General model for prediction of pressure drop and capacity of countercurrent gas/liquid packed columns

J. Stichlmair, J.L. Bravo* and J.R. Fair*

University of Essen, 4300 Essen 1, FRG
*Separations Research Program, University of Texas at Austin, Austin, TX 78712, USA

A generalized model has been developed for the prediction of pressure drop and flooding in packed columns in which gas and liquid flow countercurrently. The model has been validated for a wide variety of packings, both random and structured. A single mathematical expression is used to describe all flow regimes: dry gas, irrigated gas flow below the load point, loading region, and flooding. The approach to the model development is fundamental in character and is an improvement over models published earlier.

Keywords: countercurrent; packed columns; pressure drop; mathematical model

Nomenclature

a	Specific surface area of packing $(m^2\,m^{-3})$	S	Cross-sectional area of column (m^2)
c	Exponent in Equation (10)	U_g	Superficial gas velocity through a packed bed $(m\,s^{-1})$
C_1, C_2, C_3	Constants in Equation (7)	U_L	Superficial liquid velocity through a packed bed $(m\,s^{-1})$
d_N	Nominal diameter of packing element (mm)		
d_p	Particle diameter, $d_p \equiv 6(1-\varepsilon)/a$ (m)	U_0	Velocity for suspending a single particle $(m\,s^{-1})$
d'_p	Particle diameter including surface liquid (m)	U_S	Velocity for fluidizing a bed of particles $(m\,s^{-1})$
D_c	Column diameter (m)	Z	Total height of packing (m)
f	Friction factor for flow through a packed bed		
f_e	Friction factor for Ergun equation	**Greek letters**	
f_0	Friction factor for flow past a single particle	Δp	Pressure drop $(N\,m^{-2})$
		Δp_d	Pressure drop through an unirrigated (dry) bed $(N\,m^{-2})$
f_s	Friction factor for flow past a particle in a bed	Δp_{irr}	Pressure drop through an irrigated bed $(N\,m^{-2})$
Fr_L	Froude number for liquid [Equation (14)]	ε	Bed void fraction (porosity) $(m^3\,m^{-3})$
g	Gravitational constant $(m\,s^{-2})$	μ	Absolute viscosity $(kg\,(ms)^{-1})$
h	Liquid hold-up in a packed bed $(m^3\,m^{-3})$	σ	Surface tension $(N\,m^{-1})$
		ρ	Density $(kg\,m^{-3})$
h_0	Liquid hold-up below the loading point $(m^3\,m^{-3})$	**Subscripts**	
n	Exponent in Equation (1)	f	Flooding
p	Pressure $(N\,m^{-2}$ or bar)	g	Gas
Re_g	Reynolds number for the gas $= d_p U_g \rho_g/\mu_g$	L	Liquid

Numerous attempts have been made to describe the hydrodynamic behaviour of packed columns operating as countercurrent gas/liquid contactors. These attempts have ranged from the very empirical to the semi-empirical and have achieved moderate success for some applications within certain limited ranges of operating conditions. In

the present work, a generalized approach is developed for gas/liquid packed columns, one that covers all of the regions normally encountered in operation, from dry gas flow around a single packing particle to gas-liquid counterflow under loadings up to the flood point. The approach is fundamental in character and based on the description of the system pressure drop and liquid hold-up under varying conditions. A single mathematical expression is used to describe all regimes, and its derivative can be used to predict the flood point.

Two constraints that were placed on the development of this new approach were that the number of correlating constants should be minimized and that the fundamental geometric properties of the packings, such as surface area and void fraction, should suffice in most cases to account for differences in packing behaviour. Within these constraints the result should be a more generally applicable and rigorous approach to predicting hydrodynamic behaviour.

The two basic approaches to describing the hydrodynamics of a packed column are the channel model and the particle model. In the first, the gas is assumed to be flowing upward inside numerous small channels having some characteristic dimension; as liquid flows down the 'walls' of the same channels it reduces the available cross-sectional area for gas flow, thus causing increased pressure drop. In the particle model the gas is assumed to flow around a packing particle having a characteristic dimension and the liquid acts to increase this dimension by its adherence to the particle surface. The presence of the liquid also reduces the void fraction of the bed.

Several authors[1-3] have used the channel model to describe pressure drop in beds of random and structured packings for gas/liquid service, with some success. Single phase flow through packed beds, using the particle model, has been described by Ergun[4], Brauer[5] and Rose[6], and others. The particle model was used in this work; it has not previously been formally applied to gas/liquid contacting.

Dry pressure drop in a packed bed

The flow of a single phase through a packed bed has been studied extensively for many chemical engineering applications, particularly for the design of fixed catalytic beds. Extensive work has also been done in the area of fluidized beds where the porosity of the bed is a variable that depends on the geometry as well as the loading. Conventional packed columns for gas/liquid contacting are not fluidized beds, but they are similar in that their effective bed porosity also changes with geometry and loading because of liquid hold-up. Richardson and Zaki[7] developed the following relationship between gas velocity and porosity for fluidized beds

$$U_s/U_0 = \varepsilon^n \tag{1}$$

or

$$U_s/U_0 = f(\varepsilon, Re_g) \tag{1a}$$

where: U_s is the superficial velocity required to suspend a multitude of particles; U_0 is the superficial velocity required to suspend a single particle; ε is the void fraction of the fluidized bed; and the exponent n is a function of the particle Reynolds number as shown in *Figure 1*. In the
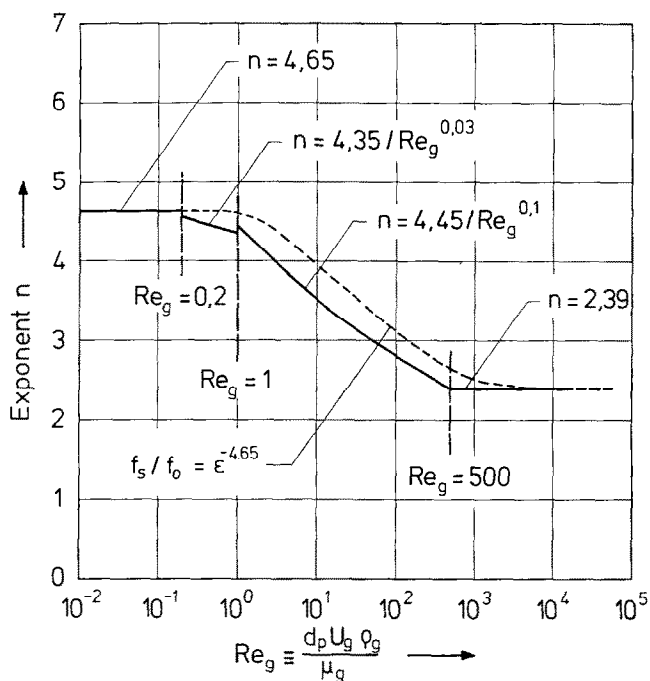

Figure 1 Values of the exponent n in Equation (1), after Richardson and Zaki[7]. ----, Equation (2) is nearly equivalent for all Reynolds numbers

laminar region, $n = 4.65$; in the turbulent region, $n \approx 4.65/2$ (see Reference 7).

