Chapter 24

GEODESY AND GRAVITY

C. Jekeli

Geodesy is a science as old as man's curiosity about the earth—its shape, position, and relation to the stars and planets; yet it has remained a new and timely science by successfully adapting to rapidly changing requirements and novel techniques to answer these requirements. Simply defined, geodesy is the branch of applied mathematics that determines, by observation and measurement, the size and shape of the earth and the precise positions of points on the earth's surface (geometric geodesy) as well as the direction and magnitude of the earth's external gravity field (physical geodesy). The continuing improvements in observational accuracy lend increasing importance to the dependence of each of these quantities on time, and the definition of geodesy now includes the determination of the temporal variations in positions and the gravity field [Mather, 1973]. More and more, geodesy is called upon to provide ancillary methods and data in the study of global, regional, and local geodynamics, as well as other geophysical phenomena. Geodetic measurements and observations contribute to the understanding of tectonic motions, earthquakes, the dynamic evolution of the ocean floor, and the earth's internal structure, as well as its motion in space. To achieve its goals, geodesy, in turn, relies on the continuing efforts in a wide spectrum of terrestrial physics of which it thus forms an integral part.

The following sections provide the nonspecialist with a cursory survey of modern geodesy, emphasizing definitions, results, achievable accuracies, and existing data, but also including a superficial discussion of methodology, when appropriate. The introduction of artificial satellites, modern computers, and other technological engineering advances have broadened the field of geodetic applications to the extent that it is virtually impossible to mention all aspects in a summary such as this. The geodetically connected fields of (plane) surveying, cartography, and photogrammetry are omitted entirely since they have attained prominence in their own right, particularly with their incorporation of new and unique instrumentation and computer capabilities. The textbooks of Mueller and Ramsayer [1979], the Manual of Photogrammetry [1980], and the annual proceedings of the American Congress on Surveying and Mapping and of the American Society of Photogrammetry are excellent sources of detailed information for the interested reader. A description of the diverse instrumentation, vital to geodesy, must also be relegated to other sources. Brief accounts of principles and accuracy can be found in textbooks such as Bomford [1971], Torge [1980], and Mueller [1969]. Other textbooks expounding the theory of geodesy, in addition to these, are by Moritz [1980], Groten [1979], Heiskanen and Moritz [1967], Kaula [1966], and Vanicek and Krakiwsky [1982].

24.1 GEOMETRIC GEODESY

The two reference surfaces in geometric geodesy are the geoid, to which all height observations are referred (vertical control), and the reference ellipsoid, where horizontal geodetic positions are determined (horizontal control). The geoid is the equipotential surface in the earth's gravity field that coincides most closely with the undisturbed mean sea level extended continuously under the continents. The direction of gravity is perpendicular to the geoid at every point. On the earth's surface, this direction is defined [Helmert, 1884] by two angles, the astronomical coordinates: The astronomical latitude is the angle Φ, $-90° \leq \Phi \leq 90°$, that the gravity vector forms with the equatorial plane; and the astronomical longitude is the angle Λ, $0° \leq \Lambda \leq 360°$ and positive eastwards, that the plane defined by this vector and the celestial pole forms with the Greenwich meridional plane. Astronomic coordinates are obtained directly from star observations using, for example, a theodolite, its vertical axis being aligned with the local gravity vector.

The reference ellipsoid is a simple mathematical figure that closely approximates the geoid, historically on a regional basis, but on a global basis for modern requirements. It is a surface of revolution formed by rotating an ellipse about its minor (vertical) axis resulting in an oblate (flattened at the poles) ellipsoid. Geodetic ellipsoids are defined by the parameters a = equatorial radius (semimajor axis) and $f = (a - b)/a$ = (polar) flattening, where b is the ellipsoid's semiminor axis. The minor axis is supposed to be parallel to the rotational axis of the earth. Points on the ellipsoid are specified by geodetic coordinates—the geodetic latitude is the angle ϕ, $-90° \leq \phi \leq 90°$, that the normal to the ellipsoid at the point in question forms with the equator; and the geodetic longitude is the angle λ, $0° \leq \lambda \leq 360°$ and positive eastwards, that the ellipsoidal meridional plane forms with the plane through the Greenwich meridian. The two angles ϕ, λ therefore also define the direction of the ellipsoidal normal.

Because of the irregular distribution of the earth's internal masses, the geoid is an irregular surface which cannot be described by a simple geometric figure. It is determined with respect to a particular ellipsoid in terms of the geoid height, or geoid undulation, which is the separation between the two surfaces at any point (Figure 24-1). Another geoid-ellipsoid relationship is the astrogeodetic deflection of the vertical. This is the angle between the local vertical and the normal to the ellipsoid through the point on the earth's surface [Helmert, 1884], (Figure 24-1). The deflection of the vertical is resolved into west-east and south-north components, which then provide the functional relationship between geodetic and astronomic coordinates.

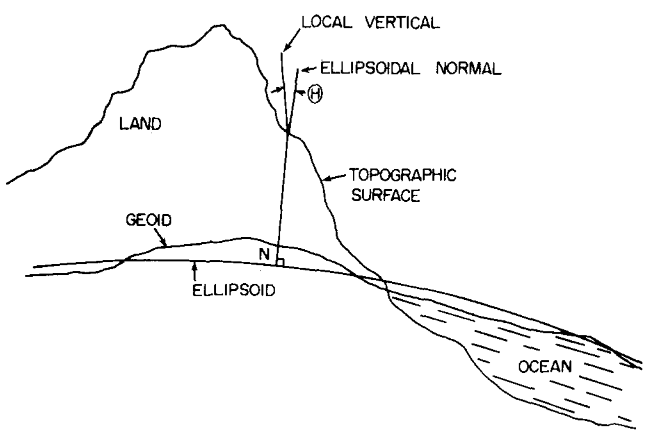

Figure 24-1. The relationship between the geoid and the ellipsoid.
N: geoid undulation
Θ: astrogeodetic deflection of the vertical

24.1.1 Horizontal Control

Geodetic surveying generally falls into one of three categories: (1) triangulation networks, whereby points on the earth's surface are connected by a network of triangles whose angles are measured instrumentally and whose sides are computed with the aid of one or more precisely measured baselines; (2) trilateration networks, in which the observations consist primarily of distance measurements of the triangle sides; and (3) the traverse, a succession of distance and direction measurements connecting a sequence of points.

The high-precision traverse in the U.S., that is, the Transcontinental Traverse (TCT), is accurate to 1 part in 10^6 and runs in essentially unidirectional east-west and north-south directions (Figure 24-2). It provides the principal framework, scale and orientation, of the National Horizontal Control Network. Additional strength in the determination of scale and orientation is obtained by incorporating satellite Doppler stations (Section 24.3.1) into these traverses. Horizontal control within the areas outlined by the TCT is accomplished by densification networks consisting of first-, second-, and third-order triangulations (Table 24-1), as well as lower accuracy local traverses, depending on economic and other needs of the area. Table 24-1 gives a general overview of the accuracies to be achieved and the purposes, uses, and configuration constraints for the several classes of horizontal control.

In most countries the reference ellipsoid, to which the horizontal control refers, is fixed to the earth's surface at an origin point in the network, where the deflection of the vertical and geoid undulation are specified by definition. All measurements to subsequent points in the network are reduced to the same ellipsoid. This entails knowing the height above the geoid (elevation), the geoid undulation, and the deflection of the vertical at each point. The ellipsoid and its relation to the geoid as specified at the origin point define a datum. The orientation of the datum with respect to the earth is determined by three rotation angles—two of these are the geodetic coordinates of the origin point, obtained from the observed astronomic coordinates and the (defined) deflection of the vertical. The remaining degree of freedom, namely a rotation about the ellipsoid normal at the origin point, is fixed by the geodetic azimuth to some other point. It is computed from its relationship to the astronomic azimuth and the deflection of the vertical which incorporates the parallelism of the minor axis of the ellipsoid and the rotational axis of the earth. Because of measuring errors that propagate through the network, additional astronomic azimuths (Laplace azimuths) and longitudes must be determined at selected points to enforce the proper orientation. The position of the center of a geodetic ellipsoid with respect to the earth's center of mass is generally unknown. Table 24-2 lists the most important datums in use throughout the world (Figure 24-3). The relative orientation, position, and scale of several datums can be obtained if the coordinates of a sufficient number of points in each relative system are also known in the other systems, or in an absolute (geocentric) system. Table 24-3 lists the relationships of several regional datums to a global datum. The global datum, in this case, is designated WN14 and is the result of a global solution of geometric observations to satellites, namely ranging and camera observations. Δx, Δy, Δz refer to the displacements of the datum center from the satellite system center with respect to a right-handed Cartesian x, y, z coordinate system (z-axis through the Conventional International Origin pole, x-axis on the equator through the zero meridian, Section 24.4); δ is the relative scale of the two systems; and α, ξ, η are the relative rotations at the origin point of the datum—α is the rotation in azimuth; ξ, η in latitude and longitude.

The U.S. National Geodetic Survey of the National Ocean Survey/National Oceanic and Atmospheric Administration (NOAA) in concert with corresponding organizations of Canada, Mexico, the republics of Central America, and Denmark are currently completing the new adjustment of the North American geodetic control network. It will en-

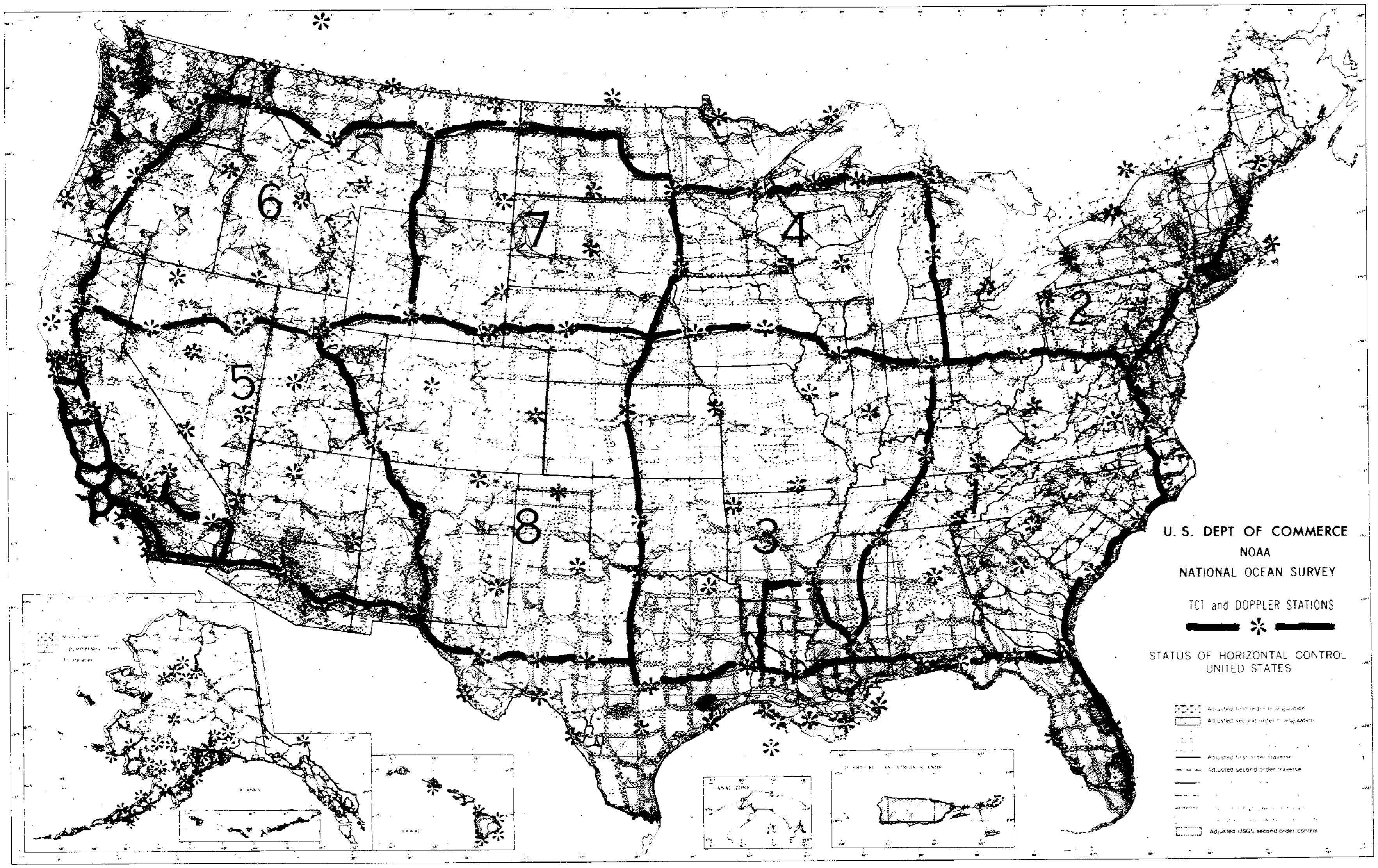

Figure 24-2. Status of horizontal control in the United States [NRC, 1978].

