Generating Equivalent Numerical Expressions

MODULE 9

CALIFORNIA

LESSON 9.1
Exponents
CA CC 6.EE.1

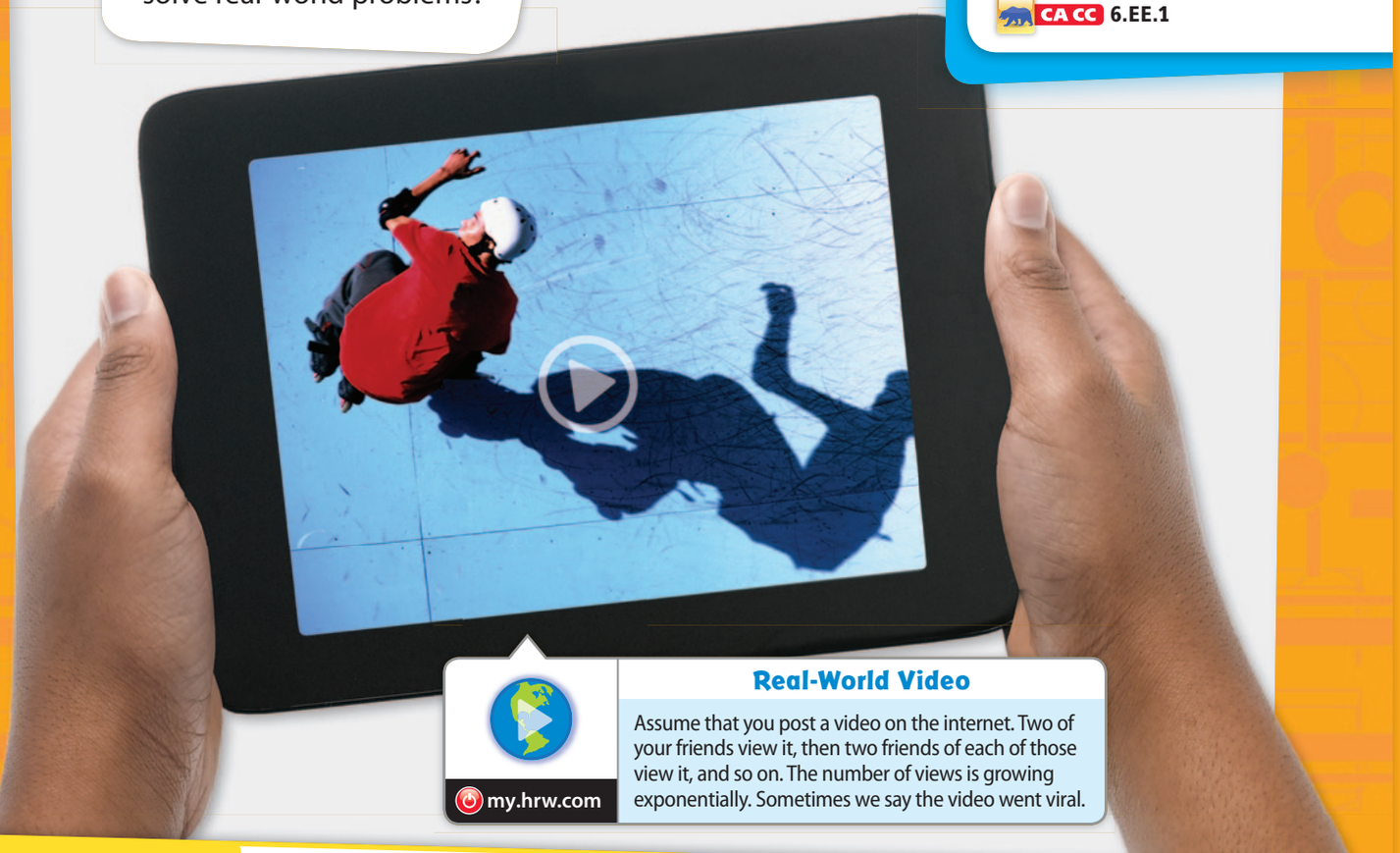
LESSON 9.2
Prime Factorization
CA CC 6.EE.1

LESSON 9.3
Order of Operations
CA CC 6.EE.1

ESSENTIAL QUESTION

How can you generate equivalent numerical expressions and use them to solve real-world problems?

You can represent real-world problems with numerical expressions and simplify the expressions by applying rules relating to exponents, prime factorization, and order of operations.



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Real-World Video

Assume that you post a video on the internet. Two of your friends view it, then two friends of each of those view it, and so on. The number of views is growing exponentially. Sometimes we say the video went viral.

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Are You Ready?

Assess Readiness

Use the assessment on this page to determine if students need intensive or strategic intervention for the module's prerequisite skills.

RtI Response to Intervention

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Intervention	Enrichment

Access Are You Ready? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

Online and Print Resources

Skills Intervention worksheets
- Skill 34 Whole Number Operations
- Skill 35 Use Repeated Multiplication
- Skill 38 Division Facts

Differentiated Instruction
- Challenge worksheets PRE-AP

Extend the Math PRE-AP Lesson Activities in TE

Real-World Video Viewing Guide

After students have watched the video, discuss the following:

- What does the exponent 3 mean in the expression 2^3? multiply 2 three times
- The expression 2^9 shows how fast Video A spread, and 9^2 shows how fast Video B spread. Which video spread more rapidly? Video A

Are YOU Ready?

Complete these exercises to review skills you will need for this module.

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Whole Number Operations

EXAMPLE 270×83

$$\begin{array}{r} 270 \\ \times\ 83 \\ \hline 810 \\ +21{,}600 \\ \hline 22{,}410 \end{array} \quad \begin{array}{l} \leftarrow 3 \times 270 \\ \leftarrow 80 \times 270 \\ \leftarrow (3 \times 270) + (80 \times 270) \end{array}$$

Find the product.

1. 992×16 — 15,872
2. 578×27 — 15,606
3. 839×65 — 54,535
4. 367×23 — 8,441

Use Repeated Multiplication

EXAMPLE $5 \times 5 \times 5 \times 5$

$5 \times 5 \times 5 \times 5$ — Multiply the first two factors.
$25 \times 5 \times 5$ — Multiply the result by the next factor.
125×5 — Multiply that result by the next factor.
625 — Continue until there are no more factors to multiply.

Find the product.

5. $7 \times 7 \times 7$ — 343
6. $3 \times 3 \times 3 \times 3$ — 81
7. $6 \times 6 \times 6 \times 6 \times 6$ — 7,776
8. $2 \times 2 \times 2 \times 2 \times 2 \times 2$ — 64

Division Facts

EXAMPLE $54 \div 9 = \blacksquare$

Think: 9 times what number equals 54?
$9 \times 6 = 54$

$54 \div 9 = 6$ — So, $54 \div 9 = 6$.

Divide.

9. $20 \div 4$ — 5
10. $21 \div 7$ — 3
11. $42 \div 7$ — 6
12. $56 \div 8$ — 7

PROFESSIONAL DEVELOPMENT VIDEO

Author Juli Dixon models successful teaching practices as she explores equivalent numerical expressions in an actual sixth-grade classroom.

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Personal Math Trainer: Online Assessment and Intervention
Assign automatically graded homework, quizzes, tests, and intervention activities. Prepare your students with updated practice tests aligned with Common Core.

Reading Start-Up EL

Have students complete the activities on this page by working alone or with others.

Strategies for English Learners

Each lesson in the TE contains specific strategies to help English Learners of all levels succeed.

Emerging: Students at this level typically progress very quickly, learning to use English for immediate needs as well as beginning to understand and use academic vocabulary and other features of academic language.

Expanding: Students at this level are challenged to increase their English skills in more contexts, and learn a greater variety of vocabulary and linguistic structures, applying their growing language skills in more sophisticated ways appropriate to their age and grade level.

Bridging: Students at this level continue to learn and apply a range of high-level English language skills in a wide variety of contexts, including comprehension and production of highly technical texts.

Active Reading

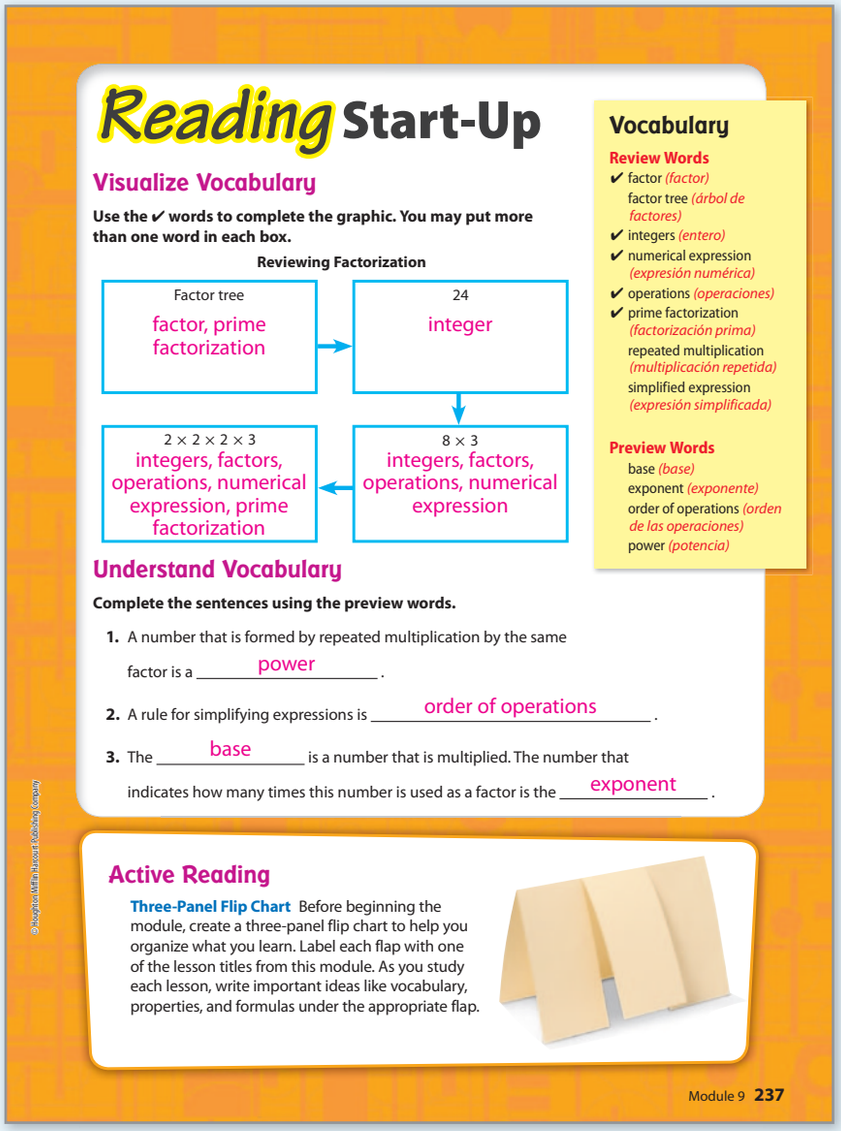
Integrating Language Arts

Students can use these reading and note-taking strategies to help them organize and understand new concepts and vocabulary.

Additional Resources

Differentiated Instruction

- Reading Strategies EL



Reading Start-Up

Visualize Vocabulary

Use the ✔ words to complete the graphic. You may put more than one word in each box.

Reviewing Factorization

Factor tree — factor, prime factorization → 24 — integer → 8 × 3 — integers, factors, operations, numerical expression → 2 × 2 × 2 × 3 — integers, factors, operations, numerical expression, prime factorization

Vocabulary

Review Words
- ✔ factor *(factor)*
- factor tree *(árbol de factores)*
- ✔ integers *(entero)*
- ✔ numerical expression *(expresión numérica)*
- ✔ operations *(operaciones)*
- ✔ prime factorization *(factorización prima)*
- repeated multiplication *(multiplicación repetida)*
- simplified expression *(expresión simplificada)*

Preview Words
- base *(base)*
- exponent *(exponente)*
- order of operations *(orden de las operaciones)*
- power *(potencia)*

Understand Vocabulary

Complete the sentences using the preview words.

1. A number that is formed by repeated multiplication by the same factor is a power.
2. A rule for simplifying expressions is order of operations.
3. The base is a number that is multiplied. The number that indicates how many times this number is used as a factor is the exponent.

