Generalized Theory of Electrical Machines- A Review

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***Abstract*:-This paper provides an overview of the Generalized Theory of Electrical Machines. The attempts to unify the piecemeal treatment of rotating electrical machines has led to generalized theory of electrical machines or two-axis theory of electrical machines. Park developed two-axis equations of the synchronous machines by making use of appropriate transformations. Park's ideas were then developed by Kron to deal with all rotating electrical machines in a systematic manner by tensor analysis. However, Gibbs et al. simplified the work of Kron by applying matrices to the electrical machines analysis. This unified treatment of rotating electrical machines, developed by Kron, is now called generalized theory of electrical machines. This theory can be appreciated from the fact that a three-phase machine requires three voltage equations whereas its generalized model requires only two voltage equations which can be solved more easily as compared to three voltage equations. Further, the circuit equations for a three-phase machine are more complicated because of the magnetic coupling amongst the three-phase windings, but this is not the case in the generalized (or two-axis) model, in which m.m.f. acting along one axis has no mutual coupling with the m.m.f. acting along the other axis. The general equations, applicable to almost all types of rotating machines, can deal comprehensively with their steady state, dynamic and transient analysis.**

I. INTRODUCTION

The Generalized Theory was developed for a basic or an idealized two-pole machine, which is closely related to an actual machine. Each part of the winding forming a single circuit, is represented by a single coil in the basic two-pole machine. Thus, a basic two-pole machine is one in which each coil represents each part of the winding forming a single circuit of actual machine. [1]-[2]

Certain conventions have been adopted in the development of generalized machine theory. A brief outline of them is given below:

1. The distribution of current and flux under one pair of poles, repeats itself under all other pairs of poles as shown in Fig. 1(a), whatever the actual number of pole-pairs may be. Hence any machine can be replaced by an equivalent two-pole machine and the generalized machine theory is developed in terms of two-pole machines.

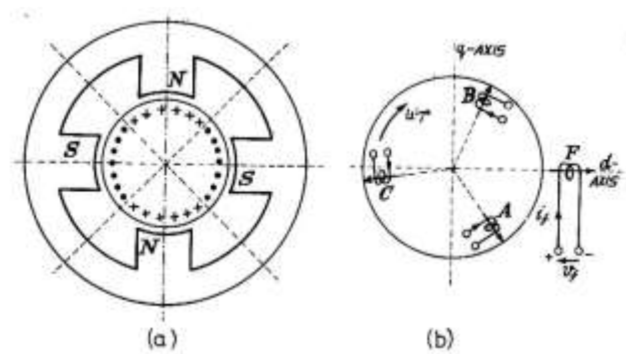

Fig. 1. (a) Four-pole electrical machine (b) Synchronous machine

2. Each winding of the actual machine or each part of the winding forming a single circuit is represented in the idealized or basic two-pole machine by a single coil. For rotor, these single coils, occupy only a part of the radius and for stator, the coils are shown on one side of the machine only. This is essential to indicate the positive direction of currents and m.m.fs. For instance, a three-phase synchronous machine without damper bars, necessitates one coil F for field winding and three coils A, B, C for three phase windings as illustrated in Fig. 1(b).
3. The axis of the poles around which the field is wound, is called the direct axis of the machine, while the axis 90^0 away from it is called the quadrature axis as shown in Fig. 1(b). It is customary to take d-axis as horizontal and q-axis as vertical.
4. Lower case *v* represents the voltage impressed on the coil from an external source and *i*, the current measured in the same direction as *v*.
5. The positive direction of rotation of rotor is taken as clockwise and torque is also taken positive when acting in the sense of positive rotation.
6. Capital L and X are used for total (i.e. self) inductance and the corresponding reactance respectively. The leakage inductance and reactance are represented by the lower case letters *l* and *x* respectively.
7. Capital letters are used to indicate the coils and the corresponding lower case letters serve as subscripts for the voltages, currents, impedances etc. For example, in

Fig. 1(b) voltage v_f and current i_f pertain to the coil f; similarly v_a, i_a would pertain to coil A.

