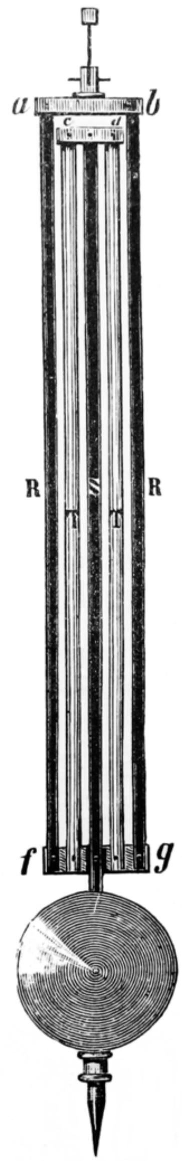
General Physics Laboratory I
PHYS 1601L

Laboratory Manual
Spring 2016



The Greek Alphabet

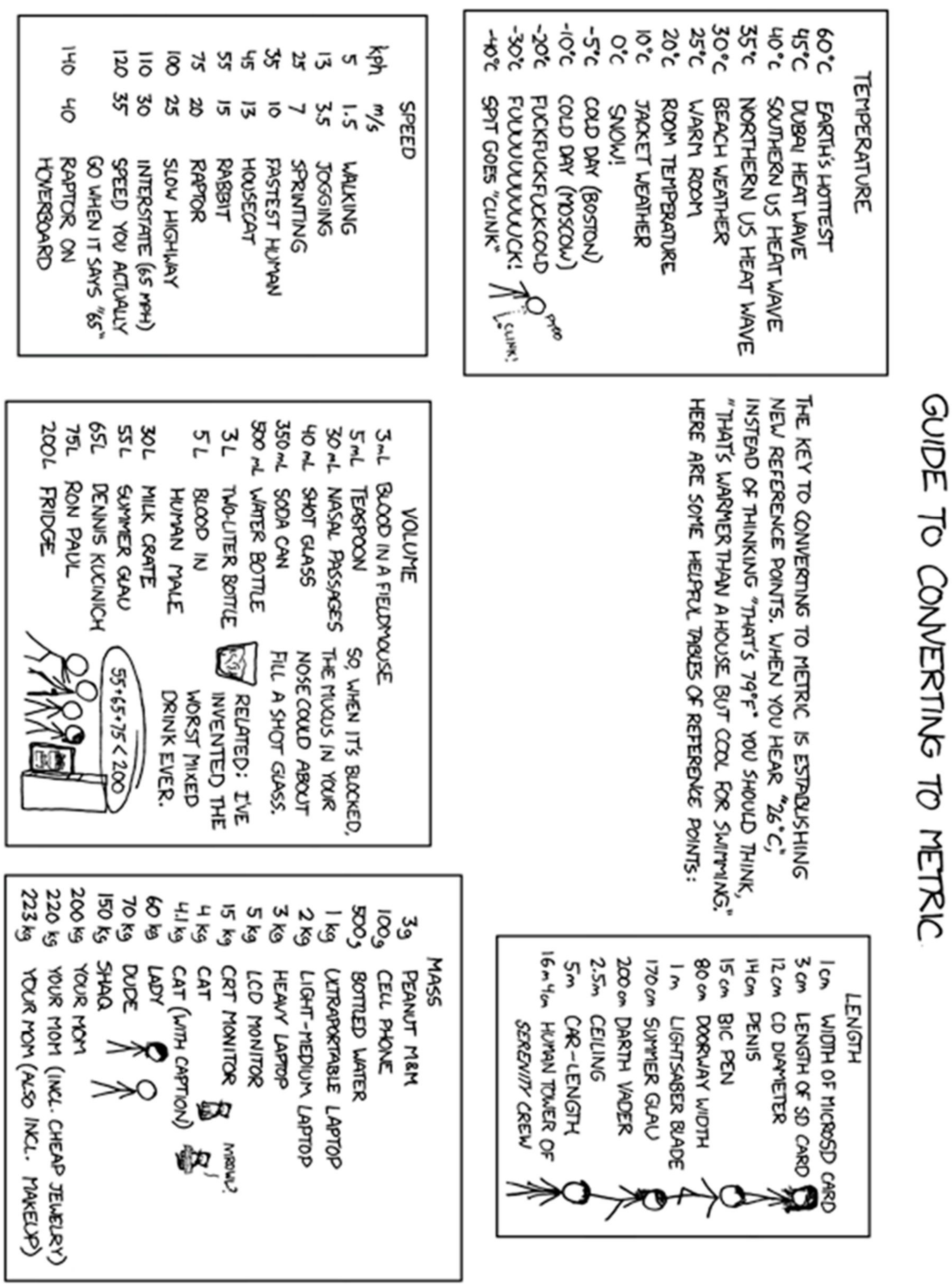
The 26 letters of the Standard English alphabet do not supply enough variables for our algebraic needs. So, the sciences have adopted the Greek alphabet as well. You will have to learn it eventually, so go ahead and learn it now, particularly the lower case letters.

Alpha	Α	α
Beta	Β	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	Ε	ε
Zeta	Ζ	ζ
Eta	Η	η
Theta	Θ	θ
Kappa	Κ	κ
Lambda	Λ	λ
Mu	Μ	μ
Nu	Ν	ν
Xi	Ξ	ξ
Omicron	Ο	ο
Pi	Π	π
Rho	Ρ	ρ
Sigma	Σ	σ
Tau	Τ	τ
Upsilon	Υ	υ
Phi	Φ	ϕ or φ
Chi	Χ	χ
Psi	Ψ	ψ
Omega	Ω	ω

Based on: RealTime Physics
David R. Sokoloff, Priscilla W. Laws, Robert K. Thornton.

Modified, with permission, by the
University of Virginia: Steve Thornton;
and Vanderbilt University: Kenneth Schriver.

Front Illustration: Diagram of a temperature compensating gridiron pendulum, invented by John Harrison, 1726.


GUIDE TO CONVERTING TO METRIC
THE KEY TO CONVERTING TO METRIC IS ESTABLISHING NEW REFERENCE POINTS. WHEN YOU HEAR "26°C," INSTEAD OF THINKING "THAT'S 79°F" YOU SHOULD THINK, "THAT'S WARMER THAN A HOUSE BUT COOL FOR SWIMMING." HERE ARE SOME HELPFUL TABLES OF REFERENCE POINTS:
TEMPERATURE
60°C EARTH'S HOTTEST
45°C DUBAI HEAT WAVE
40°C SOUTHERN US HEAT WAVE
35°C NORTHERN US HEAT WAVE
30°C BEACH WEATHER
25°C WARM ROOM
20°C ROOM TEMPERATURE
10°C JACKET WEATHER
0°C SNOW!
-5°C COLD DAY (BOSTON)
-10°C COLD DAY (MOSCOW)
-20°C FUCKFUCKFUCKCOLD
-30°C FUUUUUUUUUCK!
-40°C SPIT GOES "CLINK"
PTOO
CLINK!
SPEED
kph m/s
5 1.5 WALKING
13 3.5 JOGGING
25 7 SPRINTING
35 10 FASTEST HUMAN
45 13 HOUSECAT
55 15 RABBIT
75 20 RAPTOR
100 25 SLOW HIGHWAY
110 30 INTERSTATE (65 MPH)
120 35 SPEED YOU ACTUALLY GO WHEN IT SAYS "65"
140 40 RAPTOR ON HOVERBOARD
VOLUME
3mL BLOOD IN A FIELDMOUSE
5mL TEASPOON
30mL NASAL PASSAGES
40mL SHOT GLASS
350mL SODA CAN
500mL WATER BOTTLE
3L TWO-LITER BOTTLE
5L BLOOD IN HUMAN MALE
30L MILK CRATE
55L SUMMER GLAU
65L DENNIS KUCINICH
75L RON PAUL
200L FRIDGE
SO, WHEN IT'S BLOCKED, THE MUCUS IN YOUR NOSE COULD ABOUT FILL A SHOT GLASS.
RELATED: I'VE INVENTED THE WORST MIXED DRINK EVER.
55+65+75<200
LENGTH
1cm WIDTH OF MICROSD CARD
3cm LENGTH OF SD CARD
12cm CD DIAMETER
14cm PENIS
15cm BIC PEN
80cm DOORWAY WIDTH
1m LIGHTSABER BLADE
170cm SUMMER GLAU
200cm DARTH VADER
2.5m CEILING
5m CAR-LENGTH
16m4cm HUMAN TOWER OF SERENITY CREW
MASS
3g PEANUT M&M
100g CELL PHONE
500g BOTTLED WATER
1kg ULTRAPORTABLE LAPTOP
2kg LIGHT-MEDIUM LAPTOP
3kg HEAVY LAPTOP
5kg LCD MONITOR
15kg CRT MONITOR
4kg CAT
4.1kg CAT (WITH CAPTION)
MROWR?
60kg LADY
70kg DUDE
150kg SHAQ
200kg YOUR MOM
220kg YOUR MOM (INCL. CHEAP JEWELRY)
223kg YOUR MOM (ALSO INCL. MAKEUP)

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General Physics Laboratory I

PHYS 1601L

(Prior to Fall 2015, this lab was known as PHYS 116A.)

