

# **EnvSci 360**

# **Computer and**

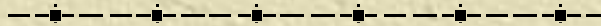
# **Analytical**

# **Cartography**

## **Lecture 3**

### ***Geodesy***

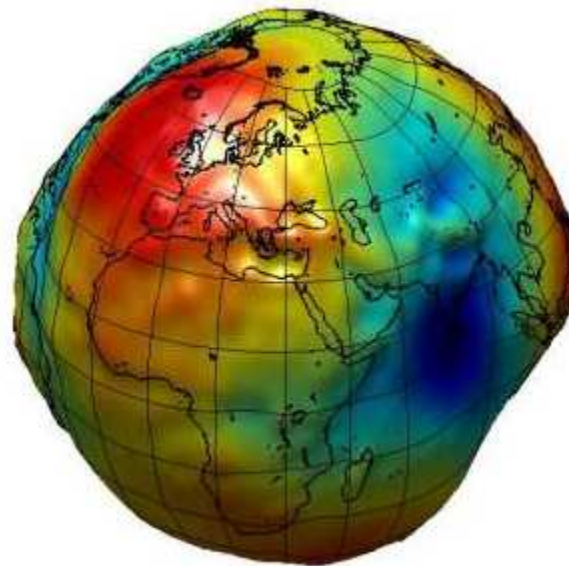
### ***Map Projections, Datums, and Coordinate Systems***



# *Geodesy*

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- ✦ The science of measuring and representing the shape and size of the earth, and the study of its gravitational and magnetic fields.
  - <http://support.esri.com/en/knowledgebase/GISDictionary/term/geodesy>



# ***Geodesy & Cartography***

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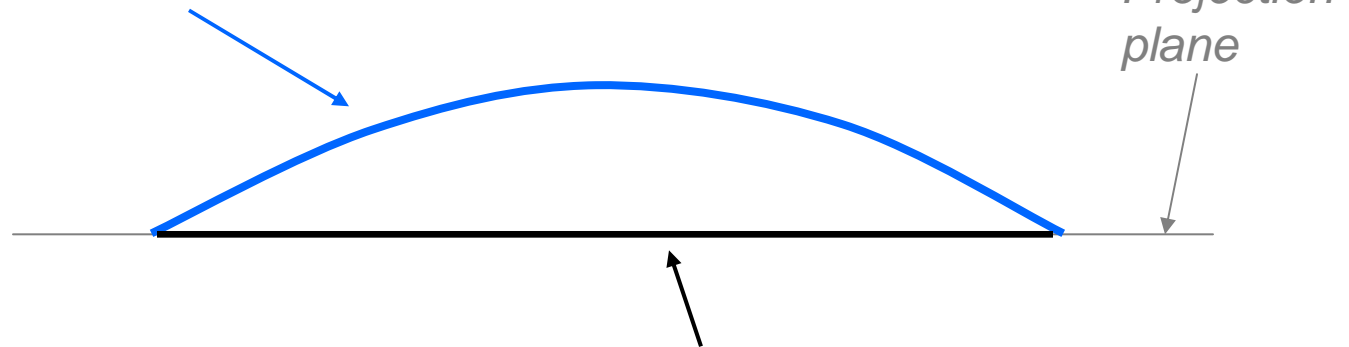
- ✦ The principles of geodesy directly relate to **map projections** and **coordinate systems** – fundamental concepts that determine how you create 2D maps based on the 3D earth.



# ***"Fitting" the Earth to a plane***

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This much Earth ...



... Has to fit on this much map surface

**The projection of a map involves the use of coordinates as defined by projection formulas**

# ***A quick look at projections***

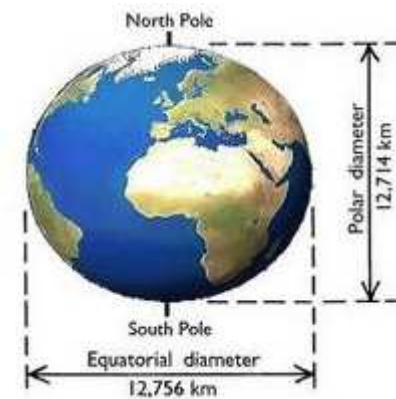
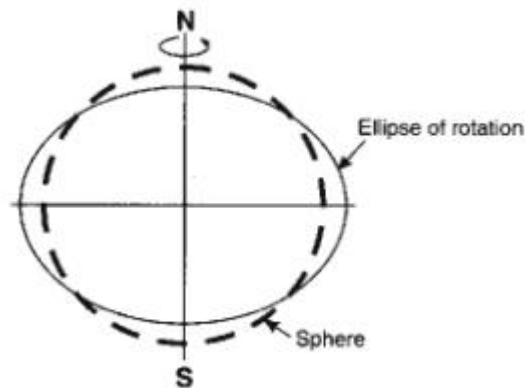
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<https://www.youtube.com/watch?v=kIID5FDi2JQ>

Also see: <http://thetruesize.com>

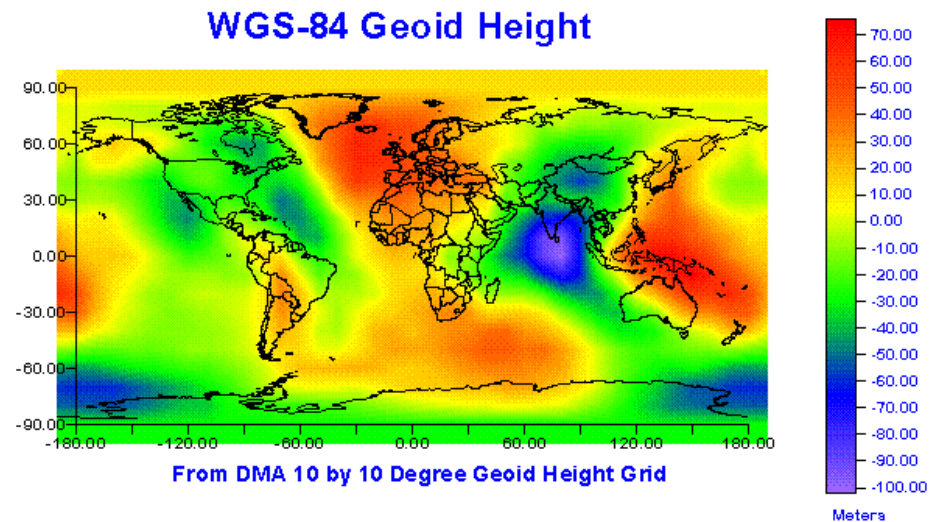
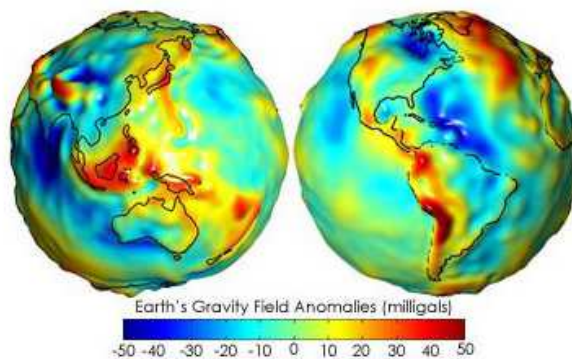
# Earth Facts

- ✦ Equatorial Diameter: 7,926.59 mi.
- ✦ Polar Diameter: 7,900.01 mi.
- ✦ Equatorial Circumference: 24,902.13 mi.
- ✦ Polar Circumference: 24,818.64 mi.
- ✦ Earth is considered an **oblate spheroid** -- a solid formed when an ellipse is rotated about its axis



# The Geoid

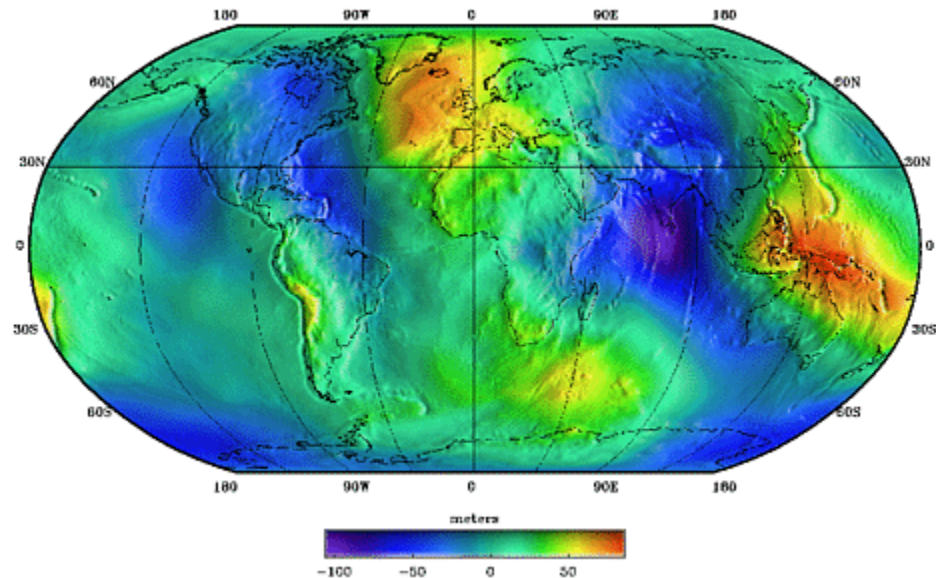
- ✦ But even if we assume the earth is an oblate spheroid, we actually model it as a **“geoid”** – the representation of the earth if the planet were covered only by the oceans.
  - Mean Sea Level (MSL) + Gravitational Pull



# *The Geoid*

- ✦ Gravity varies around the globe, because of variations in density due to magma distributions, mountain ranges, deep sea trenches, etc. So the MSL would not be even everywhere.

**EGM96 (Earth  
Gravitational  
Model 1996)**



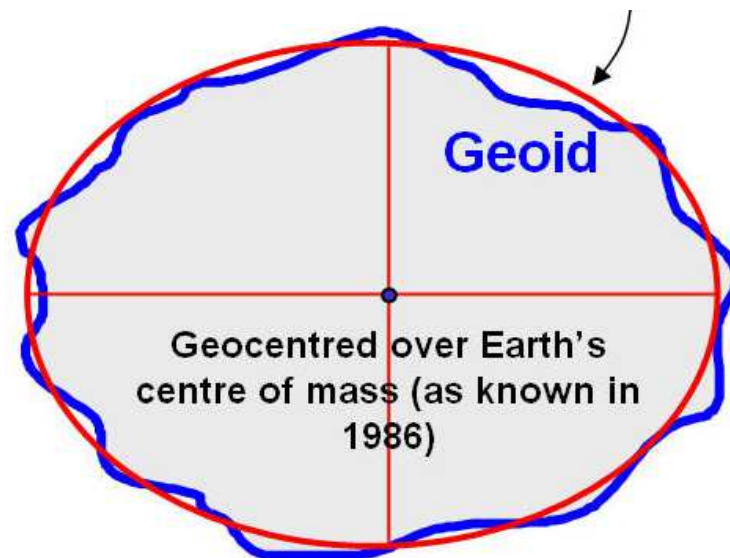


# *Ellipsoid*

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- ✦ To simplify projection math, we use a reference **ellipsoid** to model the earth for mapping - an approximation of the geoid

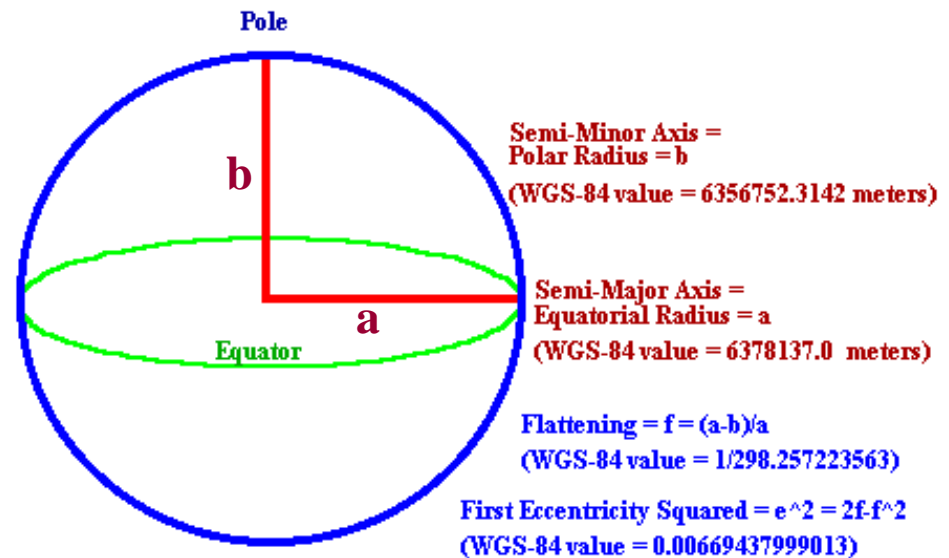
## Geodetic Reference System 1980 (GRS 80) Ellipsoid



*World Geodetic System 1984 (WGS84) refined the GRS80 for use in GPS*

# Ellipsoid

- ✦ A theoretical surface of the Earth which approximates mean sea level
- ✦ Has 2 axes:
  - Major
    - the axis with the greatest radius
  - Minor
    - the axis with the least radius



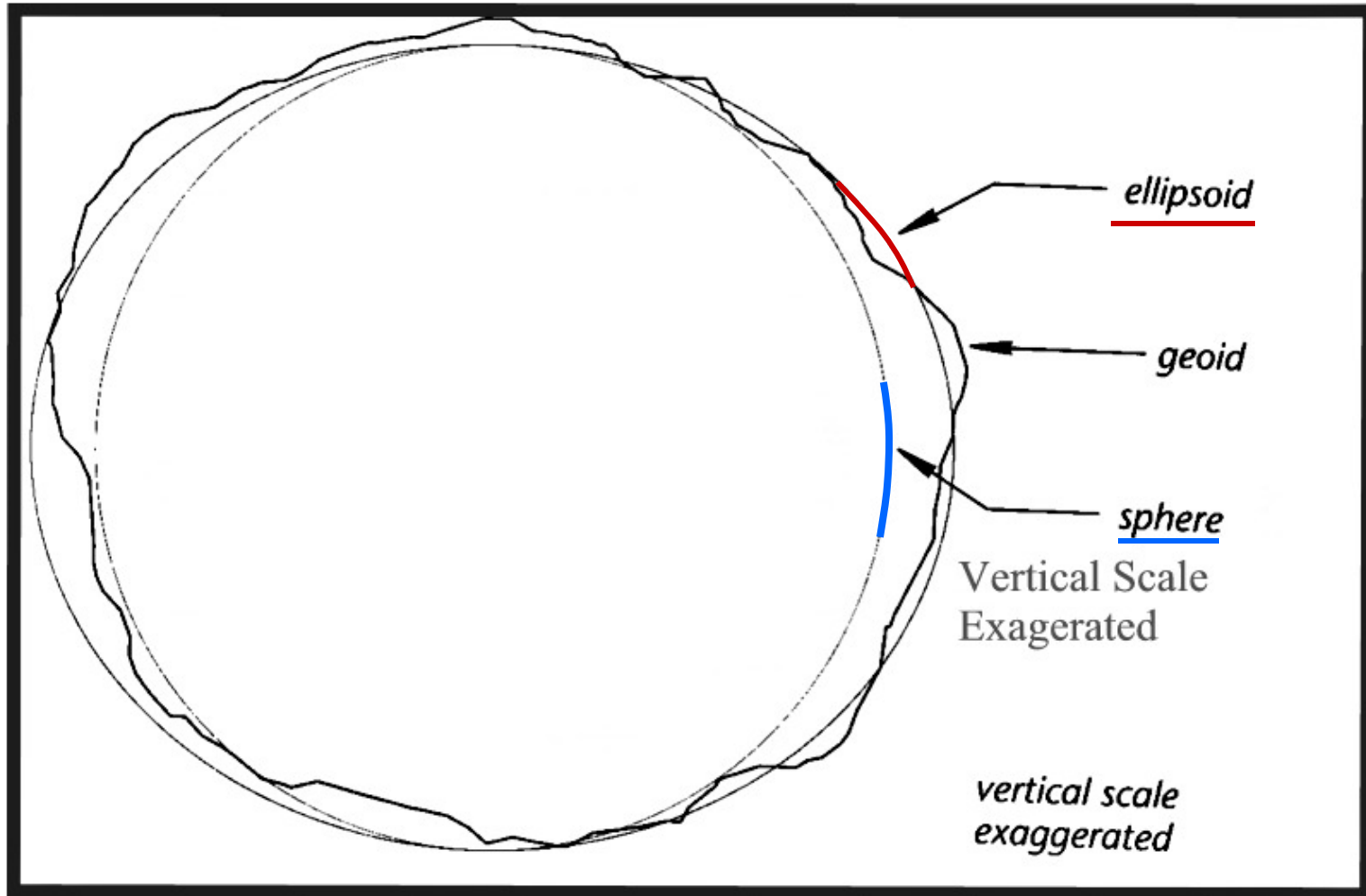
## Ellipsoidal Parameters

# Ellipsoid

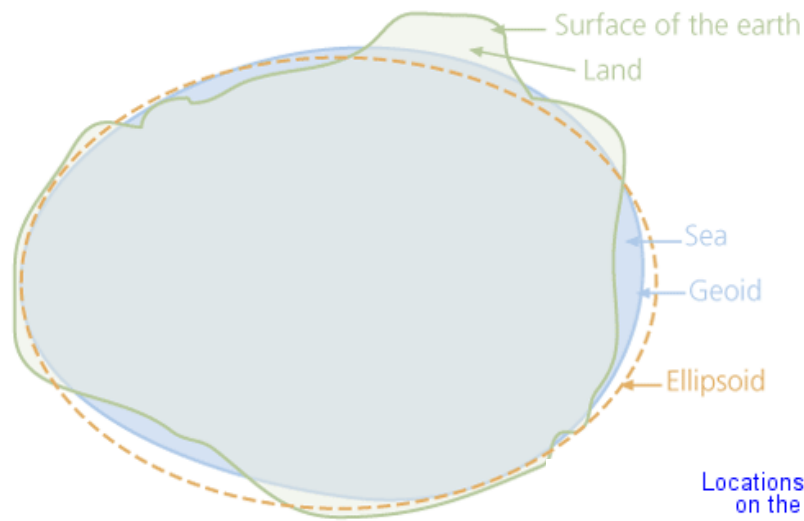
- ✦ Widely used ellipsoids adopted in the U.S.:
  - 1) Clark ellipsoid of 1866 (datum – NAD27 (*North American Datum of 1927*))
  - 2) Geodetic Reference System (GRS80) (datum – NAD83 (*North American Datum of 1983*))
    - Original basis for WGS84, which now uses the EGM96 (Earth Gravitational Model 1996) geoid, last revised in 2004

<u>Ellipsoid reference</u>	<u>Semi-major axis <math>a</math></u>	<u>Semi-minor axis <math>b</math></u>	<u>Inverse flattening (<math>1/f</math>)</u>
Clark 1866	6,378,206.4 m	6,356,583.8 m	294.9786982
GRS 80	6,378,137.0 m	≈ 6,356,752.314 140 m	298.257 222 101
WGS 84	6,378,137.0 m	≈ 6,356,752.314 245 m	298.257 223 563

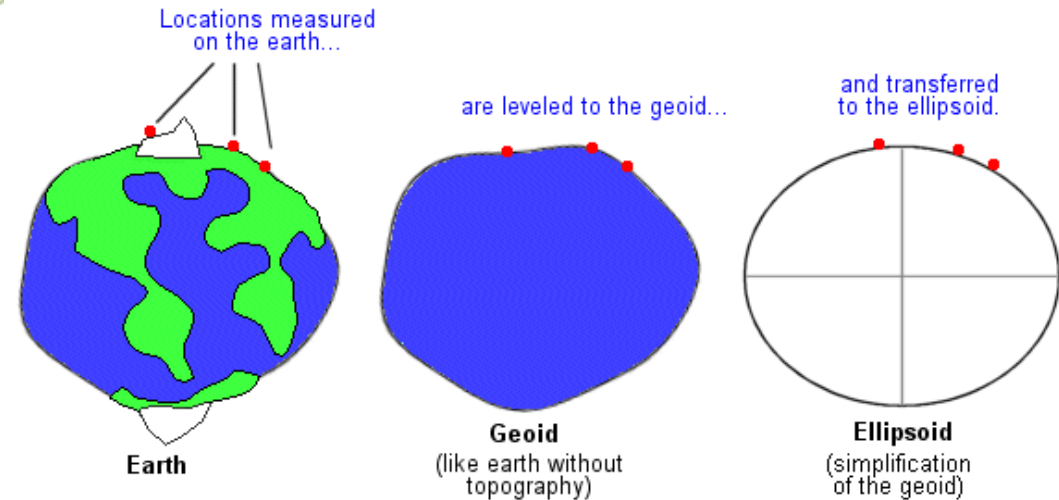
# ***Sphere, Geoid, Ellipsoid***



# Geoid, Ellipsoid, Surface



Ellipsoid aka  
“reference globe”



See [http://geology.isu.edu/geostac/Field\\_Exercise/topomaps/ref\\_datum.htm](http://geology.isu.edu/geostac/Field_Exercise/topomaps/ref_datum.htm)

# Map Projection

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✦ **The systematic mathematical transformation of the three-dimensional curved surface of the Earth to the two-dimensional flat surface of a map.**

- Basically, it is how you “show” the curved 3-D Earth on a flat map.

