Gauged Supergravity and Holographic RG Flows

Stonybrook, December, 2001

Based upon:

- hep-th/9904017; hep-th/9906194; hep-th/004063

  Freedman, S. Gubser, K. Pilch and NPW

- Recent work with R. Corrado, M. Gunaydin and M. Zagermann

  to appear

Overview

- Teaching an old dog new tricks: The AdS/CFT Correspondence
- Holgraphic Description of RG flows
- Open problems in holographic field theory and supergravity
  - Half-maximal supersymmetric models

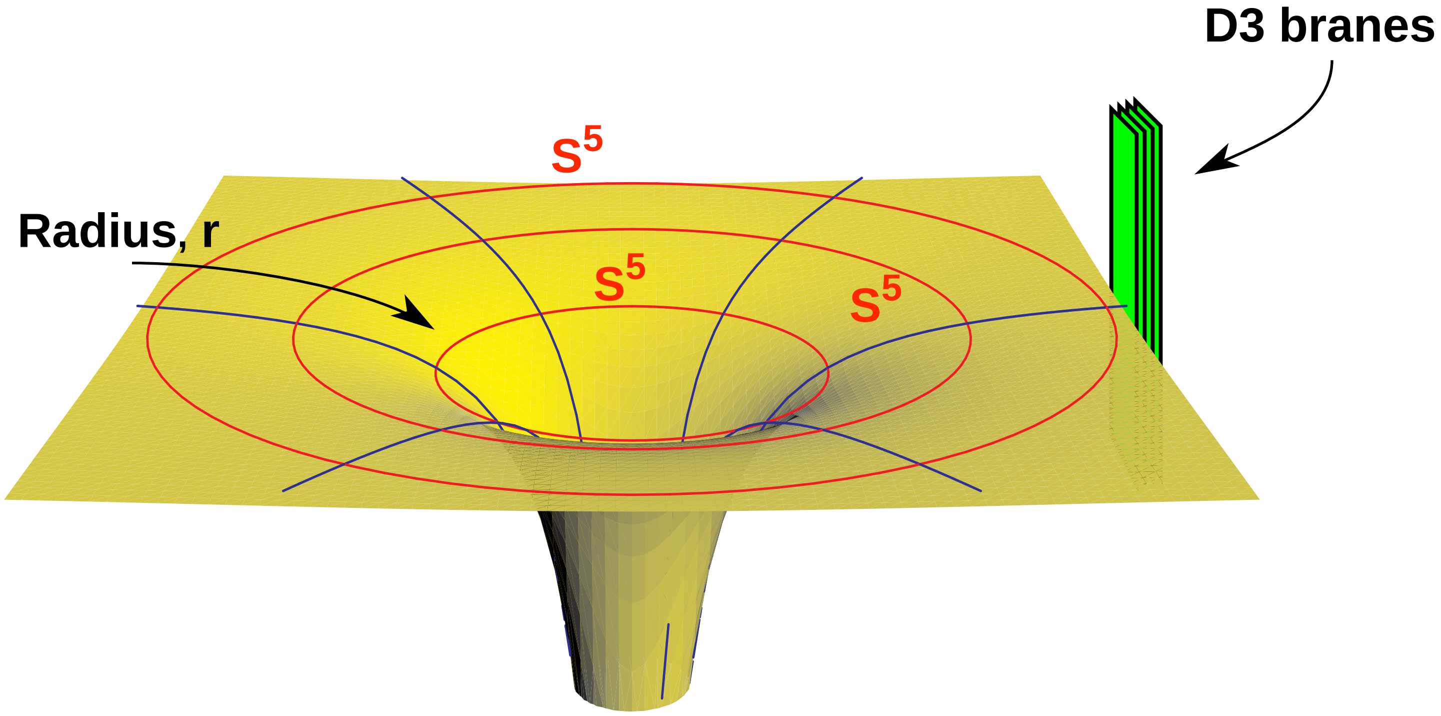
    $\mathcal{N}$ = 2 Quiver Gauge theories ↔ $\mathcal{N}$ = 4 supegravity + tensor multplets
  - Half-maximal supersymmetric flows

    $\mathcal{N}$ = 2 Seiberg Witten flows
  - Supersymmetric AdS geometries
- Other issues