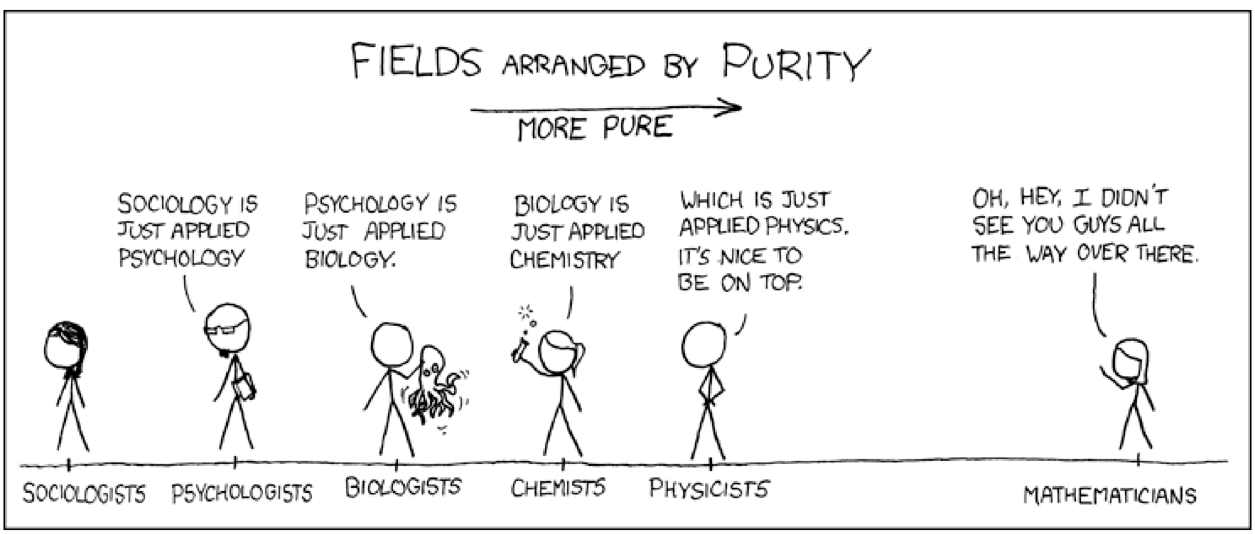
Contents

ACKNOWLEDGEMENT

The experiments presented herein are based largely on the *RealTime Physics* labs developed by Laws, Sokoloff, and Thornton, as well as some developed by Steve Thornton at the University of Virginia and A. Ramayya at Vanderbilt. They have been modified for use at Vanderbilt by Cynthia Coutre, Sherry Thompson, Ken Schriver Richard Helms, and myself. We continue to make changes based on evaluations of students' learning and feedback from graduate teaching assistants. I am pleased to have the opportunity to continue the process of refining this text in order to improve the laboratory experience for students taking introductory courses in physics.

Forrest Charnock

Physics & Astronomy
Vanderbilt University



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Introduction

The Sermon

The speed of light is 2.99792458×10^8 m/s. This is not science.

The Wikipedia entry on Newton's 2^{nd} law of motion is not science.

Nor is the periodic table of the elements.

Science is not a collection of facts. (Not even true facts!) Rather, science is a **process** for figuring out what is really going on. *What is the underlying principle here? How does this relate to some other observation?* If you are not involved in such a process, you are not doing science. A brilliant, dedicated, A+ student memorizing a list of equations is not doing science. A baby dropping peas on the floor to see what happens: now *that's science!!* (Does oatmeal fall too? Let's find out!!)

This is a science lab. I expect you to do some science in it.

"*Yeah, yeah, Dr. Charnock, I've heard this sermon before.*"

Perhaps so, but I have seen too many brilliant and dedicated students who have learned to succeed in their other science classes by learning **lots of stuff**.[*] So, they come into physics planning to memorize every equation they encounter and are completely overwhelmed. **You cannot succeed in physics by learning lots of stuff.** There are simply too many physics problems in the world; you cannot learn them all.

Instead, **you should learn as little as possible!**[†] More than any other science, physics is about fundamental principles, and those few principles[‡] must be the focus of your attention. Identify and learn those fundamental principles and how to use them. Then you can **derive** whatever solution that you need. And that **process** of derivation is the **process** of science.

"OK, thanks for the advice for the class, but this is a lab!!"

It's still about fundamental principles. Look, each week you will come to lab and do *lots of stuff*. By following the instructions and copying (. . . oh, I mean *sharing* . . .) a few answers from your lab partners, you can blunder through each lab just fine. The problem is that the following week you will have a quiz, and you will not remember everything you did in that lab the week before.

When you are doing each lab, consciously relate your experiments to the underlying principles.

How did I measure this? Where did this equation come from? Why are we doing this?

On the subsequent quiz, instead of having to **remember what you did**, you can apply the principles to **figure out what you did**. Trust me. It really is easier this way.

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[*] To get through organic chemistry, sometimes you just have to memorize all those formulas.
[†] . . . but not less.
[‡] F = ma, conservation of energy and momentum, oscillations and waves. You will learn a few more in the second semester.

GOALS AND OBJECTIVES

Physics is about the real world, not some idealized Platonic world that only exists in your head. The purpose of this lab is to relate the theories and equations you are learning in the classroom to reality. Hopefully, we'll convince you that all that physics stuff actually does work. Of course, reality can be messy, and along the way you will learn to deal with experimental uncertainty, loose cables, bad sensors, sticky wheels, temperamental software, temperamental lab partners, your own awful handwriting, and the typos in this lab book.

Welcome to experimental physics!

xkcd.com

CORRELATION WITH LECTURE

Most of the topics covered in the lab will also be covered in your lecture, although not necessarily in the same sequence or at the same time during the semester. Given the scheduling of the different lecture sections (some are MWF and some are TR), and the different lab sections (the first lab is Monday at 1 PM, the last is Thursday at 4 PM), *perfect correlation of lecture and lab topics is not possible for all students at all times.* The TA will provide a brief overview of the physics concept being explored in the lab during the first part of each lab section.

Occasionally, to improve the correlation with the lecture, the order of the labs may be changed from the sequence in this lab book. If so, you will be informed by your TA. **Check your email.***

PREPARATION

Prior to coming to lab, you should read over each experiment. Furthermore, for each laboratory, you must complete a pre-lab activity printed at the beginning of each lab in this manual. The pre-

* Being an old fart, I don't Tweet.

lab should be completed before the lab and turned in at the **beginning** of the lab. See the course syllabus for more details. In some labs, you may also be required to complete experimental predictions and enter them in your lab manual before you come to lab. Sometimes, you must watch an online video. Your TA will discuss this with you when necessary.

Bring the following to the lab:

- Your lab manual and a ring binder to hold it.
- A notebook or some extra sheets of loose leaf paper.
- Your completed pre-lab.
- A pen, pencil and an eraser.*
- A scientific calculator. Graphing calculators are nice, but overpriced and not necessary.

For some calculations, you may find a spreadsheet more appropriate. You are welcomed and encouraged to use such tools.

PROCEDURE IN THE LABORATORY

In the laboratory, you will need to be efficient in the use of your time. We encourage a free exchange of ideas between group members and among students in the section, and we expect you to share both in taking data and in operating the computer, but **you should do your own work (using your own words) in answering questions in the lab manual and on the review questions handed out in lab.**

HONOR CODE

The Vanderbilt Honor Code applies to all work done in this course. Violations of the Honor Code include, but are not limited to:

- Copying another student's answers on a pre-lab, lab questions, review questions, or quiz.
- Submitting data as your own when you were not involved in the acquisition of that data.

* You will definitely need the eraser.

- Copying data or answers from a prior term's lab (even from your own, in the event that you are repeating the course).

GRADING

Your lab reports will be graded each week and returned to you the following week. Grades (including lab and quiz grades) will be posted on OAK.

- ***Mistakes happen!*** Check that the scores on OAK are correct. If you don't do this, *no one will.*
- Retain you lab reports so that any such errors can be verified and corrected.
- Details of grading may be found on the online syllabus

SYLLABUS: available online

https://my.vanderbilt.edu/physicslabs/documents/

How to Count Significant Figures[*]

For all *measured* quantities (excepting counted quantities[†]), there will always be an associated uncertainty. For example,

height of Mt. Everest[‡] = 8844.43 m ± 0.21 m

Understanding the uncertainty is crucial to understanding the quantity. However, it is usually not necessary to provide a precise uncertainty range as shown above. The simplest way to represent uncertainty is the method significant figures. Here, the ± is dropped and the uncertainty is implied by the figures that are shown. An individual digit is usually considered significant if its uncertainty is less than ±5. In the case of Mt. Everest, the uncertainty is greater than 0.05 m; thus making the "3" uncertain. Rounding to the nearest 0.1 meter, we can write

height of Mt. Everest = 8844.4 m.

This quantity has five significant figures. (Notice that a digit does not need to be precisely known to be significant. Maybe the *actual* height is 8844.2 m. Maybe it is 8844.6 m. But the Chinese Academy of Sciences is confident that it is NOT 8844.7 m. Hence, that final "4" is worth recording.)

In general, the rules for interpreting a value written this way are

- All non-zero digits are significant
- All zeros written between non-zero digits are significant
- All zeros right of the decimal AND right of the number are significant
- Unless otherwise indicated, all other zeros are implied to be mere place-holders and are not significant.

Consider the following examples. The significant digits are underlined

<u>1023</u>

<u>1023</u>00

<u>102300.00</u>

00<u>1023.450</u>

0.00<u>10230</u>

[*] Even if you think you understand significant figures, read this anyway. Some of what you think you know may be wrong.
[†] For example: "There are **exactly** 12 eggs in that carton."
[‡] 2005, Chinese Academy of Sciences, https://en.wikipedia.org/wiki/Mount_Everest

Occasionally, a zero that appears to be a mere place-holder is actually significant. For example, the length of a road may be measured as 15000 m ± 25 m. The second zero is significant. There are two common ways two write this.

- Use scientific notation (preferable): 1.500×10^4 m
- Use a bar to indicate the least significant figure: $150\bar{0}0$ m or $150\underline{0}0$ m

Addition and Subtraction

If several quantities are added or subtracted, the result will be limited by the number with the largest uncertain decimal position. Consider the sum below:

$$\begin{array}{r} 123.4500 \\ 12.20 \\ 0.00023 \\ \hline 135.65\cancel{023} \\ 135.65 \end{array}$$

This sum is limited by 12.20; the result should be rounded to the nearest hundredth. Again, consider another example:

$$\begin{array}{r} 3210\underline{0}0 \\ 12.30 \\ -333 \\ \hline 32067\cancel{9.3} \\ 320680 \end{array}$$

In $3210\underline{0}0$, the last zero is not significant. The final answer is rounded to the ten's position.

Multiplication and Division

When multiplying or dividing quantities, the quantity with the fewest significant figures will determine the number of significant figures in the answer.

$$\frac{123.45 \times 0.0555}{22.22} = 0.308\cancel{34721} = 0.308$$

0.0555 has the fewest significant figures with three. Thus, the answer must have three significant figures.

To ensure that round off errors do not accumulate, ***keep at least one digit more*** than is warranted by significant figures during intermediate calculations. Do the final round off at the end.

How Do I Round a Number Like 5.5?

I always round up[*] (for example, 5.5 → 6), but others have different opinions[†]. Counting significant figures is literally an order-of-magnitude approximation, so it does not really matter that much.