Active Reading

Three-Panel Flip Chart Before beginning the module, create a three-panel flip chart to help you organize what you learn. Label each flap with one of the lesson titles from this module. As you study each lesson, write important ideas like vocabulary, properties, and formulas under the appropriate flap.

Module 9 237

CA CC Focus | Coherence | Rigor

Tracking Your Learning Progression

Before	In this module	After
Students understand: • operations with whole numbers, decimals, and fractions • prime numbers • order of operations	Students will learn to: • generate equivalent numerical expressions using exponents • generate equivalent numerical expressions using prime factorization • simplify numerical expressions using the order of operations	Students will connect: • order of operations and numerical expressions • numerical and algebraic expressions

GETTING READY FOR

Generating Equivalent Numerical Expressions

Use the examples on this page to help students know exactly what they are expected to learn in this module.

CA Common Core Standards

Content Areas

CA CC Expressions and Equations—6.EE

Cluster Apply and extend previous understandings of arithmetic to algebraic expressions.

Go online to see a complete unpacking of the CA Common Core Standards.

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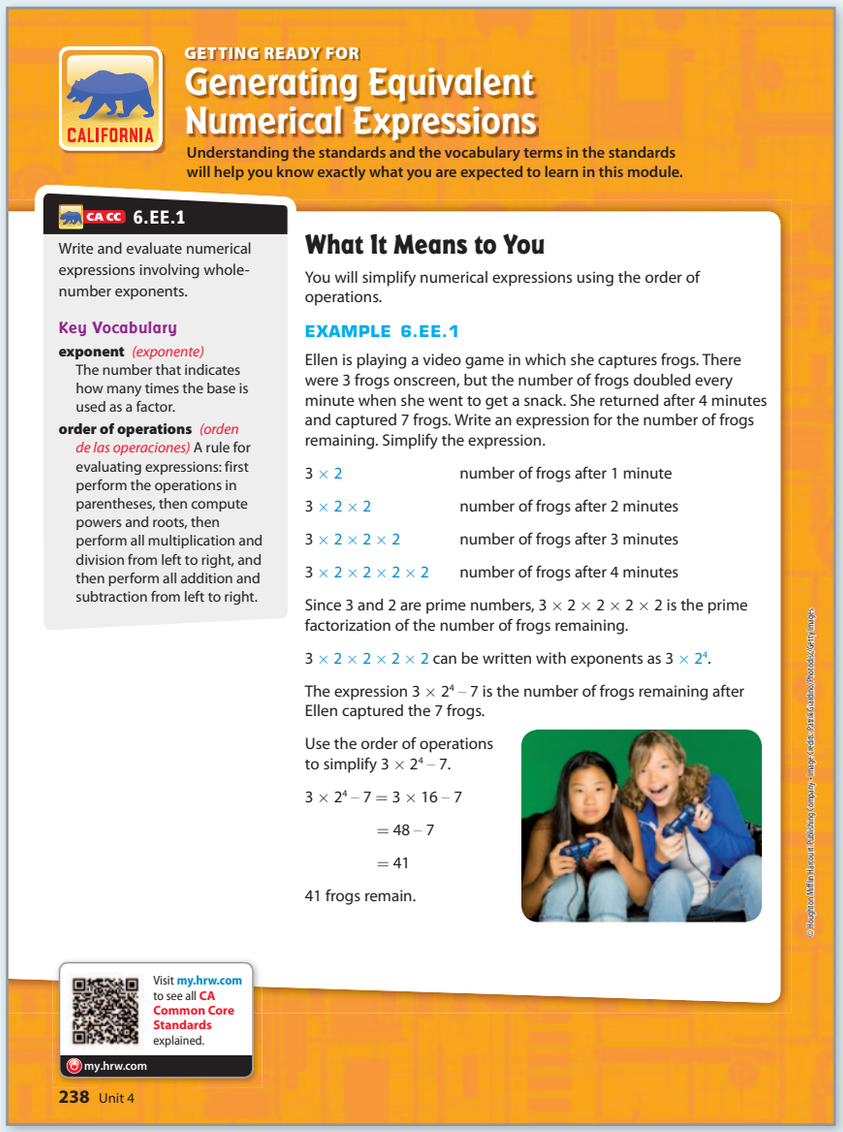

GETTING READY FOR

Generating Equivalent Numerical Expressions

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.

CA CC 6.EE.1

Write and evaluate numerical expressions involving whole-number exponents.

Key Vocabulary

exponent *(exponente)* The number that indicates how many times the base is used as a factor.

order of operations *(orden de las operaciones)* A rule for evaluating expressions: first perform the operations in parentheses, then compute powers and roots, then perform all multiplication and division from left to right, and then perform all addition and subtraction from left to right.

What It Means to You

You will simplify numerical expressions using the order of operations.

EXAMPLE 6.EE.1

Ellen is playing a video game in which she captures frogs. There were 3 frogs onscreen, but the number of frogs doubled every minute when she went to get a snack. She returned after 4 minutes and captured 7 frogs. Write an expression for the number of frogs remaining. Simplify the expression.

3×2	number of frogs after 1 minute
$3 \times 2 \times 2$	number of frogs after 2 minutes
$3 \times 2 \times 2 \times 2$	number of frogs after 3 minutes
$3 \times 2 \times 2 \times 2 \times 2$	number of frogs after 4 minutes

Since 3 and 2 are prime numbers, $3 \times 2 \times 2 \times 2 \times 2$ is the prime factorization of the number of frogs remaining.

$3 \times 2 \times 2 \times 2 \times 2$ can be written with exponents as 3×2^4.

The expression $3 \times 2^4 - 7$ is the number of frogs remaining after Ellen captured the 7 frogs.

Use the order of operations to simplify $3 \times 2^4 - 7$.

$$3 \times 2^4 - 7 = 3 \times 16 - 7$$
$$= 48 - 7$$
$$= 41$$

41 frogs remain.

Visit my.hrw.com to see all CA Common Core Standards explained.

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California Common Core Standards	Lesson 9.1	Lesson 9.2	Lesson 9.3
CA CC **6.EE.1** Write and evaluate numerical expressions involving whole-number exponents.	●	●	●

Lesson Support

Content Objective Students will learn to use exponents to represent numbers.

Language Objective Students will explain the meanings of GCF and LCM and use them, when adding, subtracting, and multiplying fractions.

California Common Core Standards

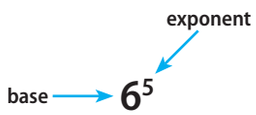
CA CC **6.EE.1** Write and evaluate numerical expressions involving whole-number exponents.

CA CC **MP.2** Reason abstractly and quantitatively.

CA CC Focus | Coherence | Rigor

Building Background

Eliciting Prior Knowledge Write $6 \times 6 \times 6 \times 6 \times 6$ on the board. Have the students read the product aloud. Tell students that an easier way to write this repeated multiplication is 6^5. Explain that 6^5 is called a power. The *base* is 6 and the *exponent* is 5, which tells how many times 6 is multiplied by itself.



Learning Progressions

In this lesson, students extend their prior knowledge of repeated multiplication and powers of 10 to using exponents and evaluating powers. Some key understandings for students include the following:

- A power is a number formed by repeated multiplication by the same factor.
- An exponent and a base are used to write a power.
- An exponent indicates how many times to use a given base as a factor in the value of a power.
- The value of any nonzero number raised to the power of 0 is 1.

Students start to incorporate whole-number exponents into numerical expressions. This prepares them to solve word problems algebraically and to use more complex expressions in later lessons.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in this cluster: ***Apply and extend previous understandings of arithmetic to algebraic expressions.***

Tell students, "Use an exponent to write these expressions and then find the value of each power: $4 \times 4 \times 4 \times 4 \times 4$ and $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}$. If the base of each power were negative, would the value of each power change? How?" Answers: 4^5, $\left(\frac{1}{3}\right)^4$; 1,024, $\frac{1}{81}$; the value 4^5 would be negative; the value $\frac{1}{81}$ would not change.

Language Support

California ELD Standards

CA ELD Emerging 2.I.8. Analyzing language choices – Explain how phrasing or different common words with similar meaning produce different effects on the audience.

CA ELD Expanding 2.I.8. Analyzing language choices – Explain how phrasing, different words with similar meaning, or figurative language produce shades of meaning and different effects on the audience.

CA ELD Bridging 2.I.8. Analyzing language choices – Explain how phrasing, different words with similar meaning, or figurative language produce shades of meaning, nuances, and different effects on the audience.

Linguistic Support

Academic/Content Vocabulary

exponents – English learners need specific instruction on how to speak about exponents, especially since there are so many ways to say them. You can create sentence frames to help students discuss them. Pair students and have them practice speaking in a variety of ways about exponents in complete sentences.

___ to the ___ power has a value of ___.

___ to the ___ equals ___.

___ raised to the ___ power is ___.

___ cubed equals ___.

___ squared has a value of ___.

Multiple Meaning Words

power/base – *Power* and *base* both have several definitions. Explain to students that the word *power* in this context does not refer to physical strength, energy/electricity, or influence/control. *Base* in this lesson does not refer to the bottom of an object or the main ingredient in a mixture. Point out the appropriate definitions in the textbook, restate them in terms that are familiar to the students, and give examples and illustrations. Have students rephrase them, illustrate them, and record them in their notebooks or math journals.

Leveled Strategies for English Learners

Emerging Have English learners at this level communicate their understanding of exponents with a visual such as an array or a multiplication equation, such as 2 to the power of 3 is $2 \times 2 \times 2 = 8$.

Expanding Students at this level may benefit from expressing their knowledge about exponents through sentence frames. They may need to use a visual as well.

Two to the third power means _____.

Bridging Use more advanced sentence frames and more specific vocabulary for this level.

_____ has a base of _____ and an exponent of _____.

Therefore, it represents the multiplication expression _____ and the product is _____.

LESSON

9.1 Exponents

CA Common Core Standards

The student is expected to:

CA CC Expressions and Equations—6.EE.1

Write and evaluate numerical expressions involving whole-number exponents.

Mathematical Practices

CA CC MP.2 Reasoning

Engage

ESSENTIAL QUESTION

How do you use exponents to represent numbers? Sample answer: You can use exponents to represent repeated multiplication. For example, in $5 \times 5 \times 5 \times 5$, the number 5 is multiplied 4 times, so you can represent it as 5^4.

Motivate the Lesson

Ask: Have you ever heard the terms *squared* or *cubed*? Both of those expressions are used to describe expressions in which a factor is multiplied by itself. Begin the Explore Activity to analyze an experiment that leads to repeated multiplication of the same factor.

Explore

EXPLORE ACTIVITY

Engage with the Whiteboard

Have students fill in the table on the whiteboard. Extend the table to include 8 hours, and have students fill in the table for 5, 6, 7, and 8 hours. Ask students if they could predict the total number of bacteria for hour 12 and hour 20. Discuss a rule that students could use to make those kinds of predictions.

Explain

EXAMPLE 1

Connect Vocabulary EL

Students may know other definitions of the words *raised* and *base*. Point out that in math, **raised** means "to multiply by itself," and **base** can mean "the foundation," or that on which something is built. In 2^4, you build on the base 2 by multiplying it by itself 4 times.

Questioning Strategies Mathematical Practices

- In B, why is the base $\frac{4}{5}$ in parentheses in the power $\left(\frac{4}{5}\right)^4$? The base is in parentheses to show that the entire fraction is used for repeated multiplication, not just the numerator.
- Can a power have a base and an exponent that are the same number? Justify your answer. Yes; for example, $6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^6$.

YOUR TURN

Avoid Common Errors

Students may want to find the product for each expression. Review the direction line. They are not asked to simplify the expression, but to write it in exponential form.

Talk About It

Check for Understanding

Ask: What is the difference between the two numbers in a power? The first number is the base, which is the number that is multiplied. The second number is the exponent, which tells how many times the base is multiplied by itself.

ADDITIONAL EXAMPLE 1

Use an exponent to write each expression.

A $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$ 7^6

B $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$ $\left(\frac{2}{3}\right)^5$

Interactive Whiteboard
Interactive example available online

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LESSON
9.1 Exponents

CA CC 6.EE.1
Write and evaluate numerical expressions involving whole-number exponents.