Yang-Mills theory and AdS/CFT

D3-branes in IIB Supergravity

$\mathcal{N} = 4$ Yang-Mills in strongly coupled, Large N limit



D3 branes + r ↔ AdS_5

Operators on the brane ↔ Sources in Supergravity

Correlators and states ↔ Classical supergravity solutions

<u>Five-dimensional perspective:</u>

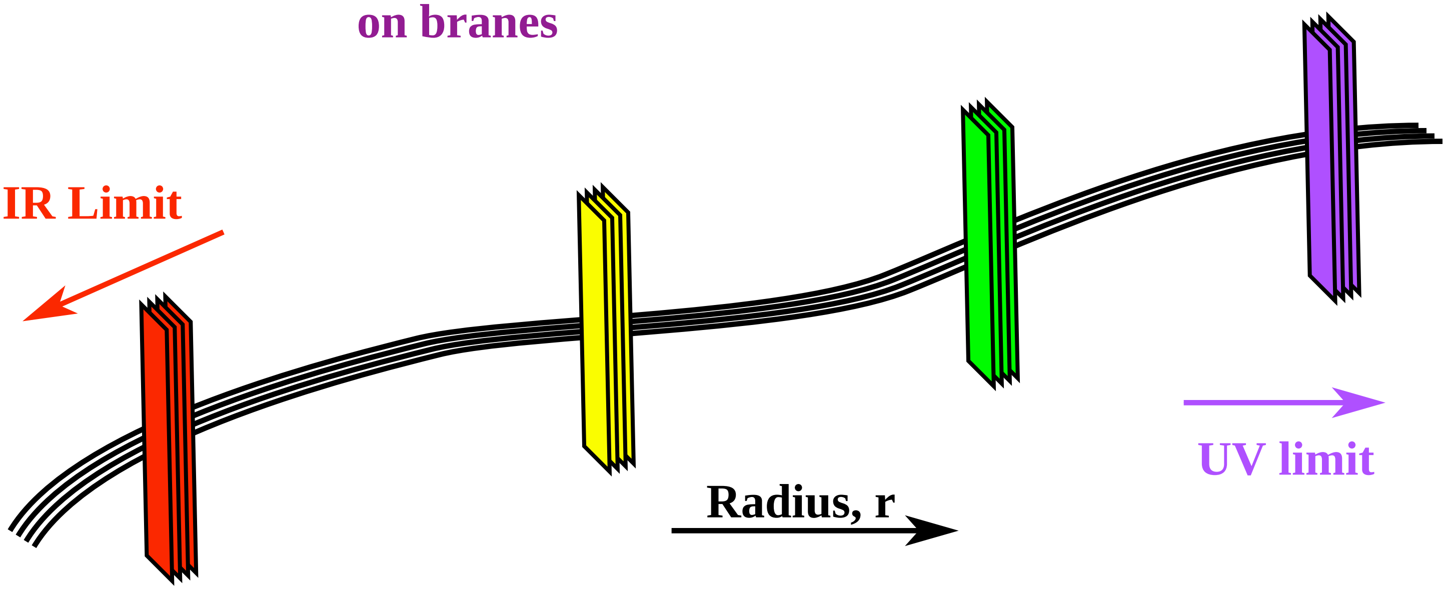
- **View S^5 as an internal space**
- **Decompose theory into generalized Fourier modes on S^5**
- **Space-time = D3 branes + radius, r**
- **Interpret r as an RG scale for theory on brane**

Interpreting the metric

$$ds_{10}^2 = \Omega^2(y)\left(e^{2A(r)}\, \underbrace{\eta_{\mu\nu}\, dx^\mu\, dx^\nu}_{\text{D3 brane}} + dr^2 \right) + \underbrace{ds_5^2\,(y)}_{\text{internal}}$$

warp factor

Physical scale on branes



- **Localize D3-branes at infinity** ↔ **Conformal, UV limit**
  Localize D3-branes finite r ↔ Explicit cut-off
- **Red shifts** ↔ **Wilsonian coarse graining**
- **Cosmological Entropy** ↔ **c-function / theorem**
- **No hair theorems** ↔ **Universality in IR limit**
- **Gauged supergravity potentials** ↔ **Phase diagrams / flows** from relevant perturbations
- **Consistent truncation** ↔ **Large N structure of OPE in E.M. supermultiplets**