How This Can Break Down

Remember, counting significant figures is NOT a perfect way of accounting for uncertainty. It is only a first approximation that is easy to implement and is ***usually*** sufficient.

For transcendental functions (sines, cosines, exponentials, *etc*.) these rules simply don't apply. When doing calculations with these, I usually keep one extra digit to avoid throwing away resolution.

However, even with simple math, naively applying the above rules can cause one to needlessly loose resolution.

> Suppose you are given two measurements 1$\underline{0}$m and 9s. You are asked to calculate the speed.
> With 1$\underline{0}$m I will assume an uncertainty of about 0.5 out of 10 or about 5%.[‡]
> With 9s you have ***almost*** the same uncertainty (0.5 out of 9), but technically we only have one significant digit instead of two.
> If I naively apply the rules . . .
>
> $$\frac{1\underline{0}m}{9s} = 1.\cancel{1111}\frac{m}{s} = 1\frac{m}{s}$$
>
> . . . my answer has an uncertainty of 0.5 out of 1!!! 50%!!!

This is what I call the **odometer problem**: When you move from numbers that are close to rolling over to the next digit (0.009, 8, 87, 9752953, etc.) to numbers that have just barely rolled over (0.001, 1.4, 105, 120258473, etc.), the estimate of the uncertainty changes by a factor of 10.[§] Here, we really need to keep a second digit in the answer.

$$\frac{1\underline{0}m}{9s} = 1.1\cancel{111}\frac{m}{s} = 1.1\frac{m}{s}$$

Notice: In the problem above, if the numbers are flipped, the odometer problem goes away:

$$\frac{9m}{1\underline{0}s} = 0.9\cancel{000}\frac{m}{s} = 0.9\frac{m}{s}$$

[*] . . ., and for good mathematical reasons, mind you. But still, it does not really matter that much.
[†] Google it, if you want to waste an hour of your life.
[‡] Of course, I don't really know what the uncertainty is. It could be much larger, but bear with me anyway.
[§] . . . and, vice-versa.

Oh Great! I thought this was supposed to be easy.

Well . . ., it is! But, you still have to use your head!

- Apply the rules.
- Look out for the “odometer problem”.
- If warranted, keep an extra digit.
- Simple!

Remember: counting significant figures is literally an order-of-magnitude approximation. So, don’t get too uptight about it. If you need something better than an order-of-magnitude approximation, see Lab 1.

What you should ***never*** do is willy-nilly copy down every digit from your calculator. If you are in the habit of doing that, STOP IT. You are just wasting your time and lying to yourself. If you ever claim that your cart was traveling at 1.35967494 m/s, expect your TA *to slap you down*. **That is just wrong!!!**

To say that Mt. Everest is about 9000 m is perfectly true.
To say that Mt. Everest is 8844.4324 m is a lie.

Forrest T. Charnock
Director of the Undergraduate Laboratories
Vanderbilt Physics

Name______________________________ Section _______ Date_____________

Pre-Lab Preparation Sheet for Lab 1:
Measurement, Uncertainty, and the Propagation of Uncertainty

(Due at the Beginning of Lab)

Directions:
Read the essay *How to Count Significant Figures*, then read over the following lab. Answer the following questions.

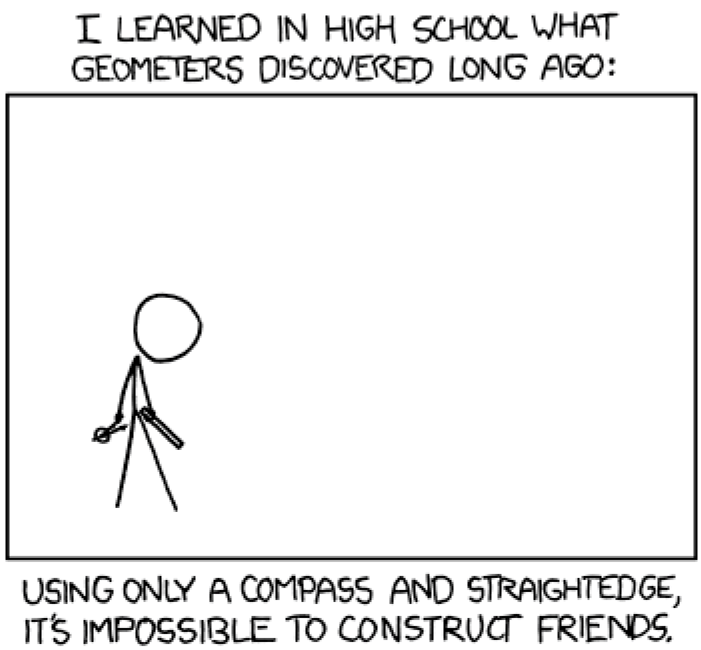
1. Applying the rules of significant figures, calculate the following

$123.4 + 120 + 4.822 - 21 =$

$$\frac{185.643 \times 0.0034}{3022} =$$

$$(523400 \times 0.0032) + 253 =$$

Modified from: *RealTime Physics*, P. Laws, D. Sokoloff, R. Thornton
and University of VA Physics Labs: S. Thornton



xkcd.com (But I want you to try anyway.)

Name ______________________ Date __________ Partners _________________
TA _________________ Section _______ _________________

Lab 1: Measurement, Uncertainty, and Uncertainty Propagation

"The first principle is that you must not fool yourself – and you are the easiest person to fool."

--Richard Feynman

Objective: To understand how to report both a measurement and its uncertainty.

Learn how to propagate uncertainties through calculations

Define, *absolute* and *relative* uncertainty, *standard deviation*, and *standard deviation of the mean*.

Equipment: meter stick, 1 kg mass, ruler, caliper, short wooden plank

DISCUSSION

Before you can really know anything, you have to measure something, be it distance, time, acidity, or social status. However, measurements cannot be "exact". Rather, all measurements have some uncertainty associated with them.* Ideally, all measurements consist of *two* numbers: the *value* of the measured quantity x and its *uncertainty*† Δx. The uncertainty reflects the reliability of the measurement. The range of measurement uncertainties varies widely. Some quantities, such as the mass of the electron $m_e = (9.1093897 \pm 0.0000054) \times 10^{-31}$ kg, are known to better than one part per million. Other quantities are only loosely bounded: there are 100 to 400 billion stars in the Milky Way.

Note that **we are not talking about "*human error*"! We are not talking about mistakes!** Rather, uncertainty is inherent in the instruments and methods that we use **even when perfectly applied.** The goddess Athena cannot not read a digital scale any better than you.

* The only exceptions are counted quantities. "There are <u>*exactly*</u> 12 eggs in that carton."

† Sometimes this is called the *error* of the measurement, but *uncertainty* is the better term. *Error* implies a variance from the correct value:

$$error = x_{measured} - x_{correct}$$

But, of course, we don't know what the correct value is. If we did, we would not need to make the measurement in the first place. Thus, we <u>cannot</u> know the error in principle! But we can measure the uncertainty.

Recording uncertainty

In general, uncertainties are usually quoted with no more precision than the measured result; and the last significant figure of a result should match that of the uncertainty. For example, a measurement of the acceleration due to gravity on the surface of the Earth might be given as

$$g = 9.7 \pm 1.2 \text{ m/s}^2 \qquad \text{or} \qquad g = 9.9 \pm 0.5 \text{ m/s}^2$$

But you should **never** write

$$g = 9.7 \pm 1.25 \text{ m/s}^2 \qquad \text{or} \qquad g = 9.92 \pm 0.5 \text{ m/s}^2.$$

In the last two cases, the precision of the result and uncertainty do not match.

Uncertainty is an inherently fuzzy thing to measure; so, it makes little sense to present the uncertainty of your measurement with extraordinary precision. It would be silly to say that I am (1.823643 ± 0.124992)m tall. Therefore, the stated uncertainty will usually have only one significant digit. For example

$$23.5 \pm 0.4 \qquad \text{or} \qquad 13600 \pm 700$$

However, if the uncertainty is between 1.0 and 2.9 (or 10 and 29, or 0.0010 and 0.0029, *etc.*) it may be better to have two significant digits. For example,

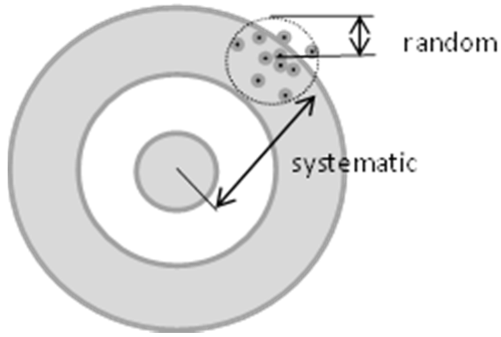
$$124.5 \pm 1.2$$

There is a big difference between saying ±1 and ±1.4. There is not a big difference between ±7 and ±7.4. (This is related to the odometer problem. See the above essay *How to Count Significant Figures*.)

Types of uncertainties

Random uncertainties occur when the results of repeated measurements vary due to truly random processes. For example, random uncertainties may arise from small fluctuations in experimental conditions or due to variations in the stability of measurement equipment. These uncertainties can be estimated by repeating the measurement many times.

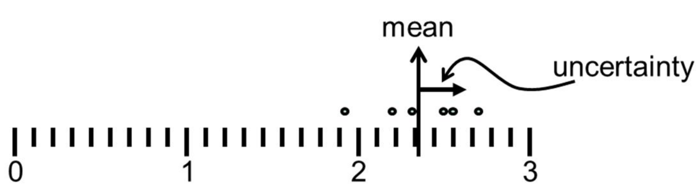
A *systematic uncertainty* occurs when all of the individual measurements of a quantity are biased by the same amount. These uncertainties can arise from the calibration of instruments or by experimental conditions. For example, slow reflexes while operating a stopwatch would systematically yield longer measurements than the true time duration.



Mistakes can be made in any experiment, either in making the measurements or in calculating the results. However, by definition, mistakes can also be avoided. Such blunders and major systematic errors can only be avoided by a thoughtful and careful approach to the experiment.

Estimating uncertainty

By repeated observation: Suppose you make repeated measurements of something: say with a stopwatch you time the fall of a ball. Due to random variations, each measurement will be a little different. From the spread of the measurements, you can calculate the uncertainty of your results.



