GCE MATHEMATICS

Ordinary Level (Syllabus 4016)

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MATHEMATICS GCE Ordinary Level (Syllabus 4016)

AIMS

The syllabus is intended to provide students with the fundamental mathematical knowledge and skills.

The general aims of the mathematics syllabuses are to enable students to:

- 1. acquire the necessary mathematical concepts and skills for continuous learning in mathematics and related disciplines, and for applications to the real world;
- 2. develop the necessary process skills for the acquisition and application of mathematical concepts and skills;
- 3. develop the mathematical thinking and problem solving skills and apply these skills to formulate and solve problems;
- 4. recognise and use connections among mathematical ideas, and between mathematics and other disciplines;
- 5. develop positive attitudes towards mathematics;
- 6. make effective use of a variety of mathematical tools (including information and communication technology tools) in the learning and application of mathematics;
- 7. produce imaginative and creative work arising from mathematical ideas;
- 8. develop the abilities to reason logically, to communicate mathematically, and to learn cooperatively and independently.

ASSESSMENT OBJECTIVES

The assessment will test candidates' abilities to:

- AO1 understand and use mathematical concepts and skills in a variety of contexts;
- **AO2** organise and analyse data and information; formulate problems into mathematical terms and select and apply appropriate techniques of solution, including manipulation of algebraic expressions;
- **AO3** solve higher order thinking problems; interpret mathematical results and make inferences; write mathematical explanation and arguments.

SCHEME OF ASSESSMENT

Paper	Duration	Description	Marks	Weighting
Paper 1	2 hours	There will be about 25 short answer questions testing more on the fundamental skills and concepts. Candidates are required to answer ALL questions.	80	50%
Paper 2	2 hours 30 minutes	There will be 10 to 11 questions of varying marks and lengths testing more on higher order thinking skills. Candidates are required to answer ALL questions.	100	50%

NOTES

- 1. Omission of essential working will result in loss of marks.
- 2. Some questions may integrate ideas from more than one topic of the syllabus where applicable.
- 3. Relevant mathematical formulae will be provided for candidates.
- 4. Scientific calculators are allowed in **both** Paper 1 and Paper 2.
- 5. Candidates should also have geometrical instruments with them for Paper 1 and Paper 2.
- 6. Unless stated otherwise within a question, three-figure accuracy will be required for answers. This means that four-figure accuracy should be shown throughout the working, including cases where answers are used in subsequent parts of the question. Premature approximation will be penalised, where appropriate. Angles in degrees should be given to one decimal place.
- 7. SI units will be used in questions involving mass and measures.

Both the 12-hour and 24-hour clock may be used for quoting times of the day. In the 24-hour clock, for example, 3.15 a.m. will be denoted by 03 15; 3.15 p.m. by 15 15, noon by 12 00 and midnight by 24 00.

- 8. Candidates are expected to be familiar with the solidus notation for the expression of compound units, e.g. 5 cm/s for 5 centimetres per second, 13.6 g/cm³ for 13.6 grams per cubic centimetre.
- 9. Unless the question requires the answer in terms of π , the calculator value for π or π = 3.142 should be used.
- 10. Spaces will be provided on the question paper of Paper 1 for working and answers.

CONTENT OUTLINE

No	Topic/Sub-topics	Content
1	NUMBERS AND ALGEBRA	
1.1	Numbers and the four operations	 Include: primes and prime factorisation finding HCF and LCM, squares, cubes, square roots and cube roots by prime factorisation negative numbers, integers, rational numbers, real numbers and their four operations calculations with the use of a calculator representation and ordering of numbers on the number line use of the symbols <, >, ≤, ≥ approximation and estimation (including rounding off numbers to a required number of decimal places or significant figures, estimating the results of computation, and concepts of rounding and truncation errors) examples of very large and very small numbers such as mega/million (10⁶), giga/billion (10⁹), tera/trillion (10¹²), micro (10⁻⁶), nano (10⁻⁹), and pico (10⁻¹²), use of standard form A × 10ⁿ, where n is an integer, and 1 ≤ A < 10 positive, negative, zero and fractional indices laws of indices
1.2	Ratio, rate and proportion	 Include: ratios involving rational numbers writing a ratio in its simplest form average rate map scales (distance and area) direct and inverse proportion problems involving ratio, rate and proportion
1.3	Percentage	 Include: expressing one quantity as a percentage of another comparing two quantities by percentage percentages greater than 100% increasing/decreasing a quantity by a given percentage reverse percentages problems involving percentages
1.4	Speed	 Include: concepts of speed, uniform speed and average speed conversion of units (e.g. km/h to m/s) problems involving speed, uniform speed and average speed