Infra-red Fixed Points

I. Perturb the $\mathcal{N} = 4$ theory: $\left(A_\mu, \ \lambda^1, \ldots, \lambda^4, \ X^1, \ldots, X^6 \right)$

Freedman, Gubser, Pilch and Warner, hep-th/9901017

Give mass to an $\mathcal{N} = 1$ chiral multiplet: $W = W_0 + m \, \mathrm{Tr}(\Phi^2)$

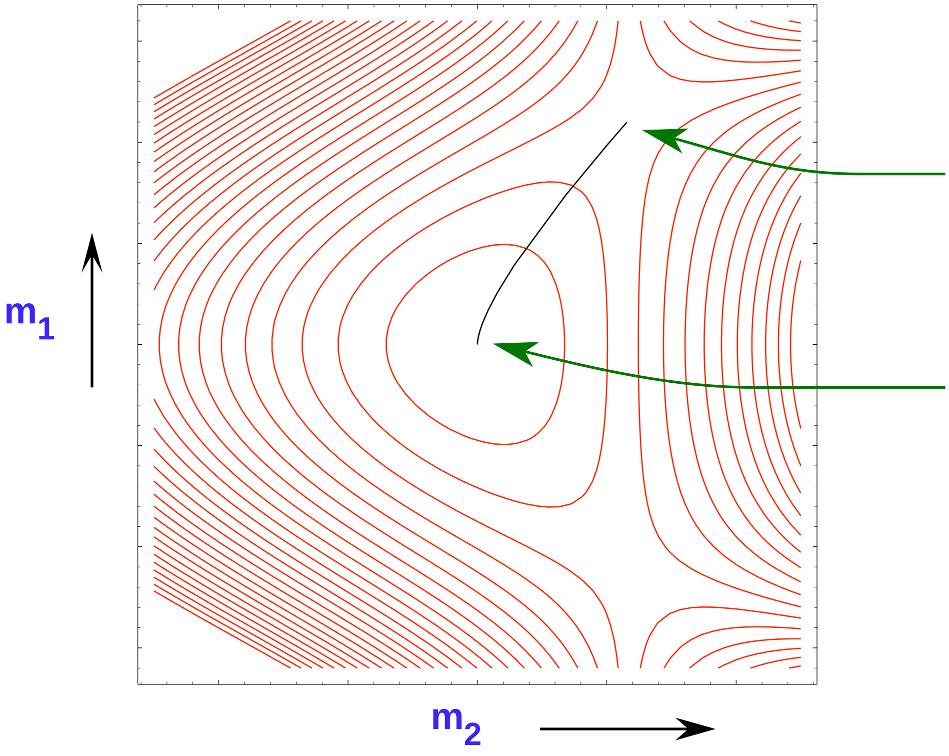
$$\Delta \mathcal{L} = m_1 \, \mathrm{Tr}(\lambda^1 \lambda^1) + m_2 \, \mathrm{Tr}((X^1)^2 + (X^2)^2); \qquad m_2 = m_1^2 = m^2$$

Non-trivial $\mathcal{N} = 1$ fixed point

(Leigh and Strassler) for flow as $m \to \infty$

$$\frac{c_{IR}}{c_{UV}} = \frac{27}{32}$$

Supergravity: steepest descent on a supergravity superpotential, $\mathcal{W}$:



$\mathcal{N} = 1$ supersymmetric critical point / phase

$\mathcal{N} = 4$ supersymmetric critical point / phase

II. Perturb an $\mathcal{N} = 2$ superconformal quiver theory:

Klebanov and Witten hep-th/9807080

Gubser, Nekrasov and Shatashvili hep-th/9811230

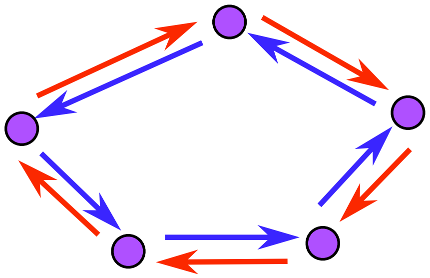
$$\frac{c_{IR}}{c_{UV}} = \frac{27}{32}$$

