Chapter 6: Fatigue Failure (Review)

I) **Sec 6-17 Road Maps and Important Equations**

II) **Loading**

Simple loading

- Axial loading

- Torsion

- Bending

Combined Loading

III) **Characterizing Stress (Fig. 6-23d, e, f)**

Completely reversed stress

Repeated stress

Fluctuating stress

IV) **Endurance Limit S'_e (Eq. 6-8)**

Correction Factors: $k_a\ k_b\ k_c\ k_d\ k_e$ (Sec. 6-9)

Corrected Endurance Limit S_e (Sec. 6-18)

V) **Equivalent Stresses**

Theoretical stress concentration factors: $k_t\ k_{ts}$ (A-15)

North sensitivity: $q\ q_{shear}$ (Sec. 6-10)

Fatigue stress concentration factors: $k_f\ k_{fs}$ (Eq. 6-32)

Von Mises Stress for alternating comp. (Eq. 6-55)

Von Mises Stress for mid-range comp. (Eq. 6-56)

VI) $\sigma'_m = 0\ (Sec. 6-8)$

If $\sigma'_a \le S_e$; infinite life and factor of safety is:

$$n = \frac{S_e}{\sigma'_a}$$

Else finite life and number of stress cycles N is by (Eq. 6-16) where σ_{rev} is set to σ'_a

$\sigma'_m > 0\ (Sec. 6-12)$

Decide what to use, Soderberg? Modified Goodman? Gerber? ASME Elliptic?

(Eq. 6-45 to Eq. 6-48) for factor of safety n.

If $n \ge 1$, finite life.

Else finite life and N is by (Eq. 6-16) but σ_{rev} is per Step 4 on pp. 340.

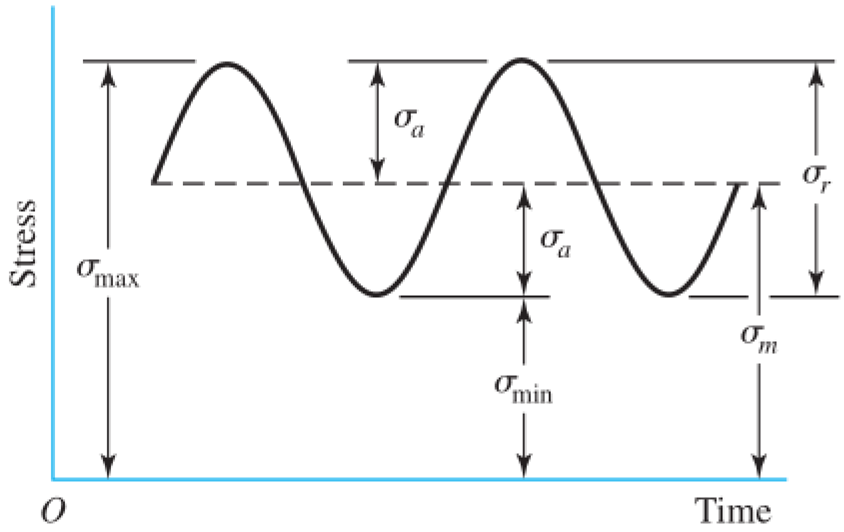


Figure 1: Fluctuating Stress

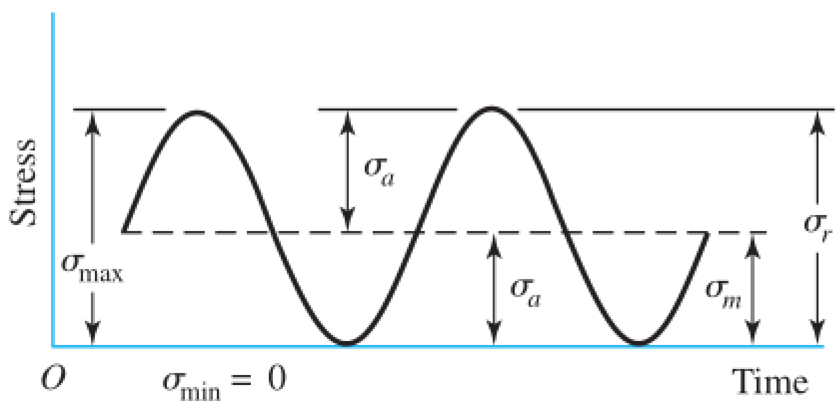


Figure 2: Repeated Stress

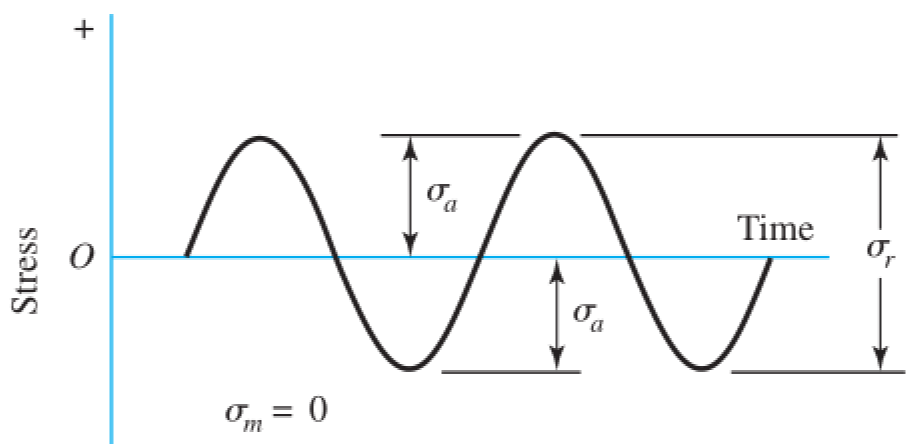


Figure 3: Completely Reversed Stress

Example (1): A low carbon steel stock is lathe-turned to have a diameter of 1” The stock has $S_{ut} = 100\ ksi$, $S_y = 76\ ksi$. Axial load varies $-10{\sim}50\ kips$. Fatigue stress concentration factor is $K_f = 1.3$. Find factor of safety n if infinite life, or number of cycles N if infinite life. Assume room temperature and 99% reliability.

Case: Simple loading, fluctuating stress

$F_{min} = -10\ kips$
$F_{max} = 50\ kips$

$$\sigma_{min} = \frac{k_f F_{min}}{A} = -16.552\ ksi$$
$$\sigma_{max} = \frac{k_f F_{max}}{A} = 82.761\ ksi$$

$\sigma_m = 33.105\ ksi$
$\sigma_a = 49.657\ ksi$

$S_e' = 0.5 S_{ut} = 50\ ksi$

$k_a = 0.797$
$k_b = 1$
$k_c = 0.85$
$k_d = 1$
$k_e = 0.814$

$S_e = 27.572\ ksi$

Criterion	Equation	n
Soderberg	6-45	0.45
Modified Goodman	6-46	0.47
Gerber	6-47	0.54
ASME-elliptic	6-48	0.54

$\therefore finite\ life$

$f = 0.845$
$a = 258.97\ ksi$
$b = -0.16213$

Criterion	σ_{rev}, ksi	N
Soderberg	87.981	779
Modified Goodman	74.231	2223
Gerber	55.769	12974
ASME-elliptic	55.166	13874

(see pp. 314 for procedure)

Chapter 7: Shafts and Shaft Components

(7-1): Introduction

Shaft Loading

- Power transmission shafting is to transmit power/motion from an input source (e.g. motors, engines) to an output work site.
- Shafts are supported by bearings, and loaded torsionally, transversely, and/or axially as the machine operates.
- Shafts can be solid or hollow, and are often stepped.
- They are widely required by virtually all types of machinery and mechanical systems.

New Shaft Design Procedure

- Conceptual sketch for shaft <u>layout</u>, based on functional spec. and system config (7-3)
- Shaft materials (7-2)
- An appropriate <u>design factor</u>
- Support reactions, bending moment diagrams (in tow planes, as well as resultant/combined), and torque diagram, critical cross-sections.
- Shaft <u>diameters</u> based on strength requirement (7-4)
- <u>Slopes and Deflections</u> at locations of interest in order to select bearings, couplings, etc.: or to ensure proper functioning of bearings, couplings, gears, etc. (7-5)
- <u>Critical Speeds</u> and other vibration characteristics (7-6)

Calculations Needed

- Shaft diameters based on strength requirement > by ANSI/ASME standard B106-1M-1985 “Design for Transmission Shafting”, or by other practices.
- Slopes and deflections > simplified / approximate geometry, numerical, graphical, FEA;
- Critical speed and other vibration characteristics > specific deflections of shaft.

(7-2): Shaft Materials

- Requiring generally/typically high strength and high modulus of elasticity
- Typical selection: low carbon steel (cold-drawn or hot-rolled) such as ANSI 1020-1050 steels
- If higher strength is required, alloy steels plus heat treatment such as ANSI 1340-50, 4140, 4340, 5140, 8650.

- Cold-drawn steel is used for diameters under 3 inches; machining is not needed where there is no fitting with other components.
- Hot-rolled steels should be machined all over.
- Stainless steels when environment is corrosive, for example.

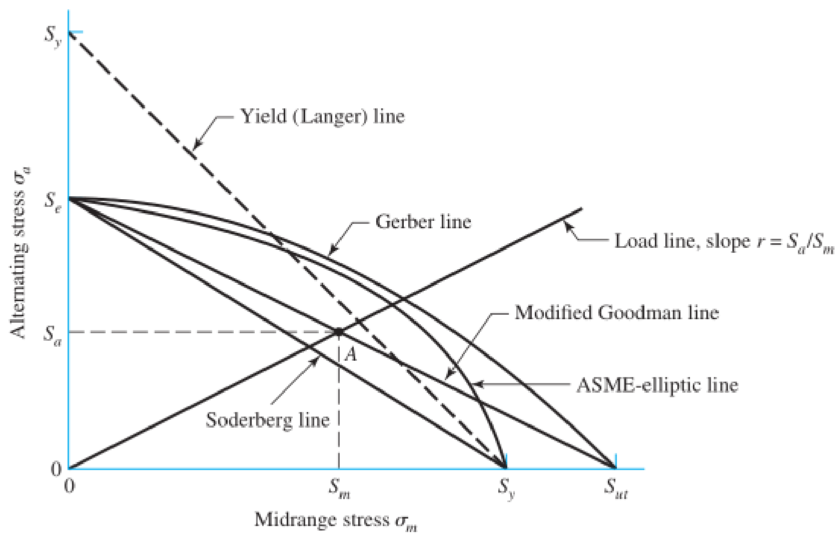
Equations for the Fatigue Failure Criteria
(6-40) Soderbeg
(6-41) Modified Goodman
(6-42) Gerber
(6-43) ASME-Elliptic

For factor of safety:
Replace S_a with $n\sigma_a$
Replace S_m with $n\sigma_m$
Resulting in Equations $(6-45) \sim (6-48)$

For σ_{rev} (which is needed for number of cycles N):
Replace S_a with σ_a
Replace S_m with σ_m
Replace S_e with σ_{rev}
and solving for σ_{rev}

Figure 6–27

Fatigue diagram showing various criteria of failure. For each criterion, points on or "above" the respective line indicate failure. Some point A on the Goodman line, for example, gives the strength S_m as the limiting value of σ_m corresponding to the strength S_a, which, paired with σ_m, is the limiting value of σ_a.



Modified-Goodman line – too dangerous, goes directly to S_{ut}
Soderberg – too conservative

Most of the time, we'll be using ASME-elliptic line, not

Example (2): A non-rotating shaft is lathe-turned to have a 1"-diameter. The shaft is subject to a torque that varies $0 \sim T_{max}(in\ lb - in)$. Determine T_{max} such that the shaft will have an infinite life with a factor of safety of 1.8. The shaft's material has $S_{ut} = 100\ ksi$ and $S_y = 76\ ksi$. Assume room temperature and 99% reliability. Fatigue stress concentration factor is $k_{fs} = 1.6$.

Answer: Based on simple loading and ASME-elliptic criterion, $T_{max} \approx 2480\ lb - in$

Design Factor vs. Factor of Safety (not from the text)

- Design factor is to indicate the level of overload that the part/component is required/intended to withstand
- Safety factor indicated how much overload the designed part will actually be able to withstand.
- Design factor is chosen, generally in advance and often set by regulatory code or an industry's general practice.
- Safety factor is obtained from design calculations.

The following are some recommended values of design factor based on strength considerations. They are valid for general applications.

- 1.25 - 1.5: for reliable materials under controlled conditions subjected to loads and stresses known with certainty;
- 1.5 – 2: for well-known materials under reasonably constant environmental conditions subjected to known loads and stresses;
- 2 – 2.5: for average materials subjected to known loads and stresses;
- 2.5 – 3: for less well-known materials under average conditions of load, stress and environment;
- 3 – 4: for untried materials under average conditions of load, stress and environment;
- 3 – 4: for well-known materials under uncertain conditions of load, stress and environment.

Courtesy of Mechanical Design, 2nd Edition, P. Childs, Elsevier Ltd. (p. 95)

(7-4): Shaft Design for Stress

Loads on a Shaft

- The primary function of a shaft is to transmit torque, typically through only a portion of the shaft;
- Due to the means of torque transmission, shafts are subject to transverse loads in two planes such that the shear and bending moment diagrams are needed in two planes.

Shaft Stress from Fatigue Perspective

- In 1985, ASME published ANSI/ASME Standard B106.1M-1985 "Design for Transmission Shafting". However, it was withdrawn in 1994.
    - Little or no axial load
    - Fully reversed bending and steady (or constant) torsion
- General case: Fluctuating bending and fluctuating torsion
    - As considered by the text

Fully reversed bending and steady torsion is in fact special cases of the general one. In reality, the general case is not as common as the typical case of fully reversed bending and steady torsion.

Critical Locations

Stresses are evaluated at the critical locations. Look or where:

- Bending moment is large
- There is torque
- There is stress concentration

Factor of Safety or Required Shaft Diameter – General Case
Generally, $\sigma_{bending} = \sigma_m$ and σ_a
And $\tau_{torsion} = \tau_m$ and τ_a

Assuming negligible axial load, due to fluctuating bending and fluctuating torsion, the amplitude and mean stresses are given by (Eq. 7-1) and (Eq. 7-2):

$$\sigma_a = K_f \frac{M_a c}{I} \qquad \sigma_m = K_f \frac{M_m c}{I} \tag{7–1}$$

$$\tau_a = K_{fs} \frac{T_a r}{J} \qquad \tau_m = K_{fs} \frac{T_m r}{J} \tag{7–2}$$

For solid shaft, stresses can be written in terms of d, the diameter, see (Eq. 7-3) and (Eq. 7-4):

$$\sigma_a = K_f \frac{32 M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32 M_m}{\pi d^3} \tag{7–3}$$

$$\tau_a = K_{fs} \frac{16 T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16 T_m}{\pi d^3} \tag{7–4}$$

Next, the equivalent (von Mises) amplitude and mean stresses are determined, resulting in (Eq. 7-5) and (Eq. 7-6):

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32 K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2} \tag{7–5}$$

$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32 K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2} \tag{7–6}$$

Now, define terms A and B (pp. 361):

$$A = \sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$

$$B = \sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}$$

Finally, the pair of equations determining factor of safety and diameter, are (DE stands for distortion energy):

- For DE-Goodman: (Eq. 7-7), (Eq. 7-8)

DE-Goodman

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}[4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \frac{1}{S_{ut}}[4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2}\right\} \qquad (7\text{–}7)$$

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}[4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \frac{1}{S_{ut}}[4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2}\right\}\right)^{1/3} \qquad (7\text{–}8)$$

- For DE-Gerber: (Eq. 7-9), (Eq-7-10)

DE-Gerber

$$\frac{1}{n} = \frac{8A}{\pi d^3 S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\} \qquad (7\text{–}9)$$

$$d = \left(\frac{8nA}{\pi S_e}\left\{1 + \left[1 + \left(\frac{2BS_e}{AS_{ut}}\right)^2\right]^{1/2}\right\}\right)^{1/3} \qquad (7\text{–}10)$$

- For DE-ASME-Elliptic: (Eq. 7-11), (Eq. 7-12)

DE-ASME Elliptic

$$\frac{1}{n} = \frac{16}{\pi d^3}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2} \qquad (7\text{–}11)$$

$$d = \left\{\frac{16n}{\pi}\left[4\left(\frac{K_f M_a}{S_e}\right)^2 + 3\left(\frac{K_{fs} T_a}{S_e}\right)^2 + 4\left(\frac{K_f M_m}{S_y}\right)^2 + 3\left(\frac{K_{fs} T_m}{S_y}\right)^2\right]^{1/2}\right\}^{1/3} \qquad (7\text{–}12)$$

- For DE-Soderberg: (Eq. 7-13), (Eq. 7-14)

DE-Soderberg

$$\frac{1}{n} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}[4(K_f M_a)^2 + 3(K_{fs}T_a)^2]^{1/2} + \frac{1}{S_y}[4(K_f M_m)^2 + 3(K_{fs}T_m)^2]^{1/2}\right\} \qquad (7\text{–}13)$$

$$d = \left(\frac{16n}{\pi}\left\{\frac{1}{S_e}[4(K_f M_a)^2 + 3(K_{fs}T_a)^2]^{1/2} + \frac{1}{S_y}[4(K_f M_m)^2 + 3(K_{fs}T_m)^2]^{1/2}\right\}\right)^{1/3} \qquad (7\text{–}14)$$

After the above fatigue-based calculations, it is customary to check against static failure.

- The factor of safety against yielding in the first loading cycle is (Eq. 7-16):

$$n_y = \frac{S_y}{\sigma'_{\text{max}}} \qquad (7\text{–}16)$$

Where the equivalent maximum stress is σ'_{max} by (Eq. 7-15)

$$\sigma'_{\text{max}} = [(\sigma_m + \sigma_a)^2 + 3(\tau_m + \tau_a)^2]^{1/2} = \left[\left(\frac{32K_f(M_m + M_a)}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs}(T_m + T_a)}{\pi d^3}\right)^2\right]^{1/2} \qquad (7\text{–}15)$$

- A quick but conservative check against yielding in the first loading cycle is, see the paragraph following (Eq. 7-16):

$$n_y = \frac{S_y}{\sigma'_a + \sigma'_m}$$

Factor of Safety or Required Shaft Diameter – Typical Cases

The typical case is defined as, fully reversed bending and steady (or constant) torsion. Therefore, setting $M_m = 0$ and $T_a = 0$ in above equations will result in what are needed.

ASME-Elliptic:

Setting $M_m = 0$ and $T_a = 0$, (Eq. 7-5) and (Eq. 7-6) and the A and B terms become:

$$\sigma'_a = \sqrt{\sigma_a^2} = \sqrt{\left(K_f\frac{32M_a}{nd^3}\right)^2} = \frac{K_f 32M_a}{nd^3}$$

$$\sigma'_m = \frac{\sqrt{3}K_{fs}16T_m}{nd^3}$$

$$A = 2K_f M_a$$

$$B = \sqrt{3}K_{fs}T_m$$

(Eq. 7-11) and (Eq. 7-12) become,

$$n = \frac{1}{\sqrt{\left(\frac{\sigma'_a}{S_e}\right)^2 + \left(\frac{\sigma'_m}{S_y}\right)^2}} = \frac{nd^3}{16}\frac{1}{\sqrt{\left(\frac{A}{S_e}\right)^2 + \left(\frac{B}{S_y}\right)^2}}$$

$$d = \sqrt[3]{\frac{16}{nd^3}\sqrt{\left(\frac{A}{S_e}\right)^2 + \left(\frac{B}{S_y}\right)^2}}$$

And (Eq. 7-15) simplified to:

$$\sigma'_{max} = \frac{16}{nd^3}\sqrt{A^2 + B^2}$$

Estimating S_e

- Shaft design equations involve S_e which in turn involves five modification factors.
- Therefore, it requires knowing the material, its surface condition, the size and geometry (stress raisers), and the level of reliability.
- Material and surface condition can be decided before the analysis.
- The sizes and geometry are however unknown in the preliminary stage of design.
- For size factor, a diameter may be determined from safety against yielding, or use 0.9 as the estimate of size factor.
- Reliability is typically set at 90%.
- Stress concentration factors K_t and K_{ts} for first iteration are given in (Table 7-1, pp. 365)

Table 7–1

First Iteration Estimates for Stress-Concentration Factors K_t and K_{ts}.

Warning: These factors are only estimates for use when actual dimensions are not yet determined. Do *not* use these once actual dimensions are available.

	Bending	Torsional	Axial
Shoulder fillet—sharp ($r/d = 0.02$)	2.7	2.2	3.0
Shoulder fillet—well rounded ($r/d = 0.1$)	1.7	1.5	1.9
End-mill keyseat ($r/d = 0.02$)	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove	5.0	3.0	5.0

Missing values in the table are not readily available.

After that (For values not in table - second round), we would have to use (A-15)

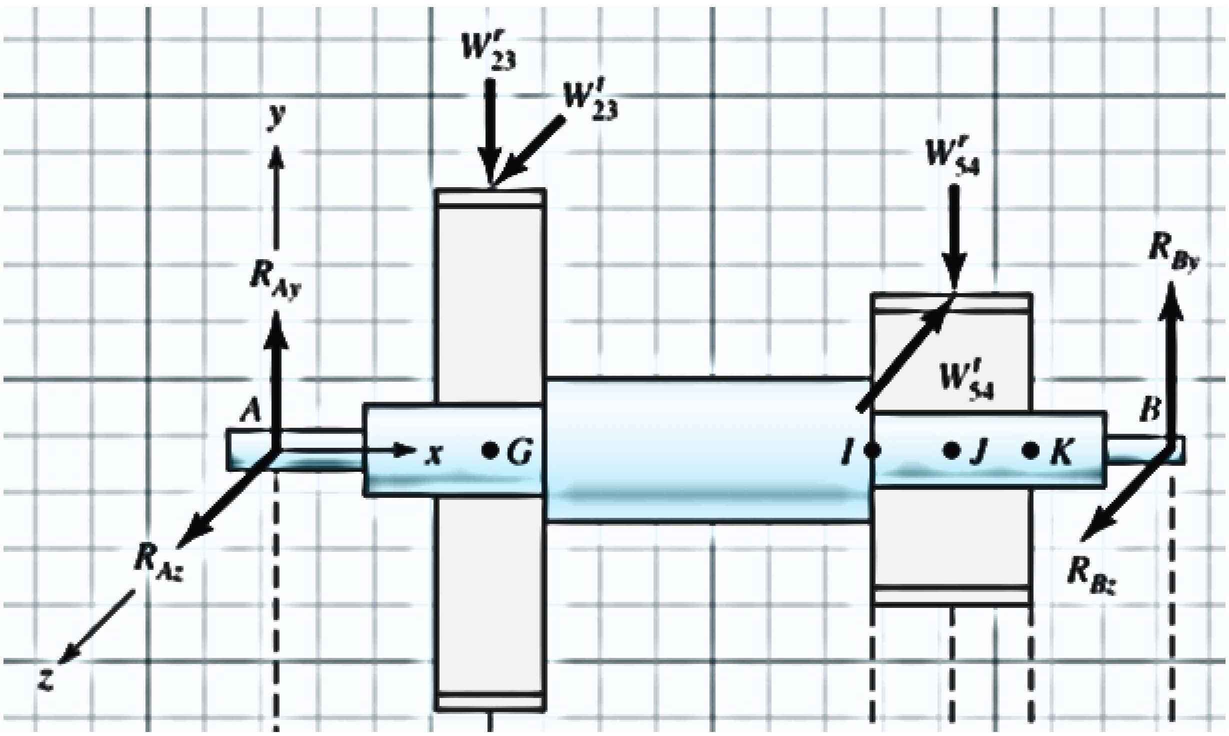
Example 7-1

Loading and design details are given. We are asked to, (1) determine n using DE-Goodman, DE-Gerber, DE-ASME, and DE-Soderberg; and (2) check against yielding failure by evaluating n_y

Example 7-2

Countershaft AB carries two spur gears at G and J; is supported by two bearings at A and B. Its layout is shown in Figure 7-10. Gear loads are

$$W_{23}^{r} = 197\ lb$$
$$W_{23}^{t} = 540\ lb$$
$$W_{54}^{r} = 885\ lb$$
$$W_{54}^{t} = 2431\ lb$$



We are to select appropriate materials and/or diameters at various cross sections, based on fatigue with infinite life. Design factor is 1.5.

The text starts with cross-section I where there are torque and bending moment, and a shoulder for stress concentration. Generous shoulder fillet (r/d = 0.1) is assumed DE-Goodman is used to determine diameters.

Example

A critical cross-section of a shat is subject to a combined bending moment of 63-lb-in and a torque of 74 lb-in. The cross-section is the seat of a rolling-element bearing, and sharp shoulder fillet is expected. Shaft material is SAE 1040 CD. Estimate the shaft's diameter at the cross-section for infinite life with $n? = 1.5$. Base calculations on ASME-Elliptic criterion. Operating conditions are typical.

Solution

(1) First iteration

$M_a = 63\ lb - in$
$T_m = 74\ lb - in$
$S_{ut} = 85\ ksi$
$S_y = 71\ ksi$
$S_e' = 42.5\ ksi$
$k_a = 2.7(85)^{-0.265} = 0.832$
$k_b = 0.9$
$k_c = 1$
$k_d = 1$
$k_e = 0.897$
$S_e = 28.55\ ksi$
$k_t = 2.7$
$k_{ts} = 2.2$

Set $q = q_{shear} = 1$, so that $K_f = K_t = 2.7$ and $K_{fs} = K_{ts} = 2.2$, and
$A = 2K_f M_a = 340.2\ lb - in$
$B = \sqrt{3}K_{fs}T_m = 282.0\ lb - in$

$$d = \sqrt[3]{\frac{16n}{\pi}\sqrt{\left(\frac{A}{S_e}\right)^2 + \left(\frac{B}{S_y}\right)^2}} = 0.458"$$

Round off $d = 12\ mm = 0.472"$

(2) Second iteration

$M_a = 63\ lb - in$
$T_m = 74\ lb - in$
$S_{ut} = 85\ ksi$
$S_y = 71\ ksi$
$S_e' = 42.5\ ksi$
$k_a = 2.7(85)^{-0.265} = 0.832$
$k_b = 0.879(0.472)^{-0.107} = 0.953$
$k_c = 1$
$k_d = 1$
$k_e = 0.897$
$S_e = 30.23\ ksi$

$\frac{r}{d} = 0.02$, then $r = 0.009$” , and $q = 0.57$, $q_{shear} = 0.6$, so that $K_f = 1.91$ and$K_{fs} = 1.66$

Finally,
$A = 2K_f M_a = 240.7\ lb - in$
$B = \sqrt{3}K_{fs}T_m = 212.8\ lb - in$

$$n = \left(\frac{\pi d^3}{16}\right)\left(\frac{1}{\sqrt{\left(\frac{A}{S_e}\right)^2 + \left(\frac{B}{S_y}\right)^2}}\right) = 2.4$$

$$\sigma'_{max} = \frac{16}{\pi d^3}\sqrt{A^2 + B^2} = 15.56\ ksi$$

$$n_y = \frac{S_y}{\sigma'_{max}} = 4.6$$

Therefore, $d = 12\ mm$ or $0.472"$ is sufficient, giving a factor of safety against fatigue at 2.4 and a factor of safety against yielding at 4.6.

(7-5) Deflection Consideration
Why deflection considerations?

- Shaft deflects transversely like a beam.
- Shaft also has torsional deflection like a torsion bar.
- Excessive deflections affect the proper functioning of gears and bearings, for example. Permissible slopes and transverse deflections are listed in Table 7-2.
- To minimize deflections, keep shaft short, and avoid cantilever or overhang.

How to Determine Beam Deflection

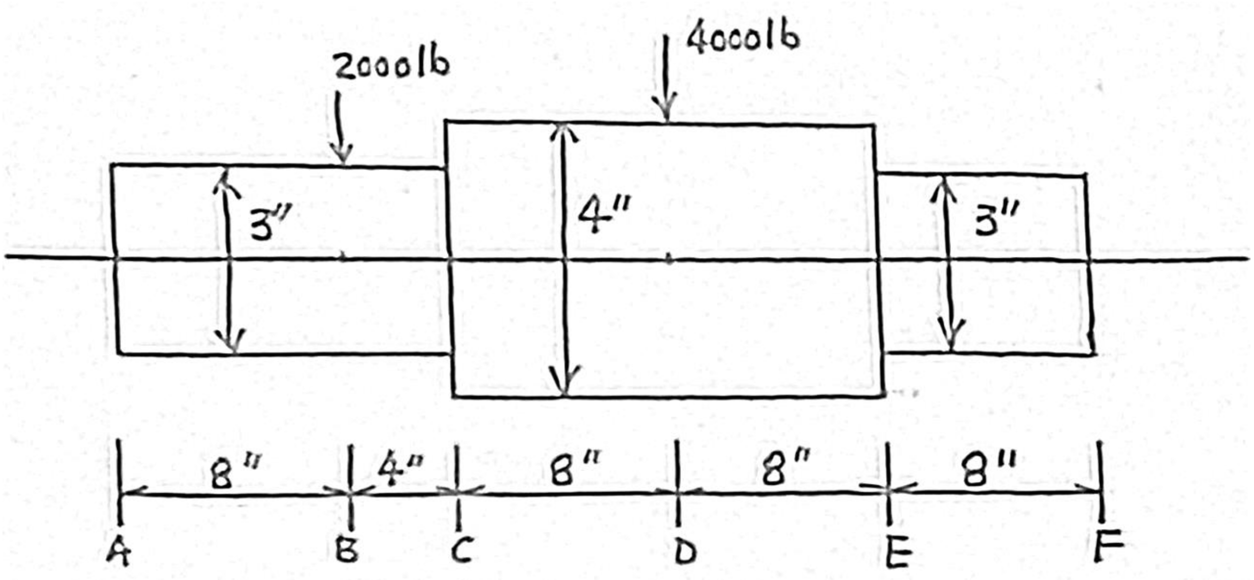
- Analytically
  Closed-form solutions (4-4)
  Superposition (4-5)
  Singularity Functions (4-6)
  Strain Energy (4-7)
  Castigliano's 2nd Theorem (4-8)
  Statically indeterminate beams (4-10)
  …..
  and so on.

Limitations**:** effective when EI=const. (but a stepped shaft does not have constant I)

- Numerical integration
  Simplified geometry; for example, small shoulders (diameter-wise and length-wise), fillets, keyways, notches, etc., can be omitted.
  May be tedious.

Example

A stepped shaft is shown below, which is supported by ball bearings at A and F. Determine its maximum (in magnitude) lateral deflection, and the slopes at A and F. Use $E = 30\ Mpsi$.



1) Plot bending moment $M(x)$

2) Plot M/d^4
$$\therefore \frac{M}{EI} = \frac{\left(\frac{64}{\pi E}\right) M}{d^4} = kM/d^4$$

3) Integrate
$$\frac{M}{d^4} => "slope"$$

4) Integrate $"slope"$
$$\frac{M}{d^4} => "deflection"$$

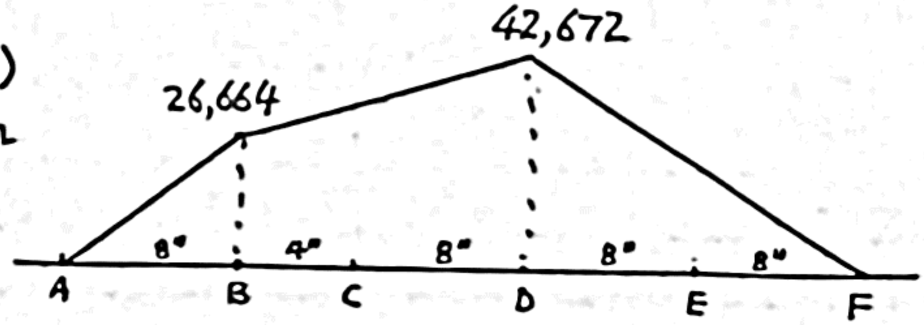
5) Baseline

6) Deflection

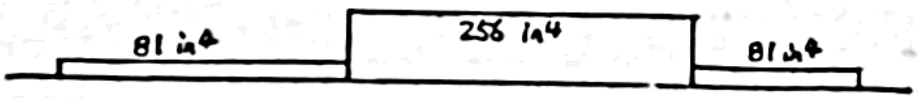
7) Slope

1) $M(x)$

lb.in

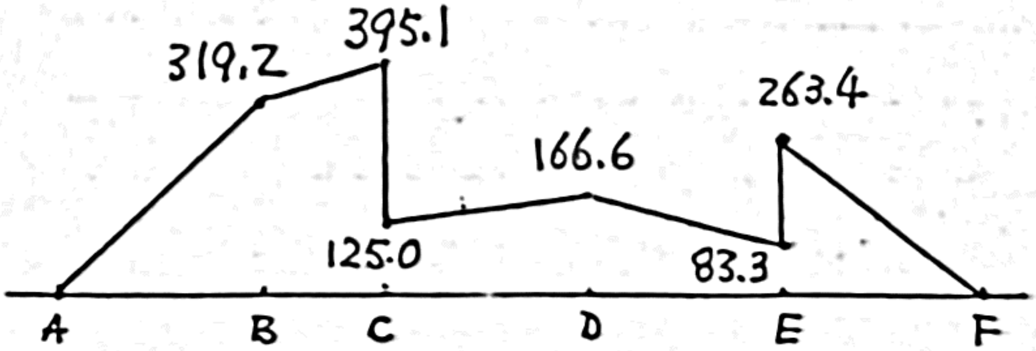


d^4



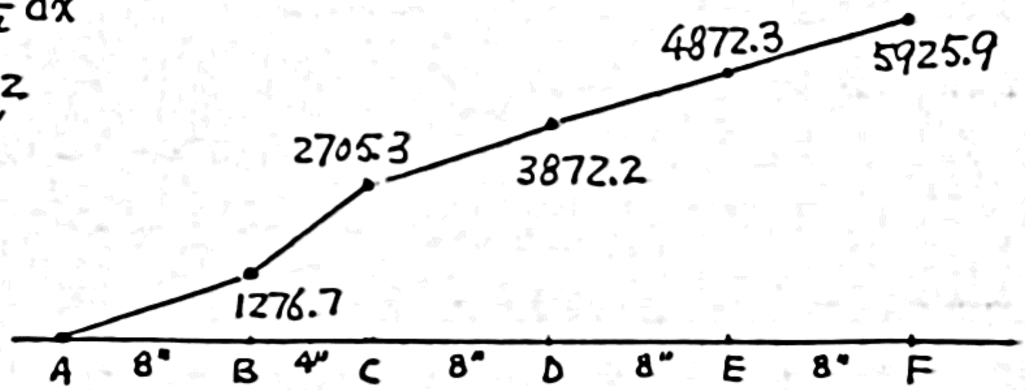
2) $\frac{M}{d^4}$

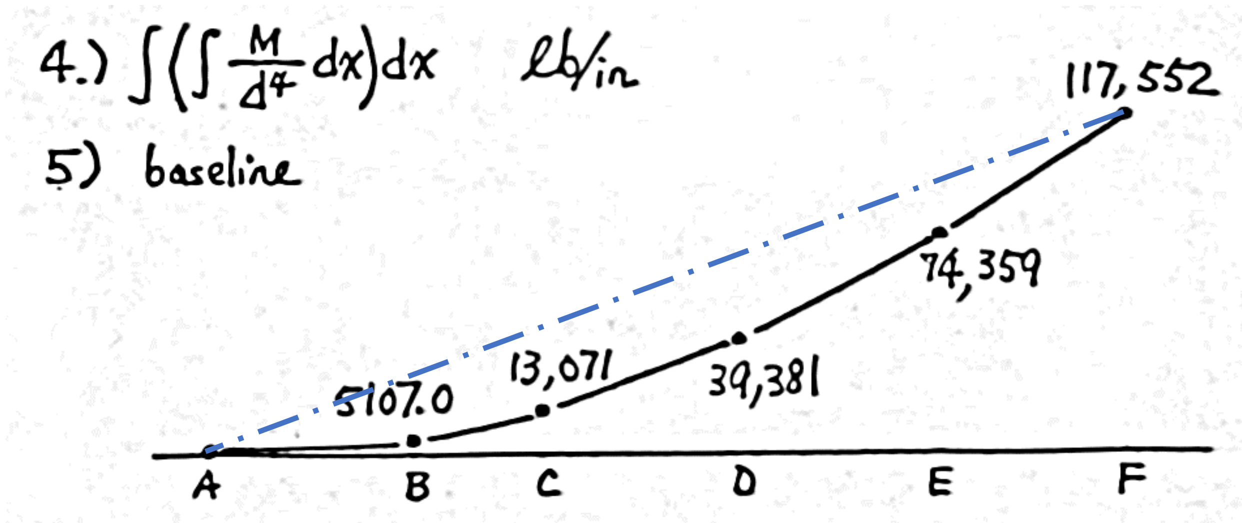
lb/in³

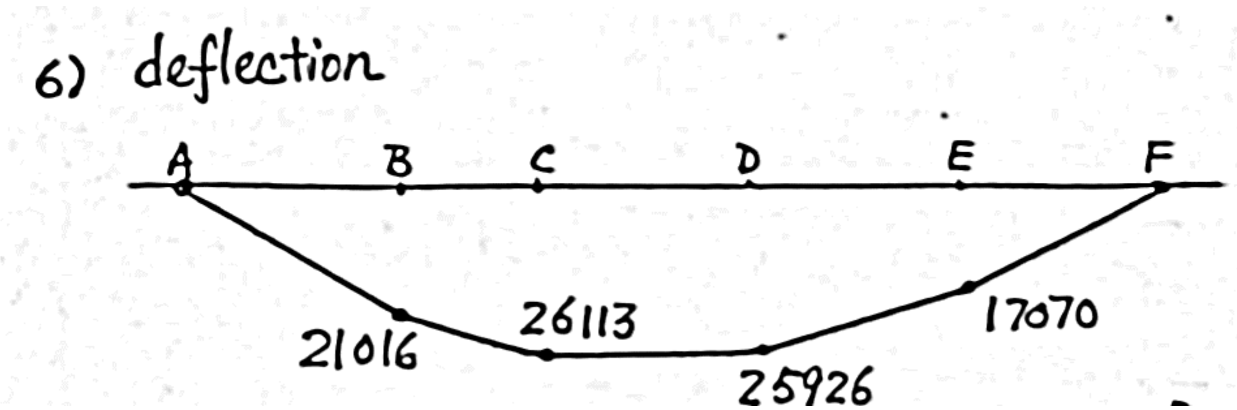


3) $\int \frac{M}{d^4}\,dx$

lb/in²





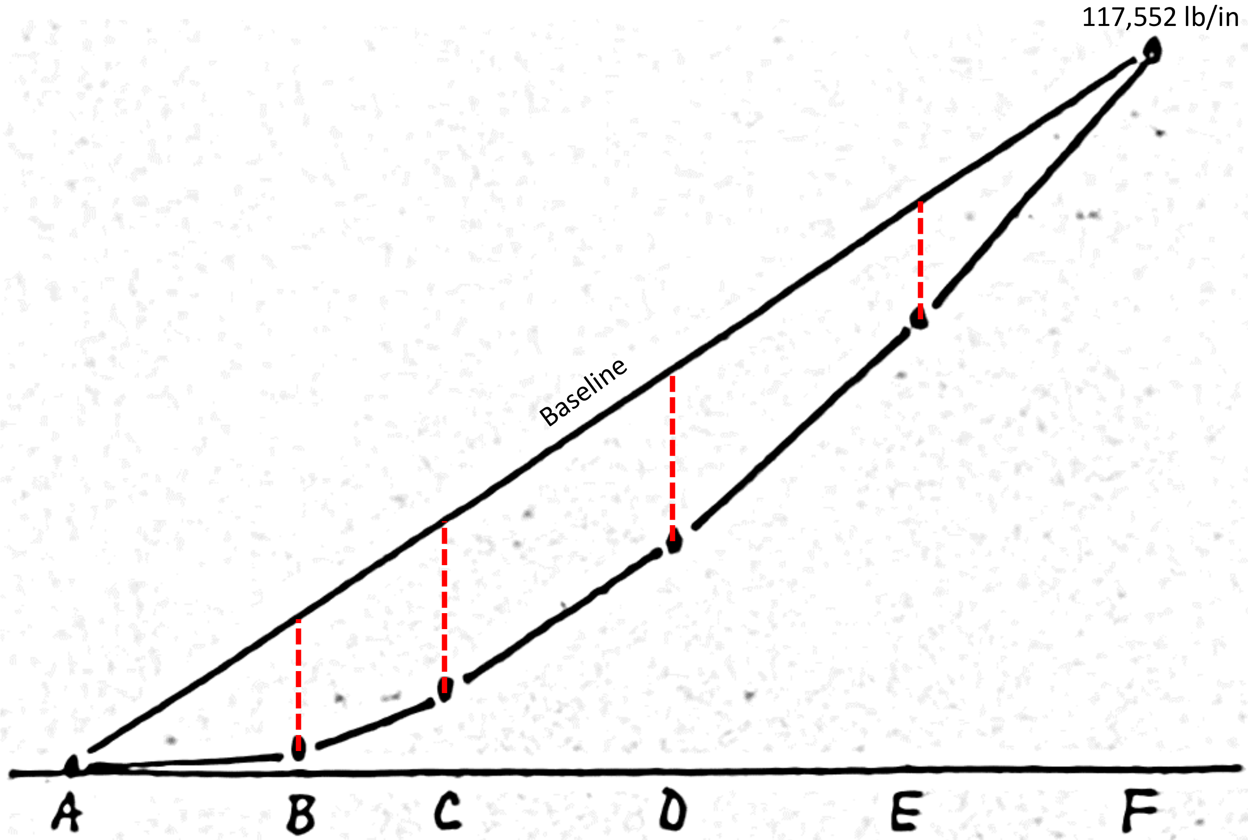


To obtain deflection curve of step (6), at any cross-section, subtract value obtained in step (4) from baseline value.