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No Topic/Sub-to	opics Content
1.5 Algebraic representatio and formulae	 interpreting notations: ab as a × b ab as a × b ab as a ÷ b a² as a × a, a³ as a × a × a, a²b as a × a × b, 3y as y + y + y or 3 × y 3[±]y/5 as (3 ± y) ÷ 5 or 1/5 × (3±y) evaluation of algebraic expressions and formulae translation of simple real-world situations into algebraic expressions recognising and representing number patterns (including
1.6 Algebraic manipulation	finding an algebraic expression for the <i>n</i> th term) Include: • addition and subtraction of linear algebraic expressions • simplification of linear algebraic expressions, e.g. -2(3x-5)+4x $\frac{2x}{3} - \frac{3(x-5)}{2}$ • factorisation of linear algebraic expressions of the form * $ax + ay$ (where <i>a</i> is a constant) * $ax + bx + kay + kby$ (where <i>a</i> , <i>b</i> and <i>k</i> are constants) • expansion of the product of algebraic expressions • changing the subject of a formula • finding the value of an unknown quantity in a given formula • finding the value of an unknown quantity in a given formula • recognising and applying the special products * $(a \pm b)^2 = a^2 \pm 2ab + b^2$ * $a^2 - b^2 = (a + b)(a - b)$ • factorisation of algebraic expressions of the form * $a^2x^2 - b^2y^2$ * $ax^2 + bx + c$ • multiplication and division of simple algebraic fractions, e.g. $\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ $\frac{3a}{4} \div \frac{9a^2}{10}$ • addition and subtraction of algebraic fractions with linear or quadratic denominator, e.g. $\frac{1}{x-2} + \frac{2}{x-3}$ $\frac{1}{x^2-9} + \frac{2}{x-3}$ $\frac{1}{x-3} + \frac{2}{(x-3)^2}$

No	Topic/Sub-topics	Content
1.7	Functions and graphs	 Include: Cartesian coordinates in two dimensions graph of a set of ordered pairs linear relationships between two variables (linear functions) the gradient of a linear graph as the ratio of the vertical change to the horizontal change (positive and negative gradients) graphs of linear equations in two unknowns graphs of quadratic functions and their properties positive or negative coefficient of x² maximum and minimum points symmetry sketching of the graphs of quadratic functions given in the form y = ±(x - p)² + q y = ±(x - a)(x - b) graphs of functions of the form y = axⁿ where n = -2, -1, 0, 1, 2, 3, and simple sums of not more than three of these graphs of exponential functions y = ka^x where a is a positive integer
1.8	Solutions of equations and inequalities	 Include: solving linear equations in one unknown (including fractional coefficients) solving simple fractional equations that can be reduced to linear equations, e.g. \$\frac{x}{3} + \frac{x-2}{4} = 3\$ \$\frac{3}{x-2} = 6\$
		 solving simultaneous linear equations in two unknowns by substitution and elimination methods graphical method solving quadratic equations in one unknown by factorisation use of formula completing the square for y = x² + px + q graphical methods solving fractional equations that can be reduced to quadratic equations, e.g. ⁶/_{x+4} = x + 3 ¹/_{x-2} + ²/_{x-3} = 5 formulating equations to solve problems solving linear inequalities in one unknown, and representing the solution set on the number line

No	Topic/Sub-topics	Content	
1.9	Applications of mathematics in practical situations	Include: • problems derived from practical situation * utilities bills * hire-purchase * simple interest and compound inter * money exchange * profit and loss * taxation • use of data from tables and charts • interpretation and use of graphs in prace • drawing graphs from given data • distance-time and speed-time graphs Exclude the use of the terms "percentage properties.	rest ctical situations
1.10	Set language and notation	 Include: use of set language and the following r Union of A and B Intersection of A and B Number of elements in set A " is an element of" " is not an element of" " is not an element of" Complement of set A The empty set Universal set A is a subset of B A is a proper subset of B A is not a subset of B union and intersection of two sets Venn diagrams 	notation: $A \cup B$ $A \cap B$ n(A) \in \notin A' \varnothing $A \subseteq B$ $A \subseteq B$ $A \subseteq B$ $A \not\subseteq B$
1.11	Matrices	 Exclude: use of n(A ∪ B) = n(A) + n(B) – n(A ∩ A) cases involving three or more sets Include: display of information in the form of a r interpreting the data in a given matrix product of a scalar quantity and a matrix problems involving the calculation of the (where appropriate) of two matrices Exclude: matrix representation of geometrical trates solving simultaneous linear equations of matrix method 	natrix of any order ix he sum and product ansformations