Shortly, we will describe the formal procedure to do this calculation. (Oddly enough, truly random uncertainties are the easiest to deal with.)

By eye or reason: Sometimes, repeated measurements are not relevant to the problem. Suppose you measure the length of something with a meter stick. Meter sticks are typically ruled to the mm; however, we can often read them more precisely that that.

Consider the figure above. Measuring from the left side of each mark and considering the position uncertainties of <u>both ends</u> of the bar, I can confidently say that the bar is (1.76 ± 0.04) cm. Perhaps your younger eyes could read it with more confidence, but when in doubt it is better to overestimate uncertainty.

Could I do a better job by measuring several times? Not always. Sometimes with repeated measurements, it still comes down to "*Looks like (1.76 ± 0.04) cm to me.*" But that's ok. Your reasoned judgment is sufficient. **Science is defined by rigorous honesty, not rigorous precision!**

Vocabulary

Here we define some useful terms (with examples) and discuss how uncertainties are reported in the lab.

Absolute uncertainty: This is the magnitude of the uncertainty assigned to a measured physical quantity. It has the same units as the measured quantity.

Example 1. Again, consider the example above:

$$L = (1.76 \pm 0.04)\ \text{cm}$$

Here, the uncertainty is given in units of length: 0.04 cm. When the uncertainty has the same dimension as the measurement, this is an *absolute uncertainty*.

Relative uncertainty: This is the ratio of the absolute uncertainty and the value of the measured quantity. It has no units, that is, it is dimensionless. It is also called the *fractional uncertainty* or, when appropriate, the *percent uncertainty*.

Example 2. In the example above the *fractional uncertainty* is

$$\frac{\Delta V}{V} = \frac{0.04\ \text{cm}}{1.76\ \text{cm}} = 0.023 \tag{1.1}$$

The *percent uncertainty* would be 2.3%.

Reducing random uncertainty by repeated observation

By taking a large number of individual measurements, we can use statistics to reduce the random uncertainty of a quantity. For instance, suppose we want to determine the mass of a standard U.S. penny. We measure the mass of a single penny many times using a balance. The results of 17 measurements on the same penny are summarized in Table 1.

Table 1. Data recorded measuring the mass of a US penny.

	mass (g)	deviation (g)
1	2.43	-0.088
2	2.49	-0.028
3	2.49	-0.028
4	2.58	0.062
5	2.52	0.002
6	2.55	0.032
7	2.52	0.002
8	2.64	0.122
9	2.55	0.032

	mass (g)	deviation (g)
10	2.46	-0.058
11	2.52	0.002
12	2.4	-0.118
13	2.58	0.062
14	2.61	0.092
15	2.49	-0.028
16	2.52	0.002
17	2.46	-0.058

The **mean** value $\overline{m}$ (that is, the average) of the measurements is defined to be

$$\overline{m} = \frac{1}{N}\sum_{i=1}^{N} m_i = \frac{1}{17}\left(m_1 + m_2 + . . . + m_{17}\right) = 2.518g \tag{1.2}$$

The **deviation** d_i of the ith measurement m_i from the mean value $\overline{m}$ is defined to be

$$d_i = m_i - \overline{m} \tag{1.3}$$

Fig. 1 shows a histogram plot of the data on the mass of a US penny. Also on the graph is a plot of the smooth bell curve (that is a *normal distribution*) that represents what the distribution of measured values would look like if we took many, many measurements. The result of a large set of repeated measurements (when subject only to random uncertainties) will always approach a normal distribution which is symmetrical about $\overline{m}$.

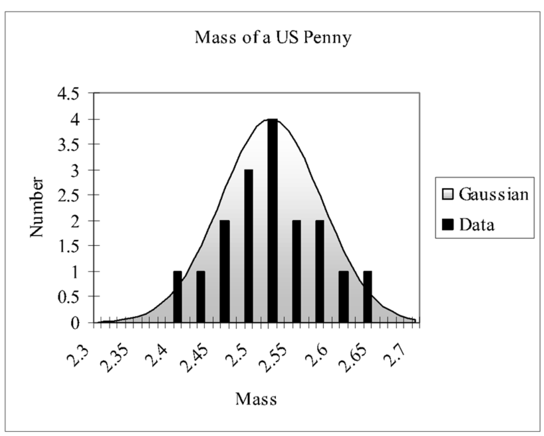


Figure 1. The Gaussian or normal distribution for the mass of a penny N=17, $\overline{m}$ =2.518 g, Δm=0.063 g.

OK, now I have all of these measurements. How accurate is any <u>one</u> of these measurements?

For this, we now define the **standard deviation** Δm as

$$\Delta m = \sqrt{\sum_{i=1}^{N} \frac{(m_i - \overline{m})^2}{(N-1)}} = \sqrt{\sum_{i=1}^{N} \frac{(m_i - \overline{m})^2}{16}} = 0.063g \tag{1.4}$$

For normal distributions, 68% of the time the result of an individual measurement would be within $\pm\,\Delta m$ of the mean value $\overline{m}$. Thus, Δm is the experimental uncertainty for an *individual* measurement of m.

*The mean $\overline{m}$ should have less uncertainty than any individual measurement. What is **that** uncertainty?*

The uncertainty of the final average is called the **standard deviation of the mean**. It is given by

$$\Delta \overline{m} = \frac{\Delta m}{\sqrt{N}} \tag{1.5}$$

With a set of N=17 measurements, our result is

$$\begin{aligned} \text{mass of a penny} &= \overline{m} \pm \Delta\overline{m} = \overline{m} \pm \frac{\Delta m}{\sqrt{N}} \\ &= 2.518g \pm \frac{0.063g}{\sqrt{17}} \\ &= (2.518 \pm 0.015)\,g \end{aligned} \tag{1.6}$$

Thus, *if our experiment is only subject to random uncertainties in the individual measurements*, we can improve the precision of that measurement by doing it repeatedly and finding the average. Note, however, that the precision improves only as $\frac{1}{\sqrt{N}}$. To reduce the uncertainty by a factor of 10, we have to make 100 times as many measurements. We also have to be careful in trying to get better results by letting $N \to \infty$, because the overall accuracy of our measurements will eventually be limited by systematic errors, which *do not cancel out* like random errors do.

Exercise 1:

a. With a caliper, measure the width and thickness of the plank. Make at least five measurements of each dimension and enter the result into Table 2.
b. With a ruler, measure the length of the wooden plank on your table as precisely as possible. Estimate the uncertainty, and enter the result into Table 2.

c. For both the width and thickness, calculate your final result (the *mean*) and the uncertainty (the *standard deviation of the mean*). Enter the final results below the table.

Table 2

width	deviation	thickness	deviation

width = ____

thickness = ____

length = ____

Propagation of uncertainties

Usually, to obtain a final result, we have to measure a variety of quantities (*say, length and time*) and mathematically combine them to obtain a final result (*speed*). How the uncertainties in individual quantities combine to produce the uncertainty in the final result is called the *propagation of uncertainty*.

Here we summarize a number of common cases. For the most part these should take care of what you need to know about how to combine uncertainties.*

Uncertainties in sums and differences:

If several quantities $x_1, x_2, x_{3.}$ are measured with absolute uncertainties $\Delta x_1, \Delta x_2, \Delta x_{3.}$ then the absolute uncertainty in Q (where $Q = x_1 \pm x_2 \pm x_3$) is

$$\Delta Q = |\Delta x_1| + |\Delta x_2| + |\Delta x_3| \tag{1.7}$$

In other words, *for sums and differences, add the* ***absolute*** *uncertainties*.

***Uncertainties in products and quotients*:**

Several quantities *x*, *y*, *z* (with uncertainties Δx, Δy, Δz,) combine to form *Q*, where

$$Q = \frac{x\,y}{z}$$

(or any other combination of multiplication and division). Then the fractional uncertainty in *Q* will be

$$\frac{\Delta Q}{|Q|} = \left|\frac{\Delta x}{x}\right| + \left|\frac{\Delta y}{y}\right| + \left|\frac{\Delta z}{z}\right| \tag{1.8}$$

In other words, *for products and quotients, add the* ***fractional*** *uncertainties*.

* These expressions for the propagation of uncertainty are an **upper limit** to the resulting uncertainty. In this case, you could actually do better. See Appendix C for details.

Exercise 2:

d. Calculate the total volume of the block and the associated uncertainty. Show your math below.

Exercise 3:

You can also find the volume of an object by measuring the volume water displaced by the object when it is submerged.

e. Measure the volume of water in the graduated cylinder and the associated uncertainty.
f. Submerge the block (holding it under the surface with a pen or pencil), and find the resulting volume and the associated uncertainty.
g. Calculate the volume of the plank and the associated uncertainty. Show your math below.
h. Is this answer consistent with your result from Exercise 3? Explain.

Exercise 4:

a. Calculate the total surface area of the block and the associated uncertainty. Show your math below.