For example, at B, step (4) has 5107.0;

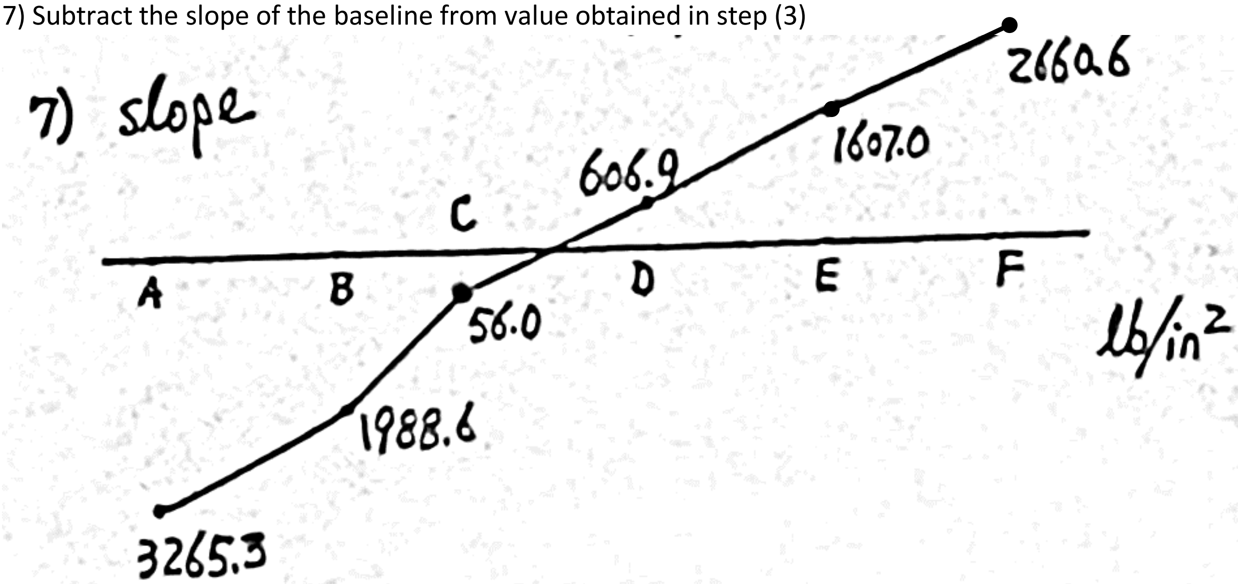
baseline value is 26,123;

$\therefore$ $21{,}106 = 26{,}123 - 5107.0$ will be used in step (6)



$$slope\ of\ baseline = m = \frac{117{,}552}{36"}\left(\frac{lb}{in}\right) = 3265.3\ lb/in^2$$

7) Subtract the slope of the baseline from value obtained in step (3)



$$k = \frac{64}{\pi E} = 0.6791 * 10^{-6}\left(\frac{in^2}{lb}\right)$$

$$\therefore \delta_{max} = \left(26{,}113\frac{lb}{in}\right)\left(0.679 * 10^{-6}\frac{in^2}{lb}\right)$$

$$= 0.0177\ in\ \downarrow$$

$$\theta_A = \left(3265.3\frac{lb}{in^2}\right)\left(0.679 * 10^{-6}\frac{in^2}{lb}\right)$$

$$= 0.00222\ rad\ \downarrow TODO\ curve$$

$$\theta_F = 0.00181\ rad\ \uparrow TODO\ curve$$

Comments regarding the numerical integration method:
Applicable to simple supports as outlined;
Fixed supports?
More divisions for better accuracy;
Vertical plane, horizontal plane, and vector sum.

An exact numerical method for determining the bending deflection and slope of stepped shafts, C.R. Mischke, in *Advanced in reliability and stress analysis*, ASME winter annual meeting, December 1978

Mechanical Design of Machine Elements and Machines, J.A. Collins, John Wiley & Sons, 2003 (Sec. 8.5)

Example (7-3)
By the end of Example 7-2, diameters D_1 through D_7 were determined. The layout is shown below (Figure 7-10). Here we are to evaluate the slopes and deflections at key locations.

The text uses “Beam 2D Stress Analysis” (a software with FEA-core) for the evaluation.

The results are verified by the above numerical integration method implemented with MATLAB.

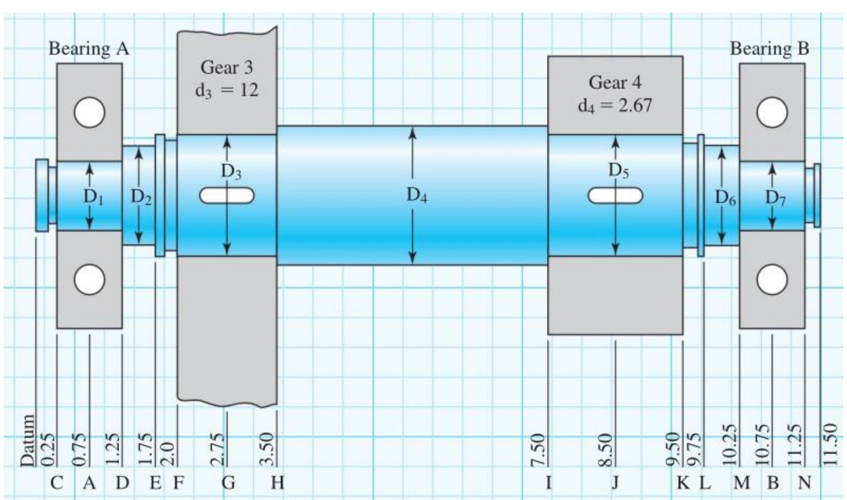


Figure 7–10

Shaft layout for Ex. 7–2. Dimensions in inches.

Diameter	$D_1 = D_7$	$D_2 = D_6$	$D_3 = D_5$	D_4
Example 7-2	1.0	1.4	1.625	2.0

Point of Interest	Example 7-3	Numerical Integration
Slope, left bearing (A)	0.000501 rad	0.000507 rad
Slope, right bearing (B)	0.001095 rad	0.001090 rad
Slope , left gear (G)	0.000414 rad	0.000416 rad
Slope, right gear (J)	0.000426 rad	0.000423 rad
Deflection, left gear (G)	0.0009155 in	0.0009201 in
Deflection, right gear (J)	0.0017567 in	0.0017691 in

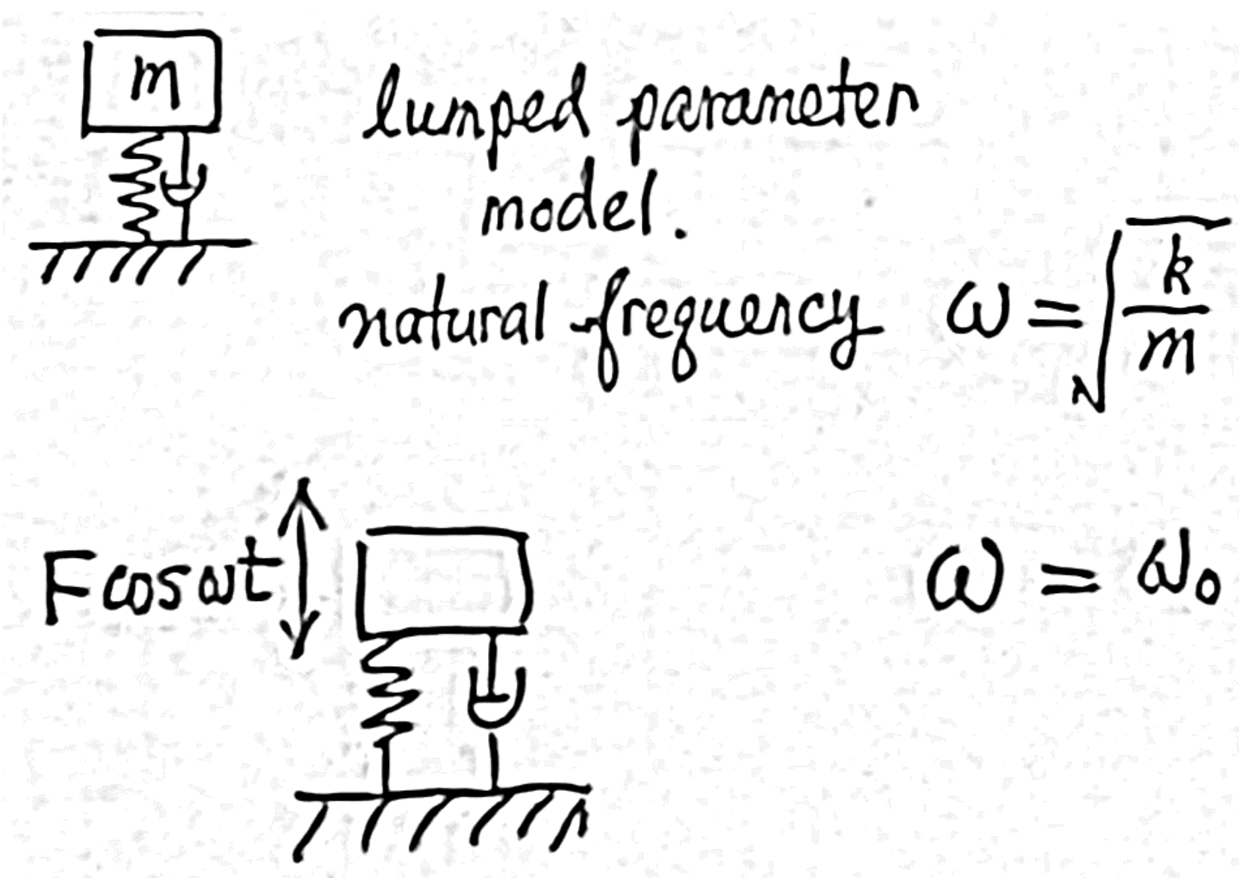
How to Determine Torsional (Angular) Deflection

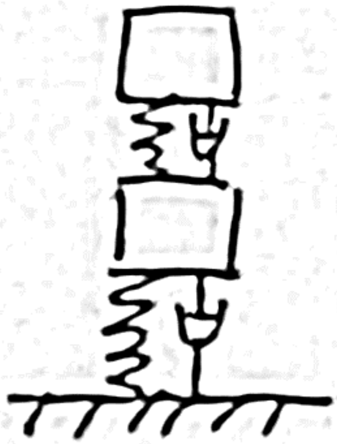
- Important for shafts carrying components that are required to function in sync with each other; for example, cam shafts;
- For a stepped shaft with individual cylinder length l_i, torque T_i and material G_i the angular deflection is,

$$\theta = \sum \theta_i = \sum \frac{T_i l_i}{G_i J_i} \tag{7-19}$$

7-6 Critical Speeds for Shafts

It is about applying knowledge of vibrations and deflections of shafts in the design of shafts.

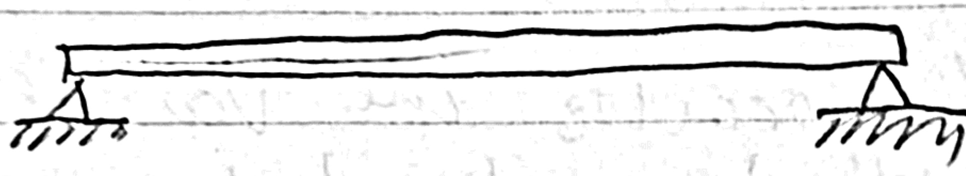




2 ω_1, ω_2

Continuum Model

∞ - DOF



$\omega_1, \omega_2, \ldots, \omega_n$

The organization of this section:

(Eq. 7-22): Exact solution of critical speed for a simply supported shaft with uniform cross section and material

$$\omega_1 = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{m}} = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{gEI}{A\gamma}} \qquad (7\text{–}22)$$

(eq. 7-23): Rayleigh's method to estimate critical speed.

$$\omega_1 = \sqrt{\frac{g \sum w_i y_i}{\sum w_i y_i^2}} \qquad (7\text{–}23)$$

(Eq. 7-24): Through (Eq. 7-32) Derivation of Dunkerley's method to estimate critical speed.

(see pp. 376-377)

Example (7-5)

Notes: (a) Rayleigh's and Dunkerley's methods only give rise to estimates; (b) They yield the upper and lower bound solutions, respectively. That is, $\omega_{1(Dunkerley)} < \omega_1 < \omega_{1(Rayleigh)}$; (c) The methods and their derivations fall under vibrations/dynamics of continuum by energy method.

Critical Speeds

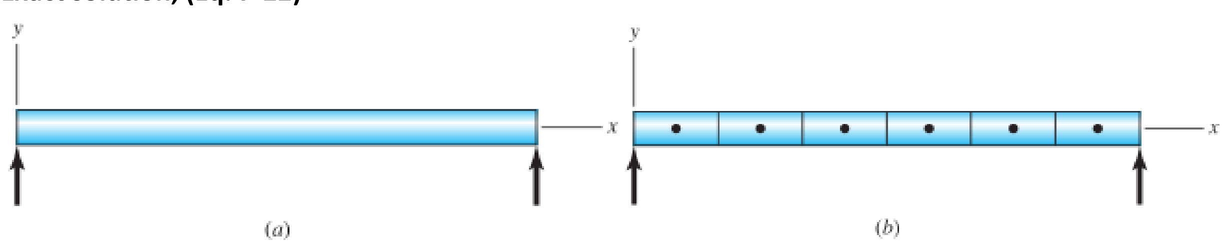
- Critical speeds refer to speeds at which the shaft becomes unstable, such that deflections (due to bending or torsion) increase without bound.
- A critical speed corresponds to the fundamental natural frequency of the shaft in a particular vibration mode.
- Three shaft vibration modes are to be concerned: lateral, vibration, shaft whirling and torsional vibration.
- Critical speeds for lateral vibration and shaft whirling are identical.
- Numerically speaking, two critical speeds can be determined, one for lateral vibration or shaft whirling, and another for torsional vibration.
- Focus will be the critical speed for lateral vibration or shaft whirling. Regarding critical speed for torsional vibration, one can reference "rotor dynamics" and the transfer matrix method.
- If the critical speed is ω_1, it is required that the operating speed ω be:

  If the shaft is rigid (shafts in heavy machinery):
  $$\frac{\omega}{\omega_1} \geq 3$$

  If the shaft is flexible (shafts that are long with small-diameters):
  $$\frac{\omega}{\omega_1} \leq 1/3$$

  The text recommends:
  $$\frac{\omega}{\omega_1} \leq 1/2$$

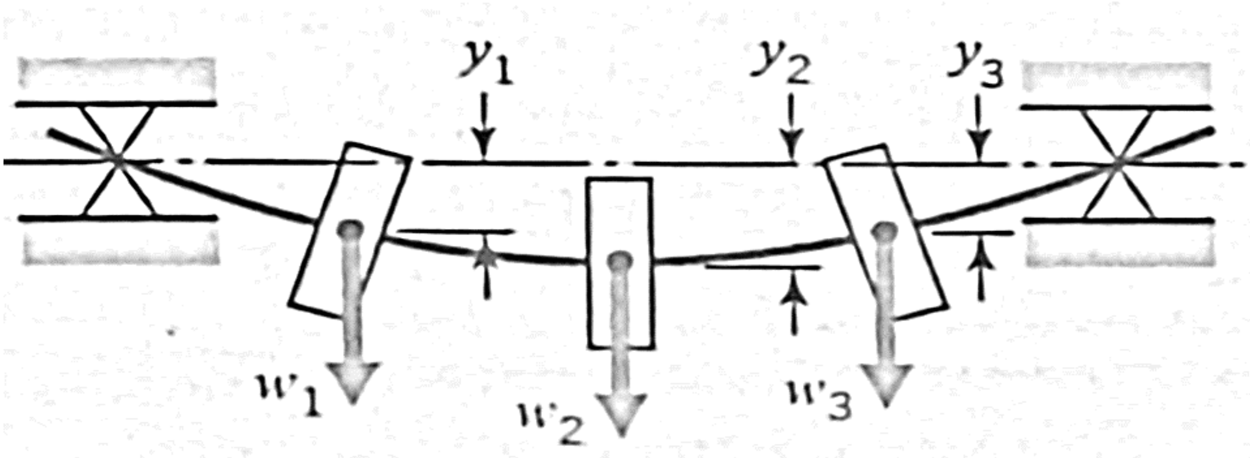
Exact solution, (Eq. 7-22)



Simply supported, uniform cross-section and material.

$$\omega_1 = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{m}} = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{gEI}{A\gamma}} \qquad (7\text{–}22)$$

Rayleigh's Method for Critical Speed, (Eq. 7-23)
Shaft is considered massless and flexible; Components such as gears, pulleys, flywheels, and so on, are treated as lumped masses; The weight of the shaft, if significant, will be lumped as a mass or masses.



$$\omega_1 = \sqrt{\frac{g \sum w_i y_i}{\sum w_i y_i^2}} \qquad (7\text{–}23)$$

Textbook equation should be updated to include the following absolute symbols:

$$\omega_1 = \sqrt{g \frac{\sum w_i |y_i|}{\sum w_i y_i^2}}$$

Where:
w_i = weight of mass i
w_i should be treated as a force with a magnitude equation the weight of mass i;

Forces w_i ($i = 1, \ldots$) should be applied in such a way that the deflection curve resembles the fundamental mode shape of lateral vibration.

y_i = lateral deflection at location i (where w_i is applied) and caused by all forces.

Dunkerley's Method for Critical Speed, (Eq. 7-32)
The model for Dunkerley's method is the same as that for Rayleigh's.

$$\frac{1}{\omega_1^2} \approx \sum_{i=1}^{n} \frac{1}{\omega_{ii}^2} \qquad (7\text{–}32)$$

and $\omega_{ii} = \sqrt{\frac{g}{|y_{ii}|}}$

Where y_{ii} is the lateral deflection at location i and caused by w_i only. ω_{ii} represents the critical speed with only w_i on the shaft.

Deflections y_i y_{ij} and δ_{ij}

y_{ij} is the deflection at location i and due to a load applied at location j. When the load at location j is a unit load, then y_{ij} is denoted by δ_{ij}. δ_{ij} is also known as the influence coefficient.

y_i is the deflection at location i and caused by all applied loads. Therefore,

$$y_i = \sum_j y_{ij}$$

unit load, then Type equation here.unit load, then y_{ij} is denoted at δ_{ij}. δij**Problem Solving**

Closed-form solutions (Sec. 4-4 and Table A-9).

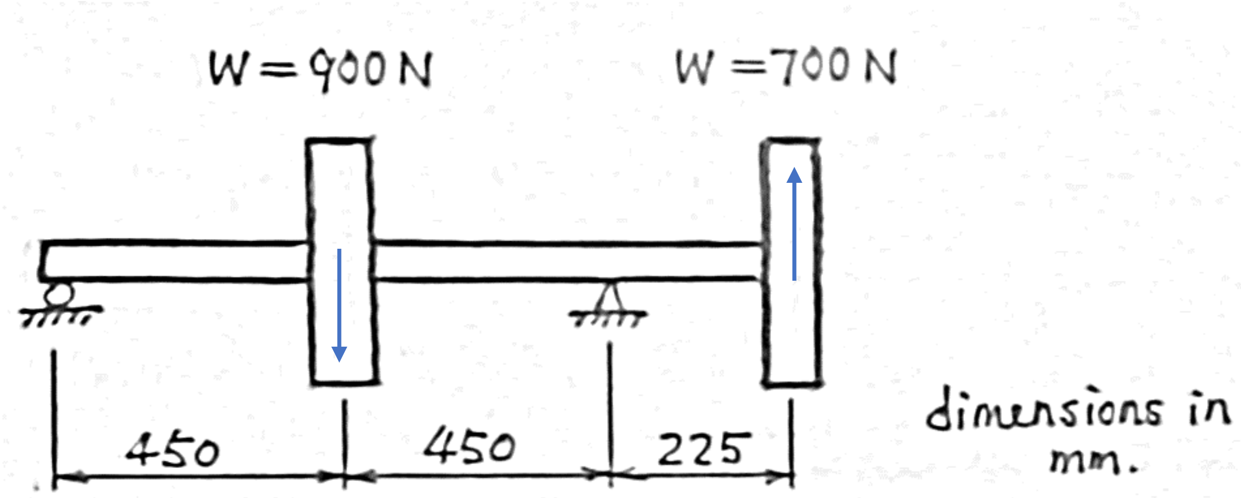
Superposition (Section 4-5);

Indexes i and $j$each runs 1 through the number of masses/forces;

y_{ij} are signed numbers;

Example:

Evaluate the range of the shaft's critical speed corresponding to its lateral vibration, in terms of EI where $EI = constant$.



$i = 1,2$

$j = 1,2$

From previous example:

$\begin{cases}\text{Table A}-9\\ \quad\text{Case 10}\end{cases}$

$$y_{12} = y_{AB}|_{x=0.45} = \frac{(-700)(0.225)(0.45)(0.9^2 - 045^2)}{(6*0.9)EI}$$

$$= -\frac{7.9738}{EI}$$

$$y_{22} = y_t = -\frac{(-700)(0.225)^2(0.9+0.225)}{3EI} = \frac{13.289}{EI}$$

$\begin{cases}\text{Table A}-9\\ \quad\text{Case 5}\end{cases}$

$$y_{11} = y_{max} = -\frac{(900)(0.9)^3}{48EI} = -\frac{(13.669)}{EI}$$

$$y_{AB} = \frac{Fx}{48EI}(4x^2 - 3l^2)$$

$$\theta_{AB} = \frac{dY_{AB}}{dx} = \frac{F}{16EI}(4x^2 - l^2) \quad ; \quad 0 \le x \le \frac{l}{2}$$

$$\theta_A = \frac{(900)(-0.9^2)}{16EI} = -\frac{45.563}{EI}$$

$$y_{21} = (-\theta_A)(0.225) = \frac{10.252}{EI}$$

Or:

$$y_{21} = (\theta_C)(0.225) = \frac{10.252}{EI}$$

$$\therefore y_1 = y_{11} + y_{12} = -\frac{21.643}{EI}$$

$$y_2 = y_{21} + y_{22} = \frac{23.541}{EI}$$

Rayleigh’s Method:

$$\therefore \omega_1 = \sqrt{\frac{g(\sum w_i|y_i|)}{(\sum w_i {y_i}^2)}}$$

$$= \sqrt{(9.81)\frac{(900)\left(\frac{21.643}{EI}\right) + (700)\left(\frac{23.541}{EI}\right)}{(900)\left(\frac{21.643}{EI}\right)^2 + (700)\left(\frac{23.541}{EI}\right)^2}}$$

$$= 0.66011\sqrt{EI}$$

Dunkerley’s Method:

$$y_{11} = -\frac{13.669}{EI}$$

$$y_{22} = \frac{13.289}{EI}$$

$$\omega_{11}^2 = \frac{g}{|y_{11}|} = 0.71768\ EI$$

$$\omega_{22}^2 = \frac{g}{|y_{22}|} = 0.73820\ EI$$

$$\therefore \frac{1}{\omega_1^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{22}^2} = \frac{2.7480}{EI}$$

$$\omega_1 = \sqrt{\frac{EI}{2.7480}} = 0.60324\sqrt{EI}$$

Take $E = 200\ GPa, d = 25\ mm$
$\sqrt{EI} = 1261.6\ (N \cdot m^2)$

$\therefore\ \omega_{1(Dunkerley)} = (0.60324)(1261.6)$
$= 761.0\ rad/s$
$Or\ n_{1(Dunkerley)} = 7267\ rpm$
$Also\ \omega_{1(Rayleigh)} = 832.8\ rad/s$
$Or\ n_{1(Rayleigh)} = 7952\ rpm$

$\therefore Operating\ Speed \leq \frac{1}{3}\omega_1$
$Or\ n \leq 2422\ rpm$

7-3 Shaft Layout

- Between a shaft and its components (e.g., gears, bearings, pulleys, etc.), the latter must be located axially and circumferentially.
- Means to provide for torque transmission
  Keys
  Splines
  Setscrews
  Pins
  Press/shrink fits
  Tapered fits
  etc.
- Means to provide for axial location – large axial load shoulders
  Shoulders
  Retaining rings
  sleeves
  Collars
  etc.
- Means to provide for axial location – small axial load
  Press/shrink fits
  Setscrews
  etc.
- Locating rolling element bearings
  See Ch. 11

7-7 Miscellaneous Shaft Components
Includes:
- Setscrews
- Keys and pins
- Retaining rings

Focus:
Keys
Example 7-6

7-8 Limits and Fits
- Fits (clearance, transition, and interference) are to ensure that a shaft and its components/attachments will function as intended.
- Preferred fits are listed in Table 7-9
- Medium drive fit and force fit will give rise to the press/shrink fits.
- Press-fit is typically for small hubs; Shrink fit (or expansion fit) it used with larger hubs
- How much diameter interference to have?
  0.001” for up to 1” of diameter
  0.002” for diameter 1” to 4”
- Press/shrink fits can be designed to transfer torque and axial load.
- Press/shrink fits are known to be associated with fretting corrosion (loss of material from the interface)
- (Section 3-14) for stress distributions in thick-walled cylinder under pressures.
- (Section 3-16) for stresses developed in the shaft and hub due to pressure induced by a press/shrink fit; or (Eq 7-39) to (Eq. 7-47)
- Axial load and torque capacities: (Eq. 7-48) and (Eq. 7-49)
- Radial interference versus diametral interference (not necessarily the same thing).

Chapter 11: Rolling-Contact Bearings

Part 1: (Introduction)
11-1 Bearing Types

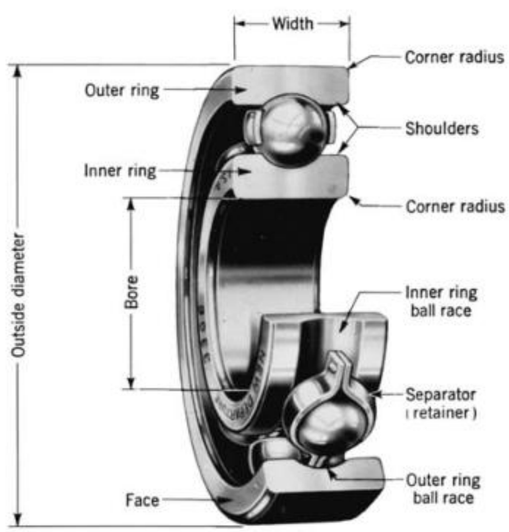
Part 2: (The Basics)
11-2 Bearing Life
11-3 Bearing Life at Rated Reliability
11- 4Reliability versus Life – The Weibull Distribution
11-5 Relating Bearing Load, Life and Reliability

Part 3: (Selection of Bearing)
11-6 Combined Radial and Thrust Bearing
11-8 Selection of Ball and Roller Bearings
11-7 Variable Loading
11-10 Design Assessment

Part 4: Others
11-12 Mounting and Enclosure
11-11 Lubrication
11-9 Selection of Tapered Roller Bearings

11-1 Bearing Types
Nomenclature
See Figure 11-1

Figure 11–1
Nomenclature of a ball bearing. *(General Motors Corp. Used with permission, GM Media Archives.)*



Classifications
- By shape of rolling elements (sphere, cylinder, tapered, etc.)
- By type of loads taken (radial only, axial only, combination)
- By permissible slope (Self-aligning, non-self-aligning)
- Sealed? Shielded?
- Figures 11-2, 11-3

Figure 11–2

Various types of ball bearings.

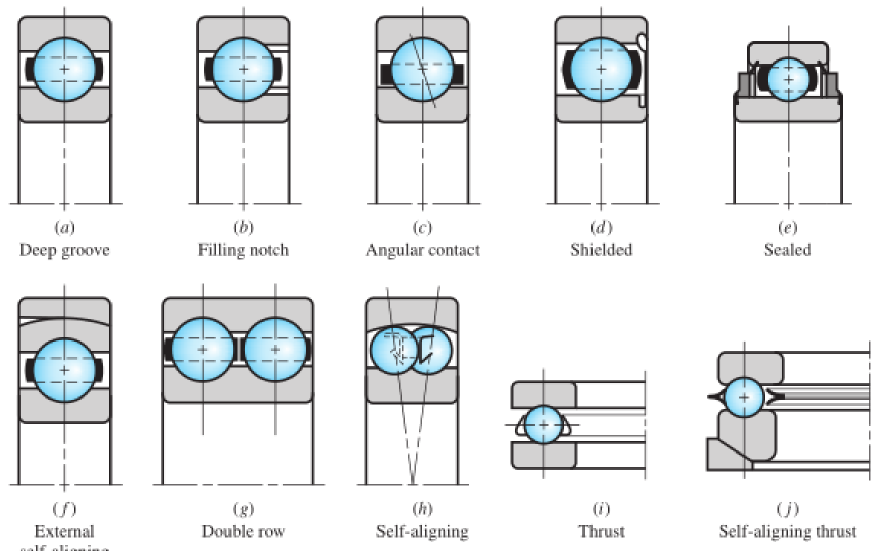
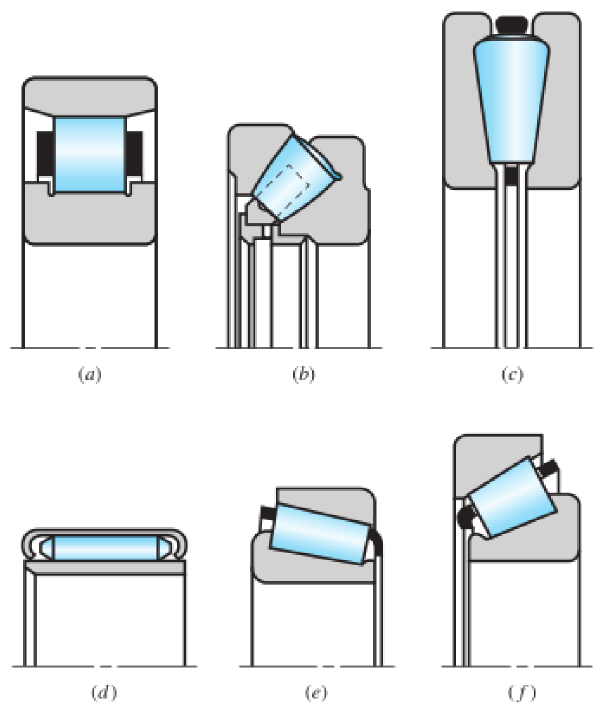


Figure 11–3

Types of roller bearings: (*a*) straight roller; (*b*) spherical roller, thrust; (*c*) tapered roller, thrust; (*d*) needle; (*e*) tapered roller; (*f*) steep-angle tapered roller. *(Courtesy of The Timken Company.)*



11-2 Bearing Life

Why Bearing Life?

- Bearings are under cyclic contact stresses (compressive as well as shear). As a result, they may experience crack, putting, spalling, fretting, excessive noise, and vibration.
- Common life measures are:
  - Number of revolutions of the inner ring (outer ring stationary) until the first evidence of failure; and
  - Number of hours of use at standard angular speed until the first evidence of fatigue
- Number of revolutions is more common

Rating Life (or **Rated Life)**

- This is the terminology used by ABMA (American Bearing Manufacturers Association) and most bearing manufacturers.
- It is defined as, of a group of nominally identical bearings, the number of revolutions that 90% of the bearings in the group will achieve or exceed, before failure occurs.
- It is denoted as L_{10} or B_{10} life.
- The typical value for L_{10} or B_{10} is 1 million.
- However, a manufacturer can choose its own specific rating life.
- For example, Timken uses 90 million for tapered roller bearings, but 1 million for its other bearings.
- Refer to bearings catalog for value(s) of L_{10}.

11-3 Bearing Life at Rated Reliability

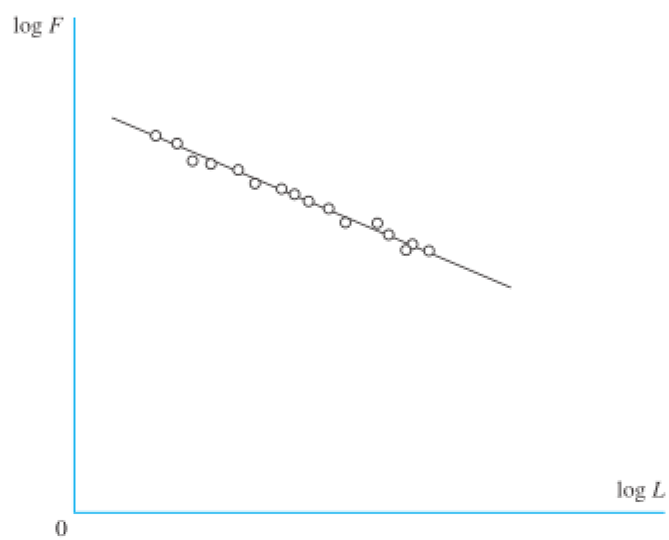
- At rated reliability of 90%, bearing life L relates to bearing's radial load F by:

$$FL^{1/a} = constant$$

Where $a = 3$ for ball bearings and $a = 10/3$ or roller bearings.
Figure 11-4 shows the meaning of (Eq. 11-1)

Figure 11–4

Typical bearing load-life log-log curve.



  - It's a straight line on log-log scales;
  - Points on the line will have the same reliability;
  - The line corresponding to 90% reliability is called the rated line.

- To determine the constant on the RHS of (Eq. 11-1), L is set to L_{10}. The corresponding F is designated as C_{10}.
  C_{10} is called the **Basic Dynamic Load Rating**, or the **Basic Dynamic Rated Load** It is defined as the radial load that causes 10% of the group of nominally identical bearings to fail at or before L_{10} (1 million, or 90 million, or revs as chosen by a manufacturer).
- (Equation 11-1) becomes:

$$F_D L_D^{1/a} = C_{10} L_{10}^{1/a} \qquad (11-3)^*$$

$$C_{10} = F_R = F_D \left(\frac{L_D}{L_R}\right)^{1/a} = F_D \left(\frac{\mathcal{L}_D n_D 60}{\mathcal{L}_R n_R 60}\right)^{1/a} \qquad (11\text{–}3)$$

Where the subscript D means design. (Equation 11-3)* is essentially (Eq. 11-3) of the text. In (Eq. 11-3), the subscript R means rated.

- Example 11-1: find C_{10} from known L_D, F_D and L_{10}

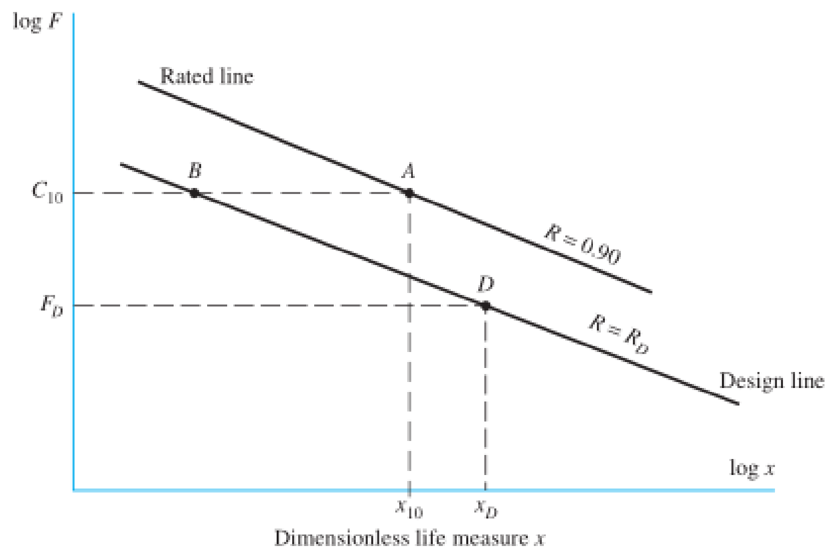
11-4 Reliability versus Life – The Weibull Distribution

11-5 Relating Load, Life and Reliability

- At 90% reliability, (Eq. 11-3*) forms the basis for selecting a bearing.
- What if the reliability is not 90%?
  Figure 11-5 shows the process of going from the rated line to a different line.
  $A \to D$, with B being the intermediary.

Figure 11–5

Constant reliability contours. Point A represents the catalog rating C_{10} at $x = L/L_{10} = 1$. Point B is on the target reliability design line R_D, with a load of C_{10}. Point D is a point on the desired reliability contour exhibiting the design life $x_D = L_D/L_{10}$ at the design load F_D.



- $A \to B$: Load is constant, and life measure (which is a random variable) follows a three-parameter Weibull distribution
- $B \to D$: Reliability is constant, and (Eq. 11-3*) is valid; The mathematics is given in (Sec. 11-4) and (Sec. 11-5).
- Another approach is to use a reliability factor a_1, the value of which depends on R, the reliability. Values of a_1 are available from a number of references.

R, %	Reliability Factor, a_1
90	1.00
95	0.62
96	0.53
97	0.44
98	0.33
99	0.21

- The factor a_1 can be determined by (courtesy of, for example SKF catalog)

$$a_1 = 4.48\left(\ln\frac{100}{R}\right)^{2/3}$$

Where R is in %, e.g., $R = 92.5$. Note: $R \leq 99$.

- Timken recommends the following formula for a_1

$$a_1 = 4.26\left(\ln\frac{100}{R}\right)^{2/3} + 0.05$$

Where $R \leq 99.9$

The Basic Bearing Equation

The basic bearing equation can now be obtained by, in (Eq. 11-3), introducing the reliability factor a_1 and a load-application factor k_a. That is,

$$F_D L_D^{1/a} = C_{10} L_{10}^{1/a}$$

Becomes,

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

k_a is given in (Table 11-5)

Table 11-5

Load-Application Factors

Type of Application	Load Factor
Precision gearing	1.0–1.1
Commercial gearing	1.1–1.3
Applications with poor bearing seals	1.2
Machinery with no impact	1.0–1.2
Machinery with light impact	1.2–1.5
Machinery with moderate impact	1.5–3.0

This basic equation can be used for bearing selection and for assessment after selection.

Example 1

A SKF deep-groove ball bearing is subjected to a radial load of 495 lb. The shaft rotates at 300 rpm. The bearing is expected to last 30,000 hours (continuous operation). Catalog shows a $C_{10} = 19.5\ kN$ on the basis of 10^6 revs. (1) is the bearing suitable for 90% reliability? (2) Also assess the bearing's reliability. Set $k_a = 1$.

Solution:

The basic bearing equation is:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

(Where a and L_{10} are generally set by the manufacturer, k is a variable we can change.)

Where:

$C_{10} = 19.5\ kN = 4387.5\ lb$; $L_{10} = 10^6\ revs$; $k_a = 1$; $a = 3$

Also $F_D = 495\ lb$; and $L_D = (30{,}000)(60)(300) = (540)(10^6)\ revs$.

Assume 90% reliability, then $a_1 = 1$. From the basic bearing equation,

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting values, $LHS = 8.92$, $RHS = 8.14$. Therefore, (L_D, F_D) is <u>not</u> on the rated line.