Table 24-1. Synopsis of horizontal control classifications [NOAA, 1980b].

Order of Surveys and Classes of Accuracy					
Attributes	Super First-Order	First-Order	Second-Order Class I	Second-Order Class II	Third-Order Class I, II
General title	Transcontinental control.	Primary horizontal control.	Secondary horizontal control.	Supplemental horizontal control.	Local horizontal control.
Purpose	Transcontinental traverses. Satellite observations. Lunar ranging. Interferometric surveying.	Primary arcs. Metropolitan area surveys. Engineering projects.	Area control. Detailed surveys in areas of very high land value.	Area control. Detailed surveys in areas of high land value.	Area control. Detailed surveys in areas of moderate and low land value.
Network design	Control develops the national network.		Control strengthens the national network.	Control contributes to the national network.	Control referenced to the national framework.
Accuracy	1:1 000 000	1:100 000	1:50 000	1:20 000	1:10 000 1:5 000
Spacing	Traverses at 750 km. Spacing-stations at 15 to 30 km or greater. Satellite as required.	Arcs not in excess 100 km. Stations at 15 km. Metropolitan area control 3–8 km.	Stations at 10 km. Metropolitan area control at 1–2 km.	As required.	As required.
Examples of use	Positioning and orientation of North American continent. Continental drift and spreading studies.	Surveys required for primary framework. Crustal movement. Primary metropolitan area control.	Metropolitan area densification. Land subdivision. Basic framework for densification.	Mapping and charting. Land subdivision. Construction.	Local control. Local improvements and developments.

compass all of North America including Greenland. The datum, however, will be a global, or absolute, datum. Its origin (geocenter) and shape and scale (flattening and semimajor axis) will be obtained by gravimetric and dynamic satellite methods, as well as from Doppler observations at over 250 stations. The orientation of this datum will be primarily controlled by Very Long Baseline Interferometry (VLBI) observations (Section 24.4.2) which are ultimately tied to an astronomic coordinate system. The new adjustment will result, in the U.S., in positions of over 180 000 stations with accuracies of about 1 part in 10^5 [Bossler, 1978]. The deflections of the vertical required for the data reduction to the reference ellipsoid were determined at every point in the network, computed in the U.S. from about 5000 astrogeodetic deflections in conjunction with gravimetric deflections of the vertical (Section 24.2.2). The gravimetric deflections were estimated to an accuracy of approximately 1″ to 2″ from about one million gravity anomalies [C.R. Schwarz, 1978].

In an attempt to construct a unified reference system for military purposes, the World Geodetic System (WGS) has evolved primarily out of improvements, through satellite techniques [NASA, 1977], in the estimation of the size and shape of the earth and its gravity field. In January 1974 the current global datum WGS72 was published by the Department of Defense and represents its most comprehensive study of the earth and its gravity field to date. Station positions can now be determined to an accuracy of 5 to 10 m [Western Space and Missile Center, 1981], thus establishing a truly worldwide geocentric system. One of the features of WGS72 is that it combined vast amounts of data consisting of different data bases (satellite, surface gravity, astrogeodetic, etc.) into a homogeneous unified solution by a large least squares adjustment.

Table 24-2. The most important datums of the world, their associated ellipsoids, and their origin points [extracted from NASA, 1973]

Datum	Reference ellipsoid	Semimajor axis [m]	l/flattening	Station	Origin point coordinates φ [°′″]	λ [°′″]
Australian Geodetic	Australian National	6378160	289.25	Johnston Geodetic Station	−25 56 54.55	133 12 30.08
Arc-Cape (South Africa)	Clarke (1880)	6378249.145	293.465	Buffelsfontein	−33 59 32.000	25 30 44.622
European 1950	International (1909)	6378388	297.	Helmertturm	52 22 51.45	13 03 58.24
Indian	Everest (1830)	6378206.4	300.8017	Kalianpur	24 07 11.26	77 39 17.57
North American Datum 1927	Clarke (1866)	6378206.4	294.978698	Meades Ranch	39 13 26.686	261 27 29.494
Pulkova 42	Krassovaki (1940)	6378245	298.3	Pulkovo Observatory	59 46 18.55	30 19 42.09
S. American 69	S. American (1969)	6378160	298.25	Chua	−19 45 41.653	311 53 55.936
Tokyo	Bessel (1841)	6377397.155	299.152813	Tokyo Observatory (old)	35 39 17.51	139 44 40.50

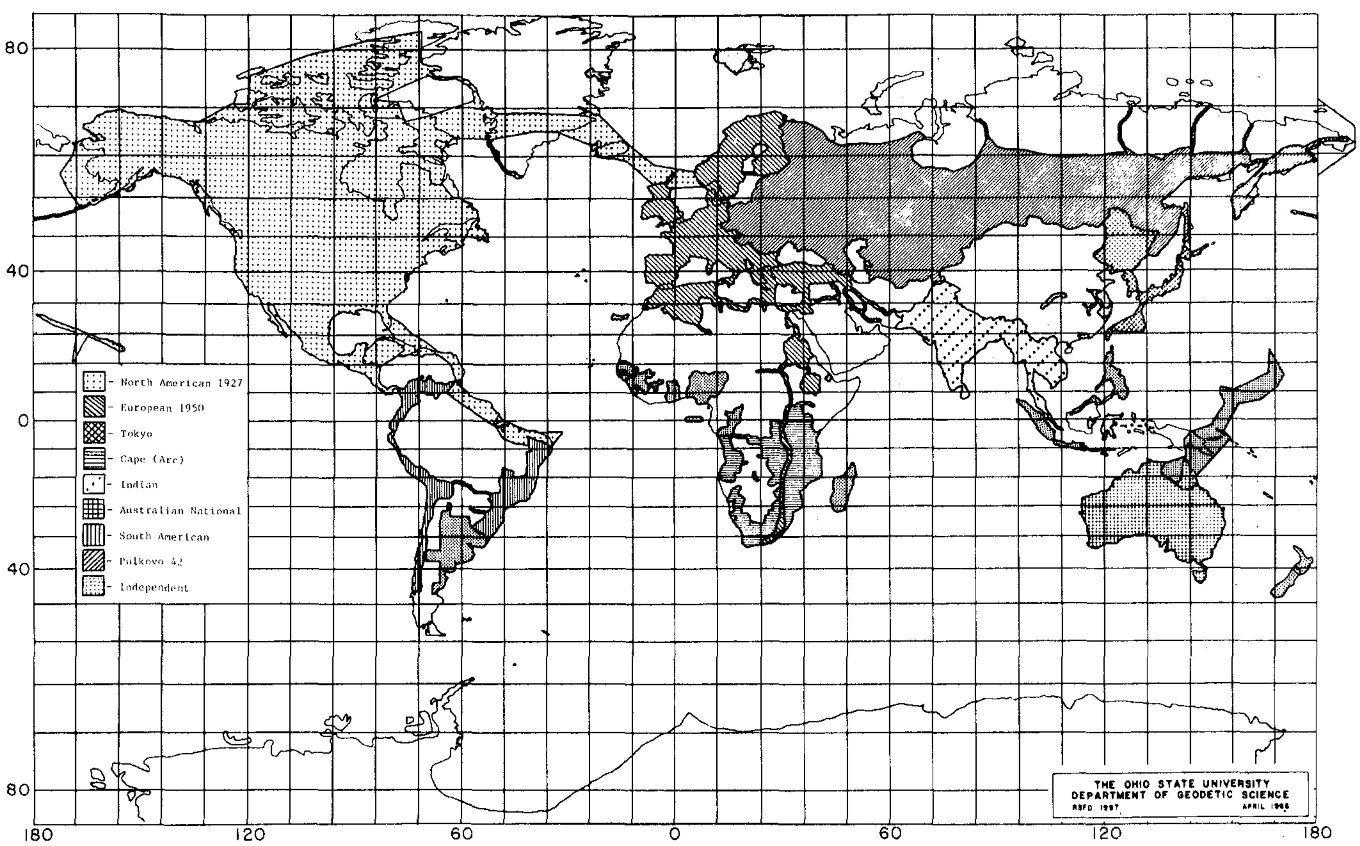

Figure 24-3. Major geodetic datum blocks [Mueller, 1975].

Table 24-3. Values of transformation parameters: Datum to Satellite WN14 System (satellite datum); parenthetical values are standard deviations [extracted from Mueller, 1975].

Datum / Transformation parameters	Australian National (3 stations)	European 1950 (16 stations)	NAD27 (21 stations)	S. American 69 (10 stations)
ΔX [m]	−157. (3.2)	−99.4 (5.0)	−31.7 (1.7)	−96.6 (4.0)
ΔY [m]	−59.1 (3.2)	−132.0 (5.0)	142.3 (1.6)	−13.7 (4.3)
ΔZ [m]	131.2 (3.6)	−116.0 (4.8)	177.3 (1.5)	−29.4 (4.1)
δ (10^{-6})	1.14 (1.83)	6.06 (2.83)	−0.96 (0.98)	−6.67 (1.41)
α [″]	−0.35 (0.38)	0.30 (0.65)	−0.33 (0.18)	−0.02 (0.29)
ξ [″]	0.49 (0.64)	−0.13 (0.98)	−0.34 (0.35)	−0.03 (0.35)
η [″]	1.31 (0.52)	0.26 (0.70)	0.84 (0.21)	−0.66 (0.43)

Values used to define the WGS72 ellipsoid and its associated parameters include the semimajor axis (a), the product of the gravitational constant (G) and the earth's mass (M), the earth's angular velocity (ω), and the second degree zonal harmonic coefficient of the geopotential ($C_{2,0}$) (Section 24.2.1).

24.1.2 Vertical Control

Vertical control is generally established by leveling networks that are tied to mean sea level. The networks consist of loops of leveling lines, the misclosure of the loops (the sum of the height differences around the loop should be zero) being indicative of the measurement errors. Since the level surfaces of the earth's gravity field are not parallel, the heights above the geoid depend also on gravity, and for precise work, cannot be deduced solely from leveling data. The height of the topographic surface above the geoid is known as the orthometric height. The orthometric correction, calculable with the aid of gravity measurements and applied to leveled heights, accounts for the nonparallelism of the level surfaces. The leveling networks are classified as first-, second-, and third-order according to the spacing, length, and method of the leveling run. First-order surveys are the most accurate and establish the primary vertical control on a national basis; see Table 24-4 for a brief summary of accuracy standards and recommended uses of the various survey classes.

The leveling networks are tied to tidal stations, these providing the only physical access to the geoid. At these stations, mean sea level is ascertained by averaging sea level recordings over several years to average both short and long period tidal fluctuations. In the U.S. and Canada over 100 000 km of first-order leveling lines, constrained by mean sea level determined at 26 tidal stations along the Pacific and Atlantic coasts and the Gulf of Mexico, were used to establish the National Geodetic Vertical Datum of 1929 [NOAA, 1980]. The readjustment of the geodetic vertical control for North America is currently in progress with an expected completion date of 1987. The data from existing surveys plus the planned new levelings combine to form a total of over 800 000 km of first- and second-order leveling lines in the U.S. alone (Figure 24-4). The gravity support for the vertical control is guided by the requirement that the effect of gravity on the observed heights should be determined to within 1 cm [Whalen, 1980].