A simpler description is achieved when the influence of the porosity, ε, on the friction factor is taken into account. It can be shown that Equation (1) can be re-written as

$$f_s/f_0 = \varepsilon^{-4.65} \tag{2}$$

In this form, the exponent does not depend on the Reynolds number as it does in Equation (1), since the friction factor, f_0, of a single particle is itself a function of the Reynolds number. Equation (2) describes the influence of void fractions on friction factor and provides the basis for the derivation of the pressure drop model. In *Figure 1*, the dashed line represents values of n obtained from Equation (2) when reverted to the form of Equation (1).

The pressure drop in a fluidized bed in equilibrium is equal to the weight of the bed

$$\Delta pS = ZS(1 - \varepsilon)\Delta\rho g \tag{3}$$

Similarly, a force balance on a single particle in the bed yields

$$\frac{\pi d_p^2}{4} f_s \frac{\rho_g U_g^2}{2} = \frac{\pi d_p^3}{6} \Delta\rho g \tag{4}$$

In Equation (4), f_s is used in the left-hand term because the force balance applies to a single particle of a swarm so that the fluid is affected by the swarm and not just by the single particle.

A pressure drop expression that utilizes the single particle friction factor as well as the bed porosity is obtained by combining Equations (2), (3) and (4)

$$\Delta p/Z = 3/4 f_0 [(1 - \varepsilon)/\varepsilon^{4.65}] \rho_g U_g^2/d_p \tag{5}$$

Equation (5) should also be valid for a fixed bed since it represents a special case of a fluidized bed, one in which

the porosity does not depend on loading. Such is also the case for a packed column operating under dry (unirrigated) conditions. This can be demonstrated by comparison of Equation (5) with the large collection of pressure drop data presented by Coulson and Richardson[8]. This comparison is shown in *Figure 2* for beds of spheres. In addition, Rumpf and Gupte[9] studied pressure drop in fixed beds with a systematic variation of porosity from 0.35 to 0.70, and found that the correction to the friction factor for a single particle was $(1 - \varepsilon)\varepsilon^{-4.55}$ which compares favourably with the form of Equation (5).

The Ergun[4] equation, generally accepted as descriptive of the pressure drop of a single phase fluid flowing through a fixed bed is, for the gas

$$\Delta p_d/Z = f_e [(1 - \varepsilon)/\varepsilon^3] \rho_g U_g^2/d_p \qquad (6)$$

The main difference between Equations (5) and (6) is the exponent on the porosity term. In the Ergun equation the porosity term results from an oversimplified model never validated experimentally, since porosity is a constant in a given packed bed. In Equation (5), however, the porosity term results from a large number of experiments in fluidized beds having a wide variation in porosity.

Because of the improved porosity term in Equation (5) the pressure drop in a bed of particles can be calculated from a knowledge of the friction factor of a single particle. Thus, there is no need for a friction factor of the bed. *Figure 2* represents an example of how the data can be correlated by a relationship of the following type

$$f_0 = C_1/Re_g + C_2/Re_g^{1/2} + C_3 \qquad (7)$$

where the constants vary with packing type as shown in *Table 1*. Equation (5) should describe the dry pressure drop in a packed column given the appropriate single packing particle friction factor and the porosity of the dry bed. In doing so, it provides the basis for modelling of pressure drop under irrigated conditions.

Pressure drop in an irrigated packing

The pressure drop of an irrigated bed is higher than that of a dry bed, as shown in *Figure 3*. This increase of pressure drop is caused by liquid being held up in the bed; this liquid changes the effective structure of the bed: porosity is decreased to ε', particle diameter is increased to d_p', and friction factor changes to f'.

The change in bed void fraction can be expressed as

$$\varepsilon' = \varepsilon - h$$

or

$$\varepsilon' = \varepsilon(1 - h/\varepsilon) \qquad (8)$$

where h is the operating hold-up (volume liquid/volume total bed). The change in particle diameter can be described by

$$(1 - \varepsilon')/d_p'^3 = (1 - \varepsilon)/d_p^3$$

or

$$d_p' = d_p\{[1 - \varepsilon(1 - h/\varepsilon)]/(1 - \varepsilon)\}^{1/3} \qquad (9)$$

The friction factor for a single wet particle will be different from that of a dry particle since the actual gas Reynolds number depends on the actual wet particle diameter. Equation (7) can be simplified (see Appendix B) to

$$f_0 \propto Re_g^c \qquad (10)$$

where

$$c = \frac{[-C_1/Re_g - C_2/(2Re_g^{1/2})]}{f_0}$$

then the change in the friction factor will be

$$f_0' = f_0 \left(\frac{d_p'}{d_p}\right)^c \qquad (11)$$

or

$$f_0' = f_0\left\{\left[1 - \varepsilon\left(1 -\frac{h}{\varepsilon}\right)\right]/(1 - \varepsilon)\right\}^{c/3}$$

Equations (8)–(11) describe the changes in the system that are caused by the presence of the liquid disregarding the effect of liquid drag on the gas flow. If these equations are substituted into Equation (5)

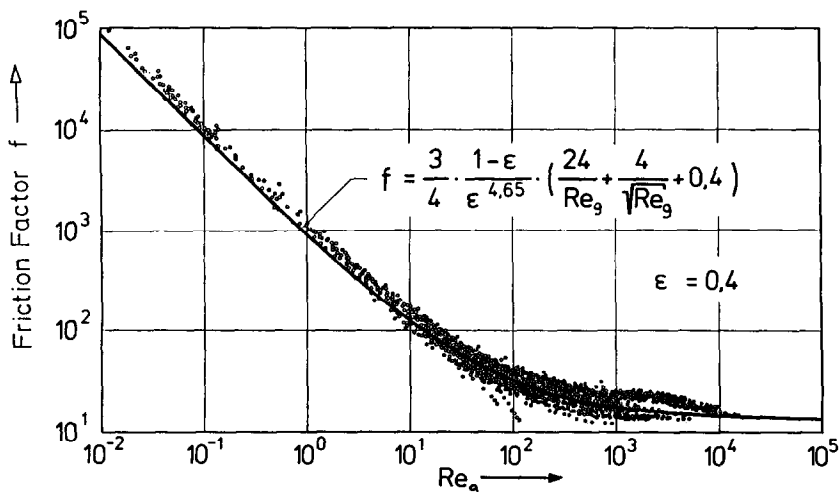$$\Delta p_{ir}/Z = 3/4 f_0' [(1 - \varepsilon')/\varepsilon'^{4.65}]\rho_g U_g^2/d_p' \qquad (12)$$