There are a number of ways to seek the answer.
90% reliability, $a_1 = 1$;

<u>The first</u>: *similar to the typical calculations done for selecting a bearing*
Set $F_D = 495\ lb$; $L_D = (540)(10^6)\ revs$; and $L_{10} = 10^6\ revs$; find C_{10} and check if it is less than the $4387.5\ lb$ that the bearing is capable of providing.
From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{C_{10}}{(1)(495)} = \left(\frac{(540)(10^6)}{(1)(10^6)}\right)^{1/3}$$

Resulting in $C_{10} = 4031\ lb$
Since it's less than the catalog's C_{10}, or $4387.5\ lb$, the selected bearing is suitable for 90% reliability.

<u>The second:</u> *can be used to select a bearing (pre-selecting a bearing, then checking to make sure it is suitable)*
Set $L_D = (540)(10^6)\ revs$, find F_D, and check is $F_D \geq 495\ lb$.
From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{(4387.5)}{(1)F_D} = \left(\frac{(540)(10^6)}{(1)(10^6)}\right)^{1/3}$$

Solving gives $F_D = 538.8\ lb$
∴ with 90% reliability and a life of $(540)(10^6)\ revs$, the bearing can take on a maximum radial load of $538.8\ lb$. Since the applied radial load is only $495\ lb$, the bearing will have better than 90% reliability.

<u>The third:</u> *similar to post-selection calculation to evaluate the life of the bearing.*
Set $F_D = 495\ lb$, find L_D, and check if $L_D \geq (540)(10^6)\ revs$.
From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{(4387.5)}{(1)(495)} = \left(\frac{L_D}{(1)(10^6)}\right)^{1/3}$$

Solving gives $L_D = (696)(10^6)\ revs$
With a radial load at $495\ lb$, the bearing has 90% chance proabability to survive at least $(696)(10^6)\ revs$. The chance of surviving only $(540)(10^6)\ revs$ is better than 90%.

(2) Set $F_D = 495\ lb$; $L_D = (540)(10^6)\ revs$
To assess reliability means to evaluate a_1. This is typically done after selection.

From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

Substituting known values:

$$\frac{(4387.5)}{(1)(495)} = \left(\frac{(540)(10^6)}{a_1(10^6)}\right)^{1/3}$$

Solving gives $a_1 = 0.775$

Finally, from:

$$a_1 = 4.26\left(\ln\frac{100}{R}\right)^{2/3} + 0.05$$

Solving for R results in $R = 93\%$.
With the radial load at $495\ lb$, there is a 7% of chance that the bearing would fail at or before 540 millions of revs.
It shows that the bearing is more than suitable for 90% reliability.

Example 2
Select bearings A and B for the shaft of Example 7-2. They are to be used for a minimum of 1,000 hours of continuous operation. Shaft rpm is 450. Radial loads are, $F_A = 375\ lb$ and $F_B = 1918\ lb$. Shaft diameter at both locations is 1" (D_1 and D_7 in Figure 7-10). Assume 90% reliability.

Solution:
Table 11-2
Table 11-3
Since there is no thrust load, deep-groove ball bearings may

Example 2

Select bearings A and B for the shaft of Example 7-2. They are to be used for a minimum of 1,000 hours of continuous operation. Shaft rpm is 450. Radial loads are, $F_A = 375\ lb$ and $F_B = 1918\ lb$. Shaft diameter at both locations is 1" (D_1 and D_7 in Figure 7-10). Assume 90% reliability.

Solution:

Since there is no thrust load, deep-groove ball bearings are first considered. Table 11-2 has a list of 02-series deep groove ball bearings.

Table 11-2

Dimensions and Load Ratings for Single-Row 02-Series Deep-Groove and Angular-Contact Ball Bearings

Bore, mm	OD, mm	Width, mm	Fillet Radius, mm	Shoulder Diameter, mm		Load Ratings, kN			
						Deep Groove		Angular Contact	
				d_S	d_H	C_{10}	C_0	C_{10}	C_0
10	30	9	0.6	12.5	27	5.07	2.24	4.94	2.12
12	32	10	0.6	14.5	28	6.89	3.10	7.02	3.05
15	35	11	0.6	17.5	31	7.80	3.55	8.06	3.65
17	40	12	0.6	19.5	34	9.56	4.50	9.95	4.75
20	47	14	1.0	25	41	12.7	6.20	13.3	6.55
25	52	15	1.0	30	47	14.0	6.95	14.8	7.65
30	62	16	1.0	35	55	19.5	10.0	20.3	11.0
35	72	17	1.0	41	65	25.5	13.7	27.0	15.0
40	80	18	1.0	46	72	30.7	16.6	31.9	18.6
45	85	19	1.0	52	77	33.2	18.6	35.8	21.2
50	90	20	1.0	56	82	35.1	19.6	37.7	22.8
55	100	21	1.5	63	90	43.6	25.0	46.2	28.5
60	110	22	1.5	70	99	47.5	28.0	55.9	35.5
65	120	23	1.5	74	109	55.9	34.0	63.7	41.5
70	125	24	1.5	79	114	61.8	37.5	68.9	45.5
75	130	25	1.5	86	119	66.3	40.5	71.5	49.0
80	140	26	2.0	93	127	70.2	45.0	80.6	55.0
85	150	28	2.0	99	136	83.2	53.0	90.4	63.0
90	160	30	2.0	104	146	95.6	62.0	106	73.5
95	170	32	2.0	110	156	108	69.5	121	85.0

$a_1 = 1;$

$k_a = 1.2\ (Table\ 11-5,\ \ commercial\ gearing,\ \ 1.1 \sim 1.3);$

$L_{10} = 10^6\ revs;$

$L_D = 1000 \cdot 60 \cdot 450 = 27 \cdot 10^6\ revs;$

Table 11–5

Load-Application Factors

Type of Application	Load Factor
Precision gearing	1.0–1.1
Commercial gearing	1.1–1.3
Applications with poor bearing seals	1.2
Machinery with no impact	1.0–1.2
Machinery with light impact	1.2–1.5
Machinery with moderate impact	1.5–3.0

Bearing B: $F_D = F_B = 1918\ lb = 8535\ N$. From:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

It's found that $C_{10} = 30{,}726\ N$. Note $a = 3$.

Table 11-2 Shows that the smallest (in dimensions) bearing meeting the requirements is the one with bore diameter of 40 mm and its C_{10} is 30.7 kN.

Switch to roller bearing (Table 11-3). With $a = 10/3$, then $C_{10} = 27{,}529\ N$.
From Table 11-3, under 02-series, the bearing with 35-mm bore has $C_{10} = 31.9\ N$;
Under 03 series, the bearing with 25-mm bore has a $C_{10} = 28.6$ kN.

Table 11–3

Dimensions and Basic Load Ratings for Cylindrical Roller Bearings

	02-Series				03-Series			
Bore, mm	OD, mm	Width, mm	Load Rating, kN C_{10}	Load Rating, kN C_0	OD, mm	Width, mm	Load Rating, kN C_{10}	Load Rating, kN C_0
25	52	15	16.8	8.8	62	17	28.6	15.0
30	62	16	22.4	12.0	72	19	36.9	20.0
35	72	17	31.9	17.6	80	21	44.6	27.1
40	80	18	41.8	24.0	90	23	56.1	32.5
45	85	19	44.0	25.5	100	25	72.1	45.4
50	90	20	45.7	27.5	110	27	88.0	52.0
55	100	21	56.1	34.0	120	29	102	67.2
60	110	22	64.4	43.1	130	31	123	76.5
65	120	23	76.5	51.2	140	33	138	85.0
70	125	24	79.2	51.2	150	35	151	102
75	130	25	93.1	63.2	160	37	183	125
80	140	26	106	69.4	170	39	190	125
85	150	28	119	78.3	180	41	212	149
90	160	30	142	100	190	43	242	160
95	170	32	165	112	200	45	264	189
100	180	34	183	125	215	47	303	220
110	200	38	229	167	240	50	391	304
120	215	40	260	183	260	55	457	340
130	230	40	270	193	280	58	539	408
140	250	42	319	240	300	62	682	454
150	270	45	446	260	320	65	781	502

Select 03-series, $bore = 25\ mm$, $OD = 62\ mm$, $width = 17\ mm$, and $C_{10} = 28.6\ kN$

Assessing reliability: $a_1 = 0.88$, and $R = 91.7$
Bearing A: $F_D = F_A = 375\ lb = 1669\ N$. Use the same bearings as at B.

Assessing reliability: $a_1 = 0.00382$, $R = 99.9975$. So $R \geq 99$.
The two bearings combined will have a reliability of $(0.917) * (0.99) = 0.91$

11-6 Combined Radial and Thrust Loading
Equivalent Radial Load F_e

- C_{10}, the Basic Dynamic Load Rating, is a radial load.
- A ball or roller bearing s typically capable of taking radial as well as some thrust loads.
- Thrust loads shortens a ball or roller bearing's life faster than radial load; that is, thrust load does more damage.
- Equivalent radial load F_e is

$$F_e = XVF_r + YF_a$$

Where:
F_r and F_a are the radial and thrust loads applied to the bearing (from FBD)
F_e is the equivalent radial load; => F_D
V, X and Y are factors whose values depend on specific bearing.
V: the rotation factor
X: the radial load factor
Y: the thrust load factor

For X and Y:
Table 11-1 lists the X and Y values for ball bearings. X and Y depend on e, which in turn depends on F_a/C_0. C_0 is the **Basic Status Load Rating.**

Table 11-1
Equivalent Radial Load Factors for Ball Bearings

		$F_a/(VF_r) \leq e$		$F_a/(VF_r) > e$	
F_a/C_0	e	X_1	Y_1	X_2	Y_2
0.014*	0.19	1.00	0	0.56	2.30
0.021	0.21	1.00	0	0.56	2.15
0.028	0.22	1.00	0	0.56	1.99
0.042	0.24	1.00	0	0.56	1.85
0.056	0.26	1.00	0	0.56	1.71
0.070	0.27	1.00	0	0.56	1.63
0.084	0.28	1.00	0	0.56	1.55
0.110	0.30	1.00	0	0.56	1.45
0.17	0.34	1.00	0	0.56	1.31
0.28	0.38	1.00	0	0.56	1.15
0.42	0.42	1.00	0	0.56	1.04
0.56	0.44	1.00	0	0.56	1.00

*Use 0.014 if $F_a/C_0 < 0.014$.

$\boxed{F_a/(VF_r) \leq e}$ $\boxed{F_a/(VF_r) > e}$

For straight or cylindrical roller bearings, $Y = 0$.

It is highly recommended to use the X and Y values published by the manufacturer. Methods or processes to evaluate X and Y vary with manufacturers.

For V:
$V = 1$ if inner ring rotates;
$V = 1.2$ if outer ring rotates
$V = 1$ if using self aligning bearings.

Basic Static Load Rating C_0

- **Basic Static Load Rating** or **Basic Static Rated Load** C_0 is defined as the load that will produce a total permanent deformation (in the rolling elements and raceways) at any contact point of 0.0001 times the diameter of the rolling elements.
- C_0 can also be defined as the load that will produce a maximum contact (compressive) stress of $4\ GPa\ (580\ ksi)$ at the contact point.
- C_0 can be exceeded. It takes about 8 times of C_0 to fracture a bearing.

11-8 Selection of Ball and Roller Bearings
General Principles/Considerations

- Always refer to a catalog and read the engineering section
- Type of load to be carried
    - Radial
    - Axial
    - Radial + Axial
- Rating Loads (dynamic and static)
- Limiting Speed
- Permissible Alignments
- Space Limitation (bore, OD, width)
- Mounting/Dismounting, and Enclosure

General Procedure (not meant to be followed mechanically)

1. Use FBD of the shaft to determine the radial load F_r, and thrust (axial) load F_a.
2. Set design life L_D; Table 11-4 lists typical values.
3. Set reliability factor, application factor (and other factors if required).
4. Select type of bearing.
5. Pre-select a bearing, say, based on bore diameter and/or permissible misalignment, and/or permissible speed, so as to facilitate selecting X and Y values.
6. Evaluate the equivalent dynamic (radial) load F_e.
7. Determine C_{10}.
8. Select appropriate bearing(s) from the catalog.
9. If necessary, iterate (back to step 6) until a suitable bearing is chosen. Note that all bearings require iterations.
10. Once a bearing is chosen, assess its reliability.

Example 3
A bearing is subject to $F_r = 5400\ N$ and $F_a = 1900\ N$. The following is known $bore = 35\ mm$, at least $30 \cdot 10^6\ revs$, 90% reliability, and $k_a = 1.5$. Select a suitable bearing from $Table\ 11 - 2$.

Solution:
From Table 11-2, 02-series single-row deep-groove ball bearing with a $35 - mm$ bore has $C_{10} = 25.5\ kN$ and $C_0 = 13.7\ kN$.
$F_a/VF_r = 0.352$ and $F_a/C_0 = 0.139$
From Table 11-1, F_a/C_0 is between 0.11 and 0.17. So, $e < 0.34$.
That means, $F_a/VF_r = 0.352 > e$, therefore $X_2 = 0.56$ and $Y_2 = 0.138$ by linear interpolation.
So, $F_e = XVF_r + YF_a = 5646\ N$.
Use the basic bearing equation to determine C_{10}:

$$\frac{C_{10}}{(1.5)(5646)} = \left(\frac{(30)(10^6)}{(1)(10^6)}\right)^{1/3}$$

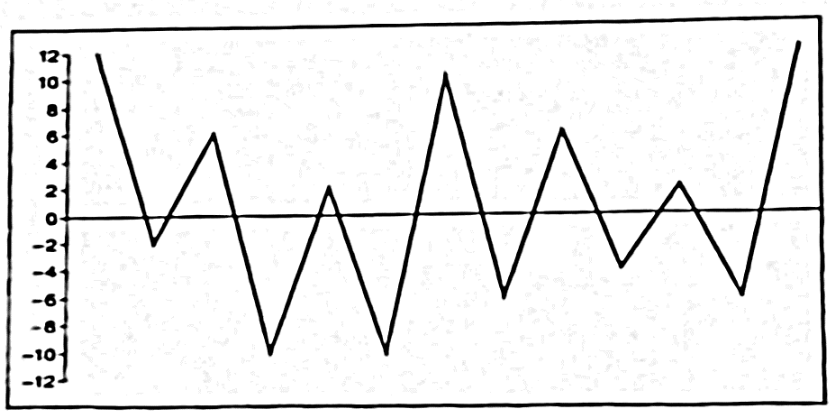
So, $C_{10} = 26{,}315\ N$. Therefore the 02-series single-row deep-groove ball bearing is not suitable.

Consider 02-series angular contact ball bearing with the same bore diameter. It has $C_{10} = 27.0\ kN$ and $C_0 = 15.0\ kN$.
Since $F_a/C_0 = 0.127$, then $e < 0.34$ and $F_a/VF_r > e$.
Then X_2 remains at 0.56, but Y_2 becomes 1.41.
Now $F_e = 5703\ N$, and C_{10} is found to be 26,581 N, which is less than the catalog's 27.0 kN.
So, the 02-series single-row angular contact ball bearing is suitable.
The actual reliability is 90.6% under the given loads and a life of 30-million revs.

11-7 Variable Loading
6-15 Cumulative Fatigue Damage
Sec. 11-7 is simply Sec. 6-15 as applied to bearings.

Cumulative Fatigue Damage
The first half of Sec 6-15 is about the *rain-flow counting technique* which is used to determine the minimum and maximum stresses at different load cycles.

The technique is aimed for irregular stress-time plots such as:



Details of the technique can be found in ASTM E1049-85(2017), "Standard Practices for Cycle Counting in Fatigue Analysis."

The outcome of rain-flow counting is a list of information such as cycle index i, maximum and minimum stresses σ_{max_i} and σ_{min_i}, and the number of cycles n_i, within a repetitive time block.

For example, the stress-time plot of $Q1$ of $A1$ can be summarized as follow:

Cycle index	σ_{max_i}	σ_{min_i}	n_i
1	8	-8	13
2	16	-16	4
3	20	-20	2
4	28	-28	1

The orders of occurrence of the stress cycles are not taken into account. This simplified the cumulative fatigue damage analysis, but is however a well recognized drawback of the analysis.

The second half of Sec. 6-15 contains the Palmgren-Miner rule (or known as Miner's rule in North America) for cumulative fatigue damage analysis.

The basic premise is, if it takes N cycles to fail (by fatigue) a component, then each cycle contributes towards the eventual failure, or does damage, by the amount of 1/N. When the damage adds up to 1 (i.e. 100%), the component fails.

The rule states that:

$$\sum \frac{n_i}{N_i} = c \tag{6–57}$$

Where n_i is the number of cycles at stress level σ_i, and N_i is the number of cycles to failure as if all cycles were loaded at stress level σ_i. n_i is determined from load-time plot; N_i is by finite life calculation.

The constant c is determined experimentally. It is found that $c = 0.7 \sim 2.2$. But $c = 1$ is typically used.

$$D = \sum \frac{n_i}{N_i} \tag{6–58}$$

where D is the accumulated damage. When $D = c = 1$, failure ensues.

When evaluating N_i, if the component is found to have infinite life, N_i is set to ∞ and $n_i/N_i = 0$.

Finally, stress level σ_i means σ_{rev_i}, the i-th σ_{rev}.

Bearings under Variable Loading

Applying Eq. (6-58) to bearings, the result is

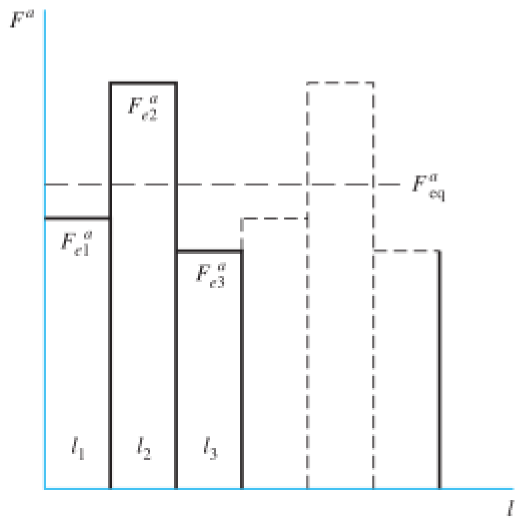
$$\sum \frac{l_i}{L_i} = 1 \tag{11–16}$$

Where L_i is the bearing life (in revs) under load level F_{ei}, and I_i is the number of revs under load F_{ei}.

- (Eq. 11-16) is only applicable to piecewise constant (including zero) loadings, see Figure 11-10.

Figure 11-10

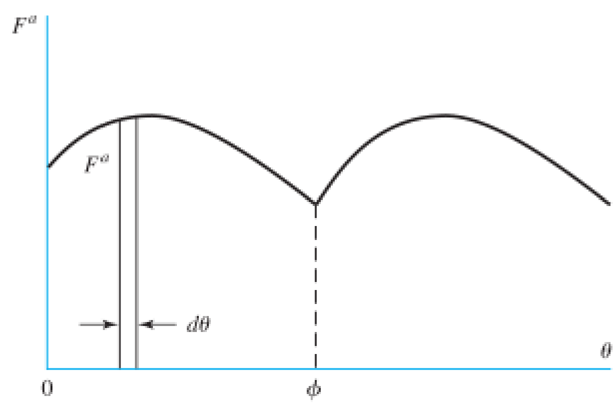
A three-part piecewise-continuous periodic loading cycle involving loads F_{e1}, F_{e2}, and F_{e3}. F_{eq} is the equivalent steady load inflicting the same damage when run for $l_1 + l_2 + l_3$ revolutions, doing the same damage D per period.



- If the load is continuous (see Figure 11-11), the summation is to be replaced by integral. Example 11-6 shows how it is done.

Figure 11-11

A continuous load variation of a cyclic nature whose period is ϕ.



- (Eq. 11-16) can be used to find
  - The life of a bearing under variable loading
  - The equivalent radial load under variable loading

$$F_{eq} = \left[\sum f_i (a_{fi} F_{ei})^a \right]^{1/a} \qquad L_{eq} = \frac{K}{F_{eq}^a} \qquad (11\text{–}15)$$

Where f_i is the fraction of revs under F_{ei}, the equivalent radial load in load cycle i; and k_{ai} is the load-application factor associated with loading condition for load cycle i.

F_{eq} is to replace all the individual $F_{ei}'s$ with the aim of simplifying the calculation. The load application factor on F_{eq} is 1.

Example 11-5

A ball bearing is run under four piecewise continuous steady loads. Information is given in tabular format, cols. (1), (2), (5) to (8) in particular. Other columns are from calculations.

(1) Time Fraction	(2) Speed rev/min	(3) Product Column	(4) Turns Fraction,	(5) F	(6) F	(7) F	(8) a_{fi}	(9) (7)x(8)
0.1	2000	200	0.077	600	300	794	1.10	873
0.1	3000	300	0.115	300	300	626	1.25	795
0.3	3000	900	0.346	750	300	878	1.10	966
0.5	2400	1200	0.462	375	300	668	1.25	835
Σ		2600	1.000					

Note:

The a_{fi} in Col. (8) are load application factors k_{ai}, which are either given, or selected from Table 11-5.
Last row of Col. (3) gives the equivalent rpm;
Col. (4) gives the f_i

Example 4

The table below lists the information relating to four piecewise constant loads that a ball bearing is subjected to.

Load case index i	f_i	F_{ei}, lb	k_{ai}	$(k_{ai}F_{ei})^a$x10^3 lb^3	$f_i(k_{ai}F_{ei})^a$x10^6 lb^3
1	0.08	794	1.10	666.25	53.30
2	0.115	626	1.25	479.13	55.01
3	0.35	878	1.10	900.87	315.3
4	0.455	668	1.25	582.18	264.9
Σ					688.5

The chosen bearing has $C_{10} = 9.56\ kN$ (rated at 10^6 revs). Determine the life of the bearing. Reliability is 95%, All other conditions (such as rotating inner ring, room temperature, non-corrosive environment, and so on) are assumed typical.

Solution:

There are two ways to find L_D.

(1) $F_{eq} \rightarrow L_D$ by the basic bearing equation. For F_{eq}, we need $f_i(k_{ai}F_{ei})^a$

(Eq. 11-15)a:

$$F_{eq} = \sqrt[3]{688.6(10^6)} = 833.0\ lb = 3929\ N \rightarrow F_D$$

Life of the bearing is then evaluated:

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{1/a}$$

$$\frac{9560}{(1)(3929)} = \left(\frac{L_D}{(0.62)(10^6)}\right)^{1/3}$$

So, $L_D = 8.93$ million revs.

(2) $F_{ei} \rightarrow L_i$ for each load case by the basic bearing equation. Then use the Miner's rule to find L_D.

Load case index i	f_i	F_{ei} lb	k_{ai}	L_i x10^6, revs
1	0.08	794	1.10	9.23
2	0.115	626	1.25	12.83
3	0.35	878	1.10	6.82
4	0.455	668	1.25	10.56

If L_D denotes the life of the bearing under the given variable loading, then $l_i = f_i L_D$

$(Eq. 11-16)$ becomes:

$$\sum \frac{l_i}{L_i} = \frac{0.08L_D}{9.23(10^6)} + \cdots + \frac{0.455L_D}{10.56(10^6)} = 1$$

So $L_D = 8.93$ million revs.

11-10 Design Assessment for Selected Bearings

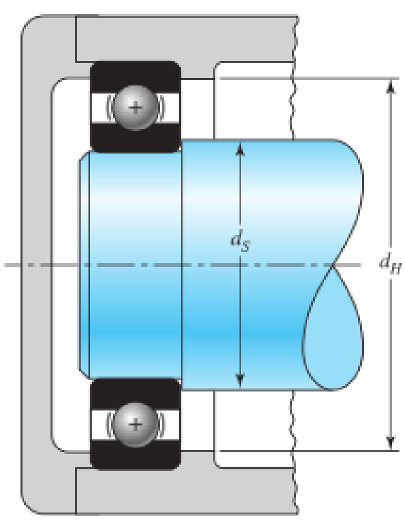
Once a selection was made, assessing for the following:

- Reliabilities of individual bearings, and of pairs of bearings;
- Shouldering on shaft and housing;
- Journal's and housing's tolerance;
- Lubrication; (see Sec. 11-11)
- Pre-load, where applicable. (see Sec. 11-12)

Reliability

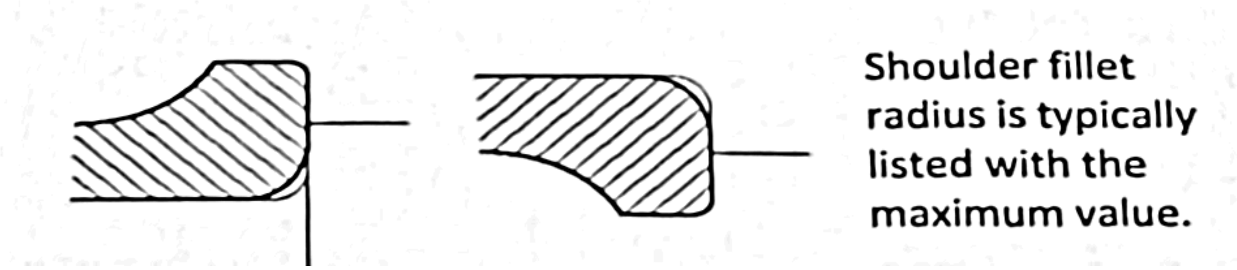
- The reliability of a pair of bearings A and B is $R = R_A R_B$;
- If $R \geq 90\%$ is required, ten one bearing (or both bearings) has (or have) to have more than 90% reliability. For example, $(0.9)(0.99) \approx 0.9$; $(0.95)(0.95) \approx 0.9$; On the other hand, $(0.9)(0.9) \approx 0.81$

Shoulders and Fillets



d_S is the shaft shoulder diameter. Catalogs may list the min and max values of d_S. If only one value is given, it is typically the minimum value.

d_H is the housing shoulder diameter. Catalogs may list the min and max values of d_H. If only one value is given, it is typically the maximum value.



Fits between Journal Bearing, and Bearing and Housing
Rotating ring: press fit;
Nonrotating ring: push fit or tap fit;

Tolerances on Journal and Housing
Bearings manufacture tolerances are specified by ABEC (Annular Bearing Engineering Committee). There are 5 AEBC grades/scales, 1, ,3 , 5, 7 and 9. The higher the grade, the higher the precision.

Catalogs provide details of bearing tolerances.

Tolerances are then assigned to the journal and housing, based on the required or chosen fits.

In general, with rotating journals, choices are j6 (or k6, m6 n6, j5) Note that these are transition fits.

For stationary housing, we may choose H7 (or G7, F7, H8). They are clearance fits.

Catalogs have detailed recommendations.

Table 2

Fits for solid steel shafts

Radial bearings with cylindrical bore

Conditions	Examples	Shaft diameter, mm Ball bearings	 Cylindrical and taper roller bearings	 CARB and spherical roller bearings	Tolerance
Rotating inner ring load or direction of load indeterminate					
Light and variable loads ($P \le 0{,}06\ C$)	Conveyors, lightly loaded gearbox bearings	(18) to 100	≤ 40	–	j6
		(100) to 140	(40) to 100		k6
Normal and heavy loads ($P > 0{,}06\ C$)	Bearing applications generally, electric motors, turbines, pumps, internal combustion engines, gearing, woodworking machines	≤ 18	–	–	j5
		(18) to 100	≤ 40	≤ 40	k5 (k6)[1)]
		(100) to 140	(40) to 100	(40) to 65	m5 (m6)[1)]
		(140) to 200	(100) to 140	(65) to 100	m6
		(200) to 280	(140) to 200	(100) to 140	n6
		–	(200) to 400	(140) to 280	p6
		–	–	(280) to 500	r6[2)]
		–	–	> 500	r7[2)]

11-11 Lubrication

- Purpose of lubrication;
- Types of lubricant (grease and oil);
- When to use what?
  See the rules listed near the end of Section 11-11.

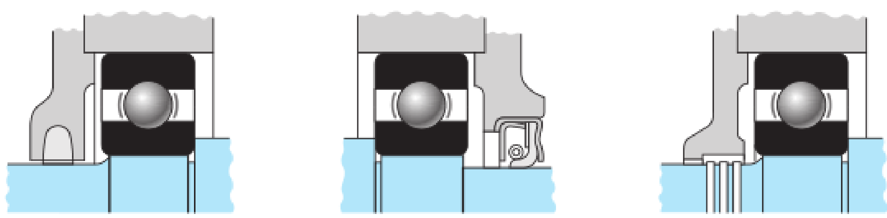
Use Grease When	Use Oil When
1. The temperature is not over 200°F.	1. Speeds are high.
2. The speed is low.	2. Temperatures are high.
3. Unusual protection is required from the entrance of foreign matter.	3. Oiltight seals are readily employed.
4. Simple bearing enclosures are desired.	4. Bearing type is not suitable for grease lubrication.
5. Operation for long periods without attention is desired.	5. The bearing is lubricated from a central supply which is also used for other machine parts.

11-12 Mounting and Enclosure

- An enclosure is used to prevent dirt and foreign matter from a entering a bearing, and to retain lubricant.
- Figure 11-26 shows typical means of external (to bearing) seals. That is, shaft and components inside the housing will be protected as well.

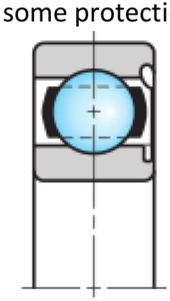
Figure 11–26

Typical sealing methods. *(General Motors Corp. Used with permission, GM Media Archives.)*



(*a*) Felt seal (*b*) Commercial seal (*c*) Labyrinth seal

- Shielded bearings (Figure 11-2d) provide some protection against dirt, but not a complete closure.



(*d*)
Shielded

- A sealed bearing, when sealed on both sides (Figure 11-2e), keeps lubricant in, for life, but can be relubricated.

Mounting

- Mounting bearings in a trouble-free and low-cost way is an important but challenging part of any design.
- Bearing catalogs area good source of information and guide, giving details for many design situations.
- The following is to be considered:
    - Axially locating a single bearing on a shaft
    - Locating and non-location arrangements of a pair of bearings
    - Misalignment
    - Preloading

Axially Locating a Single Bearing

Shoulder or spacer, and locknut or end plate or snap ring or retaining ring, for example.

Locating and Non-location Arrangements

- Such arrangements are the methods to locate the shaft as well as to allow for thermal expansion or contraction in the axial direction.
- The principle behind is: thrust load in each direction much be carried by *one* and *only one* bearing.
- Figure 11-20 and Figure 11-21 illustrate the locating and non-location arrangements, respectively, for two situations where the shaft is to be supported by two bearings, one at or near each end of the shaft.

Figure 11–20

A common bearing mounting.

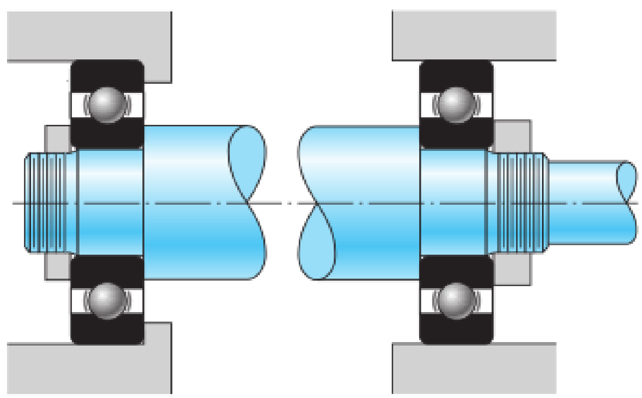
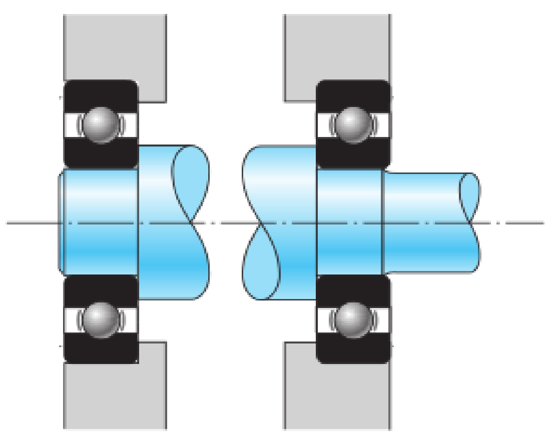


Figure 11–21

An alternative bearing mounting to that in Fig. 11–20.



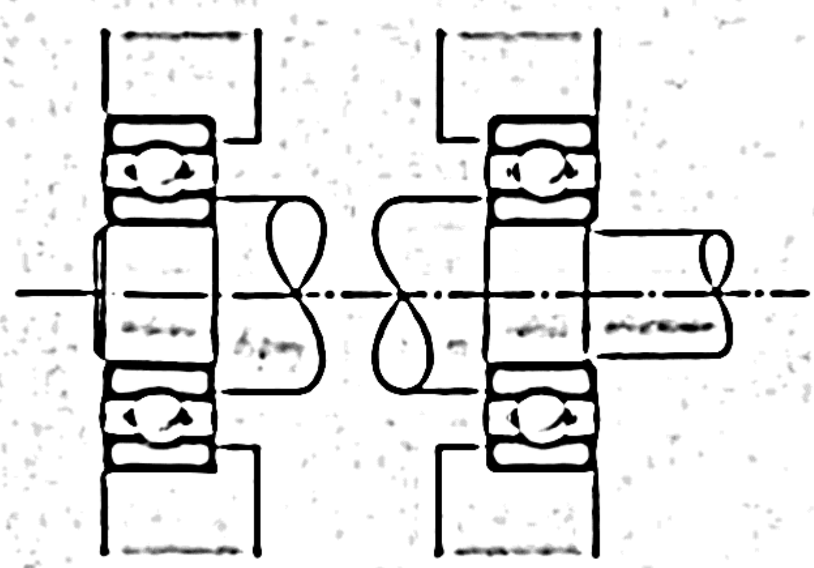


Figure 11-21: Non-locating (floating) arrangement;

Missing details?

Not to directly apply in actual designs. Refer to catalogs.

- For the so-called cantilevered shafts, refer to Figures 11-22 and 11-23, for example; or catalogs.

Misalignment
The permissible misalignment of a bearing depends on its type, and other design details of the bearing.

See Table 7-2 for typical permissible misalignments or maximum slopes.

Preloading
Although many design applications require little attention to stiffness or bearings, there are plenty of applications requiring high stiffness, high natural frequencies, low deflection, and low noise level.

Under such circumstances, preloading on bearings is to be considered.

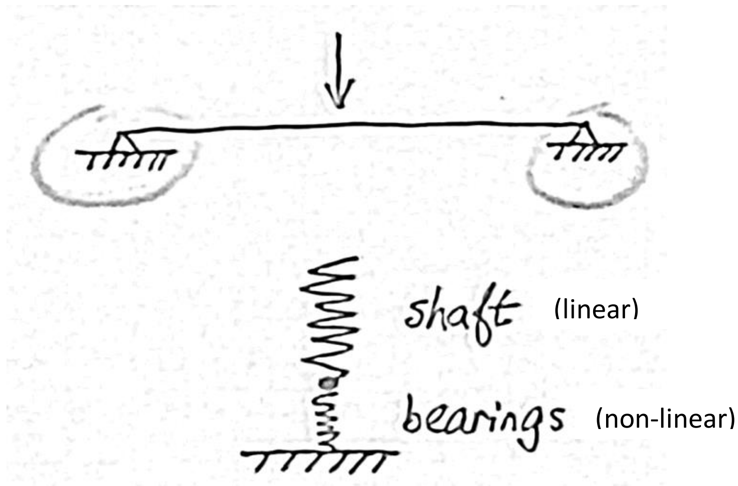
Preloading can also remove internal clearance, and increase fatigue life, amongst others.

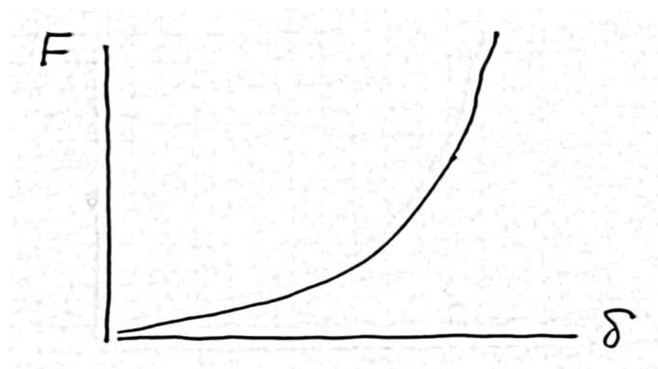
Too much preload causes early or premature failure. The key is to apply the appropriate amount of preload. Catalogs usually provide details of how much preload to apply, and how to apply.

Table 7–2

Typical Maximum Ranges for Slopes and Transverse Deflections

Slopes	
Tapered roller	0.0005–0.0012 rad
Cylindrical roller	0.0008–0.0012 rad
Deep-groove ball	0.001–0.003 rad
Spherical roller	0.026–0.052 rad
Self-align ball	0.026–0.052 rad
Uncrowned spur gear	<0.0005 rad
Transverse Deflections	
Spur gears with $P < 10$ teeth/in	0.010 in
Spur gears with $11 < P < 19$	0.005 in
Spur gears with $20 < P < 50$	0.003 in


shaft
(linear)
bearings
(non-linear)

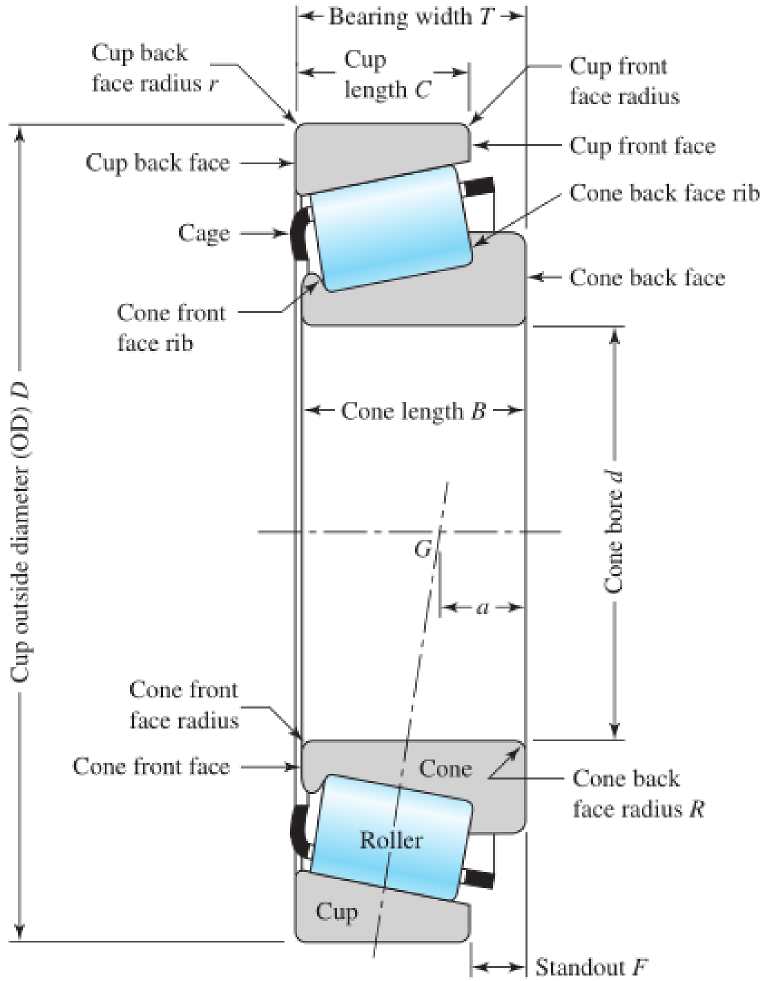

F
δ

11-9 Selection of Tapered Roller Bearings

Figure 11-13: terminology

Figure 11–13

Nomenclature of a tapered roller bearing. Point G is the location of the effective load center; use this point to estimate the radial bearing load. *(Courtesy of The Timken Company.)*



<u>A few notes about the terminology:</u>

We don't call them inner and outer rings (Inner = cone, Outer = cup)

The larger end of the cone is the back face.

The smaller end of the cone is the front face.