A problem of some concern, and which has yet to be satisfactorily resolved, is the establishment of the datum. With modern accuracy standards, the presumption that mean sea level coincides everywhere, at any time, with the geoid must yield to the realization that differences of up to 60 to 70 cm may exist in mean sea level between the Atlantic and Pacific coasts as well as 1.5 mm/yr changes in the geoid due to the worldwide change in mean sea level [Castle and Vanicek, 1980]. With the difficulty of the physical accessibility to the geoid at this level of accuracy (tens of cm), the establishment of a World Vertical Datum awaits improvements in the accuracy and extent of global and local gravity data.

24.1.3 Inertial Positioning

Inertial positioning (or survey) systems evolved in the early to mid 1970s from inertial navigation systems, used primarily for military purposes, when geodetic accuracies were warranted by the use of a stabilized platform to isolate the measuring device from the irregular motions of the vehicle. A system of gyroscopes maintains the orientation of the platform which is suspended by gimbal supports. A set of accelerometers is used to measure the accelerations of the survey vehicle in the three mutually orthogonal directions. One integration of these data yields the velocity and

Table 24-4. Standards for the Classification of Geodetic Vertical Control and principal uses [extracted from NOAA, 1980a].

Classification	Vertical Control First-Order		Second-Order		Third-Order
	Class I	Class II	Class I	Class II	
Relative accuracy between directly connected points or benchmarks (standard error)	0.5 mm $\sqrt{K}$	0.7 mm $\sqrt{K}$	1.0 mm $\sqrt{K}$ (K is the distance in kilometers between points)	1.3 mm $\sqrt{K}$	2.0 mm $\sqrt{K}$
Recommended uses	Basic framework of the National Network and metropolitan area control. Regional crustal movement studies. Extensive engineering projects. Support for subsidiary surveys.		Secondary framework of the National Network and metropolitan area control. Local crustal movement studies. Large engineering projects. Tidal projects. Tidal boundary reference. Support for lower order surveys.	Densification within the National Network. Rapid subsidence studies. Local engineering projects. Topographic mapping.	Small-scale topographic mapping. Establishing gradients in mountainous areas. Small engineering projects. May or may not be adjusted to the National Network.

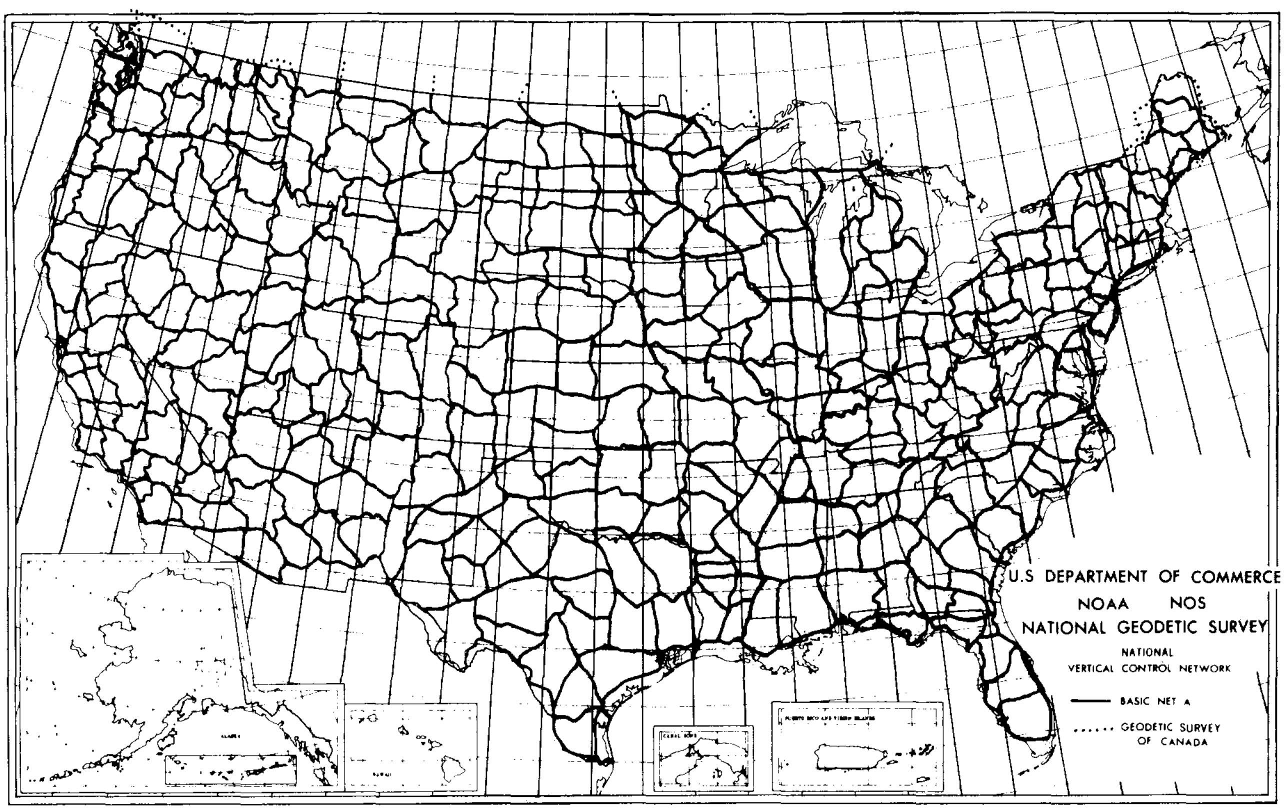

Figure 24-4. National Geodetic Vertical Control Network [NOAA, 1980b].

a second integration yields the distance traveled. Obviously, the acceleration due to gravity must be taken into account. The vehicle (for example, truck or helicopter) carrying the instrument must come to rest every 2, 3, or 4 minutes in order to impose a zero velocity on the velocity output, thus reducing the growth of systematic instrumental errors. For traverse distances of about 75 km, present instrumentation and analyses produce about 50 cm accuracy in horizontal and 40 cm in vertical position. For horizontal control, this is first-order accuracy (Table 24-1). Further improvements in hardware, as well as software, will undoubtedly increase the potential of inertial surveying for geodetic applications [Bose and Huddle, 1981].

24.2 GRAVIMETRIC GEODESY

24.2.1 The Normal Gravity Field

Just as the earth's surface is not a perfect sphere, or even an ellipsoid, the earth's gravity field is not the field generated by a homogeneous ball. Because of the overall ellipticity in the earth's shape, as well as the centrifugal force produced by its rotation, the values of gravity increase as one travels from the equator to either pole. This general trend is conveniently described by a so-called normal gravity field which then serves as the reference against which the actual gravity field is measured. The normal field is defined, in part, by requiring one of its equipotential (or level) surfaces to be an ellipsoid oriented in an earth-fixed coordinate system. Stokes proved that four quantities—the surface potential, the size of the ellipsoid, its shape, and the rotational speed (ω)—suffice to uniquely define the (exterior) normal gravity field. The size and shape of the ellipsoid are specified, for example, by its semimajor axis (a) and its flattening (f). The definition of the field retains its uniqueness if the value of the (constant) surface potential is replaced by the value of the earth's total mass (the combination GM is more accurately determined than the mass M alone, where G is the gravitational constant, $G = 6.67 \times 10^{-8}$ cm^3 g^{-1} s^{-2}), or by the value of equatorial gravity, γ_e. Similarly, f can be replaced by $C_{2,0}$, the second-degree zonal harmonic coefficient in the spherical harmonic expansion of the earth's gravity potential. Once these parameters are specified, the "normal gravity" on the ellipsoid can be calculated as accurately as desired.

The concerted effort to establish a reference ellipsoid and associated normal field that would find international acceptance and application did not materialize until well into this century culminating in the adoption by the International Union of Geodesy and Geophysics [IUGG] in 1930 of the following parameter values:

$$a = 6378388 \text{ m}$$

$$f = 1/297$$

$$\gamma_e = 9.780490 \text{ ms}^{-2}$$

$$\omega = 7.292115 \times 10^{-5} \text{ s}^{-1}$$

The inevitable improvements in observation techniques and instrumentation (particularly with the advent of satellite methods) prompted a redefinition of the reference system in 1967, and again in 1980, when the IUGG agreed on the following [Chovitz, 1981]:

$$\begin{aligned} a &= 6378137 \text{ m} && (\pm 2 \text{ m}) \\ C_{2,0} &= 0.00108263 && (\pm 1 \times 10^{-8}) \\ GM &= 3.986005 \times 10^{14} \text{ m}^3 \text{ s}^{-2} && (\pm 4 \times 10^{7} \text{ m}^3 \text{ s}^{-2}) \\ \omega &= 7.292115 \times 10^{-5} \text{ s}^{-1} && (\pm 2 \times 10^{-12} \text{ s}^{-1}) \end{aligned}$$

Parenthetical values are standard deviations. This constitutes the Geodetic Reference System 1980; GM includes the mass of the atmosphere and $C_{2,0}$ excludes the permanent tidal deformation. The gravity formula based on these parameters is given by

$$\gamma = 9.780327\,(1 + 0.00530224 \sin^2 \phi - 0.000058 \sin^2 2\phi) \text{ms}^{-2}$$

where ϕ is the geodetic latitude; it yields normal gravity accurate to six digits. The method of computing the coefficients of this formula can be found in International Association of Geodesy (IAG), [1971].

24.2.2 Disturbing Gravity Field

The normal gravity, as given by the formula above, approximates the true gravity of the earth to about 5 parts in 10^5. This is hardly adequate for geodetic and modern geophysical and navigational needs, for which the variations in the gravity field at the 10^{-4} to 10^{-6} ms^{-2} (1 part in 10^5 to 10^7) level have a significant influence. In this respect, the direction of the gravity vector is as important as its magnitude.

For calculational convenience, the true gravity field is referenced to the normal field, the difference being the "disturbing" or "anomalous" field. Thus, the disturbing potential is the difference between the actual potential and the normal potential and the gravity disturbance is the difference between magnitudes of the true gravity and the normal gravity. The gravimetric (or absolute) deflection of the vertical is the angle between the true gravity vector and the normal gravity vector; it differs from the astrogeodetic deflection in that the perpendicular to the geodetic ellipsoid generally does not coincide with the direction of normal gravity, especially if the datum to which the astrogeodetic deflection refers is not geocentric. The geoid (the equipotential surface of the true gravity field that closely approximates mean sea level) is separated from the ellipsoid by the geoid undulation, or geoid height. And finally, the gravity anomaly is the difference between the earth's gravity magnitude on the geoid and the normal gravity magnitude on the ellipsoid, the two points being connected by the perpendicular to the ellipsoid. This definition assumes the absence of masses external to the geoid. In practice, the computation of geoidal gravity anomalies requires (through various techniques) a reduction of surface measurements to the geoid, thus accounting for the intervening masses. The gravity anomaly differs from the gravity disturbance in that the normal gravity is computed on the ellipsoid instead of the geoid, the latter computation being impossible if the undulation is not known.

Assuming that the geoid and reference ellipsoid enclose the same total mass and have identical centers of mass and surface potentials, the geoid undulation is directly proportional to the disturbing potential, according to Bruns's formula [Heiskanen and Moritz, 1967]. Its global average is then zero and the global root mean square (rms) value is approximately 30 m. The sources for the geoidal variations are primarily density inhomogeneities below the earth's crust. Table 24-5 summarizes the relative distribution of the sources for the variations in the geoid (also in gravity and its gradient) according to an earth model comprising various density layers at prescribed depths. Therefore, the major features of the geoid have a long-wavelength structure, and its large-scale variations are generally not correlated with the earth's topography (Figure 24-5). On the other hand, the short wavelength constituents of the disturbing potential (hence the geoid) exhibit a definite correlation with topography (Figure 24-6). In addition to its importance in geodetic positioning, the geoid undulation plays an essential role in oceanographic studies where a knowledge of deviations of the actual sea surface from the geoid contributes to the understanding of ocean surface circulation and ocean tides. The so-called sea surface topography has magnitudes of at most 1 to 2 m, and an undulation accuracy of about 10 cm at 100 km (or lower) resolution is required to definitely discern these features [NRC, 1979]. The satellite-borne altimeter is the complementary facet to the problem of determining the sea surface topography. It provides the ocean surface height above the reference ellipsoid (Section 24.3.4). From the satellites GEOS-3 and SEASAT-1 a new wealth of geoidal data was obtained over most of the earth's ocean surface yielding an oceanic geoid at or below the 1 m accuracy level (corrections for the sea surface topography were either modeled or calculated using global tidal models, or they were neglected).