✦ Attempt to accurately depict the following characteristics:

1. Direction
2. Distance
3. Area
4. Shape

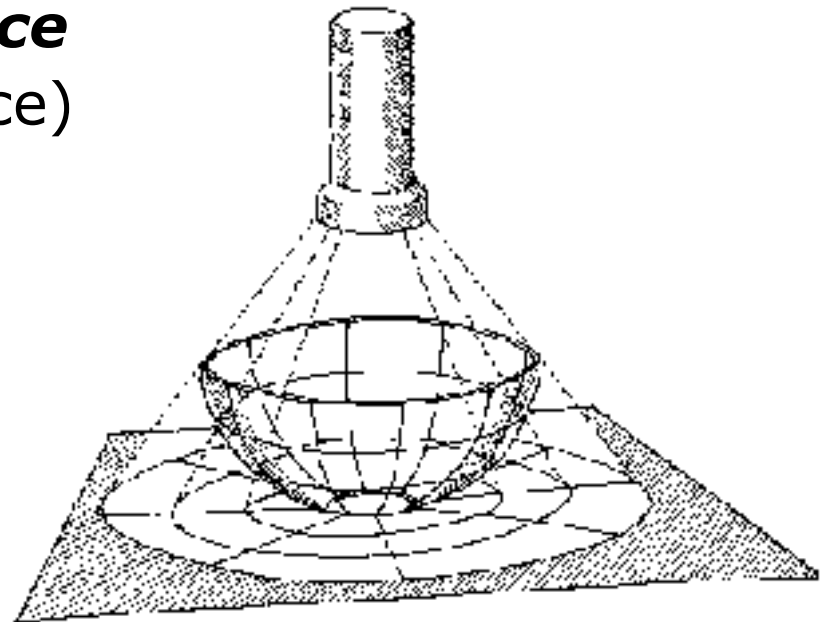
**\* No Map Projection can accurately depict all four characteristics at once.**

**Every map projection will contain some distortion.**

# ***Producing Projections***

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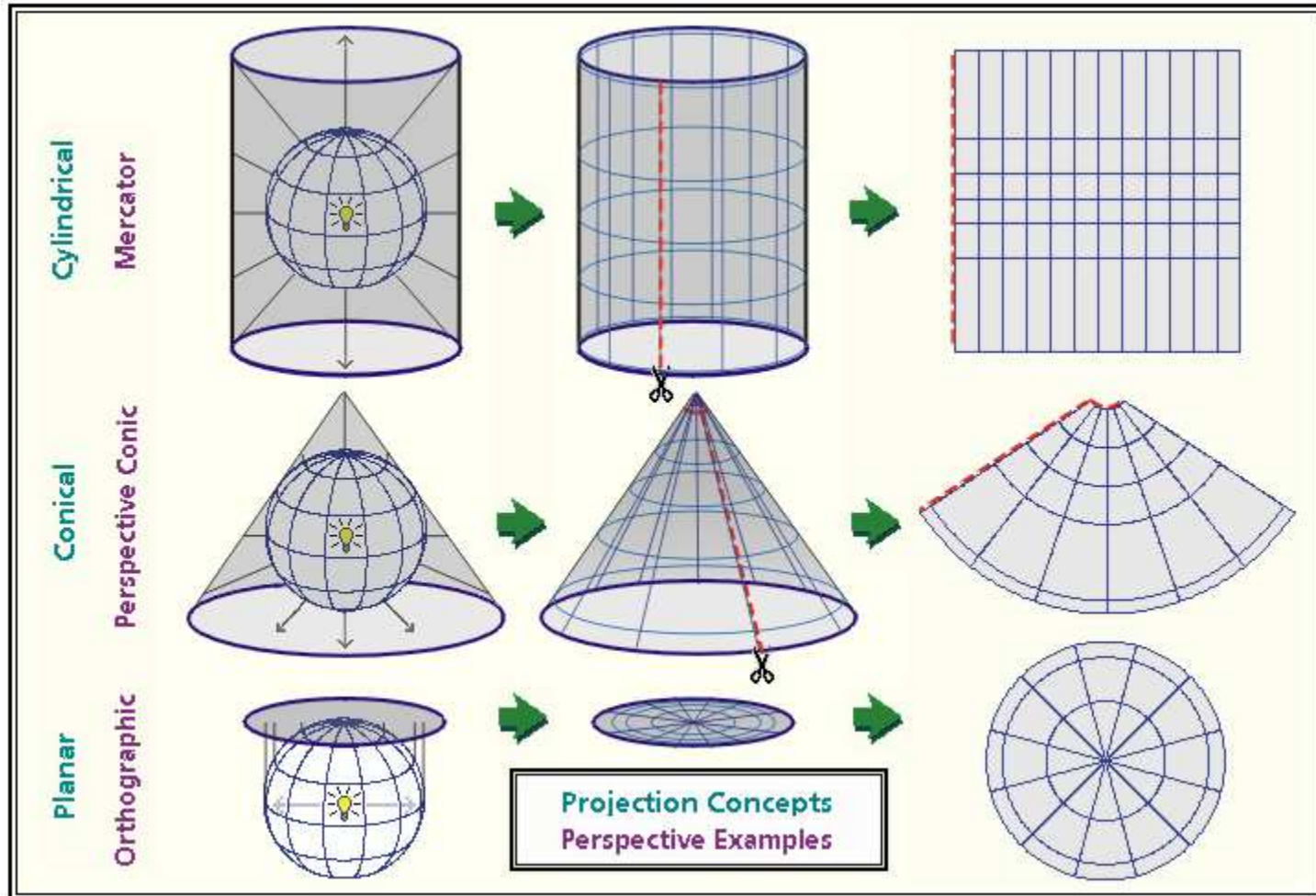
- ✦ Projections may be physically produced by “shining a light” from the center of the earth onto a *developable surface* that is being “wrapped around” or “tangent to” the reference globe
- ✦ A ***developable surface*** (aka flattenable surface) is a geometric shape that may be flattened without stretching its surface - **cones, cylinders, and planes**



# Producing Projections

Developable surfaces

Flat maps

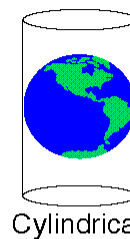




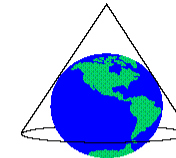
# Producing Projections

- ✦ First step is to create one or more points of contact between the developable surface and the reference globe - these are called points of tangency
- ✦ A **tangent projection** is one where the developable surface touches in one location

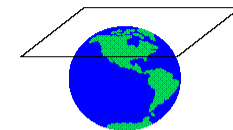
Map Projection Surfaces



Cylindrical

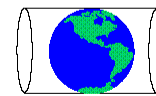


Conic

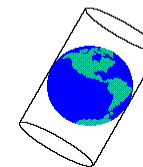


Azimuthal

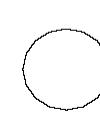
Orientation



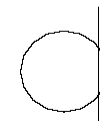
Transverse



Oblique



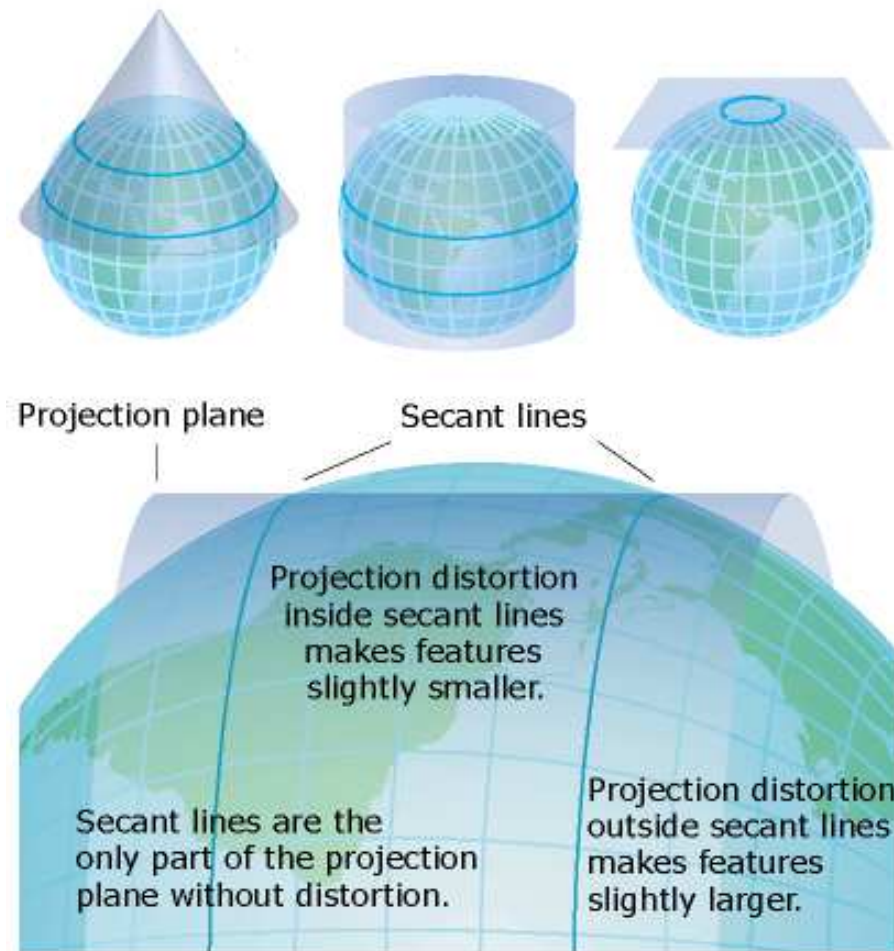
Tangent



Secant

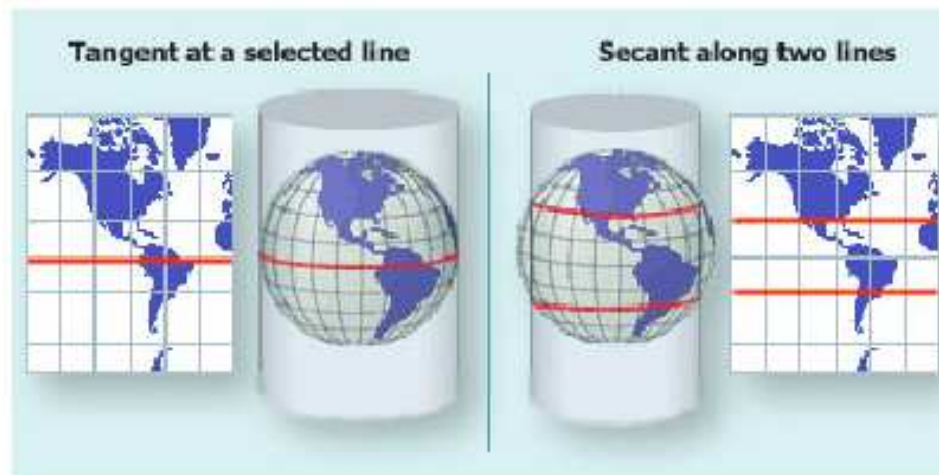
# Producing Projections

- ✦ In a **secant projection** the developable surface intersects the globe in two places



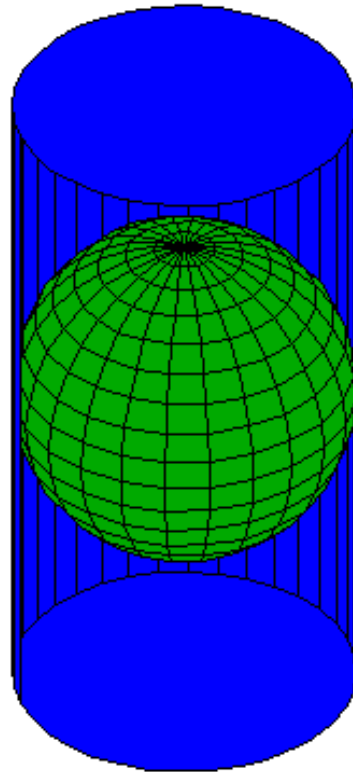
# Producing Projections

- ✦ Where the dev. surface touches the globe is important - **this is a line of true scale**, often referred to as a *standard line*, such as a line of standard latitude (**standard parallel**) or standard longitude (**central meridian**)
- ✦ **Distortion is zero** along standard lines or points; distortion increases away from these standard locations

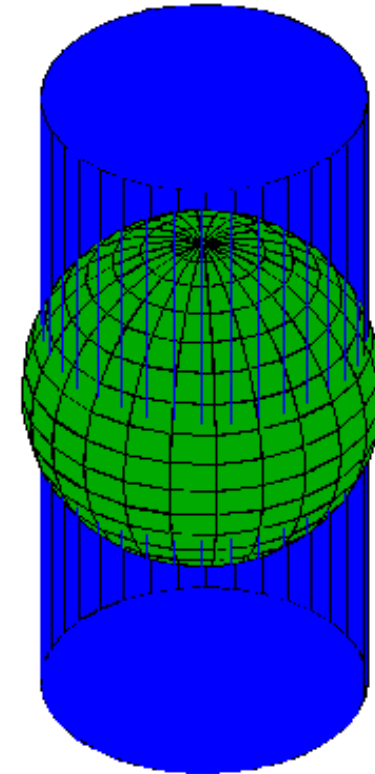


# ***Cylindrical Projection – Conceptual View***

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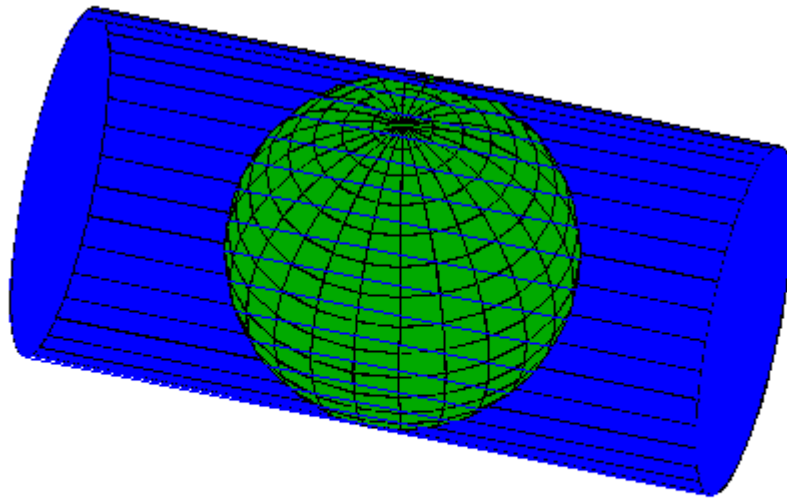
**Cylindrical Projection Surface  
(Tangent)**



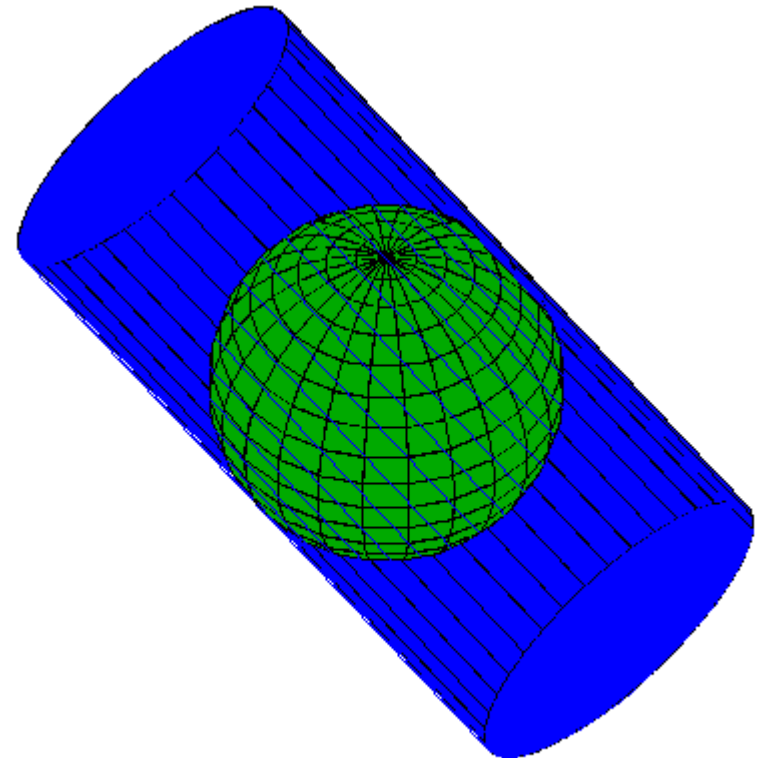
**Secant Cylindrical Projection**

# ***Cylindrical Projection – Conceptual View***

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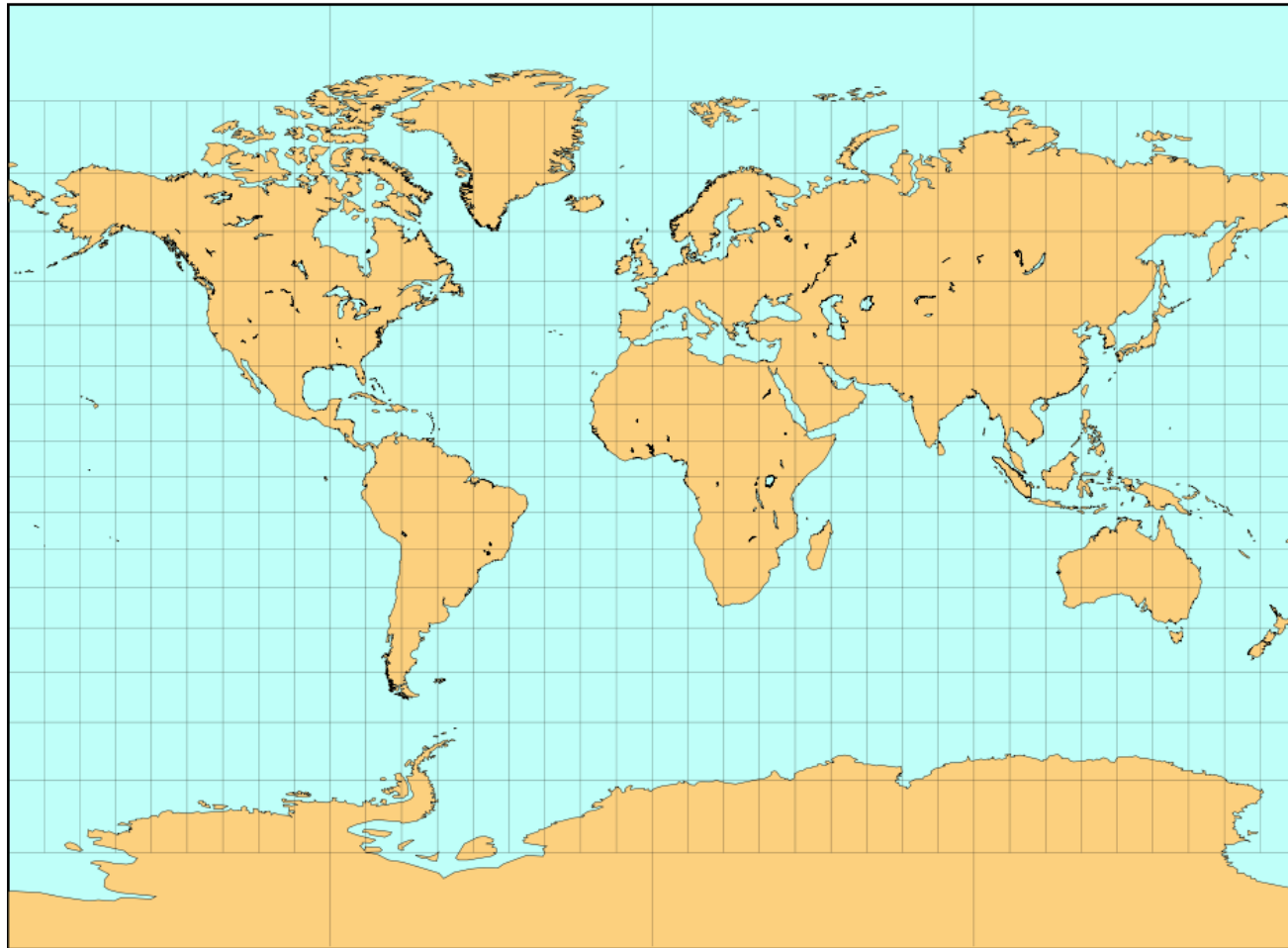
**Transverse Cylindrical  
Projection Surface**