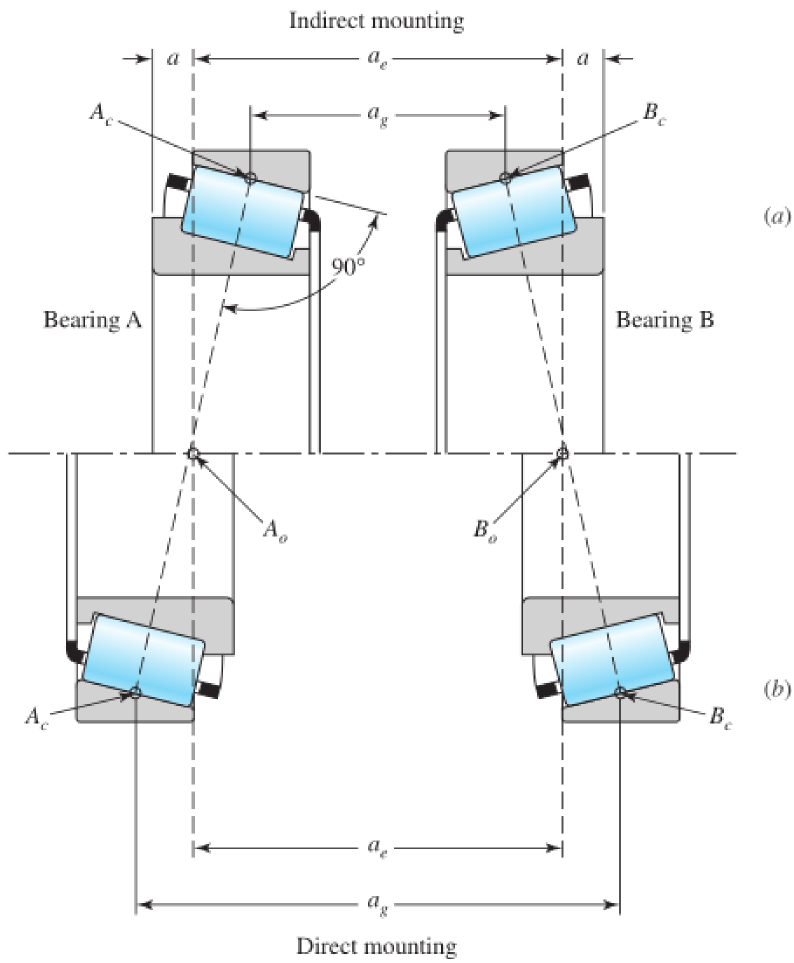
G is the point of load application (radial and axial).

When G is inside the back face of a cone as shown, a will be given a negative value.

- Figure 11-14: *direct mounting* (back-to-back in terms of cone faces and *indirect mounting* (front-to-front in terms of cone faces).
- A_0 (point G of bearing A) and B_0 (point G or bearing B) are the points of application of loads, or locations of bearings or supports for analyses; a_e is the effective length between bearings or supports.

Figure 11–14

Comparison of mounting stability between indirect and direct mountings. *(Courtesy of The Timken Company.)*

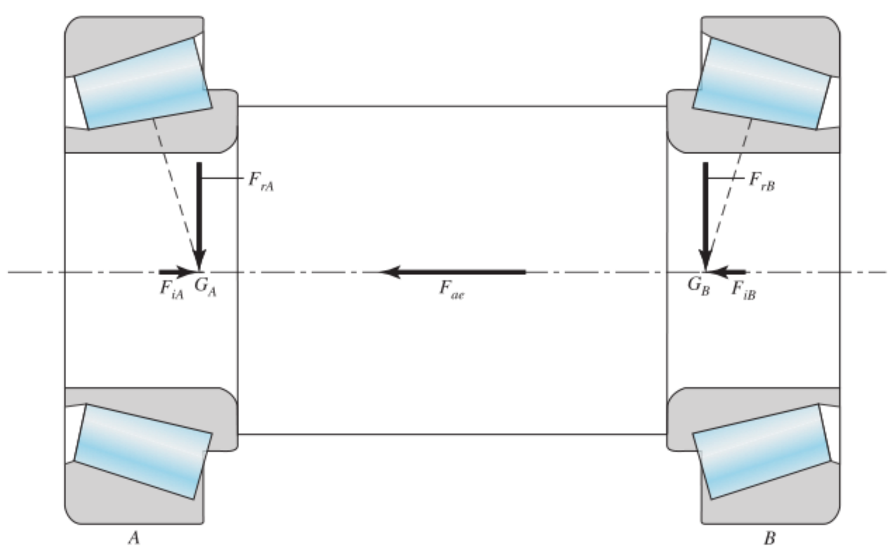


- How to select tapered roller bearings?

<u>The induced thrusts</u>, see Figure 11-16.

Figure 11–16

Direct-mounted tapered roller bearings, showing radial, induced thrust, and external thrust loads.



From FBD of the shaft: F_{rA} and F_{rB} are the radial loads, and F_{ae} is the externally applied axial load.

F_{iA} and F_{iB} are the *induced* axial loads.

The value of an induced axial load depends on the geometry of the bearing, and the radial load on it.

$$F_i = \frac{0.47F_r}{K} \qquad (11\text{–}18)$$

K is found from catalogs.

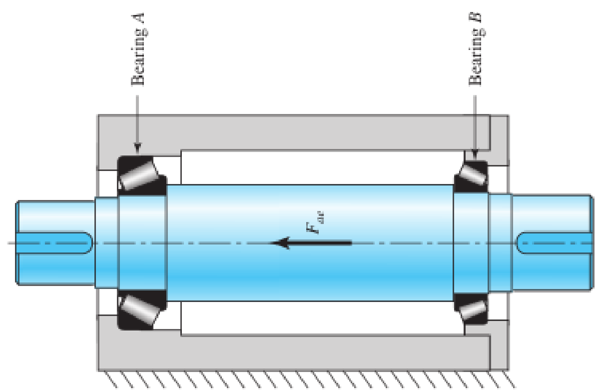
The process to determine the equivalent radial loads F_{eA} and F_{eB}:

- The pair will be labelled A and B;
- Bearing A is the one being "squeezed" by F_{ae}. Label the other at B;
- The equivalent radial loads F_{eA} and F_{eB} will then be determined by (Eq. 11-19) or (Eq. 11-20)

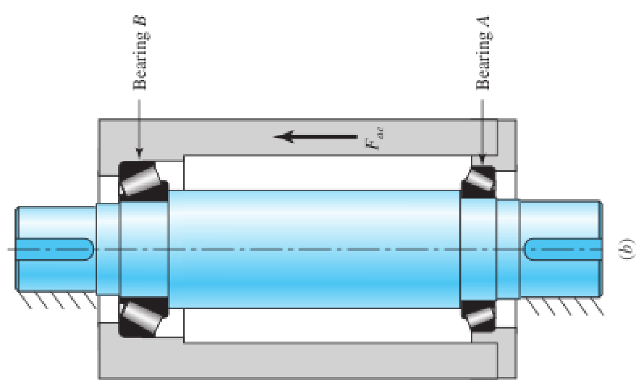
$$\text{If} \quad F_{iA} \le (F_{iB} + F_{ae}) \qquad \begin{cases} F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) & (11\text{–}19a) \\ F_{eB} = F_{rB} & (11\text{–}19b) \end{cases}$$

$$\text{If} \quad F_{iA} > (F_{iB} + F_{ae}) \qquad \begin{cases} F_{eB} = 0.4F_{rB} + K_B(F_{iA} - F_{ae}) & (11\text{–}20a) \\ F_{eA} = F_{rA} & (11\text{–}20b) \end{cases}$$

Which bearing is bearing A? Figures 11-17 and 11-19:



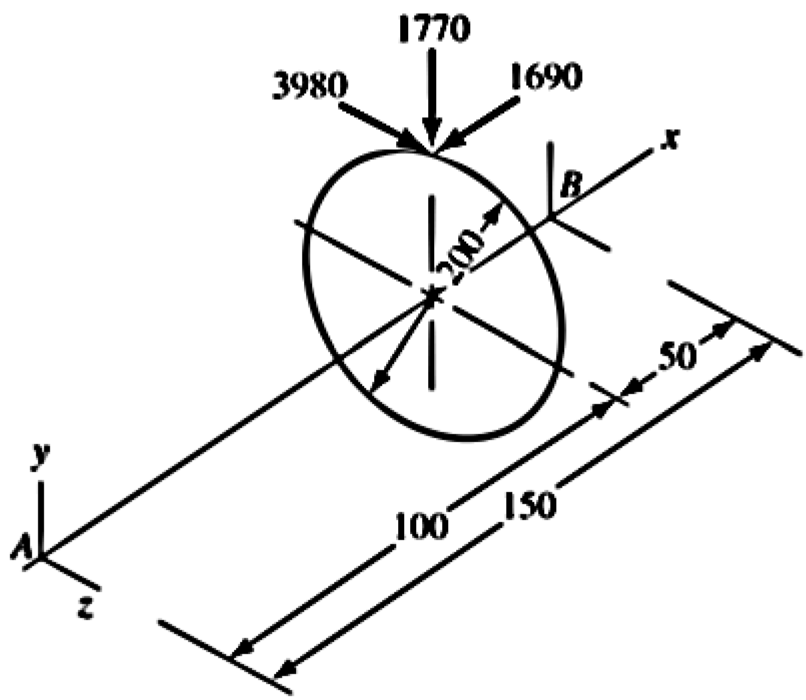
Shaft is moving, housing stationary.



Shaft is stationary, housing is moving.

- The above discussion re: induced axial load is applicable to angular-contact ball bearings. Methodology of finding F_i and F_e can be found form catalogs or manufacturers.

Example 11-8
The shaft runs at 800 rpm and is supported by two direct mounted taper-roller bearings. The design life of the bearings is to be 5000 hours. The helical gear mounted on the shaft is subject to tangential, radial, and axial loads. The reliability of the pair of bearings is set to 99% and k_a is 1.



Select suitable Timken tapered-roller bearings from Figure 11-15. Would the reversal of direction of the shaft's rotating require smaller/larger bearings?

<u>Solution:</u>
(1) Current direction of rotation

Assume 150-mm is the effective span; proceed with FBD's and finding support reactions. The vector sums of the support reactions are:

$R_A = 2170\ N$
$R_B = 2654\ N$
$F_{ae} = 1690\ N$

Also, $L_D = (800)(60)(5000) = 240(10^6)$ revs, and $L_{10} = (500)(60)(3000) = 90(10^6)$ revs

Reliabilities of the individual bearings are $\sqrt{0.99} = 0.995$.

Factor a_1 is then 0.175.

All bearings in Figure 11-15 have a bore of 25 mm. Select 07096/07196. It has $C_{10} = 6{,}990\ N$ and $K = 1.45$.

From (Eq. 11-18):

$$F_i = \frac{0.47F_r}{K} \qquad (11\text{–}18)$$

$$F_{iA} = \frac{(0.47)(2170)}{(1.45)} = 703.4\ N$$

$$F_{iB} = \frac{(0.47)(2654)}{(1.45)} = 860.3\ N$$

Since $F_{iA} \leq F_{iB} + F_{ae}$, (Eq. 11-19) is used to determine F_e.

$$\begin{Bmatrix} F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) \\ F_{eB} = F_{rB} \end{Bmatrix}$$

Therefore, $F_{eA} = (0.4)(2170) + (1.45)(860.3 + 1690) = 4566\ N$
$F_{eB} = 2654\ N$

Apply basic bearing equation for C_{10}

$$\frac{C_{10}}{k_a F_D} = \left(\frac{L_D}{a_1 L_{10}}\right)^{\frac{1}{a}}$$
$$\frac{C_{10}}{(1)(4566)} = \left(\frac{(240)(10^6)}{(0.175)(90)(10^6)}\right)^{\frac{3}{10}}$$

The result is $C_{10} = 10{,}337\ N$. So, 07096/07196 is not sufficient.

Reselect 15101/15243. $C_{10} = 12{,}100\ N$, and $K = 1.67$. Repeat above calculations:

$$F_{iA} = (0.47)\left(\frac{2170}{1.67}\right) = 519.8\ N$$
$$F_{iB} = (0.47)\left(\frac{2654}{1.67}\right) = 614.1\ N$$
$$F_{eA} = (0.4)(2170) + (1.67)(614.1 + 1690) = 4716\ N$$
$$F_{eB} = 2654\ N$$

And $C_{10} = 10{,}677\ N$, which is less than 12,100 N.

For reliability assessment, recall

$$a_1 = (4.26)\left(\ln\frac{100}{R}\right)^{\frac{2}{3}} + 0.05$$

So, bearing A has $a_1 = 0.115$, $R = 99.8$; bearing B has $a_1 = 0.0169$. Since $a_1 < 0.05$, $R \geq 99.9$; combined reliability is $(0.998)(0.999) = 0.997$, which is more than 0.99.

Therefore, 15101/15243 is sufficient for both locations.

(2) Reversal in direction of rotation

Re-label the bearings as L(eft) (used to be bearing A) and R(ight) (used to be bearing B)

FBD and calculations give rise to $R_L = 1431\ N$ and $R_R = 3516\ N$.

Bearings are 15101/15243 with $C_{10} = 12{,}100\ N$ and $K = 1.67$.

$$F_{iL} = (0.47)\left(\frac{1431}{1{,}67}\right) = 402.7\ N$$
$$F_{iR} = (0.47)\left(\frac{3516}{1.67}\right) = 989.5\ N$$

Now because the right bearing s being “squeezed”.

Because $F_{iR} \leq F_{iL} + F_{ae}$, therefore (Eq. 11-19) gives

$F_{eR} = (0.4)(3516) + (1.67)(402.7 + 1690) = 4901\ N$, and $F_{eL} = 1431\ N$

Calculating the required C_{10} for the right bearing

$$\frac{C_{10}}{(1)(4901)} = \left(\frac{(240)(10^6)}{(0.175)(90)(10^6)}\right)^{\frac{3}{10}}$$

Then $C_{10} = 11{,}096\ N$, which is less than the catalog’s $12{,}100\ N$.

So, the selection is suitable for opposite direction of the shaft’s rotation.

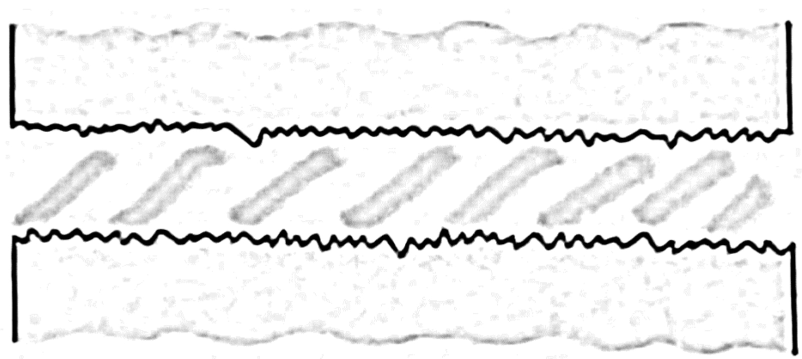
Or, assessing the bearings’ reliabilities; the left bearing has $a_1 = 0.00217$, so $R \geq 99.9$; the right bearing has $a_1 = 0.131$, $R = 99.7$; As a result, the combined reliability is $(0.999)(0.997) = 0.997$, which is more than 0.99.

So, the 15101/15243 bearings can be used in either direction of rotation.

Chapter 12
Lubrication and Journal Bearings

12-1 Types of Lubrication

- **Thick-film (full-film) lubrication:**
    - There is complete separation of contact surfaces by a relatively thick film of lubricant;
    - There is no metal-t-metal contact;
    - Typical minimal film thickness is $0.008 - 0.020\ mm$ or $0.0003 - 0.0008\ in$;
    - Resulting coefficient of friction is $0.002 - 0.01$;



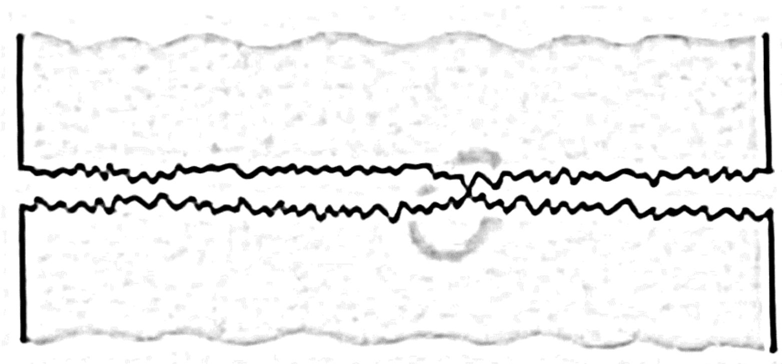
Ways to achieve thick-film lubrication:

- Hydrodynamic lubrication: The journal is lifted by the wedge-action effect of the lubricant; It requires proper lubricant, speed, and clearance; Applicable in situations of high speeds, high

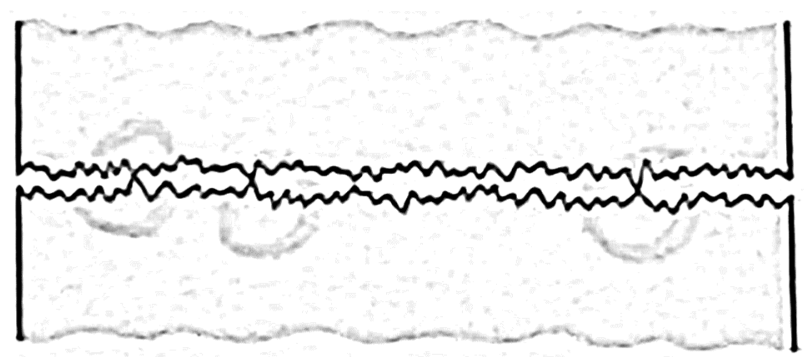
loads, high overloads, and high temperature (e.g., engines, pumps, compressors, turbines, motors, and so on).
  - Hydrostatic lubrication: Lubricant is pressured (force-fed) by external means to separate the two parts; applicable in cases of low speeds and light loads.

- **Elastohydrodynamic lubrication (EHL):**
  - Lubricant is introduced between surfaces that deform elastically;
  - Hard EHL occurs between surfaces in rolling contact, such as mating gears, cams/followers, and rolling-elements and raceways in rolling-contact-bearings;
  - Soft EHL occurs when contact region is relatively large, e.g. brushing.

- **Mixed-film lubrication:**
  - There is only partial full-film lubrication;
  - Surfaces may be in intermittent contact;
  - Coefficient of friction is $0.004 - 0.1$.



- **Boundary lubrication:**
  - It is when 90% or more of surface asperity is in contact;
  - Coefficient of friction is $0.05 - 2$.



- **Solid-film lubrication**
  - Used when bearings must be operated at high to extreme temperatures;
  - Lubricants are of powder form;
  - Graphite, molybdenum disulfide, for example;

Note: above diagrams courtesy of Fundamentals of Machine Components Design, 3rd ed. R.C. Juvinall and K.M. Marshek, Wiley & Sons, 2000.

12-2 Viscosity

- Viscosity is a measure of the internal friction resistance of the fluid.
- Units for absolute or dynamic viscosity μ:
  In ips (inch-pound-second) units: $lb \cdot s/in^2$ or reyn
  In SI units: $N \cdot s/m^2$ or $Pa \cdot s$
- Units used in journal bearing design:
  - In ips: **microreyn ($\mu reyn$)**
    $1\ \mu reyn = 10^{-6}\ reyn$
  - In SI units: **centipoise (cP)**
    $1\ cP = 1\ mPa \cdot s = 10^{-3}\ Pa \cdot s$
- Kinematic viscosity: not used in journal bearing designs

12-3 Petroff's Equation
12-4 Stable Lubrication
12-5 Thick-Film Lubrication
12-6 Hydrodynamic Theory
Available Theories/Solutions of Hydrodynamic Lubrication

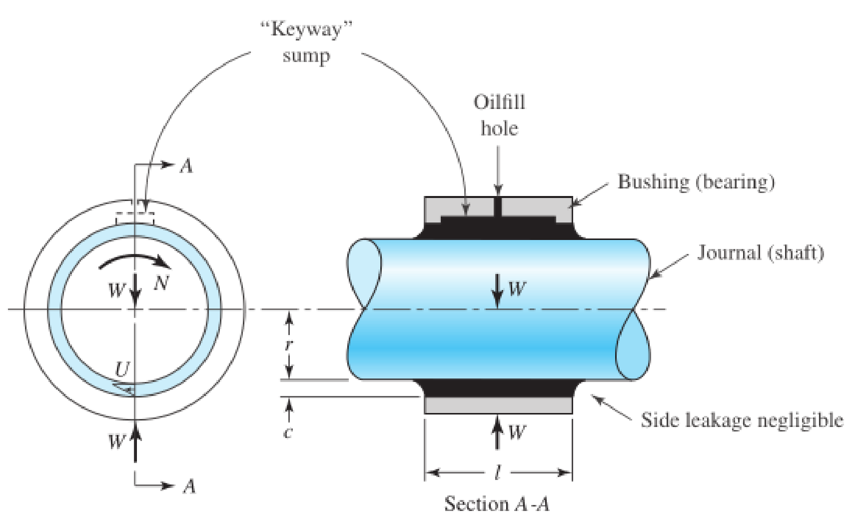
- Petroff's equation for concentric journal bearings (1883)
- Reynolds equation for eccentric journal bearings (1886)
- Sommerfeld's equation for (very) long bearings (1904)
- Ocvirk's solution for (very) short bearings (1952)
- Raimondi & Boyd's numerical solution for finite-length bearings (1958)

Petroff's Equation

- Assume that bearing and shaft are concentric; hence seldom use in design.
- But it was the first to explain the friction phenomenon in bearings.

Figure 12–3

Petroff's lightly loaded journal bearing consisting of a shaft journal and a bushing with an axial-groove internal lubricant reservoir. The linear velocity gradient is shown in the end view. The clearance c is several thousandths of an inch and is grossly exaggerated for presentation purposes.



W: bearing load, in lb or newton
l: bearing length, in inch of m

r: shaft/journal radius, in inch or m
$P = \frac{w}{2rl}$: pressure or unit load, in psi or Pa
N: rotating speed, in rps (rotation per second)
μ: absolute viscosity, in reyn or Pa·s
$\frac{\mu N}{P}$: bearing characteristic
c: radial clearance, in inch or m
$\frac{r}{c}$: radial clearance ratio

Then the coefficient of friction, f, is related to the bearing characteristic $\frac{\mu N}{P}$ and radial clearance ratio $\frac{r}{c}$ by the Petroff's equation:

$$f = 2\pi^2 \frac{\mu N}{P} \frac{r}{c} \tag{12–6}$$

- Another important parameter is the bearings characteristic number of Sommerfeld number, S

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} \tag{12–7}$$

In terms of the Sommerfeld number, Petroff's equation becomes:

$$f\frac{r}{c} = 2\pi^2 \frac{\mu N}{P}\left(\frac{r}{c}\right)^2 = 2\pi^2 S \tag{12–8}$$

Stable Lubrication

Figure 12–4

The variation of the coefficient of friction f with $\mu N/P$.

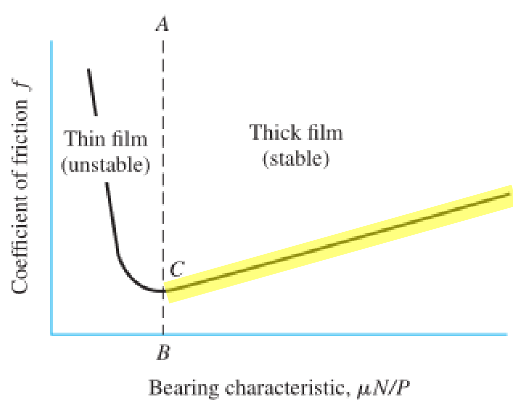


Figure 12-4 is a plot of experimental coefficient of friction in terms of bearing characteristic.

Petroff's equation assumes thick-film lubrication, is presented by the straight line to the right of transition point C. The condition for ensuring thick-film lubrication is $\frac{\mu N}{P} \geq 1.7(10^{-6})$

“Thin film” includes mixed-film and boundary lubrications. Point C represents the transition from metal-to-metal contact to thick film lubrication.

Hydrodynamic Lubrication

- Three aspects or “things” that are required to achieve hydrodynamic lubrication:
    - Relative motion of the surfaces
    - Wedge action
    - A suitable fluid
- Basic assumptions:
    - The lubricant is a Newtonian fluid
    - Inertia forces of the lubricant are negligible
    - The lubricant is an incompressible fluid
    - The viscosity is constant (true if temperature does not change much)
    - There is zero pressure gradient along the length of the bearing (true for very long bearings)
    - The radius of the journal is large compared to the film thickness
- Theory:
  (Eq. 12-10) and (Eq. 12-11) are the Reynolds equation for one-dimensional flow, and two-dimensional flow, respectively.

$$\frac{d}{dx}\left(\frac{h^3}{\mu}\frac{dp}{dx}\right) = 6U\frac{dh}{dx} \qquad (12\text{–}10)$$

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right) = 6U\frac{\partial h}{\partial x} \qquad (12\text{–}11)$$

(Eq. 12-12) represents the general form of Sommerfeld solution, which is numerical.

$$\frac{r}{c}f = \phi\left[\left(\frac{r}{c}\right)^2\frac{\mu N}{P}\right] \qquad (12\text{–}12)$$

Where ϕ represents a function. S is the Sommerfeld number. Sommerfeld found the solution for half and full bearings, under the no side leakage assumption.

12-8 The Relations of the Variable

Between 1951 and 1958, Raimondi and Boyd used the iterative technique to numerically solve the Reynolds equation.

The outcome is represented by the Raimondi-Boyd charts, Figures 12-16 through 12-22, which nomenclatures shown on Figure 12-15. Note that these charts are for full bearings.

- Sommerfeld number S is the abscissa;
- Four plots in each chart: $\frac{l}{d} = \frac{1}{4}, \frac{1}{2}$, 1, and ∞
- For other $\frac{l}{d}$ ratios, interpolation by (Eq. 12-16)

$$y = \frac{1}{r^3}\left[-\frac{1}{8}(1-r)(1-2r)(1-4r)y_\infty + \frac{1}{3}(1-2r)(1-4r)y_1 - \frac{1}{4}(1-r)(1-4r)y_{\frac{1}{2}} + \frac{1}{24}(1-r)(1-2r)y_{\frac{1}{4}}\right]$$

Where $r = \frac{l}{d}$, and $\frac{1}{4} < r < \infty$. y is the desired chart variable, and $y_\infty, y_1, y_{\frac{1}{2}}, y_{\frac{1}{4}}$ are the y values off the corresponding plots.

Or just use this:

$$y = \frac{1}{(l/d)^3}\left[-\frac{1}{8}\left(1-\frac{l}{d}\right)\left(1-2\frac{l}{d}\right)\left(1-4\frac{l}{d}\right)y_\infty + \frac{1}{3}\left(1-2\frac{l}{d}\right)\left(1-4\frac{l}{d}\right)y_1 - \frac{1}{4}\left(1-\frac{l}{d}\right)\left(1-4\frac{l}{d}\right)y_{1/2} + \frac{1}{24}\left(1-\frac{l}{d}\right)\left(1-2\frac{l}{d}\right)y_{1/4}\right] \quad (12\text{–}16)$$

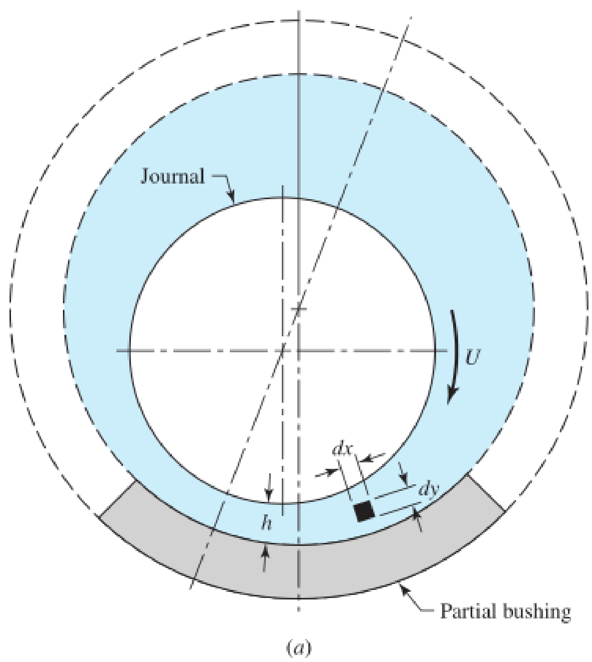


Figure 12–9

Figure 12–15

Polar diagram of the film–pressure distribution showing the notation used. *(Raimondi and Boyd.)*

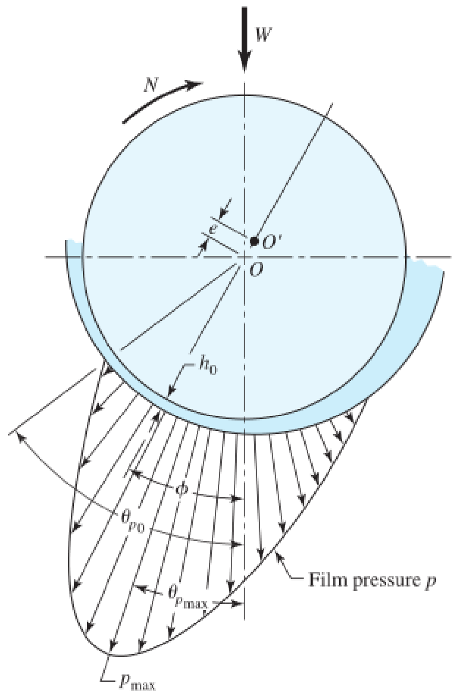
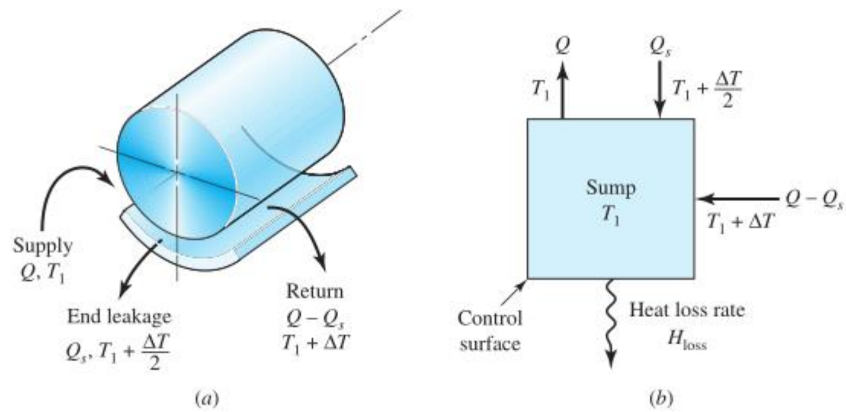


Figure 12–23

Schematic of a journal bearing with an external sump with cooling; lubricant makes one pass before returning to the sump.



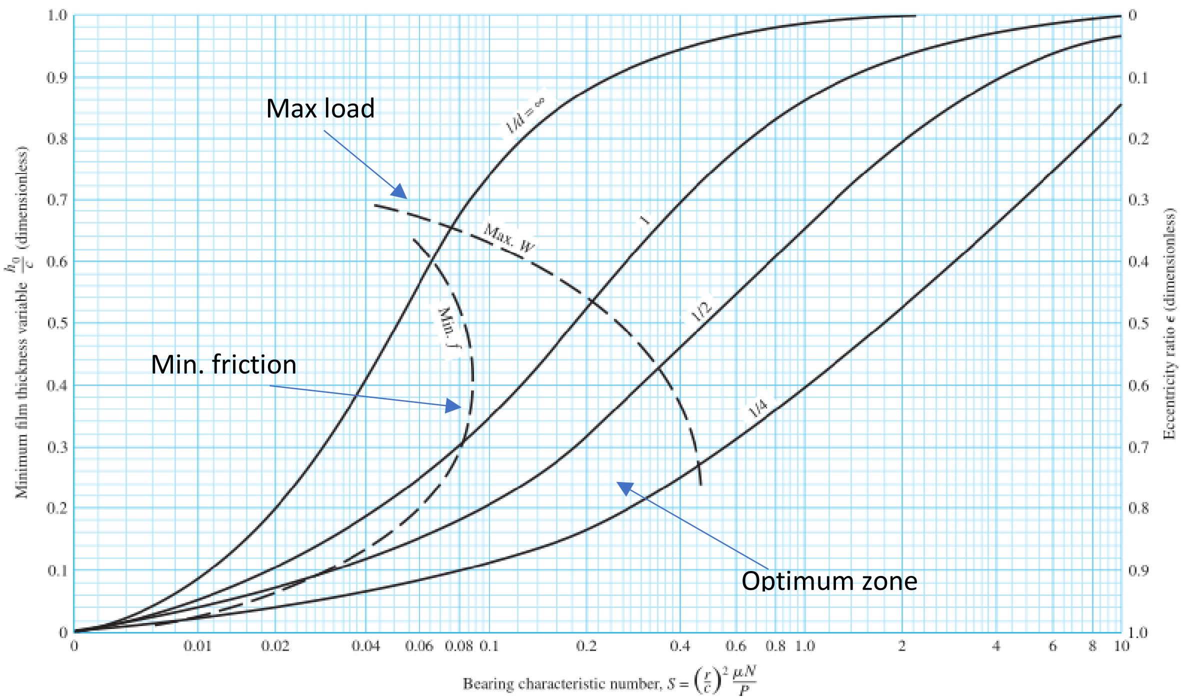


Figure 12–16

Chart for minimum film thickness variable and eccentricity ratio. The left boundary of the zone defines the optimal h_0 for minimum friction; the right boundary is optimum h_0 for load. *(Raimondi and Boyd.)*

Example 1

A journal bearing of 2"-diameter, 2"-length and 0.0015"-radial clearance is to support a steady load of 1000 lv when the shaft rotates at 3000 rpm. The lubricant is SAE 20 oil, supplied at atmospheric pressure. The average temperature of the oil is at 130 $°F$. (1) Estimate h_0, e, Q, Q_s, p_{max}, Φ, θ_{pmax} and θ_{p0}. (2) Estimate the temperature rise ΔT_F.

Solution:

Knowns:

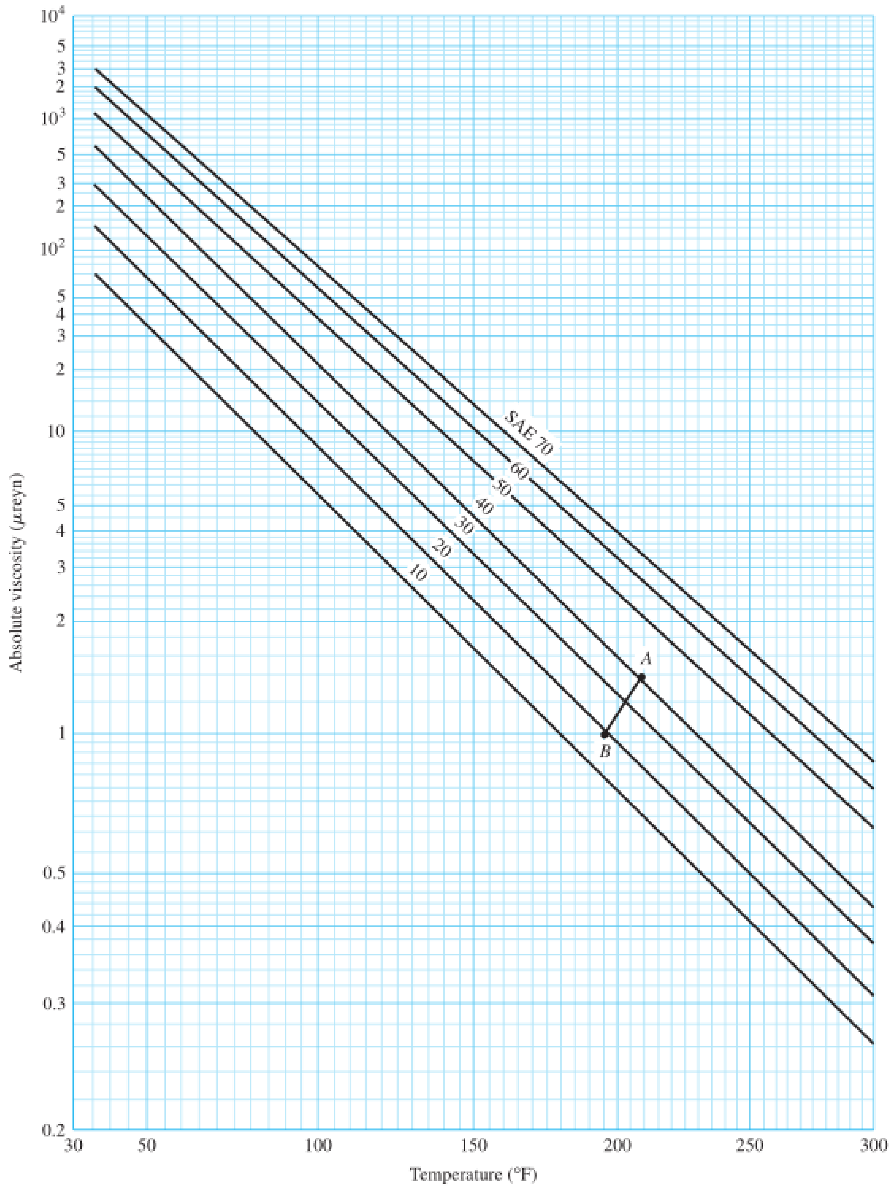
$d = 2", l = 2", c = 0.0015', w = 1000\ lb, N = \frac{3000}{60} = 50\ rps$

So, the unit load is $P = \frac{W}{ld} = 1000/(2 * 2) = 250\ psi$

Figure 12-12: $\mu = 3.7\ \mu reyn$

Figure 12–12

Viscosity–temperature chart in U.S. customary units. *(Raimondi and Boyd.)*



Sommerfeld number (or bearing characteristic number)

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{1}{0.0015}\right)^2 \frac{(3.7)(10^{-6})(50)}{250} = 0.33$$

(1) Estimate the list of variables

Figure 12-16: minimum film thickness and eccentricity

$h_0/c = 0.65$, $\varepsilon = \frac{e}{c} = 0.35$

So, $h_0 = (0.65)c = 0.000975"$, $e = (0.35)c = 0.000525"$

Note: sum of these two ratios is always unity;

Also, the bearing, as represented by $(S, h_0/c)$ is outside of the optimal zone.

