed method and the feasibility of the model.

Conclusion and future works

In the Future

❖Contract Decision for Supply Chain Optimization

Integrated supply chain management is investigated to maximize the profits, meanwhile, to decrease the energy consumption and the burden of environment

❖International Logistics

Transportation, distribution, manufacturing and the like to support the globalization of the business

❖Transportation Network Design

Reasonable routing with consideration of the future market and trade off