8. In general, it may be stated that if machine has salient poles, they are taken on the stator. [2]

Certain abbreviations used in the papers are as follows:

1) DR: Direct axis-Rotor
2) QR: Quadrature axis- Rotor
3) DS: Direct axis-Stator
4) QS: Quadrature axis-Stator
5) DSE: Direct axis-Series winding

The basic two-pole representation for a separately excited dc motor is shown in fig. 2.

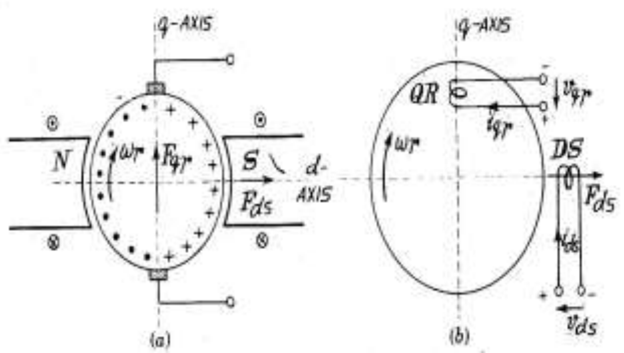

Fig. 2. (a) Separately excited d.c. motor and (b) its basic two-pole representation[2]

II. KRON'S PRIMITIVE MACHINE

The Kron's primitive machine with stationary axes is shown in Fig. 3(a). This machine consists of a stationary field winding DS in the direct axis, an independent field winding QS at right angles to d-axis, i.e. in the quadrature axis, and a rotating armature winding brought out to a commutator. Two sets of brushes are provided which are magnetically perpendicular to each other, with one brush set in the d-axis and the other in the q-axis. The idealized or basic two-pole machine diagram for this machine is shown in Fig. 3(b). This equivalent electrical network is called the "generalized machine", "Kron's primitive machine", "generalized model or two-axis model" of rotating electrical machines.

It can be noted that the equivalent armature coils DR and QR produce stationary fluxes along the brush axes, only if the brushes are stationary. If the brushes are rotating, the corresponding armature coil fluxes will also be rotating. Single armature winding is made to act as two electrically and magnetically independent armature windings in quadrature, due to the presence of two brush sets. This generalized primitive machine can be shown to be equivalent to any of the rotating electrical machines with an appropriate number of coils on each fixed axis. Some machines may require fewer than four coils to represent them, while others may require more. [2]

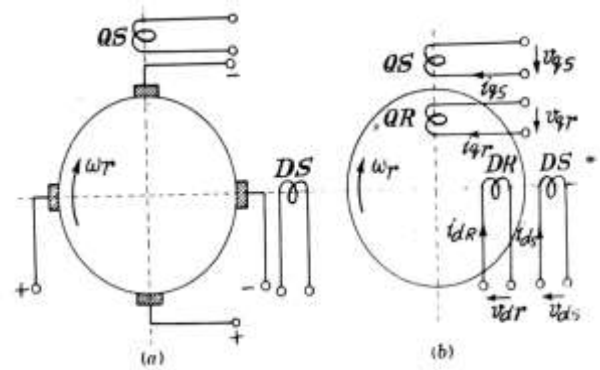

Fig. 3 (a) Kron's primitive machine (b) Its basic two-pole representation[2]

For the ordinary d.c. machine, the generalized machine diagram requires only two coils, DS for the field winding and QR for the armature. Thus the coils DR and QS will have to be omitted from Fig. 3(b), to obtain a generalized machine diagram for this machine.

Some of the basic two-pole machines are similar to the generalized or primitive machine, but if they are not, it is necessary to make a conversion. The process of conversion from the actual coils of the machine itself (i.e. the basic two-pole machine) to the equivalent d- and q–axes of the generalized machine or vice-versa is known as a transformation. For instance, in the case of a three-phase winding on the stator, a transformation must be made to the equivalent coils DS and QS, in such a manner that the magnitude and direction of m.m.f. set up by the three-phase currents i_a, i_b, i_c and the two coil currents i_{ds}, i_{qs} is identical. The rotor of this machine should not know whether the rotating m.m.f. is produced by three phase currents i_α, i_b, i_c or by two coil (or phase) currents i_{ds}, i_{qs}. Thus the generalized machine diagram for a 3-phase induction machine is as shown in Fig. 4 (a).