**Oblique Cylindrical  
Projection Surface**

# ***Cylindrical Projection - Example***

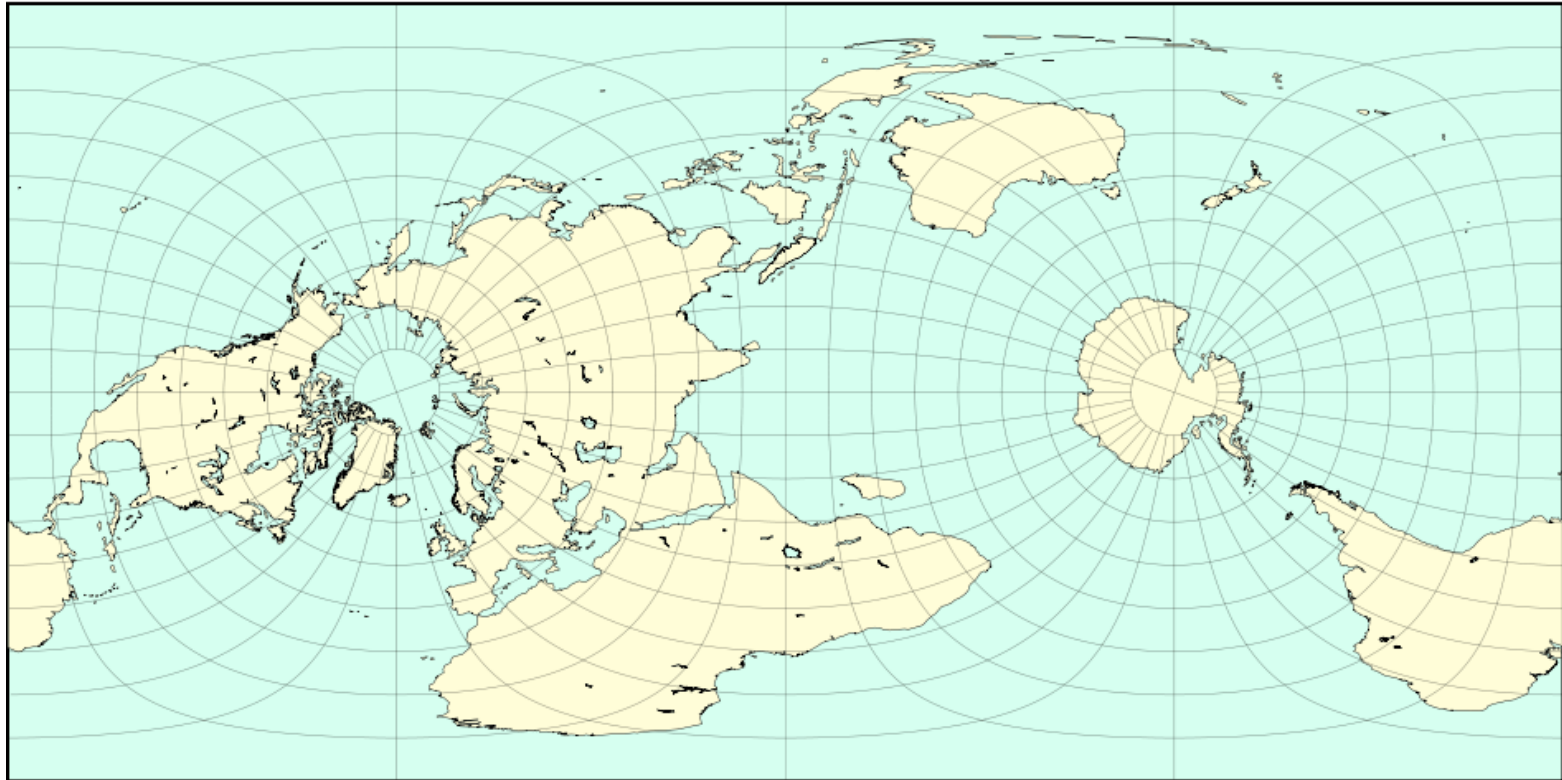
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Miller's Cylindrical Projection

# *Cylindrical Projection - Example*

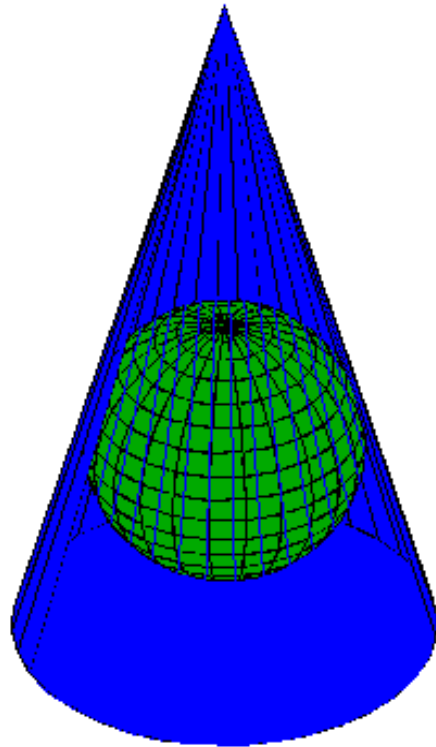
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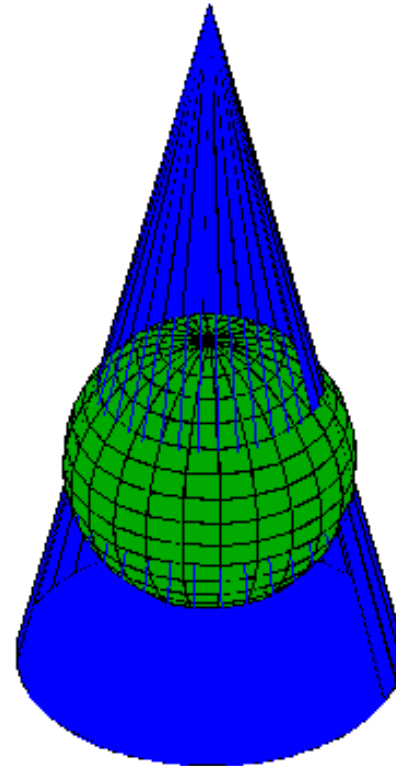
Cassini's projection is a transverse aspect,  
here with central meridians at 70°E and 110°W

# ***Conic Projection – Conceptual View***

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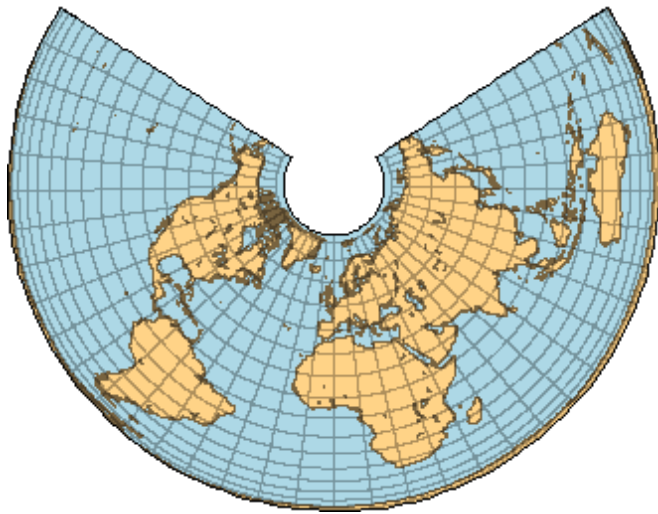
**Conical Projection Surface**



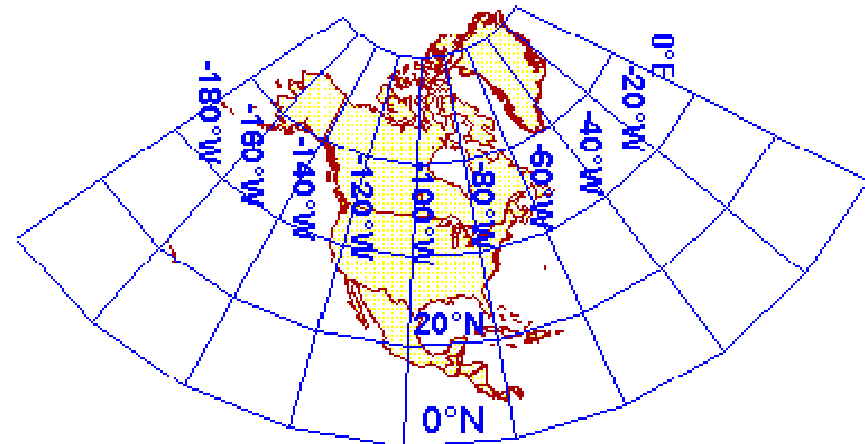
**Secant Conic Projection**



# *Conic Projection - Example*



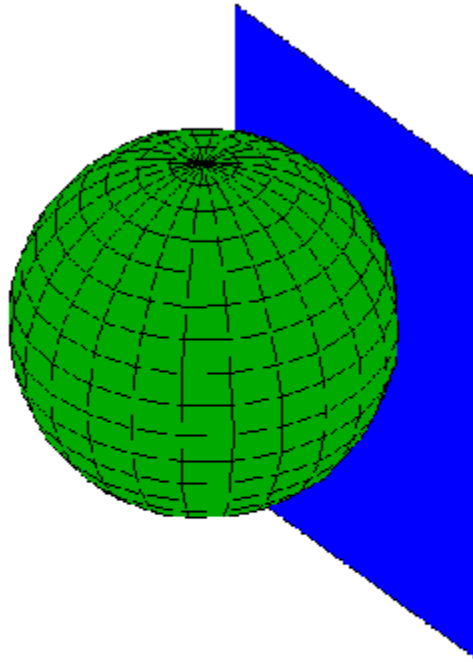
How flat maps look  
when the cone is  
“unfurled”



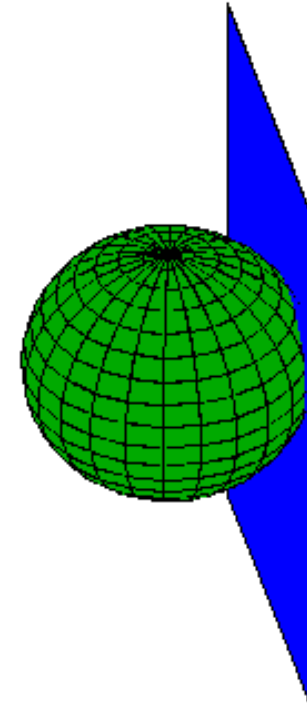
**North America**  
**Lambert Conformal Conic**  
**Origin: 23N, 96W**  
**Standard Parallels: 20N, 60N**

# ***Planar (Azimuthal) Projection – Conceptual View***

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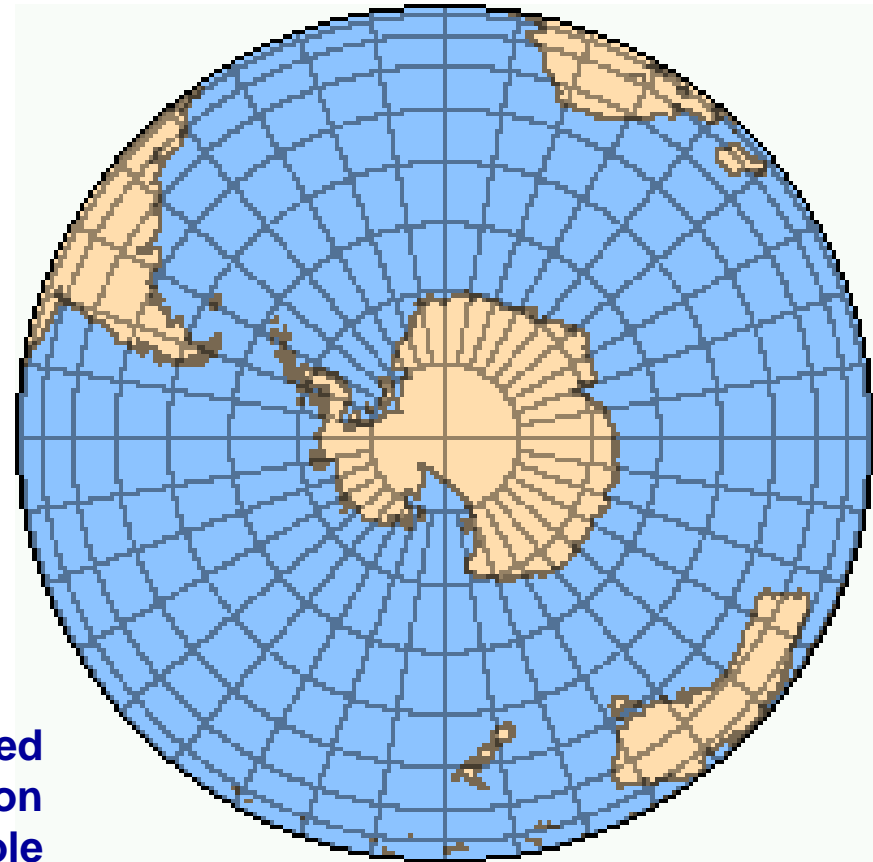
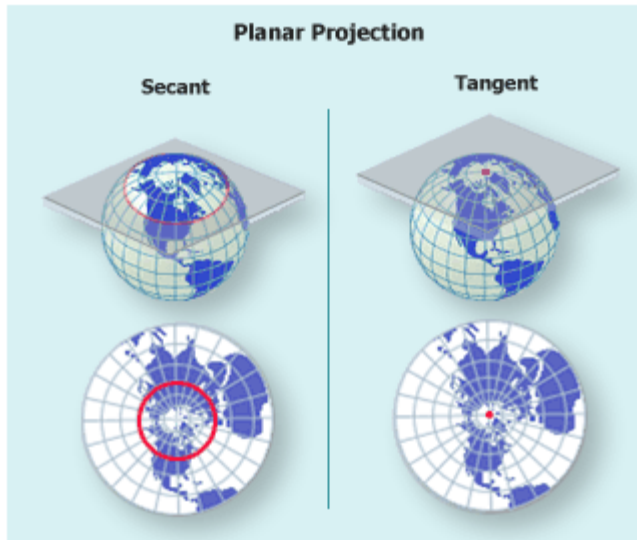


**Planar Projection Surface**



**Secant Planar Projection**

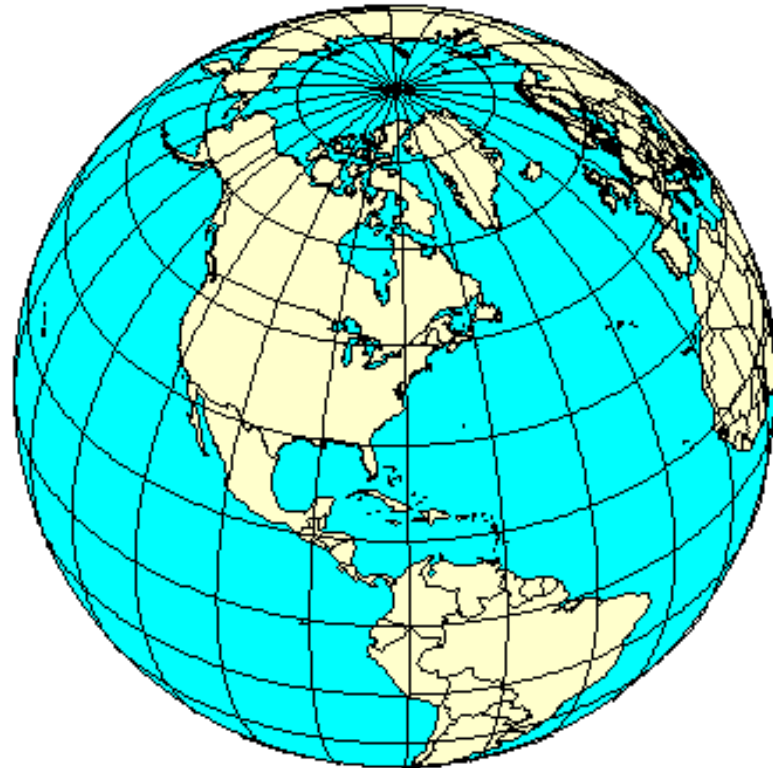
# *Planar (Azimuthal) Projection - Example*



**Example of a map produced  
with a planar projection  
centered on the South Pole**

# *Orthographic Projection*

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Perspective view that depicts a hemisphere of the globe as it appears from outer space

Orthographic Projection  
Centered on Washington, DC

# *Pseudocylindrical*

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Robinson projection;  
Pseudocylindrical;  
Neither Conformal or Equal-area;

## ***Other Ways of Categorizing Projections***

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✦ Along with using Cylinders, Cones and Planes to categorize projections, we can also categorize them based on their inherent distortions



# ***Distortion With Projections***

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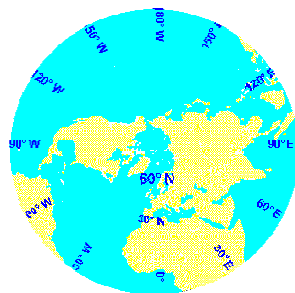
- ✦ Shape (Conformality)
- ✦ Distance
- ✦ Direction
- ✦ Area

Different map projections preserve some of these properties or attempt to reduce the distortion of some or all of these properties, but NO map projection preserves all these properties.

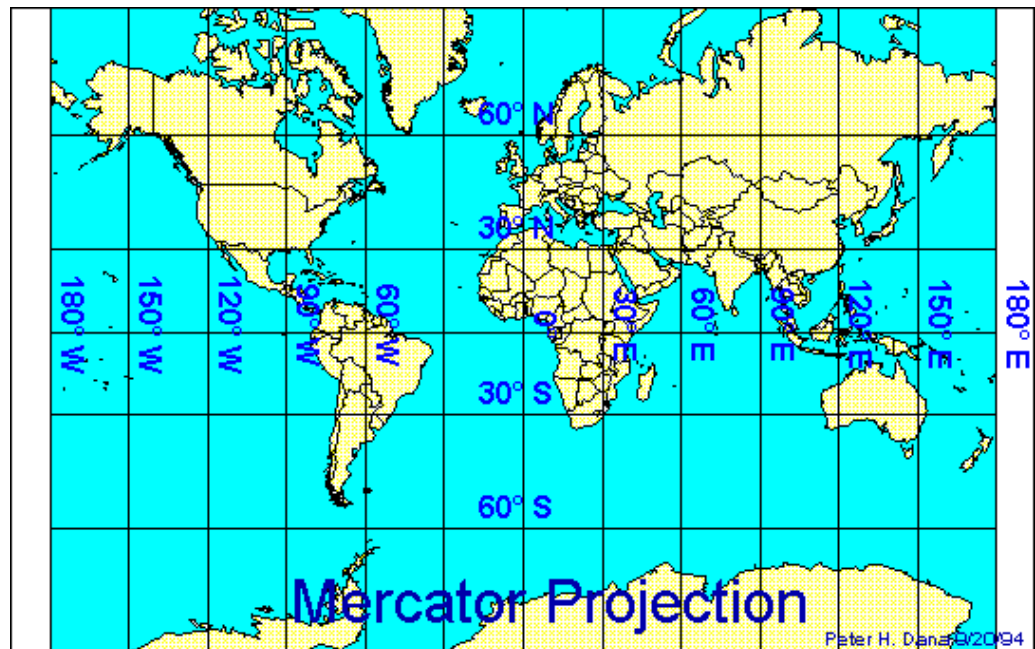
*Scale also can be distorted, or differ, throughout a single map*

# Conformal Projections

- ✦ **Conformal** projections preserve local **shape**
  - At every point the scale is the same in every direction
  - Graticule lines intersect at 90-degree angles
  - All angles between intersections of arcs are maintained
  - Shapes of very small areas and angles with very short sides are preserved.
  - Size of many areas are distorted