ESSENTIAL QUESTION How do you use exponents to represent numbers?

EXPLORE ACTIVITY Real World CA CC 6.EE.1

Identifying Repeated Multiplication

Bacteria reproduce by dividing, a process that can be observed in a microscope. Some scientists observe the hourly growth of bacteria and record their observations in a table.

Time (h)	Total bacteria
0	1
1	2
2	$2 \times 2 =$ 4
3	$2 \times 2 \times 2 =$ 8
4	$2 \times 2 \times 2 \times 2 =$ 16

After 2 hours, there are $2 \times 2 =$? bacteria.

A Complete the table. What pattern(s) do you see in the Total bacteria column?

Sample answer: Each number is 2 times the previous number.

B Complete each statement.

At 2 hours, the total is equal to the product of two 2s.

At 3 hours, the total is equal to the product of three 2s.

At 4 hours, the total is equal to the product of four 2s.

Reflect

1. **Communicate Mathematical Ideas** How is the time, in hours, related to the number of times 2 is used as a factor? Show how to find the number of bacteria at 10 hours.

The number of hours is the number of times the factor 2 is repeated. Number at 10 hours $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 1{,}024$ bacteria.

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Using Exponents

A number that is formed by repeated multiplication of the same factor is called a **power**. You can use an *exponent* and a *base* to write a power. For example, 7^3 means the product of three 7s:

$7^3 = 7 \times 7 \times 7$

The **base** is the number that is multiplied.

The **exponent** tells how many times the base appears in the expression.

Power	How to read the power
6^2	6 squared, 6 to the power of 2, 6 raised to the 2nd power
7^3	7 cubed, 7 to the power of 3, 7 raised to the 3rd power
9^4	9 to the power of 4, 9 raised to 4th power

EXAMPLE 1 CA CC 6.EE.1

Use an exponent to write each expression.

A $3 \times 3 \times 3 \times 3 \times 3 \times 3$

Find the base, or the number being multiplied. The base is 3.

Find the exponent by counting the number of 3s being multiplied. The exponent is 5.

$\underbrace{3 \times 3 \times 3 \times 3 \times 3}_{\text{5 factors of 3}} \times 3 = 3^5$

Math Talk Mathematical Practices
In the Explore Activity, how can you use an exponent to show the total bacteria at 10 hours?

Use 10 as an exponent to show that 2 is used as a factor 10 times: 2^{10}

B $\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}$

Find the base, or the number being multiplied. The base is $\frac{4}{5}$.

Find the exponent by counting the number of times $\frac{4}{5}$ appears in the expression. The exponent is 4.

$\underbrace{\frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5}}_{\text{4 factors of } \frac{4}{5}} = \left(\frac{4}{5}\right)^4$

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YOUR TURN

Use exponents to write each expression.

2. $4 \times 4 \times 4$ 4^3
3. 6 6^1
4. $\frac{1}{8} \times \frac{1}{8}$ $\left(\frac{1}{8}\right)^2$
5. $5 \times 5 \times 5 \times 5 \times 5 \times 5$ 5^6

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.2

This lesson provides an opportunity to address the Mathematical Practices standard that calls for students to reason abstractly and quantitatively. Students first use tables to identify patterns involving repeated multiplication. They then use exponents to rewrite expressions that involve repeated multiplication. Finally, students find the value of expressions that are written with exponents. This process helps students understand multiple ways to represent and use exponents.

Math Background

The use of the terms *squared* and *cubed* is directly related to the measurements of area and volume. The area of a square with sides 5 units long is found by multiplying, 5×5, or 5^2, or 5 squared. The volume of a cube with sides 5 units long is found by multiplying, $5 \times 5 \times 5$, or 5^3, or 5 cubed.

ADDITIONAL EXAMPLE 2

Find the value of each power.

A 3^5 243

B 17^0 1

C $\left(\frac{1}{7}\right)^2$ $\frac{1}{49}$

D 0.6^2 0.36

Interactive Whiteboard
Interactive example available online

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EXAMPLE 2

Avoid Common Errors

Since powers relate to multiplication, students may confuse powers with simple multiplication. After students evaluate each power, have them compare it to a simple multiplication problem to show that the two are not equal. For example, when they find that $10^4 = 10{,}000$, ask them to find $10 \times 4 = 40$. Since the two expressions have different values, it should be clear that 10^4 and 10×4 are not equivalent.

Questioning Strategies Mathematical Practices

- If a and b are positive integers and $a > b$, which is greater: 2^a or 2^b? Explain. 2^a; if a is greater than b, then the base 2 is used as a factor more times.
- If c and d are positive numbers and $c > d$, which is greater: c^3 or d^3? Explain. c^3; the exponent with the greater base has to be greater if the exponent is the same. For example, $5^3 = 5 \times 5 \times 5 = 125$, while $4^3 = 4 \times 4 \times 4 = 64$.

Integrating Language Arts EL

Encourage English learners to use the active reading strategies and the glossary as they encounter new terms and concepts.

YOUR TURN

Engage with the Whiteboard

Have students rewrite each power as repeated multiplication. Seeing the power expressed as repeated multiplication can make it easier for students to find the correct value.

Elaborate

Talk About It

Summarize the Lesson

Ask: How can you use an exponent to represent repeated multiplication? How can you find the value of a power? You can use an exponent to show repeated multiplication of the same factor: $2 \times 2 \times 2 = 2^3$. To find the value of a power, rewrite the expression without using exponents by using the base as a factor the number of times given by the exponent. For example, $5^4 = 5 \times 5 \times 5 \times 5 = 625$.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercise 1, have students complete the table on the whiteboard. Discuss different methods students may have for finding the value of each power, such as using parentheses to group the repeated multiplication or multiplying the value of the previous power by 5.

Avoid Common Errors

Exercises 2–5 Remind students that their answers should be written using exponents.

Exercises 6–20 Some students may multiply a base by its exponent instead of using the base as a factor the number of times indicated by the exponent. Remind them that 4^3 means that 4 is used as a factor 3 times ($4 \times 4 \times 4$).

Exercise 15 If students write 8 or 0 as the answer, remind them that the Property of Zero as an Exponent states that the value of *any* nonzero number raised to the power of 0 is 1.

Finding the Value of a Power

To find the value of a power, remember that the exponent indicates how many times to use the base as a factor.

Property of Zero as an Exponent

The value of any nonzero number raised to the power of 0 is 1.

Example: $5^0 = 1$

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EXAMPLE 2

CA CC 6.EE.1

Find the value of each power.

A 10^4

Identify the base and the exponent.
The base is 10, and the exponent is 4.

Evaluate: $10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000$

B 0.4^3

Identify the base and the exponent.
The base is 0.4, and the exponent is 3.

Evaluate: $0.4^3 = 0.4 \times 0.4 \times 0.4 = 0.064$

C $\left(\frac{3}{5}\right)^0$

Identify the base and the exponent.
The base is $\frac{3}{5}$, and the exponent is 0.

Evaluate.

$\left(\frac{3}{5}\right)^0 = 1$ Any number raised to the power of 0 is 1.

D $\left(\frac{2}{3}\right)^2$

Identify the base and the exponent.
The base is $\frac{2}{3}$, and the exponent is 2.

Evaluate.

$\left(\frac{2}{3}\right)^2 = \left(\frac{2}{3}\right) \times \left(\frac{2}{3}\right) = \frac{4}{9}$

Math Talk
Mathematical Practices

Is the value of 2^3 the same as the value of 3^2? Explain.

$2^3 = 2 \cdot 2 \cdot 2 = 8$ and $3^2 = 3 \cdot 3 = 9$, so the values are not the same.

YOUR TURN

Find the value of each power.

6. 3^4 81 **7.** 1^9 1 **8.** $\left(\frac{2}{5}\right)^3$ $\frac{8}{125}$ **9.** 12^2 144

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Guided Practice

1. Complete the table. (Explore Activity 1)

Exponential form	Product	Simplified product
5^1	5	5
5^2	5×5	25
5^3	$5 \times 5 \times 5$	125
5^4	$5 \times 5 \times 5 \times 5$	625
5^5	$5 \times 5 \times 5 \times 5 \times 5$	3,125

Use an exponent to write each expression. (Example 1)

2. $6 \times 6 \times 6$ 6^3 — 3 factors of 6

3. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ 10^7

4. $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4}$ $\left(\frac{3}{4}\right)^5$

5. $\frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9}$ $\left(\frac{7}{9}\right)^8$

Find the value of each power. (Example 2)

6. 8^3 512 **7.** 7^4 2,401 **8.** 10^3 1,000

9. $\left(\frac{1}{4}\right)^2$ $\frac{1}{16}$ **10.** $\left(\frac{1}{3}\right)^3$ $\frac{1}{27}$ **11.** $\left(\frac{6}{7}\right)^2$ $\frac{36}{49}$

12. 0.8^2 0.64 **13.** 0.5^3 0.125 **14.** 1.1^2 1.21

15. 8^0 1 **16.** 12^1 12 **17.** $\left(\frac{1}{2}\right)^0$ 1

18. $(13)^2$ 169 **19.** $\left(\frac{2}{5}\right)^2$ $\frac{4}{25}$ **20.** 0.9^2 0.81

ESSENTIAL QUESTION CHECK-IN

21. How do you use an exponent to represent a number such as 16?

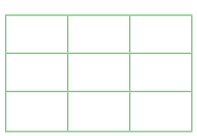
You use an exponent to write a number that can be written as a product of equal factors. $16 = 4 \times 4$ (or $2 \times 2 \times 2 \times 2$), so it can be written as 4^2 (or 2^4).

DIFFERENTIATE INSTRUCTION

Kinesthetic Experience

To help students remember the meaning of the base and the exponent in a power, have them use graph paper or square tiles to construct models of the squares of whole numbers 1–10. Label the models as shown below. A visual representation of a square number can help students remember that exponents represent repeated multiplication of the same factor.

$3^2 = 3 \times 3$



Critical Thinking

Have students explore multiplication of numbers written as powers. Partners can work together to find the values of pairs of expressions such as these:

$3^2 \times 3^3$ and $3^1 \times 3^4$

$2^2 \times 2^4$ and $2^3 \times 2^3$

$4^3 \times 4^3$ and $4^1 \times 4^5$

243 and 243; 64 and 64; 4,096 and 4,096; The values of both expressions in each pair are the same. The base is used the same number of times in each pair. The exponents in each pair have equal sums. You can add the exponents to multiply powers with the same base, for example, $2^2 \times 2^4 = 2^6$.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

9.1 LESSON QUIZ

CA CC **6.EE.1**

Use an exponent to write each expression.

1. $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7}$

2. $0.9 \times 0.9 \times 0.9 \times 0.9$

Find the value of each power.

3. 7^4

4. $\left(\frac{3}{4}\right)^3$

Lesson Quiz available online

my.hrw.com

Answers

1. $\left(\frac{3}{7}\right)^3$

2. 0.9^4

3. 2,401

4. $\frac{27}{64}$

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **6.EE.1**

Concepts & Skills	Practice
Explore Activity Identifying Repeated Multiplication	Exercises 1, 38–44
Example 1 Using Exponents	Exercises 2–5, 22–37
Example 2 Finding the Value of a Power	Exercises 6–20, 38–44

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
22–37	**2** Skills/Concepts	**MP.5** Using Tools
38–39	**2** Skills/Concepts	**MP.4** Modeling
40	**2** Skills/Concepts	**MP.2** Reasoning
41–42	**2** Skills/Concepts	**MP.4** Modeling
43	**2** Skills/Concepts	**MP.2** Reasoning
44	**3** Strategic Thinking H.O.T.	**MP.7** Using Structure
45	**3** Strategic Thinking H.O.T.	**MP.3** Logic
46–47	**3** Strategic Thinking H.O.T.	**MP.7** Using Structure
48	**3** Strategic Thinking H.O.T.	**MP.3** Logic

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

Name _____ Class _____ Date _____

9.1 Independent Practice

CA CC 6.EE.1

Personal Math Trainer — my.hrw.com — Online Practice and Help

Write the missing exponent.