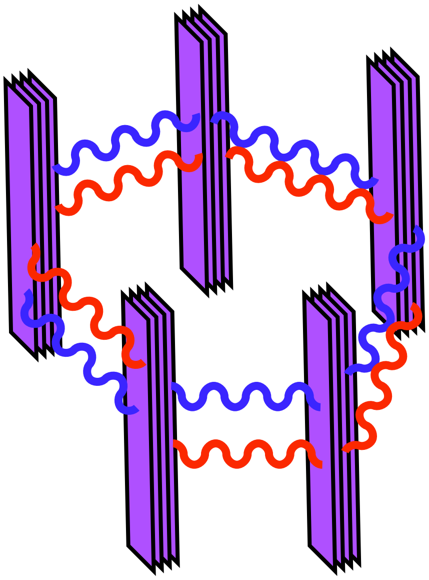
Quiver Theories and Orbifolds

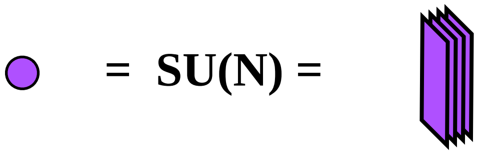
$\mathcal{N} = 2$, $(SU(N))^p$ Yang-Mills

+ bi-fundamental matter

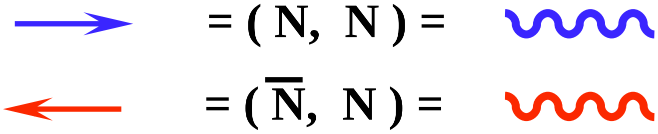
Z_p Orbifold of p x N D3-branes







= SU(N) =



= $(N, \bar{N})$ =

= $(\bar{N}, N)$ =

Superconformal with R-symmetry = SU(2) x U(1)

Untwisted sector: Vector multiplets

$(A^{(k)}_{\mu} ;\ \lambda_1^{(k)}, \ \lambda_2^{(k)} ;\ \phi^{(k)} = X_1^{(k)} + i\, X_2^{(k)})$ $\qquad k = 1, \dots, p$

Twisted sector: Hypermultiplets $(\psi_1^{(k)}, \ \psi_2^{(k)} ;\ A^{(k)} , \ B^{(k)})$

Mass terms / flow: $\Sigma_k \, m_{(k)} (\phi^{(k)})^2$

Klebanov and Witten: $m \, ((\phi^{(1)})^2 - (\phi^{(2)})^2)$ $\qquad$ p=2, $\mathcal{Z}_2$ odd

Freedman, Gubser, Pilch and Warner: $m \, ((\phi^{(1)})^2 + (\phi^{(2)})^2)$ $\qquad$ $\mathcal{Z}_2$ even

⇒ **Leigh-Strassler fixed points**

Five-dimensional, gauged supergravity and quiver flows

Gauged $\mathcal{N} = 4$ supergravity + Vector or Tensor multiplets:

Dall'Agata, Herrmann and Zagermann, hep-th/0103106

R-symmetry on the brane = Gauge group: SU(2) x U(1)

(supergravity)

$$m_{(k)}^{ab} \; \lambda_a^{(k)} \lambda_b^{(k)} \; + \text{c.c.} \qquad \longleftrightarrow \qquad 3\,(+2) \; + \; 3\,(-2)$$

$$m_{(k)} \; (\phi^{(k)})^2 \; + \; \text{c.c.} \qquad\qquad 1\,(+4) \; + \; 1\,(-4)$$

$$\tau_{(k)} = \frac{4\pi i}{g_{(k)}^{\;2}} + \frac{\theta_{(k)}}{2\pi} \qquad \longleftrightarrow \qquad 1\,(0) \; + \; 1\,(0)$$

Scalar content of a pair of five-dimensional, charged tensor multiplets

Gauged $\mathcal{N} = 4$ supergravity + 2p Tensor multiplets: charges (+2) and (-2)

Scalar Manifold (Supergravity) ⟷ Couplings in Gauge theory

$$SO(1,1) \times \frac{SO(5, 2p)}{SO(5) \times SO(2p)} \supset SO(3) \;\; \times \;\; SO(2, 2p)$$

$$SU(2)_{\mathcal{R}} \times U(1)_{\mathcal{R}} \times SU(p, 1)$$

Invariance of supergravity potential

$$\frac{SU(p, 1)}{SU(p) \times U(1)}$$ = Non-compact CP_p parametrizing couplings: $\tau_{(k)}$