Figure 2 Application of Equation (5) to friction factor data for beds of spheres as presented by Coulson and Richardson[8]

Table 1 Data base

Packing	Type/size		a [$m^2\,m^{-3}$]	$\in$ [-]	C_1	C_2	C_3	Reference
Structured packings:								
Montz	B1	300	300	0.97	2	3	0.9	18
	B1	200	200	0.98	2	4	1.0	18
	B1	100	100	0.99	3	7	1.0	18
Gempack	2A		394	0.92	3	2.4	0.31	19
	3A		262	0.93	3	2.3	0.28	19
Sulzer	Mellapak 250Y (plastic)		250	0.85	1	1	0.32	21
	Mellapak 250Y (metal)		250	0.96	5	3	0.45	20
	BX-packing		450	0.86	15	2	0.35	20
Dumped ceramic packings:								
Raschig Rings	10		472	0.655	48	8	2.0	29
	10		327	0.657	10	8	1.8	28
	15		314	0.676	48	10	2.3	23
	15		264	0.698	48	8	2.0	29
	30		137	0.775	48	8	2.0	24
	35		126	0.773	48	8	2.15	24
Pall Rings	25		192	0.742	10	3	1.2	24
	25		219	0.74	1	4	1.0	26
	35		139	0.773	33	7	1.4	24
	35		165	0.76	1	6	1.1	26
Reflux Rings	50		120	0.78	75	15	1.6	25
Hiflow Rings	20		291	0.75	10	5	1.1	27
Berl Saddles	15		300	0.561	32	6	0.9	23
	35		133	0.75	33	14	1.0	24
Intalox Saddles	20		300	0.672	30	6	1.4	23
	25		183	0.732	32	7	1.0	24
	35		135	0.76	30	6	1.2	24
Torus Saddles	25		255	0.73	19	1	0.85	26
	50		120	0.75	10	8	0.75	26
Dumped metal packings:								
Raschig Rings	12		416	0.94	60	1	7.5	29
	15		317	0.924	40	1	6	29
Pall Rings	25		215	0.94	0.05	1	3	26
	35		130	0.95	0.1	0.1	2.1	24
Bialecki Rings	25		225	0.94	50	7	2.5	22
Nutter Rings	50		96.5	0.978	1	1	2.65	31
Cascade Mini Rings	25		230	0.96	-2	-2	2	19
Supersaddles	25		165	0.978	1	1.6	2.1	24
Dumped plastic packings:								
Pall Rings	90		71	0.95	-5	-4	2.3	26
NSW-Rings	25		180	0.927	1	1	1.35	30
Leva	1		190	0.92	1	1	2.0	26
	2		143	0.94	1	1	2.3	26

The ratio of Equation (12) to Equation (5) gives, in combination with Equations (8), (9) and (11)

$$\Delta p_{irr}/\Delta p_d = \{[1 - \varepsilon(1 - h/\varepsilon)]/(1 - \varepsilon)\}^{(2 + c)/3}(1 - h/\varepsilon)^{-4.65}$$

(13)

which describes the increase in pressure drop in an irrigated packing as a function of the dry packing void fraction and the liquid hold-up. This expression should be valid for any type of packing so long as the single particle dry friction factor can be described by Equation (10). *Figure 4* shows how this expression describes various sets of experimental data for random and structured packings in the turbulent regime, where $c = 0$.

Comparison of Equation (13) with similar ones by Bemer and Kalis[1], Billet and Mackowiak[2], and Bravo *et al.*[3] clearly shows its advantages. First, there is no need for empirical packing correlation factors since the first term on the right-hand side of Equation (13) should account for all packing differences. Second, the influence of the effective void fraction on the pressure drop follows the same functionality for the wet and dry cases.

Equation (13) should be valid below as well as above the loading point provided that the liquid hold-up is known or can be accurately predicted. The excellent fit of this model exhibited in *Figure 4* demonstrates this fact; for all the points plotted there was a corresponding experimental liquid hold-up value. These experimental values
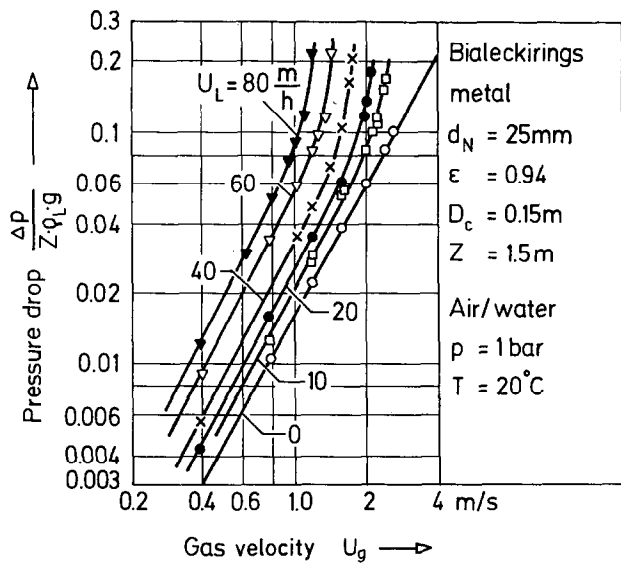
Figure 3 Dry and wet pressure for 25 mm Bialecki rings[2]. Air–water at 20°C; pressure = 1 bar; column diameter = 0.15 m
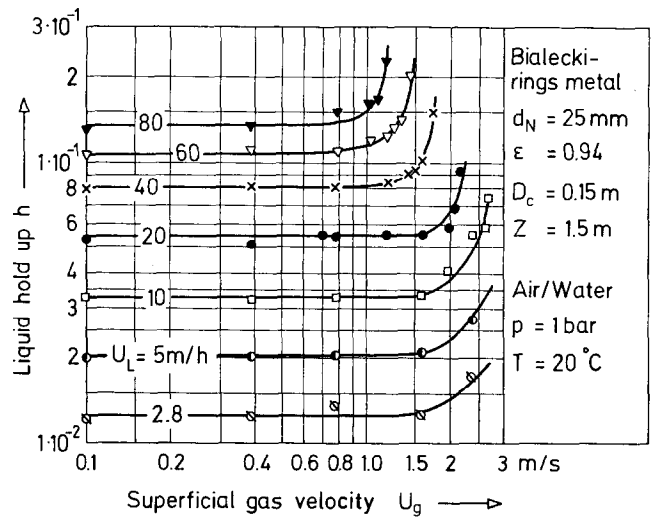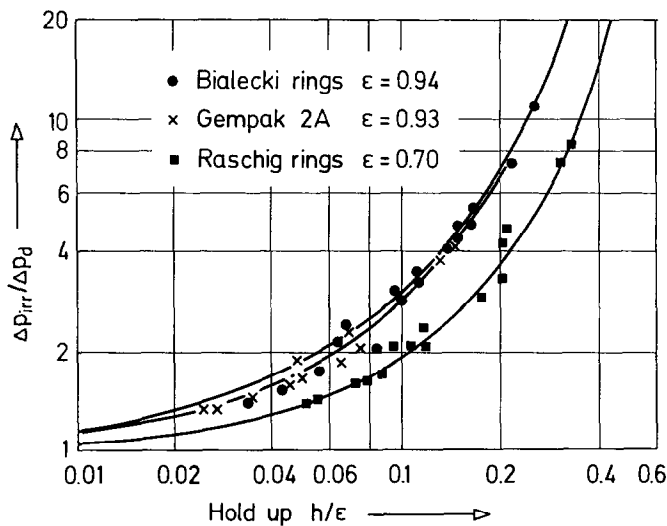

Figure 4 Validation of Equation (13), using different types of packings[2]

were the ones used in Equation (13) to plot the line representing the predicted values.