22. $100 = 10^{\boxed{2}}$ **23.** $8 = 2^{\boxed{3}}$ **24.** $25 = 5^{\boxed{2}}$ **25.** $27 = 3^{\boxed{3}}$

26. $\frac{1}{169} = \left(\frac{1}{\boxed{13}}\right)^{\boxed{2}}$ **27.** $14 = 14^{\boxed{1}}$ **28.** $32 = 2^{\boxed{5}}$ **29.** $\frac{64}{81} = \left(\frac{8}{\boxed{9}}\right)^{\boxed{2}}$

Write the missing base.

30. $1{,}000 = \boxed{10}^{3}$ **31.** $256 = \boxed{4}^{4}$ **32.** $16 = \boxed{2}^{4}$ **33.** $9 = \boxed{3}^{2}$

34. $\frac{1}{9} = \left(\frac{1}{\boxed{3}}\right)^{2}$ **35.** $64 = \boxed{8}^{2}$ **36.** $\frac{9}{16} = \left(\frac{3}{\boxed{4}}\right)^{2}$ **37.** $729 = \boxed{9}^{3}$

For Exercises 38–42, write the answer with and without using an exponent.

38. Hadley's softball team has a phone tree in case a game is canceled. The coach calls 3 players. Then each of those players calls 3 players, and so on. How many players will be notified during the third round of calls?

3^3 players, or 27 players

39. Tim is reading a book. On Monday he reads 3 pages. On each day after that, he reads 3 times the number of pages that he read on the previous day. How many pages does he read on Thursday?

3^4 pages, or 81 pages

40. The square tile shown has a side length of 10.5 inches. What power can you write to represent the area of the tile? Find the area of the tile.

$10.5^2 = 10.5 \times 10.5$; 110.25 in^2

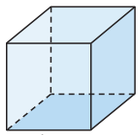
41. Antonia is saving for a video game. On the first day, she saves two dollars in her piggy bank. Each day after that, she doubles the number of dollars she saved on the previous day. How many dollars does she save on the sixth day?

2^6 dollars, or $64

42. A certain colony of bacteria triples in length every 10 minutes. Its length is now 1 millimeter. How long will it be in 40 minutes?

3^4 mm, or 81 mm

43. What power can you write to represent the volume of the cube shown? Write the power as an expression with a base and an exponent, and then find the volume of the cube.



$\frac{1}{3}$ in.

$\left(\frac{1}{3}\right)^3 = \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27}$ $in.^3$

44. Write a power represented with a positive base and a positive exponent whose value is less than the base.

Sample answer: $0.3^2 = 0.09$; $0.3 > 0.09$

H.O.T. FOCUS ON HIGHER ORDER THINKING

45. Communicate Mathematical Ideas What is the value of 1 raised to the power of any exponent? What is the value of 0 raised to the power of any nonzero exponent? Explain.

The value of 1 raised to any power is 1. 1 multiplied by itself any number of times is 1. The value of 0 raised to any power is 0. 0 multiplied by itself any number of times is still 0.

46. Look for a Pattern Find the values of the powers in the following pattern: 10^1, 10^2, 10^3, 10^4.... Describe the pattern, and use it to evaluate 10^6 without using multiplication.

Sample answer: 10; 100; 1,000; 10,000.... Each term in the pattern is a 1 followed by the same number of zeros as the exponent. $10^6 = 1{,}000{,}000$

47. Critical Thinking Some numbers can be written as powers of different bases. For example, $81 = 9^2$ and $81 = 3^4$. Write the number 64 using three different bases.

2^6, 4^3, and 8^2

48. Justify Reasoning Oman said that it is impossible to raise a number to the power of 2 and get a value less than the original number. Do you agree with Oman? Justify your reasoning.

Sample answer: Disagree; the product of a number between 0 and 1 and itself is less than the original number. For example, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, and $\frac{1}{4} < \frac{1}{2}$.

Work Area

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity Every whole number can be written as the sum of square numbers. For example:

Sum of 2 squares: $20 = 4^2 + 2^2$
Sum of 3 squares: $24 = 4^2 + 2^2 + 2^2$

Some whole numbers, such as 22, can be written as the sum of square numbers in more than one way.

Sum of 3 squares: $22 = 3^2 + 3^2 + 2^2$
Sum of 4 squares: $22 = 4^2 + 2^2 + 1^2 + 1^2$

- Write the numbers 8, 13, and 18 as the sum of 2 squares. $8 = 2^2 + 2^2$; $13 = 3^2 + 2^2$; $18 = 3^2 + 3^2$
- Can you write the number 36 as the sum of squares in more than one way?
  Yes; $36 = 3^2 + 3^2 + 3^2 + 3^2 = 9 + 9 + 9 + 9$; $36 = 4^2 + 4^2 + 2^2 = 16 + 16 + 4$
- Write a number on one side of an index card and on the reverse write the number as a sum of squares. Challenge a classmate to write the number. For example:

Write 62 as the sum of 3 squares.	$1^2 + 5^2 + 6^2$

Lesson Support

Content Objective Students will learn to write the prime factorization of a number.

Language Objective Students will create a step-by-step explanation of how to write the prime factorization of a number.

California Common Core Standards

CA CC **6.EE.1** Write and evaluate numerical expressions involving whole-number exponents.

CA CC **MP.2** Reason abstractly and quantitatively.

CA CC Focus | Coherence | Rigor

Building Background

Eliciting Prior Knowledge Ask students to write pairs of numbers that can be multiplied to get 12. Remind students that these whole numbers are called the factors of a product. Repeat the process with 48.

12	48
1×12	1×48
2×6	2×24
3×4	3×16
	4×12
	6×8

Learning Progressions

In this lesson, students connect their previous knowledge of multiplication to finding factors and the prime factorization of products. Important understandings for students include the following:

- Factors of a product are whole numbers that are multiplied to find that product.
- The prime factorization of a number is the number written as the product of its prime factors.
- Factor trees and ladder diagrams can be used find the prime factorization of a number.

Students use factors of products to solve real-world problems involving areas and employ factor trees and ladder diagrams to find prime factors. This also prepares them to work with equivalent expressions and equations in later lessons and in Grade 7.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in this cluster: ***Apply and extend previous understandings of arithmetic to algebraic expressions.***

Tell students, "Jeffrey wants to build a deck that has an area of 84 square feet. The length of the deck should be longer than the width. What are the possible whole number measurements for the length and width of the deck? Complete the table with possible measurements of the deck. If the deck had an area of 42 square feet, how would this affect the answer?"

Length	84	42	28	21	14	12
Width	1	2	3	4	6	7

Because 42 has only 8 factors, there would be fewer possible length and width measurements for the deck.

Language Support

California ELD Standards

CA ELD **Emerging 2.I.9. Presenting** – Plan and deliver brief oral presentations on a variety of topics and content areas.

CA ELD **Expanding 2.I.9. Presenting** – Plan and deliver longer oral presentations on a variety of topics and content areas, using details and evidence to support ideas.

CA ELD **Bridging 2.I.9. Presenting** – Plan and deliver longer oral presentations on a variety of topics and content areas, using reasoning and evidence to support ideas, as well as growing understanding of register.

Linguistic Support

Academic/Content Vocabulary

prime factorization – The term *prime factorization* is a complex term linguistically. Model how to restate and illustrate the definition of *prime* and *factor*. Then, explain and illustrate the terms *factor tree* and *factor ladder*. These terms are examples of **figurative language** in mathematics. Point out that these are not trees with leaves or ladders used to reach up high. They are diagrams used to help find the factors that create a larger number.

Tools and Resources

diagrams and labels – Diagrams with labels can help students visualize a mathematical concept. The labels are key to understanding and using accurate language to discuss the concept. Use the factor tree and factor ladder in this lesson and create a class chart for each one. Write labels that capture the key action for each step. Post those labels on the chart as you model each step.

Leveled Strategies for English Learners

Emerging Students at this level can describe the process of prime factorization using a diagram of a factor tree or ladder and index cards with labels for each step. They can sequence the cards to show that they know the right order for the steps.

Expanding English learners with expanding skills can demonstrate the process of using a factor tree or ladder by using the diagram and basic sentence frames as support.

Bridging At this stage, English learners are ready to use more complex sentence structures and vocabulary in their sentence frames to describe the process of using a factor tree or ladder.

Math Talk

Pair students with different language levels to allow for language modeling between peers. Model first, and then use the sentence frame:

_____ is a number that has exactly two factors because _____ is a _____ and _____ is a _____.

LESSON

9.2 Prime Factorization

CA Common Core Standards

The student is expected to:

CA CC Expressions and Equations—6.EE.1

Write and evaluate numerical expressions involving whole-number exponents.

Mathematical Practices

CA CC MP.2 Reasoning

Engage

ESSENTIAL QUESTION

How do you write the prime factorization of a number? Sample answer: Use a factor tree or a ladder diagram to find the prime factorization of the number, then write the prime factorization using exponents.

Explore

Motivate the Lesson

Draw a rectangle and explain that its area is 12 square feet. Ask students to come up with possible pairs of whole number lengths and widths. Point out that these numbers are factors of 12.

Explain

EXAMPLE 1

Focus on Reasoning Mathematical Practices

Point out to students that you can tell when you have found all the factors of a number when the factor pairs start to repeat.

Questioning Strategies Mathematical Practices

- For any number, which numbers are always factors? 1 and the number itself.
- Is it possible for a number to have all even factors? No; 1 is a factor for all numbers.

YOUR TURN

Avoid Common Errors

Remind students that when they list the factors of a number they should always begin with 1 and end with the number itself.

EXPLORE ACTIVITY 1

Connect Vocabulary EL

Remind students that a **prime number** is a number with exactly 2 factors, 1 and itself, and a **composite number** is a number that has more than 2 factors.

Engage with the Whiteboard

Have students make alternate factor trees for 240 on the whiteboard, next to the given factor tree. Have students start with the following pairs: 8 and 30; 24 and 10; and 12 and 20. Point out to students that while the order of the factors in a factor tree may differ, the prime factors of a number are always the same.

Questioning Strategies Mathematical Practices

- When you choose the first factor pair for the branches of the factor tree for 240, does one of the factors have to be a prime number? Explain. No. A factor tree can start with any factor pair.

ADDITIONAL EXAMPLE 1

Rayshawn is designing a mural. The mural must have an area of 42 square yards. What are the possible whole number lengths and widths for the mural? The possible lengths and widths are listed:

Length (yd)	42	21	14	7
Width (yd)	1	2	3	6

Interactive Whiteboard
Interactive example available online

my.hrw.com

Animated Math
Prime Factorization

Students use an interactive factor tree to find prime factors of composite numbers.

my.hrw.com

LESSON
9.2 Prime Factorization

CA CC 6.EE.1
Write and evaluate numerical expressions involving whole-number exponents

ESSENTIAL QUESTION

How do you write the prime factorization of a number?

Finding Factors of a Number

Whole numbers that are multiplied to find a product are called factors of that product. A number is divisible by its factors. For example, 4 and 2 are factors of 8 because $4 \times 2 = 8$, and 8 is divisible by 4 and by 2.

Math On the Spot
my.hrw.com

EXAMPLE 1

Real World

CA CC Prep for 6.EE.1

Ana wants to build a rectangular garden with an area of 24 square feet. What are the possible whole number lengths and widths of the garden?

STEP 1 Recall that area = length × width. For Ana's garden, 24 ft^2 = length × width.

STEP 2 List the factors of 24 in pairs. List each pair only once.

$24 = 1 \times 24$
$24 = 2 \times 12$
$24 = 3 \times 8$
$24 = 4 \times 6$

$4 \times 6 = 6 \times 4$, so you only list 4×6.

You can also use a diagram to show the factor pairs.

1 2 3 4 6 8 12 24

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

STEP 3 The possible lengths and widths are:

Length (ft)	24	12	8	6
Width (ft)	1	2	3	4

Math Talk
Mathematical Practices

Give an example of a whole number that has exactly two factors. What type of number has exactly two factors?

Sample answer: 13; its factors are 1 and 13; prime number

YOUR TURN

List all the factors of each number.

1. 21 1, 3, 7, 21
2. 37 1, 37
3. 42 1, 2, 3, 6, 7, 14, 21, 42
4. 30 1, 2, 3, 5, 6, 10, 15, 30

Personal Math Trainer
Online Practice and Help
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EXPLORE ACTIVITY 1 CA CC 6.EE.1

Finding the Prime Factorization of a Number

Animated Math
my.hrw.com

The prime factorization of a number is the number written as the product of its prime factors. For example, the prime factors of 12 are 3, 2, and 2.