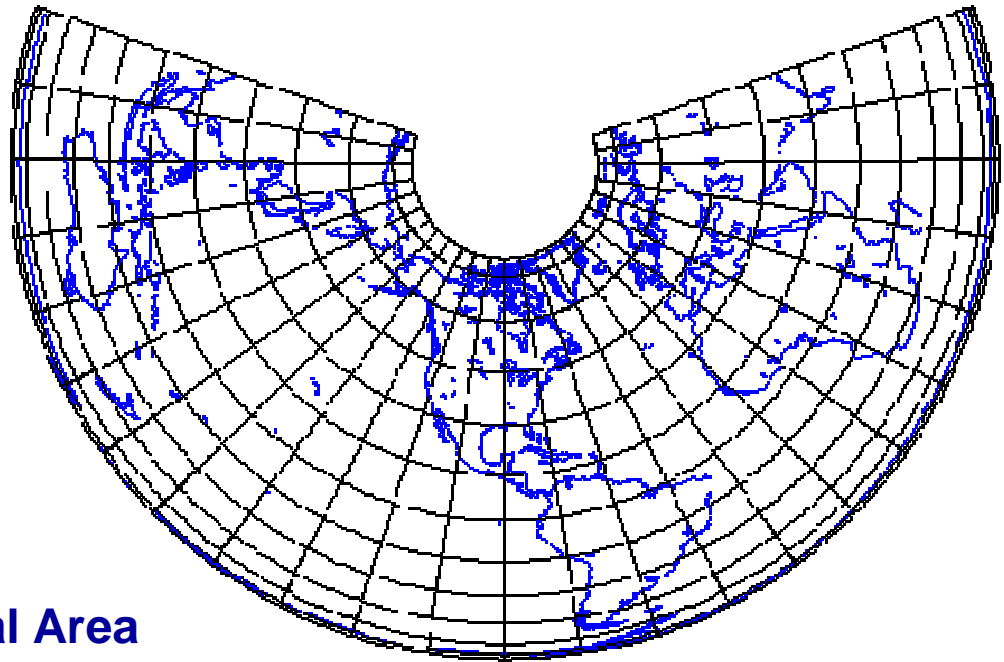
Stereographic  
North Polar Aspect





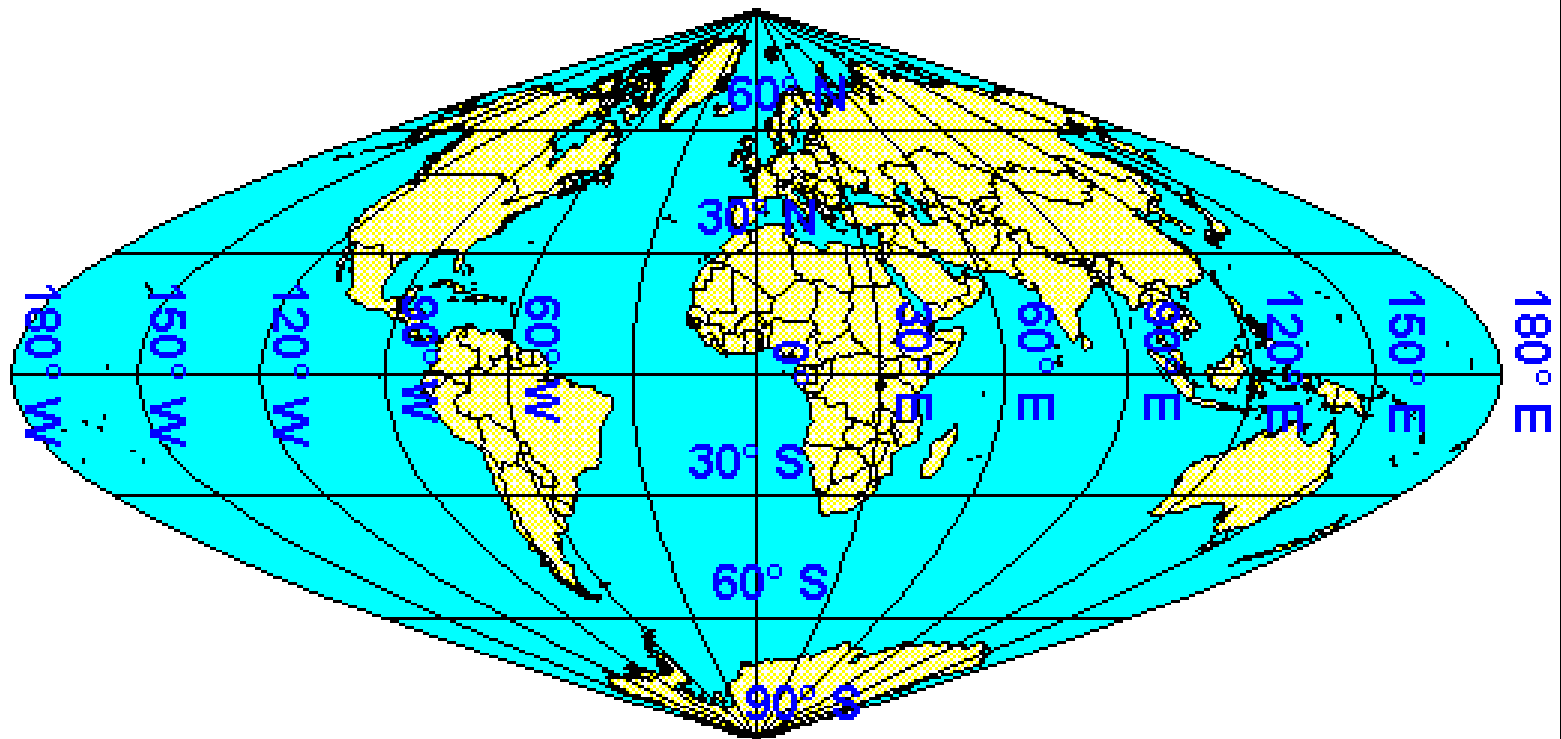
# Equal-area Projections

- ✦ **Equal-area** projections preserve the **relative area** of displayed features
  - Every part on the map, as well as the whole, has the same area as the corresponding part on the Earth, at the same reduced scale
  - Shape, angles, scale may be distorted
  - Graticule lines may not meet at 90-degree angles



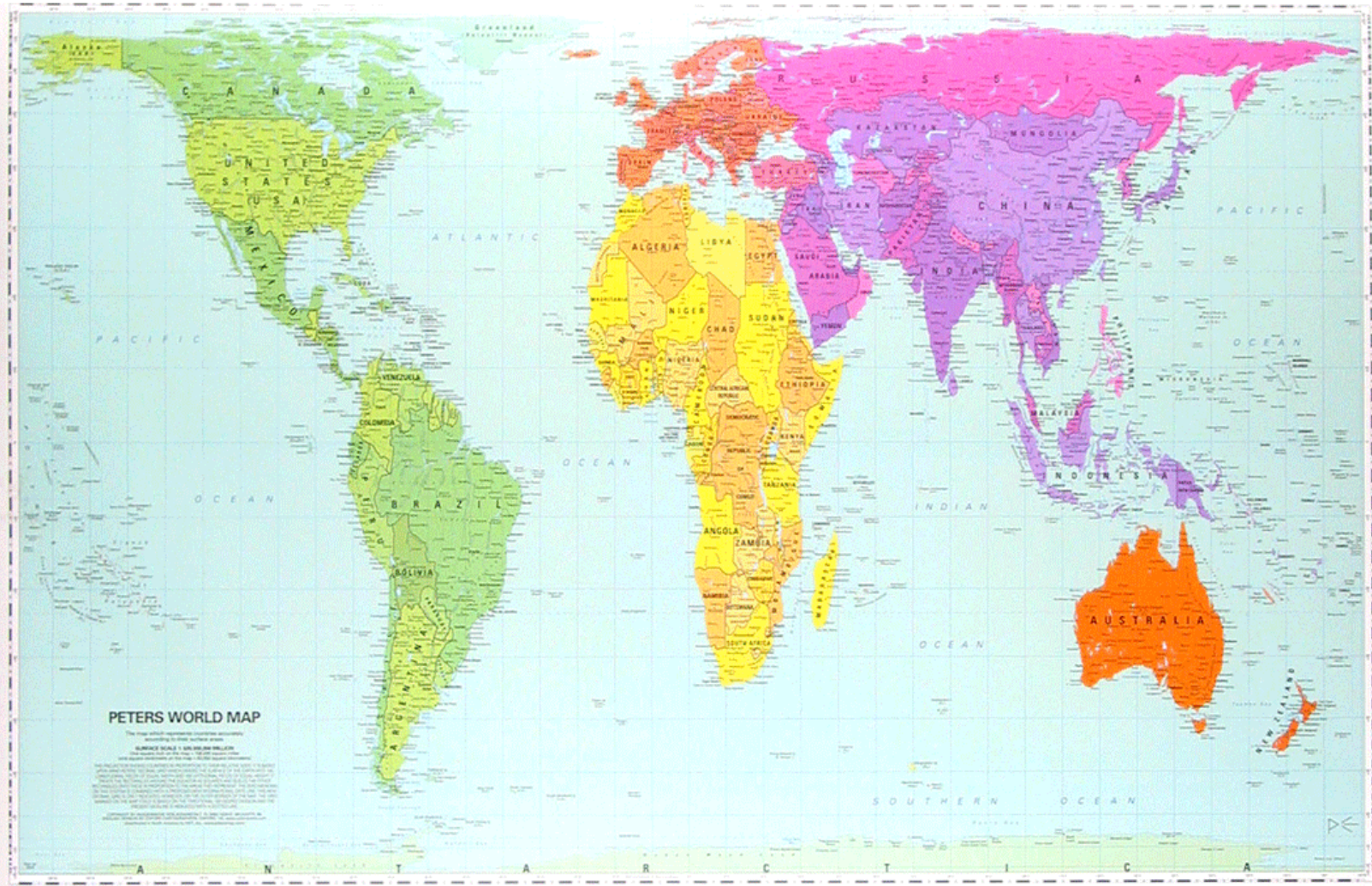
Albers Equal Area

# *Equal-area Projections*



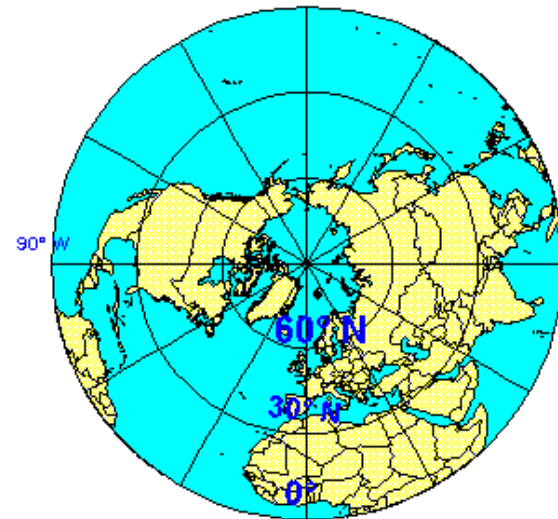
**Sinusoidal Equal-Area**

# *Equal-area Projections*



# ***Equidistant Projections***

- ✦ **Equidistant** projections preserve the **distances** between certain points
  - Scale is maintained along certain lines on map in relation to its reference globe; the distances along these lines are *true*
  - True distances only from the center of the projection or along special lines
  - No projection is equidistant to and from all points on a map



**Azimuthal Equidistant**

# ***True-direction Projections***

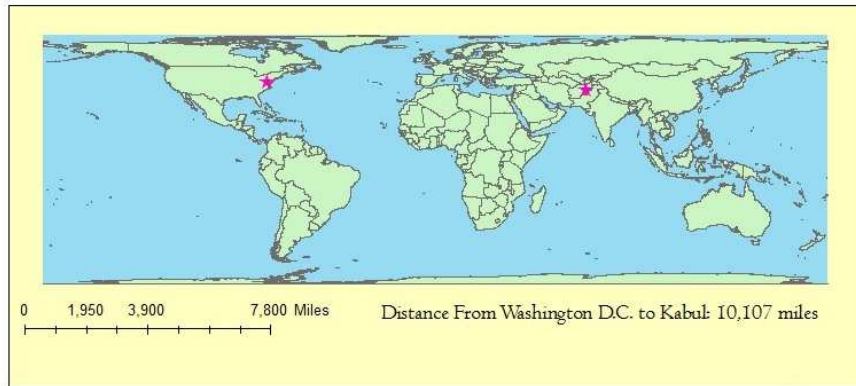
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- ✦ **True-direction** or azimuthal projections preserve the direction of specified points on a great circle
  - The shortest route between two points on a curved surface such as the earth is along the spherical equivalent of a straight line on a flat surface - called the great circle
  - Great circle arcs are *rectified*, or shown as straight lines
  - Azimuths (angles from a point on a line to another point) are portrayed correctly in all directions

See <http://www.colorado.edu/geography/gcraft/notes/mapproj/mapproj.html> for a good list and examples of different projections

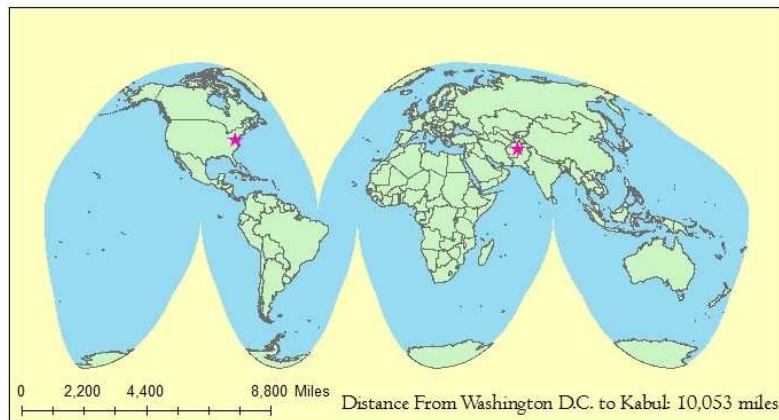
# ***Distortion With Projections***

Cylindrical Equal Area



Both of these maps are based on equal-area projections, but distances between the same two locations on the globe are not equal

Goode Homolosine Land

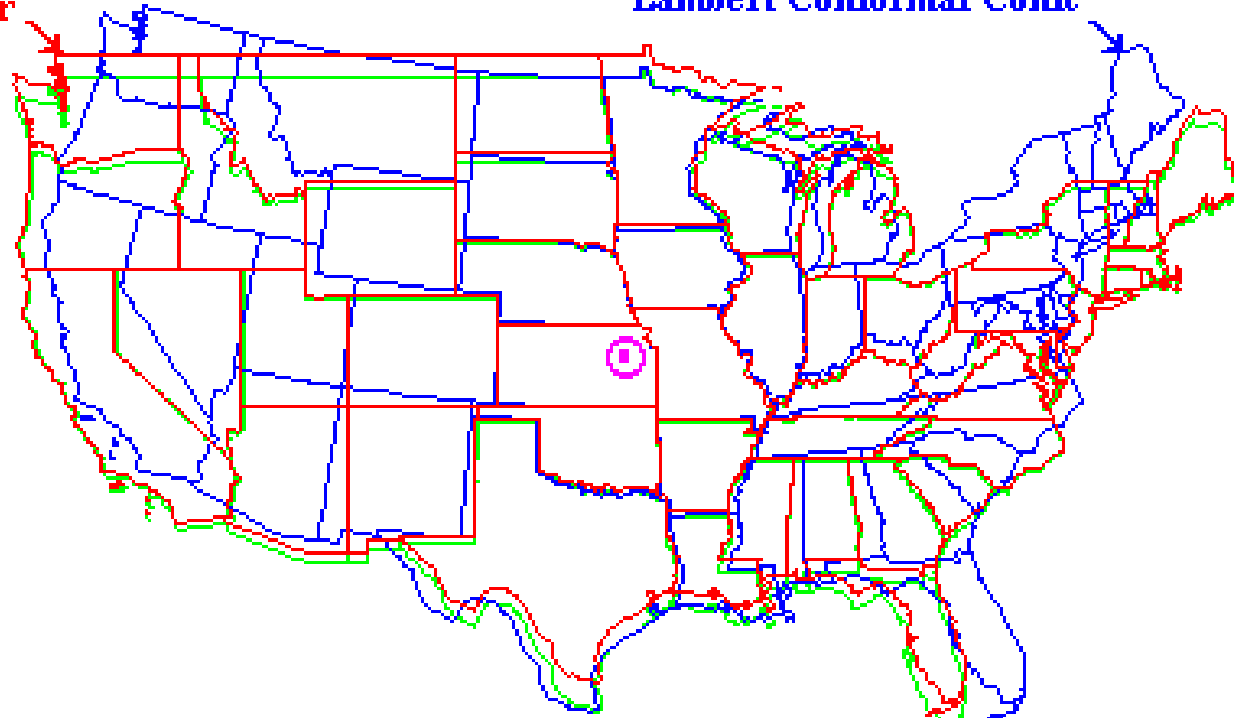


# Comparing Projections

## Three Map Projections Centered at 39 N and 96 W

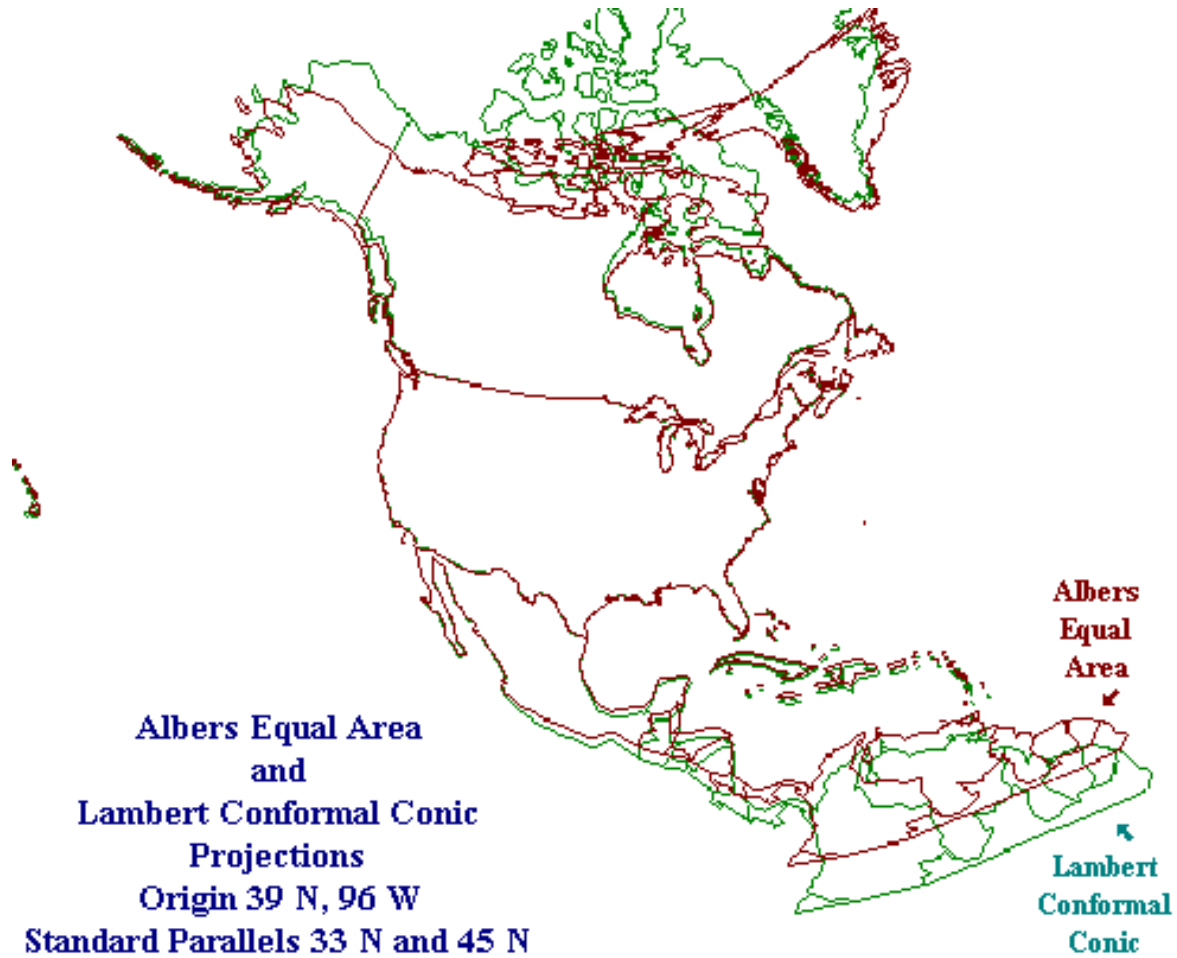
**Mercator**

**Lambert Conformal Conic**



**Un-Projected Latitude and Longitude**

# Comparing Projections





# ***Which Projection to Use?***

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- ✦ Computers do the transformations, but the cartographer's primary task is to choose the proper projection
- ✦ Ask -
  - what is the purpose of the map?
  - Will the projection influence the map's effectiveness?

# ***Which Projection to Use?***

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## ✦ Guidelines:

- How the projection's major property (conformality, equivalence, etc.) relates to the map's purpose
- Amount and arrangement of distortion
- Is the map part of a series?
- The overall shape of the projected area - N/S, E/W?
- The size of the mapped area - world? town?
- Location of the area - Polar? Mid-latitude? Equatorial?