Liquid hold-up

The typical behaviour of liquid hold-up in a packed column for different liquid and gas loadings is shown in *Figure 5* for a random packing. Below the loading point, the hold-up is a function only of the liquid rate; above the loading point the hold-up also depends on the gas rate. The region where there is an influence of gas rate is commonly known as the loading region.

Hold-up below the loading point

Numerous attempts have been made to describe the dependence of hold-up on liquid velocity below the loading point. Hold-up measurements by Billet[10], for eight different packings, have been plotted in *Figure 6* and may be correlated by

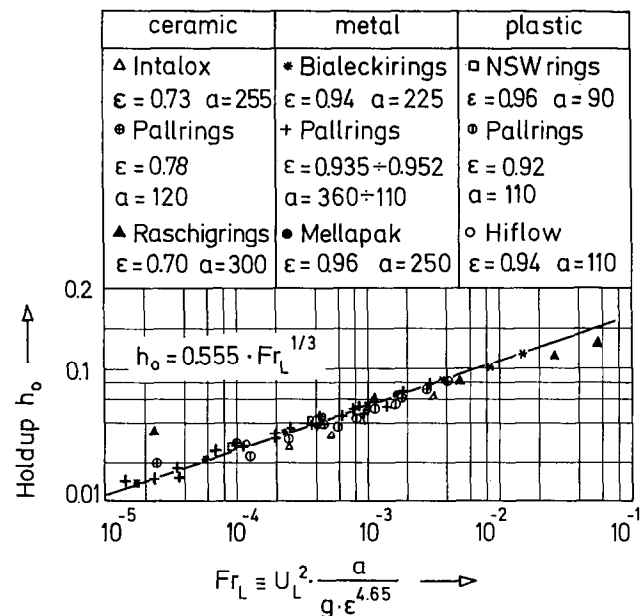$$h_0 = 0.555\, Fr_L^{1/3} \tag{14}$$

where the Froude number is defined as

Figure 5 Hold-up of Bialecki rings, conditions as for *Figure 3*[2]

ceramic	metal	plastic
△ Intalox ε = 0.73 a=255	* Bialeckirings ε = 0.94 a= 225	◻ NSW rings ε =0.96 a = 90
⊕ Pallrings ε = 0.78 a = 120	+ Pallrings ε =0.935÷0.952 a= 360÷110	⊕ Pallrings ε = 0.92 a = 110
▲ Raschigrings ε =0.70 a=300	● Mellapak ε = 0.96 a=250	○ Hiflow ε =0.94 a=110

Figure 6 Correlation of liquid hold-up below the loading point (data by Billet[10])

$$Fr_L \equiv U_L^2 \cdot \frac{a}{g\varepsilon^{4.65}}$$

The correlation does not take into account any properties of the liquid and has been validated for air/water only. The definition of the Froude number includes the term $\varepsilon^{4.65}$ because it intuitively follows Equation (5). It was found that this definition provides an excellent correlation of the hold-up data.

Figure 7 shows that Equation (14) is applicable for liquid viscosities up to about 5 centipoises, based on the data of Buchanan[11]. The influence of higher viscosities on hold-up is not fully accounted for. Nevertheless, most distillation, absorption and stripping applications of commercial importance exhibit viscosities in the range of applicability of Equation (14).

Still another effect that is not taken into account in Equation (14) is surface tension of the irrigating liquid. Mersmann and Deixler[12] developed a correlation for small Raschig rings that incorporates the effect of surface tension as indicated in *Figure 8*. It would appear that surface tension is important at low liquid loadings but
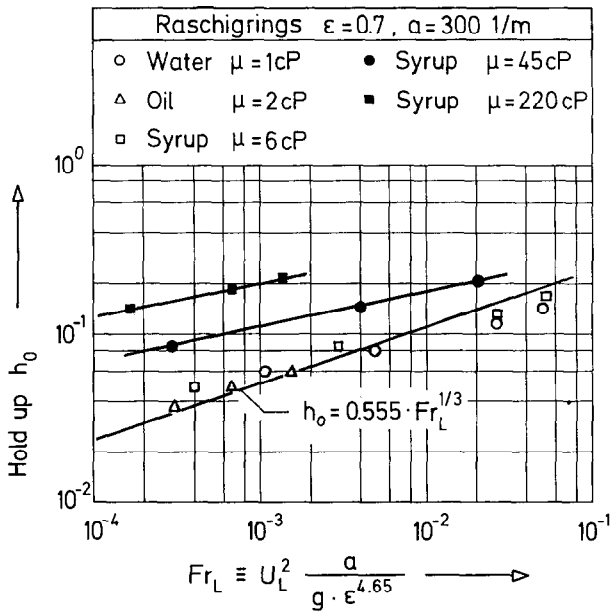
Figure 7 Liquid hold-up below the loading point for different liquid viscosities[11]

viscous forces are more important at higher loadings. It appears also that for a given loading, liquid hold-up increases with surface tension. This result is somewhat similar to findings of Bravo and Fair[13] which suggests that the effective interfacial area for mass transfer in random packings increases with increasing surface tension.