Generalizes:

SU(1,1)/U(1) of IIB supergravity ⟷ $\mathcal{N} = 4$ Yang-Mills Coupling, τ

Unexpected Consequences: SU(p) Symmetry

Untwisted scalar sector in $\mathcal{N} = 8$ supergravity = $\mathcal{N} = 4$ supergravity + two charged tensor mutiplets ⟹ $\dfrac{SO(5, 2)}{SO(5) \times SO(2)}$

+

Twisted sector scalars from 2(p-1) additional tensor multiplets = (p-1) x (5,2) of SO(5) x SO(2) ⟹ $\dfrac{SO(5, 2p)}{SO(5) \times SO(2p)}$

Flow of FGPW: $m\,((\phi^{(1)})^2 + (\phi^{(2)})^2)$

lies entirely in scalars of untwisted sector

Flow of KW: $m\,((\phi^{(1)})^2 - (\phi^{(2)})^2)$

lies entirely in scalars of twisted sector

U(p) symmetry of $\mathcal{N} = 4$ supergravity potential

SU(p) symmetry ⟹ **Critical point of FGPW extends to a (p-1)-dimensional critical surface**

Field Theory:

Flows driven by $\Sigma_k\ m_{(k)}(\phi^{(k)})^2$ go to a smooth, SU(p) symmetric family of IR fixed points parametrized by $\tau_{(k)}^2/m_{(k)}$ considered as homogeneous coordinates on CP_{p-1}.

IIB Supergravity: Bizarre

- FGPW flow: topologically trivial flux on $S^5/\mathcal{Z}_2$
- KW flow: topologically non-trivial flux on blow-up of A_1 singularity on $S^5/\mathcal{Z}_2$

U(p)

Comments:

- **Family of equivalent fixed points with same central charge: parametrized by initial "velocities", $m_{(k)}$, of flow.**

- **This is a "large N result:" the U(p) symmetry is broken by anomalies at finite N.**

  Residual Symmetry $U(p) \supset \overset{\text{finite}}{\mathcal{G}} \supset (Z^{2N})^p$

- **Discrete symmetry at finite N between FGPW and KW flow?**

- **Singular behavior at finite N when one or more $m_{(k)}$ vanish?**

- **Does consistent truncation work for $\mathcal{N} = 4$ supergravity?**
  - **Unknown territory...**
  - **We have solutions in both five and ten dimensions.**
    - **Solutions coincide at linearized level**
    - **Same initial velocities**
    - **Same supersymmetry**
    - **Same $\mathcal{R}$-symmetry**

- **Try to find the SU(2) family of solutions in IIB supergravity that interpolates between a topologically trivial flux on $S^5/\mathbb{Z}_2$ and a topologically non-trivial flux on the blow-up of an A_1 singularity on $S^5/\mathbb{Z}_2$.**

Corrado, Pilch and Warner: work in progress

Another duality that trades 3-form flux for Kähler moduli ....

Open Problems: Supersymmetric *RR*-Geometries

Geometry of supersymmetric solutions *with background RR-fluxes?*

Ricci flat manifolds + $\mathcal{N} = 2$ supersymmetry: Kähler

Ricci flat manifolds + $\mathcal{N} = 4$ supersymmetry: Hyperkähler

What is the analogue of this for holographic RG flow solutions?

The issue for flow solutions in IIB supergravity:

- **The flows generally start from AdS$_5$, with a background 4-form RR-tensor gauge field**
- **Softly broken $\mathcal{N} = 4$ Yang-Mills: fermion masses are holographically dual to 2-form tensor gauge fields**
- **The dilaton and axion are dual to the Yang-Mills gauge couplings, and thus generically run.**

⇒ **Interesting supersymmetric flow solutions involve fluxes for all the background tensor gauge fields**

Classification theorems for such solutions: very few, and none of them relevant to the important physical holographic flows.