The gravity anomaly, being essentially the derivative of the disturbing potential, is a more local phenomenon than the geoid undulation. Gravity anomalies are generated primarily by shallow density variations, that is, in the earth's crust. Considered over the entire earth, they have an rms value of about 40 to 45 mgal (1 mgal = 10^{-3} gal = 10^{-5} ms^{-2}). Classical formulas exist that relate global sets of gravity anomaly data to both the geoid undulation and the (gravimetric) deflection of the vertical (Stokes and Vening-Meinesz formulas). In the last decade new, alternative estimation techniques (least squares collocation) have been fully developed to provide speed, efficiency, and versatility to the problem of determining undulations and deflections from gravimetry. Aside from purely geodetic interests, gravity anomaly data have found wide application in geological studies ranging from the determination of geologic strata configurations, including isostatic compensation models, to the investigations of tectonic motions and mantle convective processes. The increase in the quality and quantity gravity measurements at sea provides an improved understanding of the origin, properties, and continuing evolution of the oceanic lithosphere. The accuracy currently required for these applications is generally less than 10 mgal with a resolution of about 100 km [NRC, 1979]. A detailed global map of gravity data has been constructed by the Defense Mapping Agency, Aerospace Center [DMAAC, 1973] which has compiled a 1° × 1° mean anomaly field from various sources, comprising a total of 27 441 (as of 1973) values and their estimated accuracies. However, gravity anomalies in unsurveyed areas, particularly in the southern hemi-

Table 24-5. Contributions of each density layer to variances of undulation, anomaly and gradient [Jordan, 1978].

Depth of Density Contrast	Undulation Model		Free-Air Gravity Anomaly Model		Vertical-Vertical Gravity Gradient Model	
[km]	rms Value [m]	Percent of Total Variance	rms Value [mgal]	Percent of Total variance	rms Value [E]	Percent of Total Variance
0.15	0.4	. . .	24.4	33	28.7	99
	3.2	1	28.5	44	3.2	1
	7.2	6	14.7	12	0.1	. . .
350	20.4	44	12.9	9	0.2	. . .
2880	21.5	49	5.4	2	. . .	. . .
	30.7†		42.7†		28.9†	

†Root-sum-square values

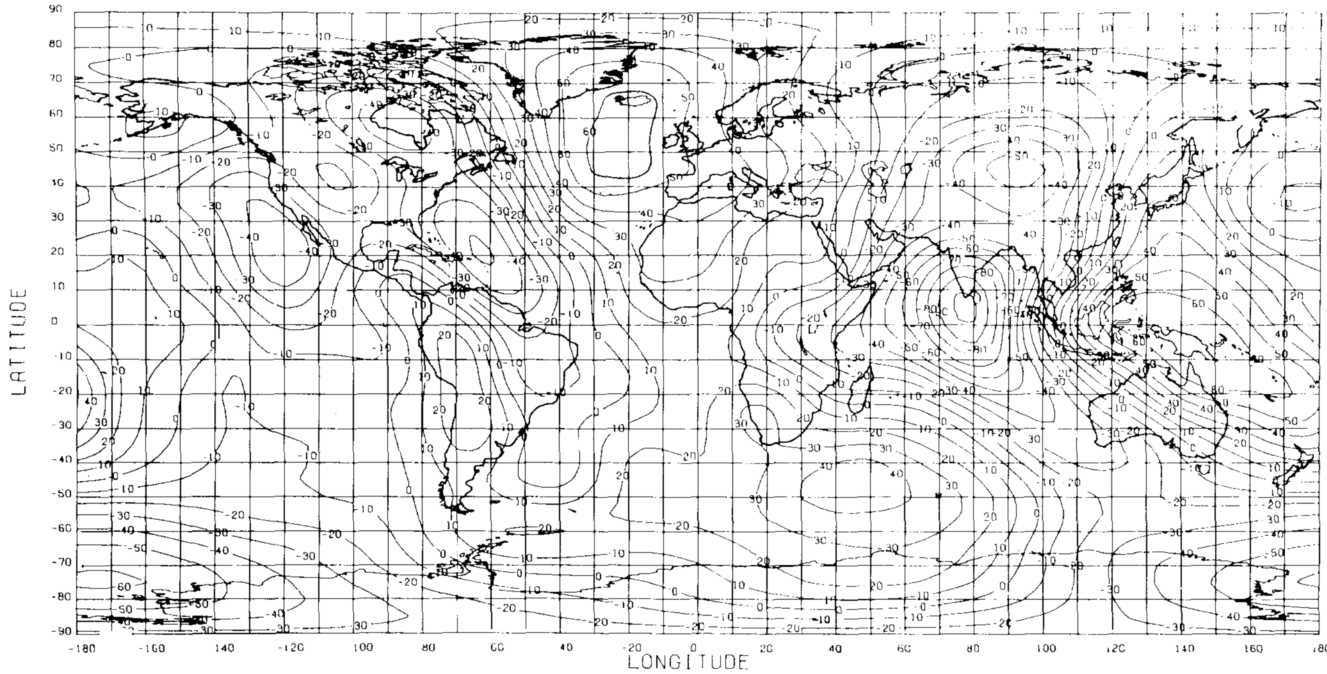

Figure 24-5. Long-wavelength geoid undulation contours (contour interval = 10 m) from (14,14) solution using weighted GEM 10 coefficients and altimetric data; reference flattening = 1./298.25 [Hadgigeorge, 1981].

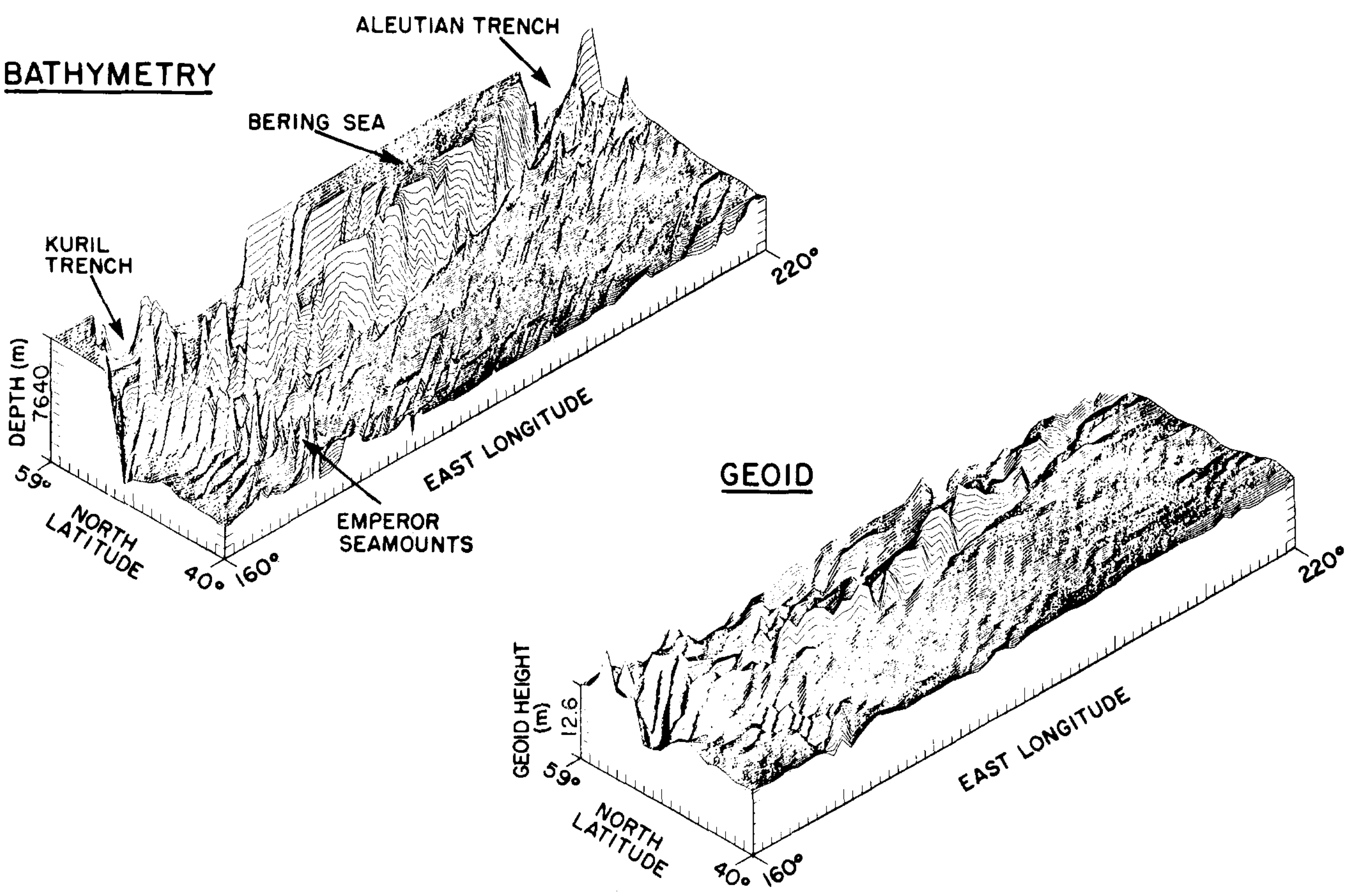

Figure 24-6. Short-wavelength geoid versus bathymetry.

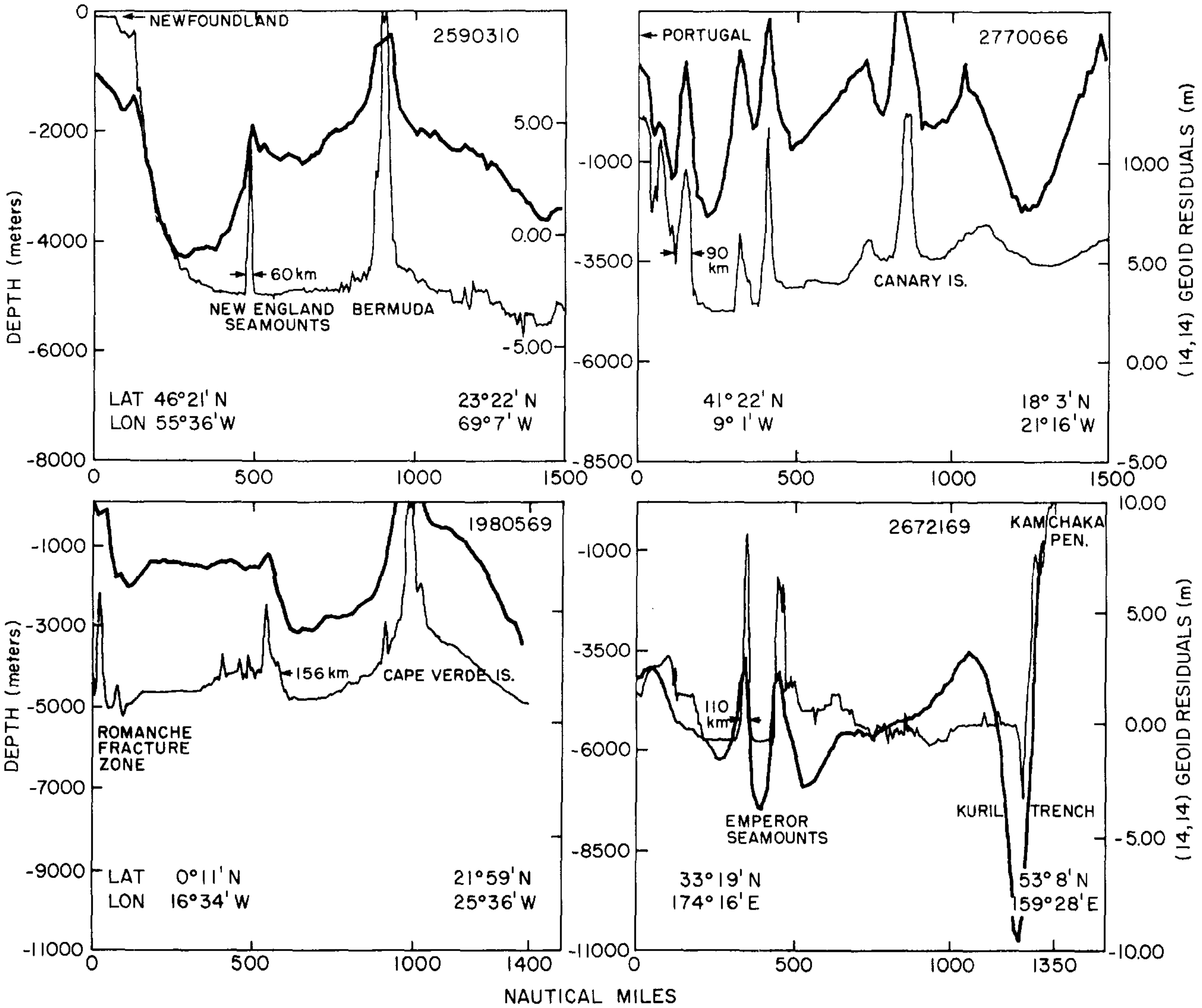

Figure 24-6. (*Continued*)