Figure 12-17: position of the minimum film thickness $\Phi = 70°$

Figure 12–17

Chart for determining the position of the minimum film thickness h_0. *(Raimondi and Boyd.)*

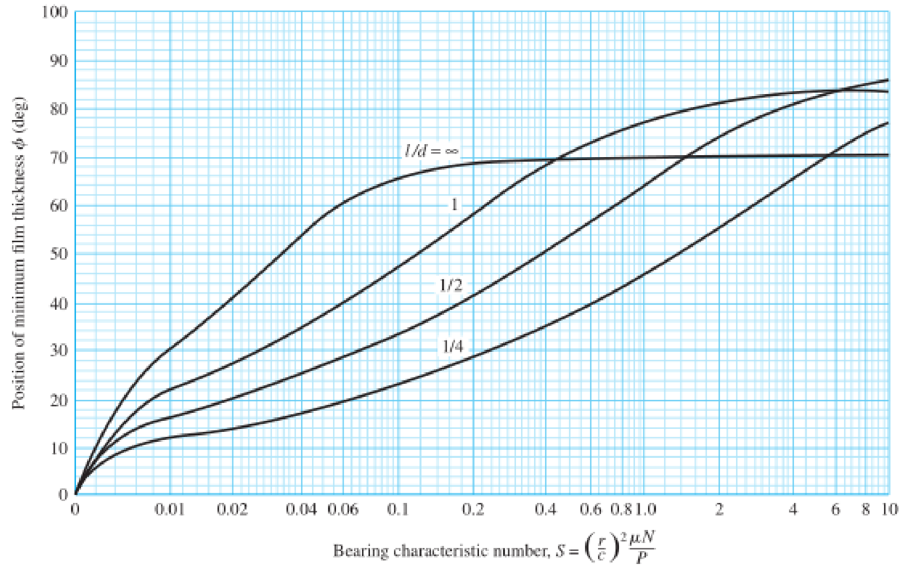


Figure 12-18: coefficient of friction

$$\frac{r}{c} f = 7; so\ f = 7\left(\frac{0.0015}{1}\right) = 0.0105$$

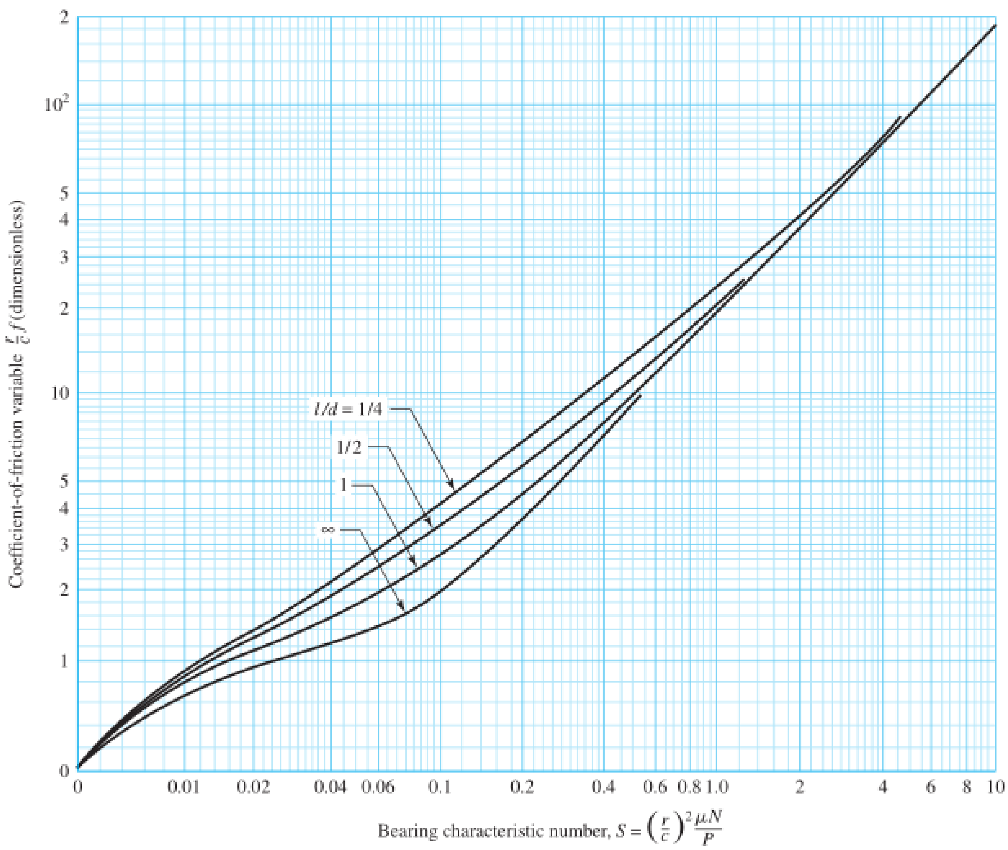


Figure 12–18

Chart for coefficient-of-friction variable; note that Petroff's equation is the asymptote. *(Raimondi and Boyd.)*

Figure 12-19 and 12-20: total flow rate and side flow rate of lubricant.

$\frac{Q}{rcNl} = 3.85$, so $Q = (3.85)(1)(0.0015)(50)(2) = 0.5775\ in^3/s$

$\frac{Q_s}{Q} = 0.45$, so $Q_s = (0.45)(0.5775) = 0.2599\ in^3/s$

Figure 12–19

Chart for flow variable. *Note:* Not for pressure-fed bearings. *(Raimondi and Boyd.)*

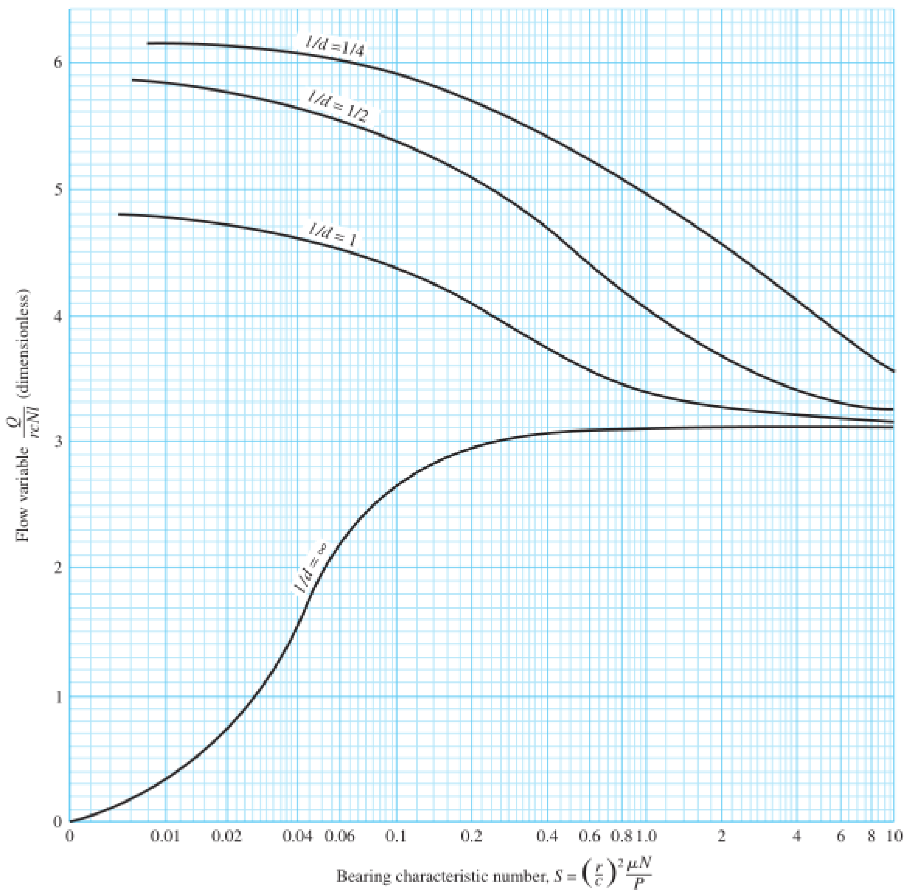
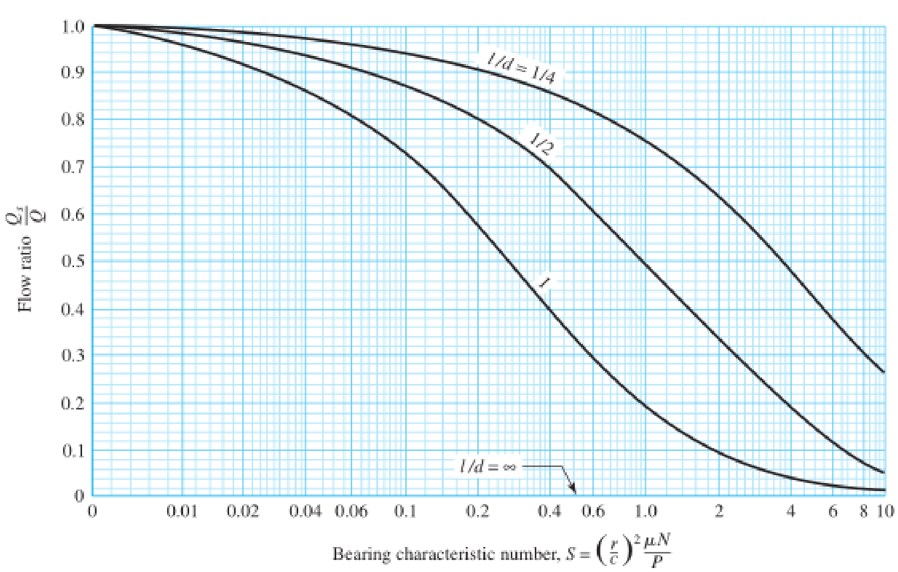


Figure 12–20

Chart for determining the ratio of side flow to total flow. *(Raimondi and Boyd.)*



Figures 12-21 and 12-22: maximum film pressure and its position, and film's termination position:

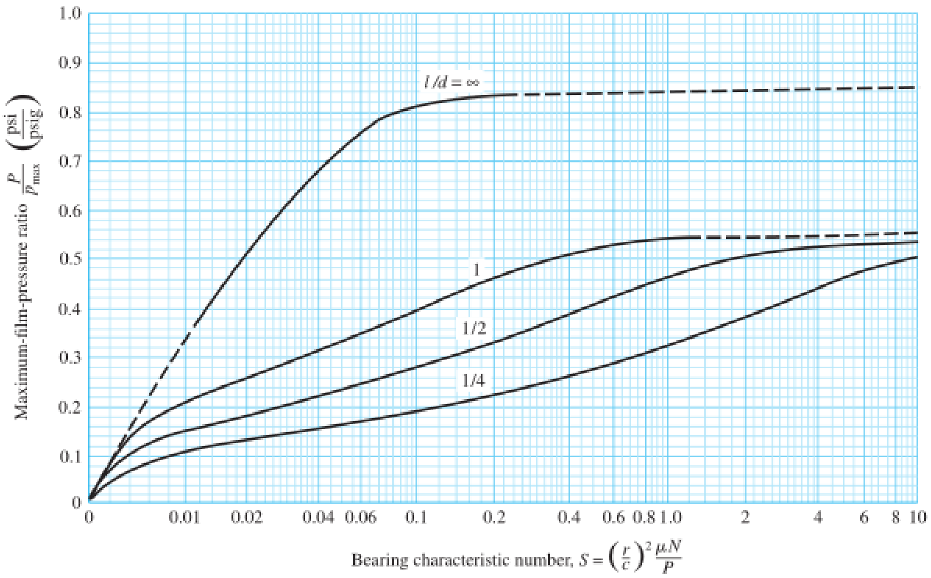
$$\frac{P}{p_{max}} = 0.5, so\ p_{max} = \frac{250}{0.5} = 500\ psi$$

$$\theta_{pmax} = 15°$$

$$\theta_{p0} = 90°$$

Figure 12–21

Chart for determining the maximum film pressure. *Note:* Not for pressure-fed bearings. *(Raimondi and Boyd.)*



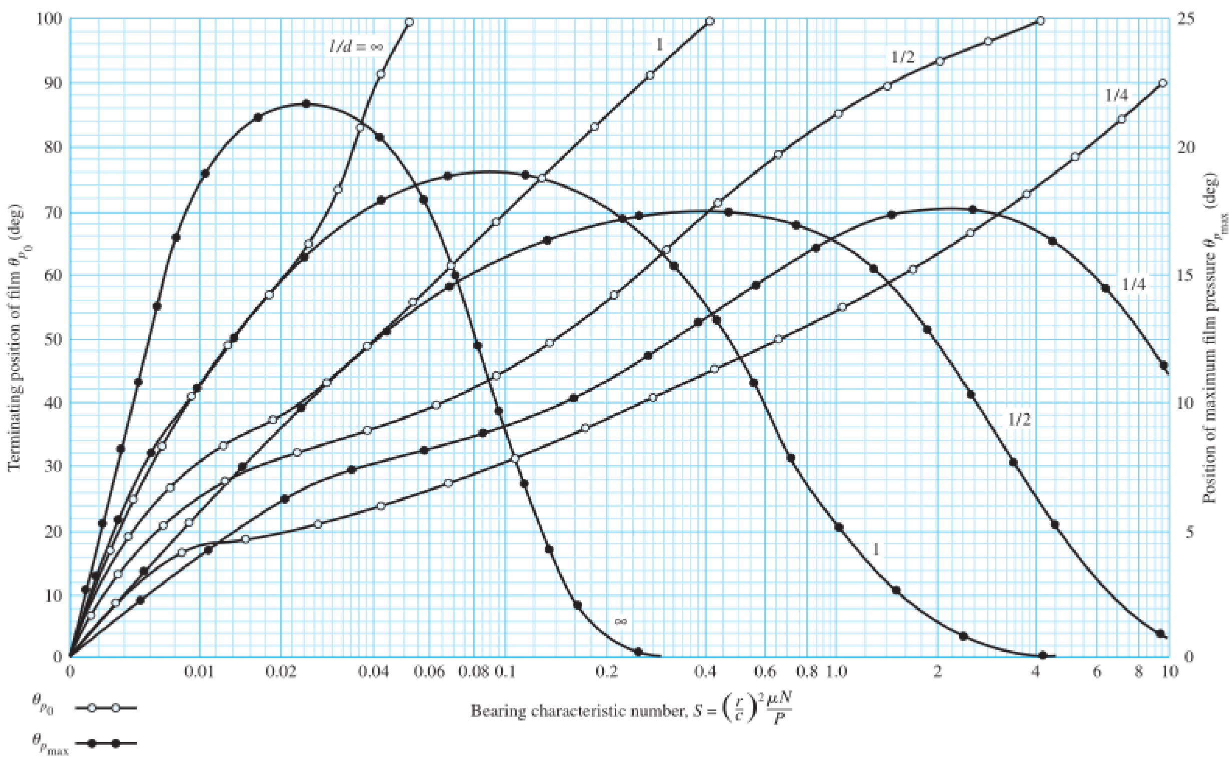


Figure 12–22

Chart for finding the terminating position of the lubricant film and the position of maximum film pressure. *(Raimondi and Boyd.)*

(2) There are two ways to determine temperature rise.

$$\frac{9.70\Delta T_F}{P_{psi}} = \frac{1}{1-0.5\left(\frac{Q_s}{Q}\right)} \frac{\frac{r}{c}f}{\frac{Q}{rcNl}} = \frac{1}{1-(0.5)(0.45)} \frac{7}{3.85} = 2.3460$$

$So, \Delta T_F = (2.3460)(250)/(9.70) = 60.5\ °F$

Or, Figure 12-24 (noting $S = 0.33$)

$$\frac{9.70\Delta T_F}{P_{psi}} = 0.349109 + 6.00940S + 0.047467S^2 = 2.3374$$

So, $\Delta T_F = (2.3374)(250)/(9.70) = 60.2\ °F$

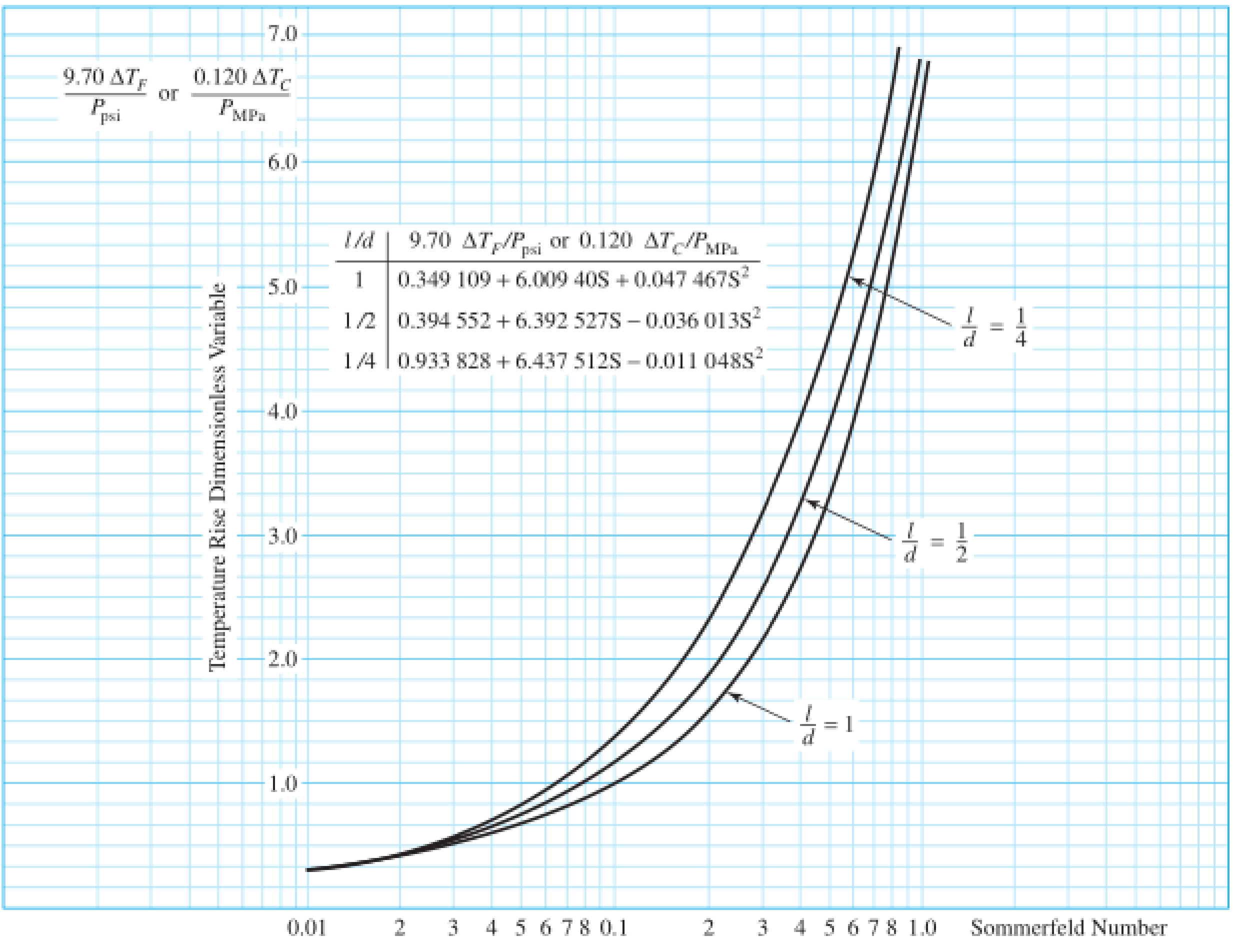


Figure 12–24

Figures 12–18, 12–19, and 12–20 combined to reduce iterative table look-up. *(Source: Chart based on work of Raimondi and Boyd boundary condition (2), i.e., no negative lubricant pressure developed. Chart is for full journal bearing using single lubricant pass, side flow emerges with temperature rise $\Delta T/2$, thru flow emerges with temperature rise ΔT, and entire flow is supplied at datum sump temperature.)*

Note: Figure 12-24 combines Figures 12-18, 12-19, 12-20, to facilitate the evaluation of ΔT_F or ΔT_C, which in turn is to expedite the iterative process of journal bearing design.
The metric version of the temperature rise formula is:

$$\frac{0.120\Delta T_C}{P_{MPa}} = \frac{1}{1 - 0.5\left(\frac{Q_s}{Q}\right)} \frac{\frac{r}{c} f}{\frac{Q}{rcNl}}$$

12-12 Loads and Materials
12-13 Bearing Types
Unit Loads in Typical Applications: Table 12-5.

Table 12-5

Range of Unit Loads in Current Use for Sleeve Bearings

Application	Unit Load psi	Unit Load MPa
Diesel engines:		
Main bearings	900–1700	6–12
Crankpin	1150–2300	8–15
Wristpin	2000–2300	14–15
Electric motors	120–250	0.8–1.5
Steam turbines	120–250	0.8–1.5
Gear reducers	120–250	0.8–1.5
Automotive engines:		
Main bearings	600–750	4–5
Crankpin	1700–2300	10–15
Air compressors:		
Main bearings	140–280	1–2
Crankpin	280–500	2–4
Centrifugal pumps	100–180	0.6–1.2

Bearing Materials
Table 12-6 lists bearing alloys for applications involving high speed, high temperature, and high varying loads:

- Automotive engines (e.g., connecting rod, crankshaft)
- Turbo machinery

Why Alloys?
Babbitt metals are tin- or lead-based bearing materials, named after Babbitt, who came up with some tin-based alloys (~80% tin, ~10% antimony, ~10% copper).

Lead-based: ~80% lead, ~15% antimony, ~5% tin

Lead-based Babbittt can compensate for reasonable shaft misalignment and deflections.

Tin-based and Lead-based Babbitt metals allow foreign particles to become embedded not the bearing to prevent scratching of journal and bearing.

Trimetals typically have layers of materials such as Babbitt, copper or lead, and steel backing.

Trimetals have high fatigue strength to support compressive cyclic loading. In addition, metals like lead, copper and so on, have high thermal conductivity to remove heat rapidly from the bearing.

Bearing Types:
See Figures 12-32 to 12-34, from (Very simple) solid bushing to two-piece designs with elaborate groove pattern.

12-7 Design Variables

- **Controlling Variables** (variables under the control of the designer)
  Viscosity of the lubricant μ;
  Unit load $P = W/(2rl)$; ($W = radial\ load, d = 2r$)
  Speed N (rps);
  Bearing dimensions: r (journal radius), c (radial clearance), and l (bearing length).
- **Dependent Variables** (variables determined by the charts; they indicate how well the bearing performs, hence performance variables)
  The coefficient of friction;
  The temperature rise;
  The maximum film pressure and location;
  The flow rate of lubricant:
  The minimum film thickness and location; and so on

Recommended Bearing Dimensions

- $l/(2r) = 0.25 \sim 1.5$ (not to exceed 2.0);
  Shorter bearings place less stringent requirement on shaft deflection and misalignment; longer bearings on the other hand have less end leakage.
- $c/r = 0.001 \sim 0.0015$;
  The lower end value 0.001 is for precision bearings; the higher end value 0.0015 is for less precise bearings.

Minimum Film Thickness

- $h_{min} = 0.0002 + 0.000\ 04d$ (British units, d and h_{min} in mm)
  or
  $h_{min} = 0.005 + 0.000\ 04d$ (Metric units, d and h_{min} in mm)
- The first term on the RHS represents the peak-valley roughness of finely ground journal surface. Smaller h_{min} would require more expensive manufacturing.
- The second term on the RHS represents the influence of size (tolerance increases with size)

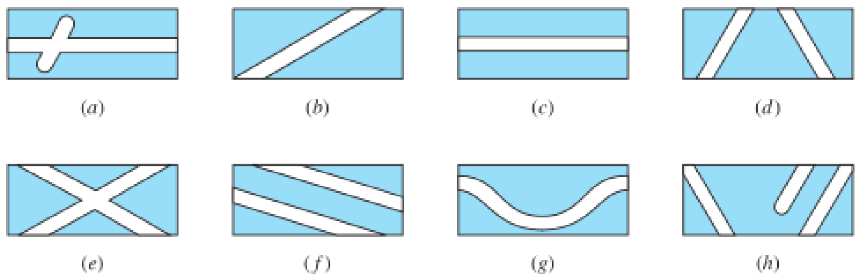
What Constitutes a Good Design?

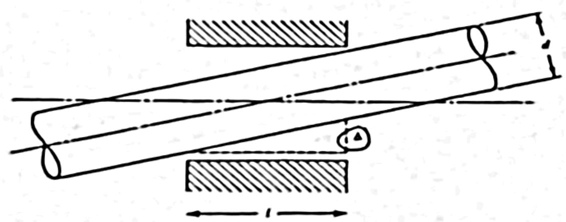
- Minimum film thickness: $h_0 \geq h_{min}$
- Friction: as low as possible ($f < 0.01$)
- Maximum temperature: $T_{max} \leq 250°F$
- In the "optimal zone" (as shown in Figure 12-16)

- Minimum film thickness is greater than shaft deflection across the length of bearing (to prevent binding between shaft and bearing; see diagram below)
- Using a suitable bearing material (see Sec. 12-12)
- Able to accommodate changes in clearance, temperature, viscosity, etc. (this will be discussed in Sec. 12-10)
- Allowing lubricant to be distributed over the surface (see Figure 12-34 regarding groove pattern)

Figure 12–34

Developed views of typical groove patterns. *(Courtesy of the Cleveland Graphite Bronze Company, Division of Clevite Corporation.)*





Δ: shaft deflection across the bearing; *l*: length of bearing.

Courtesy of "Machine Design Databook", Ch.23, K. Lingaiah, 2nd Ed.,

Procedure

- There is not a *typical* procedure.
- The common theme amongst the procedures: most involve assumptions and iterations, and do not guarantee convergence in the end, not to mention a solution/design in the optimal zone and meeting other requirements.

The first part of the process includes,

1. Choose bearing dimensions r, l, and c, making reference to Table 12-5: unit load for various applications.
2. Determine significant angular speed N by (Eq. 12-13); and
3. Select a lubricant

$$N = |N_j + N_b - 2N_f| \qquad (12\text{–}13)$$

where N_j = journal angular speed, rev/s
N_b = bearing angular speed, rev/s
N_f = load vector angular speed, rev/s

Where N_j is the speed of the journal; N_b is the speed of the bushing and N_f is the speed of load W as a vector. And all speeds are in rps. Figure 12-11 shows examples of applying (Eq. 12-13)

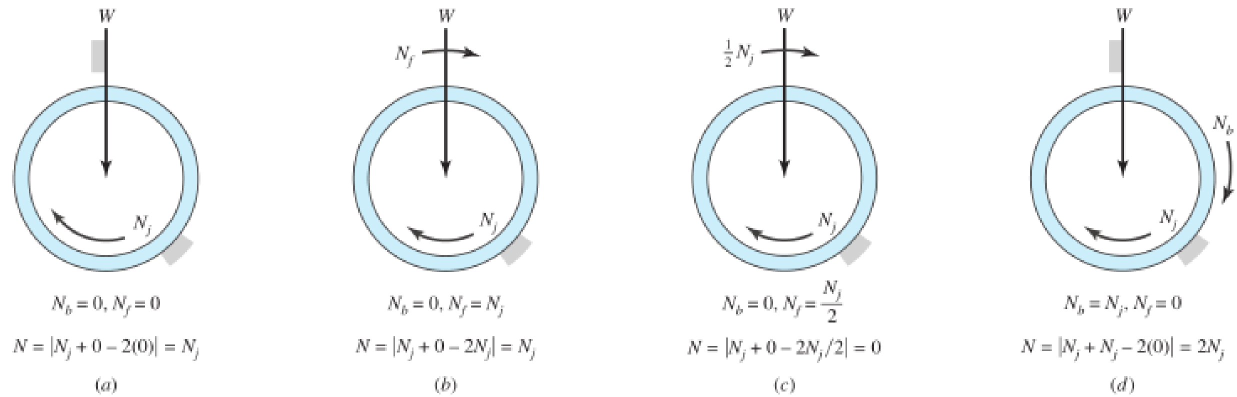


Figure 12–11

How the significant speed varies. (*a*) Common bearing case. (*b*) Load vector moves at the same speed as the journal. (*c*) Load vector moves at half journal speed, no load can be carried. (*d*) Journal and bushing move at same speed, load vector stationary, capacity halved.

The second part involves assumptions, iterations and so on. It ends when a converged solution/design is found. For example,

> Assume a temperature rise ΔT; determine the lubricant's temperature rise ΔT_F (in Fahrenheit) or ΔT_C (in Celsius); compare the assumed value with the calculated value; and if necessary, assume a new ΔT or select a different lubricant, iterate until the assumed ΔT and calculated ΔT_F or ΔT_C are close; (p. 624 of text, last paragraph)

For other ways to iterate for a converged solution/design, it's recommended that you reference other sources.

The last part is to evaluate the performance variables via Figures 12-16 through 12-22, and check against design requirements such as, in the optimal zone, $T_{max} \leq 250°F$, and so on. Typically, power loss due to friction, or rate of heat loss, is also determined.

Power loss H_{loss}

British units:

$$H_{loss} = \frac{fWrN}{1050}$$

Where H_{loss} is in hp; W is in lb; r is in inch; and N is in rps.

Metric units:

$$H_{loss} = \frac{fWrN}{9549}$$

Where H_{loss} is in kW; W is in newton; r is in m; and N is in rpm.

Example 2

A journal bearing has $d = l = 1.5"$, $c = 0.0015"$, $W = 500\ lb$, and $N = 30\ rps$. Lubricant is SAE20 oil with an inlet temperature of $100°F$. Design and evaluate the bearing.

Solution:

The first part of the process is not necessary since d, l , c, and N are given and lubricant is chosen. So,

$$P = \frac{W}{ld} = \frac{500}{1.5 \cdot 1.5} = 222.2\ psi, N = 30\ rps, \frac{l}{d} = 1, and\ \frac{r}{c} = \frac{0.75}{0.0015} = 500$$

Also, $T_1 = 100°F$

The second part is completed as follows:

Select $\mu = 4\ \mu reyn$

Sommerfeld number is:

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = (500)^2 \frac{(4)(10^{-6})(30)}{222.2} = 0.135$$

Temperature rise is, by Figure 12-24

$$\frac{9.70\Delta T_F}{P_{psi}} = 0.349109 + 6.00940S + 0.047467S^2 = 1.161$$

$$\Delta T_F = \frac{(1.161)(222.2)}{9.70} = 26.6°F$$

And the average temperature is $T_{av} = T_1 + \frac{\Delta T_F}{2} = 113.3°F$

On Figure 12-12, plot ($T_{av},\ \mu$). It is located below the SAE20 line.

Select $\mu = 7\ \mu reyn$

Sommerfeld number is:

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = (500)^2 \frac{(7)(10^{-6})(30)}{222.2} = 0.236$$

Temperature rise is, by Figure 12-24

$$\frac{9.70\Delta T_F}{P_{psi}} = 1.773$$

$$\Delta T_F = \frac{(1.773)(222.2)}{9.70} = 40.6°F$$

And the average temperature is $T_{av} = T_1 + \frac{\Delta T_F}{2} = 120.3°F$

Plot the second point ($T_{av},\ \mu$), making sure it is above the SAE20 line. Otherwise, assume a different viscosity until ($T_{av},\ \mu$) is above the line.

Draw a straight line between the two points of (T_{av}, μ). The intersecting point with the SAE20 line is (117, 5.2).

So, $T_{av} = 117°F$ or $\Delta T_F = 34\ °F$ and $5.2\ \mu reyn$
(Keep in mind every time we have a new viscosity, we have to find a new Sommerfeld number)

The last part is as follows:
$T_{max} = 134°F < 250°F$

Figure 12-15: $\frac{h_0}{c} = 0.49$
$h_0 = (0.49)(0.0015) = 0.000\ 735"$
$h_{min} = 0.000\ 2 + 0.000\ 04d = 0.000\ 26"$

So, $h_0 > h_{min}$ and $(S, h_0/c)$ is inside the optimal zone.
(Where $S = 0.176$ and $h_0/c = 0.49$)

Figure 12-17: $(r/c)f = 4.2$
$$f = \frac{4.2}{500} = 0.0084 < 0.01$$

Figure 12-20: $\frac{P}{p_{max}} = 0.45$
$$p_{max} = \frac{222.2}{0.45} = 494\ psi$$

Power loss:
$$H_{loss} = \frac{fWrN}{1050} = \frac{(0.0084)(500)(0.75)(30)}{1050} = 0.0897\ hp$$

Example 3
A journal bearing has $r = l = 1.5"$, $c = 0.0015"$, $W = 1000\ lb$, and $N = 30\ rps$. Lubricant's inlet temperature is assumed to be $120°F$. The bearing is to be designed with high load capacity. Select a lubricant, and design and evaluate the bearing.

Solution:
The first part of the process is as follows:
$P = \frac{W}{ld} = \frac{1000}{1.5*3} = 222.2\ psi$, $N = 30\ rps$, $\frac{l}{d} = \frac{1}{2}$, and $\frac{r}{c} = \frac{1.5}{0.0015} = 1000$
Also, $T_1 = 120°F$

For the second part,
Figure 12-16: on the plot of $\frac{l}{d} = ½$, select a point located close to the "Max W" edge, say $S = 0.3$
$$S = 0.3 = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = (1000)^2 \frac{\mu(30)}{222.2}$$
So, $\mu = 2.22\ \mu reyn$

Figure 12-24 with $S = 0.3$:
$$\frac{9.70\Delta T_F}{p_{psi}} = 0.394\ 552 + 6.392\ 527S - 0.036\ 013S^2 = 2.309$$
$\Delta T_F = (2.309)(222.2)/(9.70) = 52.9°F$

So, $T_{av} = T_1 + \frac{\Delta T_2}{2} = 146°F$

Figure 12-12: locate the point (146, 2.22). It is below the SAE20 line. On the line, $\mu = 2.5\ \mu reyn$, which gives a Sommerfeld number $S = 0.34$. The design remains inside the optimal zone.

Or, the average temperature needs to be $153°F$ for the lubricant to have a viscosity of 2.22 $\mu reyn$. That means inlet temperature needs to be at $126.5°F$.

The last part gives the following results: (based on $S = 0.34$)
$h_0 = (0.43)c = 0.000\ 625"$
$h_{min} = 0.000\ 2 + 0.000\ 04d = 0.000\ 32" < h_0$

$f = (8.5)/(r/c) = 0.0085 < 0.01$

$Q = (4.8)/(rcNl) = 1394\ in^3/s$
$Q_s = 0.7\dot{2} \cdot Q = 1004\ in^3/s$

$p_{max} = P/0.375\ = 593\ psi$

12-10 Clearance
Why this section?

- Clearance c has a range due to manufacture and assembly
- It tends to increase due to wear

How to take into consideration change in clearance?
A suitable fit is assigned between journal and bushing. For example, H8,f7 (close to running fit) or H9/d9 (free running);

Then the range of clearance is determined (see Table 12-3, for example);

Performance of the bearing (As indicated by h_0, T_2 the outlet temperature, Q, and H the power loss, for example) is calculated and plotted against clearance c. See Figure 12-25.

The initial clearance band (i.e., the tolerance specified for manufacturing) should be located to the left of the peak of the $h_0 - c$ plot.

Chapter 13
Gears - General

Part 1: Geometry and Tooth System

13-1 … 13.8:

Types of Gears, …, The Forming of Gear Teeth

13-12: Tooth Systems

Part 2: Kinematics

13-13: Gear Trains

Part 3: (to be discussed with Chapters 14 and 15)

13-19 … 13-11:

Bevel Gears, Parallel Helical Gears, Worm Gears

13-14 … 13-17:

Force Analysis – Spur, Bevel, Helical and Worm Gears

13-1 Types of Gears

Why Gears?

Of constant-speed mechanical transmission elements, the frequencies of usage are:

- Gears: 50%
- Couplings: ~20%
- Chain Drives: 10−20%
- Belt Drives: 10-12%
- Power screws, wire ropes, friction wheels, etc.: 5-10%

Types of Gears:

- Spur gears: most common; transmit power between two parallel shafts
- Helical gears: between two intersecting shafts
- Bevel gears: between intersecting shafts
- Worm gear sets: between non-parallel and non-intersecting shafts
- And many other types

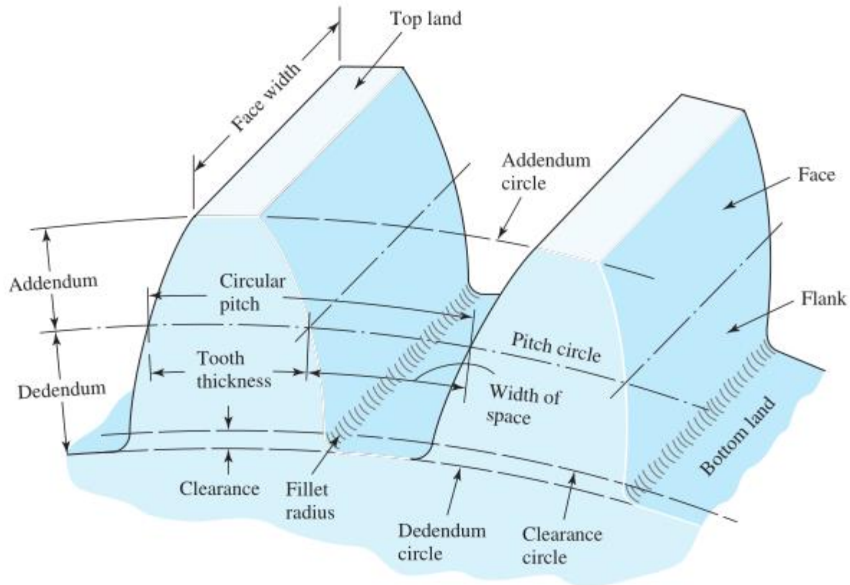
13-2 Nomenclature

13-12 Tooth Systems

Nomenclature: Figure 13-5, for spur gears only.

Figure 13–5

Nomenclature of spur-gear teeth.



Tooth System: refers to the standard that specifies the tooth geometry and so on. There are the metric system and the US customary system.

- Pitch circle
- Circular pitch p, pitch diameter d, in in or mm
- Number of teeth N
- Diametral pitch $P = N/d$, in teeth/in; or
- Module $m = d/N$, in mm
- P and m are standardized, see Table 13-2
- Metric gears and US customary gears are NOT interchangeable

Tables 13-1, 13-3, 13-4: formulas for spur gears, 20° straight bevel gears, and helical gears

Table 13-5: information for worm gearing

Typical values for face width

$$\frac{3\pi}{P} \leq F \leq \frac{5\pi}{P}$$

Or

$$3\pi m \leq F \leq 5\pi m$$

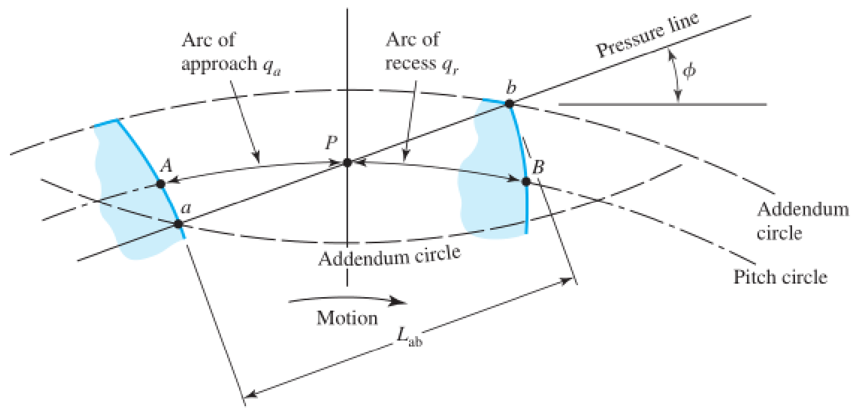
13-3 Conjugate Action
13-4 Involute Properties
13-5 Fundamentals

- The fundamental Law of Gearing:
  Angular velocity ratio between the gears of a gearset must remain constant throughout the mesh
- Involute tooth form meets the fundamental law, and has the advantage that error in center-to-center distance will not affect the angular velocity ratio.

13-6 Contact Ratio

Figure 13–15

Definition of contact ratio.



- Addendum circles
- Pressure line (passing through pitch circles)
- A part of gear teeth enters into contact at point a and exits from contact at point b on the same line
- The distance between these points in the length of action L_{ab} also labelled as Z.

Gear contact ratio m_c defines the average number of teeth that are in contact at any time,

$$m_c = \frac{L_{ab}}{p_b} = \frac{L_{ab}}{pcos\phi}$$

Where $p_b = p \cdot cos\phi$ is the base pitch and ϕ is the pressure angle.

Significance:

For a pair of gears the mesh properly, their diametral pitch or module, and pressure angle must be the same. In addition, contact ratio must meet certain requirements.

$$m_c = \frac{L_{ab}}{pcos\phi} = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag}^2 - r_{bg}^2} - csin\phi}{pcos\phi}$$

Or

$$m_c = \frac{L_{ab}}{\pi mcos\phi} = \frac{\sqrt{r_{ap}^2 - r_{bp}^2} + \sqrt{r_{ag^2} - r_{bg}^2} - csin\phi}{\pi mcos\phi}$$

Where:

r_{ap}, r_{bp}: radii of addendum circle and base circle of the pinion;
r_{ag}, r_{bg}: radii of addendum circle and base circle of the gear;
c: center-to-center distance;

The radius of base circle of a gear is $r_b = \frac{d}{2}cos\phi$, with d being the pitch diameter.

Example 1

A gear set has diametral pitch of $P = 10\ teeth/in$ and pressure angle of 20°. Teeth numbers are $N_p = 30$ and $N_g = 75$. Determine the contact ratio of the set. Assume full depth tooth profile.

Solution:

	Pinion	**Gear**
Pitch radius, in	1.5	3.75
Addendum radius, in	1.6	3.85
Base radius, in	1.4095	3.5238
Center-to-centre distance, in	5.25	

$$L_{ab} = 0.7572 + 1.5509 - 17956 = 0.5125"$$
$$p = \frac{\pi}{P} = 0.3142"$$
$$m_c = \frac{0.5125}{0.3142 \cdot \cos(20°)} = 1.74$$

Example 2

A set of stub-profiled gear has, $N_p = 18$ and $N_g = 72$. Module is $m = 5\ mm$. Determine the set's contact ratio. Pressure angle is 22.5°.

