ipdae INSTITUTE FOR THE PROFESSIONAL DEVELOPMENT OF ADULT EDUCATORS

GED® Preparation Lesson Plan

Module: Mathematical Reasoning

Lesson Title: Ratios, Proportions, and Scale Factors in the Real World

Standards: (AGE) Curriculum Framework GED® Comprehensive Preparation

Prerequisite Skills ABE Florida Curriculum Framework 2017-2018	Mathematical Reasoning 2014 GED® Assessment Targets Quantitative Problem Solving Standards and Content Indicators
Develop and understanding of ratio concepts and use ratio reasoning to solve problems. (CCR.MA.ABE.8.3.1) Explain ratio concepts and use ratio reasoning to solve problems (CCR.MA.ABE.8.4.1) Analyze proportional relationships and use them to solve mathematical and real-world problems. (CCR.MA.ABE.8.4.1)	Solve multistep , arithmetic, real-world problems using ratios or proportions including those that require converting units of measure (Q.3.c)

Objectives of the Lesson

Students will:

- Review vocabulary related to the lesson and discuss real-world applications of ratio and rate
- Express ratios and rates as fractions
- Solve equations using ratio and proportional concepts
- Apply proportional relationships to convert units of measurement

Materials

- Handout #1: Prep Activity, Ratio and Proportion Vocabulary Definitions
- Handout #2: Prep Activity, Where Do You See Ratios and Proportions in the Real World?
- You Tube videos:
  - https://www.youtube.com/watch?v=RQ2nYUBVvqI (Ratios and Rates) mathantics.com- 8.49 minutes
  - https://www.youtube.com/watch?v=USmit5zUGas (Proportions) mathantics.com- 10.29 minutes
- Handout #3: Ratios and Rates - https://www.math-aids.com'

- Handout #4: Proportions - https://www.math-aids.com
- Handout #5: Using Proportions to Convert Measurements
- Handout #6: Using Proportions to Convert Measurements in Real-World Situations
- Computer, internet access, projector

Instructional Plan

Overview

Solving multistep problems using ratio and proportional relationships can be difficult for students, especially problems that involve converting units of measurement. By building on the understanding of these relationships, this lesson demonstrates how these concepts can be used mathematically and connected to real-world situations. GED Testing Service® has identified this assessment target as a high impact indicator that describes some of the critical thinking skills students need to be successful in college, career training, and the workforce.

Process

Introduce the lesson by asking the following questions and discussing as a class:

- How are fractions, ratios, and proportions related?
- Where do you see ratios and proportions in the real world?

Write the following words/terms on the board:

- Ratio
- Proportion
- Reducing a fraction
- Solving equations by cross multiplying

Review vocabulary definitions on Handout #1 and briefly discuss the relationship between fractions and ratios. Ask students to create a ratio using an example from their own life and share with class, i.e. John has 6 pairs of sneakers in his closet, 2 white pairs, 1 red pair, and 3 black pairs. What is the ratio of white sneakers to black ones? What is the ratio of black sneakers to total sneakers? Discuss proportion and demonstrate the cross multiplying process to solve for *x*.

Pass out Handout #2 and briefly discuss each example of ratios and proportions in the real world. Have students examine the newspaper (the sports pages are a great section to use for this activity) for opportunities to express numbers in ratio or proportional form. Ask students for examples of when they might use a proportion when they are shopping, cooking, or performing another activity. After these prep activities, you may want to show the following video to deepen understanding:

- *Ratios and Rates* at https://www.youtube.com/watch?v=RQ2nYUBVvqI from *https://www.mathantics.com*

Using Handout #3 on *Ratios and Rates* from *https://www.math-aids.com*, have students set up the following ratios and reduce to simplest terms.

- 20 blue cars out of 25 cars = $\frac{20}{25}$ -:- $\frac{5}{5}$ = $\frac{4}{5}$
- 4 dimes to 40 dimes = $\frac{4}{40}$ -:- $\frac{10}{10}$ = $\frac{1}{10}$

Have students work in pairs or individually to complete remainder of handout and discuss answers. Once students demonstrate proficiency with ratios, introduce proportional relationships by viewing video https://www.youtube.com/watch?v=USmit5zUGas on *Proportions* from *https://www.mathantics.com* to refresh understanding of equivalent fractions and proportions and how to use the cross-multiplying process to find an unknown value. Solve a few problems on the board to demonstrate this process. Be sure to demonstrate that the unknown value can be in different positions. Remind students to imagine the crisscross, as shown in the video, to set up the proportion