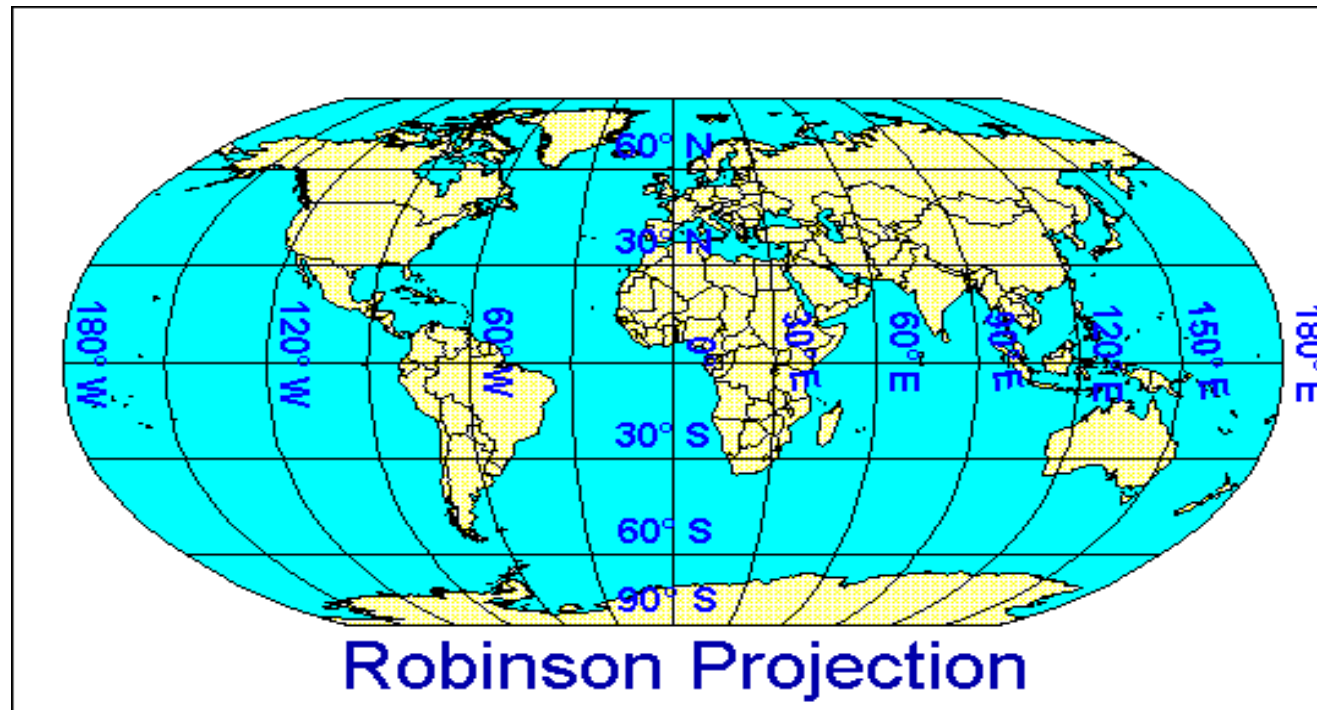
# ***Which Projection to Use?***

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- ✦ Equal-area for thematic or distribution maps (statistical mapping)
- ✦ True-direction and/or equidistant for navigational
- ✦ Conformal for presentation maps

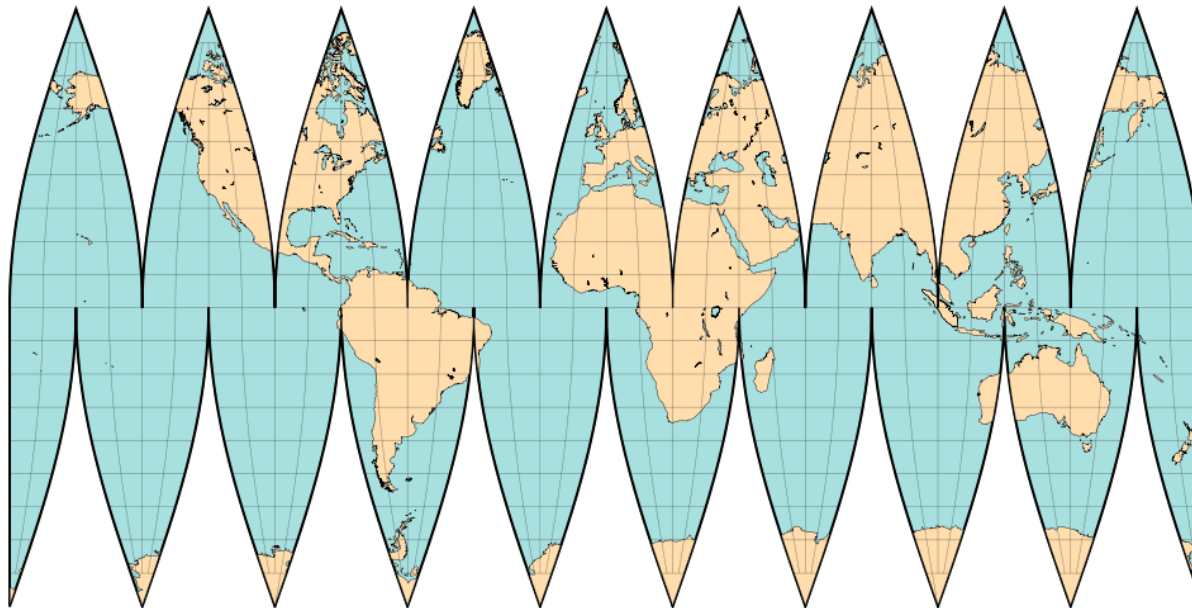
# "Compromise" Projections

- ✦ Neither true shapes, true directions, true areas, nor true distances, but a compromise among these properties
- ✦ Attempts to minimize the distortions inherent in some of these properties when others are made to be true



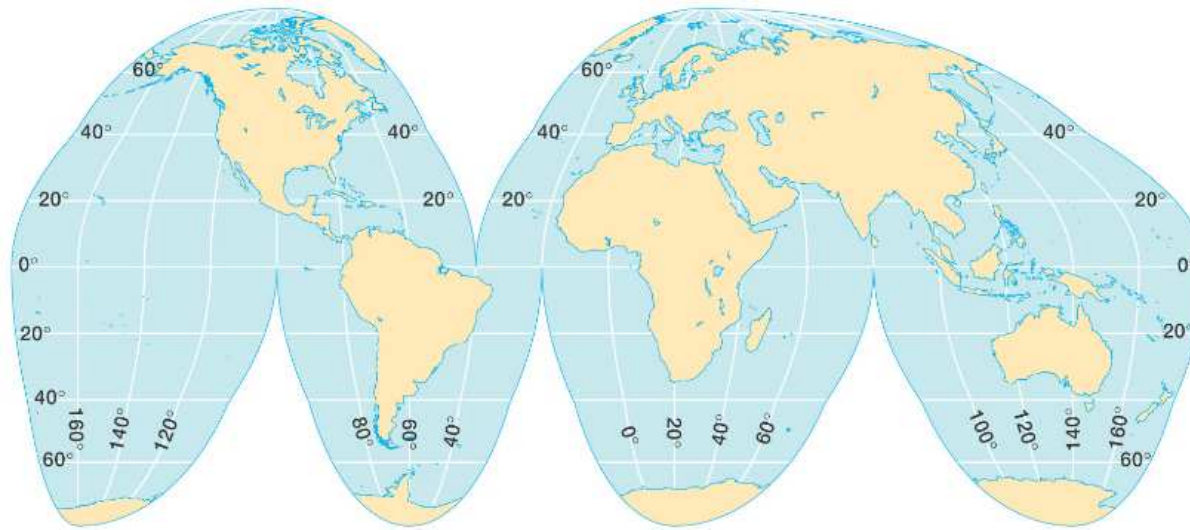
# *Interrupted Projections*

- ✦ A compromise projection, "cutting" the earth's surface along arbitrarily chosen lines and projecting each section separately, which results in less stretching.

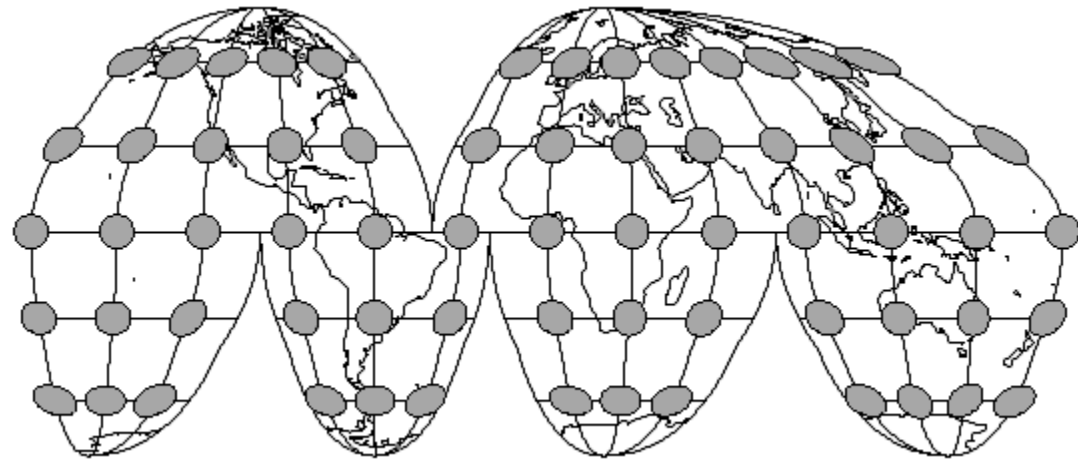


**Interrupted  
sinusoidal  
map**

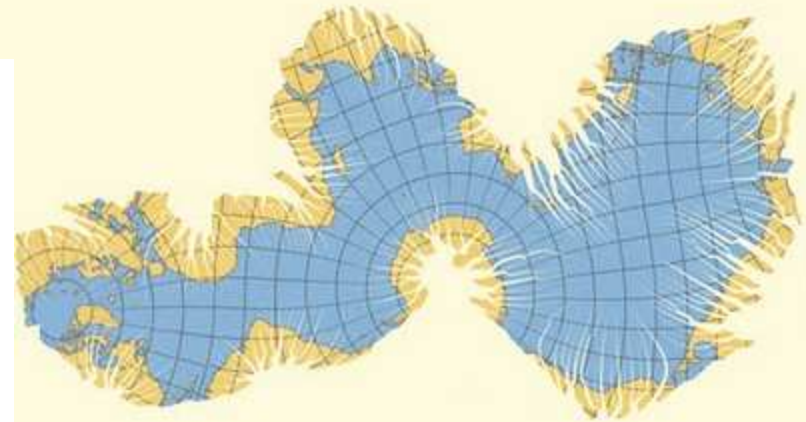
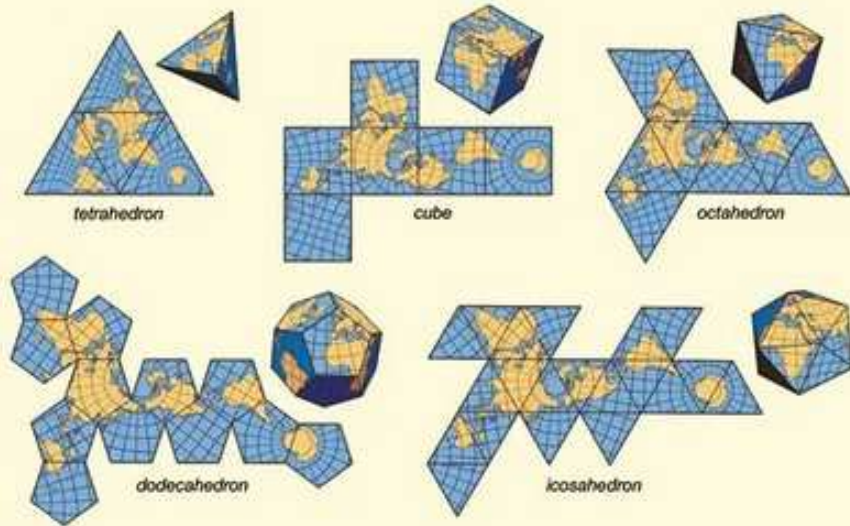
# *Interrupted Projections*



**Goode's  
Homolosine  
Projection**

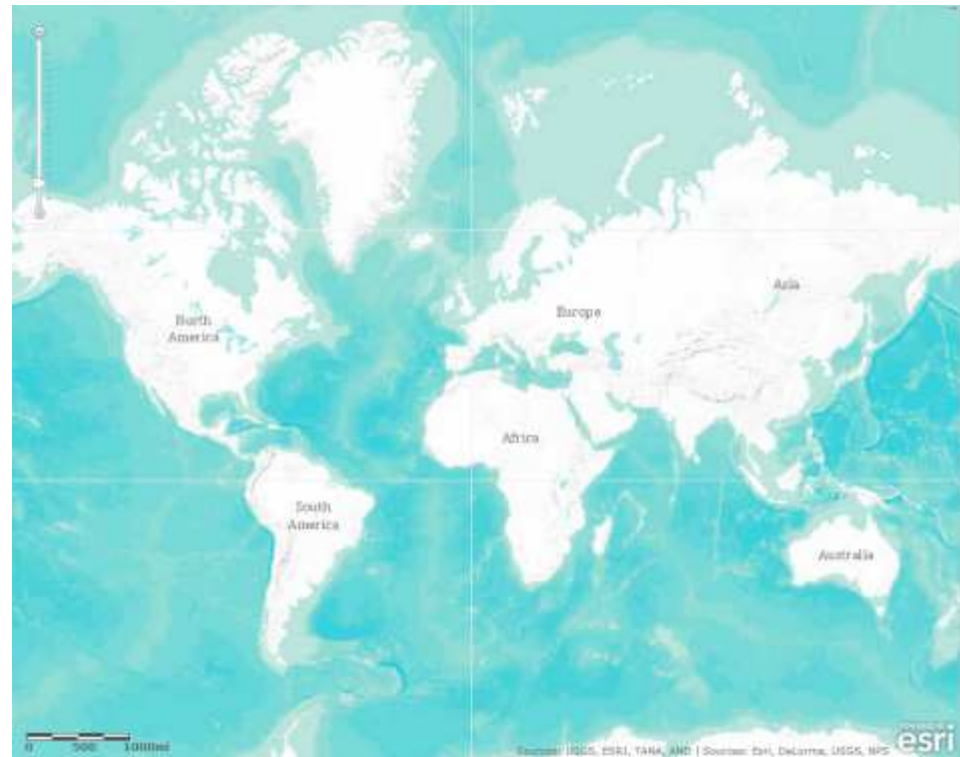


# *Interrupted Projections*



# Web Mercator

- ✦ Standard for web-based mapping
- ✦ Used by Microsoft Bing Maps, Google Maps, and ESRI ArcGIS Online



See

<http://www.esri.com/events/seminars/bettermaps/materials/~media/files/pdfs/events/seminars/bettermaps/materials/pdfs/webmercatorsmnrbrochure.pdf>

<http://www.hydrometronics.com/downloads/Web%20Mercator%20-%20Non-Conformal,%20Non-Mercator%20%28notes%29.pdf>

<http://gis.stackexchange.com/questions/12646/why-has-web-mercator-auxiliary-sphere-become-the-web-map-standard>

<http://msdn.microsoft.com/en-us/library/bb259689.aspx>



# *Horizontal Control*

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- ✦ In order for a projection to be used as a map, a **datum** must be declared.
- ✦ Datum - a set of quantities to serve as a referent (or point of reference)
  - have an origin
  - measurement based on a datum must be repeatable

# *Horizontal Control*

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- ✦ **Datum** - sets of parameters defining a coordinate system, and a set of control points whose geometric relationships are known, either through measurement or calculations.
- ✦ Simply put, a datum is the **mathematical model of the Earth we use to calculate the coordinates on any map, chart, or survey system**. All coordinates reference some particular set of numbers (datum) for the size and shape of the Earth.

# *Horizontal Control*

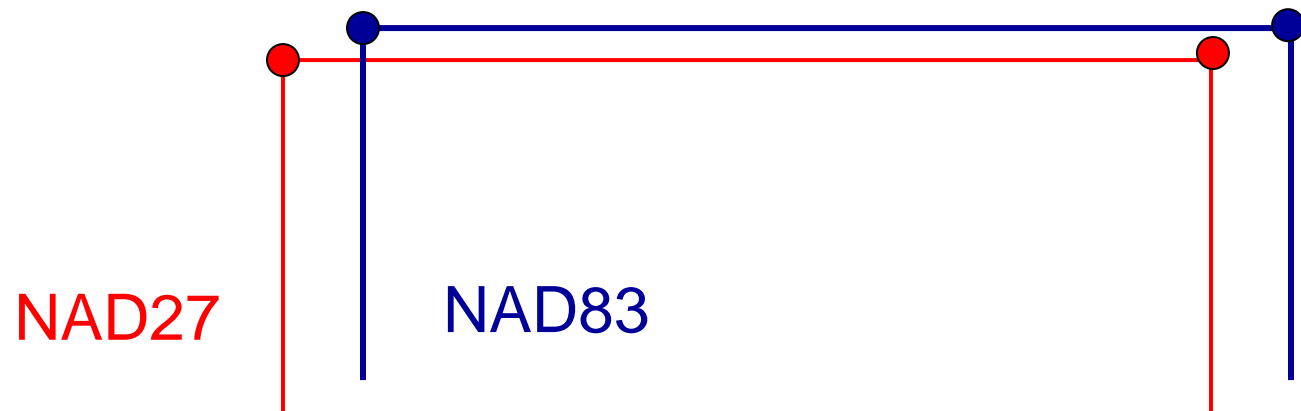
---

- ✦ In North America: two geodetic datum are commonly used for displaying spatial data:
  - 1. North American Datum of 1983 (NAD83)** - based on 250,000 satellite measurements and GRS-80 Ellipsoid & more accurate; has been refined a few times since (e.g. NAD 83( CORS 96) and NAD 83(NSRS2007))
    - Units: Meters
  - 2. North American Datum of 1927 (NAD27)** - based on 25,000 ground surveys and Clarke 1866 Ellipsoid & less accurate.
    - Units: Feet

# ***NAD27 vs. NAD83***

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✦ Shift is shown on USGS Quad maps and will result in map projections from one datum to another.



# ***Geo-Referencing Systems***

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- ✦ As mentioned previously, all data displayed and analyzed in a GIS have a spatial component (geographic reference – a reference to a location on the earth)
- ✦ In order to display and analyze multiple layers of spatial data for the same geographic area in a GIS, the multiple data layers must be associated with a geo-referencing system (or coordinate system)
- ✦ A coordinate system is closely related to a datum (has an origin)

# ***Geo-Referencing Systems***

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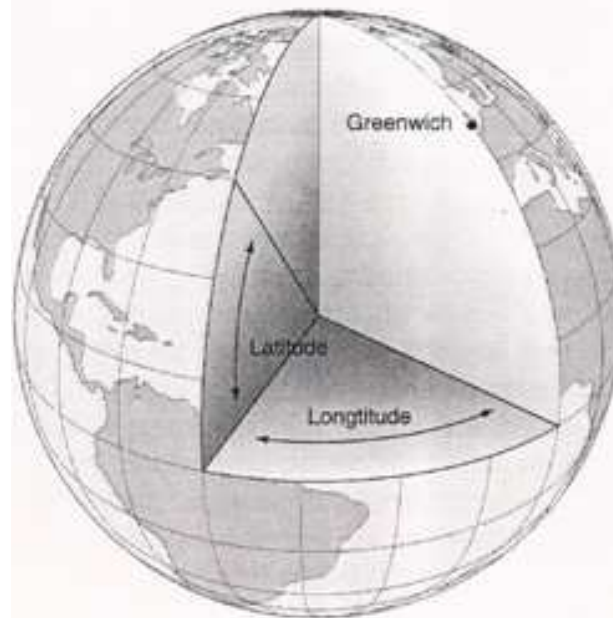
- ✦ Locate points precisely
- ✦ 2-D (X,Y) or 3-D (X,Y,Z)
- ✦ Two major types used today
  - Geographical (latitude/longitude) - curved Earth
  - Rectangular or Plane (aka Projected)- based on a grid with an origin (i.e. Cartesian coords) - flat map

# Geo-Referencing Systems



## Geographic Grid

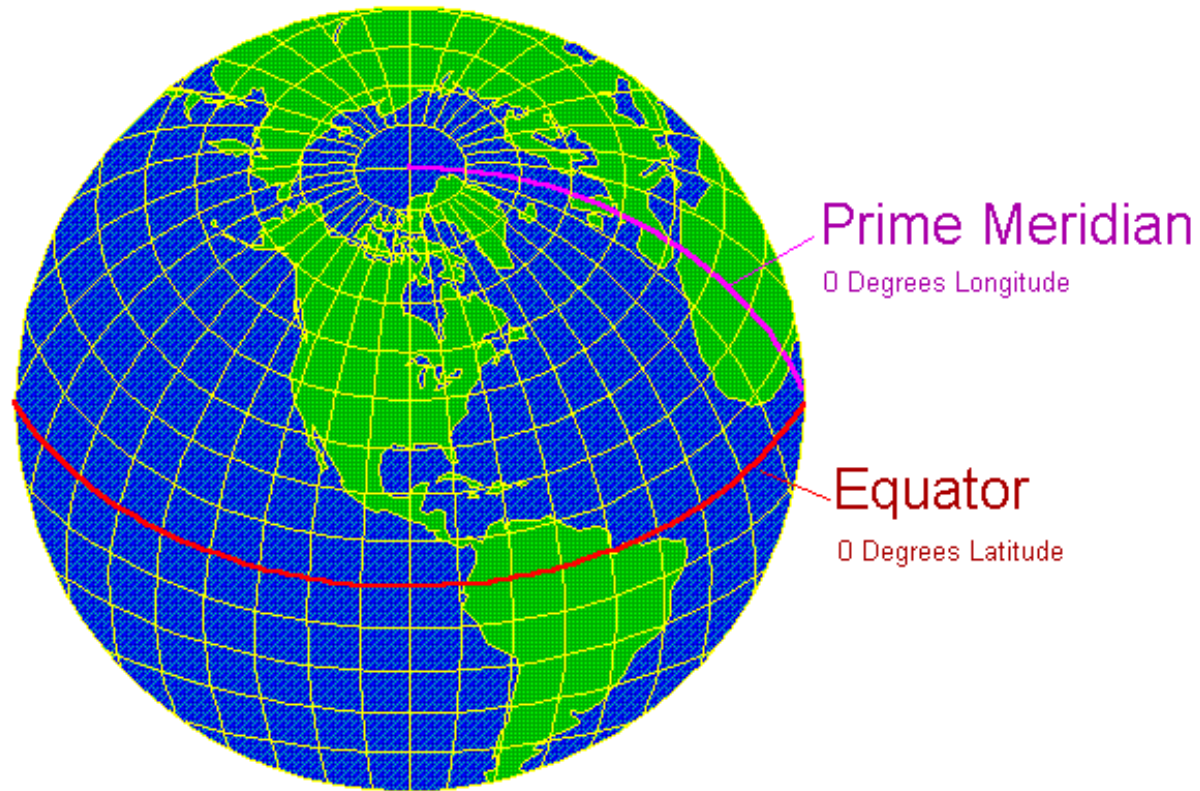
- a.k.a Latitude & Longitude
- Continuous, 3-D
- is geo-referenced... but not suitable for 2D mapping (not planar)



**Figure 2.5 The geographic grid.** Spherical grid system showing parallels and meridians. Parallels allow us to measure angular distance north and south (latitude) from the equator (0 degrees latitude) up to a maximum of 90 degrees north (North Pole) and 90 degrees south (South Pole). Meridians start at the prime meridian and allow us to measure angular distance east and west (longitude) up to a maximum of 180 degrees where they would meet at the international date line. Source: Robinson et al., *Elements of Cartography*, 6th ed., John Wiley & Sons, Inc., New York, © 1995, modified from Figure 4.4, page 47. Used with permission.