sphere, in inaccessible areas of the major continents, and in the polar regions, have a relatively large uncertainty since in these areas they are derived through special estimation techniques including the assumed correlation of gravity with geologic formations. The accuracies in the entire set range from a few mgal to tens of mgal.

The principal geodetic application of the gravimetric deflection of the vertical is in the reduction of horizontal control data (specifically directions) to the geocentric reference ellipsoid (absolute datum; Section 24.1.1). The deflection can be computed from gravimetric data to an accuracy of about 1″ to 3″, or better, depending on the density, accuracy, and extent of available data, the estimation technique, and the terrain [Lachapelle, 1978; Tscherning and Forsberg, 1978]. Since the vertical deflection defines the direction of the true gravity vector with respect to the normal gravity vector, due consideration of the deflection is required in any precise inertial navigation as well as positioning system where the platform is aligned with respect to the local vertical. Conversely, differences in the deflection of the vertical can be determined accurately (anticipated accuracy $< \pm 1''$) using inertial positioning data [Schwarz, K.-P., 1978].

For global descriptions of the earth's potential field, it is convenient to use an infinite series of spherical harmonic functions in which the coefficients, regarded as parameters of the field, have been determined by observing the perturbations of satellites in their orbits. For example, one such solution is the Goddard Earth Model 9 (GEM 9), complete to degree and order 20 [Lerch et al., 1979]. Additional terrestrial data were used, including altimetry, to develop GEM 10B, yielding coefficients complete to degree and order 36. With the near global and very dense coverage of altimetry data, solutions to degree and order 180 and higher have been developed [Rapp, 1979]. Not including inaccuracies in the determined coefficients, a rough estimate of the error in the corresponding geoid undulation models is given by the rule 64/n [meters], where n is the highest degree of the coefficients, and represents the neglect of higher-degree information (omission error). For example, the GEM 9 geoid undulations, apart from a formal standard error of about 1.9 m in the expansion to degree 20, contain an omission error of about 3 m.

24.2.3 Gravimetry

Gravity measurements are characterized as either relative or absolute. Absolute measurements result in the total magnitude of the gravity vector and involve the determination of length and time. For example, the acceleration due to gravity is determined from the length of a pendulum arm and its period of oscillation. With the best pendulum instruments and technique, this method gives a few tenths of a mgal accuracy [Torge, 1980]. In the modern, more sophisticated, and accurate methods, the apparatus consists of a proof mass which moves (falls) in the gravity field. The distance through which it moves is measured by interferometric methods (Michelson interferometer), while the corresponding time is determined electronically with an atomic frequency standard. The accuracy today is about 10 μgal (1 μgal = 10^{-3} mgal = 10^{-8} ms^{-2}) for laboratory instruments, but formal precisions of 1 to 3 μgal have been reported [Sakuma, 1976].

Because of the exacting demands in stability of the measurement environment, absolute determinations are not practical or economical for widespread surveys, although transportable instruments have been developed by various organizations [Hammond and Iliff, 1978]. Instead, with a much simpler apparatus and less time one can measure the relative gravity, that is, the change in gravity between stations. In principle, only one absolute site is necessary to fix the "origin" of an entire network of relative measurements. The measurement principle of the gravimeter can be stated as determining the displacement of a mass attached to a spring while the instrument is moved from one station to the next. The best achievable accuracy in measuring gravity differences with today's instruments is several μgal [Harrison and LaCoste, 1978] and depends primarily on correct calibration models.

In 1971 the International Gravity Standardization Network (IAG 71) was completed and adopted by the IUGG. It comprises gravity values at 1854 stations around the world; at ten stations absolute measurements were made. After a final adjustment of the entire network, the standard errors of the gravity values were estimated to be about 0.1 mgal. Further regional densifications and improvements are in progress using transportable absolute gravimeters.

24.2.4 Gradiometry

Gradiometry involves the measurement of gradients of components of the gravity vector, that is, the change in the gravity components with distance. The principle of measurement is based either on determining the difference in outputs of two accelerometers separated by a known distance, or sensing the ensuing torquing action on specific known proof mass configurations. Precise gradiometers have been in the development stage for many years, but only since 1982 have they begun to prove their viability in routine geodetic applications. The essential technological difficulties include eliminating external vibrations and thermal noise. Also, because the gradiometer senses primarily shortwavelength gravity variations, the determination of the total gravity field requires supplemental long-wavelength information.

The vertical gradient of normal gravity is about 0.31 mgal/m; however, of greater interest is the gradient of the gravity anomaly. This is an extremely local phenomenon, and it is unavailing to attempt to quantify it using a global rms value. Any component can vary from a few Eötvös (1 Eötvös = 1 E = 0.1 mgal/km = 10^{-9} s^{-2}) to several hundred or even a thousand Eötvös, depending on the terrain and the underlying mass anomalies.

Some development and analysis has been oriented toward a satellite-borne gradiometer to be used in a global gravity mapping mission; however, the realization of this concept lies some years in the future. A renewed effort is underway to develop the airborne gradiometer for the purpose of providing, in an efficient manner, a detailed survey of the gravity field in local areas. This device should be operational, attaining an expected precision of a few E, within the next years. In order to account for the varying effects of gravity, the gradiometer has also been proposed as a means of improving the accuracy of inertial navigation systems [Doukakis, 1979; Wells, 1981].

24.3 SATELLITE GEODESY

The two basic methods of tracking artificial earth satellites from earth stations are the dynamic method and the geometric method. The purpose of the dynamic method is to establish an ephemeris for the satellites; that is, to enable the prediction of the satellite's position in its orbit as a function of time. This entails solving (from worldwide tracking data, such as Doppler Ranging) for the parameters of the perturbing forces, such as the gravitational force, atmospheric drag, lunar-solar attraction, and solar radiation pressure of which the orbital elements are a function.

The perturbing forces cause secular and long- and short-periodic variations in the normal (elliptical) Keplerian orbit that would be described by the satellite if the earth were a point mass in an otherwise empty universe. Table 24-6 lists

Table 24-6. Disturbing forces on earth-orbiting satellite [Blitzer, 1970].

Source	Disturbing force per unit mass [m/s²] Altitude 150 km	Altitude 750 km	Altitude 1500 km	Synchronous Altitude
Central Gravity	9.35	7.85	6.42	0.22
Earth Oblateness (J_2)	30×10^{-3}	20×10^{-3}	14×10^{-3}	160×10^{-7}
(J_3)	0.09×10^{-3}	0.06×10^{-3}	0.04×10^{-3}	0.08×10^{-7}
(J_4)	0.07×10^{-3}	0.04×10^{-3}	0.02×10^{-3}	0.01×10^{-7}
Principal Tesseral Harmonic (J_{22})	0.09×10^{-3}	0.07×10^{-3}	0.04×10^{-3}	0.5×10^{-7}
Atmospheric Drag Vanguard I*	3×10^{-3}	10^{-7}	—	—
Echo I**	1300×10^{-3}	500×10^{-7}	—	—
Lunar-Solar Attraction ***	10×10^{-7}	10×10^{-7}	10×10^{-7}	70×10^{-7}
Solar Radiation Pressure Vanguard I*	10^{-7}	10^{-7}	10^{-7}	10^{-7}
Echo I**	500×10^{-7}	500×10^{-7}	500×10^{-7}	500×10^{-7}

*Area/Mass = 2.12×10^{-2} m²/kg
**Area/Mass = 10.2 m²/kg
***This is not the direct attraction but the effective disturbing force.

a coarse, comparative summary of typical values of the various perturbing effects (central gravity is the force per unit mass due to the earth considered as a point mass—it is not a perturbing element). J_2, J_3, J_4, and $J_{2,2}$ are coefficients of the expansion into spherical harmonic functions of the earth's gravitational potential. Particularly noteworthy is the importance of the area-to-mass ratio of a satellite with respect to atmospheric drag and solar radiation pressure, the latter predominating with increasing altitude. The overall design of a satellite orbit is therefore contingent on the purpose of the satellite mission. High orbits of small, but massive, satellites (such as LAGEOS) are designed for station positioning or for the determination of polar motion and variation of earth rotation, while low orbits are required to detect variations in the earth's gravitational field or to study other specific perturbing effects such as atmospheric drag.

With the geometric method, the tracking data are obtained simultaneously from several stations and used to determine relative station positions (satellite triangulation); accurate knowledge of the satellite's orbit is not required. Various intermediate methods exist; for example, the short arc method, where observations are generally limited in duration (approximately 1/4 orbit), and either a fixed (non-perturbed) or computed orbit is used. For this method, the relative station positions become a part of the set of unknowns to be determined. In point positioning, observations from a single station determine its position in a coordinate system defined by the satellite ephemeris.

24.3.1 Doppler Positioning

Of the several new techniques of observing satellites, the Doppler method has been the most successful in achieving worldwide and routine application. By measuring the change in frequency (corresponding to the Doppler shift) of a signal transmitted from a satellite with respect to a reference frequency on the ground, one can deduce the range and range rate to the satellite. Usually two signals at different frequencies are transmitted simultaneously to essentially eliminate the frequency dependent effect of the ionosphere on the propagation of the signal. Most satellites now transmit in the hundreds of MHz, but to further reduce the ionospheric effect this is increased (for example, for NAVSTAR) into the GHz range.

A system of six satellites, the Navy Navigation Satellite System (NNSS) in operation since 1971, has provided a global positioning capability within two hours of observation. These satellites transmit a "broadcast ephemeris," which provides the observer with the satellite's position as a function of time. This information is updated and loaded into the satellite's memory once or twice a day and is used for real-time positioning. A "precise ephemeris" is available for more accurate (but delayed in time) position fixes. The three-dimensional spatial position and the time synchronization offset are obtained by the simultaneous observation of four satellites with a corresponding positional accuracy of about 25 m, sufficient for some, but certainly not all, navigational requirements. With longer periods of observation (six to eight days), combining data from many passes, and using the precise ephemeris, accuracies below the 1 m level can be obtained [Brown, 1979]. Even greater accuracy comes from the use of the short arc method to deduce relative positions (baseline accuracy of 20 to 30 cm). The NNSS is to be superseded by the NAVSTAR/GPS (Global Positioning System) in the late 1980s. This system will consist of 18 satellites, 6 each in 3 orbital planes, providing a four-satellite configuration observable anywhere on the earth at almost all times. The accuracy for point positioning using one hour's observations and the broadcast ephemeris is an estimated 10 m [Fell, 1980], while observations spanning five days encompassing many passes in conjunction with the precise ephemeris yield geodetically useful accuracies of about 50 cm. The main source of error is the uncertainty in the satellite's position owing, in part, to an inadequate knowledge of the earth's external gravitational field. Baseline determinations of 300 km or less in length, for which inaccuracies in the determination of the satellite's orbit are less detrimental, are anticipated to be accurate to less than 10 to 20 cm, approaching the 1 cm level with improved tropospheric error modeling [Anderle, 1980].