- $\frac{n}{10} = \frac{2}{5}$ 10 x 2 = 5 x n 20 = 5n to solve for n $\frac{20}{5} = \frac{5n}{5}$ 20 -:- 5 = 4 n = 4 the 5's cancel out
- $\frac{3}{12} = \frac{b}{8}$ 3 x 8 = 12 x b 24 = 12b to solve for b $\frac{24}{12} = \frac{12b}{12}$ 24 -:- 12 = 2 b = 2 the 12's cancel out
- $\frac{14}{10} = \frac{7}{c}$ 14 x c = 7 x 10 14c = 70 to solve for c the 14's cancel out $\frac{14c}{14} = \frac{70}{14}$ 70 -:- 14 = 5 c = 5

Use Handout #4 on proportion. Have students work in pairs or individually to complete the handout. Check the answers and share a few with the group. Provide some real-world examples of problems involving proportions to ensure students understand the concept.

- If Gustav can finish 5 pages in his workbook in 2 days, if he keeps up the same rate for the remaining 12 days of the month, how many pages will he finish in total? $\frac{5 \text{ pages}}{2 \text{ days}} = \frac{n \text{ pages}}{12 \text{ days}}$
- 5 x 12 = 2 x n 60 = 2n to solve for n $\frac{60}{2} = \frac{2n}{2}$ 60 -:- 2 = 30 n = 30 pages in 12 days the 2's cancel out

- Maggie is making a cookie recipe that calls for 1.5 cups of sugar for every 2 cups of flour. If she uses 4 cups of flour, how many cups of sugar will she need? $\frac{1.5 \text{ c sugar}}{2 \text{ c flour}} = \frac{n \text{ c sugar}}{4 \text{ c flour}}$

- $1.5 \times 4 = 2 \times n \quad 6.0 = 2n$ to solve for n $\frac{6}{2} = \frac{2n}{2}$ 6 -:- 2 = 3 n = 3 cups of sugar
  the 2's cancel out

Once students have an understanding of the proportional process, introduce Handout #5 and review all measurement units in the colored boxes. Explain to students that since there is a relationship between different units of measurement, they can use a proportion to convert. Complete a few examples of converting units of measurement as a class, and then have students complete Handout #5. Check the answers as a group.

- How many ounces are in 5 cups? $\frac{1 \text{ cup}}{8 \text{ ounces}} = \frac{5 \text{ cups}}{n \text{ ounces}}$ $1 \times n = 5 \times 8$ n = 40

- 9 pints = how many quarts? $\frac{9 \text{ pints}}{n \text{ quarts}} = \frac{2 \text{ pints}}{1 \text{ quart}}$ $9 \times 1 = 2 \times n$ $\frac{9}{2} = \frac{2n}{2}$ n = 4.5 quarts

Once students have conquered using proportional relationships to convert units of measurement, introduce Handout #6. Model the process of selecting relevant information from a few problems, set up proportions and solve. Students may complete the handout in pairs. Go over answers and then have students brainstorm additional examples and share with group.

Sample Debriefing Questions

- How are fractions and ratios the same?
- Give an example of equivalent fractions.
- What is the relationship between ratios and proportions?
- Describe the cross-multiplying process to solve for an unknown value in a proportion.
- Why is it important to convert units of measurement? How can you use a proportion?
- What would happen if you try to solve a proportion, without using the same units of measurement for both ratios?

Modifications for Different Levels

To modify instruction, you may need to review the process of reducing fractions for the ratio lessons. If needed, provide students with index cards, have them write out the units of measurement as they may not be given this information for test questions. Since this lesson teaches several concepts, including ratios, rates, proportions, the cross-multiplying process, as well as converting units of measurement, you may want to consider breaking it up into several shorter lessons.

Assessments/ Extensions

Throughout the lesson, be sure to monitor and check for student understanding for each new concept. Allowing students to work in pairs helps to build confidence and share knowledge. There are several resources available on the internet, as well in the GED® preparation materials, which will give students more practice with GED® type questions that deal with proportion.

A Few Real World Problems to Get Started

For the following problems set up a proportion and solve.

1. A town in Iowa with a population of 2,000 people has a daily milk consumption rate of 500 gallons. If the daily mild consumption is about the same in the next largest town of 5,000, how many gallons of milk would be consumed there?

2. If the high speed train can travel 150 miles in 90 minutes, how many miles can it travel in 4 hours? (**Hint-** First convert the hours to minutes, so you have the same units of measurement)

3. The ratio of the number of males to females in the adult education class is 2 to 3 (2 males for every 3 females). If there are 18 females in the class, how many males are there?