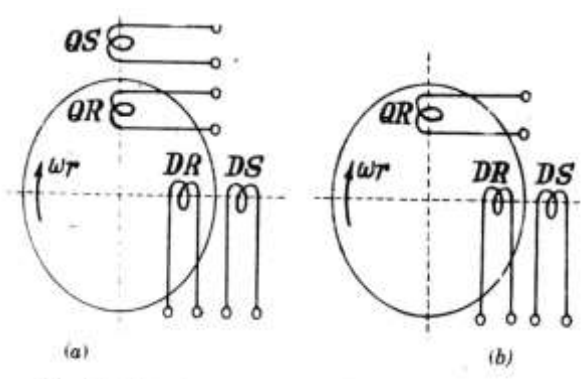

Fig. 4 Primitive (or generalized) machine diagrams (a) induction machine (b) synchronous machine (without dampers)

Similarly the primitive machine diagram for a synchronous machine (without damper bars) is as shown in Fig. 4 (b).

III. VOLTAGE EQUATIONS

At this stage, the voltage equations for the primitive machine of Fig. 3 (b), can be written as follows:

a) Stator field coil DS: Its resistance is r_{ds} and total inductance is $L_{ds} = (M_d + l_{ds})$. Here L_{ds} is self-inductance of the coil DS, M_d is the mutual inductance between the coils DR and DS, and l_{ds} is the leakage inductance of coil DS. It is mutually coupled with any other d-axis coil, i.e. when the voltage equation for the DS circuit is to be written, the mutual (i.e. transformer) effect of other d-axis coils must be considered. Q-axis coils can have no effect on DS and no rotational voltages appear in it, since the coil DS is on the stationary element. Thus the applied voltage v_{ds} must account for the components given by

$$v_{ds} = r_{ds}i_{ds} + L_{ds}pi_{ds} + M_dpi_{dr} \qquad .....(1)$$

b) Stator field coil QS: This coil is identical with coil DS, so that applied voltage v_{qs} is given by

$$v_{qs} = r_{qs}i_{qs} + L_{qs}pi_{qs} + M_qpi_{qr} \qquad .....(2)$$

*c) Armature coil DR***:** The armature coil DR and QR are the pseudo-stationary coils and they have rotational voltages induced in them.

$$v_{dr} = r_{dr}i_{dr} + L_{dr}pi_{dr} + M_dpi_{ds} - e_{dr}$$

(induced rotational voltage)

The negative sign before the induced rotational voltage in the above equation is due to the fact that, if other voltage drops are neglected, the induced voltage is equal and opposite to the applied voltage. Now using $e_d = +\omega_r\psi_{mq}$ and noting that rotational emf in coil DR appears due to *q*-axis flux, we get

$$e_{dr} = +\,\omega_r\psi_q$$

In the rotational emf relation e_{dr} as given above, note that total armature flux linkage ψ_q is taken into consideration as assumed before.

Substitution of the value of e_{dr} in the expression for v_{dr} gives

$$v_{dr} = r_{dr}i_{dr} + L_{dr}pi_{dr} + M_dpi_{ds} - \omega_r\psi_q$$

Here $\omega_r\psi_q$, the total flux linkages with the armature in *q*-axis, is given by,

$$\psi_q = M_q(i_{qs} + i_{qr}) + l_{qr}i_{qr}$$

Thus $v_{dr} = r_{dr}i_{dr} + L_{dr}pi_{dr} + M_dpi_{ds} - \omega_r M_q (i_{qs} + i_{qr}) - \omega_r l_{qr}i_{qr}$

$$= r_{dr}i_{dr} + L_{dr}pi_{dr} + M_dpi_{ds} - \omega_rL_qri_{qr} - \omega_rM_qi_{qs} \qquad .....(3)$$

d) Armature coil QR: This coil is similar to coil DR and, therefore adopting the above procedure, the applied voltage v_{qr} must have the components:

$$v_{qr} = r_{qr}i_{qr} + L_{qr}pi_{qr} + M_qpi_{qs} + \omega_rL_{dr}i_{dr} + \omega_rM_di_{ds} \qquad .....(4)$$