# ***Geographical Coordinates***

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Peter H. Dana 9/1/94



# ***Geographic Coordinates***

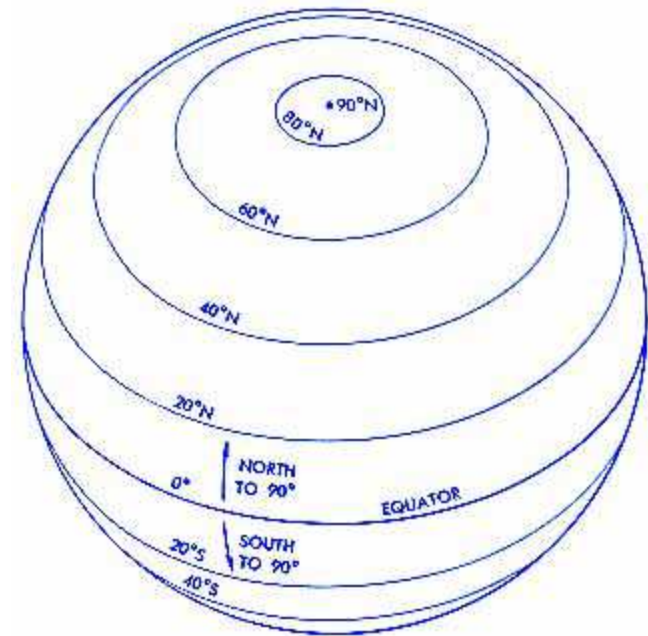
---

- ✦ **Latitude** ( $y$ ) - north-south distance from the equator - the "origin". Also called parallels.
- ✦ **Longitude** ( $x$ ) - east-west angular distance from a prime meridian - the "origin". Also called meridians.
- ✦ Not a map projection - a set of spherical coordinates used to reference positions on the curved surface of the Earth for use in map projections.
- ✦ Basis for projected coordinate systems

# *Latitude*

---

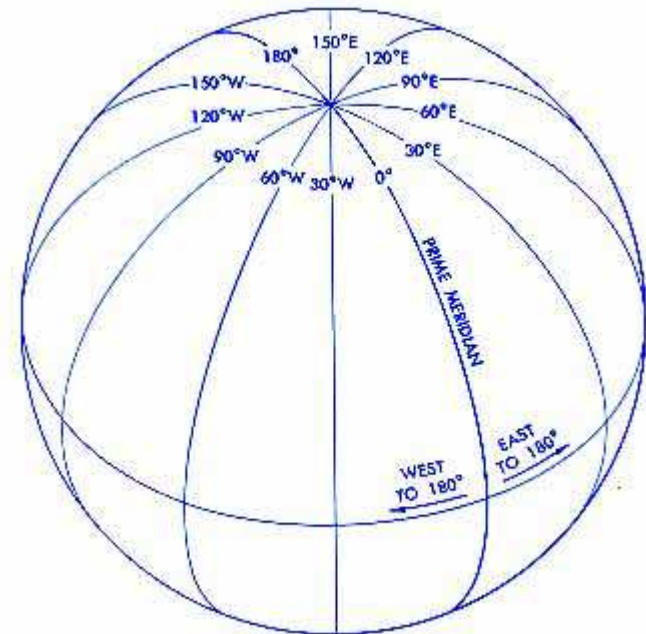
- ✧ A function of the angle between the horizon and the North Star
- ✧ Range: 0 – 90 degrees
- ✧ Origin: The Equator
- ✧ Direction: North and South of the origin



# *Longitude*

---

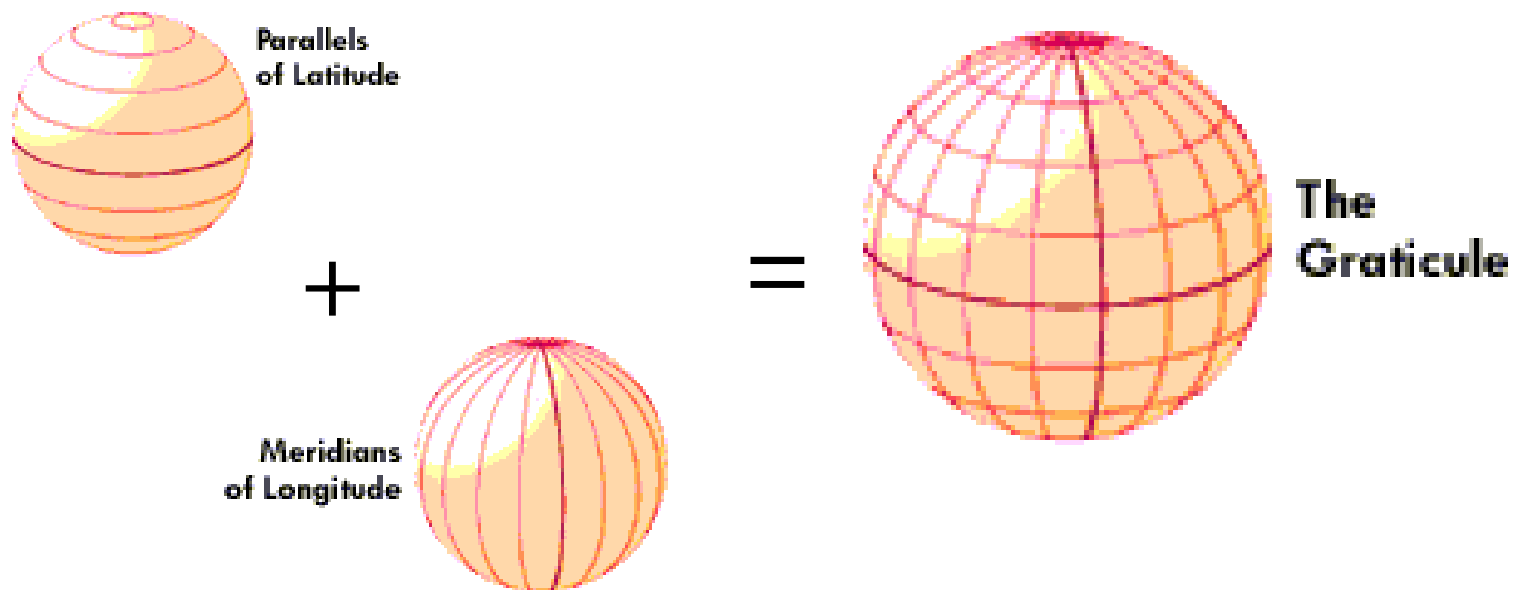
- ✦ Comprises meridians (half circles)
- ✦ Range: 0 – 180 degrees
- ✦ Origin: The Prime Meridian
- ✦ Direction: East and West of the origin



# *The Graticule*

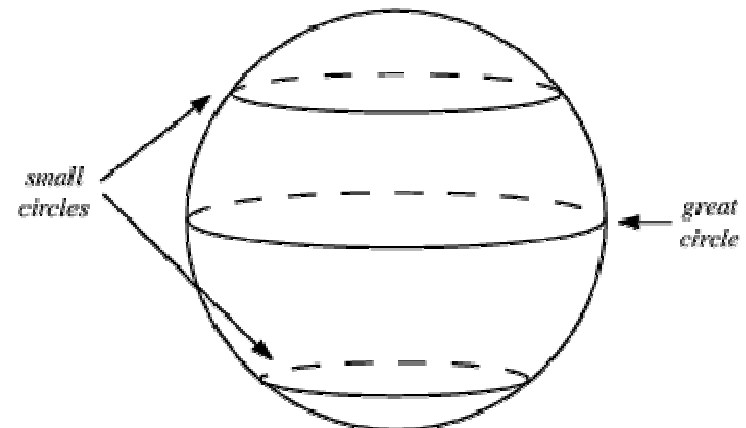
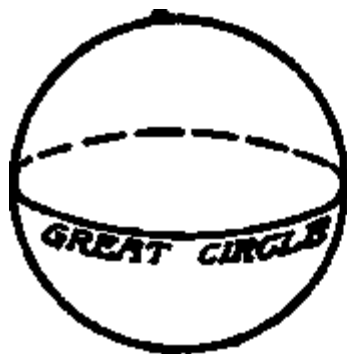
---

- ✦ The **graticule** is the gridded network of latitude and longitude, the pattern that the meridians of longitude and the parallels of latitude form on the surface of the earth.



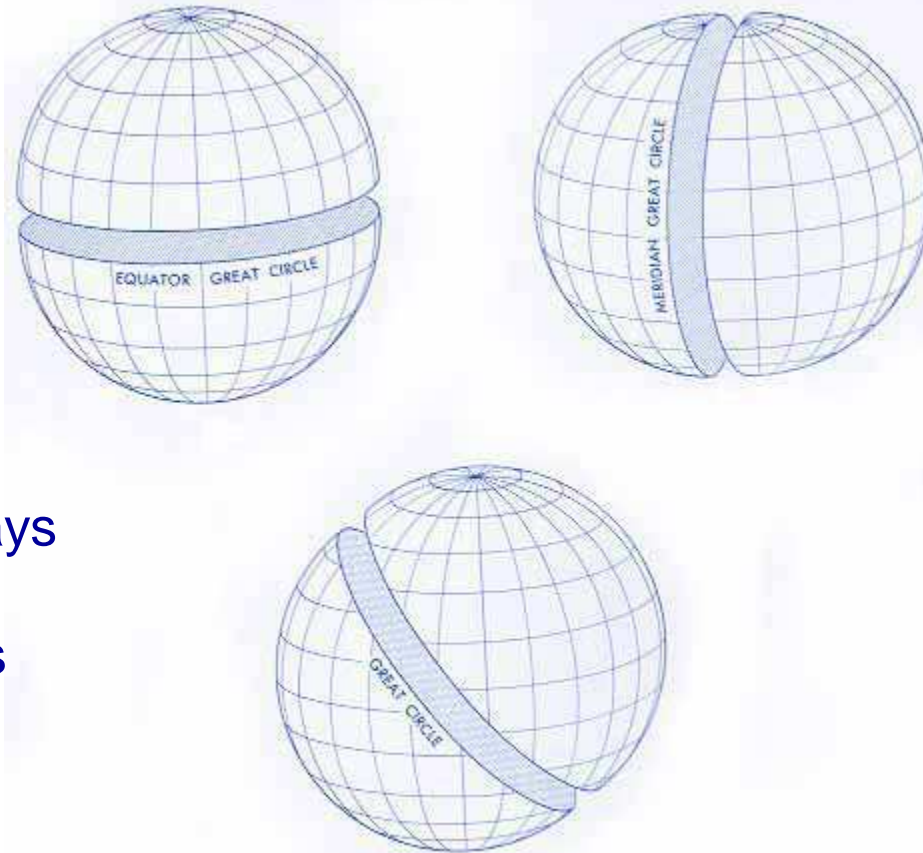
# Properties of the Graticule

- ✦ **Great Circle** - circle created by a plane that intersects 2 points and the center of the Earth (i.e. meridians or the equator)
- ✦ **Small Circle** - any circle created by an intersecting plane that doesn't cut through the center of the Earth (any parallel other than the equator)



# *Properties of the Graticule*

---

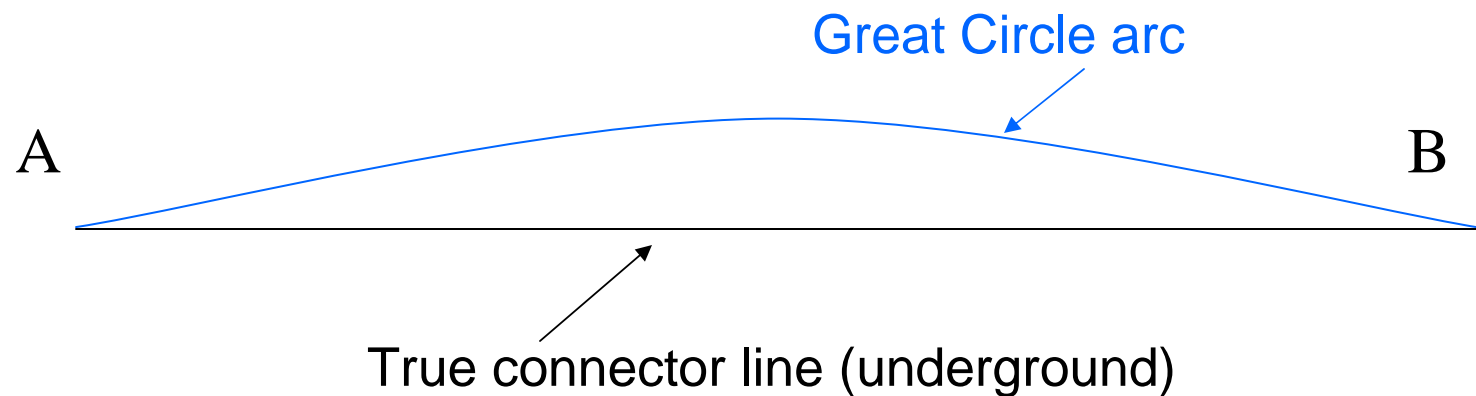


Different ways  
of creating  
great circles

See <http://www.csulb.edu/~rodrigue/geog140/lectures/geographicgrid.html>

# Measuring Distance

- ✦ The shortest distance between 2 points on a curved surface is the great circle arc above the true "connector line"



# ***Direction on the Graticule***

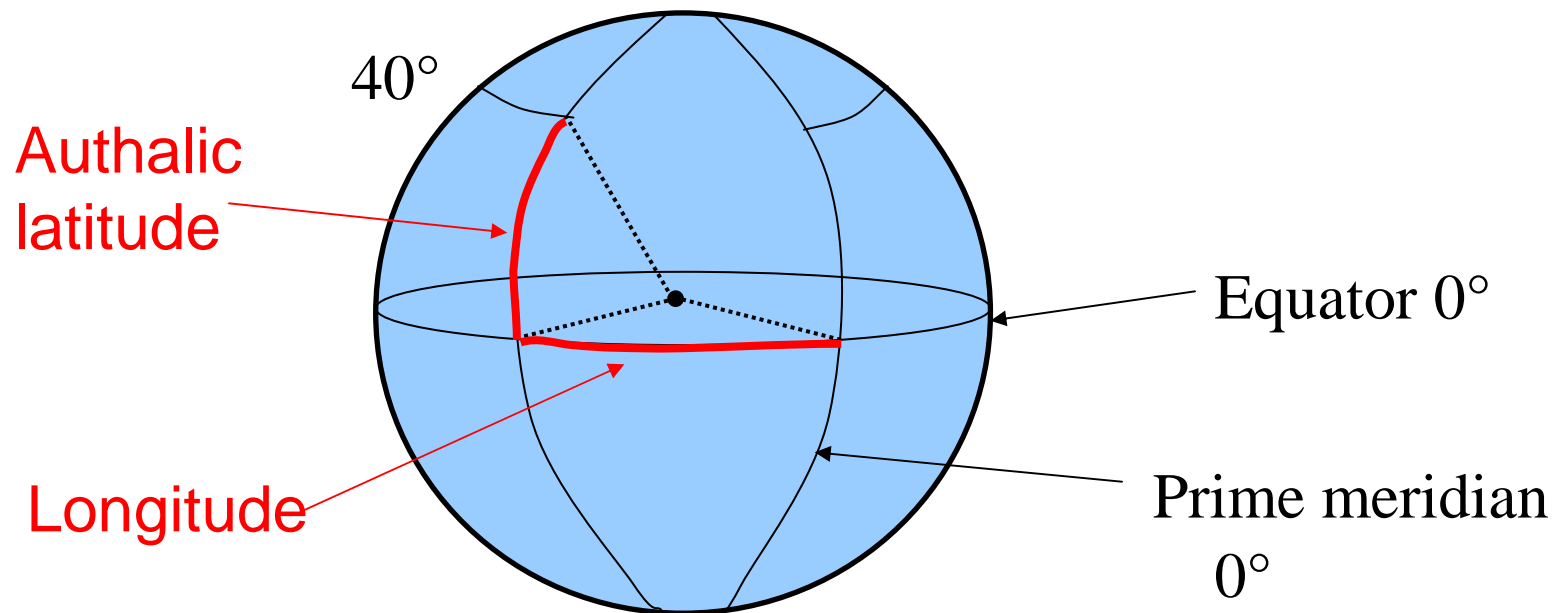
---

- ✦ Bearing, course, heading, flightline, azimuth are other terms for direction
- ✦ **True azimuth** - cross meridians at different angles
- ✦ **Constant azimuth** (rhumb line) - cross meridians at the same angle
- ✦ Direction is important to commercial flight maps



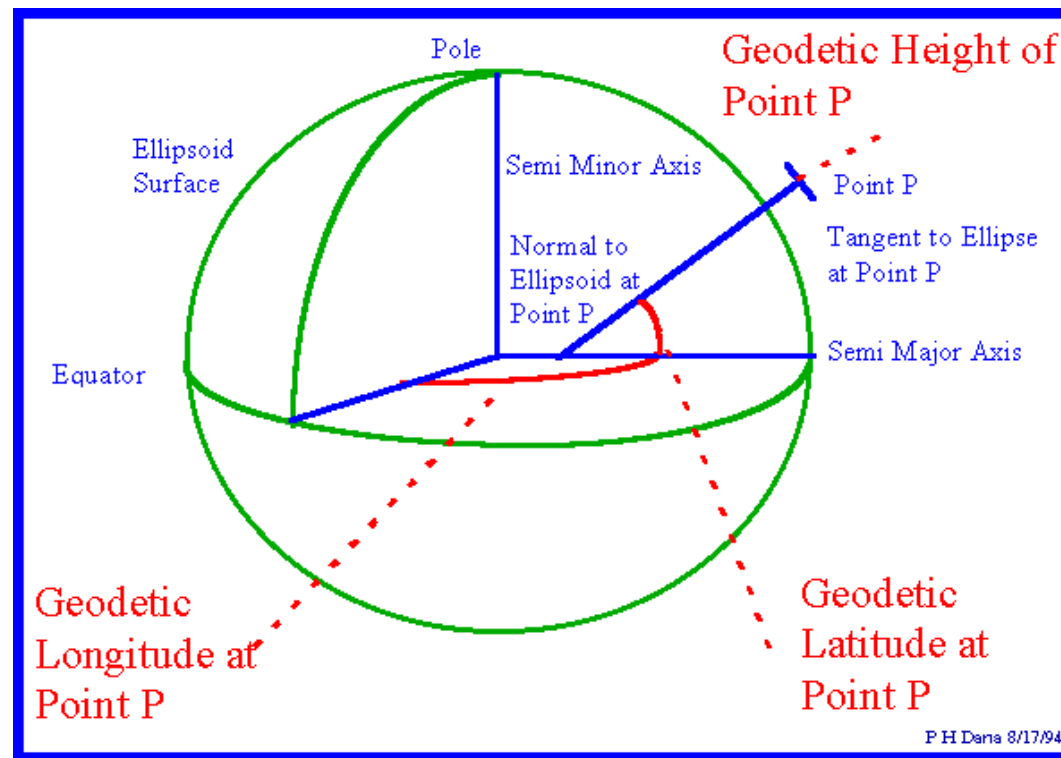
# ***Types of Latitude and Longitude***

- ✦ **Authalic** - measured from the center of the earth; measured on a sphere



# Types of Latitude and Longitude

- ✦ Geodetic - measured perpendicular to the surface; measured on an ellipsoid



# *Latitude/Longitude Coordinates*

---

## ✦ 1 - Degrees, minutes, seconds (DMS)

**dddmmssD Or ddd mm'ss" D**

where ddd = 1-3 digits for degrees, mm = 2 digits for minutes, ss = 2 digits for seconds and D = N,S,E, or W. The seconds and special characters (spaces, apostrophes, quotes) are all optional in this format. This leads to quite a large number of possible valid formats.  
Ex. 42° 37' 30" N 71° 00' 50" W

## ✦ 2 -Decimal degrees (DD)

**ddd.ffffD**

where ddd = 0-3 digits, ffff = 0-10 digits and D = N,S,E, or W. This format represents a decimal number of degrees. If the number of degrees is a whole number, the decimal point is optional.  
Ex. 42.625° N 71.014° W

# Conversion of DMS and DD

---

- ✳ 360 degrees in a circle
- ✳ 60 minutes to a degree
- ✳ 60 seconds to a minute, so ...