Solution:

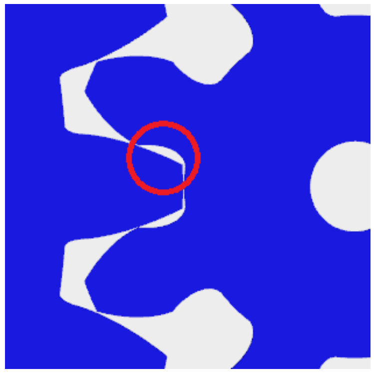
	Pinion	**Gear**
Pitch radius, mm	45	180
Addendum radius, mm	49	184
Base radius, mm	41.575	166.298
Center-to-centre distance, mm	225	

$$L_{ab} = 25.933 + 78.746 - 86.104 = 18.575\ mm$$
$$m_c = \frac{18.575}{n \cdot \pi \cdot \cos(22.5°)} = 1.28$$

13-8 The Forming of Gear Teeth

13-7 Interference

- Mainly, there are form cutting and generating cutting
  - Form cutting: the cutter is the exact shape of the tooth space; expensive
  - Generating cutting: the cutter has a shape different from the tooth space; more common
- Of interest to discussing interference is generating cutting which includes,
  - Shaping: pinion cutting (Figure 13-17) and rack cutting (Figure 13-18)
  - Hobbing: using hob, a worm-like cutting tool, to cut a blank (Figure 13-19)
- Interference refers to contact taking place on the non-involute portion of the tooth profile (inside base circle)
- Undercut refers to the removal of interfering material during generating cutting.



- To avoid interference, the pinion requires a minimum number of teeth ,while the gear has a restriction on maximum number of teeth.
- The text has three equations, (Eq. 13-10), (Eq. 13-11) and (Eq. 13-13) for determining the minimum number of teeth to avoid interference.
- (Eq. 13-11) is for general cases, and recommended. For a pinion-gear set, the minimum number of teeth on pinion without interference is, $(Eq. 13-11)$:

$$N_P = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right) \qquad (13\text{–}11)$$

Where:
m is the gear ratio $m = N_G/N_p . m > 1$.
$k = 1$ for full-depth teeth, and 0.8 for stub teeth
ϕ is the pressure angle

- (Eq. 13-10) is for cases of one-to-one gear ratio.

$$N_P = \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right) \qquad (13\text{–}10)$$

- (Eq. 13-13) is for cases of pinion meshing with a rack.

$$N_P = \frac{2(k)}{\sin^2\phi} \qquad (13\text{–}13)$$

- Maximum number of teeth on a gear mating with a specific pinion is determined by (Eq. 13-12)

$$N_G = \frac{N_P^2\sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi} \qquad (13\text{–}12)$$

- A number of examples are shown within the section.

1st example: a set of gears, shapes by pinion cutter, 20° pressure angle, full-depth teeth; then smallest N_p is 1 3and largest N_G is 16.

2nd example: a set of gears, 20° pressure angle, full-depth teeth, cut by hobbing; then smallest N_p is 17 and largest N_G is 1309.

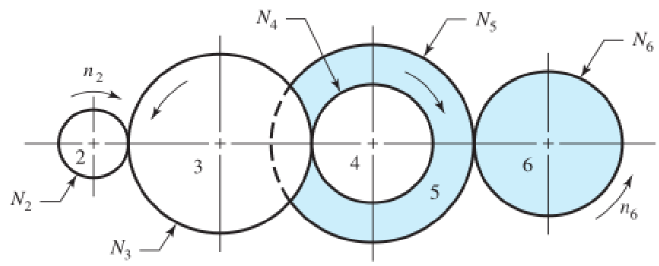
13-13 Gear Trains

Types of Gear Trains

- Simple of series trains, See Figure 13-27

Figure 13–27

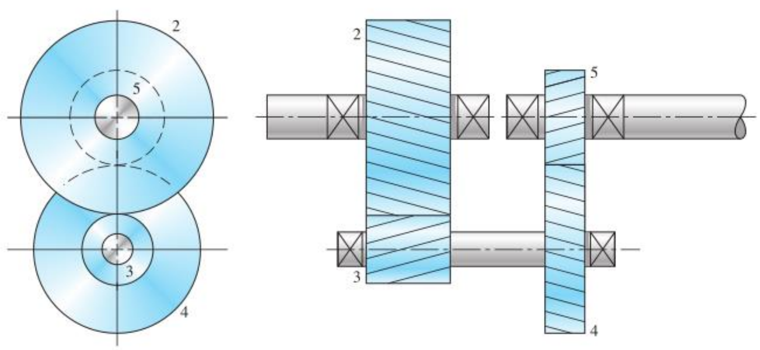
A gear train.



- Compound trains
    - Reverted (Figure 13-29)

Figure 13–29

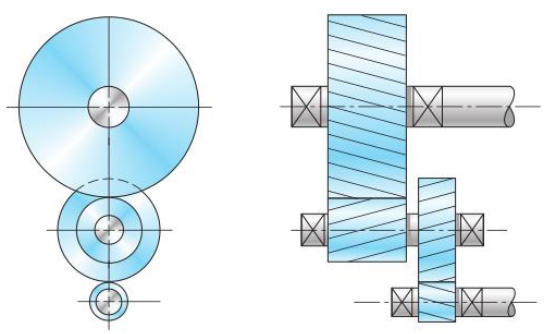
A compound reverted gear train.



-
    - Non-reverted (Figure 13-28)

Figure 13–28

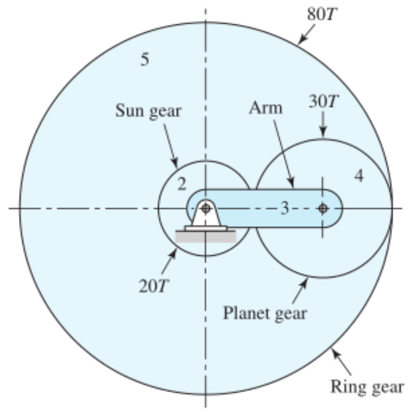
A two-stage compound gear train.



- Planetary or epicyclic trains (Figure 13-30)

Figure 13–30

A planetary gear train.



Train Value, Speed Ratio, Gear Ratio, and so on

- In the text, train value e is used. It is defined as

$$e = \pm \frac{product\ of\ driving\ tooth\ numbers}{product\ of\ driven\ tooth\ numbers}$$

  e is positive if the last gear rotates in the same sense as the first, and negative if the last gear rotates in the opposite sense.
  e is also the ratio of n_L, the speed of the last gear, over the speed of the first gear n_F.

$$e = \frac{n_L}{n_F}$$

- Speed ratio = velocity ratio = transmission ratio – train value.
- Gear ratio is commonly used in daily conversions. Gear ratio $= 1/e$

Problem-Solving

- Given a train, find velocity ratio;
- Given required gear ratio, determine the type of train and teeth numbers.
  See Examples $13-3 \sim 13-5$.

Velocity Ratio – Planetary Gear Trains

- Tabular method see "Dynamics of Machinery", R. L. Norton.
  - Follow the power flow
  - Velocity difference equation

$$\omega_{gear} = \omega_{arm} + \omega_{gear/arm}$$

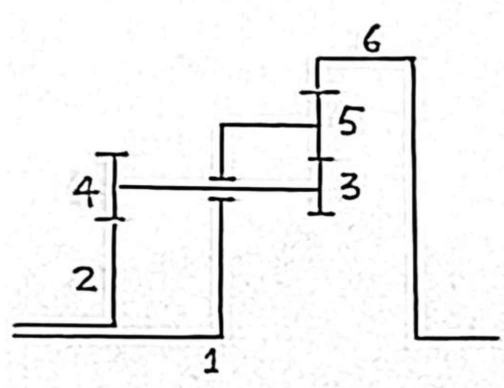
    (NOTE: The last term represents the velocity of gear relative to the arm.)
  - Relative velocity obeys

$$VR = \frac{[\omega_{gear/arm}]_{driven}}{[\omega_{gear/arm}]_{driver}} = \pm \frac{N_{driver}}{N_{driven}}$$

    (NOTE: Where the " $+$ " is used with internal set and " $-$ " is used with external set.)

Example 3:

The schematic of a planetary gear train is shown below, with "1" being the arm. Gears 2 and 6 rotate about the same axis as the arm. Input is to Gear 2.



(1) Given $N_2 = 30$, $N_3 = 25$, $N_4 = 45$, $N_5 = 30$, $N_6 = 160$, $\omega_2 = 50\ rad/s$, $\omega_{arm} = -75\ rad/s$
Find ω_6.
(2) Given $N_2 = 30$, $N_3 = 25$, $N_4 = 45$, $N_5 = 50$, $N_6 = 200$, $\omega_2 = 50\ rad/s$, $\omega_6 = 0$
Find ω_{arm}, ω_3, ω_4, ω_5

Solution:
(1): Power flow: 2 → 4 & 3 → 5 → 6

Gear	ω_{gear} =	ω_{arm} +	$\omega_{gear/arm}$	VR
2	50	-75	125	$-N_2/N_4$
4		-75	$(125)\left(-\frac{N_2}{N_4}\right)$	
3				$-N_3/N_5$
5		-75	$(125)\left(-\frac{N_2}{N_4}\right)\left(-\frac{N_3}{N_5}\right)$	
6	ω_6	-75	$(125)\left(-\frac{N_2}{N_4}\right)\left(-\frac{N_3}{N_5}\right)\left(\frac{N_5}{N_6}\right)$	N_5/N_6

$\therefore \omega_6 = -61.98\ rad/s$

(2): Power flow: 2 → 4 & 3 → 5 → 6

Gear	ω_{gear} =	ω_{arm} +	$\omega_{gear/arm}$	VR
2	50	x		
4		x		
3				
5		x		
6	0	x		

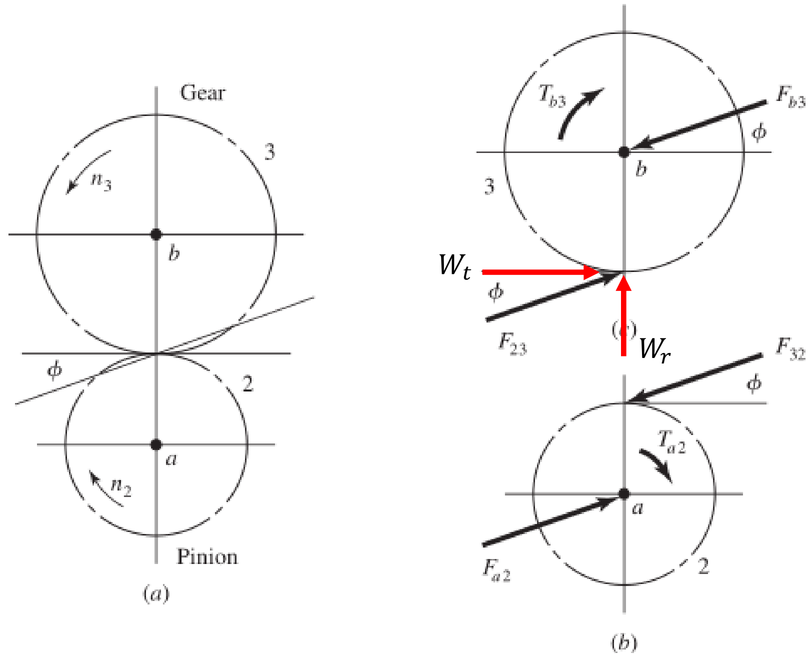
$\omega_{arm} = x = -4.545\ rad/s$
$\omega_3 = \omega_4 = -40.91\ rad/s$
$\omega_5 = 13.64\ rad/s$

Spur Gears

13-14 Force Analysis – Spur Gearing

Figure 13–32

Free-body diagrams of the forces and moments acting upon two gears of a simple gear train.



F_{23} and F_{32} are action and reaction forces;
Transmitted load: $W_t = F_{23}cos\phi = F_{32}cos\phi$;
Radial load (separating force): $W_r = F_{23}sin\phi = F_{32}sin\phi$

Determination of Transmitted Load

US-customary units

Pitch line velocity

$$V = \pi dn/12 \qquad (13–34)$$

Where d and n are pinion pitch diameter (in in) and pinion speed (in rpm), or gear pitch diameter (in in) and gear speed (in rpm). V is pitch line velocity in ft/min.

Transmitted Load W_t (or W^t):

$$W_t = 33\,000\frac{H}{V} \qquad (13–35)$$

Where H is power in hp; V is pitch-line speed in ft/min; and W_t is in lb.

SI Units

$$W_t = \frac{60\,000H}{\pi dn} \qquad (13–36)$$

Where H is power in kW; d is pitch diameter in mm (of pinion or gear); n is speed (of pinion or gear) in rpm; and W_t is in kN.

Gear Materials
Information from the 6th Ed.

- **Through-hardening:** (by annealing, normalizing and annealing, and quench and temper)
  1040, 1060, 1335, 3135, 4037, 4140, 4340, 5150, 8640, and 8740, with 4140 and 4340 being the most commonly used.
- **Case hardening**
  By carburization (up to 600 HB):
  4118, 4320, 4620, 4720, 4820, 5120, and 8620
  By nitrization:
  4140, 4340

Information from AGMA

- AGMA recommends to specify the following for gear materials:
  - Material designator or stress grade
  - Material cleanliness
  - Surface and core hardness
  - AGMA quality level
- Gear materials are given a stress grade $0 \sim 3$.
  0: Ordinary quality. No gross defects <u>but</u> no close control of quality;
  1: Good quality. Modest control of most important quality items; used in typical industrial applications;
  2: Premium quality. Close control of all critical quality items; improved quality/performance but increased material cost;
  3: Superior quality. Absolute control of all critical quality items; ultimate performance but high material cost.

- Material *cleanliness*
  This is by AMS, Aerospace Material Specification.
  Grade 3 materials: call for AMS 2300 (Premium Aircraft-Quality Steel Cleanliness, Magnetic Particle Inspection).
  Grade 2 materials: call for AMS 2301 (Cleanliness, Aircraft Quality Steel Magnetic Particle Inspection Procedure).
  Grade 1 materials: not required to adhere to any AMS specification.
- AGMA *quality* (or accuracy) *level:*

Q_V	Descriptive	Manufacture	Applications
14-15	Highest level	Trade secret	Highest load & reliability; high speed
12-13	High level	Grinding, shaving	Aerospace turbo-machinery
10-11	Relatively high	Grinding, shaving	Mass production; automotive vehicles; automotive
8-9	Good	Hobbing, shaping	Automotive; electric motor; industrial
6-7	Nominal	Hobbing, shaping (by older machines)	Low speed gears
4-5	Minimal	Casting, molding	Slow speed gears; toys, gadgets

Chapter 14

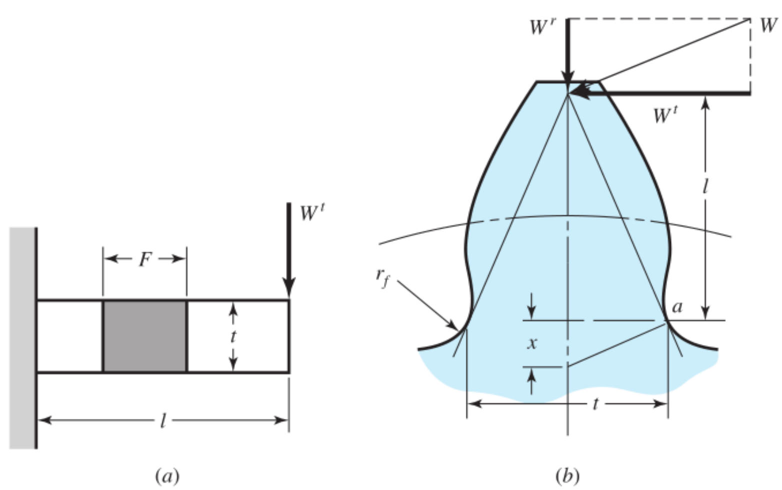
14-1: The Lewis Bending Equation
14-2: Surface Durability
Main failure modes of gearing

- Bending fatigue → Breakage of the tooth
- Contact fatigue → Pitting and spalling
- Wear (due to adhesion, abrasion, corrosion, scoring, scuffing …)

Bending Fatigue is caused by excessive dynamic bending stress at the base of the tooth;

Surface Fatigue is caused by repeated applications of loads on the surface.

Figure 14-1 and the Lewis Bending Equation



A tooth is considered a cantilever beam;

Transmitted and radial loads are labelled W^t and W^r;

Span of beams depends on tooth geometry;

Bending stress at "fixed end": by (Eq. 14-2) or (Eq. 14-3);

$$\sigma = \frac{W^t P}{FY} \tag{14–2}$$

$$Y = \frac{2xP}{3} \tag{14–3}$$

Dynamic effects: (Eq. 14-7) or (Eq. 14-8);

$$\sigma = \frac{K_v W^t P}{FY} \tag{14–7}$$

$$\sigma = \frac{K_v W^t}{FmY} \tag{14–8}$$

Example 14-1: Applying above equations to determine required horsepower;
Example 14-2: Fatigue due to bending stress for infinite life;

Drawbacks: compressive stress due to W^r is not considered.

Surface Durability
At the contact point, each tooth is considered part of a cylindrical surface;

Hertz contact theory: see Sec. 3-19;

Contract stress: (Eq. 14-14);

$$\sigma_C = -C_p \left[\frac{K_v W^t}{F \cos \phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2} \tag{14–14}$$

Example 14-3: Applying the above equation;

Drawbacks: fatigue due to contact stress is not considered.

14-3: AGMA Stress Equations
Stress Numbers

- In AGMA terminology, a stress caused by an applied load is a stress number.
- There are two stress numbers: bending and contact.

Bending Stress Number (for spur and helical gears)

$$\sigma = \begin{cases} W^t K_o K_v K_s \dfrac{P_d}{F} \dfrac{K_m K_B}{J} & \text{(U.S. customary units)} \\ W^t K_o K_v K_s \dfrac{1}{bm_t} \dfrac{K_H K_B}{Y_J} & \text{(SI units)} \end{cases} \tag{14–15}$$

Contact Stress Number (for spur and helical gears)

$$\sigma_c = \begin{cases} C_p \sqrt{W^t K_o K_v K_s \dfrac{K_m}{d_P F} \dfrac{C_f}{I}} & \text{(U.S. customary units)} \\ Z_E \sqrt{W^t K_o K_v K_s \dfrac{K_H}{d_{w1} b} \dfrac{Z_R}{Z_I}} & \text{(SI units)} \end{cases} \tag{14–16}$$

14-4 AGMA Strength Equations

Allowable Stress Numbers

- In AGMA terminology, an allowable stress is an allowable stress number.
- There are two allowable stress numbers: allowable bending and allowable contact.

Allowable Bending Stress Number (for spur and helical gears)

$$\sigma_{\text{all}} = \begin{cases} \dfrac{S_t}{S_F}\dfrac{Y_N}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_t}{S_F}\dfrac{Y_N}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \tag{14–17}$$

(Where S_F is factor of safety)

Allowable Contact Stress (for spur and helical gears)

$$\sigma_{c,\text{all}} = \begin{cases} \dfrac{S_c}{S_H}\dfrac{Z_N C_H}{K_T K_R} & \text{(U.S. customary units)} \\ \dfrac{S_c}{S_H}\dfrac{Z_N Z_W}{Y_\theta Y_Z} & \text{(SI units)} \end{cases} \tag{14–18}$$

(Where S_H is factor of safety)

14-18 Analysis

Figures 14-17 and 14-18

Summary of above formulas (US-customary units only), including where to find the factors.

Overload Factors K_o

Table of Overload Factors, K_o

	Driven Machine		
Power source	**Uniform**	**Moderate shock**	**Heavy shock**
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Examples of power sources and driven machines in each “shock” category (courtesy Mechanical Design of Machine Elements and Machines, A Failure Prevention Perspective, J.A. Collins, Wiley & Sons, 2003).

Power sources:

- Uniform: electric motors, steam turbines, gas turbines;
- Light shock: multi-cylinder engines;
- Medium Shock: single-cylinder engines.

Driven machines:

- Uniform: generators; uniformly loaded conveyors;
- Medium Shock: centrifugal pumps, reciprocating pumps and compressors, heavy-duty conveyors, main drives of machine tools;
- Heavy Shock: punch press, crushers, shears, power shovels.

Example 14-4: Spur gears
Example 14-5: Helical gears

14-19 Design of a Gear Mesh
Initial Steps

- Choose a diametral pitch; $(P = 8, or\ 10\ teeth/in)$
- Select face width and material;
- Decide on core and surface hardness for pinion and gear, and other details such as reliability, etc.

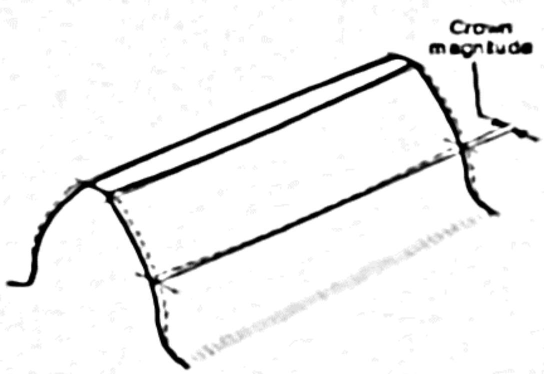
With the second and third steps above, iteration back to the first step may be necessary.

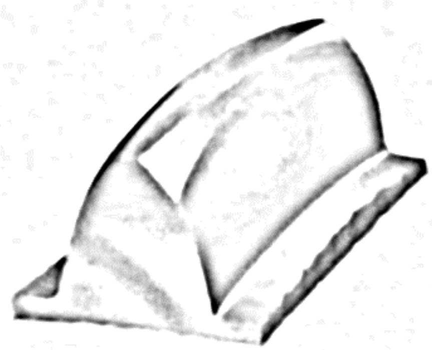
Detailed Calculations

- Referring to page 768;
- It shows all factors, and four factors of safety. They are, one for pinion bending, one for gear bending, one for pinion contact, and one for gear contact.

Example:
A gearbox contains a set of spur ears. IT is driven by a single cylinder engine, and to drive a reciprocating compressor. Output shaft rotates at 1500 rpm, with a maximum torque of 550 lb-in. Gear ratio is 2.5:5 to 1. Pitch diameter of pinion Is expected to be around 3.25”. Assume:

(1) AGMA grade 1 steel through hardened to $H_B = 350$ for pinion and 280 for gear;
(2) 20° full depth and uncrowned teeth;
(3) $Q_V = 10$;
(4) 10-year life of 8-hour shift continuous operation;
(5) Gears are located in the mid-span of their respective shafts;
(6) 99% reliability.
(7) Oil temperature is less than $250°F$





Solution

1. Choose $P = 10\,\frac{teeth}{in}$

   $N_P = P \cdot d_p = (10)(3.25) = 32.5$ - Select $N_p = 33$

   $N_G = (2.5)(33) = 82.5$ - Select $N_G = 83$.

   So $N_P = 33, d_p = 3.3$, N_G =83, d_G=8.3, 20° full depth and uncrowned teeth.

   Contact ratio is:

$$m_c = \frac{0.81145 + 1.6896 - 1.9837}{\left(\frac{\pi}{10}\right)\cos(20°)} = 1.7525$$

2. Face width $F = (3 \sim 5)p = 4p = 1.26"$; Choose $F = 1.25"$.

3. Transmitted load

$$W^t = \frac{T_{max}}{r_G} = \frac{500}{\left(\frac{8.3}{2}\right)} = 132.5\ lb$$

4. AGMA allowable stress numbers

   Tables 14-3 and 14-6 indicate what to use, based on gear materials.

   Pinion: $H_B = 350$, $S_t = 39{,}855\ psi$, $S_c = 141{,}800\ psi$

   Gear: $H_B = 280$, $S_t = 34{,}444\ psi$, $S_c = 119{,}260\ psi$

5. Geometry factors

   For bending: $J_P = 0.4, J_G = 0.445$

   For contact: use (Eq. 14-23) where m_N is the load sharing ratio, and m_G is the speed ratio.

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G + 1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G - 1} & \text{internal gears} \end{cases} \qquad (14\text{–}23)$$

   That is, $m_N = 1$ (for spur gears), and $m_G = \frac{83}{33} = 2.515$.

   So, $I = 0.115$

6. Elastic coefficient

   $C_P = 2300\sqrt{psi}$

Table 14–8

Elastic Coefficient C_p (Z_E), $\sqrt{\text{psi}}$ ($\sqrt{\text{MPa}}$) *Source: AGMA 218.01*

Pinion Material	Pinion Modulus of Elasticity E_p psi (MPa)*	Gear Material and Modulus of Elasticity E_G, lbf/in^2 (MPa)* Steel 30×10^6 (2×10^5)	Malleable Iron 25×10^6 (1.7×10^5)	Nodular Iron 24×10^6 (1.7×10^5)	Cast Iron 22×10^6 (1.5×10^5)	Aluminum Bronze 17.5×10^6 (1.2×10^5)	Tin Bronze 16×10^6 (1.1×10^5)
Steel	30×10^6 (2×10^5)	2300 (191)	2180 (181)	2160 (179)	2100 (174)	1950 (162)	1900 (158)
Malleable iron	25×10^6 (1.7×10^5)	2180 (181)	2090 (174)	2070 (172)	2020 (168)	1900 (158)	1850 (154)
Nodular iron	24×10^6 (1.7×10^5)	2160 (179)	2070 (172)	2050 (170)	2000 (166)	1880 (156)	1830 (152)
Cast iron	22×10^6 (1.5×10^5)	2100 (174)	2020 (168)	2000 (166)	1960 (163)	1850 (154)	1800 (149)
Aluminum bronze	17.5×10^6 (1.2×10^5)	1950 (162)	1900 (158)	1880 (156)	1850 (154)	1750 (145)	1700 (141)
Tin bronze	16×10^6 (1.1×10^5)	1900 (158)	1850 (154)	1830 (152)	1800 (149)	1700 (141)	1650 (137)

Poisson's ratio = 0.30.

*When more exact values for modulus of elasticity are obtained from roller contact tests, they may be used.

7. Dynamic factor
   $Q_V = 10$; So $B = 0.39685, A = 83.776$
   Pitch line velocity $V = \pi d_G n_G / 12 = 3259\ ft/min$
   (Eq. 14-29): max. pitch line velocity $V_{max} = 8240\ ft/min$

$$(V_t)_{\max} = \begin{cases} [A + (Q_v - 3)]^2 & \text{ft/min} \\ \dfrac{[A + (Q_v - 3)]^2}{200} & \text{m/s} \end{cases} \qquad (14\text{–}29)$$

   (Eq. 14-27): $K_V = 1.229$

$$K_v = \begin{cases} \left(\dfrac{A + \sqrt{V}}{A}\right)^B & V \text{ in ft/min} \\ \left(\dfrac{A + \sqrt{200V}}{A}\right)^B & V \text{ in m/s} \end{cases} \qquad (14\text{–}27)$$

8. Overload factor
   $K_o = 1.75$

9. Surface-condition factor
   $C_r = 1$ (currently as a place holder)

10. Size factor

$$K_s = \frac{1}{k_b} = 1.192\left(\frac{F\sqrt{Y}}{P}\right)^{0.0535}$$

   $F = 1.25"$, $P = 10\ teeth/in$, Y is the Lewis form factor from Table 14-2. By linear interpolation,
   $Y_P = 0.368, Y_G = 0.439$.
   So, $K_{sP} = 1.038, K_{sG} = 1.043$
   Note: if calculated value is less than 1, set $K_s = 1$.

11. Load distribution factor

Sec. 14-11 lists conditions under which to use (Eq. 14-30 through (Eq. 14-35).

$$C_{mc} = \begin{cases} 1 & \text{for uncrowned teeth} \\ 0.8 & \text{for crowned teeth} \end{cases} \qquad (14\text{–}31)$$

then $C_{mc} = 1$

$$C_{pf} = \begin{cases} \dfrac{F}{10d_P} - 0.025 & F \le 1 \text{ in} \\ \dfrac{F}{10d_P} - 0.0375 + 0.0125F & 1 < F \le 17 \text{ in} \\ \dfrac{F}{10d_P} - 0.1109 + 0.0207F - 0.000\,228F^2 & 17 < F \le 40 \text{ in} \end{cases} \qquad (14\text{–}32)$$

then $C_{pf} = 0.01600$

$$C_{pm} = \begin{cases} 1 & \text{for straddle-mounted pinion with } S_1/S < 0.175 \\ 1.1 & \text{for straddle-mounted pinion with } S_1/S \ge 0.175 \end{cases} \qquad (14\text{–}33)$$

then $C_{pm} = 1$

$$C_{ma} = A + BF + CF^2 \qquad \text{(see Table 14–9 for values of } A, B, \text{ and } C) \qquad (14\text{–}34)$$

then $C_{ma} = 0.1466\ (commercial, enclosured\ units)$

$$C_e = \begin{cases} 0.8 & \text{for gearing adjusted at assembly, or compatibility} \\ & \text{is improved by lapping, or both} \\ 1 & \text{for all other conditions} \end{cases} \qquad (14\text{–}35)$$

then $C_e = 1$

(Eq. 14-30): $K_m = 1.163$

$$K_m = C_{mf} = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \qquad (14\text{–}30)$$

12. Hardness-ratio factor

$\dfrac{H_{BP}}{H_{BG}} = \dfrac{350}{280} = 1.25$

$A' = 0.002935$

(Eq. 14-36): $C_{Hg} = 1.004$; but $C_{HP} = 1$.

13. Stress-cycle factors

For pinion, $n_p = \left(\frac{83}{33}\right) \cdot n_G = 3772.7\ rpm$

Pinion's life $N = (10)(365)(8)(60)(3772.7) = 6.610(10^9)$ revs

Figure 14-14: $Y_{NP} = 1.6831 \cdot N^{-0.0323} = 0.8108$

Figure 14-15: $Z_{NP} = 2.466 \cdot N^{-0.056} = 0.6951$

Figure 14–14

Repeatedly applied bending strength stress-cycle factor Y_N. *(ANSI/AGMA 2001-D04.)*

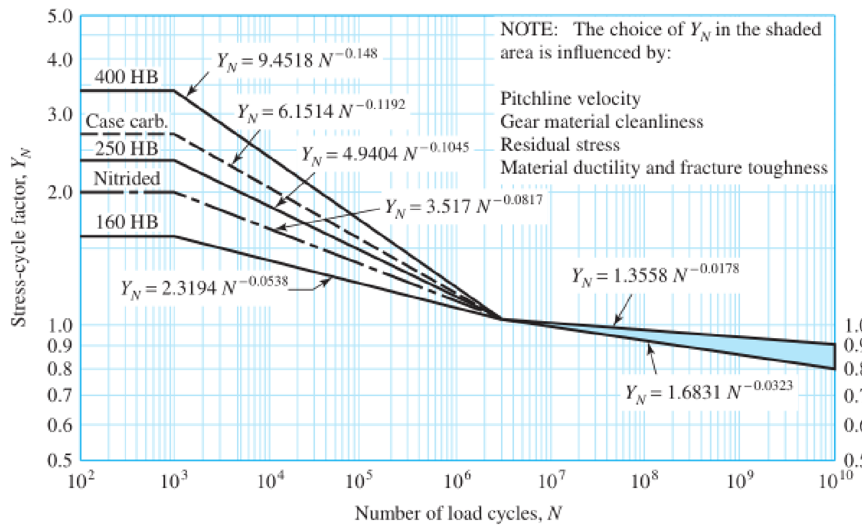
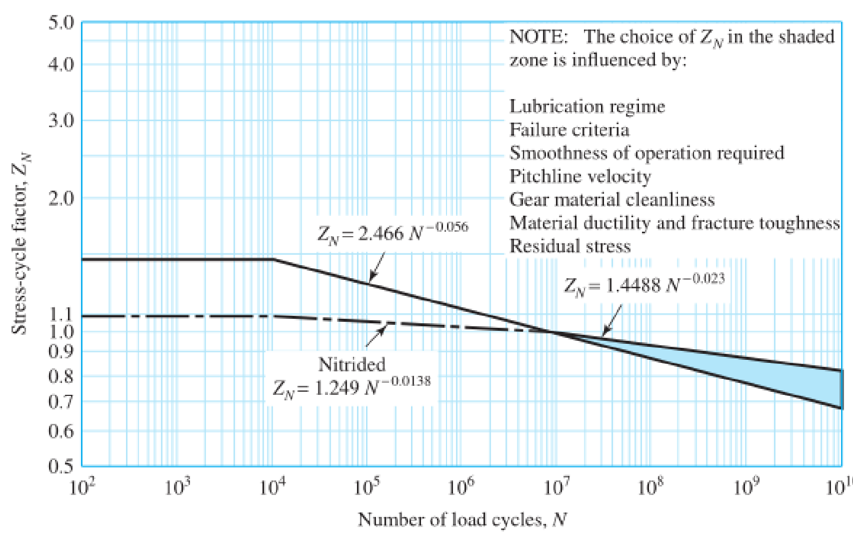


Figure 14–15

Pitting resistance stress-cycle factor Z_N. *(ANSI/AGMA 2001-D04.)*



Gears life $N = (10)(365)(8)(60)(1500) = 2.628(10^9)$ revs

Figure 14-14: $Y_{NG} = 1.6831 \cdot N^{-0.0323} = 0.8353$

Figure 14-15: $Z_{NG} = 2.466 \cdot N^{-0.056} = 0.7320$

14. Reliability factor
    Table 14-10 or (Eq. 14-38):
    $K_R = 1$

$$K_R = \begin{cases} 0.658 - 0.0759 \ln(1 - R) & 0.5 < R < 0.99 \\ 0.50 - 0.109 \ln(1 - R) & 0.99 \leq R \leq 0.9999 \end{cases} \qquad (14\text{–}38)$$

15. Temperature factor
    Just use $K_T = 1$ (Valid for processes up to $250°F$)

16. Rim-thickness factor
    (Eq. 14-40): $K_B = 1$ (but need to ensure $m_B \geq 1.2$)

$$K_B = \begin{cases} 1.6 \ln \dfrac{2.242}{m_B} & m_B < 1.2 \\ 1 & m_B \geq 1.2 \end{cases} \qquad (14\text{–}40)$$

17. Safety factor S_F and S_H
    They are to be determined from AGMA equations.
    $\sigma_P = 6{,}880\ psi$
    $\sigma_G = 6{,}214$ psi

    $\sigma_{all,P} = \dfrac{32{,}314}{S_{F,P}}$
    $\sigma_{all,G} = \dfrac{28{,}771}{S_{F,G}}$

    $S_{F,P} = 4.70$
    $S_{F,G} = 4.63$

    Similarly,
    $S_{H,P} = 1.49$
    $S_{H,G} = 1.41$

SPUR GEAR BENDING
Based on ANSI/AGMA 2001-D04 (U.S. customary units)

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

1 [or Eq. (a), Sec. 14–10]; p. 751

$$W^t = \frac{33\,000\ \mathrm{H}}{V}$$

Eq. (14–30); p. 751

Eq. (14–40); p. 756

Gear bending stress equation Eq. (14–15)

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

Fig. 14–6; p. 745

Eq. (14–27); p. 748

Table below

${}_{0.99}(S_t)_{10^7}$ Tables 14–3, 14–4; pp. 740, 741

Fig. 14–14; p. 755

Gear bending endurance strength equation Eq. (14–17)

$$\sigma_{\text{all}} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

Table 14–10, Eq. (14–38); pp. 756, 755

1 if $T < 250°\mathrm{F}$

Bending factor of safety Eq. (14–41)

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma}$$

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

SPUR GEAR WEAR

Based on ANSI/AGMA 2001-D04 (U.S. customary units)

$$d_P = \frac{N_P}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33\,000\text{ H}}{V}$$

Gear contact stress equation Eq. (14–16)

$$\sigma_c = C_p \left(W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I} \right)^{1/2}$$

- C_p: Eq. (14–13), Table 14–8; pp. 736, 749
- W^t: $W^t = \frac{33\,000\text{ H}}{V}$
- K_o: Table below
- K_v: Eq. (14–27); p. 748
- K_s: 1 [or Eq. (a), Sec. 14–10]; p. 751
- K_m: Eq. (14–30); p. 751
- C_f: 1
- I: Eq. (14–23); p. 747

Gear contact endurance strength Eq. (14–18)

$$\sigma_{c,\text{all}} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

- S_c: ${}_{0.99}(S_c)_{10^7}$ Tables 14–6, 14–7; pp. 743, 744
- Z_N: Fig. 14–15; p. 755
- C_H: Section 14–12, gear only; pp. 753, 754
- K_T: 1 if $T < 250°\text{F}$
- K_R: Table 14–10, Eq. (14–38); pp. 756, 755

Wear factor of safety Eq. (14–42)

$$S_H = \frac{S_c Z_N C_H/(K_T K_R)}{\sigma_c}$$

- C_H: Gear only

Remember to compare S_F with S_H^2 when deciding whether bending or wear is the threat to function. For crowned gears compare S_F with S_H^3.

Table of Overload Factors, K_o

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Assessments

- Refer to pp. 768-769, and p. 772
- Ideally, the four factors of safety should be at the same level, between 1 and 3;
- Factors of safety $S_{F,P}$ and $S_{F,G}$ are directly proportional to transmitted load W^t;
- Factors of safety $S_{H,P}$ and $S_{H,G}$ are proportional to the square root or cubic root of the transmitted load W^t;
- So S_F should be compared with S_H^2, or S_H^3 if teeth are crowned;
- For the spur gearset example, calculated factors of safety are:

	Pinion	Gear		Pinion	Gear
S_F	4.70	4.63	S_H	1.59	1.41

Comparison should be done as:

	Pinion	Gear		Pinion	Gear
S_F	4.70	4.63	S_H^2	2.53	1.99

- For steels with HB < 500, good balance between bending and contact can be achieved by going for higher P (or finer pitch);
- For steels with HB ≥ 500, it is suggested to start with P = 8 $(teeth/in)$
- Remember that a broken tooth is more dangerous than a worn tooth;

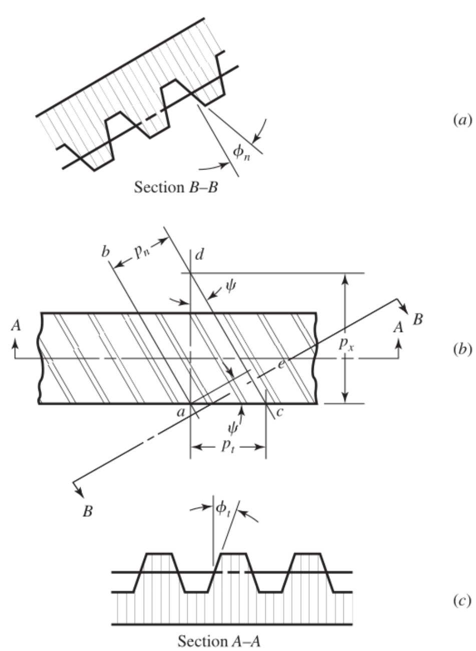
13-10: Parallel Helical Gears

13-12 Tooth Systems

- Shaping and gobbing are the two commonly used gear manufacturing methods.
- Depending on the manufacturing method, there are two tooth systems for helical gears.
- Shaped helical hears follow the transverse tooth system, but hobbed helical gears follow the normal tooth system.
- **Figure 13-22:** Transverse plane A-A and normal plane B-B

Figure 13–22

Nomenclature of helical gears.



- Transverse tooth system vs. normal tooth system

  They refer to the fact that tooth proportion (addendum and dedendum) is determined in the transverse plane for shaped gears, and in the normal plane for hobbed gears.

- **Table 13-4:** standard tooth proportions for hobbed US-customary helical gears.

  Helix angle ψ, not standardized but $< 30°$; a value between $15°$ and $30°$ is common.

  Standard values are applied to:
  - Normal pitch P_n; and
  - Normal pressure angle φ_n

  Transverse circular pitch p_t ↔ Normal circular pitch p_n: (Eq. 13-16)

$$p_n = p_t \cos \psi \qquad (13\text{–}16)$$

  Transverse pitch P_t ↔ Normal pitch P_n: (Eq. 13-18)

$$P_n = \frac{P_t}{\cos \psi} \qquad (13\text{–}18)$$

  Transverse pressure angle φ_t ↔ Normal pressure angle φ_n: (Eq. 13-19)

$$\cos \psi = \frac{\tan \phi_n}{\tan \phi_t} \qquad (13\text{–}19)$$

  Axial (circular pitch) p_x: (Eq. 13-17)

$$p_x = \frac{p_t}{\tan \psi} \qquad (13\text{–}17)$$

  Face width $\geq$ 2x axial pitch
  Example $13 - 2$ on applying the above.