The above four equations from (1) to (4) can be written down rapidly in matrix form as illustrated below:

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{dr} \\ v_{qr} \end{bmatrix} = \begin{array}{c|cccc} & ds & qs & dr & qr \\ \hline ds & r_{ds} + L_{ds}p & & M_{dP} & \\ qs & & r_{qs} + L_{qs}p & & M_q\,p \\ dr & M_dp & -M_q\omega_r & r_{dr} + L_{dr}\,p & -\omega_rL_{qr} \\ qr & \omega_r M_d & M_qp & \omega_rL_{dr} & r_{qr} + L_{qr}p \end{array} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix} \qquad .....(5)$$

IV. ELECTRICAL PERFORMANCE EQUATIONS OF THREE-PHASE INDUCTION MOTORS

For applying generalized theory to poly-phase induction machines, it is essential to have *d-q* axes fixed in the stator structure. Since the stator 3-phase winding A, B, C and *d*, *q* axes are stationary with respect to each other, the transformation matrix from ABC to *d-q* variables, or vice-versa, must have constant coefficients[3]-[7]. The transformation equations relating the three-phase ABC and *d-q* currents and voltages on the stator can be obtained using Park's transformation. Therefore,

$$\begin{bmatrix} i_{ds} \\ i_{qs} \\ I_{os} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos 0 & \cos\frac{2\pi}{3} & \cos\frac{2\pi}{3} \\ \sin 0 & \sin\frac{2\pi}{3} & \sin\frac{2\pi}{3} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} \qquad .....(6a)$$

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ v_{os} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos 0 & \cos\frac{2\pi}{3} & \cos\frac{2\pi}{3} \\ \sin 0 & \sin\frac{2\pi}{3} & \sin\frac{2\pi}{3} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad \text{.....(6b)}$$

In Eq. (6), subscript s stands for stator and has been added to indicate that *d-q* currents and voltages are on the stator. Let the three-phase currents (voltages) be given by

$$i_A = I_m \cos\ (\omega t + \alpha)$$
$$i_B = I_m \cos\left(\omega t + \alpha - \frac{2\pi}{3}\right)$$
$$\text{and} \quad i_C = I_m \cos\left(\omega t + \alpha - \frac{4\pi}{3}\right)$$

where $I_m = \sqrt{2}$ (rms value of current, I), and α = phase angle of i_A with time origin.

The value of i_{ds}, from Eq. (6), is given by

$$i_{ds} = \sqrt{\frac{2}{3}}\,.\,I_m\left[\cos 0.\cos(\omega t + \alpha) + \cos\frac{2\pi}{3}\cos\left(\omega t + \alpha - \frac{2\pi}{3}\right) + \cos\frac{4\pi}{3}\cos\left(\omega t + \alpha - \frac{4\pi}{3}\right)\right]$$

$$= \sqrt{\frac{2}{3}}\ I_m \cos(\omega t + \alpha) \quad \text{..... (7a)}$$

Also,
$$v_{ds} = \sqrt{\frac{2}{3}}\ V_m \cos(\omega t + \alpha) \quad \text{..... (7b)}$$

Similarly,

$$i_{qs} = \sqrt{\frac{2}{3}}\,.\,I_m \sin(\omega t + \alpha) \quad \text{..... (8a)}$$

$$v_{qs} = \sqrt{\frac{2}{3}}\,.\,V_m \sin(\omega t + \alpha) \quad \text{..... (8b)}$$

Eqs. (7) and (8) reveal that *d-q* axes components of currents (voltages) are also functions of time and are displaced from each other by a time phase angle of 90^0. This shows that i_{ds}, i_{qs} constitute a two-phase system of currents having a frequency equal to the three-phase supply frequency. The mmfs produced by two-phase currents i_{ds}, i_{qs} and three-phase currents i_A, i_B, i_C are identical in all respects.