There are several different ways to write the prime factorization of 12. Some examples are shown below. The symbol · means "times."

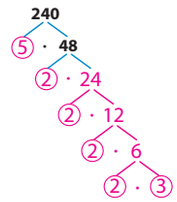
$2 \times 2 \times 3$ $2 \cdot 2 \cdot 3$ $2^2 \cdot 3$

Use exponents to show repeated factors.

Use a factor tree to find the prime factorization of 240.

A List the factor pairs of 240.

$1 \cdot 240, 2 \cdot 120, 3 \cdot 80, 4 \cdot 60, 5 \cdot 48, 6 \cdot 40,$
$8 \cdot 30, 10 \cdot 24, 12 \cdot 20, 15 \cdot 16$



B Choose any factor pair to begin the tree. If a number in this pair is prime, circle it. If a number in the pair can be written as a product of two factors, draw additional branches and write the factors.

C Continue adding branches until the factors at the ends of the branches are prime numbers.

D Write the prime factorization of 240.

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

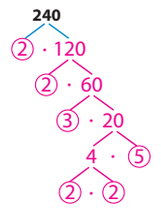
Then write the prime factorization using exponents.

$2^4 \cdot 3 \cdot 5$

Reflect

5. **What If?** What will the factor tree for 240 look like if you start the tree with a different factor pair? Check your prediction by creating another factor tree for 240 that starts with a different factor pair.

Sample answer: The intermediate steps on the factor tree will be different but the final prime factorization will be the same.



PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.2

This lesson provides an opportunity to address this Mathematical Practice standard. It calls for students to create and use representations to organize, record, and communicate mathematical ideas. Students use diagrams and factor trees to organize factor pairs of a number to find prime factorizations. They also use ladder diagrams to find prime factorizations, with the "ladder" as the means of recording and communicating the prime factorization.

Math Background

Mathematicians in ancient Greece observed that there were an infinite number of prime numbers and that there were irregular gaps between successive prime numbers.

In 1984, Samuel Yates coined the term *titanic prime*. He used this term to refer to any prime number with 1,000 digits or more. When he first defined a titanic prime, only 110 of them were known. Today more than 110,000 titanic primes have been identified.

EXPLORE ACTIVITY 2

Engage with the Whiteboard

Have students complete the ladder diagram starting with different combinations of prime factors. Point out to students that while the order in which they used the prime factors may differ, the prime factors of a number are always the same.

Focus on Modeling Mathematical Practices

Students who find mental math difficult may find ladder diagrams to be challenging. Model other methods for dividing by 2, such as using long division or a calculator, to show that the ladder diagram is a useful organizational tool. Emphasize that they can use a combination of division methods when using ladder diagrams.

Questioning Strategies Mathematical Practices

- How do you know when 2 is a factor of a number? How do you know when 2 is *not* a factor of a number? Even numbers have 2 as a factor. Odd numbers do not.
- Why do you have to divide by prime numbers when using the ladder diagram? The divisors on the left show the prime factorization, so all of them must be prime numbers.

Focus on Communication Mathematical Practices

Discuss with students ways to check that $2 \cdot 3 \cdot 3 \cdot 3$ is the prime factorization of 54. Students should understand that they can check their work by making sure that every number in the prime factorization is prime and then multiplying the expression $2 \cdot 3 \cdot 3 \cdot 3$ to verify that the product is 54.

Elaborate

Talk About It

Summarize the Lesson

Ask: What is the prime factorization of a number, and how can you find the prime factorization of a number? The prime factorization of a number is an expression that shows the number as the product of its prime factors. You can use a factor tree or a ladder diagram to find the prime factorization of a number.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercises 1–2, have students draw a diagram to list the factors of each number on the whiteboard.

For Exercise 5, have students make several different factor trees on the whiteboard.

Avoid Common Errors

Exercise 3 Remind students that you can tell when you have found all the factors of a number when the factor pairs start to repeat. Also remind them that writing factor pairs in order makes it easier to check that all the factor pairs are listed.

Exercises 4–7 Remind students that they can check their work by multiplying the factors in each prime factorization to make sure the product is the original number.

EXPLORE ACTIVITY 2 CA CC 6.EE.1

Using a Ladder Diagram

A ladder diagram is another way to find the prime factorization of a number.

Use a ladder diagram to find the prime factorization of 132.

A Write 132 in the top "step" of the ladder. Choose a prime factor of 132 to write next to the step with 132. Choose 2. Divide 132 by 2 and write the quotient 66 in the next step of the ladder.

B Now choose a prime factor of 66. Write the prime factor next to the step with 66. Divide 66 by that prime factor and write the quotient in the next step of the ladder.

C Keep choosing prime factors, dividing, and adding to the ladder until you get a quotient of 1.

D What are the prime factors of 132? How can you tell from the ladder diagram?

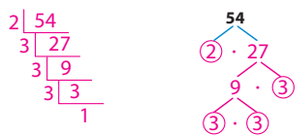
The prime factors are 2, 2, 3, and 11. They are written to the left of the steps of the ladder.

E Write the prime factorization of 132 using exponents.

$2^2 \cdot 3 \cdot 11$

Reflect

6. Complete a factor tree and a ladder diagram to find the prime factorization of 54.



prime factorization: $2 \cdot 3^3$

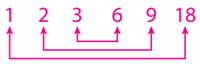
7. **Communicate Mathematical Ideas** If one person uses a ladder diagram and another uses a factor tree to write a prime factorization, will they get the same result? Explain.

Yes; there is only one unique prime factorization for every integer greater than 1.

Guided Practice

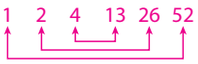
Use a diagram to list the factor pairs of each number. (Example 1)

1. 18



1, 2, 3, 6, 9, 18

2. 52



1, 2, 4, 13, 26, 52

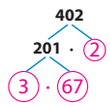
3. Karl needs to build a stage that has an area of 72 square feet. The length of the stage should be longer than the width. What are the possible whole number measurements for the length and width of the stage? (Example 1)

Complete the table with possible measurements of the stage.

Length	72	36	24	18	12	9
Width	1	2	3	4	6	8

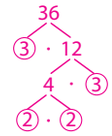
Use a factor tree to find the prime factorization of each number. (Explore Activity 1)

4. 402



$2 \cdot 3 \cdot 67$

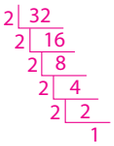
5. 36



$2^2 \cdot 3^2$

Use a ladder diagram to find the prime factorization of each number. (Explore Activity 2)

6. 32



2^5

7. 27

$3 \cdot 3 \cdot 3$ or 3^3

ESSENTIAL QUESTION CHECK-IN

8. Tell how you know when you have found the prime factorization of a number.

Sample answer: when all the factors are prime and their product is the original number

DIFFERENTIATE INSTRUCTION

Kinesthetic Experience

Students can use a method called the sieve of Eratosthenes to help identify prime numbers. Start with a 10 × 10 grid showing the numbers 1 to 100. Cross out 1, because 1 is not a prime number. Circle 2 because it is prime. Then cross out all multiples of 2, because multiples of 2 are not prime. Circle the next prime number, 3, and cross out all multiples of 3. Repeat the process until all numbers are circled or crossed out. Students can refer to this chart when deciding whether a number is prime or composite.

Critical Thinking

Discuss with students how the prime factorization of a number can be used to find all the factors of a number, by using the Associative and Commutative properties. For example, the prime factorization of 30 is $2 \cdot 3 \cdot 5$, which can be expressed as $2 \cdot (3 \cdot 5)$, $(2 \cdot 3) \cdot 5$, or $(2 \cdot 5) \cdot 3$.

When you multiply the numbers inside the parentheses, the expressions simplify to $2 \cdot (15)$, $(6) \cdot 5$, and $(10) \cdot 3$.

These numbers along with the factor pair $1 \cdot 30$, give all the factors for 30: 1, 2, 3, 5, 6, 10, 15, 30.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

9.2 LESSON QUIZ

CA CC **6.EE.1**

1. Find all the factors of 54.
2. Find the prime factorization of 54, and then write it using exponents.
3. Find all the factors of 60.
4. Find the prime factorization of 60, and then write it using exponents.
5. Chanasia has 30 beads. She wants to put them in boxes, so that each box will contain the same whole number of beads. Use factors to list all the different ways she can put the beads into boxes.

Lesson Quiz available online

my.hrw.com

Answers

1. 1, 2, 3, 6, 9, 18, 27, 54
2. $2 \cdot 3 \cdot 3 \cdot 3 = 2 \cdot 3^3$
3. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
4. $2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$
5. 1 box with 30 beads
   2 boxes with 15 beads each
   3 boxes with 10 beads each
   5 boxes with 6 beads each
   6 boxes with 5 beads each
   10 boxes with 3 beads each
   15 boxes with 2 beads each
   30 boxes with 1 bead each

Evaluate

CA CC Focus | Coherence | Rigor

GUIDED AND INDEPENDENT PRACTICE

CA CC **6.EE.1**

Concepts & Skills	Practice
Example 1 Finding Factors of a Number	Exercises 1–3, 9–10
Explore Activity 1 Finding the Prime Factorization of a Number	Exercises 4–5, 12–15, 17–19
Explore Activity 2 Using a Ladder Diagram	Exercises 6–7, 16

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
9	**2** Skills/Concepts	**MP.4** Modeling
10	**2** Skills/Concepts	**MP.4** Modeling
11	**3** Strategic Thinking H.O.T.	**MP.3** Logic
12–15	**2** Skills/Concepts	**MP.5** Using Tools
16	**3** Strategic Thinking H.O.T.	**MP.6** Precision
17	**3** Strategic Thinking H.O.T.	**MP.5** Using Tools
18	**3** Strategic Thinking H.O.T.	**MP.6** Precision
19	**3** Strategic Thinking H.O.T.	**MP.7** Using Structure
20	**3** Strategic Thinking H.O.T.	**MP.3** Logic
21–22	**3** Strategic Thinking H.O.T.	**MP.7** Using Structure

Additional Resources

Differentiated Instruction includes:

- Leveled Practice Worksheets

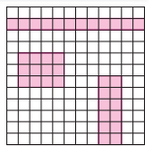
Name _____ Class _____ Date _____

9.2 Independent Practice

CA CC 6.EE.1

Personal Math Trainer — my.hrw.com — Online Practice and Help

9. **Multiple Representations** Use the grid to draw three different rectangles so that each has an area of 12 square units and they all have different widths. What are the dimensions of the rectangles?



1 × 12; 2 × 6; 3 × 4

10. Brandon has 32 stamps. He wants to display the stamps in rows, with the same number of stamps in each row. How many different ways can he display the stamps? Explain.

6 different ways; 1 row of 32 stamps; 2 rows of 16; 4 rows of 8; 32 rows of 1; 16 rows of 2; 8 rows of 4

11. **Communicate Mathematical Ideas** How is finding the factors of a number different from finding the prime factorization of a number?

When you find the factors of a number, you find all factors, some of which are prime; when you find the prime factorization, you find only the prime factors.

Find the prime factorization of each number.

12. 891 $3^4 \cdot 11$

13. 504 $2^3 \cdot 3^2 \cdot 7$

14. 23 23

15. 230 $2 \cdot 5 \cdot 23$

16. The number 2 is chosen to begin a ladder diagram to find the prime factorization of 66. What other numbers could have been used to start the ladder diagram for 66? How does starting with a different number change the diagram?

3 and 11 can be chosen because they are prime factors. The intermediate steps would be different, but the prime factorization is the same.

17. **Critical Thinking** List five numbers that have 3, 5, and 7 as prime factors.

Sample answer: 105, 315, 525, 735, 945

18. In a game, you draw a card with three consecutive numbers on it. You can choose one of the numbers and find the sum of its prime factors. Then you can move that many spaces on a game board. You draw a card with the numbers 25, 26, 27. Which number should you choose if you want to move as many spaces as possible? Explain.

26; the prime factors of 25 are 5 and 5, the prime factors of 26 are 2 and 13, and the prime factors of 27 are 3, 3, and 3. The sums are 10, 15, and 9. The greatest sum is 15, so choose 26 to move 15 spaces.