Hold-up above the loading point

In this region liquid hold-up is affected by gas velocity and increases with gas rate at a constant liquid rate. The liquid in the packing is held back by friction forces imposed on it by the gas as well as by the static pressure gradient produced by the pressure drop. Buoyant forces also come into play in this region but are only significant in high pressure systems where the densities of gas and liquid are somewhat similar. The influence of gas friction as well as the effect of the pressure gradient can be combined in a single dimensionless pressure drop term of the form $\Delta p_{irr}/(Z\rho_L g)$. This term relates the actual pressure drop in the system to the maximum potential head available for liquid flow down the packing.
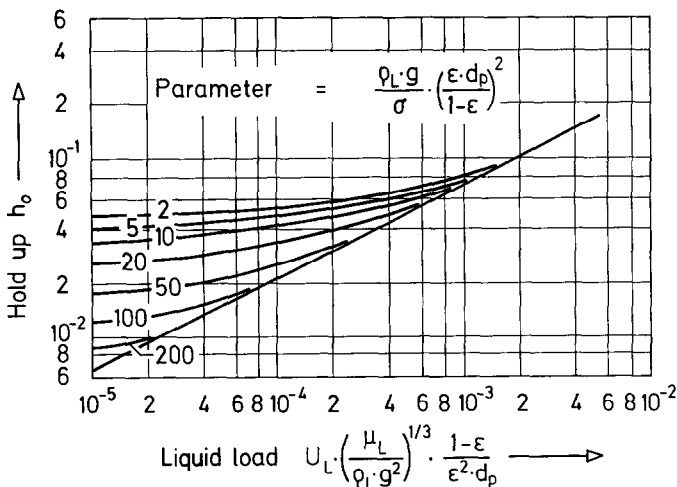

Figure 8 Correlation of liquid hold-up below the loading point, for small Raschig rings[12]
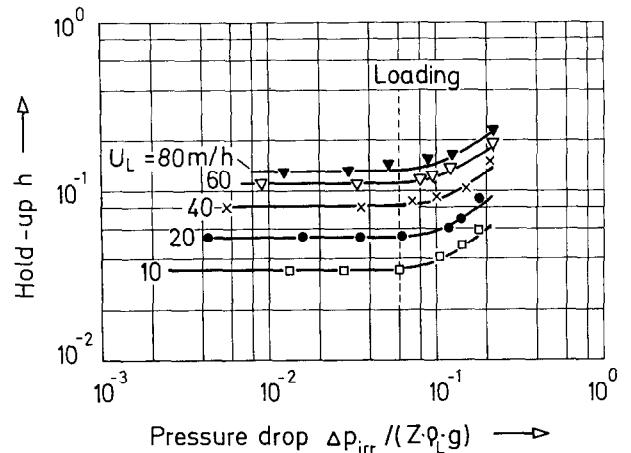

Figure 9 New plot of liquid hold-up; data according to *Figure 5*

The influence of gas rate on hold-up in the loading region is complex, as indicated in *Figure 5*. However, if the hold-up is plotted *versus* pressure drop, as in *Figure 9*, its increase is uniform for all liquid loads. From *Figure 9* the following relationship can be developed

$$h = h_0 [1 + 20(\Delta p_{irr}/(Z\rho_L g))^2] \tag{15}$$

Flooding of packed columns

Knowledge of the flood point is essential for column dimensioning in that the flooding condition establishes the theoretical minimum column diameter. There exist several empirical or semi-empirical correlations for the flood point, such as those developed by Sherwood *et al.*[14], Lobo *et al.*[15], Mersmann[16] and Eckert[17]. In some cases, the graphical representations of these methods include curves for pressure drop prediction. The Mersmann correlation of Reference 16 is an example of the latter.

The model developed in the present work allows calculation of the pressure drop even above the loading point. Combining Equations (13) and (15) provides the basic relationship

$$\Delta p_{irr}/\Delta p_d = \tag{16}$$

$$\frac{\{1 - \varepsilon[1 - h_0/\varepsilon(1 + 20[\Delta p_{irr}/(Z\rho_L g)]^2)](1 - \varepsilon)^{-1}\}^{(2 + c)/3}}{\{1 - h_0/\varepsilon(1 + 20[\Delta p_{irr}/(Z\rho_L g)]^2)\}^{4.65}}$$

Equation (16) is implicit in the irrigated pressure drop term and it also depends on an accurate value for the liquid hold-up (below the loading point) at the given liquid rate. It also provides a valuable tool for determining the capacity limits or flood point of a packed column, as shown below.

At the flood point the pressure drop increases infinitely with increasing gas load. If the gas load is represented by the dry pressure drop, the flooding condition is represented by

$$\frac{\partial \Delta p_{irr}}{\partial \Delta p_d} = \infty \tag{17}$$

or

$$\frac{\partial \Delta p_d}{\partial \Delta p_{irr}} = 0$$

Differentiation of Equation (16) then yields

$$\left(\frac{\Delta p_{irr}}{Z\rho_L g}\right)_f^{-2} - \frac{40\dfrac{2+c}{3}h_0}{1 - \varepsilon + h_0\{1 + 20[\Delta p_{irr}/(Z\rho_L g)]_f^2\}}$$

$$- \frac{186\,h_0}{\varepsilon - h_0\{1 + 20[\Delta p_{irr}/(Z\rho_L g)]_f^2\}} = 0 \qquad (18)$$

The solution of this equation yields the pressure drop at the flood point. In most cases this pressure drop has a value between $[\Delta p_{irr}/(Z\rho_L g)]_f = 0.1-0.3$, which is in good agreement with practical experience.

The essential quantity for the calculation of the flood point is liquid hold-up. The data published thus far (mostly for air/water) are not sufficient to develop a correlation valid for all types of packing and all systems. In particular, experimental hold-up data are required for systems with low surface tensions and low viscosities.

Validation of the model

It seems clear that for the complex contacting mechanics in an irrigated packed column, the most meaningful method of validation is to compare the predicted parameters against those measured under test conditions. Liquid hold-up is not normally reported, and few reliable data are available; thus for this parameter the empirical correlation [Equation (14)] is used to support estimates of pressure drop and flooding.

Many pressure drop data have been reported, often for wide ranges of liquid and gas rates, and these permit comparisons between dry and irrigated pressure drops, as shown in *Figure 3*. When the loadings have been carried to high values, flooding conditions are obtainable from the same sets of data. The data bank that we have used for validation is shown in *Table 1*. It contains entries for structured as well as random packings. Data sources are given as part of the table.

For dry pressure drop, data were chosen carefully from graphed results unless specific values were reported.
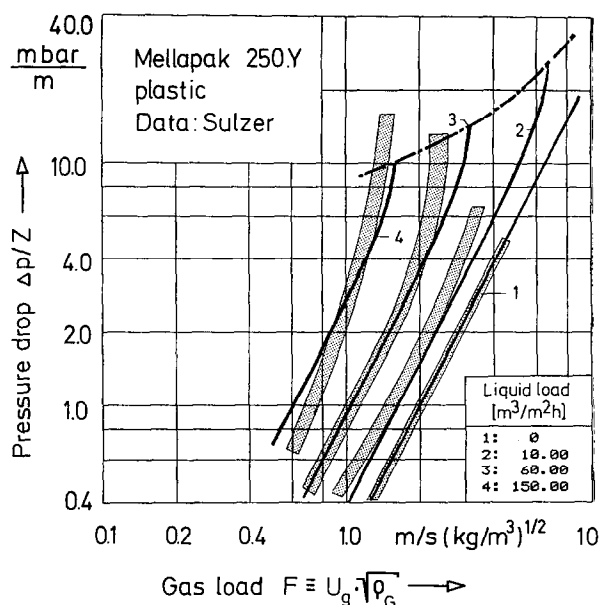
Equations (15) and (16) were used to determine irrigated pressure drops. Comparisons between prediction and measurement were made in plots, such as those shown in *Figures 10-12*. Because of the huge amount of data, a single parity plot was not feasible.