4. A picture frame has a width of 3 inches and a length of 5 inches. If the picture is enlarged to be 15 inches wide, how long will it be?

5. If a 12 pack of soda costs $4.50, how much will 30 cans cost at the same rate?

6. The scale on a map is ½ inch = 15 miles. How far apart are two towns that are 2 inches apart on the map?

Answers: A Few Real World Problems to Get Started

1. 1,250 gallons
2. 400 miles
3. 12 males
4. 25 inches
5. $11.25
6. 6 miles

Vocabulary

Handout #1

1. **Ratio:** A ratio compares one number to another. A ratio can be written three different ways. It can be written as a fraction, with a colon (:), or with the word "to." The order of the numbers should follow the order of the words. The first number becomes the numerator (top number of the fraction), and the second number becomes the denominator (bottom number of the fraction).

   For example:
   Leo has 15 coins, 5 quarters and 10 dimes. What is the ratio of quarters to dimes?

   $\frac{5}{10}$ **5:10** **5 to 10**

   Just like in fractions, ratios have to be **reduced**, so these ratios must be reduced by dividing by 5 since the 5 and the 10 can both be divided by 5. The following represents the reduced ratios for this question.

   $\frac{1}{2}$ **1:2** **1 to 2**

   *If a ratio is an improper fraction, **do not** write it as a mixed number. For example, if Maria drove 55 miles in 1 hour, you would leave the ratio as:

   $\frac{55}{1}$ **55:1** **55 to 1**

2. **Proportion:** A statement that two ratios are equal. For example,

   $$\frac{6}{8} = \frac{3}{4}$$

   Proportions are used to solve many types of word problems. For example, if 12 inches = 1 foot, then how many inches are in 5 feet?

   $\frac{12}{1} = \frac{x}{5}$ **Then you cross multiply to solve for x**

   $1 \cdot x = 12 \cdot 5$

   x = 60 inches

Where Do You See Ratios and Proportions in the Real World? Handout #2

Some places where we see ratios used are in measurements, sports, and business. In real life, you see ratios used every day. Look at the headlines of any newspaper. Some of the headlines seem straight forward – "One in Seven 2-Year Olds in Boston Drinks Coffee" or "One in Four Deaths in America is Due to Heart Disease."

Other times, you will read things like, "Congress reaches deal on 1600-page, $1.1 trillion spending bill" from the Washington Times. These kinds of numbers can seem too large to think about or too complex to handle. But through ratios, we can say that this bill proposes spending on a scale of about $700 million per page. By reducing the ratio, we can begin to understand the scale.

Grocery Shopping
When going shopping, children often look at the prices of various groceries. A parent can easily explain ratios to his or her child using two different boxes of cereal. For example, if a 10-ounce box of cereal costs $3 and a 20-ounce box of cereal costs $5, the 20-ounce box is the better value because each ounce of cereal is cheaper. This relationship is produced by dividing the number of ounces of cereal by the price. For the smaller box of cereal, each ounce costs 30 cents; for the larger box of cereal, each ounce of cereal costs 25 cents.

Builders and Contractors
As a contractor or builder, it is imperative to know how many pounds of weight each beam can support and to make sure that buildings are made to code. Codes or regulations include ratios like: thickness to height ratios; occupancy to area; and height to length (the slope of ramps). Contractors also need to understand ratios so that they can order the correct number of parts or estimate additional costs. If every additional wall requires 3 sheets of plywood and 84 nails (3 sheets: 84 nails reduces to 1 sheet: 28 nails), a builder can extrapolate the quantities and costs of the project.

Baking and Cooking
Have you ever dreamed of baking or being the next Gordon Ramsay or Rachel Ray? The truth is that making delicious food not only requires a great palate, but also using the right ratio of sweet to savory, water to flour, or oil to vinegar. Many a master chef has tanked because they did not use the right ratios in their dishes.

Ratios in Sports
We use ratios and odds in sports to describe anything from basketball free throws (free throws made/free throws taken) to yards per play in football. Understanding these numbers allows coaches to figure out which player to put in the game and that player's odds of winning or achieving a certain objective. The movie *Moneyball*, starring Brad Pitt, is a great example of how coaches use these ratios. The movie profiles Billy Beane's use of various ratios to help recruit for the Oakland A's team on a lean budget.

Ratios in Business
If you want to operate in business, you will need to be able to understand ratios. The Sharks on ABC's show *Shark Tank* often want to know what the average cost per unit item will be. Additionally in business, you will see ratios in time units such as units produced per day or profit per quarter.