The Doppler principle also has applications in space, providing the means for one satellite to track another either in a high-low configuration (the high [geosynchronous] satellite tracks the low satellite) or a low-low configuration where one satellite follows the other in the same orbit. The analysis of satellite-to-satellite tracking has thus far been oriented toward determinations of the gravity field. The basic measurement is the relative velocity between the two satellites which is indicative of the difference in gravitational potentials [Rapp and Hajela, 1979].

24.3.2 Satellite Interferometry

Radio interferometry using satellite signals is one of the newest techniques to make geodetic linear measurements. The length of baseline vectors between pairs of survey marks ranges from 0.1 to 200 km (short-baseline interferometry) and is determined by radio-interferometric observations of the signals broadcast by satellites of the GPS. The signals are much stronger than the quasar sources used in VLBI (Section 24.4.2). Errors can be reduced to the 1 cm level, or less, by properly accounting for the tropospheric effects [Rogers et al., 1978; Counselman and Shapiro, 1979; MacDoran, 1979; Anderle, 1981].

24.3.3 Laser Ranging

Just emerging from the exploratory and experimental stage, satellite laser ranging promises to become a valuable asset for geodetic applications. Basically, the ranges are deduced from the measured travel time of laser pulses. The overriding advantage of the laser is its tremendous accuracy, achieved by utilizing shortpulsed (~ 1 ns) lasers to enhance the definability of the returning pulse. Also, the satellite need only be equipped with retroreflectors, while for Doppler ranging, an on-board power supply is required. In contrast to Doppler ranging, however, laser ranging is weather-dependent. Ranging data to the satellite LAGEOS have been used to determine baseline lengths; for example, Kolenkiewicz and Ryan [1981] (using observations from a three month period) report a 6 cm agreement with the VLBI determined length of a 3930 km baseline connecting Westford, Mass. and Owens Valley, Calif. The analysis of LAGEOS data has also yielded determinations of polar motion and length-of-day variations with respective formal accuracies of about 0.″01 and 0.3 ms [Tapley, 1982]. As with all transmissions through the atmosphere, the random effects of the troposphere and ionosphere limit the achievable accuracy. Improvements in laser ranging will result from better atmospheric density models and by upgrading the basic measurement (travel-time of pulse) accuracy.

24.3.4 Satellite Altimetry

The satellite borne radar altimeter has had an exceptional impact on the continuing efforts to improve our knowledge of the earth's gravity field. The distance of the satellite above the ocean surface is obtained simply by measuring the travel time of a reflected radar pulse. This distance and the given position of the satellite (its orbit must be either known or solved for) in a geocentric coordinate system yield directly the distance of the ocean surface from the geocenter, or equivalently, from the reference ellipsoid (sea surface height). That is, one obtains an estimate for the geoid undulation if the sea surface topography (at most 1 or 2 cm) is neglected. The altimeter measuring accuracy was better than 1 m for the GEOS-3 satellite and its orbital position was obtained to within a few meters. Imposing the constraint of equal altimeter heights at track crossings (track crossing adjustments) and accounting for atmospheric and sea surface conditions, the GEOS-3 geoid could be determined to an optimal accuracy of 60 cm to 1 m.

The subsequent (short-lived) satellite SEASAT-1 carried an altimeter emitting a shorter pulse at a more rapid rate with a consequent 5 to 10 cm sensitivity. These data are currently being processed with expected accuracies of about 20 cm or better on a regional basis [Marsh et al., 1981; Anderle, 1980]. Figure 24-7 shows the oceanic geoid recovered from 1° × 1° mean SEASAT geoid heights. See

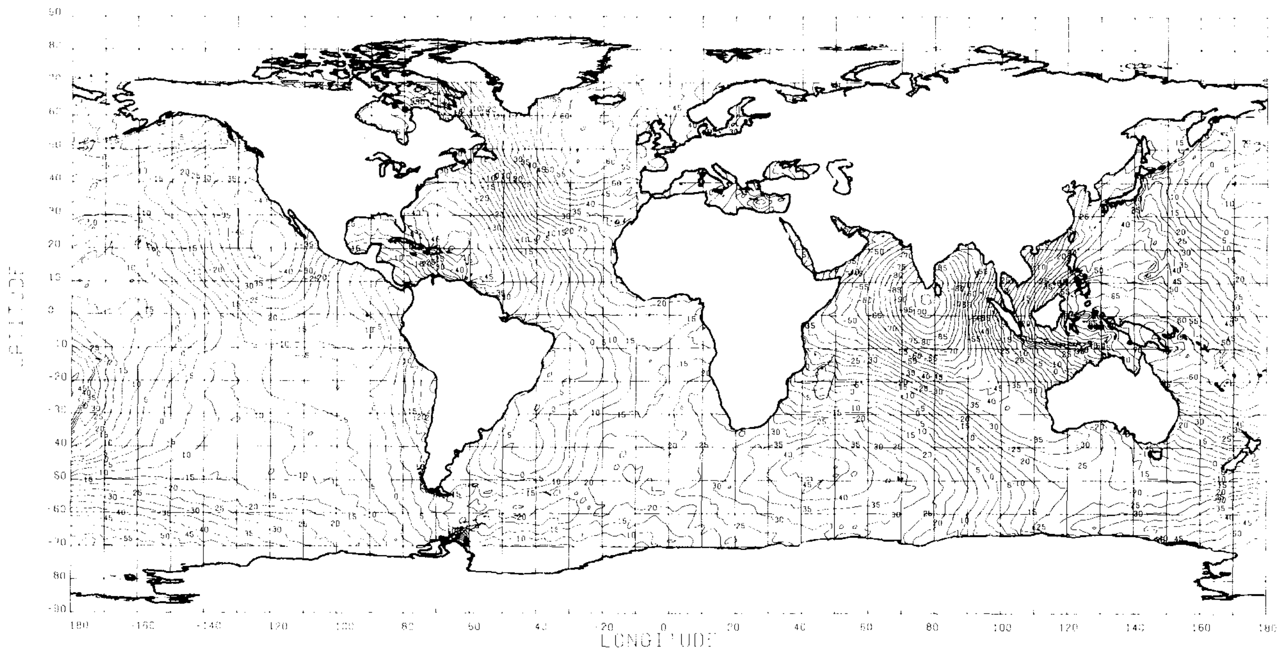

Figure 24-7. Global Seasat 1° × 1° observed geoid (contour interval = 5m); reference flattening = 1/298.25.

also Rooney et al. [1980]. The satellite GEOSAT carrying a similar altimeter is to be launched by DoD in 1985.

One of the ultimate objectives of satellite altimetry is the capability to determine and monitor ocean circulation. In this case, the sea surface topography is the object of measurement. This requires prior knowledge of the geoid, unless only temporal changes in sea surface topography are to be ascertained. The proposed five-year NASA satellite mission for this project is TOPEX; its anticipated error budget is given in Table 24-7.

Table 24-7. TOPEX Error Budget [extracted from NASA, 1981].

Source of Error	Cause of Error	Uncorrected Error [cm]	Corrected Error [cm]	Wavelength [km]
Altimeter	Altimeter noise		1.5	20
Altimeter	Ocean waves	4	1.0	1000
Troposphere	Mass of air	240	0.7	1000
Troposphere	Water vapor	20	1.0	50–500
Troposphere	Rain	—	—	
Ionosphere	Free electrons	10	0.5	50–10 000
Orbital error	Gravity	5 (km)	0.7	10 000
Orbital error	Drag	5	3.6	10 000
Orbital error	Solar radiation	30	7.0	10 000
Orbital error	Earth radiation	3	1.0	10 000
Orbital error	Station location	10	3.0	10 000
Orbital error	Timing		0.2	10 000
Sea level	Weight of air	50	3.0	200–1000
Sea level	Geoid	100 (m)	1.5	200

The table summarizes the errors in satellite measurements of ocean surface topography. This brief summary assumes (a) a dense spherical satellite orbiting at a height of 1300 km tracked by a realistic network of laser stations, (b) accurate measurements of earth's gravity from the GRAVSAT mission, (c) additional data from a dual-frequency radiometer and dual-frequency altimeter on the spacecraft, (d) surface pressure with an accuracy of 3 millibars from global weather charts, (e) a spacecraft clock having an accuracy of 100 μs, (f) an average wave height on the sea surface of two meters and a wave skewness of less than 0.1, and (g) no data collected in heavy rain. The corrected error is the error in measurements made along a single satellite pass crossing the ocean basin. We expect that long-term averages of many passes will substantially reduce the random error.

24.4 GEODETIC ASTRONOMY

The position of the earth's rotational axis varies in time with respect to inertial space, as well as with respect to the earth's crust. The variations in inertial space are usually termed nutations, specifically, either forced or force-free nutations, depending on whether or not they are generated by the gravitational attraction of an extra-terrestrial disturbing body such as the sun or moon. Similarly, the variation of the spin axis with respect to the earth's crust, called polar motion, is either forced of force-free motion. The mechanisms for polar motion are generally of a geophysical nature, such as the elasticity of the mantle and the liquidity of the outer core. The force-free polar motion of the instantaneous rotation axis is also known as wobble; the predominant variation is called the Chandler wobble and has a period of about 435 days. If the rotational axis serves as the third axis in both an earth-fixed coordinate system, as well as in the celestial (inertial) system, then nutations change the declinations of stars and polar motion affects the latitudes of terrestrial stations. This permits the observational separability of the two types of variations in the spin axis. Tables 24-8 and 24-9 list the amplitudes, periods, and causes of the various motions. With the increase in accuracy of astronomical observations, the earth cannot be considered a rigid body. One must account for the effects on the motions of the rotational axis that are attributable to the earth's elasticity, liquid core, redistributions of internal mass, and plate tectonics.

The variations of latitude (polar motion) are monitored by the International Polar Motion Service (IPMS) (formerly the International Latitude Service (ILS)) and the Bureau

Table 24-8. Spectrum of changes in earth's rotation [Rochester, 1973].

A. Inertial Orientation of Spin Axis	B. Terrestrial Orientation of Spin Axis (Polar Motion)	C. Instantaneous Spin Rate ω about Axis
1. Steady precession: amplitude $23^{\circ}.5$; period $\simeq$ 25 700 years.	1. Secular motion of pole: irregular $\simeq 0''.2$ in 70 years.	1. Secular acceleration: $\dot{\omega}/\omega \simeq -5 \times 10^{-10}$/yr.
2. Principal nutation: amplitude $9''.20$ (obliquity); period 18.6 years.	2. 'Markowitz' wobble: amplitude $\simeq 0''.02$(?); period 24–40 years(?).	2. Irregular changes: (a) over centuries $\dot{\omega}/\omega \leqslant \pm 5 \times 10^{-10}$/yr; (b) over 1–10 years $\dot{\omega}/\omega \leqslant \pm 80 \times 10^{-10}$/yr; (c) over a few weeks or months ('abrupt'), $\dot{\omega}/\omega \leqslant \pm 500 \times 10^{-10}$/yr.
3. Other periodic contributions to nutation in obliquity and longitude: amplitude $<1''$; periods 9.3 years, annual, semiannual, and fortnightly.	3. Chandler wobble: amplitude (variable) $\simeq 0''.15$; period 425–440 days; damping time 10–70 years(?).	
4. Discrepancy in secular decrease in obliquity: $0''.1$/century(?).	4. Seasonal wobbles: annual, amplitude $\simeq 0''.09$; semiannual, amplitude $\simeq 0''.01$.	3. Short-period variations: (a) biennial, amplitude $\simeq$ 9 ms; (b) annual, amplitude $\simeq$ 20–2 ms; (c) semiannual, amplitude $\simeq$ 9 ms; (d) monthly and fortnightly, amplitudes $\simeq$ 1 ms.
	5. Monthly and fortnightly wobbles: (theoretical) amplitudes $\simeq 0''.001$.	
	6. Nearly diurnal free wobble: amplitude $\leqslant 0''.02$(?); period(s) within a few minutes of a sidereal day.	
	7. Oppolzer terms: amplitudes $\simeq 0''.02$; periods as for nutations.	