Table 13–4

Standard Tooth Proportions for Helical Gears

Quantity*	Formula	Quantity*	Formula
Addendum	$\frac{1.00}{P_n}$	External gears:	
Dedendum	$\frac{1.25}{P_n}$	Standard center distance	$\frac{D+d}{2}$
Pinion pitch diameter	$\frac{N_P}{P_n \cos \psi}$	Gear outside diameter	$D + 2a$
Gear pitch diameter	$\frac{N_G}{P_n \cos \psi}$	Pinion outside diameter	$d + 2a$
Normal arc tooth thickness†	$\frac{\pi}{P_n} - \frac{B_n}{2}$	Gear root diameter	$D - 2b$
Pinion base diameter	$d \cos \phi_t$	Pinion root diameter	$d - 2b$
		Internal gears:	
Gear base diameter	$D \cos \phi_t$	Center distance	$\frac{D-d}{2}$
Base helix angle	$\tan^{-1}(\tan \psi \cos \phi_t)$	Inside diameter	$D - 2a$
		Root diameter	$D + 2b$

*All dimensions are in inches, and angles are in degrees.

†B_n is the normal backlash.

Example 1: Determine the pitch diameter, radius of addendum circle, radius of base circle, and face width of a helical gear (18-teeth, 20°-pressure angle, 22.5°-helical angle, and full-depth tooth profile) when It is cut by a hob with $P_n = 8\ teeth/in$.

Solution:

	Transverse Plane	Normal Plane
Addendum, in		0.125
Dedendum, in		0.1563
Transverse pitch, $teeth/in$	7.391	
Transverse pressure angle, °	21.50	
Pitch diameter, in	2.435	
Radius of addendum circle, in	1.343	
Radius of base circle, in	1.133	
Transverse circular pitch, in	0.4251	
Transverse base circular pitch, in	0.3955	
Axial pitch, in	1.026	
Face width, in	≥ 2.052	

- **Configuration of helical gears**

  For a set of helical gears to mesh, they must have the same diametral pitch and pressure angle.

  If the helical gears have helix angle ψ_1 and ψ_2, respectively, the angle between the shafts is $\sum = \psi_1 + \psi_2$.

  When $\sum \neq 0$ (that is, $\psi_2 \neq -\psi_1$), the helical hears form the so-called cross helical gears.

Crossed helical gears are in point contact, and may have low efficiency due to sliding motion between the teeth in contact. The load carrying capacity is limited to $400\ N$ or so.

Parallel helical gears: this is when $\psi_2 = -\psi_1$ and $\sum = 0$. That is, the gears have the same helical angle, but the helices are of opposite hands.

Left-handed, right-handed, standard configuration

Crossed configuration

- **Contact ratios**
  A helical gearset has transverse, normal and axial contact ratios.

  Transverse contact ratio, m_P, is determined in the same way as the m_c for a pair of spur gears.

  Face or axial contact ratio is $m_F = F/p_x$.

  Total contact ratio is $m_P + m_F$.

Example 2: Determine the total contact ratio of a set of hobbed helical gears ($P_n = 8\ teeth/in$, 20° pressure angle, 22.5° helical angle, and full-depth tooth profile). Given $N_p = 18$, $N_G = 35$, and face width is 2.5".

Solution:

	Pinion	Gear
Addendum, in	0.125	
Dedendum, in	0.1563	
Transverse pitch, $teeth/in$	7.391	
Transverse pressure angle, °	21.50	

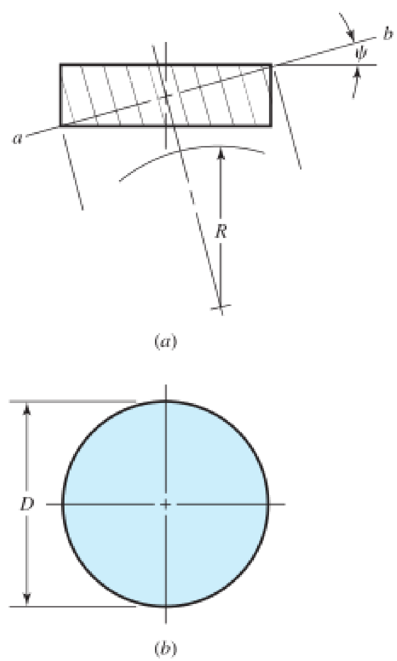
Pitch diameter, in	2.435	4.732
Radius of addendum circle, in	1.343	2.491
Radius of base circle, in	1.133	2.201
Transverse circular pitch, in	0.3955	
Transverse base circular pitch, in	0.4251	
Axial pitch, in	1.026	
Face width, in	2.5	
Centre-to-centre distance, in	3.584	

$$L_{ab} = 0.721 + 1.166 - 1.314 = 0.5730"$$
$$m_p = \frac{0.5730}{0.3955} = 1.449$$
$$m_F = \frac{2.5}{1.026} = 2.437$$

Total contact ratio = 3.866

Figure 13–23

A cylinder cut by an oblique plane.



ab is the normal plane. ab "cuts" an ellipse.

Radius of curvature of the ellipse at contact point is R.

$$R = \frac{D}{2\cos^2\psi} \geq \frac{D}{2}$$

Imagine a pitch cylinder of radius R and placing teeth on it.

Virtual number of teeth N' is the number of teeth that can be placed on a pitch cylinder of radius R.

$$N' = \frac{N}{\cos^3\psi}$$

N' is used to determine the Lewis form factor Y.

And the minimum number of teeth to avoid interference applied to N' for helical gears.

For example, $N_{min} = 17$ for hobbed spur gears; for helical gears, with $\psi = 30°$, $N' = 17$, and $N = N'\cos^3\psi = 11.04$

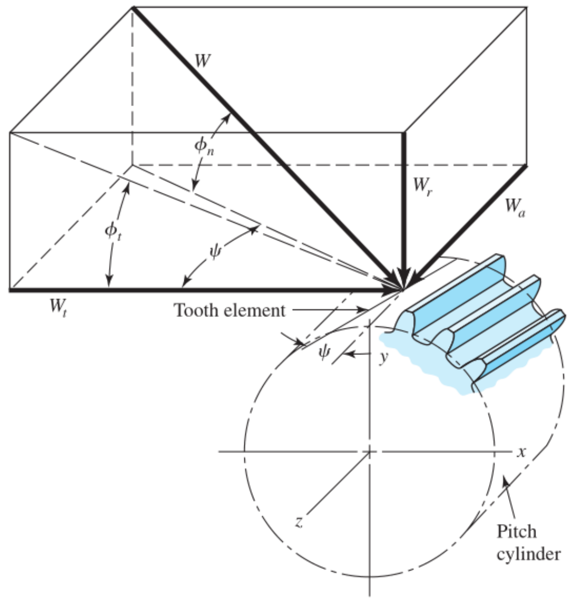
- Advantages/Disadvantages of helical gears over spur gears
  - Smoother and quieter operations (due to large total contact ratio)
  - Higher load-carrying capacity
  - Allowing for higher pitch line velocity

- Smaller minimum number of actual teeth to avoid interference
- Presence of axial thrust
- Double helical cancels out thrust but increases cost

13-16 Force Analysis – Helical Gearing

Figure 13–37

Tooth forces acting on a right-hand helical gear.



Transmitted load W_t: determined from given power or torque in the same way as for spur gears
Radial load W_r
Axial load (or separating force) W_a

$$\begin{aligned} W_r &= W \sin \phi_n \\ W_t &= W \cos \phi_n \cos \psi \\ W_a &= W \cos \phi_n \sin \psi \end{aligned} \tag{13–39}$$

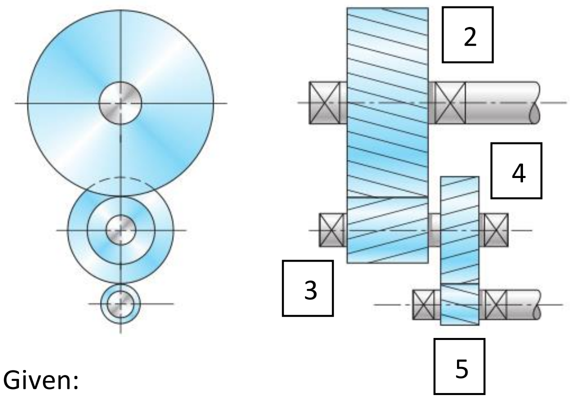
W_t, W_r, W_a in terms of W, the total force

$$\begin{aligned} W_r &= W_t \tan \phi_t \\ W_a &= W_t \tan \psi \\ W &= \frac{W_t}{\cos \phi_n \cos \psi} \end{aligned} \tag{13–40}$$

W_r, W_a and W in terms of W_t

LH versus RH, and Force Components and Directions

Example 3 (Figure 13-28 for diagram)



Given:
$N_2 = 96$
$N_3 = 16$
$N_4 = 80$
$N_5 = 15$
$P_n = 8\frac{teeth}{in}$
$\varphi = 20°$ for all gears
Helix angle is 25° for N_2 and N_3
Helix angle is 15° for N_4 and N_5

Determine, draw and label the W^t, W^r and W^a at each of the contact points, in terms of input torque T_2. Gear 2 rotates CCW. The unit of T_2 is in $lb \cdot in$.

<u>Solution:</u>
(1) Transverse diametral pitch and pitch diameter
$P_{t2,3} = 7.250\ teeth/in$
$P_{t4,5} = 7.727\ teeth/in$
$d_2 = 13.241"$
$d_3 = 2.207"$
$d_4 = 10.353"$
$d_5 = 2.071"$

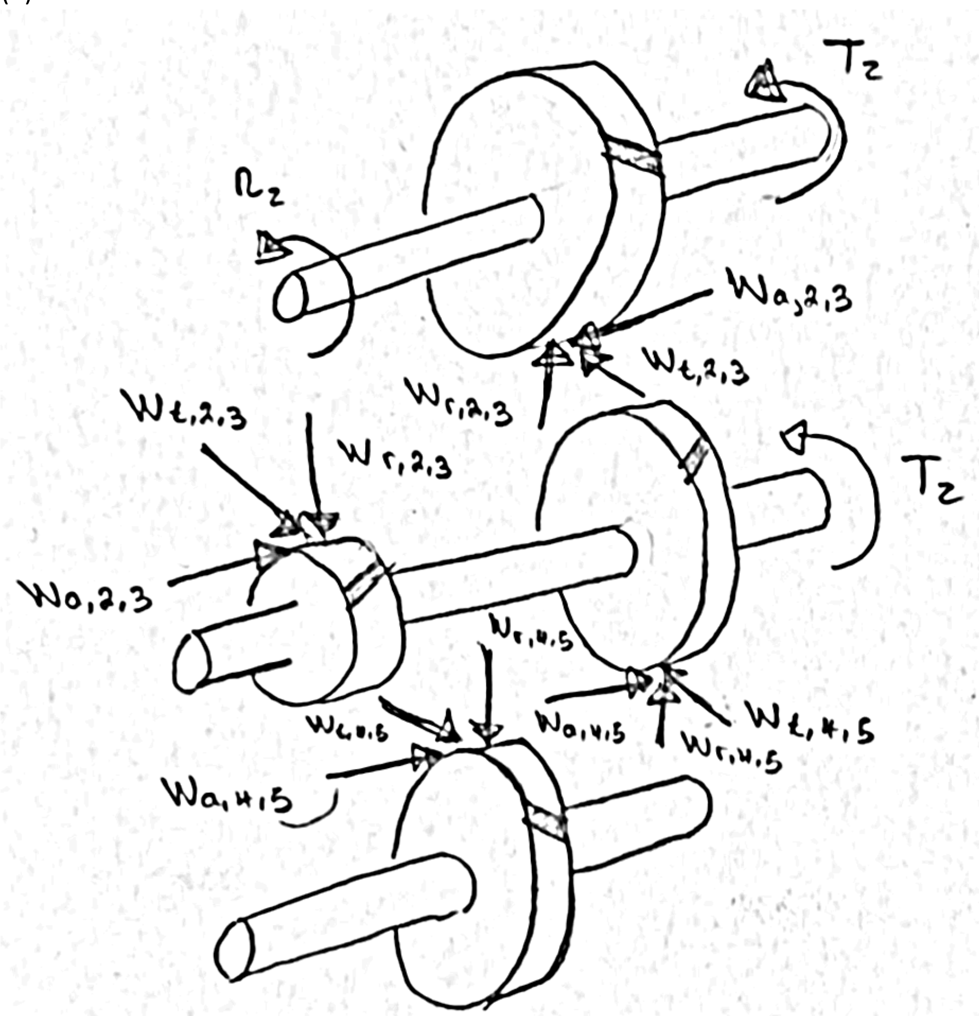
(2) Transverse pressure angle
$\varphi_{t2,3} = 21.880°$
$\varphi_{t4,5} = 20.647°$

(3) Forces between Gears 2 and 3, per (Eq. 13-40)
$W_{2,3}^t = T_2/r_2 = 0.1510\ T_2$
$W_{2,3}^r = W_{2,3}^t tan\varphi_{t2,3} = 0.06064\ T_2$
$W_{2,3}^a = W_{2,3}^t tan\psi_{2,3} = 0.07041\ T_2$

(4) Forces between Gears 4 and 5, per (Eq 13-40)
Torque balance requires $W_{2,3}^t r_3 = W_{4,5}^t r_4$
$W_{4,5}^t = W_{2,3}^t r_3/r_4 = 0.03219\ T_2$
$W_{4,5}^r = W_{4,5}^t tan\varphi_{t4,5} = 0.01213\ T_2$

$W^a_{4,5} = W^t_{2,3} tan\psi_{4,5} = \ 0.008625\ T_2$

(5) Draw and label the forces



14-18 Analysis
14-19 Design of a Gear Mesh

Example 14-5 (helical gears)

Example 4
A gearbox contains a set of hobbed helical gears. It is driven by a single cylinder engine, and to drive a reciprocating compressor. Output shaft rotates at $1500\ rpm$, with a maximum torque of $550\ lb - in$. Gear ratio is 2.5 to 1. Pitch diameter of pinion must be 3.25 ($\pm$ 1%), and $N_p = 31$. The seven assumptions are the same as the spur gearset example.

Solution:

1. Choose $P_n = 10\ teeth$/in.

$$d_p = \frac{N_p}{P_t} = \frac{N_p}{(P_n\ cos\psi)};$$

$$3.25 = \frac{31}{(10\ cos\psi)}$$

Solving for helix angle $\psi = 17.475°$.

$N_G = (2.5)(31) = 77.5$. Select 78.

So $\psi = 17.475°, N_P = 31, d_P = 3.250$" (within 3.25" $\pm$ 0.0325"), $N_G = 78$, $d_G = 8.177$", and use 20° full-depth and uncrowned teeth.

2. Other geometric quantities, including contact ratio

	Pinion	Gear
Addendum, in	0.1	
Dedendum, in	0.125	
Pitch diameter, in	3.250	8.177
Radius of addendum circle, in	1.725	4.189
Radius of base circle, in	1.519	3.820
Transverse base circular pitch, in	0.3077	
Axial pitch, in	1.041	
Face width, in	2.1	
Center-to-center distance, in	5.7135	
Virtual number of teeth	35.7	89.9

$L_{ab} = 0.8175 + 1.7191 - 2.0373 = 0.4993$"

$m_p = 0.4993/0.3077 = 1.623$

$m_F = 2.1/1.041 = 2.017$

Total contact ratio $= 3.631$

3. Transmitted load

$$W^t = \frac{T_{max}}{r_G} = \frac{550}{\left(\frac{8.177}{2}\right)} = 134.5\ lb$$

5. Geometry factors

<u>For bending:</u> Figures 14-7 and 14-8 (instead of Figure 14-6)

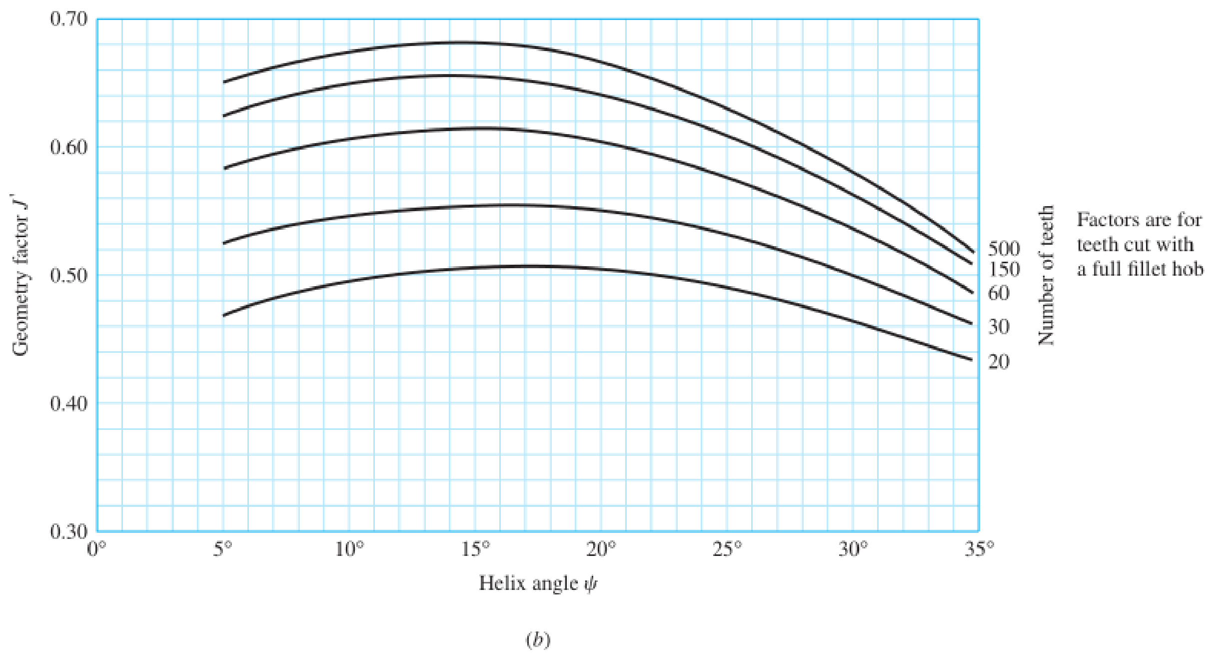
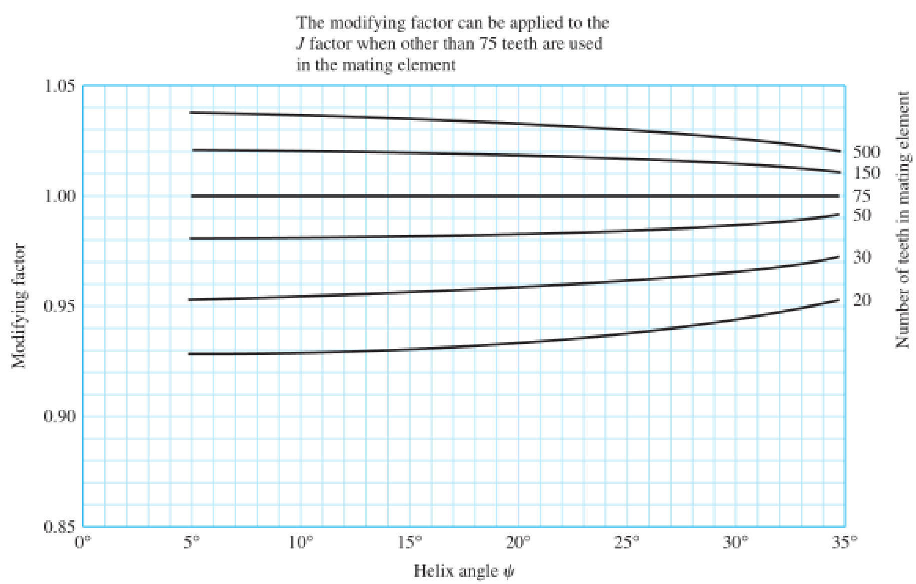


Figure 14–7

Helical-gear geometry factors J'. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.

Figure 14–8

J'-factor multipliers for use with Fig. 14–7 to find J. *Source:* The graph is from AGMA 218.01, which is consistent with tabular data from the current AGMA 908-B89. The graph is convenient for design purposes.



Limitations are, $\varphi_n = 20°$ and $m_F \geq 2$

$J_P = (0.56)(0.96) = 0.54$

$J_G = (0.62)(1.00) = 0.62$

For contact: (Eq 14-21)

$$m_N = \frac{p_N}{0.95Z} \qquad (14\text{–}21)$$

$$\left(m_N = \frac{p_N}{0.95\, L_{ab}}\right)$$

Where p_N is the normal base circular length pitch, $p_N = (\pi/P_n)\cos(\varphi_n)$, and $Z = L_{ab}$ is the length of action. p_N and Z are given by (Eq. 14-24) and (Eq. 14-25)

$$p_N = p_n \cos \phi_n \qquad (14\text{–}24)$$

$$Z = [(r_P + a)^2 - r_{bP}^2]^{1/2} + [(r_G + a)^2 - r_{bG}^2]^{1/2} - (r_P + r_G)\sin\phi_t \qquad (14\text{–}25)$$

Therefore, $p_N = \left(\frac{\pi}{10}\right)\cos(20°) = 0.2952"$, $L_{ab} = 0.4993"$, and $m_N = 0.6223$.

Geometry factor I is by (Eq. 14-23) where $m_N = 0.6223$, $m_G = \frac{78}{31} = 2.516$, $\varphi = 20.89°$. So, I = 0.1915

$$I = \begin{cases} \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G+1} & \text{external gears} \\ \dfrac{\cos\phi_t \sin\phi_t}{2m_N}\dfrac{m_G}{m_G-1} & \text{internal gears} \end{cases} \qquad (14\text{–}23)$$

10. Size factor

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P_n}\right)^{0.0535}$$

$F = 2.1"$, $P_n = 10$ teeth/in, Y is the Lewis form factor from Table 14-2 where "Number of Teeth" means N'. By linear interpolation, $Y_p = 0.376, Y_G = 0.442$

So, $K_{sP} = 1.068$, $K_{sG} = 1.073$

Note: If calculated value is less than 1, set $K_s = 1$.

17. Safety factors S_F and S_H

Bending: $\sigma_P = 3.156\ psi$;
$\sigma_G = 2{,}771\ psi$;
$S_{F,P} = 10.2$
$S_{F,G} = 10.4$

Surface Contact:
$\sigma_{C,P} = 38{,}120\ psi$
$\sigma_{C,G} = 38{,}209\ psi$
$S_{H,P} = 2.6$
$S_{H,G} = 2.3$

18. Assessment

If T_{max} is increased to, say, 3.5 times of the current level, the resulting factors of safety are, assuming all else being the same

$S_{F,P} = 2.9$
$S_{F,G} = 3.0$

$S_{H,P} = 1.4$
$S_{H,G} = 1.2$

Chapter 15: Bevel and Worm Gears

15-1 Bevel Gearing – General

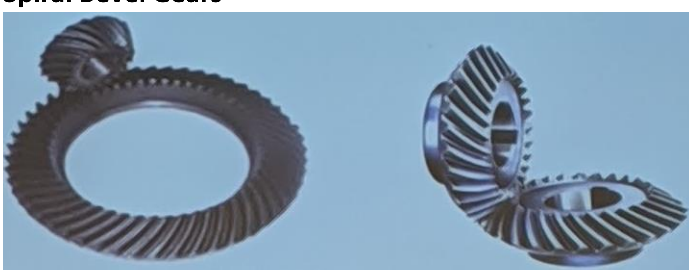
5 Types of Bevel Gearing

- Straight bevel gears
- Spiral bevel gears
- Zerol bevel gears
- Hypoid gears
- Spiroid gears

Straight Bevel Gears

Used to transmit power between two intersecting shafts, at any angle (except 0 and 180 degrees).

Spiral Bevel Gears



Teeth are curved and oblique;
Used to transmit power between two intersecting shafts;
The difference between straight and spiral bevels is similar to the difference between spur and helical gears.

Zerol Gears

Teeth are curved;
Used to transmit power between two intersecting shafts.

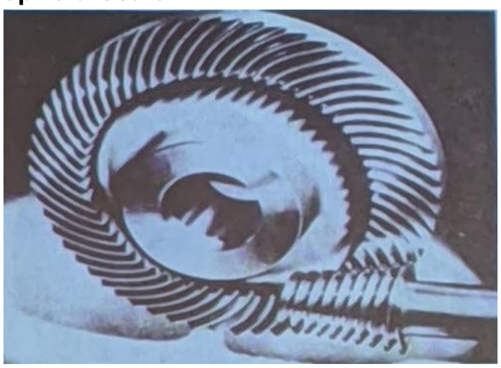
Hypoid Gears

Teeth are curved and oblique

Used to transmit power between two offset shafts at any angles;

Used in automotive differentials.

Spiroid Gears



Spiroid gears are seen as the combination of spiral bevel and worm gears;

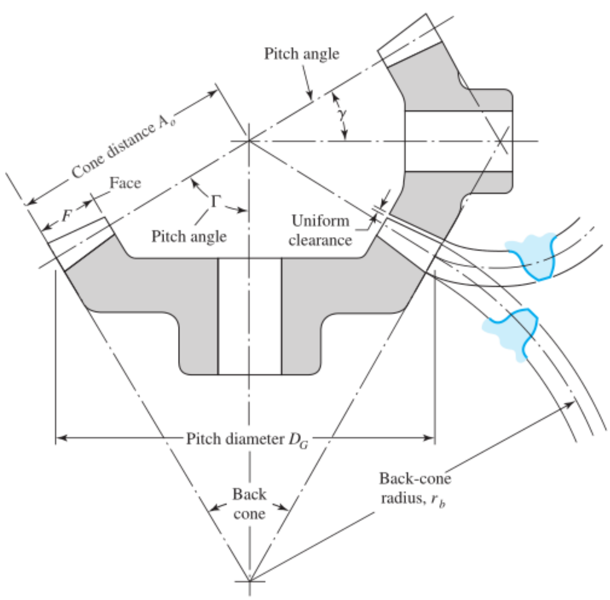
Used in heavy-duty vehicles.

13.19: Straight Bevel Gears

13-12: Tooth Systems

Figure 13–20

Terminology of bevel gears.



Pitch angles: γ and Γ

Shaft angle $\sum = \gamma + \Gamma$ can be angle, except 0° and 180°;

If $\sum = 90°$, $\tan \gamma = N_P/N_G$ and $\tan \Gamma = N_G/N_P$

$$\tan\gamma = \frac{\sin\sum}{\frac{N_G}{N_P} + \cos\sum}$$
$$\tan\Gamma = \frac{\sin\sum}{\frac{N_P}{N_G} + \cos\sum}$$

For any $\sum > 90°$:

$$\tan\gamma = \frac{\sin(180° - \sum)}{\frac{N_G}{N_P} + \cos(180 - \sum)}$$
$$\tan\Gamma = \frac{\sin(180° - \sum)}{\frac{N_P}{N_G} + \cos(180 - \sum)}$$

Small end and large end of tooth;
Pitch cone and back cone:
Pitch cone length (distance): A_0, which should be the same for both the pinion and the gear;
Back cone (large end) radi: r_{bP} and r_{bG};
Pitch and pressure angle are defined at the large end; As a result, a bevel gear's size and shape are defined at the large end;
Face width: F, which is the lesser of $0.3A_0$ or $10/P$;
Virtual numbers of teeth: $N_P' = \frac{2\pi r_{bP}}{p}$ and $N_G' = \frac{2\pi r_{bG}}{p}$, where p is the circular pitch measured at the large end of the teeth.

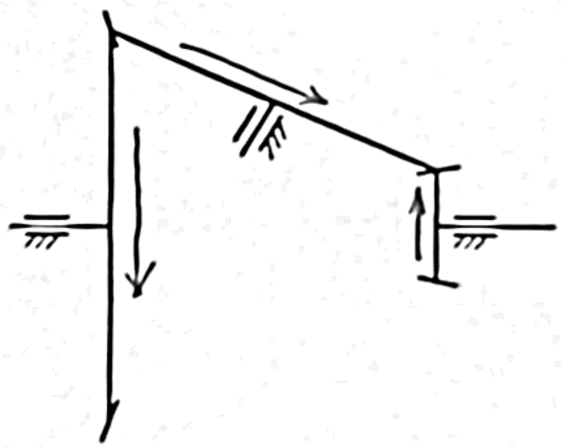
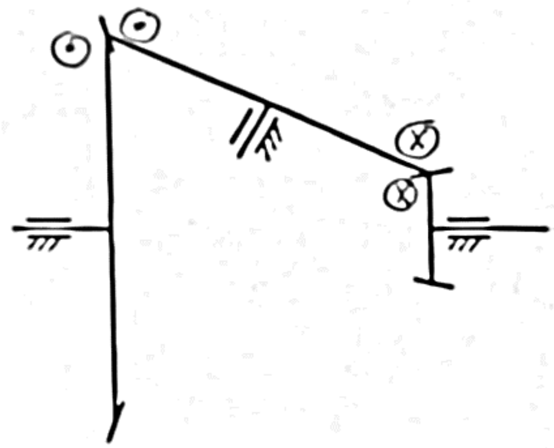
Table 13-3: Tooth proportions, 20° straight tooth bevel gears
Revision is needed for equivalent 90° ratio.

Table 13–3

Tooth Proportions for 20° Straight Bevel-Gear Teeth

Item	Formula
Working depth	$h_k = 2.0/P$
Clearance	$c = (0.188/P) + 0.002$ in
Addendum of gear	$a_G = \dfrac{0.54}{P} + \dfrac{0.460}{P(m_{90})^2}$
Gear ratio	$m_G = N_G/N_P$
Equivalent 90° ratio	$m_{90} = m_G$ when $\Sigma = 90°$
	$m_{90} = \sqrt{m_G \dfrac{\cos\gamma}{\cos\Gamma}}$ when $\Sigma \neq 90°$
Face width	$F = 0.3A_0$ or $F = \dfrac{10}{P}$, whichever is smaller
Minimum number of teeth	Pinion: 16, 15, 14, 13; Gear: 16, 17, 20, 30

- Velocity ratio: $|VR| = N_P/N_G$ regardless or $\sum$. The sign, however, is not as simple as "-" for external set and "+" for internal set.

Example 1: Determine the dimensions of the bevel gearset ($N_P = 21$, $N_G = 35$, $P = 4$ teeth/in, 20° pressure angle, and straight teeth). Shaft angle is $(1) \sum = 90°$ and $(2) \sum = 75°$.

Solution:

$(1) \sum = 90°$

	Pinion	Gear
Pitch angle, °	30.964	59.036
Gear ratio, m_G	1.667	
Equivalent 90° ratio, m_{90}	1.667	
Addendum, in	0.1764	
Working depth, in	0.5	
Pitch diameter, in	5.25	8.75
Cone distance, in (A_0)	5.102	5.102
Back cone radius, in	3.061	8.503
Virtual number of teeth	24.5	68.0
Face width, in	The lesser of 1.53 or 2.5; so $F = 1.53$	

$(2) \sum = 75°$

	Pinion	Gear
Pitch angle, °	26.641	48.359
Gear ratio, m_G	1.667	
Equivalent 90° ratio, m_{90}	1.497	
Addendum, in	0.1860	
Working depth, in	0.5	
Pitch diameter, in	5.25	8.75
Cone distance, in (A_0)	5.854	5.854
Back cone radius, in	2.937	6.584
Virtual number of teeth	23.5	52.7
Face width, in	The lesser of 1.76 or 2.5; so $F = 1.76$	

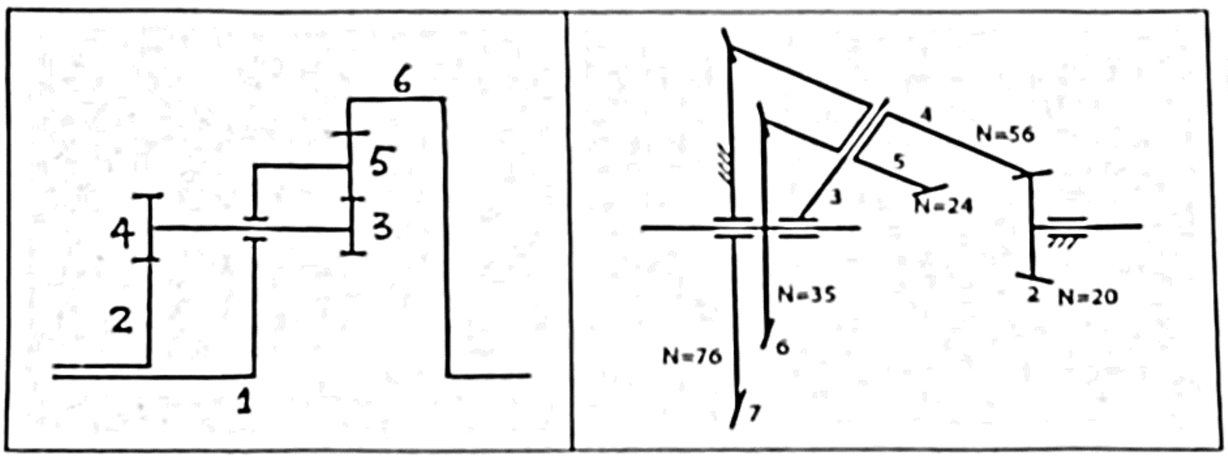
Planetary Bevel Gear Train
The tabular method can be used to analyze planetary gear trains consisting of bevel gears, but with modifications.

Recalling the following for planetary gear trains involving spur gears (and parallel helical gears):

$$VR = \frac{\left[\omega_{gear/arm}\right]_{driven}}{\left[\omega_{gear/arm}\right]_{driver}} = \pm\frac{N_{driver}}{N_{driven}}$$

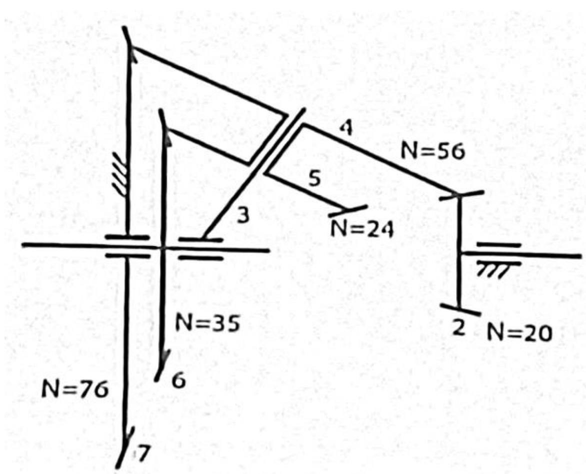
Where "+" is used with internal set and "-" with the external set.

The reasons behind the convention for the "+" and the "-" are,
(1) The angular velocities, as vectors, are parallel to each other; and
(2) An internal set of gears rotate with the same sense while an external set rotate with opposite senses.



When bevel gears are involved in a planetary gear train, angular velocities, as vectors, are not always parallel to each other. So the signs must be determined manually, on gear-to-gear basis.

Example 2: The Humpage gear train.
The schematic of the Humpage train is given below. Find the train's velocity ratio.



Solution:
$N_2 = 20$
$N_4 = 56$
$N_5 = 24$
$N_6 = 35$
$N_7 = 76$

Power flows 2 → 4 → 7 and 2 → 4 & 5 → 6
“3” is the arm
Assume the input speed is n_2.

Gear	n_{gear} =	n_{arm} +	$n_{gear/arm}$	$\lvert VR \rvert$
2	n_2	n_3	$n_2 - n_3$	$\left(\frac{N_2}{N_4}\right)\left(\frac{N_4}{N_7}\right) = 0.2631$
4	(left blank)			
7		n_3	$(n_2 - n_3)(-0.2631)$	

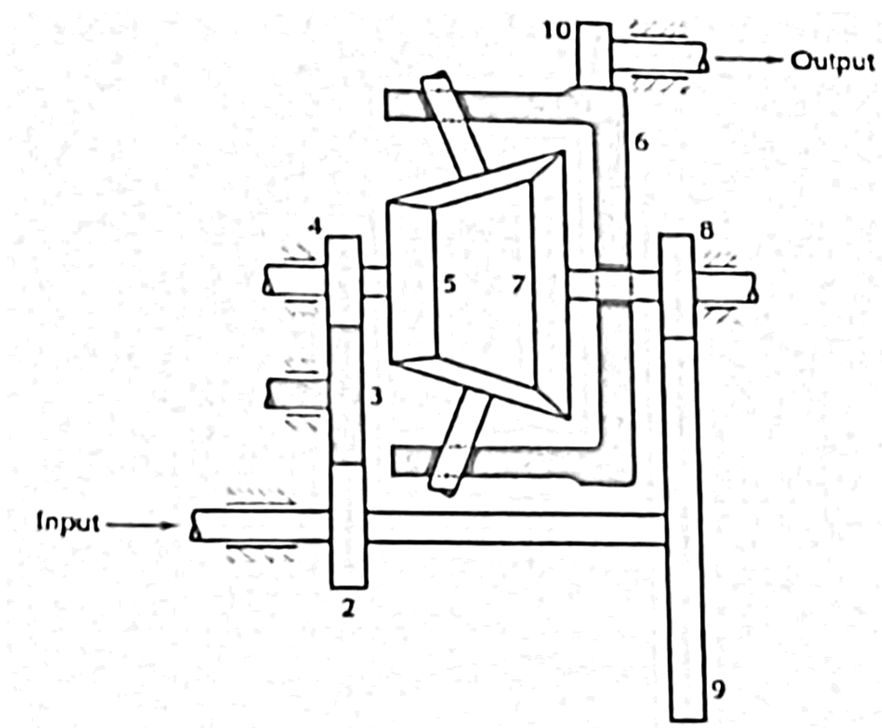
From $n_3 + (n_2 - n_3)(-0.2631) = 0 \rightarrow n_3 = 0.2083n_2$

Gear	n_{gear} =	n_{arm} +	$n_{gear/arm}$	$\lvert VR \rvert$
2	n_2	n_3	$n_2 - n_3$	$\left(\frac{N_2}{N_4}\right)\left(\frac{N_5}{N_6}\right) = 0.2449$
4	(left blank)			
5	(left blank)			
7	n_6	n_3	$(n_2 - n_3)(-0.2449)$	

From $n_6 = n_3 + (n_2 - n_3)(-0.2449) = 0.1441n_2$

So, $VR = 0.01441$

Example 3: Given $\omega_2 = 100\ rad/s$, $N_2 = 40$, $N_4 = 30$, $N_5 = 25$, $N_6 = 120$, $N_7 = 50$, $N_8 = 20$, $N_9 = 70$, $N_{10} = 20$. Determine ω_{10}.



5 and 7 are the sun gears;
Planetary gears are not labeled, and teeth numbers not given;
6 is the arm.

Solution:
(1) From $\omega_2 = 100\ rad/s$, ω_4 and ω_8 can be determined.
$\omega_4 = \omega_2 N_2/N_4 = 133.3\ rad/s$
$\omega_8 = \omega_2 N_9/N_8 = -350\ rad/s$

(2) The planetary gear train consists of gears 5, the upper planetary gear and 7, with gear 6 being the arm.

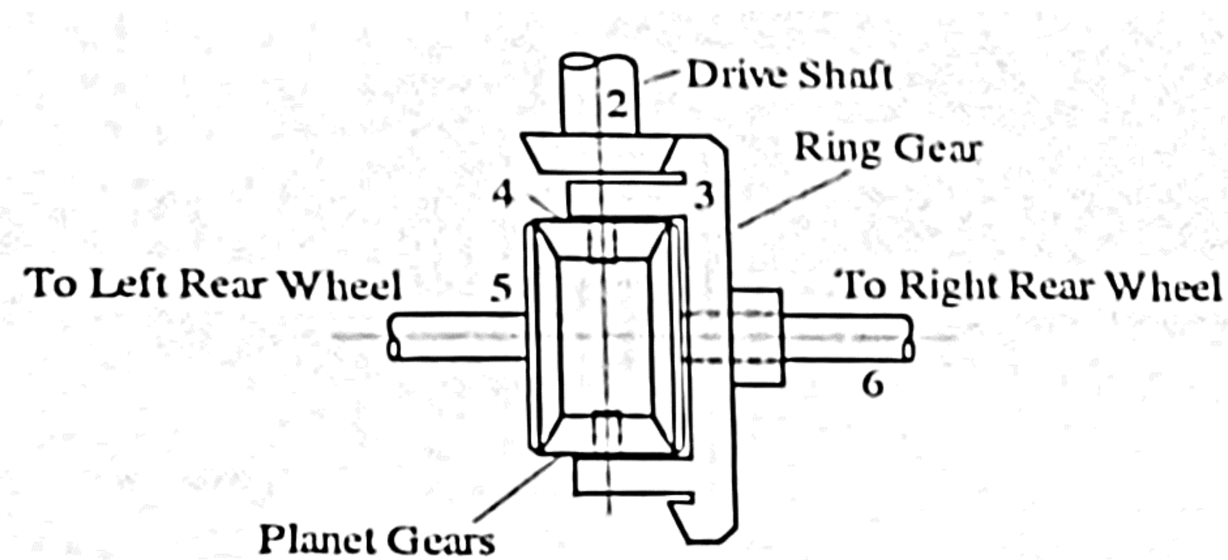
Power flows 4 & 5 → 7 & 8.