Name________________________ Section _______ Date_____________

Pre-Lab Preparation Sheet for Lab 2:
Position, Velocity, and Acceleration in one-dimensional motion

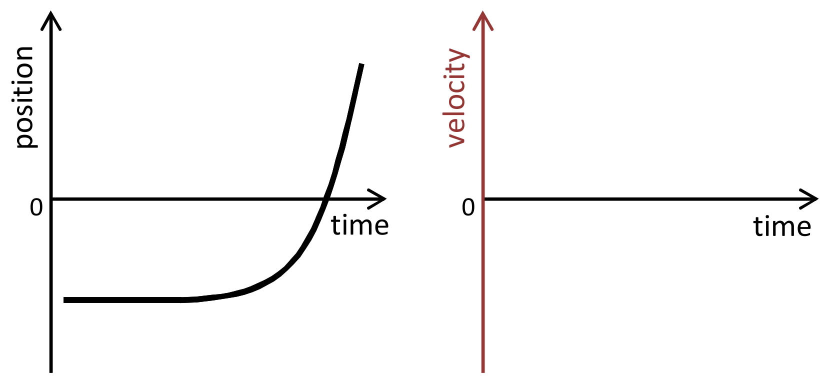
(Due at the Beginning of Lab)

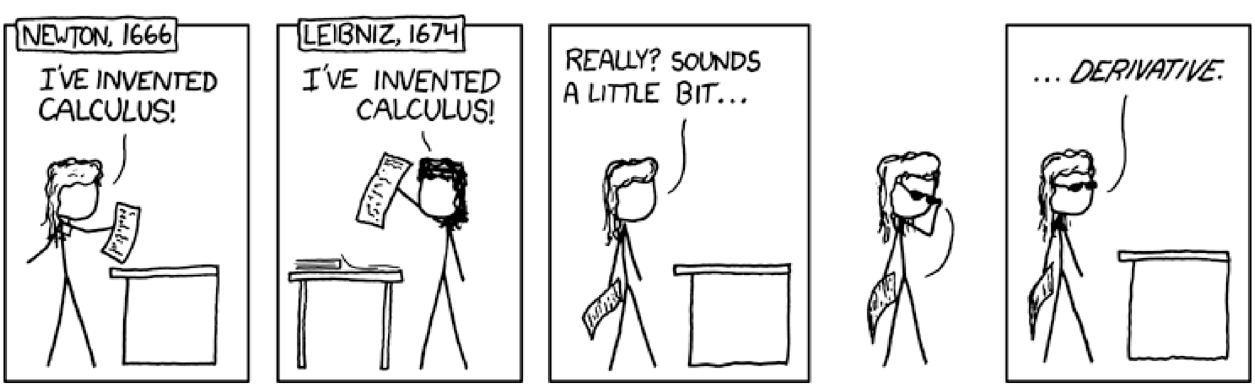
Watch the video *Introduction to Optimization and Curve Fitting* found at the following site:

https://my.vanderbilt.edu/physicslabs/videos/

Read over the lab and then answer the following questions

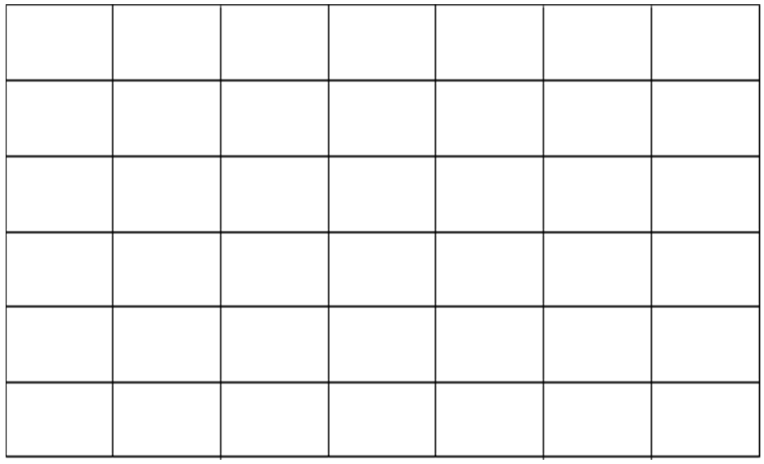
1. Given the following position curve, sketch the corresponding velocity curve.

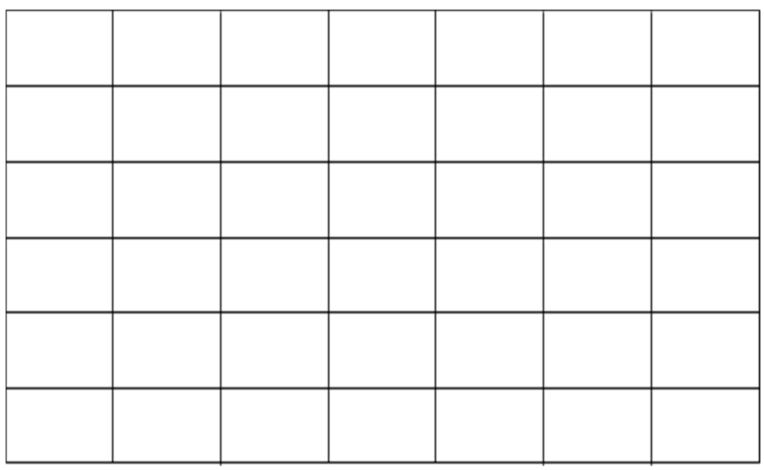




xkcd.com

2. Imagine kicking a box across the floor: it suddenly starts moving then slides for a short distance before coming to a stop. Make a sketch of the position and velocity curves for such motion.





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Lab 2: Position, Velocity, and Acceleration in one-dimensional motion

"God does not care about our mathematical difficulties. He integrates empirically."
--Albert Einstein

Objectives:
- To understand graphical descriptions of the motion of an object.
- To understand the mathematical and graphical relationships among position, velocity and acceleration

Equipment:
- 2.2-meter track w/ adjustable feet and end stop
- A block to raise one end of the cart
- Motion sensor
- Torpedo level
- PASCO dynamics cart and friction cart

DISCUSSION

Velocity is the rate of change or time derivative of position.

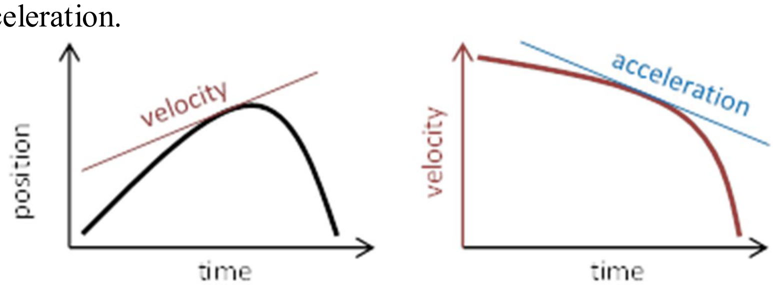
$$\vec{v} = \frac{d\vec{x}}{dt} \tag{2.1}$$

On a Cartesian plot of position vs. time, the slope of the curve at any point will be the instantaneous velocity.

Likewise, acceleration is the rate of change or time derivative of velocity (the 2nd derivative of position).

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2} \tag{2.2}$$

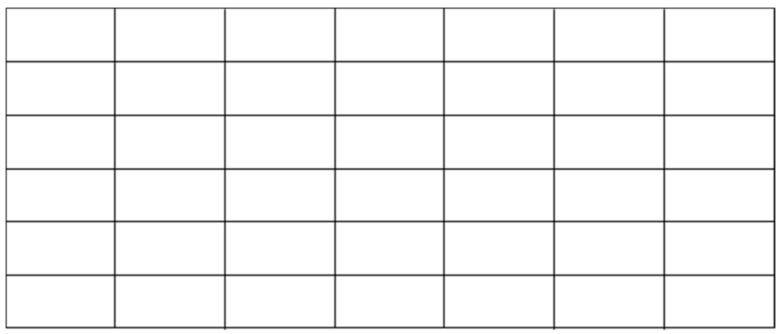
On a Cartesian plot of velocity vs. time, the slope of the curve at any point will be the instantaneous acceleration.



Thus, the shape of any one curve (position, velocity, or acceleration) can determine the shape of the other two.

Exercise 1: Back and Forth

a. Place the friction cart on the track. (That is the one with the friction pad on the bottom. Without letting go of the cart, quickly push it toward the detector by about a foot, then stop it for 1 or 2 seconds. Then quickly but smoothly return the cart to the starting point. Note the distance it travels, and sketch the position vs. time curve for the block on the plot below.



b. Now, open the Labfile directory found on your computer's desktop. Navigate to

A Labs/Lab2

and select the program Position. The PASCO Capstone program should open and present you with a blank position vs. time graph.

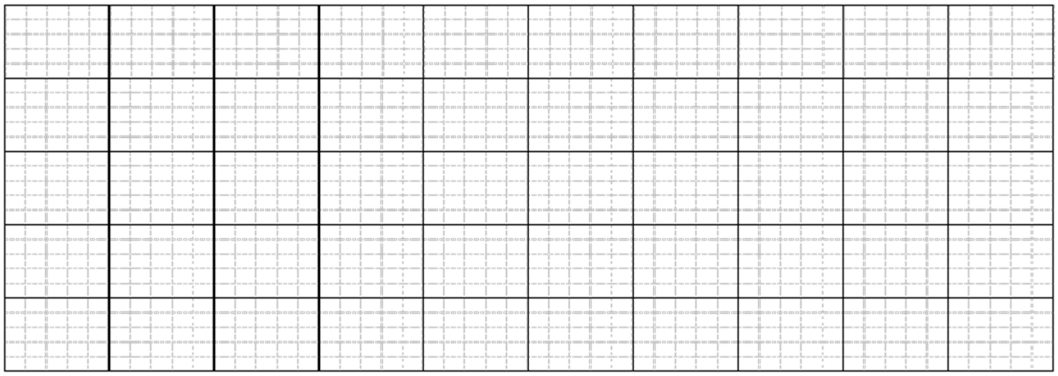
c. Click the **Record**  button (lower left side of the screen), and repeat the experiment above. Click **Stop** to cease recording data. Note how the PASCO plot compares to yours.

Note: The cart may bounce or stutter in its motion. If you don't get a smooth curve, delete the data[*] and repeat the run with more Zen[†].

d. By clicking the scaling icon  (top left corner of the Graph window) you can better fill the screen with the newly acquired data.

e. Select the slope icon . A solid black line will appear on the screen. By dragging this line to points along the plot, you can measure the slope of the curve at those points. Using this tool, find the steepest part of the curve (that is, the largest velocity). Then, sketch the velocity curve for the block in the graph below. Add appropriate numbers to the x and y axes.