Table 24-9. Mechanisms with effects now distinguishable on the earth's rotation [after Rochester, 1973].

Mechanism	Effect*
Sun	A,B7,C1,C3c
Gravitational torque	C2c(?)
Solar wind torque	
Moon	A,B7,C1,C3c-d
Gravitational torque	
Mantle	B1,B3-4,C1-2a,C3c-d
Elasticity	B1,B3
Earthquakes	B3(?),C1
Solid friction	C2a
Viscosity	
Liquid core	A2-3,B2,B6
Inertial coupling	C2b-c(?)
Topographic coupling	A4(?),B3,C2
Electromagnetic coupling	
Solid inner core	B2(?)
Inertial coupling	
Oceans	B1,B3,B5,C2a
Loading and inertia	B3(?),C1
Friction	
Groundwater	B4
Loading and inertia	
Atmosphere	B4
Loading and inertia	Cd2c,C3a-c
Wind stress	C1
Atmospheric tide	

*Numbers refer to Table 24-8.

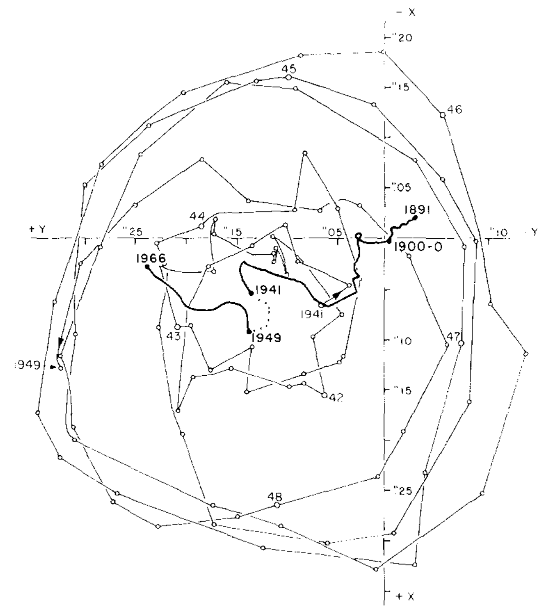

Figure 24-8. Polar wander: 1891–1966
Polar wobble: 1941–1949

International de l'Heure (BIH). These organizations coordinate the observations obtained throughout the world using classical methods (for example, zenith telescopes), as well as implementing the satellite Doppler, VLBI, LLR, etc., techniques. The results are published annually by both organizations.

Figure 24-8 shows the motion of the pole of rotation over a period of 8 years, as well as the motion (polar wander) of the average pole over a period of 75 years. The coordinate system with respect to which polar motion is measured is centered at the Conventional International Origin (CIO). This is the mean position of the pole of 1903 as defined by the latitudes of the five observatories of the ILS. Figure 24-9 shows the most recent motions (1978–1980) of the pole as determined by the BIH from worldwide observations. The origin in this case is defined by the BIH on the basis of the latitudes of all contributing observatories.

Time determination, coordination and dissemination are, on an international scale, three of the principal functions of the BIH. Because of the excellent long-term stability of atomic frequency standards (1 part in 10^{13}), atomic time (AT) has virtually replaced ephemeris time (ET) as the basic time scale. ET refers to the time variable in the theory of motion of the sun, moon, and planets and is less accurate and more difficult to determine through observations [Winkler and VanFlandern, 1977]. Several observatories around the world maintain their own atomic time; in the U.S. it is maintained by the U.S. Naval Observatory (USNO) and the National Bureau of Standards (NBS). These are pooled and intercompared by the BIH resulting in International Atomic

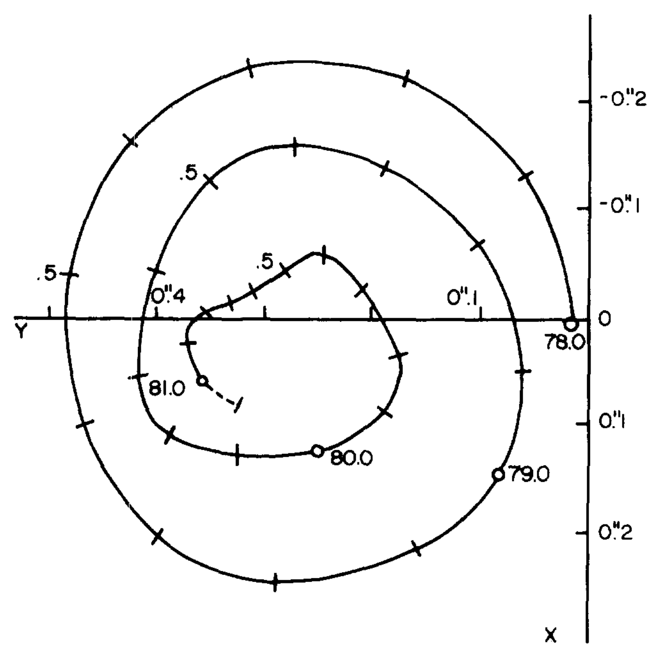

Figure 24-9. Path of the pole from 1978.0 to 1981.0 [BIH, 1981, p. B-50].

Table 24-10. Difference between the earth rotation parameters by various services [extracted from Guinot, 1981].

Services	Interval	A_x	B_x	A_y	B_y
ILS-BIH	62–78	36 ± 4	−0.2 ± 0.9	−2 ± 3	1.0 ± 0.5
	67–78	39 ± 4	2.3 ± 1.3	−4 ± 3	−0.2 ± 0.8
IPMS-BIH	62–78	29 ± 2	0.3 ± 0.3	2 ± 2	−1.1 ± 0.5
	67–78	29 ± 1	0.0 ± 0.4	1 ± 3	−0.4 ± 0.7
DMA-BIH	72–79	−15 ± 2	−1.6 ± 0.6	−2 ± 3	1.6 ± 1.1

Service—BIH = A + B (T − 1975.0)
Units are: 0."0001 for x and y.
The variations B are yearly rates.

Time (TAI). Coordinated Universal Time (UTC) is based on the same atomic time scale as AT, but is offset occasionally, by so-called leap seconds, to maintain a close relation to the actual spin rate of the earth, called UT1. UT1 is Universal Time referring to the CIO, while UTO refers to the rotational rate of the instantaneous rotation axis. Several observatories throughout the world determine UTO through astronomical observations. The longitudes of these stations are used to define the Mean Greenwich Meridian which together with the CIO provides the orientation of the average terrestrial coordinate system. Table 24-10 compares the BIH determinations of polar motion (x and y components) with the determinations of several other organizations, IPMS, ILS, and DMA which used satellite Doppler observations.

24.4.1 Lunar Laser Ranging

Retroreflectors were placed on the moon by the Apollo 11, 14, and 15 and the Soviet Luna 17 and 21 missions. Ranging to all of these reflectors began shortly after each mission, and is still continuing, primarily and regularly at the McDonald Observatory, Fort Davis, Texas. The ranging is done with high energy pulsed lasers fired through large aperture telescopes which also serve as the receiving optics for the reflected return images. Photo-electric devices record the photon receptions against a precise time record. The resulting accurate linear measurements (±15 cm) between the observing site and the reflectors on the lunar surface are well suited to studies of lunar rotation as well as irregularities in the earth's rate of rotation (determination of UTO) and polar motion (however, the determination of the polar motion component perpendicular to the meridian requires more than one observing station). Accuracies in the time determination of about 0.5 to 0.7 ms have been stated [McCarthy, 1979], but compared to other determinations, such as by the BIH, the root mean square difference can be 1 to 2 ms [Langley et al., 1981].

24.4.2 Very Long Baseline Interferometry

The primary observable in VLBI for geodetic and astrometric studies is the measured time interval between the arrival of a radio signal, from some extragalactic source such as a quasar, at one end of the interferometer and its arrival at the other end. This interval is called the delay and its derivative is the delay rate. From a sufficient set of these data as functions of time, the geometry of the interferometer baseline and the position of the observed radio sources can be determined. Figure 24-10 shows, in simplified form, a typical conventional interferometer with two antenna-receiver systems. At each antenna, the radio frequency signal from the observed source is converted to a lower "intermediate" frequency by mixing with a local oscillator (LO) signal. The LO signals are supplied to the mixers at both antennas independently, thus allowing large separations between the receiving stations. The resulting signals at each

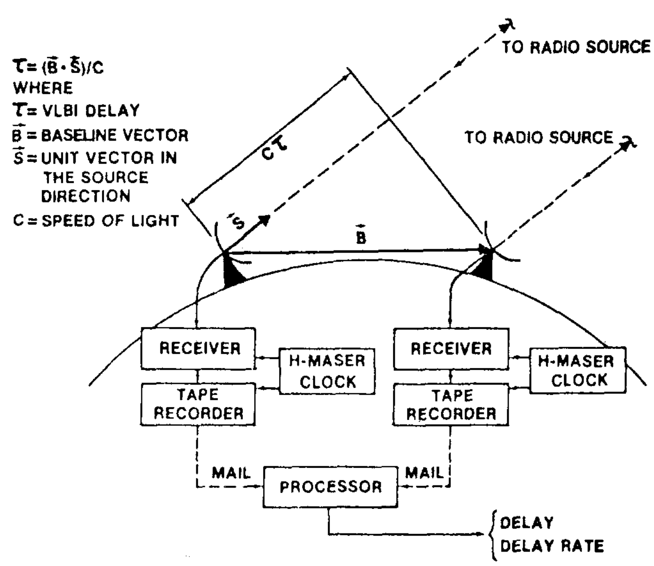

Figure 24-10. Very Long Baseline Interferometry (VLBI) observation geometry [Ryan et al., 1978].

station are recorded and time-tagged on magnetic tape. The two tapes are later cross-correlated to obtain the phase delay, which translates into time delay apart from the indeterminable multiple of $2\pi\lambda$, where λ is the wavelength of the signal. This ambiguity is resolved by combining data at several frequencies if the source has a polychromatic spectrum (bandwidth synthesis technique) [Rogers, 1970].

The state-of-the-art VLBI system incorporates the Mark III recorder, developed by the Northeast Radio Observatory Corporation (Haystack Observatory), which has an increased frequency bandwidth recording capability compared to earlier devices, and thus decreases the effect of random errors. Another parameter affecting system noise is the efficiency of collecting the signal; this depends on the size and accuracy of the antenna surface, as well as the signal strength. Transportable systems require smaller antennas and the resulting degradation in accuracy can be compensated for by increasing the recording rates. Other errors are caused by clock instabilities; inaccurate definition, location, and lack of strength of the extragalactic signal; random atmospheric disturbances; and imperfect (or lack of) modeling of geophysical effects, such as solid earth tides, crustal motion, and earth rotation. Precisely because these latter contribute to the error budget of VLBI (that is, instrumental and other environmental factors do not predominate), the continual monitoring of these geodynamic phenomena with VLBI will improve the associated models. For example, a formal precision of $0\overset{''}{.}002$ in polar motion components and 0.1 ms in UT1 was reported by Ma [1981] using 14 days of observations. The published proceedings of a VLBI conference held at the Massachusetts Institute of Technology, Cambridge, Mass., in 1979 [NASA, 1980] provide a comprehensive documentation of the technology, geodetic applications, and future capabilities of VLBI.

REFERENCES

Anderle, R.J., "Advanced Satellite Systems and Their Applications," *Manuscripta Geodaetica,* **5**:257–281, 1980.

Anderle, R.J., "Applications of NAVSTAR GPS Geodetic Receiver to Geodesy and Geophysics," *Am. Geophys.,* **37**:37–46, 1981.

Blitzer, L., *Handbook of Orbital Perturbations,* TRW Systems Group, Redono Beach, Calif, 1970.