No	Topic/Sub-topics	Content
2	GEOMETRY AND MEASUREMENT	
2.1	Angles, triangles and polygons	 Include: right, acute, obtuse and reflex angles, complementary and supplementary angles, vertically opposite angles, adjacent angles on a straight line, adjacent angles at a point, interior and exterior angles angles formed by two parallel lines and a transversal: corresponding angles, alternate angles, interior angles properties of triangles and special quadrilaterals classifying special quadrilaterals on the basis of their properties angle sum of interior and exterior angles of any convex polygon properties of regular pentagon, hexagon, octagon and decagon properties of perpendicular bisectors of line segments and angle bisectors construction of simple geometrical figures from given data (including perpendicular bisectors and angle bisectors) using compasses, ruler, set squares and protractor, where appropriate
2.2	Congruence and similarity	Include: congruent figures and similar figures properties of similar polygons: corresponding angles are equal corresponding sides are proportional enlargement and reduction of a plane figure by a scale factor scale drawings determining whether two triangles are congruent similar ratio of areas of similar plane figures ratio of volumes of similar solids solving simple problems involving similarity and congruence
2.3	Properties of circles	 Include: symmetry properties of circles: equal chords are equidistant from the centre the perpendicular bisector of a chord passes through the centre tangents from an external point are equal in length the line joining an external point to the centre of the circle bisects the angle between the tangents angle properties of circles: angle in a semicircle is a right angle angle between tangent and radius of a circle is a right angle angle at the centre is twice the angle at the circumference angles in the same segment are equal angles in opposite segments are supplementary

No	Topic/Sub-topics	Content
2.4	Pythagoras' theorem and trigonometry	 Include: use of Pythagoras' theorem determining whether a triangle is right-angled given the lengths of three sides use of trigonometric ratios (sine, cosine and tangent) of acute angles to calculate unknown sides and angles in right-angled triangles extending sine and cosine to obtuse angles use of the formula ¹/₂ <i>ab</i> sin <i>C</i> for the area of a triangle use of sine rule and cosine rule for any triangle problems in 2 and 3 dimensions including those involving angles of elevation and depression and bearings Exclude calculation of the angle between two planes or of the angle between a straight line and a plane.
2.5	Mensuration	 Include: area of parallelogram and trapezium problems involving perimeter and area of composite plane figures (including triangle and circle) volume and surface area of cube, cuboid, prism, cylinder, pyramid, cone and sphere conversion between cm² and m², and between cm³ and m³ problems involving volume and surface area of composite solids arc length and sector area as fractions of the circumference and area of a circle area of a segment use of radian measure of angle (including conversion between radians and degrees) problems involving the arc length, sector area of a circle and area of a segment
2.6	Coordinate geometry	 Include: finding the gradient of a straight line given the coordinates of two points on it finding the length of a line segment given the coordinates of its end points interpreting and finding the equation of a straight line graph in the form y = mx + c geometric problems involving the use of coordinates Exclude: condition for two lines to be parallel or perpendicular mid-point of line segment finding the area of a quadrilateral given its vertices

No	Topic/Sub-topics	Content
2.7	Vectors in two	Include:
dimensions		• use of notations: $\begin{pmatrix} x \\ y \end{pmatrix}$, \overrightarrow{AB} , a , $ \overrightarrow{AB} $ and $ \mathbf{a} $
		directed line segments
		 translation by a vector
		position vectors
		• magnitude of a vector $\begin{pmatrix} x \\ y \end{pmatrix}$ as $\sqrt{x^2 + y^2}$
		• use of sum and difference of two vectors to express given vectors in terms of two coplanar vectors
		 multiplication of a vector by a scalar
		 geometric problems involving the use of vectors
		Exclude:
		 expressing a vector in terms of a unit vector
		mid-point of line segment
		 solving vector equations with two unknown parameters