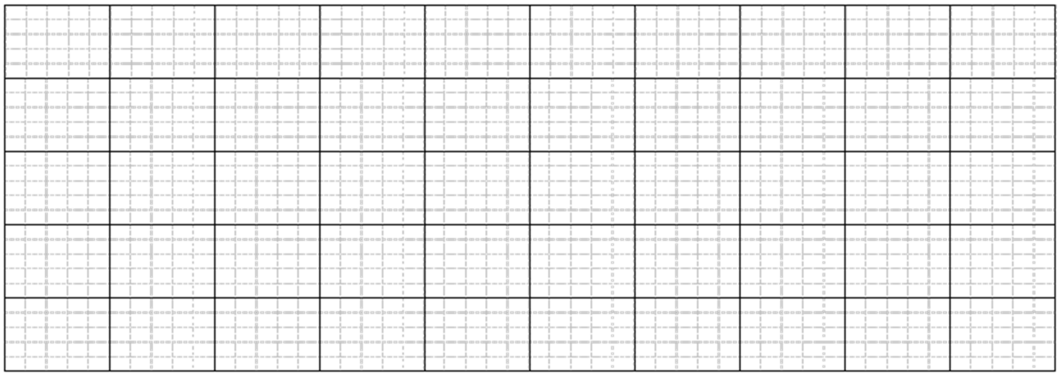
[*] To delete data: click the icon on the bottom of the screen.
[†] "*This time, let go your conscious self and act on instinct.*" Obi-Wan Kenobi



f. How does the shape of the <u>position curve</u> determine the sign of the <u>velocity curve</u>?

g. Now, let's see how well you drew it! Double-click on the new plot icon  (middle top of the screen) and select **Velocity (m/s)** for the y-axis. Note the shape and position of the curve and see how well it matches your sketch. Also note how it aligns with the position curve.

h. Use the slope tool to find the changing slope along the velocity curve. With this information, sketch the acceleration curve for the block. Again, appropriately mark the axes.



i. Let's see what PASCO says about the acceleration. Again, create a new plot  and select **Acceleration (m/s²)** for the y-axis. Compare it to your acceleration curve and PASCO's velocity curve.
j. How does the shape of the position curve determine the sign of the acceleration curve?

k. Print out the three PASCO plots. Annotate these plots to show the times when the push began, when the push ended, when it was slowing, and when it stopped.

Exercise 2: Skidding to a Stop

Delete your previous runs. (Top bar, Experiment, Delete ALL Data Runs). With a left click of the mouse, you can remove the slope tools.

a. Move the cart to end of the track opposite the detector.
b. Start recording data, then give the cart a quick, firm push so that it slides a few feet before coming to rest. Stop the data acquisition.

   By clicking the scaling icon , you can better fill the screen with the newly acquired data. You can also adjust the scale by clicking and dragging along the x or y axis, or zooming with the scroll wheel.

c. Again, if the data is not reasonably smooth, delete the data and repeat the experiment with more Zen.
d. Print out the curves and annotate on the graphs with the times when the push began, when the push ended, and when the cart was sliding on its own.

You should notice that as the cart is slowing down, the acceleration curve is nearly a constant flat line.

e. Given constant acceleration, what mathematical expression describes the velocity?

f. What mathematical expression describes the position?

You can verify that these expressions work by numerically fitting the data.

g. Click on the highlight tool  and a box for selecting data will appear on the screen. Adjust the size and position of the box to highlight the region of the velocity curve where the cart is slowing down. Then, select the fitting tool  and choose the appropriate expression to describe the data. Record the results of the fit below. (Note the uncertainty provided by the fit.)

h. Similarly, apply a numerical fit to the position data. Record the results below. Are the results consistent with the velocity and acceleration curves? (That is, do the uncertainties overlap?)

i. Similarly, find the average acceleration of this region.

Exercise 3: Up and Down

a. Place a block under one of the track stands to form a ramp. The detector must be on the raised end.
b. Place a low friction cart on the track and give it a push so that it rolls a few feet up the incline and then rolls back. After a few practice runs, run the detector and acquire motion data.
c. With a click and drag of the mouse, highlight that section of the data where the cart is freely rolling along the track. Then use the scaling tool  to zoom-in on that section of the data.
d. Print out these plots and annotate the graphs with the following information.
   When and where does the velocity of the cart go to zero?
   What is the acceleration when the velocity is zero?
   When and where does the acceleration of the cart go to zero?

e. Find the average acceleration going up the slope and down the slope. Record the results below.

f. How does the acceleration up the slope compare with the acceleration down the slope? What might account for the difference?

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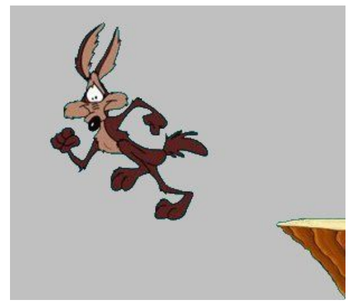
Pre-Lab Preparation Sheet for Lab 3: Momentum

(Due at the Beginning of Lab)

Directions:
Read over the following lab, then answer the following questions.



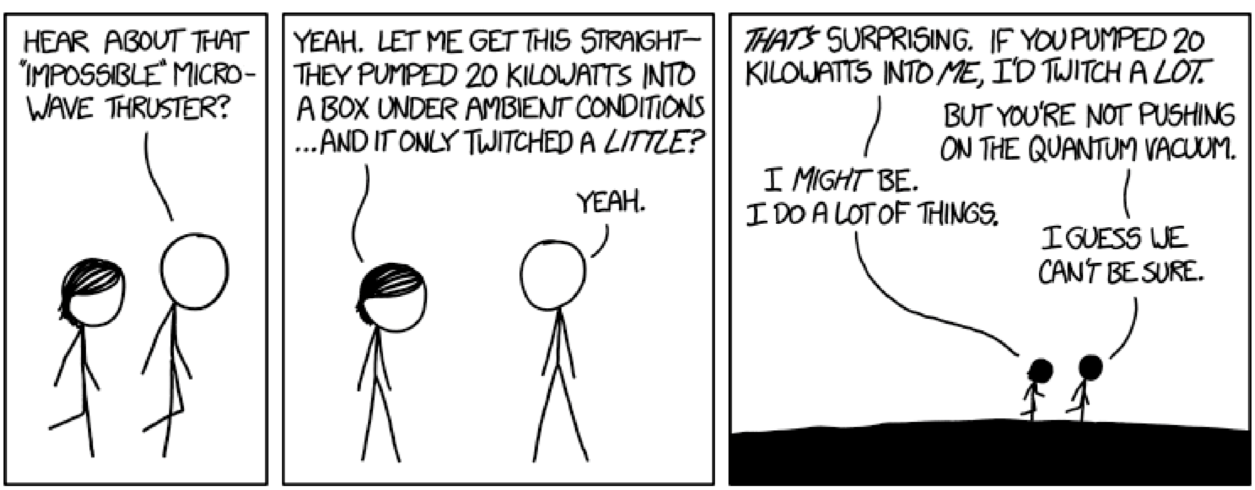
2. Wile E. Coyote steps off a cliff edge. He reasons that
    a. his momentum is zero,
    b. momentum is conserved,
        therefore . . .
    c. he cannot fall.

    Correct his reasoning. How is momentum conserved in this case?

3. Define *closed system*.

4. Two carts collide. Afterward, they have the same velocity. What kind of collision occurred?

5. Two carts have equal mass and form a closed system.
   When t = 0, they have velocities v_1 and v_2.
   When t = 1s, they have the same velocity v.
   Assume both kinetic energy and momentum are conserved.
   a. How are v_1, v_2 and v related?
   b. What does this mean?



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Lab 3: Momentum

When he was within six miles of the place,
There Number Four stared him straight in the face.
He turned to his fireman, said Jim you'd better jump,
For there're two locomotives that are going to bump.

--The Ballad of Casey Jones, variant by Eddie Newton and T. Lawrence Seibert

Objective: To observe conservation of momentum and conservation of kinetic energy (or not).

Equipment: 2m track, magnetic cart, plunger cart, two motion sensors, 2 (½ kg) cart masses

Introduction

The momentum of an object is

$$\vec{p} = m\vec{v} \tag{3}$$

Momentum is a vector. The direction of motion matters. The total momentum of a closed system is always conserved**.** No exceptions. If momentum of your system is changing, your system is not closed. Something else is meddling with it

The kinetic energy of an object is

$$K = \tfrac{1}{2}mv^2 \tag{4}$$

Kinetic energy is a scalar. The direction of motion is irrelevant.

The total energy of a closed system is always conserved. No exceptions. If the energy of your system is changing, your system is not closed. Something else is meddling with it.

But, energy has this inconvenient tendency of transforming from one type of energy to another. Kinetic energy may be transformed into potential energy, or chemical energy, or thermal energy, or magical energy[*] or . . . , and *vice versa*. While the total energy is conserved, the amount of any given species of energy may not be conserved.

Momentum, on the other hand, remains politely consistent. There is no *kinetic momentum*, or *potential momentum*, or . . . whatever. Just *momentum*, and it does not turn into anything else.[†]

Types of collisions

Object *a* (mass m_a, velocity $\vec{v}_{ai}$) collides with object *b* (mass m_b, velocity $\vec{v}_{bi}$). After the collision, the velocities are $\vec{v}_{af}$ and $\vec{v}_{bf}$.

We can define three basic types of collisions:

- **Totally inelastic collision**: Two or more objects collide and stick together after the collision. That is, their *relative* velocity is zero.
$$\vec{v}_{af} - \vec{v}_{bf} = 0$$
- **Totally elastic collision**: Two objects collide. Afterward, their *relative* velocity is unchanged, and kinetic energy is conserved.
$$\vec{v}_{ai} - \vec{v}_{bi} = \vec{v}_{af} - \vec{v}_{bf}$$
- **Partially elastic collision**: While the objects do not stick together after the collision, their relative velocities have changed, and kinetic energy is not conserved.
$$\vec{v}_{ai} - \vec{v}_{bi} \neq \vec{v}_{af} - \vec{v}_{bf} \neq 0$$

Exercise 1: Inventory

1. Mass each of the following:

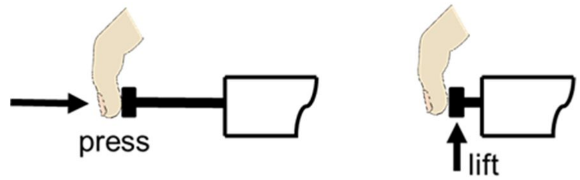
Cart 1: Cart 2:

Bar 1: Bar 2:

2. Level the track, then tighten the nut on each foot.
3. One of your carts contains a plunger. The plunger can be cocked by pressing down and lifting up.

[*] *Aresto Momentum!*
[†] Angular momentum (which you will study later) never transforms into momentum, and momentum never transforms into angular momentum.

Pressing the top pin releases the plunger.



4. Magnets are embedded on the ends of the carts. This can cause the carts to be attracted or repelled. Place the carts on the track and play with them to see which ends attract and which repel. Note that there are no magnets on the end with the plunger.