Bose, S.C. and J.R. Huddle, "Regional Adjustment of Inertial Gravity Disturbance Vector Measurements by Optimal Two-Dimensional Smoothing," *Bull. Geodesique,* **55**:340–353, 1981.

Bomford, G., *Geodesy,* Clarendon Press, Oxford, 1971.

Bossler, J.D., "Status of the New Adjustment in the United States," in *Proceedings Second International Symposium on Problems Related to the Redefinition of North American Geodetic Networks,* Arlington, Va, April 24–28, 1978. (avail. from NTIS as PB 299048/4)

Brown, D.C., "Positioning by Satellites," *Rev. Geophys.* **17**:1424–1430, 1979.

Bureau Internationale de l'Heure, *Annual Report for 1980,* Paris, 1981.

Castle, R.O. and P. Vanicek, "Interdisciplinary Considerations in the Foundation of the New North American Vertical Datum," in *Proceedings Second International Symposium on Problems Related to the Redefinition of North American Vertical Geodetic Networks,* Ottawa, Canada, May 26–30, 1980, Canadian Institute of Surveying, Ottawa, 1980.

Chovitz, B.H., "Modern Geodetic Earth Reference Models," *Eos,* Trans. AGU, **62** (7), Feb. 17, 1981.

Counselman, C.C. and I.I. Shapiro, "Miniature Interferometer Terminals for Earth Surveying," *Bull. Geodesique,* **53**:139–163, 1979.

DMAAC, "1° × 1° Mean Free-Air Gravity Anomalies," Reference Publication 73-0002, DMAAC, St. Louis, Mo, Dec. 1973.

Doukakis, E., "Evaluation of the Performance of Gradiometer-Assisted Inertial Navigation Systems," Technical Translation DMAAC-TC-2892K, DMAAC, St. Louis, Mo, 1982.

Fell, P.J., "Geodetic Positioning Using a Global Positioning System of Satellites," Report 299, Department of Geodetic Science, Ohio State University, 1980.

Groten, E., *Geodesy and the Earth's Gravity Field,* F. Dummler, Bonn, 1979.

Guinot, B., "Comments on the Terrestrial Pole of Reference, the Origin of Longitudes, and on the Definition of UT1," in *Reference Coordinate Systems for Earth Dynamics,* edited by E.M. Gaposchkin and B. Kolaczek, D. Reidel, Dordrecht, Holland, 1981.

Hadgigeorge, G., "Seasat Altimeter Reductions for Detailed Determinations of the Oceanic Geoid," *Ann. Geophys.,* **37**:123–132, 1981.

Hammond, J.A. and R.L. Iliff, "The AFGL Absolute Gravity Program," in *Proceedings Ninth Geodesy/ Solid Earth and Ocean Physics (GEOP) Research Conference,* Columbus, Ohio, Oct. 2–5, 1978, Report 280, Department of Geodetic Science and Surveying, Ohio State University, 1978.

Harrison, J.C. and L.J.B. LaCoste, "The Measurement of Surface Gravity," in *Proceedings Ninth Geodesy/Solid Earth and Ocean Physics (GEOP) Research Conference,* Columbus, Ohio, Oct. 2–5, 1978, Report 280 Department of Geodetic Science, Ohio State University, 1978.

Heiskanen, W.A. and H. Moritz, *Physical Geodesy,* Freeman, San Francisco, 1967.

Helmert, R.F., *Die Mathematischen und Physikalischen Theorien der Hoheren Geodesie, Vols. 1 and 2,* Teubner, Leipzig, Germany, 1884.

I.A.G., "Geodetic Reference System 1967," Special Publication No. 3, International Association of Geodesy, Paris, 1971.

I.A.G., "The International Gravity Standardization Net 1971," Special Publication No. 4, Bureau Central de l'Association Internationale de Geodesie, Paris, 1971.

Jordan, S.K., "Statistical Model for Gravity, Topography, and Density Contrasts in the Earth," *J. Geophys Res,* **83**:1816–1824, April 10, 1978.

Kaula, W.M., *Theory of Satellite Geodesy,* Blaisdell, London, 1966.

Kolenkiewicz, R. and J.W. Ryan, "Ten Lageos Laser Determined Continental United States Baselines with some VLBI Comparisons," paper presented at the Annual Spring Meeting of the American Geophysical Union, Baltimore, May 25–29, 1981.

Lachapelle, G., "Estimation of Geoid Deflection Components in Canada," in *Proceedings Second Symposium on Problems Related to the Redefinition of North American Geodetic Networks,* Arlington, Va, April 24–28, 1978. (avail. from NTIS as PB 2990484/4)

Langley, R.B., R.W. King, and I.I. Shapiro, "Earth Rotation from Lunar Laser Ranging," *J. Geophys. Res.,* **86**:11913–11918, 1981.

Leick, A., "The Observability of the Celestial Pole and Its Nutation," Report 262, Department of Geodetic Science, Ohio State University, 1978.

Lerch, F.J., S.M. Klosko, R.E. Laubscher, and C.A. Wagner, "Gravity Model Improvement Using GEOS-3 (GEM 9 and 10)," *J. Geophys. Res.,* **84**:3897–3916, 1979.

Ma, C., "Geodesy by Radio Interferometry: Polar Motion and UT1 from MERIT," paper presented at the Annual Spring Meeting of the American Geophysical Union, Baltimore, May 25–29, 1981.

MacDoran, P.F., "Satellite Emission Radio Interferometric Earth Surveying, Series — GPS Geodetic System," *Bull. Geodesique,* **53:** 117–138, 1979.

Manual of Photogrammetry, 4th Edition, edited by C.L. Slama, American Society of Photogrammetry, Falls Church, Va., 1980.

Marsh, J.G., R.E. Cheney, V. Grano, and T.V. Martin, "Mean Sea Surface Computations Based Upon Seasat Altimeter Data," paper presented at the Annual Spring Meeting of the American Geophysical Union, Baltimore, May 25–29, 1981.

Mather, R.S., "Four Dimensional Studies in Earth Space," *Bull. Geodesique*, **108:**187–209, 1973.

McCarthy, D.D., "Polar Motion and Earth Rotation," *Rev. Geophys. Space Phys.* **17:**1397–1403, 1979.

Moritz, H., *Advanced Physical Geodesy,* Abacus Press, Kent, UK; Heyden, Philadelphia, 1980.

Mueller, I.I., *Spherical and Practical Astronomy,* Ungar, New York, 1969.

Mueller, I.I., "Tracking Station Positioning from Artificial Satellite Observations," *Geophys. Surveys,* **2:**243–276, 1975.

Mueller, I.I. and K.H. Ramsayer, *Introduction to Surveying,* Ungar, New York, 1979.

NASA, *NASA Directory of Observation Station Locations,* vol.1, 3rd edition, Goddard Space Flight Center, Greenbelt, Md, Nov. 1973.

NASA, "National Geodetic Satellite Program," Special Publication 365, parts I and II, Washington, D.C., 1977.

NASA, "Radio Interferometry Techniques for Geodesy," Proceedings of a conference held at Massachusetts Institute of Technology, Mass., and at the Northeast Radio Observatory, Westford, Mass., June 19–21, 1979, NASA Conference Publication 2115, 1980.

NASA, "Satellite Altimetric Measurements of the Ocean," Report of the TOPEX Science Working Group, Jet Propulsion Laboratory, Pasadena, 1981.

NOAA, "Classifications, Standards of Accuracy, and General Specifications of Geodetic Control Surveys," Federal Geodetic Control Committee, NOS/NOAA, U.S. Department of Commerce, Rockville, Md, 1980a.

NOAA, "Specifications to Support Classifications, Standards of Accuracy, and General Specifications of Geodetic Control Surveys," Federal Geodetic Control Committee, NOS/NOAA, U.S. Department of Commerce, Rockville, Md, 1980b.

National Research Council, *Geodesy: Trends and Prospects*, National Academy of Sciences, Washington, D.C., 1978.

National Research Council, *Applications of a Dedicated Gravitational Satellite Mission*, National Academy of Sciences, Washington, D.C., 1979.

Rapp, R.J., "Potential Coefficient and Anomaly Degree Variance Modelling Revisited," Report 293, Department of Geodetic Science, Ohio State University, 1979.

Rapp, R.H. and D.P. Hajela, "Accuracy Estimates of 1° × 1° Mean Anomaly Determinations from a High-Low SST Mission," Report 295, Department of Geodetic Science, Ohio State University, 1979.

Rochester, M.G., "The Earth's Rotation," *Eos,* Trans. AGU, **54:**763–780, Aug. 1973.

Rogers, A.E.E., "Very-Long-Baseline-Interferometry with Large Effective Bandwidth for Phase Delay Measurements," *Radio Sci.* **5:**1239–1243, 1970.

Rogers, A.E.E., C.A. Knight, H.F. Hinteregger, A.R. Whitney, C.C. Counselman III, I.I. Shapiro, S.A. Guorevitch, and T.A. Clark, "Geodesy by Radio Interferometry: Determination of a 1.24-km Base Line Vector With ~5-mm Repeatability," *J. Geophys. Res.*, **83:**325–334, 1978.

Rooney, T., G. Hadgigeorge, and G. Blaha, "Analysis of Residuals from Short-Arc Adjustment of Seasat Altimetry," paper presented at the Fall Meeting of the American Geophysical Union, San Francisco, Dec. 8–12, 1980.

Ryan, W.J., T.A. Clark, R. Coates, B.E. Corey, W.D. Cotton, C.C. Counselman, H.F. Hinteregger, C.A. Knight, C. Ma, D.S. Robertson, A.E.E. Rogers, I.I. Shapiro, A.R. Whitney, and J.J. Wittels, "Precision Surveying Using Radio Interferometry," *J. Surveying and Mapping Division* (Proc. Am. Soc. Civil Engineers), **104,** (SU1): 25–34, Nov. 1978.

Sakuma, A., "Absolute Measurements of Gravity," Report of Special Study Group 3.18, *Travaux de l'Association Internationale de Geodesie*, Paris, **25:**141–144, 1976.

Schwarz, C.R. "Deflection Computations for Network Adjustment in the United States," in *Proceedings Second International Symposium on Problems Related to the Redefinition of North American Geodetic Networks,* Arlington, Va, April 24–28, 1978.

Schwarz, K.-P., "Accuracy of Vertical Deflection Determination by Present-Day Inertial Instrumentation," in *Proceedings Ninth Geodesy/Solid Earth and Ocean Physics (GEOP 9) Research Conference,* Columbus, Ohio, Oct. 2–5, 1978, Report 280, Department of Geodetic Science, Ohio State University, 1978. (avail. from NTIS as PB 29904/4)

Tapley, B.D., "Polar Motion and Earth Rotation from Lageos Laser Ranging," in *High-Precision Earth Rotation and Earth-Moon Dynamics,* edited by O. Calame, D. Reidel, Dordrecht, Holland, 1982.

Torge, W. *Geodesy,* Walter de Gruyter, Berlin, 1980.

Tscherning, C.C. and R. Forsberg, "Prediction of Deflections of the Vertical," in *Proceedings Second International Symposium on Problems Related to the Redefinition of North American Geodetic Networks,* Arlington, Va, April 24–28, 1978. (avail. from NTIS as PB 29904/4)

Vanicek, P. and E.J. Krakiwsky, *Geodesy: The Concepts,* Elsevier, Amsterdam, 1982.

Wells, E.M., "A Priori and Real Time Use of a Gravity Gradiometer to Improve Inertial Navigation System Accuracy," Report 530, Department of Aeronautics and Astronautics, Stanford University, Calif., 1981.

Western Space and Missile Center, "Western Test Range Geodetic Coordinates Manual," Vandenberg Air Force Base, Calif., Feb. 1981.

Whalen, C.T., "Status of the National Geodetic Vertical Control Network in the United States," in *Proceedings Second International Symposium on Problems Related to the Redefinition of North American Vertical Geodetic Networks,* Ottawa, Canada, May 26–30, 1980, Canadian Institute of Surveying, Ottawa, 1980.

Winkler, G.M.R. and T.C. VanFlandern, "Ephemeris Time, Relativity, and the Problem of Uniform Time in Astronomy," *Astron. J.*, **82:**84–92, Jan. 1977.