No	Topic/Sub-topics	Content
3	STATISTICS AND PROBABILITY	
3.1	Data handling	Include: • data collection methods such as: * taking measurements * conducting surveys * classifying data * reading results of observations/outcomes of events • construction and interpretation of: * tables * bar graphs * pictograms * line graphs * pie charts * histograms • purposes and use, advantages and disadvantages of the different forms of statistical representations • drawing simple inference from statistical diagrams Exclude histograms with unequal intervals.
3.2	Data analysis	 Include: interpretation and analysis of: dot diagrams stem-and-leaf diagrams mean, mode and median as averages purposes and use of mean, mode and median calculation of the mean for grouped data quartiles and percentiles range, interquartile range and standard deviation as measures of spread for a set of data interpretation and analysis of: cumulative frequency diagrams box-and-whisker plots calculation of the standard deviation for a set of data (grouped and ungrouped) using the mean and standard deviation to compare two sets of data
3.3	Probability	 Include: probability as a measure of chance probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability) probability of simple combined events (including using possibility diagrams and tree diagrams, where appropriate) addition and multiplication of probabilities mutually exclusive events and independent events Exclude use of P(A ∪ B) = P(A) + P(B) - P(A ∩ B).

MATHEMATICAL FORMULAE

Compound interest

Total amount =
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone
$$= \pi r l$$

Surface area of a sphere $= 4\pi r^2$

Volume of a cone
$$=\frac{1}{3}\pi r^2 h$$

Volume of a sphere $=\frac{4}{3}\pi r^3$

Area of triangle
$$ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

Sector area $=\frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$Mean = \frac{\sum fx}{\sum f}$$

Standard deviation =
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

MATHEMATICAL NOTATION

The list which follows summarises the notation used in Cambridge's Mathematics examinations. Although primarily directed towards A Level, the list also applies, where relevant, to examinations at all other levels.

1. Set Notation

E	is an element of
∉	is not an element of
$\{x_1, x_2,\}$	the set with elements x_1, x_2, \ldots
{ <i>x</i> :}	the set of all x such that
n(A)	the number of elements in set A
Ø	the empty set
C	the universal set
A'	the complement of the set A
Z	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
\mathbb{Z}^+	the set of positive integers, {1, 2, 3,}
Q .	the set of rational numbers
\mathbb{Q}^+	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
\mathbb{Q}_0^+	the set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \ge 0\}$
R .	the set of real numbers
\mathbb{R}^+	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
\mathbb{R}^+_0	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \ge 0\}$
\mathbb{R}^{n}	the real <i>n</i> -tuples
\mathbb{C}	the set of complex numbers
\subseteq	is a subset of
С	is a proper subset of
⊈	is not a subset of
¢	is not a proper subset of
U	union
\cap	intersection
[<i>a</i> , <i>b</i>]	the closed interval { $x \in \mathbb{R}$: $a \le x \le b$ }
[<i>a</i> , <i>b</i>)	the interval { $x \in \mathbb{R}$: $a \leq x < b$ }
(<i>a</i> , <i>b</i>]	the interval { $x \in \mathbb{R}$: $a < x \le b$ }
(<i>a</i> , <i>b</i>)	the open interval { $x \in \mathbb{R}$: $a < x < b$ }

MATHEMATICAL NOTATION

2. Miscellaneous Symbols

=	is equal to
<i>≠</i>	is not equal to
≡	is identical to or is congruent to
≈	is approximately equal to
x	is proportional to
<	is less than
≤;≯	is less than or equal to; is not greater than
>	is greater than
≥; <	is greater than or equal to; is not less than
∞	infinity

3. Operations

a+b	a plus b
a-b	a minus b
$a \times b, ab, a.b$	a multiplied by b
$a \div b, \frac{a}{b}, a/b$	a divided by b
<i>a</i> : <i>b</i>	the ratio of a to b
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \dots + a_n$
\sqrt{a}	the positive square root of the real number a
a	the modulus of the real number a
<i>n</i> !	n factorial for $n \in \mathbb{Z}^+ \cup \{0\}$, (0! = 1)
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$, for $n, r \in \mathbb{Z}^+ \cup \{0\}, 0 \le r \le n$
	$rac{n(n-1)(n-r+1)}{r!}$, for $n\in\mathbb{Q},$ $r\in\mathbb{Z}^+$ U {0}