Figures 10-12 demonstrate the ability of the model to predict the wet pressure drop for a variety of packing types. The comparison makes it clear that the pressure drop in the loading region can be predicted satisfactorily. *Figures 13-16* are parity plots for flooding conditions. The agreement between experiment and model is satisfactory for all four data groups. The structured packings have larger deviations, particularly at combinations of high gas rates and low liquid rates. However, it is in this region that flooding velocities are difficult to measure and thus the basic data tend to be unreliable.

The validation of the model will continue as more data become available. Future adjustments may well be required. Still, the model currently represents the best
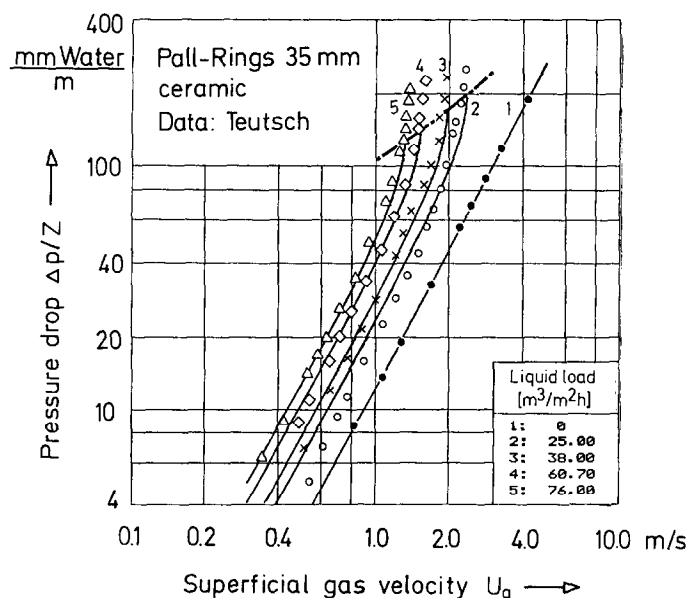
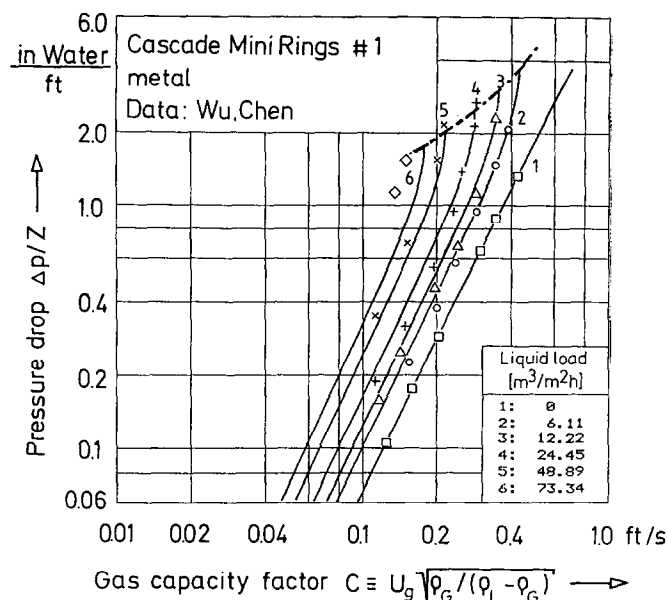**Figure 11** Comparison of experimental (points) and predicted (lines) pressure drop values, for 35 mm ceramic Pall rings

Figure 10 Comparison of experimental (shaded areas) and predicted (lines) pressure drop values, for plastic Mellapak 250Y packing

Figure 12 Comparison of experimental (points) and predicted (lines) pressure drop values, for no. 1 metal cascade minirings
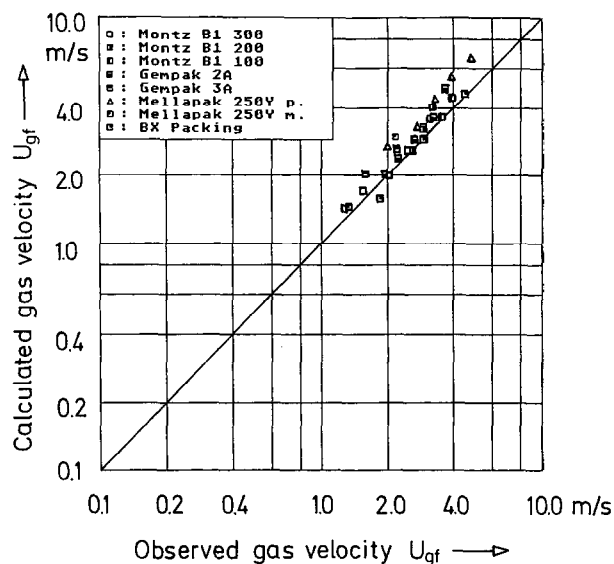
Figure 13 Parity plot for gas velocities at the flood point; structured packings (see *Table 1*)
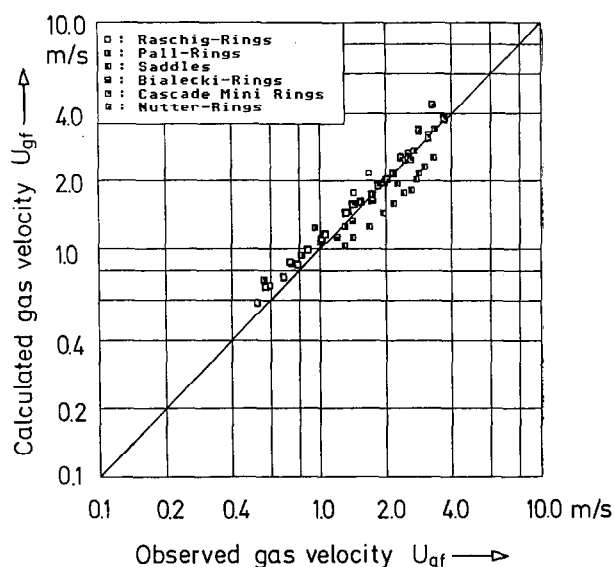