## DMS to DD

$$42^{\circ} 37' 30'' \text{ N} = 42.625^{\circ} \text{ N}$$

$$30/60 = .5 \rightarrow 37.5$$

$$37.5/60 = .625$$

## DD to DMS

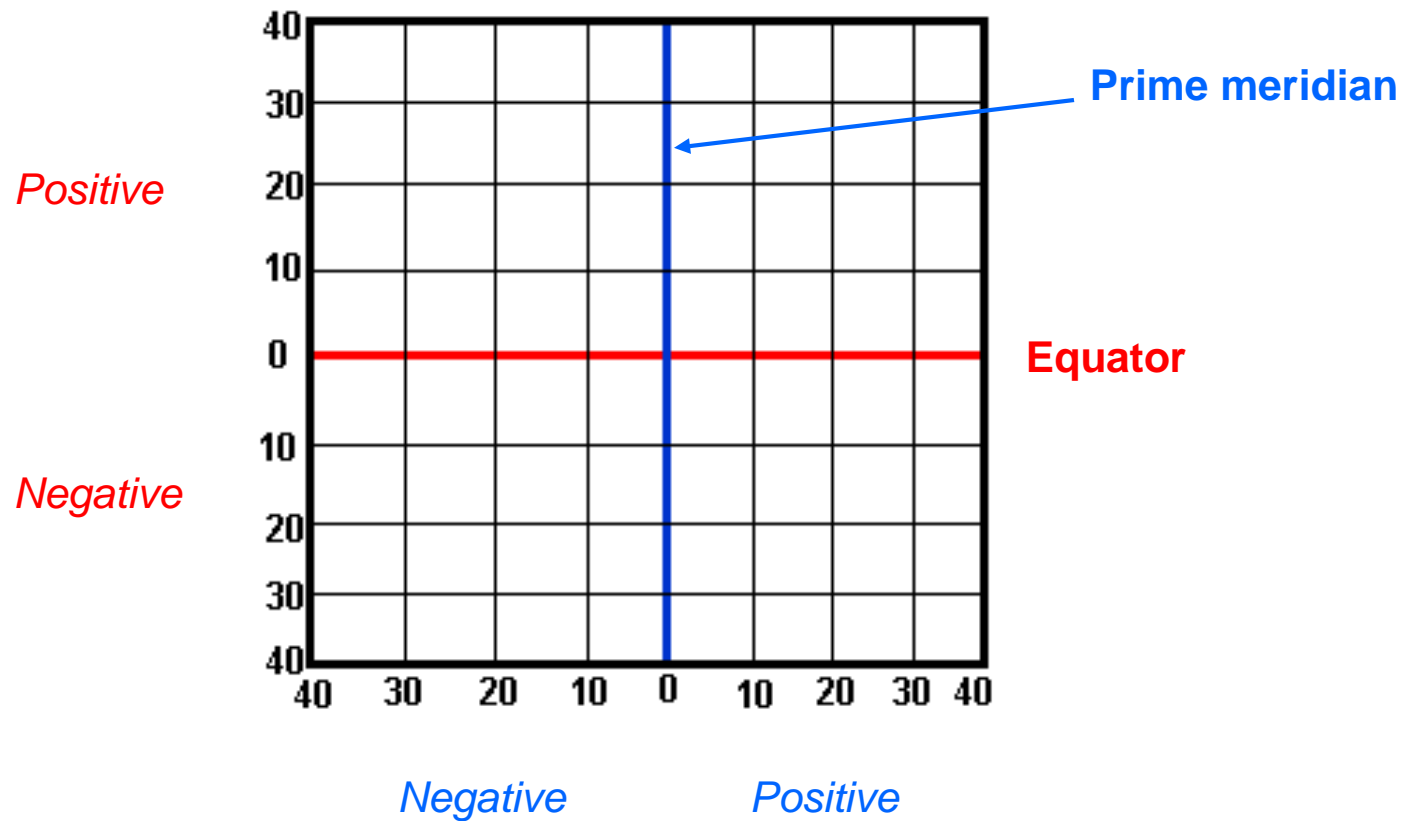
$$42.625^{\circ} \text{ N} = 42^{\circ} 37' 30'' \text{ N}$$

$$.625 \times 60 = 37.5$$

$$.5 \times 60 = 30$$

# *Latitude/Longitude Coordinates*

✦ Remember - positive or negative??

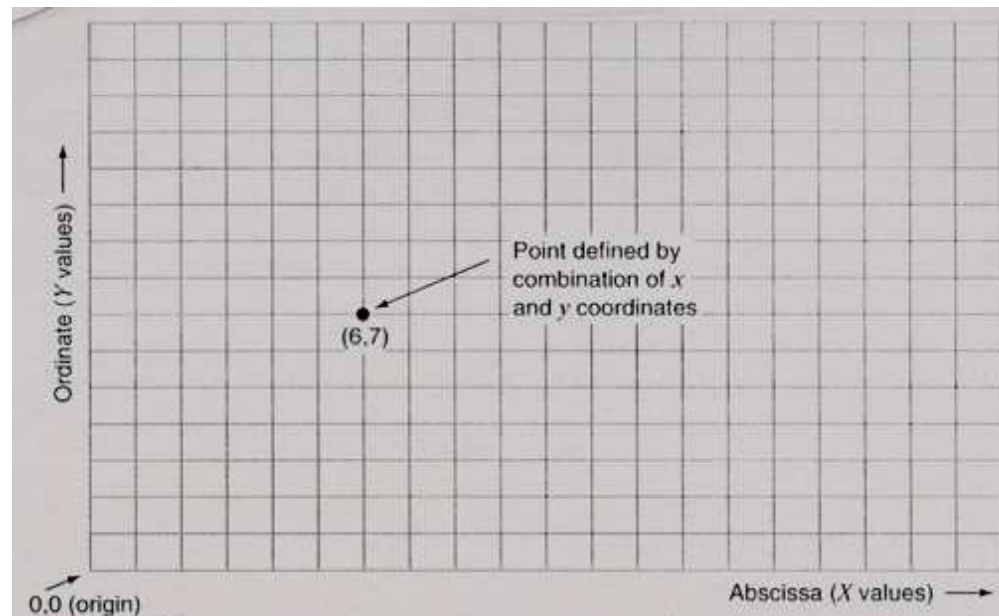


# Geo-Referencing Systems



## Cartesian

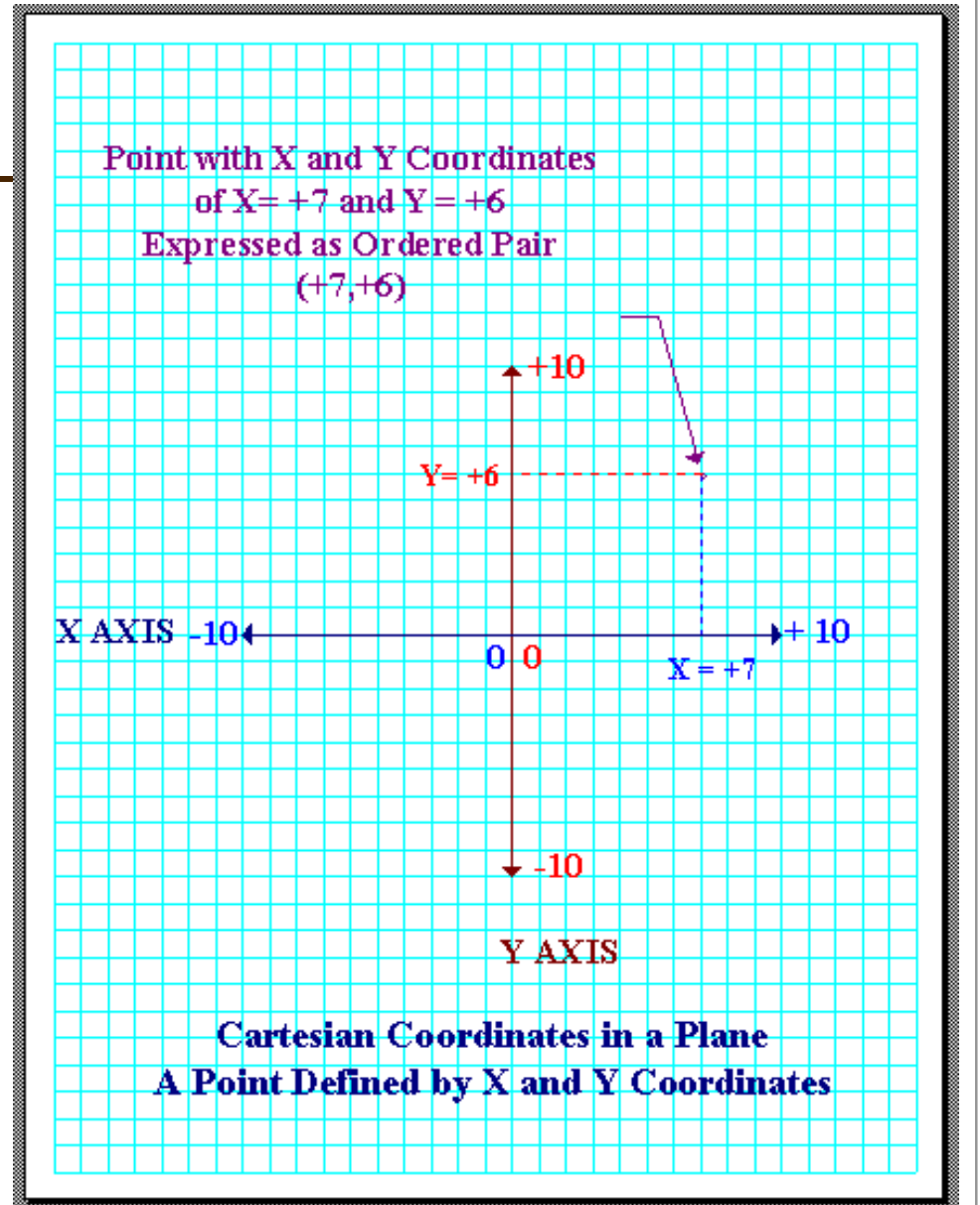
- A planar coordinate system (X and Y axis)
- Geo-referenced only when related to the graticule and real-world units are used



**Figure 3.5** A cartesian coordinate system. Classic rectangular grid system used to represent the number system. The abscissa (X values) is horizontal and the ordinate (Y values) is vertical. Any point can be defined by combinations of these two.

# Cartesian Coordinates

- ✦ Based on an origin (0,0) and 2 axes (x and y)
- ✦ Read x first, y second
- ✦ x = easting  
y = northing



# *Coordinate Systems*

---

✦ In GIS, the two most common rectangular coordinate systems used for spatial data in the U.S. are:

1. Universal Transverse Mercator (UTM)
2. State Plane Coordinate System (SPC)

Note: Underlying these planar coordinate systems is the latitude/longitude graticule



# ***UTM Coordinate System***

---

## ✦ **Universal Transverse Mercator**

- ✦ World divided into north/south columns, each  $6^\circ$  longitude wide
- ✦ Origin is central meridian and equator
- ✦ Most widely used coordinate system in the world
- ✦ Primarily used for land masses extending in a north-south direction
- ✦ Modified for use on USGS Quad maps
  - central meridian of each zone given false easting of 500000
  - equator given false northing of 0

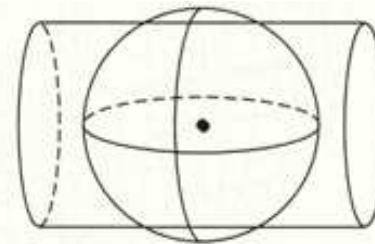
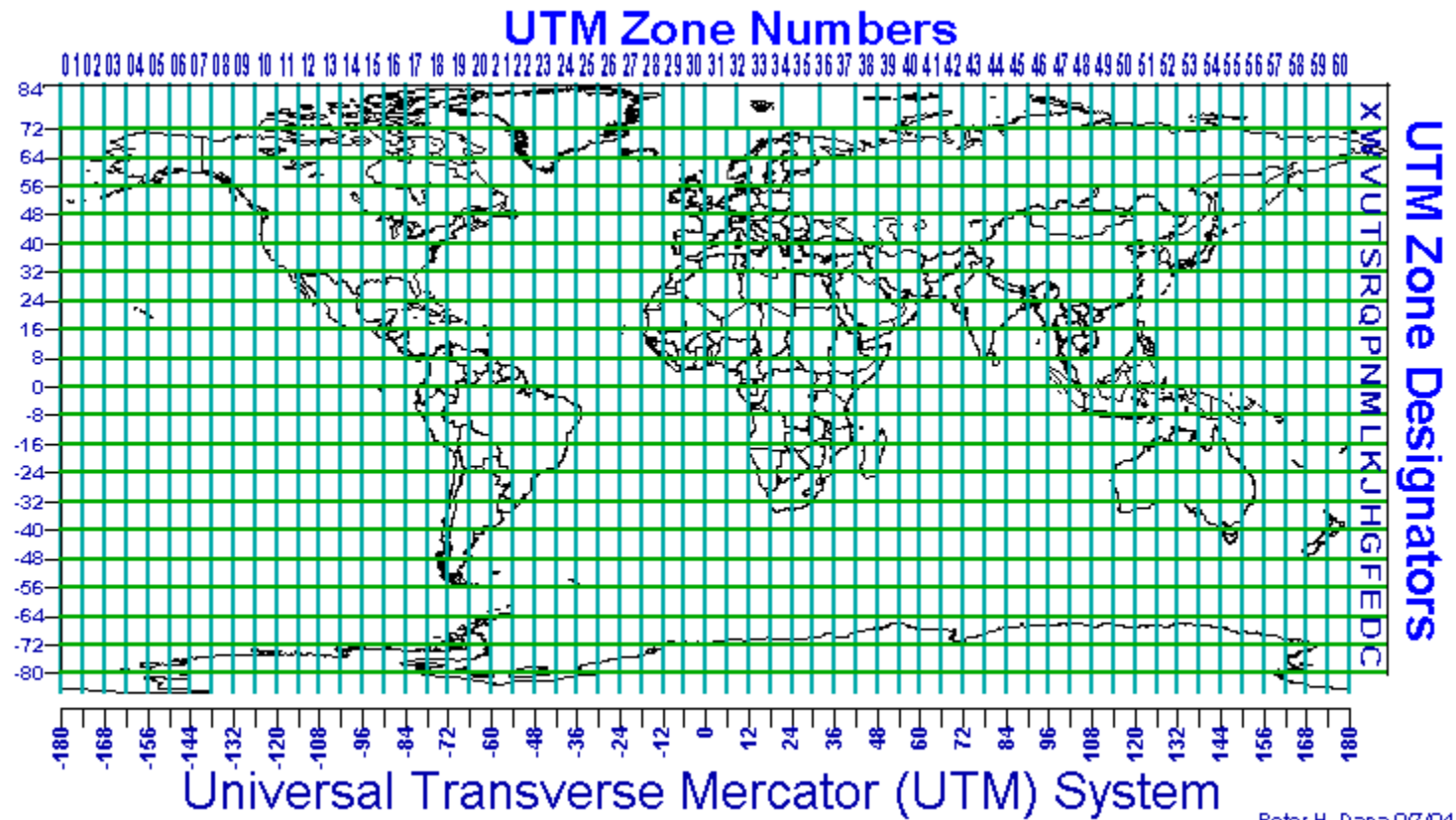


Figure 6.4

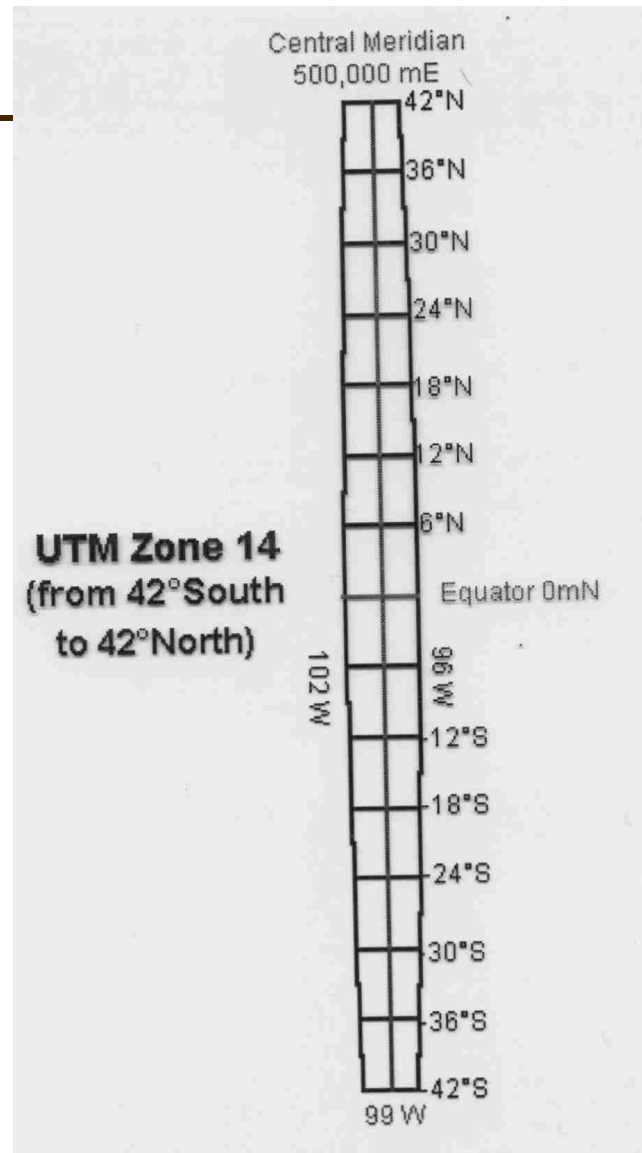
UTM is based on a cylindrical projection cutting through the globe. The zero point for the x axis is located on the equator, and that for the y axis, to the meridian through Greenwich.