4. Functions

f	the function f
$\mathbf{f}(x)$	the value of the function f at x
f: $A \rightarrow B$	f is a function under which each element of set A has an image in set B
f: $x \mapsto y$	the function f maps the element x to the element y
\mathbf{f}^{-1}	the inverse of the function ${f f}$
g o f, gf	the composite function of f and g which is defined by $(g \circ f)(x)$ or $gf(x) = g(f(x))$
$\lim_{x \to a} f(x)$	the limit of $f(x)$ as x tends to a
Δx ; δx	an increment of x
$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
$\frac{\mathrm{d}^n y}{\mathrm{d} x^n}$	the <i>n</i> th derivative of y with respect to x
$f'(x), f''(x),, f^{(n)}(x)$	the first, second, <i>n</i> th derivatives of $f(x)$ with respect to x
$\int y \mathrm{d}x$	indefinite integral of y with respect to x
$\int_{a}^{b} y \mathrm{d}x$	the definite integral of y with respect to x for values of x between a and b
<i>x</i> , <i>x</i> ,	the first, second, derivatives of x with respect to time

5. Exponential and Logarithmic Functions

e	base of natural logarithms
e^x , exp x	exponential function of x
$\log_a x$	logarithm to the base a of x
ln x	natural logarithm of x
lg x	logarithm of x to base 10

6. Circular Functions and Relations

sin, cos, tan, cosec, sec, cot	the circular functions
\sin^{-1} , \cos^{-1} , \tan^{-1} \csc^{-1} , \sec^{-1} , \cot^{-1}	the inverse circular functions

7. Complex Numbers

i	the square root of -1	
Ζ	a complex number,	z = x + iy
		$= r(\cos \theta + \mathrm{i} \sin \theta), r \in \mathbb{R}_0^+$
		$= r \mathrm{e}^{\mathrm{i}\theta}, r \in \mathbb{R}_{0}^{+}$
Re z	the real part of z , Re (x	(x + iy) = x

Im z	the imaginary part of z, $\text{Im}(x + iy) = y$
Z	the modulus of z, $ x + iy = \sqrt{x^2 + y^2}$, $ r(\cos\theta + i\sin\theta) = r$
arg z	the argument of z, $\arg(r(\cos\theta + i\sin\theta)) = \theta$, $-\pi < \theta \le \pi$
Z*	the complex conjugate of z, $(x + iy)^* = x - iy$

8. Matrices

Μ	a matrix M
\mathbf{M}^{-1}	the inverse of the square matrix ${f M}$
\mathbf{M}^{T}	the transpose of the matrix ${f M}$
det M	the determinant of the square matrix ${f M}$

9. Vectors

a	the vector a
\overrightarrow{AB}	the vector represented in magnitude and direction by the directed line segment AB
â	a unit vector in the direction of the vector a
i, j, k	unit vectors in the directions of the Cartesian coordinate axes
a	the magnitude of a
$\left \overrightarrow{AB}\right $	the magnitude of \overrightarrow{AB}
a·b	the scalar product of ${f a}$ and ${f b}$
a×b	the vector product of ${f a}$ and ${f b}$

MATHEMATICAL NOTATION

10. Probability and Statistics

1 P C ata	
A, B, C, etc.	events
$A \cup B$	union of events A and B
$A \cap B$	intersection of the events A and B
$\mathbf{P}(A)$	probability of the event A
A'	complement of the event A , the event 'not A '
$P(A \mid B)$	probability of the event A given the event B
<i>X</i> , <i>Y</i> , <i>R</i> , etc.	random variables
<i>x</i> , <i>y</i> , <i>r</i> , etc.	value of the random variables X, Y, R , etc.
x_1, x_2, \dots	observations
$f_1, f_2,$	frequencies with which the observations, x_1, x_2 occur
p (<i>x</i>)	the value of the probability function $P(X=x)$ of the discrete random variable X
p_1, p_2, \dots	probabilities of the values x_1, x_2, \dots of the discrete random variable X
f(x), g(x)	the value of the probability density function of the continuous random variable X
F(x), G(x)	the value of the (cumulative) distribution function $P(X \le x)$ of the random variable X
E(X)	expectation of the random variable X
E[g(X)]	expectation of $g(X)$
Var(X)	variance of the random variable X
B(n, p)	binominal distribution, parameters n and p
Ρο(μ)	Poisson distribution, mean μ
$N(\mu, \sigma^2)$	normal distribution, mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
\overline{x}	sample mean
s^2	unbiased estimate of population variance from a sample,
	$s^2 = \frac{1}{n-1} \sum (x - \overline{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $N\left(0,1 ight)$
Φ	corresponding cumulative distribution function
ρ	linear product-moment correlation coefficient for a population

r linear product-moment correlation coefficient for a sample