Physics 202, Lecture 4

Today's Topics

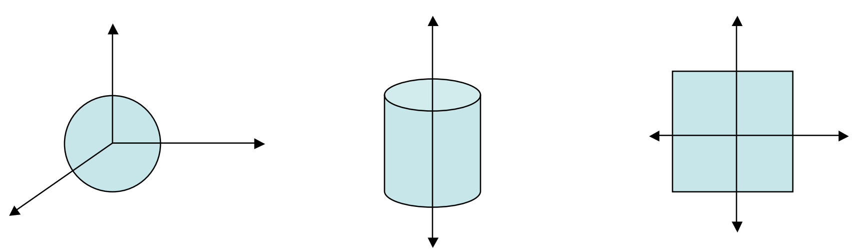
- Review: Gauss's Law
- Electric Potential (Ch. 25-Part I)
  - Electric Potential Energy and Electric Potential
  - Electric Potential and Electric Field
- Next Tuesday: Electric Potential (Ch. 25-Part II)
- Homework #1 due tomorrow (9/14) at 10 PM
  Homework #2 (now on WebAssign) due 9/24 at 10 PM

Gauss's Law: Review

$$\Phi_E = \oint \vec{E} \bullet d\vec{A} = \frac{\sum q_{in}}{\varepsilon_0}$$

Fundamental equation of electrostatics
(equivalent to Coulomb's Law)

Can use it to obtain E for highly symmetric charge distributions.
Method: evaluate flux over carefully chosen "Gaussian surface":



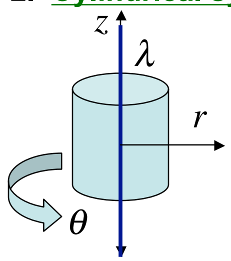
spherical — (point chg, uniform sphere, spherical shell,…)

cylindrical — (infinite uniform line of charge or cylinder…)

planar — (infinite uniform sheet of charge,…)

Gauss's Law: Examples

1. Spherical symmetry (last lecture).
2. Cylindrical symmetry. Example: infinite uniform line of charge.



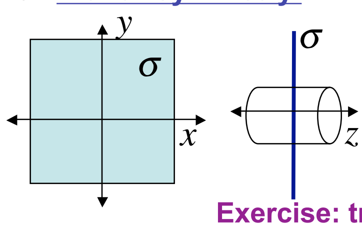
Symmetry: E indep of z, θ , in radial direction

Gaussian surface: cylinder of length L

$$\oint \vec{E}\bullet d\vec{A} = E(r)2\pi rL = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

$$\Rightarrow \quad \vec{E}(r) = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$

3. Planar symmetry. Example: infinite uniform sheet of charge.



Symmetry: E indep of x,y, in z direction
Gaussian surface: pillbox, area of faces=A

$$\oint \vec{E}\bullet d\vec{A} = 2EA = \frac{q_{in}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} \quad \Rightarrow \quad \vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}$$

Exercise: try for E field just outside of a conductor

Electric Potential Energy and Electric Potential

Review: Conservation of Energy (particle)

- Kinetic Energy (K) $K = \frac{1}{2}mv^2$
- Potential Energy U: conservative forces (work independent of path) $U(x,y,z)$
- If only conservative forces present in system, conservation of mechanical energy: $K + U =$ constant
- Examples of conservative forces:
  - Springs: elastic potential energy $U = k_{spring}x^2/2$
  - Gravity: gravitational potential energy
  - Electrostatic: electric potential energy (today)
- Examples of nonconservative forces
  - Friction, viscous damping (terminal velocity)

Electric Potential Energy (I)

Compare with gravitational force (Ch. 13):

$$\vec{F}_{12} = -G\frac{m_1 m_2}{r^2}\hat{r}_{12} \quad \Rightarrow \quad W = \int_{path} \vec{F} \bullet d\vec{s} = \frac{Gm_1 m_2}{r_f} - \frac{Gm_1 m_2}{r_i}$$

➔**Gravitational Potential energy:**

path independent!

$$U = -\frac{Gm_1 m_2}{r}$$

- **Electric Force:**

$$\vec{F}_{12} = k_e\frac{q_1 q_2}{r^2}\hat{r}_{12}$$

Electric Potential Energy ⟹

$$U = \frac{k_e q_1 q_2}{r}$$

Electric Potential Energy (II)

Given two positive charges q and q_0:

Initially charges very far apart: $U_i = 0$

(we are free to define the potential energy zero somewhere)

To push particles together requires work (they want to repel).

Final potential energy will increase! $\Delta U = U_f - U_i = \Delta W$

Now, suppose q is fixed at the origin. What is work required to move q_0 from infinity to a distance r away from q?

$$\Delta W = \int_{\infty}^{r} \vec{F}_{us} \bullet d\vec{s} = -\int_{\infty}^{r} \vec{F}_e \bullet d\vec{s} = -\int_{\infty}^{r} \frac{k_e q q_0}{r'^2} dr' = \frac{k_e q q_0}{r}$$

Note: if q negative, final potential energy negative

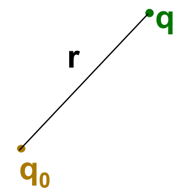
Particles will move to minimize their final potential energy!