Important Example: (unsolved)

Find the Holographic Dual of the $\mathcal{N} = 2$ Seiberg-Witten effective action

Holography and the $\mathcal{N}$ = 2 Seiberg-Witten effective action

Simplest version:

$\mathcal{N}$ = 4 Yang-Mills → $\mathcal{N}$ = 2 Yang-Mills + massive hypermultiplet

Coulomb branch: $u_n = \mathrm{Tr}\,(\phi^n)$

↓

D3-brane distribution **Source:** $\rho(y) = \Sigma\, u_n\, y^n$

Complex scalar, ϕ → Complex coordinate, y

Problem: Find the general $\mathcal{N}$ = 4 supersymmetric flow solution determined by an arbitrary source function, $\rho(y)$, of two variables

What is known: the solution for one point in the moduli space

$$\rho(y) = (\, a^2 - |y|^2\,)^{1/2}$$

A uniform disk-like distribution of D3-branes

Pilch and Warner; hep-th/0006066
Buchel, Peet and Polchinski; hep-th/0008076
Evans, Johnson and Petrini; hep-th/0008081

The general $\mathcal{N}$ = 8 supersymmetric flow solution (the Coulomb branch of $\mathcal{N}$ = 4 Yang-Mills) is well-known, and is determined by an arbitrary harmonic source function, $H(y)$, of six variables. The half-maximal supersymmetric solutions shouldn't be much more difficult....

Harder problem: Find the general $\mathcal{N}$ = 4 supersymmetric flow solution for the quiver gauge theories

Other Issues

- $\mathcal{N} = 1$ supersymmetric flows: generalized Kähler structure in the presence of *RR* fluxes?
  - Brane probe results find the Kähler structure on the moduli space of the probe.

    Johnson, Lovis, C. Page, hep-th/0107261

- Non-compact gaugings: Holographic interpretation???
  - Vacua: De-Sitter space → All supersymmetry broken
  - Domain wall solutions can be (half-maximally) supersymmetric

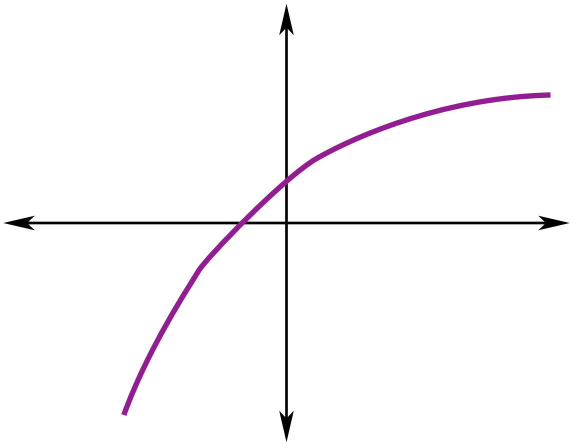
    Hull, hep-th/0110048; Gibbons and Hull hep-th/0111072

- Potentials without critical points:

  Domain wall solutions...

  ⇓ ???

  Gauged Supergravity description + Generalizations and variations on Duality Cascade of Klebanov and Strassler? hep-th/0007191



- Most of the exactly known flow solutions have been obtained by lifting solutions of gauged supergravity
  - Systematics of lifting
  - General methods of construction in 10 or 11 dimensions

Conclusions

- **Gaused supergravity is a very valuable tool in the study of holographic RG flows**

- **$\mathcal{N} = 4$ gauged supegravity coupled to tensor multiplets can be used to study a class of $\mathcal{N} = 1$ supersymmetric flows in large N, $\mathcal{N} = 2$ quiver gauge theories**

- **The large N, $\mathcal{Z}_p$ quiver models have a (p-1)-dimensional surface, CP_{p-1}, of $\mathcal{N} = 1$ supersymmetric fixed points, and all these fixed point theories are equivalent under the action of an SU(p).**

- **In ten dimensions, this SU(p) must act as a "generalized duality symmetry", mapping compactifications with fluxes on $S^5/\mathcal{Z}_p$ to compactifications with blow-ups of the A_{p-1} singularity:**

Flux ⟷ **Kähler Moduli**

- **Important open question in holographic RG flows:**

Geometry of supersymmetric compactifications with *RR* fluxes???

  - **Holographic description of Seiberg-Witten actions**
  - **Holographic interpretation of the many supersymmetric domain-wall solutions?**