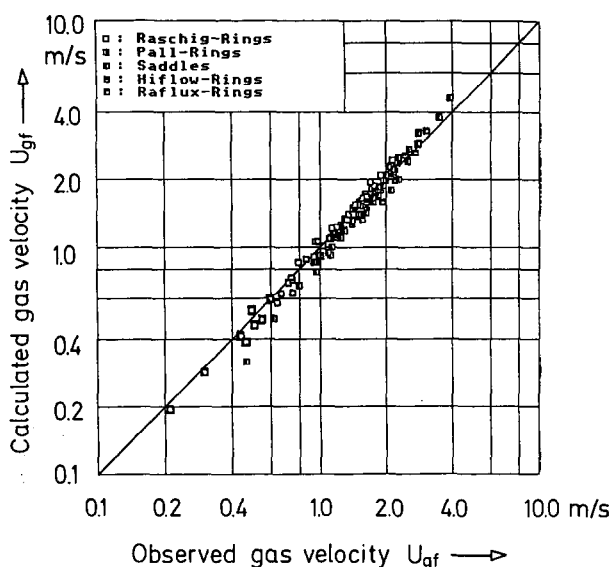**Figure 15** Parity plot for gas velocities at the flood point; dumped metal packings (see *Table 1*)

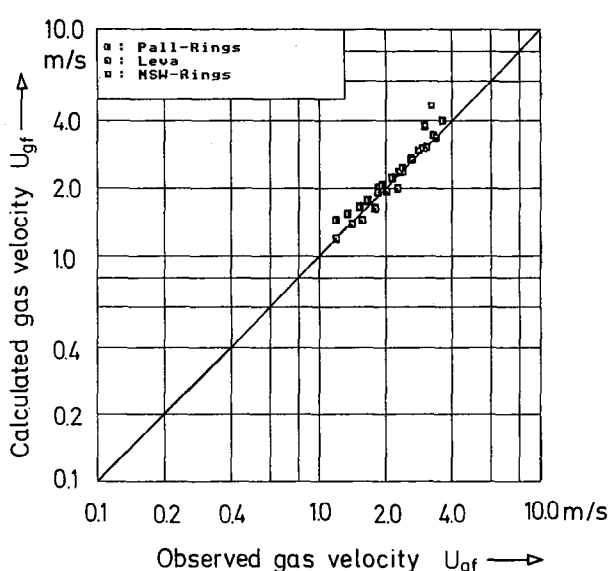**Figure 14** Parity plot for gas velocities at the flood point; dumped ceramic packings (see *Table 1*)

Figure 16 Parity plot for gas velocities at the flood point; dumped plastic packings (see *Table 1*)

available approach for estimating pressure drop in gas–liquid packed columns, particularly those which operate on systems other than air–water.

References

1 Bemer, G.G. and Kalis, G.A.J. *Trans Inst Chem Eng* (1978) **56** 200

2 Billet, R. and Mackowiak, J. *Fette Seifen Anstrichmittel* (1984) **86** 349

3 Bravo, J.L., Rocha, J.A. and Fair, J.R. *Hydrocarbon Proc* (1986) **65**(3) 45

4 Ergun, S. *Chem Eng Progr* (1952) **48**(2) 89

5 Brauer, H. *Grundlagen Der Einphasen — und Mehrphasen — Stromungen* Verlag Sauerlander, Frankfurt, FRG (1971)

6 Rose, H.E. *Inst Phys Symp* (1950) 136

7 Richardson, J.F. and Zaki, W.N. *Trans Inst Chem Engrs* (1954) **32** 35

8 Coulson, J.M. and Richardson, J.F. *Chemical Engineering. Vol. 2, Unit Operations* Pergamon Press, Oxford, UK (1976)

9 Rumpf, H. and Gupte, A.R. *Chem-Ing-Tech* (1971) **43** 367

10 Billet, R *Packed Column Analysis and Design* Proceedings of the First Glitsch Packed Column Workshop, Glitsch, Inc., Dallas, USA (1985)

11 Buchanan, J.E. *Ind Eng Chem Fundam* (1967) **6** 400

12 Mersmann, A. and Deixler, A. *Chem-Ing-Tech* (1986) **58**(1) 19

13 Bravo, J.L. and Fair, J.R. *Ind Eng Chem Process Des Develop* (1982) **21** 162

14 Sherwood, T.K., Shipley, G.H. and Holloway, F.A.L. *Ind Eng Chem* (1938) **30** 765

15 Lobo, W.E., Friend, L., Hashmall, F. and Zenz, F.A. *Trans Am Inst Chem Eng* (1945) **41** 693

16 Mersmann, A. *Chem-Ing-Tech* (1965) **37**(3) 218

17 Eckert, J.S. *Chem Eng Progr* (1970) **66**(3) 39

18 Julius Montz GmbH, Hilden, FRG

19 Wu, K.Y. and Chen, G.K. Large-scale pilot columns and packed column scale-up *Inst Chem Eng Symp Ser* **104** (1987) B225

20 Sulzer Brothers, Winterthur, Switzerland *Publication A/22.13.06-V.82* (1982)

21 Meier, W., Hunkeler, R. and Stöcker, D. Performance of the new regular tower packing mellapack *Inst Chem Eng Symp Ser* **56** (1979) 3.3/1

22 Mackowiak, J.K. *Habilitationsschrift* Bochum, FRG (1985)

23 Wiggert, K. *Interne Mitteilungen der forschungsgruppe für Wärme- und Kältetechnik* Max Plant Institut für Strömungsforschung, Publication No. 92 Göttingen, FRG (1959)

24 Teutsch, T. *Dissertation* TH München, FRG (1962)

25 Geipel, W. and Ullrich, H. *Füllkörpertaschenbuch der Firma Rauschert* Steinwiesen, FRG (1985)

26 Raschig GmbH, Ludwigshafen, FRG (1983 and 1985)
27 Billet, R. *Chem Tech* (1985) 14(5) 195
28 Schrader, H. *Kältetechnik* (1958) 10(9) 290
29 Reichelt, W. *Dissertation* TU Clausthal, FRG (1970)
30 Billet, R. and Schultes, M. Capacity studies of gas–liquid two-phase countercurrent-flow columns *Inst Chem Eng Symp Ser 104* (1987) B225
31 Nutter, D.E. A random packing developed for consistent performance *Inst Chem Engr Symp Ser 104* (1987) A129

Appendix A

Example calculation

Calculate the pressure drop and flood point for an irrigated packing under the following conditions:

Gas velocity: $U_G = 0.4$ m s^{-1}
Liquid loading: $U_L = 5 \times 10^{-3}$ m^3 (m^2 s)$^{-1}$
Properties: gas density = 5 kg m^{-3}
        liquid density = 1200 kg m^{-3}
        kinematic viscosity of gas = 10^{-5} m^2 s^{-1}
        kinematic viscosity of liquid = 2×10^{-6} m^2 s^{-1}
Packing: Berl saddles, 25 mm ($a = 260$ m^{-1}; $\varepsilon = 0.68$)