Electric Potential Energy: Summary

- Electric potential energy between two point charges:

$$U(r) = \frac{k_e q_0 q}{r}$$

  - U is a scalar quantity, can be + or -
  - convenient choice: U=0 at r= ∞
  - SI unit: Joule (J)



- Electric potential energy for system of multiple charges: sum over pairs:

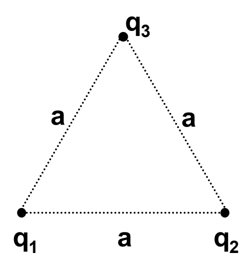
$$U(r) = \sum_{i<j}\sum_{j} \frac{k_e q_i q_j}{r_{ij}}$$

Integral if continuous distribution

Example: Three Charge system

- What is work required to assemble the three charge system as shown? ($q_1=q_2=q_3=Q$)

Answer: $k_e\ 3Q^2/a$ (see board)



- What if $q_1=q_2=Q$ but $q_3=-Q$?

Answer: $-k_eQ^2/a$

Electric Potential Energy: Charge In An Electric Field

- Charge q_0 is subject to Coulomb force in electric field **E**:

$$\vec{F} = q_0 \vec{E}$$

- Work done by electric force:

$$W = \int_i^f \vec{F} \bullet d\vec{s} = q_0 \int_i^f \vec{E} \bullet d\vec{s} = -\Delta U$$

$$\Delta U = U_f - U_i = -q_0 \int_i^f \vec{E} \bullet d\vec{s}$$

independent of q_0

Electric Potential Difference

- Electric Potential Energy: q_0 In a Generic E. Field

$$\Delta U = U_B - U_A = -q_0 \int_A^B \vec{E} \bullet d\vec{s} = q_0 \Delta V$$

system potential energy; test; source

- Electric Potential Difference

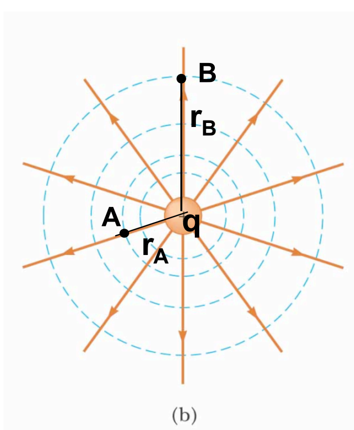
$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \vec{E} \bullet d\vec{s} = V_B - V_A$$

Properties of the Electric Potential

- Results from conservative nature of the electric force
- associated with source field only (indep. of test charge)
- units: J/C ≡ Volt (V)
- often called potential, but meaningful only as potential difference V_B-V_A.
  A convenient point (∞, earth..) typically chosen as "ground" → ΔV=V-(V_A=0) =V
- scalar quantity (no vector operations necessary!)
- related to electric potential energy by $\Delta U=q_0\Delta V$

Exercise 2: E. Potential and Point Charges

In the configuration shown, find V_B-V_A



Answer:

V_B-V_A = $k_e(q/r_B - q/r_A)$

(See board)

Exercise 1: Uniform E. Field

In the uniform electric field shown:

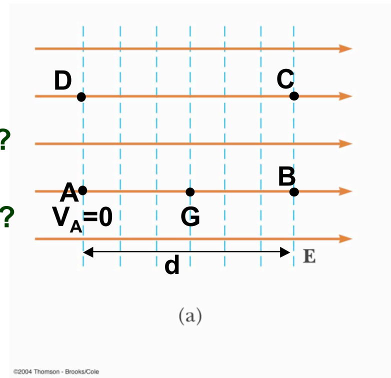
1. Find potential at B,C,D,G

2. If a charge +q is placed at B, what is the potential energy U_B?

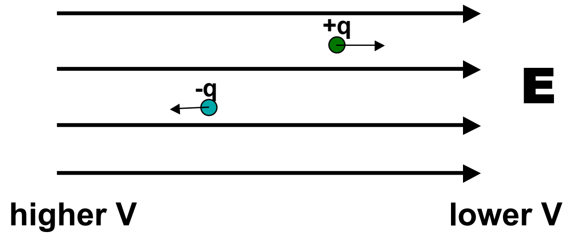
3. If now a –q is at B, what is U_B?

4. If a -q is initially at rest at G, will it move to A or B?

5. What is the kinetic energy when it reaches A?



A Picture to Remember



- Field lines always point towards lower electric potential
- In an electric field:
  - positive charges are always subject to a force in the direction of field lines, towards lower V
  - negative charge is always subject a force in the opposite direction of field lines, towards higher V

Obtaining the Electric Field From the Electric Potential

- Three ways to calculate the electric field
  - Coulomb's Law $\mathbf{E}=\Sigma\mathbf{E}_i$
  - Gauss's Law
  - Derive from electric potential ←
- Formalism

$$\Delta V = -\int_A^B \vec{E}\bullet d\vec{s}$$

$$dV = -\vec{E}\bullet d\vec{s} = -E_x dx - E_y dy - E_z dz$$

$$E_x = -\frac{\partial V}{\partial x} \quad , E_y = -\frac{\partial V}{\partial y} \quad , E_z = -\frac{\partial V}{\partial z} \quad or \quad \vec{E} = -\nabla V$$