Gear	ω_{gear}	$= \omega_{arm}$	$+ \omega_{gear/arm}$	$\lvert VR \rvert$
4	133.3	x	$(133.3 - x)$	$\left(\frac{N_5}{N_7}\right) = 0.5$
5				
7	-350	x	$(133.3 - x)(-0.5)$	
8				

Since $-350 = x - (133.3 - x)(0.5)$, solving leads to $x = -188.9\ rad/s$

(3) $\omega_6 = x = -188.9 \frac{rad}{s}$
$\omega_{10} = -\omega_6 N_6/N_{10} = 1133\ rad/s$

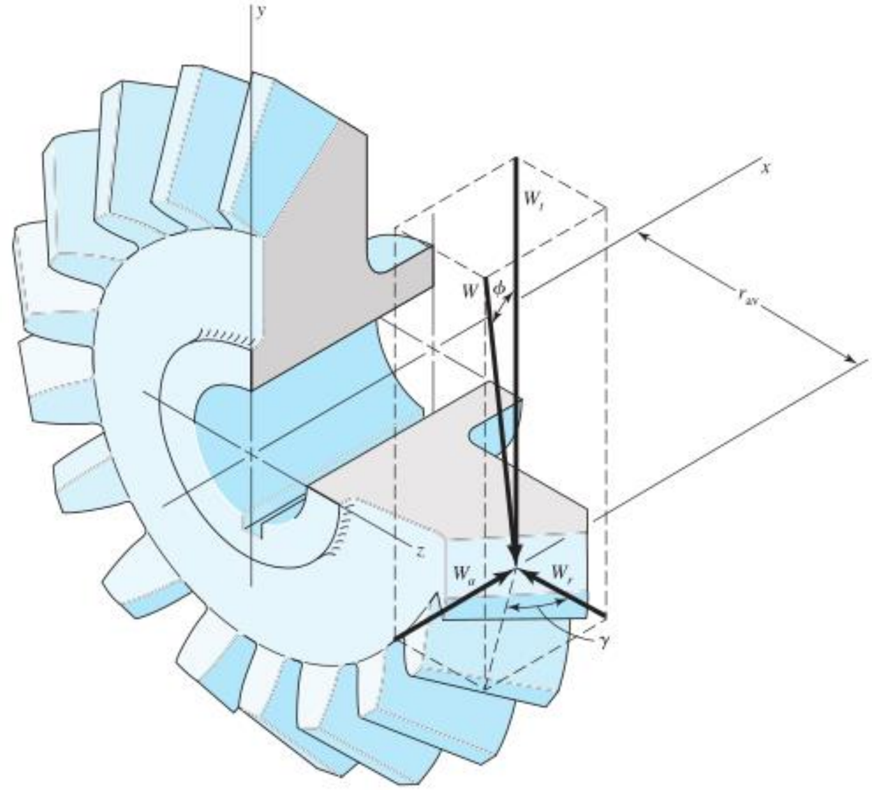
Example 4: The differential



13-15 Force Analysis – Bevel Gearing

Figure 13–35

Bevel-gear tooth forces.



Point of application of W: the actual point of application is somewhere between the midpoint and large end of a tooth, but it is typically assumed that W is applied at the midpoint, with a radius r_{av}

$$(r_{av})_P = r_P - \frac{F}{2} sin\gamma$$
$$(r_{av})_G = r_G - \frac{F}{2} sin\Gamma$$

Transmitted load W_t: determined from known power and pitch line velocity, or from known torque, in the same way as for spur gears, but replacing d (pitch diameter) with $2r_{av}$ average diameter.

Radial load W_r
Axial load W_a

(Eq. 13-37): W_t from torque

$$W_t = \frac{T}{r_{\text{av}}} \tag{13–37}$$

(Eq. 13-38) W_r and W_a from W_t

$$\begin{aligned} W_r &= W_t \tan\phi \cos\gamma \\ W_a &= W_t \tan\phi \sin\gamma \end{aligned} \tag{13–38}$$

The relation $|(W_t)_P| = |(W_t)_G|$ is always true;

But $|(W_r)_P| = |(W_a)_G|$ and $|(W_a)_P| = |(W_r)_G|$ are only true when the shaft angle is $\sum$ is 90°.

Example 5: Determine the force components acting on the bevel pinion and gear, respectively. Then show the components on an isometric drawing of the gears. Given $N_P = 21$, $N_G = 35$, $P = 4$ teeth/in, 20° pressure angle, and straight teeth. The bevel gearset is to transmit 25 hp. Pinion speed is 500 rpm. Shaft angle is 75°.

Solution:

(1) Geometric quantities are, from Example 1

	Pinion	Gear
Pitch angle,	26.641	48.359
Pitch diameter, in	5.25	8.75
Face width, in	$F = 1.76$	
Average radius, in	2.230	3.717

(2) Pinion

$V = \pi 2 (r_{av})_P n_p / 12 = 583.8\ ft/min$

$W_t = 33{,}000 \cdot H/V = 1413\ lb$

$W_r = W_t \tan\phi \cos\gamma = 459.7\ lb$

$W_a = W_t \tan\phi \sin\gamma = 230.6\ lb$

$W = 1504\ lb$

(3) Gear

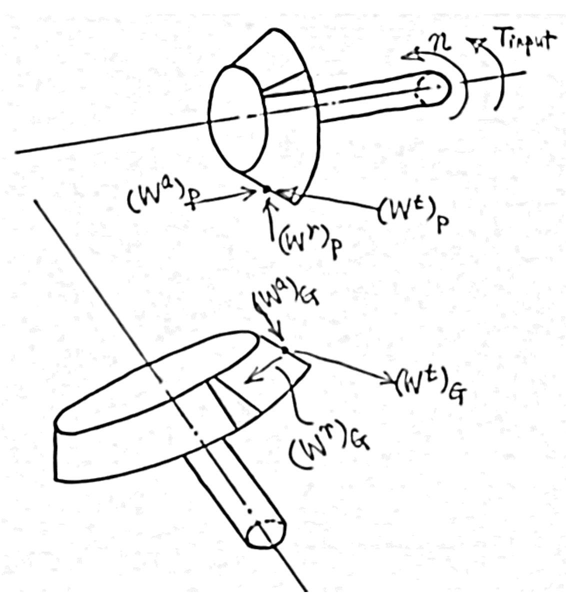
$W_t = 1413\ lb$

$W_r = W_t \tan\phi \cos\Gamma = 341.7\ lb$

$W_a = W_t \tan\phi \sin\Gamma = 384.3\ lb$

$W = 1504\ lb$

(4) Draw and label the forces



NOTE:

Radial force always points towards central axis

Axial force always points towards the larger face

Tangential force is directed to counteract input rotation and torque

15-2 Bevel Gear Stresses and Strengths
Contact stress, (Eq. 15-1)
Allowable contact stress, (Eq. 15-2)
Bending stress, (Eq. 15-3)
Allowable bending stress, (Eq. 15-4)

15-3 AGMA Equation Factors
15-4 Straight Bevel Gear Analysis
Example 15-1

15-5 Design of a Straight-Bevel Gear Mesh
Decisions made beforehand and during design.

Example 6: A gearbox contains a set of bevel gears. It is driven by a single cylinder engine, and to drive a reciprocating compressor. Output shaft rotates at 1500 rpm, with a maximum torque of 550 lb-in. Teeth numbers are 33 and 83, with P = 10 teeth/in, 20° pressure angle, and straight teeth.

Assumptions (1) through (4), and (6) through (7) are the same as the spur gearset example. Assumption (5) becomes that both shafts are straddle mounted.

<u>Solution</u>
1. $d_p = N_P/P = 33/10\ 3.3$, $d_G = N_G/P = 83/10 = 8.3$
Use 20° full depth, straight teeth

Bevel gears of straight teeth are typically somewhat crowned during manufacture; but when assessing factors of safety, S_F it is compared with S_H^2.

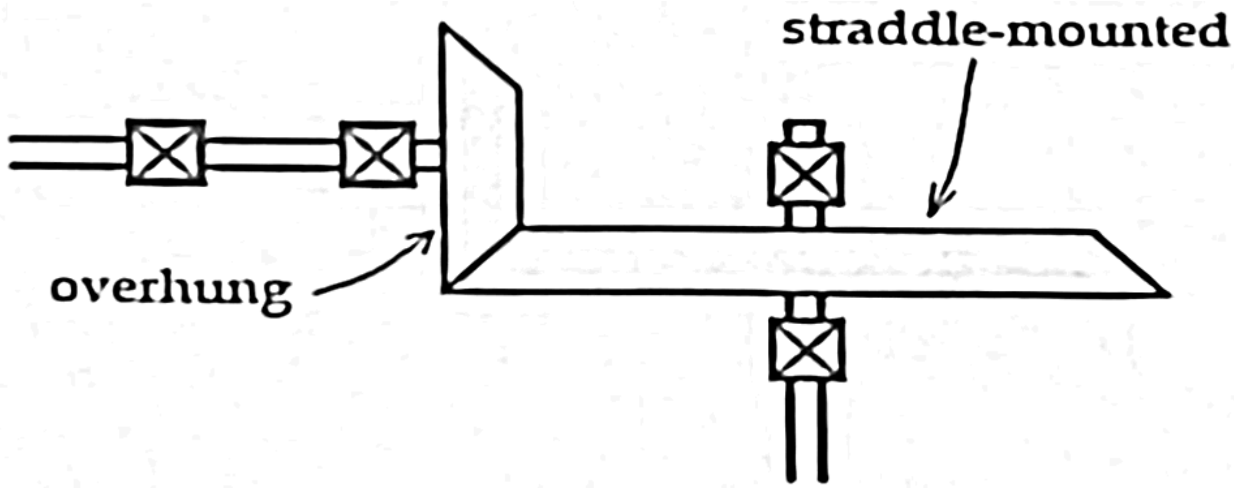
2. Other geometric quantities. (Note that not all listed quantities are needed for applying AGMA equations.)

	Pinion	Gear
Pitch angle, °	21.682	68.318
Addendum, in	0.06127	
Working depth, in	0.2	
Pitch diameter, in	3.3	8.3
Cone distance, in	4.466	4.466
Face width, in	The less of 1.34 and 1; $F = 1$	
Average radius, in	1.465	3.685

3. Transmitted load
$W^t = T_{max}/(r_{av})_G = 550\ /\ 3.685 = 149.3\ lb$
$V = \pi d_G n_G/12 = 3259\ ft/min$ (use pitch diameter here)

4.-17. Factors
- Geometry factors are from charts: $I = 0.1$, $J_P = 0.295$, $J_G = 0.255$
- Size factor is from a chart as well;
- Load distribution factor K_m: depends on the mounting of the gears

Mounting can be any combinations of straddle mounted and overhung.



- Hardness-ratio factor C_H
    - As with spur gearsets, $C_H = 1$ for pinion, and C_H for gear is determined.

(Eq. 15-16) or Figure 15-10 is for through-hardened steels
(E1. 15-17) or Figure 15-11 is for surface-hardened steels
(Eq. 15-16), N/n means N_G/N_P.

	Pinion	Gear		Pinion	Gear
W^t	149.3		W^t	149.3	
$P_d = P$	10		d_P	3.3	
F	1		F	1	
K_o	1.75		K_o	1.75	
K_v	1.229		K_v	1.229	
K_m	1.0036		K_m	1.0036	
K_s	0.5082		C_s	0.5625	
K_x	1		C_{xc}	1.5	
J	0.295	0.255	C_p	2290	
			I	0.1	
σ	5501	6364	σ_c	65734	65734
S_{at}	17500	14420	S_{ac}	142970	119100
K_L	0.9066	0.9126	C_L	0.8927	0.9355
K_T	1		K_T	1	
K_R	1		K_R	1	
			C_H	1	1.004
σ_{all}	$15866/S_F$	$13289/S_F$	$\sigma_{c,all}$	$127629/S_H$	$111864/S_H$
S_F	2.88	2.09	S_H	1.94	1.70

Worm Gears

13-11: Worm Gears
13-12: Tooth Systems

- **Features of Worm Gearing**

It is used to transmit power between two non-intersecting shafts;

Power is typically transmitted from worm to worm gear;

A set of worm and worm gear can be designed to self-lock; that is, the worm gear can't back-drive the worm;

Being able to have large speed and torque ratios, but low efficiency; the ratios can be as high as 360, but are commonly up to about 100;

Another key feature is the compact design.

- **Configurations**

Single enveloping
Double enveloping

- **Geometry Figures (13-24 and 13-25)**

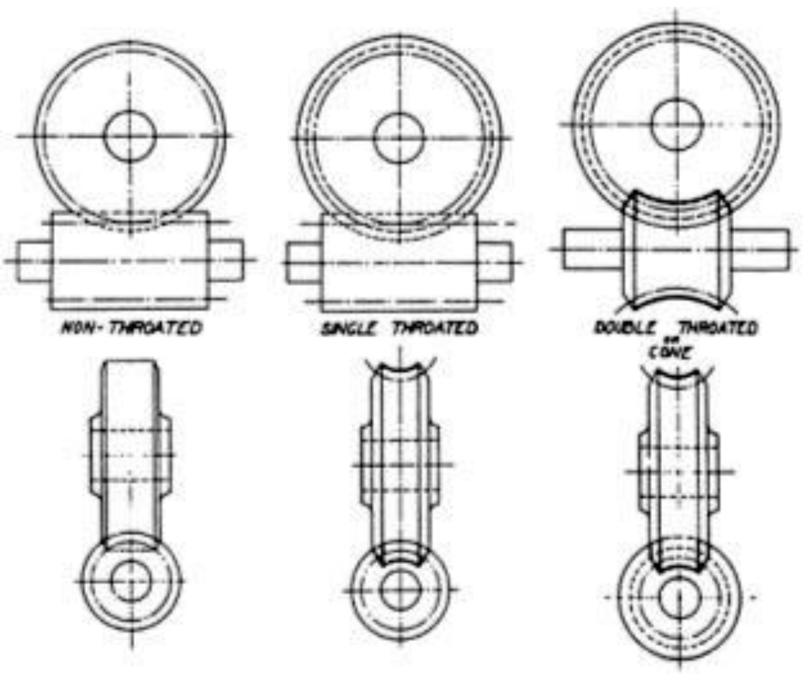
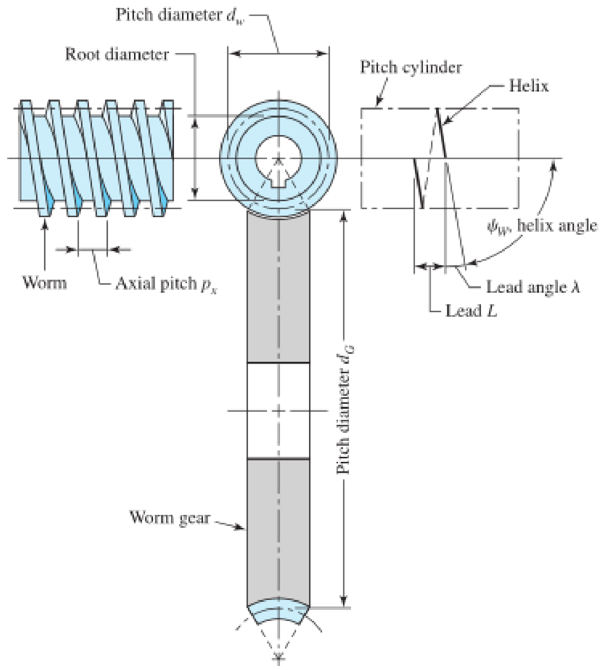


Figure 1: Three Worm Gear Designs

- Left is not used at all,
- Middle is single enveloping
- Right is double enveloping

Figure 13–24

Nomenclature of a single-enveloping worm gearset.



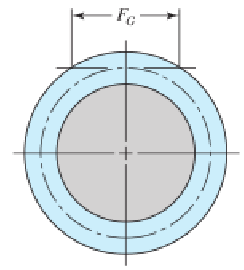


Figure 13–25

A graphical depiction of the face width of the worm of a worm gearset.

In principle, worm and worm gear work like a pair of helical gears in crossed configuration.

The worm and worm gear have helices of the same hand.

The worm has a large helix angle while the worm gear has a small one (e.g., 85° versus 5°).

Shaft angle $\sum$ does not have to be 90°, although 90° is typical, and the scope of Chapters 13 and 15.

Due to the large helix angle of the worm, it can be thought of as a power screw. Terminologies for power screws are adopted for the worm.

Helix angle on worm is ψ_W. If lead angle on worm is λ, then $\lambda = 90° - \psi_W$.

Helix angle on worm gear is ψ_G. For 90° shaft angle, then $\psi_G = \lambda$

Number of teeth of worm, or number of threads (or starts) on worm is, N_W. Typically, $N_W \leq 4$, a recommendation found in many references. However, $N_W \leq 10$ is recommended by Norton (Machine Design – An Integrated Approach, 3rd Ed., Pearson Hall, 2006).

In terms of N_W, and N_G (number of teeth on worm gear), the following is recommended:

- $N_W = 1$ if velocity ratio > 30, $N_W > 1$ if velocity ratio ≤ 30; and $N_G \geq 24$ (recommended by many references);
- $N_G + N_W > 40$. (Machine Design – An Integrated Approach, 3rd Ed., R.L. Norton, Pearson Prentice Hall, 2006).

Axial (circular) pitch p_x for worm; and transverse (circular) pitch for worm gear p_t; Proper meshing requires $p_x = p_t$

Pitch diameter of worm gear:

$$d_G = \frac{N_G p_t}{\pi} \tag{13–25}$$

Pitch diameter of worm d_w is determined by (Eq. 13-25) or (Eq. 15-27) which is recommended by AGMA for optimal power transmission capacity.

$$\frac{C^{0.875}}{3.0} \leq d_W \leq \frac{C^{0.875}}{1.7} \tag{13–26}$$

$$\frac{C^{0.875}}{3} \leq d \leq \frac{C^{0.875}}{1.6} \tag{15–27}$$

The C in (Eq. 13-25) is the center-to-center distance and

$$C = \frac{d_W + d_G}{2}$$

Velocity ratio

$$\left|\frac{\omega_W}{\omega_G}\right| = \frac{N_G}{N_W} \neq \frac{d_G}{d_W}$$

Face width of the worm gear is F_G

Face width of the worm is F_W

- **Tooth Systems**

Worm and worm gear are not as highly standardized as spur/helical/bevel gears.

Table 13-5: A list of “recommended” normal pressure angle, addendum and dedendum. Addendum and dedendum are in terms of p_X.

Table 13–5

Recommended Pressure Angles and Tooth Depths for Worm Gearing

Lead Angle λ, deg	Pressure Angle ϕ_n, deg	Addendum a	Dedendum b_G
0–15	$14\frac{1}{2}$	$0.3683p_x$	$0.3683p_x$
15–30	20	$0.3683p_x$	$0.3683p_x$
30–35	25	$0.2865p_x$	$0.3314p_x$
35–40	25	$0.2546p_x$	$0.2947p_x$
40–45	30	$0.2228p_x$	$0.2578p_x$

But really, **Table 15-5** should be used instead:

Table 15-5: a list of addendum and dedendum for single-enveloping worm and worm gear.

Table 15–5

Allowable Contact Stress Number for Iron Gears, s_{ac} ($\sigma_{H\,\mathrm{lim}}$) *Source: ANSI/AGMA 2003-B97.*

Material	Material Designation ASTM	Material Designation ISO	Heat Treatment	Typical Minimum Surface Hardness	Allowable Contact Stress Number, s_{ac} ($\sigma_{H\,\mathrm{lim}}$) lbf/in^2 (N/mm^2)
Cast iron	ASTM A48	ISO/DR 185			
	Class 30	Grade 200	As cast	175 HB	50 000 (345)
	Class 40	Grade 300	As cast	200 HB	65 000 (450)
Ductile	ASTM A536	ISO/DIS 1083			
(nodular)	Grade 80-55-06	Grade 600-370-03	Quenched	180 HB	94 000 (650)
iron	Grade 120-90-02	Grade 800-480-02	and tempered	300 HB	135 000 (930)

More dimensions calculations are given within **15-6 AGMA Equations**

The uploaded formulas may help.

Example 1

A worm and worm gear set is used as a speed reducer to drive a winch lift. Velocity ratio is 75. Center-to-center distance is to be around 5.5”. Normal pressure angle is 20°. Determine the dimensions of the worm and worm gear.

Solution:

(1) Choose $N_W = 1$ since velocity ratio is greater than 30. Then $N_G = 75$.

(2) Based on (Eq. 13-26) or (Eq. 15-27), d_w is in the range of $C^{0.875}/3$ and $C^{0.875}/1.6$, or $1.48" \ \leq \ d_w \leq 2.78"$ s

So, choose $d_w = 2"$. Also $d_G = 2C - d_w = 9"$

(3) But $d_G = \frac{N_G p_t}{\pi}$, so $p_t = \frac{\pi(9)}{75} = 0.377$”. So choose $p_t = p_x = 3/8" \ = \ 0.375"$

(4) As a result, $d_G = (75)(0.375)/\pi = 8.95"$, and $C = (2 + 8.95)/2 = 5.475"$

(5) Dimensions are tabulated as follow.

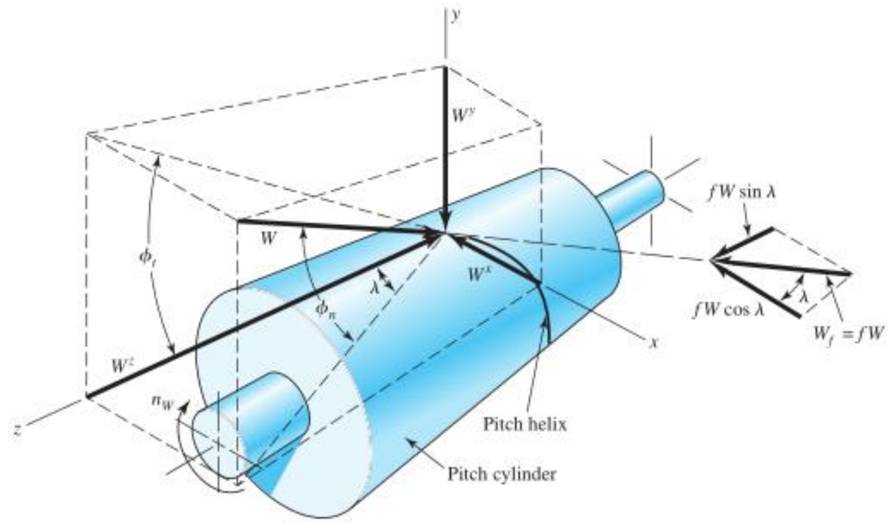
	Worm	Worm-gear
Number of teeth	1	75
Pitch diameter, in	2	8.95
Lead, in	0.375	
Lead angle, °	3.416	
Helix angle, °	86.584	3.416
Addendum, in	0.1194	
Dedendum, in	0.1381	
Clearance, in	0.0187	0.0187
Outside diameter, in	2.239	
Throat diameter, in		9.189
Root diameter, in	1.729	8.674
Mean diameter, in	2	8.95
Face width, in	$\leq$ 2.924	1.333

13-17 Force Analysis – Worm Gearing

(*) Worm/worm gear is for sure on final exam

Figure 13–40

Drawing of the pitch cylinder of a worm, showing the forces exerted upon it by the worm gear.



There's friction components now (on the right)

Friction must be taken into consideration;

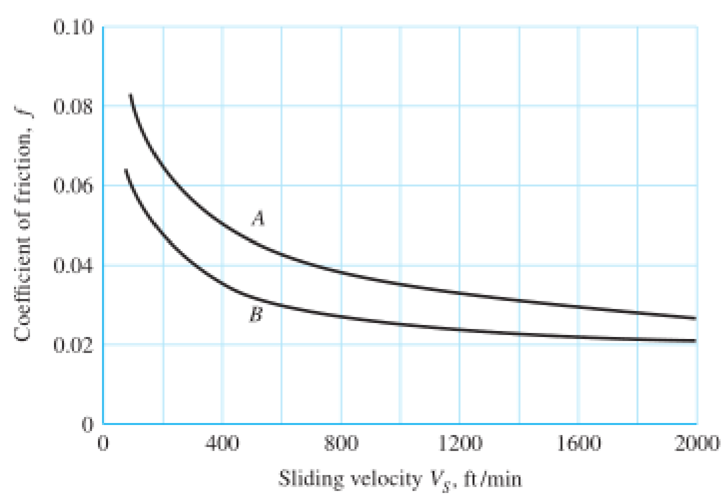
Transmitted load on worm W_w^t: determined based on the torque on the worm, or the horsepower and pitch line velocity of the worm. See Example 13-10;

Efficiency η: by (Eq. 13-46) where the coefficient of friction f is by Figure 13-42 or (Eq. 15-43a)

$$\eta = \frac{\cos \phi_n - f \tan \lambda}{\cos \phi_n + f \cot \lambda} \tag{13–46}$$

Figure 13–42

Representative values of the coefficient of friction for worm gearing. These values are based on good lubrication. Use curve *B* for high-quality materials, such as a case-hardened steel worm mating with a phosphor-bronze gear. Use curve *A* when more friction is expected, as with a cast-iron worm mating with a cast-iron worm gear.



AGMA reports the coefficient of friction f as

$$f = \begin{cases} 0.15 & V_s = 0 \\ 0.124 \exp(-0.074 V_s^{0.645}) & 0 < V_s \le 10 \text{ ft/min} \\ 0.103 \exp(-0.110 V_s^{0.450}) + 0.012 & V_s > 10 \text{ ft/min} \end{cases} \qquad (15\text{–}38)$$

Total force W is (reworked Eq. 13-43a):

$$W = \frac{W_W^t}{\cos\phi_n \, sin\lambda + f cos\lambda}$$

Transmitted load on worm gear W_G^t = axial load on worm W_W^a by (Eq.13-43c)

$$W^z = W(\cos\phi_n \cos\lambda - f \sin\lambda)$$

Radial load on worm W_W^r = radial load on worm gear W_G^r by by (Eq.13-43b)

$$W^y = W \sin\phi_n \qquad (13\text{–}43)$$

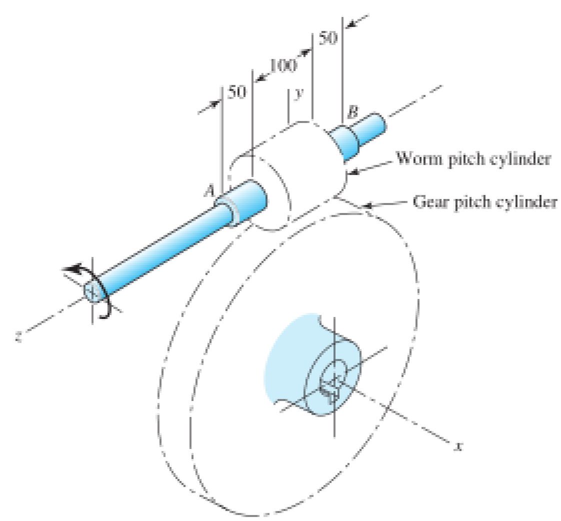
Axial load on worm gear W_G^a = transmitted load on worm W_W^t

Friction $W_f = f \cdot W$;

Note: the above discussions deal with the magnitudes only

Example 2

The worm in Example 1 is left-hand, and runs at 1750 rpm (see Figure for Prob. 13-51 as a reference). The winch requires a torque of 8000 $lb - in$. Determine and visualize the forces acting on the worm and worm gear. What is the efficiency of the set? What is the efficiency of the set? What is the friction?



Solution:

From Example 1,

	Worm	Worm-gear
Number of teeth	1	75
Pitch diameter, in	2	8.95
Lead angle, °	3.416	

Pitch line velocity of the worm:
$V_w = \pi d_w n_w/12 = \pi(2)(1750)/12 = 916.3\ ft/min$

Sliding velocity, by (Eq. 13-47):
$V_s = V_W/\cos\lambda = 916.3/\cos 3.416° = 917.9\ ft/min$

$f = 0.0216$ (Eq. 15-38)

$\eta = 0.721$ (Eq. 13-46)

Torque on the worm: $T_W = T_G/75/\eta = 147.9\ lb - in$

$W_W^t = T_W/(d_w/2) = 147.9\ lb;$

$W = 1907\ lb$

$W_f = 41.19\ lb$

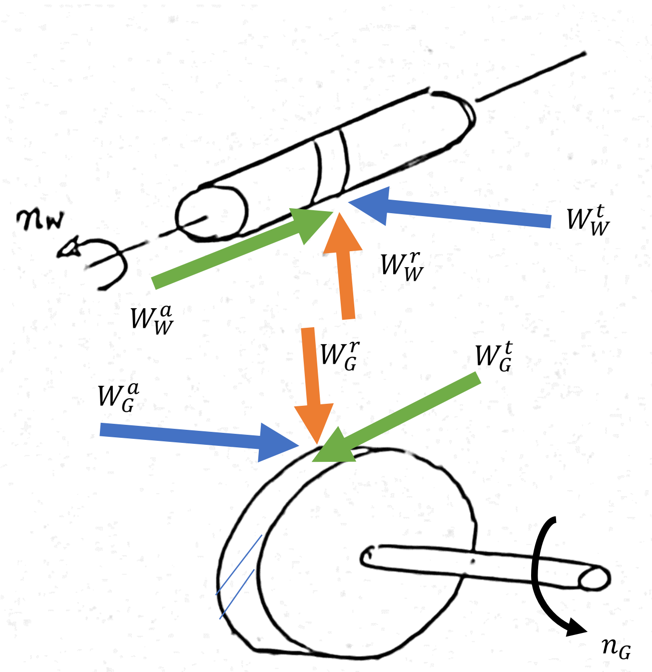
$W_W^r = W_G^r = 652.2\ lb$

$W_W^a = W_G^t = 1789\ lb$

$W_G^a = W_W^t = 147.9\ lb$

To verify, torque on the worm gear $= (1789)(8.95/2) = 8006\ lb - in$ (Compared with the given torque of $8000\ lb - in$)

Visualization:



Materials
Worms:
Low-carbon steels (1020, 1117, 8620 and 4320), case-hardened to HRC58-62.

Medium-carbon steels (4140 and 4150), induction or flame hardened to surface hardness of HRC 58-62.

Grinding or polishing of surfaces may be required.

Note: the above is taken from *Machine Design: An Integrated Approach,* 3rd Ed., R.L. Norton. Pearson Prentice Hall, 2006.

Worm gears:
Bronzes (sand-cast, chill-cast, centrifugal-cast, or forged).

Sec. 15-8 has details.

The first two columns in Table 15-11 show some typical combinations of materials for worm and worm gear.

15-6 Worm Gearing AGMA Equations
15-9 Buckingham Wear Load
Allowable Transmitted Load, (Eq. 15-28)

$$(W^t)_{\text{all}} = C_s D_m^{0.8} F_e C_m C_v \tag{15–28}$$

Note: if W_W^t or W_G^t is less than $(W^t)_{all}$, it means that the worm and worm gear under consideration will last at least 25,000 hours.

Temperature rise, (Eq. 15-51)

$$t_s = t_a + \frac{H_{\text{loss}}}{\hbar_{\text{CR}} A} = \frac{33\,000(1 - e)(H)_{\text{in}}}{\hbar_{\text{CR}} A} + t_a \tag{15–51}$$

Where H_{loss} is the rate of heat dissipation, in $ft - lb/min$.

$$H_{loss} = 33{,}000(1 - \eta)H_{in}$$

Note: t_s and t_a are the oil sump temperature, and ambient temperature, respectively. It is recommended that $t_s < 160 - 200\ °F$

Other Calculations include Buckingham's equation for dynamic (i.e. fatigue) bending stress, see (Eq. 15-53); and Buckingham's wear load, see (Eq. 15-64) and Sec. 15-9.

$$\sigma_a = \frac{W_G^t}{p_n F_e y} \tag{15–53}$$

$$(W_G^t)_{all} = K_w d_G F_e \tag{15–64}$$

Buckingham's wear load is considered the predecessor to the AGMA equation.

Only limited data are available for the factor y in (Eq. 15-53)

15-7 Worm Gear Analysis
15-8 Designing a Worm Gear Mesh
Worm and worm gear mesh has lower efficiency due to friction.

Power consumed by friction can be determined by (Eq. 15-63).

$$H_f = \frac{|W_f| V_s}{33\,000} \text{ hp} \tag{15–63}$$

Cooling, natural or using cooling fans, may be required.

Multi-start worms reduce cooling requirement and reduce d_w as well.

Self-locking means that the worm gear can't drive the worm; self-locking is necessary, even critical, for some applications.

To ensure self-locking, it is required $f_{static} > cos\phi_n tan\lambda$.

Example 15-3
Example 15-4

Example 3
Design the set of worm and worm gear of Example 1. The worm is left-hand, and runs at 1750 rpm. It will be made of carbon steel, case hardened to HRC 58. The material for the worm gear has been chosen to be sand cast bronze. Velocity ratio is 75. Center-to-center distance is to be around 5.5". The winch requires a torque of 8000 lb-in. Self locking is a must-have safety requirement. Temperature rise should not exceed $80°\ F$. Set normal pressure angle to 20°. Assume no cooling fan on the worm shaft.

Solution:
(1) Geometry,
This has been completed in Example 1

(2) Transmitted loads, sliding velocity, friction, efficiency, etc.
They were calculated in Example 2.

(3) Allowable transmitted load is, (Eq. 15-28)

$$(W^t)_{all} = C_s D_m^{0.8} F_e C_m C_v$$

$D_m = 8.95$", the mean diameter of worm gear

C_s is the materials factor, obtained by one of (Eq. 15-32) through (Eq. 15-35); $C_s = 736.0$

C_m is the ratio correction factor. (Eq. 15-36) gives $C_m = 1.309$

C_v is the velocity factor. Since $V_s = 917.9\ ft/min$, $C_v = 0.2891$, see (Eq. 15-376)

$$C_v = \begin{cases} 0.659 \exp(-0.0011 V_s) & V_s < 700 \text{ ft/min} \\ 13.31\ V_s^{-0.571} & 700 \le V_s < 3000 \text{ ft/min} \\ 65.52\ V_s^{-0.774} & V_s > 3000 \text{ ft/min} \end{cases} \tag{15–37}$$

F_e is the effective face width of the worm gear. Actual face width is $F_G = 1.3"$. Also, $0.67 d_m = 1.34"$. So, effective face width is $F_e = 1.3$. Also, $0.67 d_m = 1.34"$. So effective face width is $F_e = 1.3$.

Therefore, $(W^t)_{all} = 2091\ lb$

$(W^t)_{all}$ is greater than $W_W^t = 147.9\ lb$, and $W_G^t = 1789\ lb$. So the worm and worm gear will last at least 25,000 hours.

(4) Powers transmitted by the worm and worm gear, and consumed by friction.

(Eq. 15-59) and (Eq. 15-60) give powers transmitted by the worm and worm gear respectively, and in hp. Results are,

$$H_W = \frac{W_W^t V_W}{33\,000} = \frac{\pi d_W n_W W_W^t}{12(33\,000)} \tag{15–59}$$

$$H_G = \frac{W_G^t V_G}{33\,000} = \frac{\pi d_G n_G W_G^t}{12(33\,000)} \tag{15–60}$$

$$H_W = 4.11\ hp$$
$$H_G = 2.96\ hp$$

(Eq. 15-63) shows power consumed by friction, in hp. The result is, $H_f = 1.15\ hp$

$$H_f = \frac{|W_f| V_s}{33\,000} \text{ hp} \tag{15–63}$$

It is seen that $H_G + H_f = H_W$

In general, $H_G + H_f \approx H_W$ due to rounding.

(5) Temperature rise

(Eq. 15-49) shows the rate of heat loss, from the casing/housing, in $ft - lb/min$

$$H_{\text{loss}} = 33\,000(1 - e)H_{\text{in}} \tag{15–49}$$

H_{in} is the input power in hp. $H_{in} = H_W$.

So, $H_{loss} = 37841\ ft - lb/min$.

Temperature rise is by (Eq. 15-51)

$$t_s = t_a + \frac{H_{\text{loss}}}{\hbar_{\text{CR}} A} = \frac{33\,000(1 - e)(H)_{\text{in}}}{\hbar_{\text{CR}} A} + t_a \qquad (15\text{–}51)$$

Where A is the lateral area of the casing/housing, and h_{CR} is the combined convective and radiative coefficient of heat transfer. (NOTE: In our notes e is written as η)

The lateral area of the casing typically includes the external surface area, for example, the 6 rectangular areas of a cube that is the casing/housing.

By (Eq. 15-50), $h_{CR} = 0.3995\ ft - lb/(min \cdot in^2 \cdot °F)$. To limit temperature rise to 80 $°F$, the required lateral area is $A = H_{loss}/(t_S - t_a)/h_{cr} = 1184\ in^2$.

$$\hbar_{\text{CR}} = \begin{cases} \dfrac{n_W}{6494} + 0.13 & \text{no fan on worm shaft} \\ \dfrac{n_W}{3939} + 0.13 & \text{fan on worm shaft} \end{cases} \qquad (15\text{–}50)$$

Discussions: (Eq. 15-52) shows the AGMA recommended minimum area A_{min}.

$$A_{\min} = 43.20C^{1.7} \qquad (15\text{–}52)$$

$$A_{min} = 43.20\ C^{1.7} = (43.20)(5.475)^{1.7} = 777.6\ in^2$$

With $A = A_{min}, t_s - t_a = \frac{H_{loss}}{h_{CR}A} = 121.8\ °F$, and $t_s \approx 191.8\ °F$.

If it is desired to keep the surface area at A_{min}, while still limit the temperature rise to 80 $°F$, cooling fans may be installed; cooling fins may be incorporated. Finally, an external heat exchanger may be considered.

(6) Self-locking

Since $cos\phi_n tan\lambda = 0.056$, self locking requires $f_{static} > 0.056$.

The static COF between steel and bronze is, from engineersedge.com, 0.16 when lubricated. Static friction seems enough. However, a brake, for instance, is advised.

Ch. 12 - Lubrication and Journal Bearings

- Charts and their usages
- Interpolation
- What constitutes a Good Design?
  - $h_o \geq h_{min}$, $f > 0.01$, optimal zone, ΔT;
  - Minimum film thickness is greater than shaft deflection across the length of bearing (to prevent binding between shaft and bearing);
  - Suitable materials;
  - Etc.

Ch. 13, 14, 15 – Gearing

- Types
  - Spur, helical, bevel, worm-worm gear
    - Bevel gears: spiral, hypoid, spiroid, etc.
  - Features, from the perspectives of
    - Applications
    - Geometry
    - Analyses
    - …
  - Configurations
    - Helical: parallel, crossed
    - Bevel: shaft angle = 90°, $\neq 90°$
    - Worm-wormgear: single enveloping, double enveloping
  - Geometry, tooth systems, standard values
    - Spur gears: US-customary vs. metric; full-depth vs. stub-profiled; L_{ab}
    - Helical gears: normal plane vs. transverse plane; L_{ab}; m_p m_F
    - Bevel gears: large end, pitch cone, back cone, etc.;
    - Do practise problems posted
  - Planetary gear trains
    - Spur and helical gears: "+" for internal sets "-" for external sets
    - Bevel gears: how to deal with the signs;
  - Force components and visualization
  - Materials
  - AGMA equations

    Spur gears: the basic, and foundation for helical and bevel gears;
    Helical gears: Geometry factors I and J;
    Bevel gears:
    Comparing S_F and S_H;
  - Worms-wormgears

    AGMA equation: allowable transmitted load;
    Friction, efficiency, temperature rise, cooling considerations;
    Self-locking;

Ch. 7 – Shafts and Shaft Components

- Input(s) and power take-off(s) of individual shafts;
- Gears' axial loads → beams FBD;
- Support reactions, shear force diagrams, bending moment diagrams, combined bending moment diagram, torque diagram;
- Static-failure;
- Fatigue-failure;
- Deflection/slope in relation to bearings and gears, and by applying Table A-15;
- Miscellaneous components (keys, pins, etc.)