Ratios and Rates Handout #3

Worksheet Created at Math-Aids.Com - *https://www.math-aids.com*

Name : _____ Score : _____

Teacher : _____ Date : _____

Ratios and Rates

Express each ratio as a fraction in the simplest form.

1) 20 blue cars out of 25 cars _____
2) 4 dimes to 40 dimes _____
3) 20 dimes out of 60 coins _____
4) 49 quarts to 63 quarts _____
5) 30 pounds to 42 pounds _____
6) 70 red bikes out of 84 bikes _____
7) 15 gallons to 20 gallons _____
8) 8 pennies to 20 pennies _____
9) 16 rainy days out of 32 days _____
10) 20 cups to 36 cups _____
11) 18 inches to 42 inches _____
12) 21 cakes out of 30 cakes _____
13) 30 feet out of 36 feet _____
14) 5 snow days out of 35 days _____
15) 14 miles out of 22 miles _____
16) 3 beetles out of 24 insects _____
17) 36 pints to 54 pints _____
18) 21 footballs to 84 footballs _____
19) 10 points out of 16 points _____
20) 12 nickels to 22 nickels _____

Ratios and Rates – Answer Key

Handout #3

Worksheet Created at Math-Aids.Com - *https://www.math-aids.com*

Express each ratio as a fraction in the simplest form.

1) 20 blue cars out of 25 cars $\frac{4}{5}$

2) 4 dimes to 40 dimes $\frac{1}{10}$

3) 20 dimes out of 60 coins $\frac{1}{3}$

4) 49 quarts to 63 quarts $\frac{7}{9}$

5) 30 pounds to 42 pounds $\frac{5}{7}$

6) 70 red bikes out of 84 bikes $\frac{5}{6}$

7) 15 gallons to 20 gallons $\frac{3}{4}$

8) 8 pennies to 20 pennies $\frac{2}{5}$

9) 16 rainy days out of 32 days $\frac{1}{2}$

10) 20 cups to 36 cups $\frac{5}{9}$

11) 18 inches to 42 inches $\frac{3}{7}$

12) 21 cakes out of 30 cakes $\frac{7}{10}$

13) 30 feet out of 36 feet $\frac{5}{6}$

14) 5 snow days out of 35 days $\frac{1}{7}$

15) 14 miles out of 22 miles $\frac{7}{11}$

16) 3 beetles out of 24 insects $\frac{1}{8}$

17) 36 pints to 54 pints $\frac{2}{3}$

18) 21 footballs to 84 footballs $\frac{1}{4}$

Solve the Equations Handout # 4

Worksheet Created at Math-Aids.Com - *https://www.math-aids.com*

Name : _____ Score : _____

Teacher : _____ Date : _____

Solve the Equations

Round your answers to the nearest hundredth if needed.

1) $\frac{z}{4} = \frac{1}{16}$

2) $\frac{19}{9} = \frac{5}{h}$

3) $\frac{4}{9} = \frac{c}{26}$

4) $\frac{1}{k} = \frac{9}{8}$

5) $\frac{27}{8} = \frac{5}{a}$

6) $\frac{2}{7} = \frac{v}{25}$

7) $\frac{23}{7} = \frac{3}{r}$

8) $\frac{2}{5} = \frac{y}{17}$

9) $\frac{x}{8} = \frac{7}{27}$

10) $\frac{d}{8} = \frac{7}{17}$

Solve the Equations – Answer Key Handout # 4

Worksheet Created at Math-Aids.Com - *https://www.math-aids.com*

Name : _____ Score : _____

Teacher : _____ Date : _____

Solve the Equations

Round your answers to the nearest hundredth if needed.

1) $\frac{z}{4} = \frac{1}{16}$

z = 0.25

2) $\frac{19}{9} = \frac{5}{h}$

h = 2.37

3) $\frac{4}{9} = \frac{c}{26}$

c = 11.56

4) $\frac{1}{k} = \frac{9}{8}$

k = 0.89

5) $\frac{27}{8} = \frac{5}{a}$

a = 1.48

6) $\frac{2}{7} = \frac{v}{25}$

v = 7.14

7) $\frac{23}{7} = \frac{3}{r}$

r = 0.91

8) $\frac{2}{5} = \frac{y}{17}$

y = 6.80

9) $\frac{x}{8} = \frac{7}{27}$

x = 2.07

10) $\frac{d}{8} = \frac{7}{17}$

d = 3.29

Using Proportions to Convert Measurements

Handout # 5

In this concept, you will learn to convert customary units of measurement.