19. **Explain the Error** When asked to write the prime factorization of the number 27, a student wrote 9 · 3. Explain the error and write the correct answer.

9 is not a prime number; prime factorization of $27 = 3^3$.

H.O.T. FOCUS ON HIGHER ORDER THINKING

20. **Communicate Mathematical Ideas** Explain why it is possible to draw more than two different rectangles with an area of 36 square units, but it is not possible to draw more than two different rectangles with an area of 15 square units. The sides of the rectangles are whole numbers.

36 has five factor pairs, so five different rectangles can be drawn. 15 has only two factor pairs, so only two different rectangles can be drawn.

21. **Critique Reasoning** Alice wants to find all the prime factors of the number you get when you multiply 17 · 11 · 13 · 7. She thinks she has to use a calculator to perform all the multiplications and then find the prime factorization of the resulting number. Do you agree? Why or why not?

Disagree; the factors that are being multiplied are all prime numbers, so the prime factorization of the number is 17 · 13 · 11 · 7.

22. **Look for a Pattern** Ryan wrote the prime factorizations shown below. If he continues this pattern, what prime factorization will he show for the number one million? What prime factorization will he show for one billion?

$10 = 5 \cdot 2$

$100 = 5^2 \cdot 2^2$

$1{,}000 = 5^3 \cdot 2^3 = 1{,}000$

one million: $5^6 \cdot 2^6$; one billion: $5^9 \cdot 2^9$

Work Area

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity In mathematics, a **perfect number** is a number that is equal to the sum of all its factors (excluding the number itself).

The number 6 is an example of a perfect number. The factors of 6 are 1, 2, 3, and 6. The sum of the factors excluding 6 is $1 + 2 + 3 = 6$.

- Find the next largest perfect number, and show why it is perfect.
  28; the factors of 28 are 1, 2, 4, 7, 14, and 28, and $1 + 2 + 4 + 7 + 14 = 28$.
- A student claims that 128 is a perfect number. Prove or disprove the student's claim.
  False; The factors of 128 are 1, 2, 4, 8, 16, 32, 64, and 128. Their sum, excluding the number itself, is 127.
- Another student says that 496 is a perfect number. Prove or disprove the student's claim. True; The factors of 496 are 1, 2, 4, 8, 16, 31, 62, 124, 248, and 496. Their sum, excluding the number itself, is 496.

Lesson Support

Content Objective Students will learn to use the order of operations to simplify expressions with exponents.

Language Objective Students will explain the process of using the order of operations to evaluate expressions with exponents.

California Common Core Standards

CA CC **6.EE.1** Write and evaluate numerical expressions involving whole-number exponents.

CA CC **MP.5** Use appropriate tools strategically.

CA CC Focus | Coherence | Rigor

Building Background

Eliciting Prior Knowledge Have students write a complex expression that includes addition or subtraction, multiplication or division, an exponent, and an operation in parentheses, such as $20 - 5(8^3 \div 32)$. Ask students how they would simplify that expression.

$$\begin{aligned} 20 - 5(8^3 \div 16) &= 20 - 5(512 \div 16) \\ &= 20 - 5(32) \\ &= 20 - 160 \\ &= -140 \end{aligned}$$

Learning Progressions

In this lesson, students apply the order of operations to evaluate numerical expressions involving whole-number exponents. Important understandings for students include the following:

- The order of operations can be used to simplify numerical expressions.
- Operations inside parentheses should be performed first when simplifying expressions.
- Next the value of numbers with exponents should be found.
- Then multiply or divide from left to right.
- Finally, add or subtract from left to right.

Students use the order of operations to interpret and simplify expressions that include whole-number exponents. This prepares students for simplifying general linear expressions with rational coefficients in Grade 7.

Cluster Connections

This lesson provides an excellent opportunity to connect ideas in this cluster: ***Apply and extend previous understandings of arithmetic to algebraic expressions.***

Give students the following prompt: "Simplify the expression $32 \div (12 - 8)^2 + 7$ using the order of operations. Describe the steps you took." Sample answer: 9; First, I subtracted inside the parentheses: $12 - 8 = 4$. Next, I found the value of the number with the exponent: $4^2 = 16$. Then I divided $32 \div 16 = 2$. Finally, I added $2 + 7 = 9$.

Then have students find as many possible mistakes as they can that could have been made during simplifying (for example, squaring both 12 and 8, or adding 7 to the total in parentheses before dividing).

Language Support

California ELD Standards

CA ELD Emerging 2.I.12a. Selecting language resources – Use a select number of general academic words and domain-specific words to create some precision while speaking and writing.

CA ELD Expanding 2.I.12a. Selecting language resources – Use a growing set of academic words, domain-specific words, synonyms, and antonyms to create precision and shades of meaning while speaking and writing.

CA ELD Bridging 2.I.12a. Selecting language resources – Use an expanded set of general academic words, domain-specific words, synonyms, antonyms, and figurative language to create precision and shades of meaning while speaking and writing.

Linguistic Support

Academic/Content Vocabulary

PEMDAS – The mnemonic device ***P****lease* ***E****xcuse* ***M****y* ***D****ear* ***A****unt* ***S****ally* is often used to help students remember the order of operations, but it may not be very helpful for English learners. Have students create their own memory device with PEMDAS. Here is a possible mnemonic device in Spanish:

P **P**aréntesis primero

E **E**xponentes (potencias y raíces cuadradas, etc.)

MD **M**ultiplicación y **D**ivisión (de izquierda a derecha)

AS **A**dición y **S**ustracción (de izquierda a derecha)

Multiple Meaning Words

This lesson contains words with multiple meanings. In the Explore Activity, it describes *waves*. Point out that, in this context, *waves* does not mean hand gestures or ocean movements. It means a *stage, phase,* or *turn*. Look for other multiple meaning words and clarify their definition in this context for students.

Leveled Strategies for English Learners

Emerging Check that students know the order of operations by asking them to write and name the symbols associated with each step: ( ), ^{2}, $\times$, $+$, $-$.

Expanding Have students name the order of operations by using sequence words.

First, _____. ***Next,*** _____. ***Then,*** _____. ***Finally,*** _____.

Bridging Have students demonstrate understanding of the order of operations by explaining to a partner the mnemonic PEMDAS.

Math Talk To help English learners talk about mathematics, preview lessons for multiple meaning words or cultural influences with which they may be unfamiliar, and provide sentence frames for them to use to make complete sentences.

LESSON

9.3 Order of Operations

CA Common Core Standards

The student is expected to:

CA CC Expressions and Equations—6.EE.1

Write and evaluate numerical expressions involving whole-number exponents.

Mathematical Practices

CA CC MP.5 Using Tools

Engage

ESSENTIAL QUESTION

How do you use the order of operations to evaluate expressions with exponents? Sample answer: Find the value of any expressions within parentheses first. Then evaluate all powers. Then multiply or divide in order from left to right, and finally, add or subtract in order from left to right.

Motivate the Lesson

Ask: Previously you've used the order of operations to evaluate expressions that involve multiplication, division, addition, and subtraction. Begin the Explore Activity to find out how to use the order of operations to evaluate expressions that involve exponents.

Explore

EXPLORE ACTIVITY

Engage with the Whiteboard

Work with students to complete the diagram and the table in Steps A and B on the whiteboard. You may want to extend the diagram and the table to include a third wave of e-mails.

Explain

EXAMPLE 1

Avoid Common Errors

Students may incorrectly perform operations from left to right as they appear in an expression, rather than use the order of operations. Remind students that using the order of operations correctly ensures that everyone who simplifies the same expression will get the same answer.

Questioning Strategies **Mathematical Practices**

- Can you use the order of operations with expressions that have no parentheses? Explain. Yes. The order of operations tells the order in which operations should be performed but does not require that an expression include parentheses, exponents, or all the operations.
- Are the expressions $3 + 5 \cdot 2$ and $3 + (5 \cdot 2)$ equivalent? Explain. Yes. In both expressions $5 \cdot 2$ should be evaluated first, and then 3 should be added to the product.

YOUR TURN

Talk About It

Check for Understanding

Ask: In the expression $220 - 450 \div 3^2$, which operation should you perform first? Explain. Evaluate the power. This expression has no parentheses so the first operation to perform is to evaluate the powers.

ADDITIONAL EXAMPLE 1

Evaluate each expression.

A $30 - 3 \times 2^3$ 6

B $15 + \frac{9^2}{3}$ 42

C $5 \times 3^2 \div 5 + 7$ 16

Interactive Whiteboard
Interactive example available online

my.hrw.com

LESSON
9.3 Order of Operations

CA CC 6.EE.1
Write and evaluate numerical expressions involving whole-number exponents.

ESSENTIAL QUESTION How do you use the order of operations to evaluate expressions with exponents?

EXPLORE ACTIVITY Real World CA CC 6.EE.1

Exploring the Order of Operations

Order of Operations

1. Perform operations in parentheses.
2. Find the value of numbers with exponents.
3. Multiply or divide from left to right.
4. Add or subtract from left to right.

Amy and three friends launch a new website. Each of the four friends e-mails the web address to three new friends. These new friends forward the web address to three more friends. If no one receives the e-mail more than once, how many people will receive the web address in the second wave of e-mails?

A Use a diagram to model the situation for Amy. Each dot represents one e-mail. Complete the diagram to show the second wave.

B Complete the table to show how many e-mails are sent in each wave of Amy's diagram.

Wave	Number of e-mails	Power of 3
1st	3	3^1
2nd	9	3^2

C Amy is just one of four friends initiating the first wave of e-mails. Write an expression for the total number of e-mails sent in the 2nd wave.

number of people × number of e-mails in 2nd wave written as a power

4×3^2

D Identify the computation that should be done first to simplify the expression in **C**. Then simplify the expression.

Multiply 4 and 3 / Find the value of 3^2

The value of the expression is $4 \times$ 9 $=$ 36.

EXPLORE ACTIVITY *(cont'd)*

Reflect

1. In **B**, why does it makes sense to write the numbers of e-mails as powers? What is the pattern for the number of e-mails in each wave for Amy?

Sample answer: By writing the values as powers, you can see the exponent is equal to the wave number. The pattern would be 3^1, 3^2, 3^3, 3^4, and so on.

Math On the Spot my.hrw.com

Evaluating Numerical Expressions

A numerical expression is an expression involving numbers and operations. You can use the order of operations to evaluate numerical expressions.

EXAMPLE 1 CA CC 6.EE.1

Evaluate each expression.

A $5 + 18 \div 3^2$

$5 + 18 \div 3^2 = 5 + 18 \div 9$ Evaluate 3^2.

$= 5 + 2$ Divide.

$= 7$ Add.

The fraction bar means that you should divide the numerator by the denominator.

B $21 + \frac{3^2}{3}$

$21 + \frac{3^2}{3} = 21 + \frac{9}{3}$ Evaluate 3^2.

$= 21 + 3$ Divide.

$= 24$ Add.

Math Talk Mathematical Practices
How do you know you must divide in **B**?

C $6 \times 2^3 \div 3 + 1$

$6 \times 2^3 \div 3 + 1 = 6 \times 8 \div 3 + 1$ Evaluate 2^3.

$= 48 \div 3 + 1$ Multiply.

$= 16 + 1$ Divide.

$= 17$ Add.

Personal Math Trainer
Online Practice and Help
my.hrw.com

YOUR TURN

Evaluate each expression using the order of operations.

2. $7 + 15 \times 9^2 =$ 1,222 **3.** $220 - 450 \div 3^2 =$ 170

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices MP.5

This lesson provides an opportunity to address the Mathematical Practices standard that calls for students to use appropriate tools strategically. Students work with pencil and paper using the order of operations to evaluate expressions. Since use of the order of operations can involve many steps, students use mental math and number sense in situations such as finding the sum of two numbers with unlike signs and raising a negative number to a positive power.

Math Background

Parentheses and brackets were first used as grouping symbols by mathematicians as early as the sixteenth century. A work published in 1556, *General trattato di numeri e misure* by Niccolò Tartaglia, is one of the first in which parentheses are used. Brackets have been found in a manuscript dating from 1550.

ADDITIONAL EXAMPLE 2

Evaluate each expression using the order of operations.