Exercise 2: Inelastic Collisions

Data Acquisition

5. Depress and cock the plunger. (For now, this is just to keep it out of the way.)

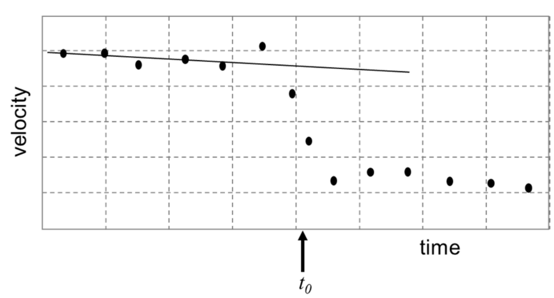
6. Orient the carts on the track so they are attracted to each other. Place a mass bar in one of the carts.
7. Run the Capstone program Momentum. Start recording.
8. With one cart stationary, have the second cart collide into it. The carts should stick together.

Caution: Do not let the carts collide with the detectors!

At this point, you could just get some points before and after the collision and see how the momenta compare. But, if you look closely at the data, you will notice several problems.

- There is some noise in the data, but with a single point, you cannot estimate your uncertainty.
- Your carts are not a truly isolated system. Friction is steadily slowing the carts both before and after the collision.
- You only really care about the moments just before and just after the collision. However, you don't actually have good velocity data around the time of the collision.

You can compensate for this by fitting the good data with a straight line. Then, extrapolate to find the velocity at the moment of the collision.



9. Find the initial velocity (including the uncertainty) at the moment just before the collision. (For the stationary cart, you can assume the initial velocity is exactly zero.)
10. Repeat to find the final velocity. Record your results on the tables below.

Data Analysis

11. Given your measured values for the initial velocity, calculate what the final velocity (and the associated uncertainty) should be if momentum is conserved. Is this calculated value consistent with the measured velocity? (Show your work on a separate sheet.) Calculate the kinetic energy before and after the collision. What fraction of the initial kinetic energy is lost in the collision?

Explosion

12. Orient the carts so they are attached and the plunger is in the middle. Place a mass bar in one of the carts.
13. Give the connected carts an initial velocity
14. While the carts are moving, use the flat of a ruler to apply a quick blow to the release pin. Acquire the velocity data and repeat the analysis.

Exercise 3: Elastic Collisions

15. Orient the carts so they repel each other. Remove any mass bars.
16. With one cart stationary, gently* collide the second cart into it. Acquire and analyze the data as above.
17. Add a mass bar to the stationary cart, then repeat the above experiment.
18. Within the resolution of the experiment, are momentum and kinetic energy conserved?

* Don't push so hard that the carts physically touch.

Discussion

19. If the momentum does not appear to be conserved within the uncertainty of you measurements, what could explain this? Be specific.

20. The position sensors work by assuming that the speed of sound is 344 m/s and measuring the time for the echo to bounce off the target. Doubtless, this speed is off by a bit. How would this effect your observation of the conservation of momentum or kinetic energy? Explain.

Collision

$m_1 =$ $m_2 =$

$v_{1i} =$ $v_{2i} =$

$v_{1f} =$ $v_{2f} =$

$p_i =$ $p_f =$

$K_i =$ $K_f =$

Calculated final velocity = Fractional change of kinetic energy =

Explosion

$m_1 =$ $m_2 =$

$v_{1i} =$ $v_{2i} =$

$v_{1f} =$ $v_{2f} =$

$p_i =$ $p_f =$

$K_i =$ $K_f =$

Calculated final velocities = Fractional change of kinetic energy =

Bounce 1

$m_1 =$ $m_2 =$

$v_{1i} =$ $v_{2i} =$

$v_{1f} =$ $v_{2f} =$

$p_i =$ $p_f =$

$K_i =$ $K_f =$

Bounce 2

$m_1 =$ $m_2 =$

$v_{1i} =$ $v_{2i} =$

$v_{1f} =$ $v_{2f} =$

$p_i =$ $p_f =$

$K_i =$ $K_f =$

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Pre-Lab Preparation Sheet for Lab 4: Force, Mass, and Acceleration

(Due at the Beginning of Lab)

Directions:

Read over the lab and then answer the following questions.

1. Consider the experimental configuration shown in Figure 1. Starting from Newton's 2nd law, show that the acceleration of the cart is on the string is given by:

$$a = \frac{mg}{m+M}.$$

Modified from: *RealTime Physics*, P. Laws, D. Sokoloff, R. Thornton
and University of VA Physics Labs: S. Thornton

2. In the limit where $M >> m$, what is the acceleration?

3. In the limit where $M << m$, what is the acceleration?

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Lab 4: Force, Mass, and Acceleration

"Well, the Force is what gives a Jedi his power . . . . It surrounds us and penetrates us. It binds the galaxy together."

-- Obi-Wan Kenobi on Newton's 2nd Law

Equipment

Motion sensor	Force sensor
Low-friction cart	2.2 meter track
Torpedo level	Low –friction pulley
Foam crash pad	0.02, 0.05, 0.10, and 0.20 kg hooked mass

Note: The acceleration due to gravity varies with location. Here at Vanderbilt, this acceleration has been measured as

$$g = \left(9.7943 \pm 0.0032\right)\frac{m}{s^2}$$

Use this value throughout the semester.

Introduction

Newton's 2nd Law is the most important concept you will learn in this class:

$$\vec{F}_{net} = m\vec{a}$$

If you know the **mass** and **net force** on an object, you know the **acceleration** of the object.

If you know the **acceleration** of an object and its **initial velocity** and its **initial position**, you know the complete **trajectory** of the object.

Consider the problem illustrated below: a frictionless wheeled cart is pulled by string attached to a falling mass.

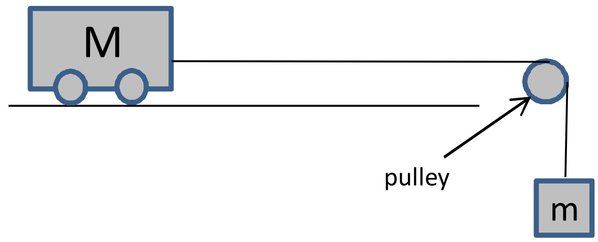


Fig. 1

It can be shown that the acceleration a is given by

$$a = \frac{mg}{M + m} \tag{1}$$

Of course, that is the theory. In the real world, things can get messy. Welcome to experimental physics.

Exercise 1: Data Acquisition

1. Label the individual forces on the diagram above.
2. Mass the combined cart and force sensor with the electronic scale

$M =$ ______________________________

Note: Before each measurement with the scale, you should *tare* the scale. Empty the scale, then press the Z or TARE button found on the panel. This resets the zero point of the scale.

You will be using a set of hooked weights for you falling masses. These have nominal values of 0.020 kg, 0.050 kg, 0.100 kg, 0.200 kg.

3. Measure their precise masses with the digital scale and record the results and associated uncertainties on Table 1 below.

You will use the PASCO Force Sensor to measure the tension in the string. Note the sign convention for the direction of the force on the hook.

Assemble the cart, force sensor, string, and falling mass. The falling mass will be one of four hooked weights: 0.020 kg, 0.050 kg, 0.100 kg, 0.200 kg. **Verify that a crash pad is positioned underneath the falling mass.**

4. Run the Capstone program Velocity&Force.

    Note: Before each measurement with the force sensor, you must *tare* the force sensor. Remove any force from the hook, then press the TARE button found on the side of the sensor. This will ensure that when the force is zero, the device returns zero.

5. Holding cart stationary, measure the static tension T_{static} on the string and the corresponding uncertainty. Record the result in Table 1 below. Briefly explain how you determined the uncertainty below.

    Note: Do <u>not</u> assume that every digit which Capstone reports is significant.

Table 1: Measured values

m	T_{static}	$T_{dynamic}$	a

6. **Dedicate one of you members to catching the cart before it crashes into the pulley.** Please, do not turn this into a projectile motion lab!
7. Start recording, then release the cart. Record the dynamic tension on the string $T_{dynamic}$, the acceleration a, and the associated uncertainties. Justify your determination of each of these uncertainties below.

Exercise 2: Data Analysis

8. From the pre-lab, what is the acceleration in the limit of $m << M$? Is this confirmed in your observations? Using Excel, plot your data in way to support your argument.

While you have measured the acceleration above, it can also be calculated from Equ. 1, or from the measured tension of the string and the cart mass.

9. Fill in the table below. Include the associated uncertainties

Table 2: Calculated accelerations

$\frac{mg}{M+m}$	$\frac{T_{dynamic}}{M}$

10. Are the measurements in Table 2 consistent with each other? Explain.

11. Are the measurements of $\frac{mg}{M+m}$ consistent with the measured accelerations a in Table 1? Explain.

12. Thus far, we have been blithely ignoring friction of the cart. Given that friction is present, how would that effect the acceleration of the cart?

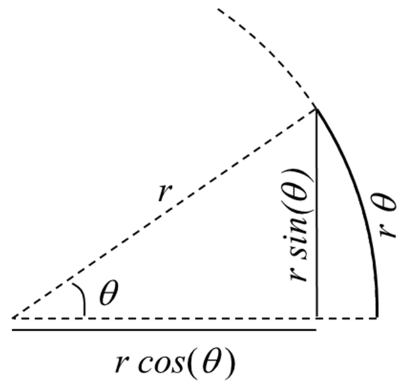
13. Would the addition of a constant friction force resolve any discrepancy between the observed acceleration and $\frac{mg}{M+m}$? Explain.

Appendix A: The Small Angle Approximation

Often, you will find yourself dealing with small angles. In such cases, the following approximations can make your calculations a lot easier.