Customary and Metric Systems

In the United States, you often use the **customary system** of measurement. The customary system is made up of units such as inches, feet, cups, gallons, and pounds. This type of measurement is often referred to as Imperial measure.

You may have also heard of the **Metric System** of measurement. The metric system is often used in countries outside of the United States, and in science measurements. Here, you will learn how to convert between different customary units of measurement.

First, let's look at some of the units in the customary system of measurement.

Customary Units of Measurement

Now, let's look at how you convert among customary units of measurement. Because there is a relationship between different units of measure, you can use proportions to help you convert between customary units of measurement.

Example: How many feet are in 5 yards?

First, set up a proportion.

$$\frac{1\ yard}{3\ feet} = \frac{5\ yards}{x\ feet}$$

Next, cross multiply to solve for *x*.

$1 \cdot x = 3 \cdot 5$

x = 15 feet

The answer is 15.

Therefore, 5 yards is equal to 15 feet.

Review

Solve each problem by converting among customary units of measurement.

1. 102 inches=–––– feet
2. 25 pounds=–––– ounces
3. 160 cups=–––– gallons
4. 150 pounds=–––– tons
5. 6 feet=–––– inches
6. 360 inches=–––– feet
7. 5.5 feet=–––– inches
8. 900 inches=–––– feet
9. 32 ounces=–––– pounds
10. 320 ounces=–––– pounds
11. 6 pounds=–––– ounces
12. 15 pounds=–––– ounces
13. 6 cups=–––– pints
14. 3 gallons=–––– quarts
15. 8 quarts=–––– pints
16. 24 pints=–––– quarts

Review (Answers)

1. 8.5 ft	2. 400 ounces	3. 10 gallons	4. .075 tons
5. 72 inches	6. 30 ft	7. 66 inches	8. 75 ft
9. 2 pounds	10. 20 pounds	11. 96 ounces	12. 240 ounces
13. 3 pints	14. 12 quarts	15. 16 pints	16. 12 quarts

Using Proportions for Converting Measurements in Real-World Situations

In this concept, you will learn to convert customary units of measurement in real-world situations.

Customary System

The **customary system**, also known as the Imperial System, is made up of units such as inches, feet, cups, gallons and pounds. Let's look at conversions within the customary system of measurement.

Customary Units of Measurement

Example:

Karin has a recipe that calls for 3 gallons of cider. How many quarts will she need?

First, set up a proportion.

$$\frac{1\ gallon}{4\ quarts} = \frac{3\ gallons}{x\ quarts}$$

Next, cross multiply to solve for *x*.

$1 \cdot x = 3 \cdot 4$

$x = 12$

The answer is 12.

Karin will need 12 quarts of cider.

Review

Solve each problem.

1. Justin ran 3 miles. How many feet did he run?
2. If the flour weighed four pounds, how many ounces did it weigh?
3. How many pounds are equal to 4 tons?
4. Mary needs 3 cups of juice for a recipe. How many ounces does she need?
5. Jess bought 3 quarts of pineapple juice. How many pints did she purchase?
6. If Karen bought 16 quarts of ice cream, how many gallons did she buy?
7. The length of the garden is four yards. How many feet is the length of the garden?
8. If the width of the garden is 4 yards, how many inches is the width?
9. Will eight cups of water fit into a two-quart saucepan?
10. A recipe calls for 2 pints of milk. If Jorge cuts the recipe in half, how many cups of milk will he need?
11. Audrey is making brownies for a bake sale. The recipe calls for 8 ounces of flour for every 24 brownies. If she makes 96 brownies, how many pounds of flour will she need?
12. Two buildings are 5 inches apart on a map. The scale on the map is 14 inch=1 mile. What is the actual distance between the two buildings?
13. The length of a classroom on a floor plan is 2.5 inches. The scale of the map is 12 inches=5 feet. What is the actual length of the classroom in inches?
14. A scale model of a mountain is 2.75 feet tall. The scale of the model is 14 inch=50 feet. What is the actual height of the mountain in feet?
15. A scale drawing of a town includes a park that measures 0.5 inch by 1.5 inches. If the scale of the map is 0.5 inches=1 mile, what is the area of the park in square feet?

Review (Answers)

1. 15,840 feet	2. 64 ounces	3. 8,000 pounds
4. 24 ounces	5. 6 pints	6. 4 gallons
7. 12 feet	8. 144 inches	9. yes
10. 2 cups	11. 2 pounds	12. 20 miles
13. 25 feet	14. 6,600 feet	15. 7,920 sq. ft.