A $128 \div (4 \times 2)^2$ 2

B $8^2 - (5 + 3)^2$ 0

C $15 - \frac{(1 + 3)^2}{2}$ 7

Interactive Whiteboard
Interactive example available online

my.hrw.com

EXAMPLE 2

Avoid Common Errors

Point out to students that some symbols have more than one purpose. Parentheses can indicate multiplication and/or act as grouping symbols. The fraction bar is a grouping symbol and also can indicate division. Remind students to read expressions carefully to determine how the symbols are being used.

Engage with the Whiteboard

Cover up the blue text in each part and have students circle the operation to be performed for each step on the whiteboard. Ask students to explain their choices. Then discuss the choices with the class.

Questioning Strategies Mathematical Practices

- In A, what operation would you perform first if there were no parenthetical groupings? You would evaluate the power first.
- In C, how do you evaluate $\frac{4^2}{2}$? First evaluate the expression in the numerator: $4^2 = 16$. Then divide: $\frac{16}{2} = 8$.

YOUR TURN

Focus on Communication

Some students may think that all multiplication is done before division and all addition is done before subtraction. Remind them that this is not the case. Multiplication and division should be done from left to right as ordered in the problem. Similarly, addition and subtraction should be done from left to right as ordered in the problem. Stress that these pairs of operations should be performed in the order they occur unless they are within grouping symbols.

Elaborate

Talk About It

Summarize the Lesson

Ask: Why is it important to use the order of operations? The order of operations is important because correctly using the order or operations ensures that everyone who evaluates the same expression will get the same answer.

GUIDED PRACTICE

Engage with the Whiteboard

For Exercise 1, have students complete the diagram on the whiteboard. Then have them number the branches at each level to show the numbers of each type of fish that can be formed.

Avoid Common Errors

Exercise 3 Remind students that when they work with an expression that has both multiplication and division, they should perform those operations in order from left to right.

Exercises 4–5 Remind students that when an expression inside parentheses has more than one operation, they need to perform those operations according to the order of operations.

Using Exponents with Parentheses

Remember to perform operations inside parentheses first when you evaluate expressions.

EXAMPLE 2

CA CC 6.EE.1

Evaluate each expression using the order of operations.

A $4 \times (9 \div 3)^2$

$4 \times (9 \div 3)^2 = 4 \times 3^2$ Perform operations inside parentheses.

$= 4 \times 9$ Evaluate 3^2.

$= 36$ Multiply.

B $5^3 + (12 - 2)^2$

$5^3 + (12 - 2)^2 = 5^3 + 10^2$ Perform operations inside parentheses.

$= 125 + 100$ Evaluate powers.

$= 225$ Add.

C $8 + \frac{(12 - 8)^2}{2}$

$8 + \frac{(12 - 8)^2}{2} = 8 + \frac{4^2}{2}$ Perform operations inside parentheses.

$= 8 + \frac{16}{2}$ Evaluate 4^2.

$= 8 + 8$ Divide.

$= 16$ Add.

Reflect

4. **Critique Reasoning** John wants to evaluate the expression $(5 + 3)^2$. As a first step, he writes $5^2 + 3^2$. Will he get the correct value for the expression? If not, what should he do to evaluate the expression?

No; $5^2 + 3^2 = 25 + 9 = 36$, which is incorrect. John needs to follow the order of operations and first add inside the parentheses: $(5 + 3)^2 = 8^2 = 64$.

YOUR TURN

Evaluate each expression using the order of operations.

5. $5 \times (20 \div 4)^2 =$ 125

6. $8^2 - (5 + 2)^2 =$ 15

7. $7 - \frac{(63 \div 9)^2}{7} =$ 0

Math On the Spot my.hrw.com

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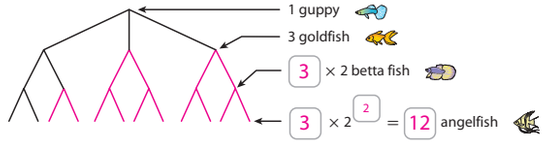
My Notes

Guided Practice

1. In a video game, a guppy that escapes a net turns into three goldfish. Each goldfish can turn into two betta fish. Each betta fish can turn into two angelfish. Complete the diagram and write the number of fish at each stage. Write and evaluate an expression for the number of angelfish that can be formed from one guppy. (Explore Activity)



$3 \times 2^2 = 12$ angelfish

Complete to evaluate each expression. (Examples 1 and 2)

2. $89 - 4^2 \times 4 + 12 = 89 - \underline{16} \times 4 + 12$

$= 89 - \underline{64} + 12$

$= \underline{25} + 12$

$= \underline{37}$

3. $6 \times (36 \div 12)^2 + 8 = 6 \times (\underline{3})^2 + 8$

$= 6 \times \underline{9} + 8$

$= \underline{54} + 8$

$= \underline{62}$

4. $12 \times \left(\frac{(4 + 2)^2}{4}\right) - 7 = 12 \times \left|\frac{(\underline{6})^2}{4}\right| - 7$

$= 12 \times \left|\frac{\underline{36}}{4}\right| - 7$

$= 12 \times \underline{9} - 7$

$= \underline{108} - 7$

$= \underline{101}$

5. $320 \div \left(\frac{(11 - 9)^3}{2}\right) \times 8 = 320 \div \left|\frac{(\underline{2})^3}{2}\right| \times 8$

$= 320 \div \left|\frac{\underline{8}}{2}\right| \times 8$

$= 320 \div \underline{4} \times 8$

$= \underline{80} \times 8$

$= \underline{640}$

ESSENTIAL QUESTION CHECK-IN

6. How do you use the order of operations to evaluate expressions with exponents?

Find the value of any expressions within parentheses first. Then evaluate all powers. Then multiply or divide from left to right, and finally add or subtract from left to right.

DIFFERENTIATE INSTRUCTION

Cooperative Learning

Have students work in groups to decide which operation signs to use to make the number sentences true. They may need to use operations more than once in each number sentence.

1. Operation signs: +, −, ·

$12 \square 4 \square 6 \square 3 \square 7 = 37$

$12 \cdot 4 - 6 \cdot 3 + 7 = 37$

2. Operation signs: +, −, ÷

$18 \square 2 \square 24 \square 12 \square 4 = 22$

$18 + 2 - 24 \div 12 + 4 = 22$

Cognitive Strategies

Students may be familiar with the abbreviation PEMDAS (or **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally) even before being introduced to the order of operations. But its abbreviation may give the impression that multiplication is always done before division and that addition is always done before subtraction. You may wish to present the mnemonic as P E M/D A/S

The slashes between the M and the D, and the A and the S, can help students remember that from left to right either operation can be performed first.

Additional Resources

Differentiated Instruction includes:

- Reading Strategies
- Success for English Learners EL
- Reteach
- Challenge PRE-AP

Personal Math Trainer

Online Assessment and Intervention

Online homework assignment available

my.hrw.com

9.3 LESSON QUIZ

CA CC **6.EE.1**

Evaluate each expression using the order of operations.

1. $3 + (7 - 5)^2 \times 6$

2. $\frac{2^5}{(4 + 4)} \times 2$

3. $8 - 6 \div 2 + 3 \times 5$

4. $7 \times 3 - \frac{3^2 + 6}{5}$

5. $8 + 2(1 + 12 \div 2)^2$

Lesson Quiz available online

my.hrw.com

Answers

1. 27

2. 8

3. 20

4. 18

5. 106

Evaluate

CA CC **Focus | Coherence | Rigor**

GUIDED AND INDEPENDENT PRACTICE

CA CC **6.EE.1**

Concepts & Skills	Practice
Explore Activity Exploring the Order of Operations	Exercise 1
Example 1 Evaluating Numerical Expressions	Exercises 2, 7–8, 11, 15–16
Example 2 Using Exponents with Parentheses	Exercises 9–10, 12, 13–14

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
7–12	**2** Skills/Concepts	**MP.5** Using Tools
13	**3** Strategic Thinking H.O.T.	**MP.7** Using Structure
14	**3** Strategic Thinking H.O.T.	**MP.3** Logic
15–16	**2** Skills/Concepts	**MP.4** Modeling
17–19	**3** Strategic Thinking H.O.T.	**MP.7** Using Structure

Additional Resources

Differentiated Instruction includes:

- Leveled Practice worksheets

CLUSTER CONNECTION

Exercise 17 combines concepts from the California Common Core cluster "Apply and extend previous understandings of arithmetic to algebraic expressions."

Name _____ Class _____ Date _____

9.3 Independent Practice

CA CC 6.EE.1

Evaluate each expression using the order of operations.

7. $5 \times 2 + 3^2$ 19

8. $15 - 7 \times 2 + 2^3$ 9

9. $(11 - 8)^3 - 2 \times 6$ 15

10. $6 + 3(13 - 2) - 5^2$ 14

11. $12 + \frac{9^2}{3}$ 39

12. $\frac{8 + 6^2}{11} + 7 \times 2$ 18

13. **Explain the Error** Jay evaluated the expression $3 \times (3 + 12 \div 3) - 4$. For his first step, he added $3 + 12$ to get 15. What was Jay's error? Find the correct answer.

Jay worked inside the parentheses first, but he should have performed the division $12 \div 3 = 4$ first; 17.

14. **Multistep** A clothing store has the sign shown in the shop window. Pani sees the sign and wants to buy 3 shirts and 2 pairs of jeans. The cost of each shirt before the discount is \$12, and the cost of each pair of jeans is \$19 before the discount.

a. Write and evaluate an expression to find the amount Pani pays if a \$3 discount is applied to her total.

$3 \times 12 + 2 \times 19 - 3$; \$71

b. Pani says she should get a \$3 discount on the price of each shirt and a \$3 discount on the price of each pair of jeans. Write and evaluate an expression to find the amount she would pay if this is true.

$3 \times (12 - 3) + 2 \times (19 - 3)$; \$59

c. **Analyze Relationships** Why are the amounts Pani pays in **a** and **b** different?

In **a**, the \$3 discount is applied 1 time; in **b** it is applied 5 times.

d. If you were the shop owner, how would you change the sign? Explain.

Sample answer: If the shop owner wants to make more money, the sign should say "\$3 off your entire purchase." If customers can take the discount off every item, a lot more money is discounted from each purchase.

15. Ellen is playing a video game in which she captures butterflies. There are 3 butterflies onscreen, but the number of butterflies doubles every minute. After 4 minutes, she was able to capture 7 of the butterflies.

a. **Look for a Pattern** Write an expression for the number of butterflies after 4 minutes. Use a power of 2 in your answer.

$3 \times 2 \times 2 \times 2 \times 2 = 3 \times 2^4$

b. Write an expression for the number of butterflies remaining after Ellen captured the 7 butterflies. Evaluate the expression.

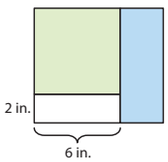
$3 \times 2^4 - 7 = 3 \times 16 - 7 = 48 - 7 = 41$; 41 butterflies remain.

16. Show how to write and evaluate an expression to represent and solve this problem: Jeff and his friend each text four classmates about a concert. Each classmate then texts four students from another school about the concert. If no one receives the message more than once, how many students from the other school receive a text about the concert?

$2 \times 4^2 = 32$; 32 students receive a text.

H.O.T. FOCUS ON HIGHER ORDER THINKING

17. **Geometry** The figure shown is a rectangle. The green shape in the figure is a square. The blue and white shapes are rectangles, and the area of the blue rectangle is 24 square inches.



a. Write an expression for the area of the entire figure that includes an exponent. Then find the area.

$6^2 + 2 \times 6 + 24 = 72$ square inches

b. Find the dimensions of the entire figure.

8 in. by 9 in.

18. **Explain the Error** Rob and Lila try to simplify $18 \times 4^2 + (9 - 3)^2$. Rob evaluates the expression and gets 360. Lila evaluates it and gets 324. Which student is correct? What error did the other student make?

Lila is correct. Rob squared each of the numbers in the parentheses and then subtracted, instead of subtracting first and then squaring the difference.

19. **Persevere in Problem Solving** Use parentheses to make this statement true: $8 \times 4 - 2 \times 3 + 8 \div 2 = 25$

$8 \times 4 - (2 \times 3 + 8) \div 2$

Work Area

EXTEND THE MATH PRE-AP

Activity available online my.hrw.com

Activity The expression $(4 \times 4 - 4) \times 4$ uses exactly 4 fours. When simplified, its value is 48.