If an angle θ is measured in radians, the arc length of the corresponding circle segment s is given by

$$s = r\theta \ .$$



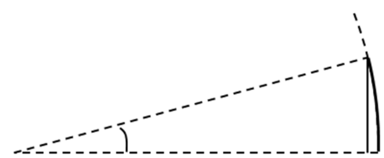
By definition

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} \ .$$

Inspecting the diagram, it is obvious that for small angles

$$r\sin(\theta) \approx r\theta$$

$$\boxed{\sin(\theta) \approx \theta} \ .$$



It is also obvious that for small angles

$$r\cos(\theta) \approx r$$

$$\boxed{\cos(\theta) \approx 1}$$

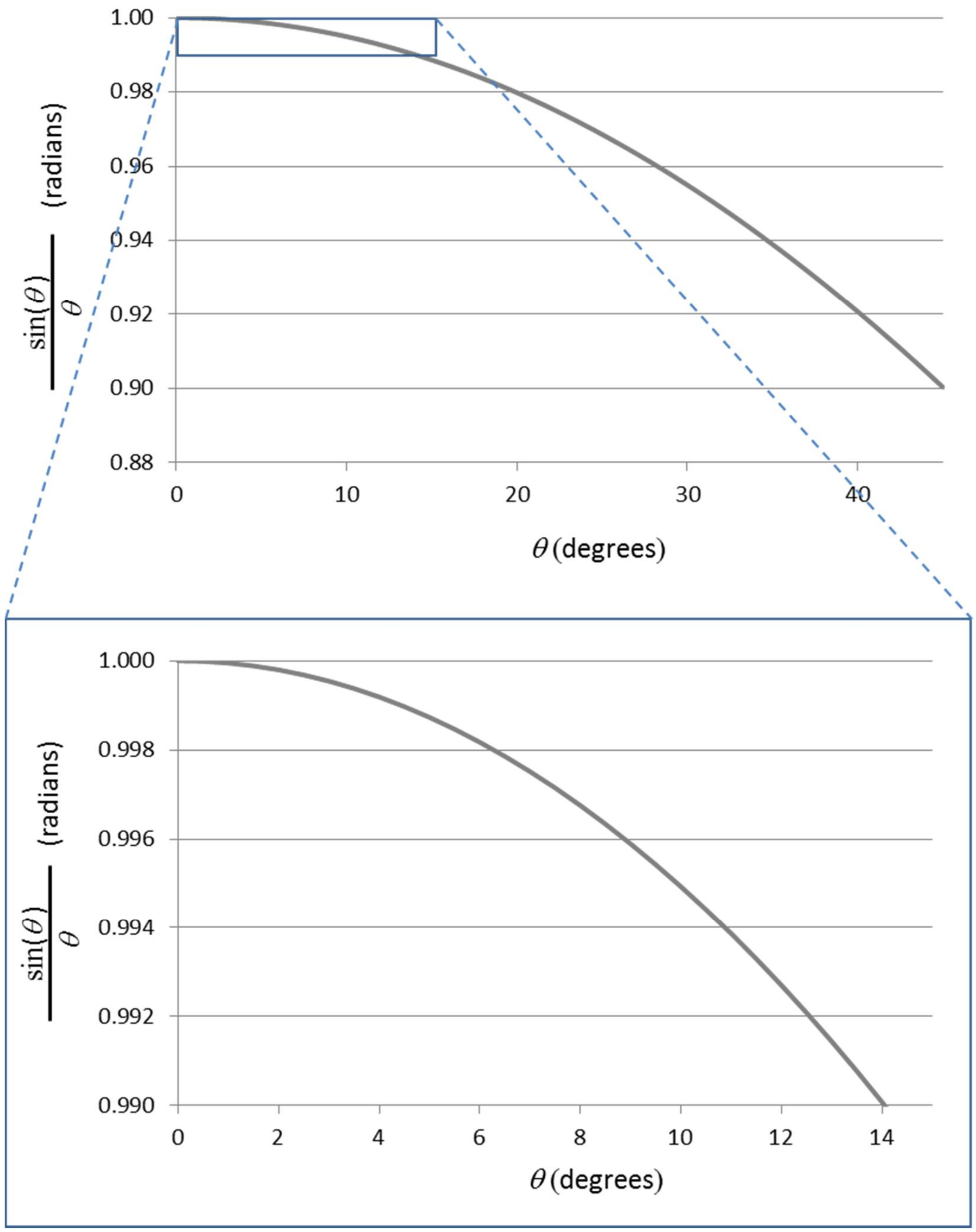
Thus,

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\boxed{\begin{aligned} \tan(\theta) &\approx \sin(\theta) \\ &\approx \theta \end{aligned}}$$

So, for what angles does this approximation work? Well, it depends on how accurate you need to be. The graph below is a useful guide. The closer $\frac{\sin(\theta)}{\theta}$ is to 1, the better the approximation.

Note: The angles on the x-axis are in degrees. The angles on the y-axis are calculated in radians.

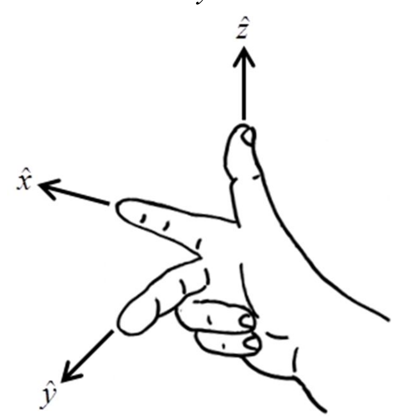


Appendix B: The Right Hand Rule and Right Handed Coordinates

Many of the equations used in this lab and the associated class require that you use a *Right Handed Coordinate System*, particularly those equations involving cross products. Using the wrong coordinates can lead to considerable confusion and grief; so, **get in the habit of drawing your coordinate axes correctly!**

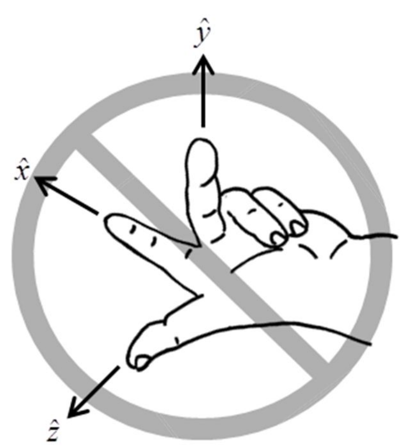
Consider the illustration of a Right Handed Coordinate axes below. The unit vectors $\hat{x}$, $\hat{y}$, and $\hat{z}$ correspond to the index finger, middle finger, and thumb respectively. Here

$$\hat{x} \times \hat{y} = \hat{z}$$



Right Handed Coordinates

Below is an example of a Left Handed Coordinate system where $\hat{x} \times \hat{y} = -\hat{z}$, and *that's wrong*!!



Left Handed Coordinates

Modified from: *RealTime Physics*, P. Laws, D. Sokoloff, R. Thornton
and University of VA Physics Labs: S. Thornton

Appendix C: Advanced Propagation of Uncertainties

Suppose you had measured three different rods:

$x = (2.34 \pm 0.03)$ m $\quad y = (5.43 \pm 0.05)$ m $\quad z = (8.31 \pm 0.02)$ m

What would be the length of these three rods placed end to end?

In Lab 1, you were instructed to combine the uncertainties by simply adding the absolute uncertainties.

$$\begin{aligned} L &= x + y + z \\ &= (2.34 + 5.43 + 8.31)\ \text{m} \ \pm\ (0.03 + 0.05 + 0.02)\ \text{m} \\ &= (16.08 \pm 0.10)\ \text{m} \end{aligned}$$

In fact, this is an upper limit of the uncertainty. In this particular case, we can actually do a bit better. The logic goes like this:

If you make several <u>*independent*</u> *measurements subject to random uncertainties, we expect some measurements will be a little bit high and some to be a little bit low. These variations should partially cancel out.*

I'll spare you the ugly details of the math, but it boils down to this:

When adding several <u>independent</u> measurements

$$Q = q_1 + q_2 + \ldots + q_n$$

the corresponding absolute uncertainties are added in quadrature.*

$$\Delta Q = \sqrt{\Delta q_1^{\,2} + \Delta q_2^{\,2} + \ldots + \Delta q_n^{\,2}}$$

Thus, in the example above we have

$$\begin{aligned} L &= x + y + z \\ &= (2.34 + 5.43 + 8.31)\ \text{m} \ \pm\ \left(\sqrt{0.03^2 + 0.05^2 + 0.02^2}\right)\ \text{m} \\ &= (16.08 \pm 0.06)\ \text{m} \end{aligned}$$

* That is, added like the Pythagorean Theorem.

Note that for this to apply, we must be adding <u>independent</u> measurements. Consider for a moment a different problem:

> *You are given a rod of length r* = (1.54 ± 0.04)m, *What would be the length of 3 similar rods placed end to end?*

Here, **we only have a measurement for a single rod**; so, there is no longer a partial canceling out of uncertainties. In this case, the best we can do is to simply add the uncertainties:

$$\begin{aligned}3r &= (1.54+1.54+1.54)\,\text{m} \pm (0.04+0.04+0.04)\,\text{m}\\ &= 3\,(1.54 \pm 0.04)\,\text{m}\\ &= (4.62 \pm 0.12)\,\text{m}\end{aligned}$$

What about when multiplying (or dividing) measurements? The method is similar:

When multiplying (or dividing) several <u>independent</u> measurements

$$Q = q_1 \times q_2 \times \ldots \times q_n$$

the corresponding relative uncertainties are added in quadrature.

$$\Delta Q = \sqrt{\left(\frac{\Delta q_1}{q_1}\right)^2 + \left(\frac{\Delta q_2}{q_2}\right)^2 + \ldots + \left(\frac{\Delta q_n}{q_n}\right)^2}$$

So, if I were to take the measurements in the first example and multiply them together (say, to find a volume), the result would be

$$\begin{aligned}V &= x \times y \times z\\ &= (2.34 \times 5.43 \times 8.31)\ \text{m}^3\ \pm \left(\sqrt{\left(\frac{0.03}{2.34}\right)^2 + \left(\frac{0.05}{5.43}\right)^2 + \left(\frac{0.02}{8.31}\right)^2}\right) 100\%\\ &= 16.08\ \text{m}^3 \pm (0.016)100\%\\ &= 16.08\ \text{m}^3 \pm 1.6\%\end{aligned}$$

But, suppose I had a measured only one rod and imagined constructing a cube 3 rods long by 2 rods tall and 1 rod deep. What would be the volume of this structure?

Now with only one actual measurement

$$r = (1.54 \pm 0.04)\text{m}$$

the relative uncertainties are simply added:

$$\begin{aligned}3r \times 2r \times r &= \left(6 \times 1.54^3\right)\text{m}^3 \pm \left(\frac{3 \times 0.04}{3 \times 1.54} + \frac{2 \times 0.04}{2 \times 1.54} + \frac{0.04}{1.54}\right) \times 100\% \\ &= 21.91\,\text{m}^3 \pm (0.08) \times 100\% \\ &= 21.91\,\text{m}^3 \pm 8\%\end{aligned}$$

Moral of the story:

Independent measurements: add uncertainties in quadrature.

Non-independent measurements: simply add the uncertainties.