1   Calculation of the dry pressure drop

   (a)   Equivalent diameter of the packing

$$d_p = \frac{6(1 - \varepsilon)}{a}$$

$$= \frac{6(1 - 0.68)}{260} = 7.39 \times 10^{-3} \text{ m}$$

   (b)   Friction factor for a single particle [Equation (7)]

$$f_0 = \frac{C_1}{Re_g} + \frac{C_2}{Re_g^{1/2}} + C_3$$

where

$$Re_g = \frac{0.4 \times 7.39 \times 10^{-3}}{10^{-5}} = 295.4$$

Constants for the packing (*Table 1*)

$$C_1 = 32$$
$$C_2 = 7$$
$$C_3 = 1$$

$$f_0 = \frac{32}{295.4} + \frac{7}{(295.4)^{1/2}} + 1 = 1.5156$$

   (c)   Dry pressure drop [Equation (5)]

$$\Delta p_{dry} = \frac{3}{4} f_0 \frac{1 - \varepsilon}{\varepsilon^{4.65}} \rho_G \frac{Z}{d_p} U_G^2$$

$$= \frac{3}{4} (1.5156) \frac{1 - 0.68}{0.68^{4.65}} (5) \frac{1}{7.39 \times 10^{-3}} (0.4)^2$$

$$= 236.81 \text{ N m}^{-2} \text{ per metre of packed height}$$

dimensionless group:

$$\frac{\Delta p_{dry}}{\rho_L g Z} = \frac{236.81}{1200 \times 9.81 \times 1} = 0.0201$$

2   Pressure drop for irrigated packing

   (a)   Liquid hold-up, below the loading point [Equation (14)]

$$h_0 = 0.555 \, Fr_L^{1/3}$$

$$= 0.555 \left(\frac{(5 \times 10^{-3})^2 (260)}{(9.81) \, 0.68^{4.65}} \right)^{1/3}$$

$$= 0.088 \text{ m}^3 \text{ m}^{-3}$$

where

$$Fr_L = \frac{U_{La}^2}{g \varepsilon^{4.65}}$$

   (b)   Exponent c for calculation of the irrigated pressure drop [Equation (10)]

$$c = \frac{\partial \ln(f_0)}{\partial \ln(Re)_g}$$

$$= \frac{-C_1/Re_g - C_2/(2Re_g^{1/2})}{f_0}$$

$$= \frac{-(32/295.2) - [7(2(295.2)^{1/2})^{-1}]}{1.5156}$$

$$= -0.20584$$

   (c)   Irrigated pressure drop. The irrigated pressure drop is obtained by iterative calculations. As a starting point, assume that the irrigated pressure is equal to the dry pressure drop. Equation (16) is used:

$$\frac{\Delta p_{irr}}{\rho_L g Z} = \frac{\Delta p_{dry}}{\rho_L g Z}$$

$$\times \frac{\left\{ 1 - \varepsilon \left[1 - \frac{h_0}{\varepsilon} \left[1 + 20 \left(\frac{\Delta p_{irr}}{\rho_L g Z} \right)^2 \right] \right] \right\}^{(2 + c/3)}}{1 - \varepsilon}$$

$$\times \left[1 - \frac{h_0}{\varepsilon} \left[1 + 20 \left(\frac{\Delta p_{irr}}{\rho_L g Z} \right)^2 \right] \right]^{-4.65}$$

Results of the iterative calculations

$$\frac{\Delta p_{irr}}{\rho_L g Z} = 0.0459$$

$$\frac{\Delta p_{irr}}{Z} = \frac{\Delta p_{irr}}{\rho_L g Z} \rho_L g = 539.81 \text{ N m}^{-2} \text{ m}^{-1}$$

The pressure drop of the gas, for the given liquid and gas loadings, is 539.81 N m^{-2} per metre of packed height.

3 Flood point [Equation (18)]. The flood point is calculated from Equation (18)

$$\frac{1}{\left[\dfrac{\Delta p_{\text{irr}}}{\rho_L g Z}\right]_f^2} - \frac{40\left(\dfrac{2+c}{3}\right)h_0}{1 - \varepsilon + h_0\left[1 + 20\left(\dfrac{\Delta p_{\text{irr}}}{\rho_L g Z}\right)_f^2\right]}$$

$$- \frac{186\,h_0}{\varepsilon - h_0\left[1 + 20\left(\dfrac{\Delta p_{\text{irr}}}{\rho_L g Z}\right)_f^2\right]} = 0$$

The procedure for calculating the flood point is as follows:

(a) assume a gas rate;
(b) calculate the dry pressure drop;
(c) for the same gas rate and a fixed liquid rate, calculate the irrigated pressure drop; and
(d) using Equation (18), check to see whether the assumed condition gives closure. If not, assume a new gas rate and repeat the calculations.

Results

U_G, flooding = 0.64 m s^{-1} (for $U_L = 5 \times 10^{-3}$ m^3 m^{-2} s^{-1})

Δp_d, flooding = 555.23 N m^{-2} m^{-1}

Δp_{irr}, flooding = 1976.52 N m^{-2} m^{-1}

Thus, the design gas rate is 0.4/0.64 = 0.625 = 62.5% of flooding.

Appendix B

Derivation of Equation (10)

We have defined the friction factor, f_0, such that

$$f_0 = \frac{C_1}{Re} + \frac{C_2}{Re^{1/2}} + C_3 \tag{1}$$

We propose that

$$f_0 \propto Re^c$$

or

$$f_0 = A Re^c \tag{2}$$

Taking logarithms

$$\ln f_0 = \ln A + c \ln Re \tag{3}$$

Taking the derivative of Equation (3)

$$\frac{\mathrm{d}\ln f_0}{\mathrm{d}\ln Re} = c \tag{4}$$

Simplifying the differentials in Equation (4)

$$\frac{\dfrac{1}{f_0}\,\mathrm{d}f_0}{\dfrac{1}{Re}\,\mathrm{d}Re} = \frac{\mathrm{d}\ln f_0}{\mathrm{d}\ln Re} = c = \frac{Re}{f_0}\frac{\mathrm{d}f_0}{\mathrm{d}Re} \tag{5}$$

Differentiating Equation (1) with respect to Re

$$\frac{\mathrm{d}f_0}{\mathrm{d}Re} = -\frac{C_1}{Re^2} - \frac{1}{2}\frac{C_2}{Re^{3/2}} \tag{6}$$

Substituting Equation (6) into Equation (5)

$$c = \frac{Re}{f_0}\left[-\frac{C_1}{Re^2} - \frac{1}{2}\frac{C_2}{Re^{3/2}}\right] \tag{7}$$

or

$$c = -\frac{1}{f_0}\left[\frac{C_1}{Re} + \frac{1}{2}\frac{C_2}{Re^{1/2}}\right] \tag{7a}$$

Equation (7a) is the same as Equation (10) in the paper. Thus, if Equations (1) and (2) are true, then Equation (7a) is true.