- Write 10 expressions that use exactly 4 fours and that equal each of the whole numbers from 0 to 9. Use what you know about the order of operations to write the expressions. You can use addition, subtraction, multiplication, division, parentheses, and exponents in the expressions.
- Justify the expressions you have written by showing how to simplify them.

Possible answers:

$4 + 4 - 4 - 4 = 0$

$4 \div 4 + 4 \div 4 = 2$

$4 - (4 - 4) \times 4 = 4$

$4 + \frac{(4 + 4)}{4} = 6$

$\frac{(4 \times 4)}{4} + 4 = 8$

$(4 + 4) \div (4 + 4) = 1$

$(4 + 4 + 4) \div 4 = 3$

$\left(\frac{4}{4}\right)^4 + 4 = 5$

$4 + 4 - \left(\frac{4}{4}\right) = 7$

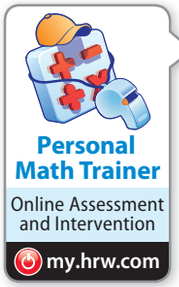
$4 + 4 + \left(\frac{4}{4}\right) = 9$

Ready to Go On?

Assess Mastery

Use the assessment on this page to determine if students have mastered the concepts and standards covered in this module.

RtI Response to Intervention



Intervention	Enrichment

Access Ready to Go On? assessment online, and receive instant scoring, feedback, and customized intervention or enrichment.

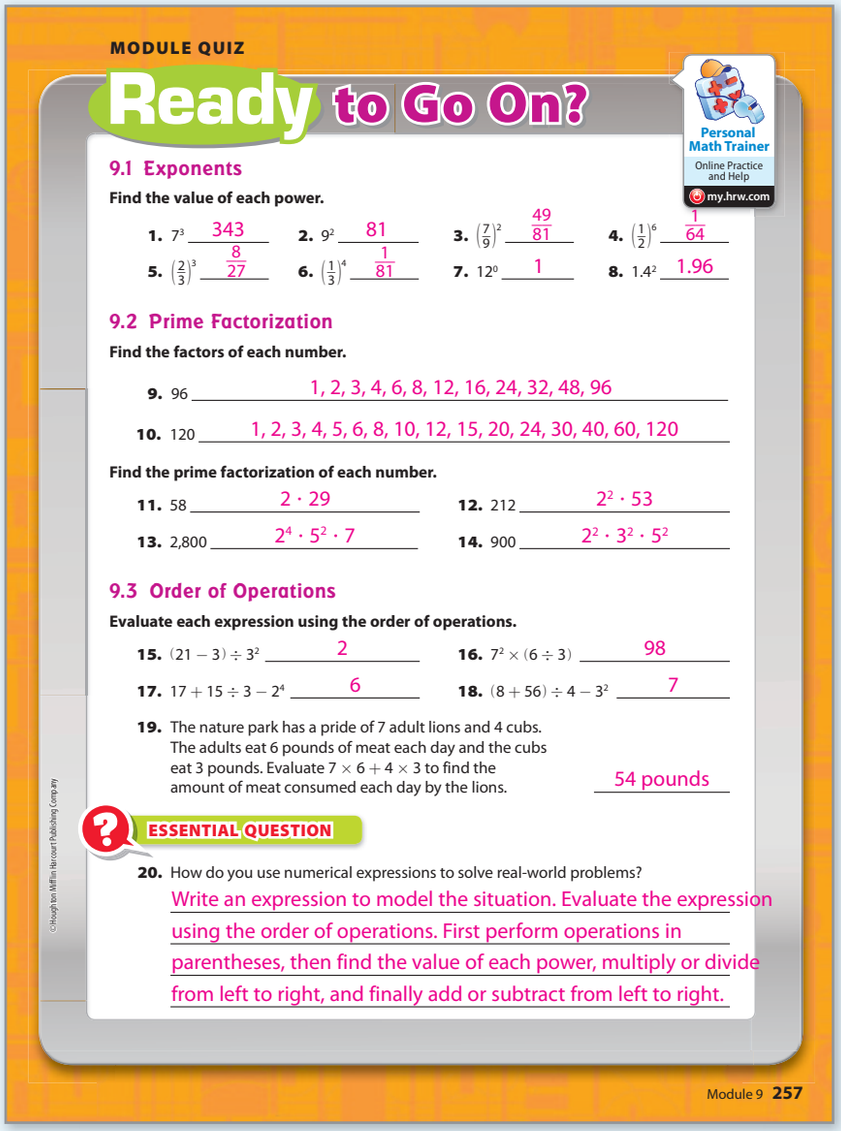
Online and Print Resources

Differentiated Instruction	*Differentiated Instruction*
• Reteach worksheets	• Challenge worksheets PRE-AP
• Reading Strategies EL	Extend the Math PRE-AP Lesson Activities in TE
• Success for English Learners EL	

Additional Resources

Assessment Resources includes:

- Leveled Module Quizzes



MODULE QUIZ

Ready to Go On?

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9.1 Exponents

Find the value of each power.

1. 7^3 343
2. 9^2 81
3. $\left(\frac{7}{9}\right)^2$ $\frac{49}{81}$
4. $\left(\frac{1}{2}\right)^6$ $\frac{1}{64}$
5. $\left(\frac{2}{3}\right)^3$ $\frac{8}{27}$
6. $\left(\frac{1}{3}\right)^4$ $\frac{1}{81}$
7. 12^0 1
8. 1.4^2 1.96

9.2 Prime Factorization

Find the factors of each number.

9. 96 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96
10. 120 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120

Find the prime factorization of each number.

11. 58 $2 \cdot 29$
12. 212 $2^2 \cdot 53$
13. 2,800 $2^4 \cdot 5^2 \cdot 7$
14. 900 $2^2 \cdot 3^2 \cdot 5^2$

9.3 Order of Operations

Evaluate each expression using the order of operations.

15. $(21 - 3) \div 3^2$ 2
16. $7^2 \times (6 \div 3)$ 98
17. $17 + 15 \div 3 - 2^4$ 6
18. $(8 + 56) \div 4 - 3^2$ 7
19. The nature park has a pride of 7 adult lions and 4 cubs. The adults eat 6 pounds of meat each day and the cubs eat 3 pounds. Evaluate $7 \times 6 + 4 \times 3$ to find the amount of meat consumed each day by the lions. 54 pounds

ESSENTIAL QUESTION

20. How do you use numerical expressions to solve real-world problems?
Write an expression to model the situation. Evaluate the expression using the order of operations. First perform operations in parentheses, then find the value of each power, multiply or divide from left to right, and finally add or subtract from left to right.

Module 9 257

California Common Core Standards

Lesson	Exercises	Common Core Standards
9.1	1–8	**6.EE.1**
9.2	9–14	**6.EE.1**
9.3	15–19	**6.EE.1**

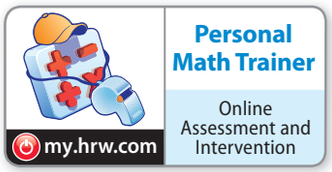
Assessment Readiness

Scoring Guide

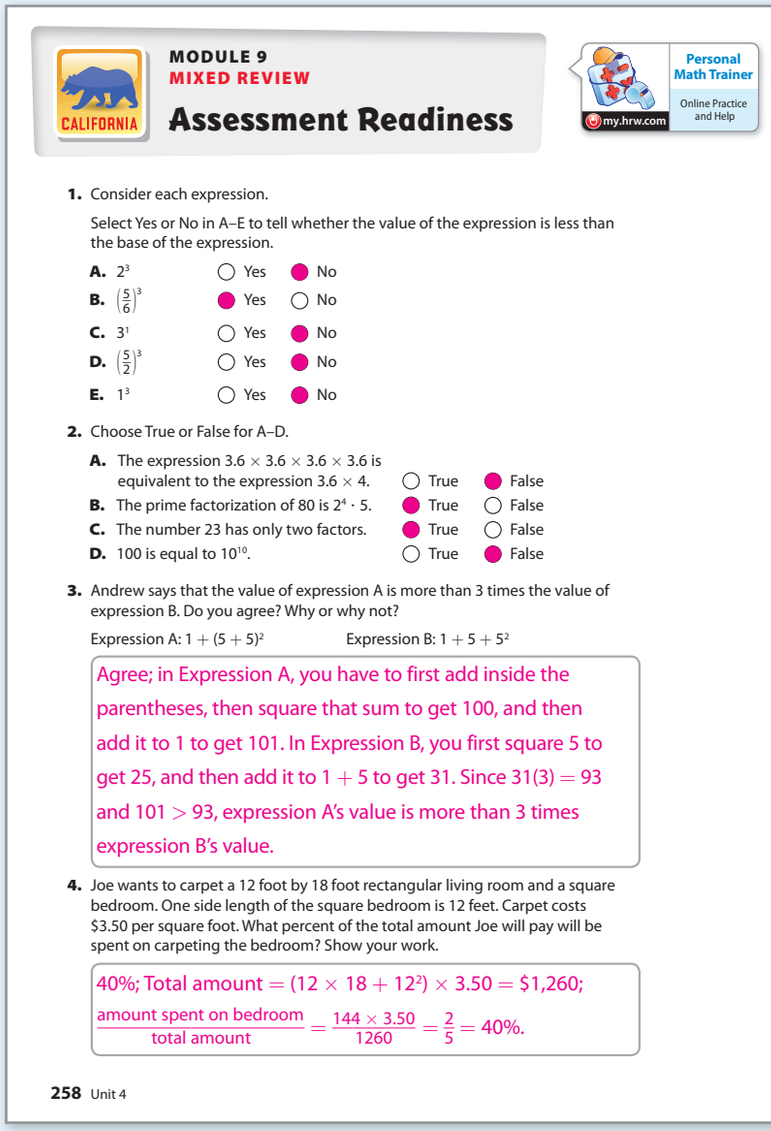
Item 3 Award the student 1 point for knowing that Andrew is correct and 1 point for correctly explaining how to evaluate Andrew's claim.

Item 4 Award the student 1 point for writing and evaluating the expression correctly and 1 point for finding the correct percent.

Additional Resources



To assign this assessment online, login to your Assignment Manager at **my.hrw.com**.



MODULE 9
MIXED REVIEW

Assessment Readiness

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1. Consider each expression.

Select Yes or No in A–E to tell whether the value of the expression is less than the base of the expression.

A.	2^3	○ Yes	● No
B.	$\left(\frac{5}{6}\right)^3$	● Yes	○ No
C.	3^1	○ Yes	● No
D.	$\left(\frac{5}{2}\right)^3$	○ Yes	● No
E.	1^3	○ Yes	● No

2. Choose True or False for A–D.

A.	The expression $3.6 \times 3.6 \times 3.6 \times 3.6$ is equivalent to the expression 3.6×4.	○ True	● False
B.	The prime factorization of 80 is $2^4 \cdot 5$.	● True	○ False
C.	The number 23 has only two factors.	● True	○ False
D.	100 is equal to 10^{10}.	○ True	● False

3. Andrew says that the value of expression A is more than 3 times the value of expression B. Do you agree? Why or why not?

Expression A: $1 + (5 + 5)^2$ Expression B: $1 + 5 + 5^2$

Agree; in Expression A, you have to first add inside the parentheses, then square that sum to get 100, and then add it to 1 to get 101. In Expression B, you first square 5 to get 25, and then add it to $1 + 5$ to get 31. Since $31(3) = 93$ and $101 > 93$, expression A's value is more than 3 times expression B's value.

4. Joe wants to carpet a 12 foot by 18 foot rectangular living room and a square bedroom. One side length of the square bedroom is 12 feet. Carpet costs \$3.50 per square foot. What percent of the total amount Joe will pay will be spent on carpeting the bedroom? Show your work.

40%; Total amount $= (12 \times 18 + 12^2) \times 3.50 = \$1{,}260$;
$\frac{\text{amount spent on bedroom}}{\text{total amount}} = \frac{144 \times 3.50}{1260} = \frac{2}{5} = 40\%$.

California Common Core Standards

Items	Grade 6 Standards	Mathematical Practices
1	6.EE.1	MP.1, MP.4
2	6.EE.1	MP.2, MP.4
3*	6.EE.1, 6.RP.3c	MP.4
4	6.EE.1.1	MP.1

* Item integrates mixed review concepts from previous modules or a previous course.