For rotor, three-phase to d-q axes transformation is given by eq. (9).

$$i_d = \sqrt{\frac{2}{3}}\left[i_a \cos\theta + i_b \cos\left(\theta - \frac{2\pi}{3}\right) + i_c \cos\left(\theta - \frac{4\pi}{3}\right)\right] \quad \text{.....(9)}$$

The d-q axes rotor currents i_{dr}, i_{qr} must be at line frequency because coils DR, QR representing the rotor are stationary with respect to the stator.

The primitive machine diagram for a polyphase induction machine requires four coils as shown in fig. 4 (a). The two coils DS and QS on the stator represents polyphase stator windings of both three-phase and two-phase induction machines, while coils DR and QR on the rotor represent the wound-rotor as well as the squirrel-cage rotor. For cage rotor machines, the two coils DR and QR may be thought of as equivalent to all the squirrel-cage bars [8]-[9]. The voltage equation of a wound rotor three-phase induction motor is thus the same as given by equation (5).

The same equation is valid for a three-phase squirrel cage induction motor after some modifications caused by the fact that the rotor is now short circuited. This causes v_{dr} and v_{qr} to become zero along with reversal of direction of currents i_{dr} and i_{qr} as shown in equation (9).

$$\begin{bmatrix} v_{ds} \\ v_{qs} \\ 0 \\ 0 \end{bmatrix} = \begin{array}{c} ds \\ qs \\ dr \\ qr \end{array} \overset{\begin{array}{cccc} ds & qs & dr & qr \end{array}}{\begin{bmatrix} r_{ds} + L_{ds}p & & -M_d p & \\ & r_{qs} + L_{qs}p & & -M_q p \\ M_d p & -M_q\omega_r & -(r_{dr} + L_{dr}p) & \omega_r L_{qr} \\ \omega_r M_d & M_q p & -\omega_r L_{dr} & -(r_{qr} + L_{qr}p) \end{bmatrix}} \begin{bmatrix} i_{ds} \\ i_{qs} \\ i_{dr} \\ i_{qr} \end{bmatrix} \quad \text{.....(9)}$$

V. CONCLUSION

The generalized theory of electrical machines, emphasizing the basic similarities of all the machines, now forms the basis of mathematical-machine modelling. The generalized approach for poly-phase synchronous and induction machine is simpler than the coupled circuit approach. The generalized theory eminently succeed not only in lending rigor and conciseness to the treatment of the subject but provide powerful methods of analyses to the researcher and have largely eliminated the necessity of studying several theories in the historical sequence of the development of the subjects.

The generalized theory makes use of simple concepts of electric circuits and provides a link between a study of one machine and another and demonstrates that some physical principles will show up again and again in different guises in the apparently dissimilar rotating machines [10]-[11]. The more rewarding aspects of such a theory are a confidence that the researcher gains in handling the subject and reduction of mental strain to remember several independents theories.

REFERENCES

[1]. Adkins, B., The General Theory of Electrical Machines, Chapman and Hall
[2]. Bimbhra, P.S., Generalized Theory of Electrical Machines, Khanna Publishers, 1987.
[3]. Gibbs, W. J., Tensors in Electrical Machine Theory, Chapman and Hall.
[4]. Gibbs, W. J., Electric Machine Analysis Using Matrices, Pitman.
[5]. Govinda Rao, C. V. and Mathur, R. M., Generalised Circuit Theory of Electrical Machines, Part I, Journal of Institution of Engineers (India), August 1963.
[6]. Govinda Rao, C. V. and Mathur, R. M., Generalised Circuit Theory of Electrical Machines, Part II, Journal of Institution of Engineers (India), August 1963.
[7]. Govinda Rao, C. V., Generalised Circuit Theory of Electrical Machines, Part III, Journal of Institution of Engineers (India), Vol. 44, February 1964.
[8]. Govinda Rao, C. V., Effects of Short Circuited Coils on the Performance of Shrage Motor, Journal of Institution of Engineers (India), February 1966.
[9]. Lewis, W. A., A Basic Analysis of Synchronous Machines, Transactions of the American Institute of Electrical Engineers, 1950.
[10]. Seely, S., Electromechanical Energy Conversion, McGraw Hill Book Co., 1962
[11]. White, D. C. and Woodson, H. H.: Electromechanical Energy Conversion, John Wiley.