# UTM Coordinate System



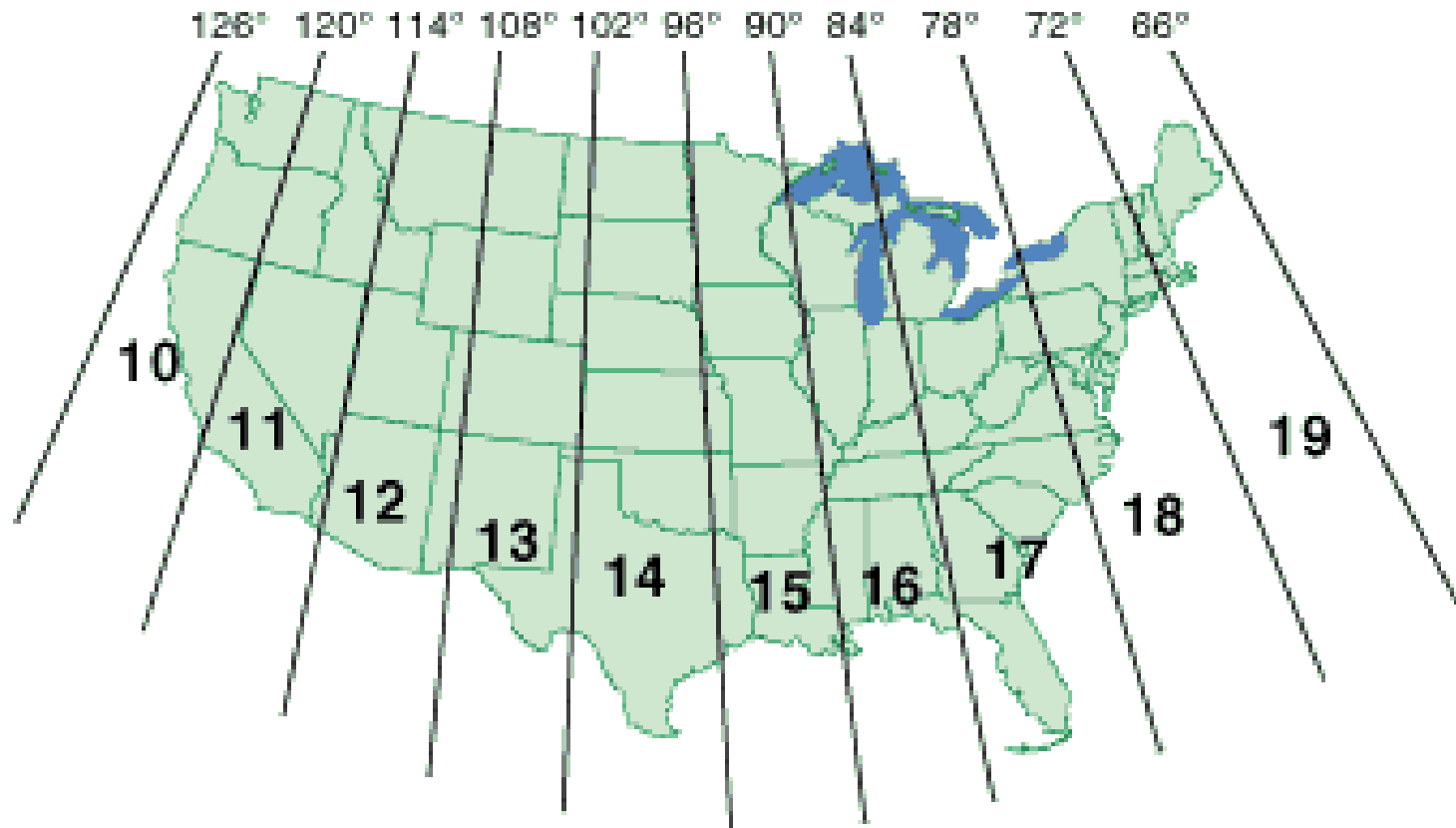
## UTM Coordinate System

- ✦ 60 zones (begin at 180W)
- ✦ Each zone:
  - Comprises 6 Degrees of Longitude
  - Has its own origin
- ✦ In the Northern Hemisphere:
  - origin is at the Equator and 500,000 (m) west of the Central Meridian



# ***UTM Zones in USA***

---



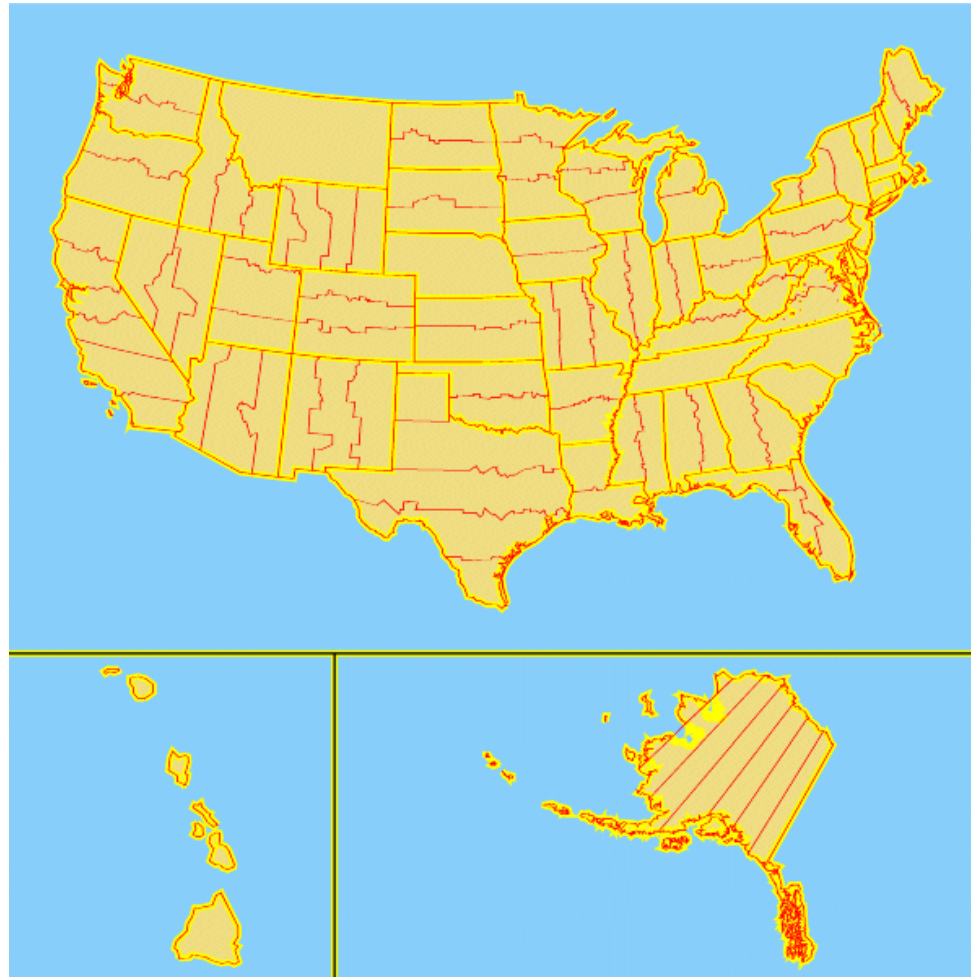
# ***State Plane Coordinates (SPC)***

---

- ✦ Rectangular grid, based on X, Y and false eastings and northings
- ✦ Developed in order to provide local reference systems that were tied to a national datum (i.e. NAD83).
- ✦ A coordinate system, not a projection
- ✦ Some states may have many zones
  - Mass. has Mainland and Island

# ***State Plane Coordinates (SPC)***

---



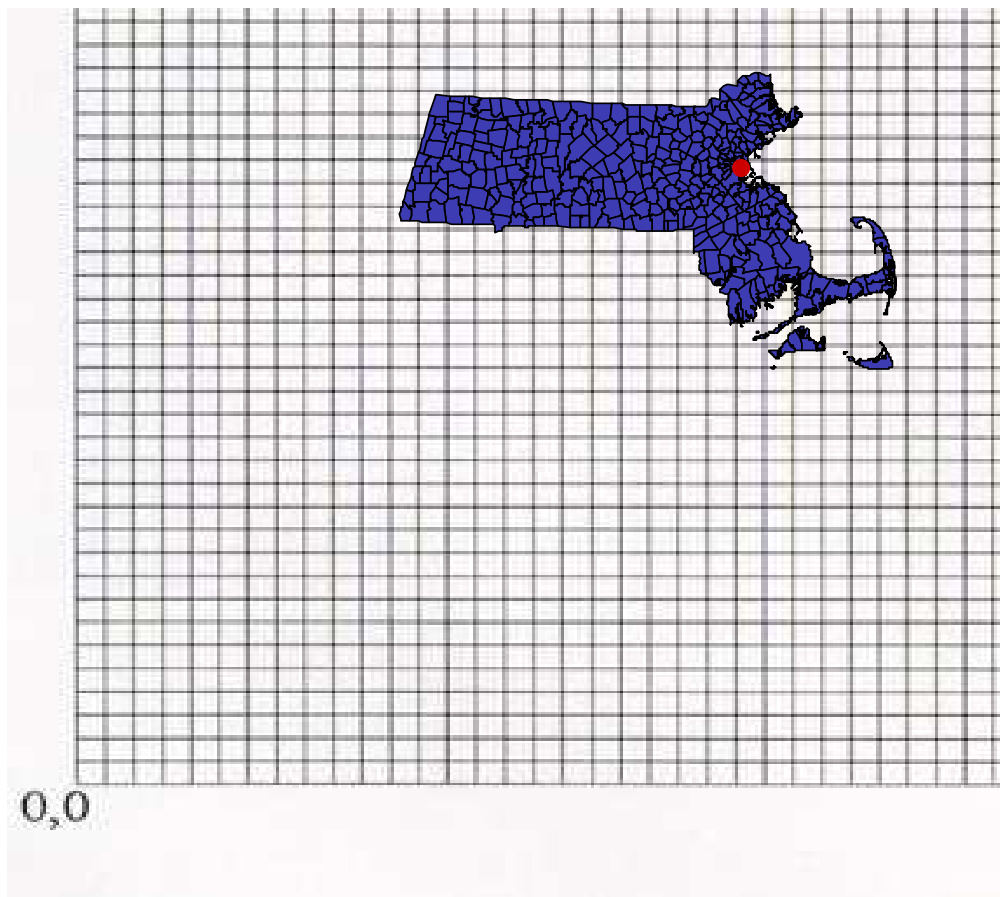
# ***False Easting and Northing***

---

- ✳ Arbitrary large values given to x and y axes so that all values in the grid will be positive
- ✳ The origin (0,0) moves to the southwest

# Massachusetts State Plane

Full domain of  
MASPC, with  
false easting  
and northing

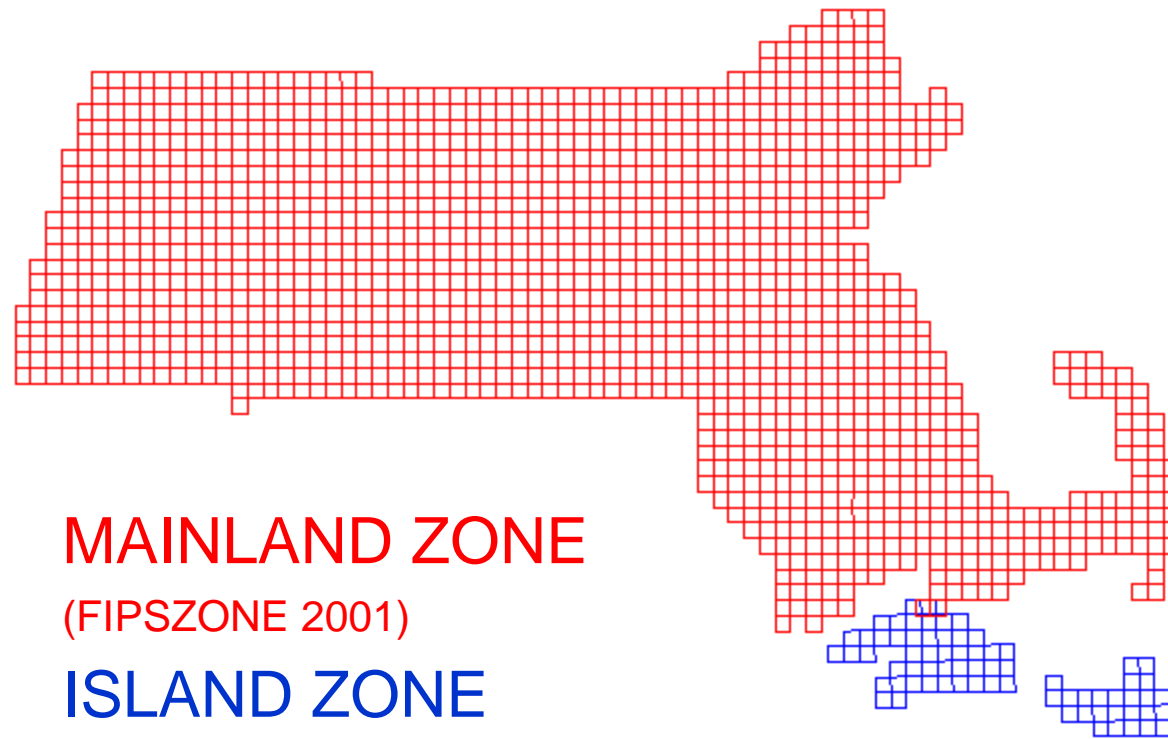


UMB Science Building: X: 238,100m Y: 895,960m



# *Massachusetts State Plane*

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**MAINLAND ZONE**

(FIPZONE 2001)

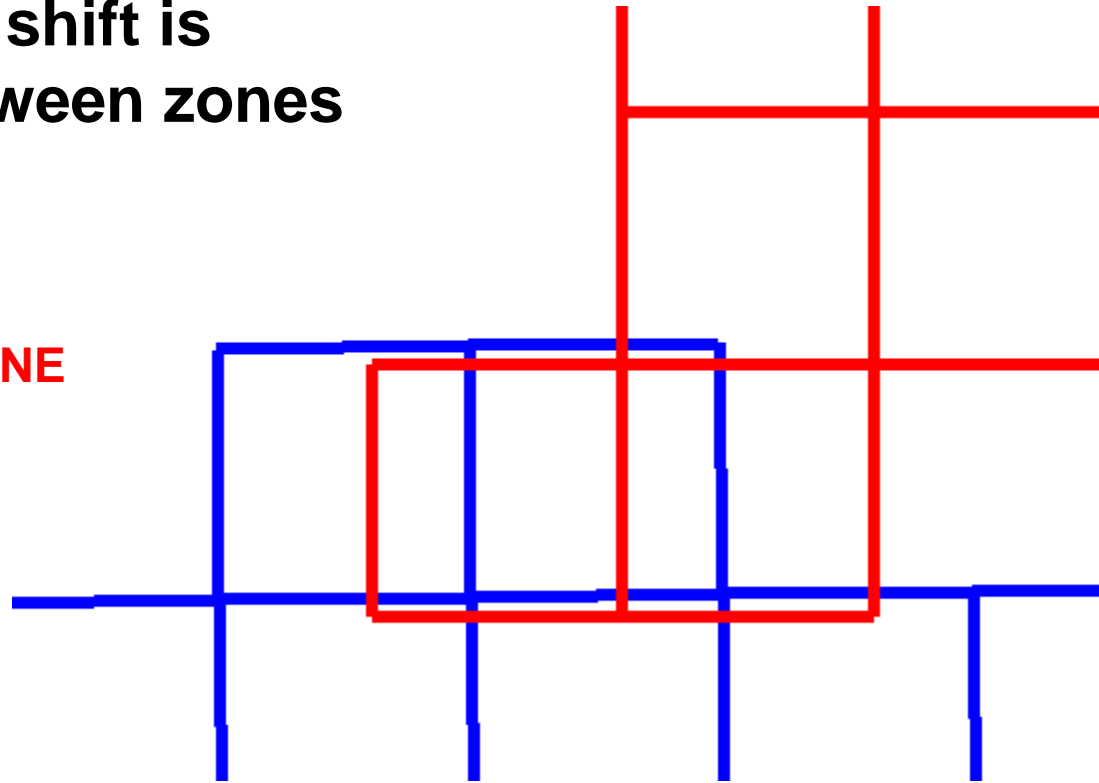
**ISLAND ZONE**

(FIPZONE 2002)

# Massachusetts State Plane

Zoomed in, shift is evident between zones

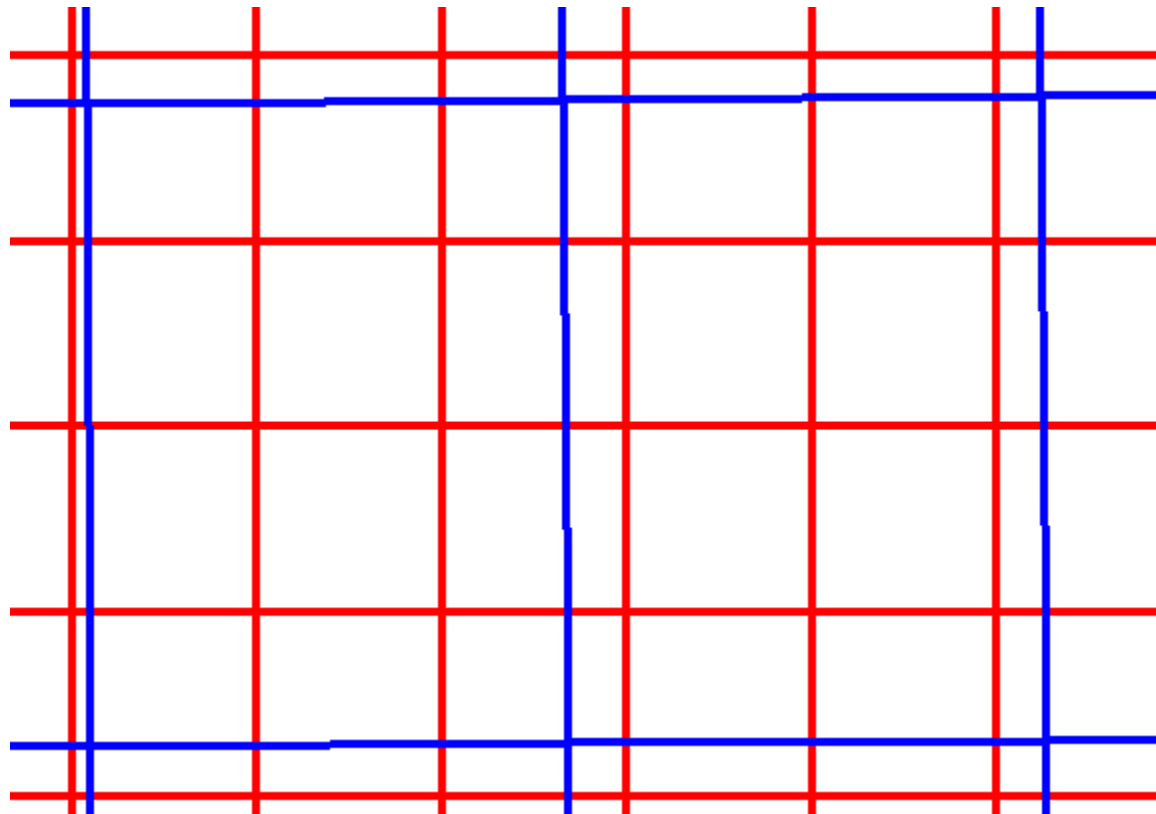
MAINLAND ZONE  
ISLAND ZONE



*The coordinate system used for this map image is Mainland Zone*

# ***Geographic vs. State Plane***

---

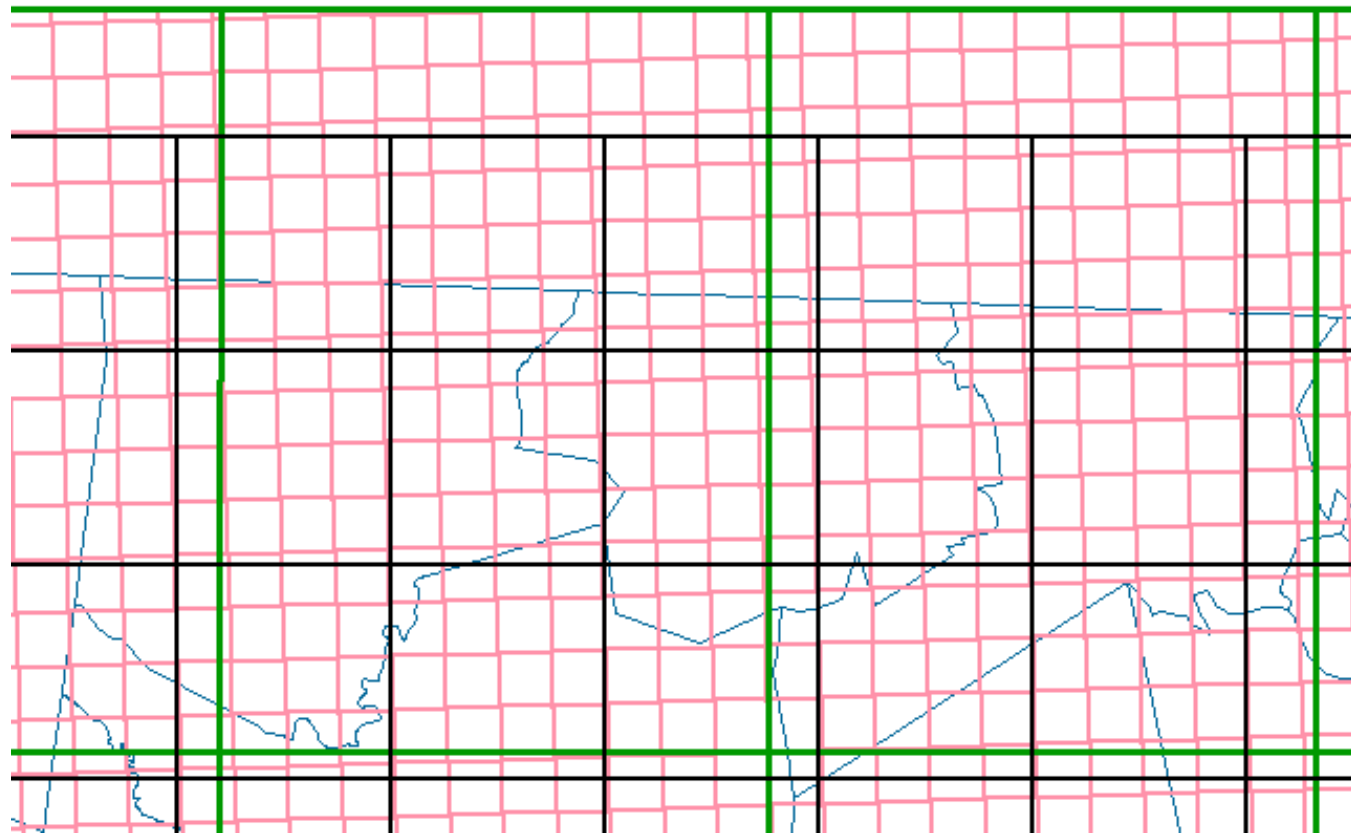


**MAINLAND ZONE SPC**

**USGS QUADS (Geographic)**

*The coordinate system used for this map image is SPC Mainland Zone*

# 3 Grids in Mass.



State Plane

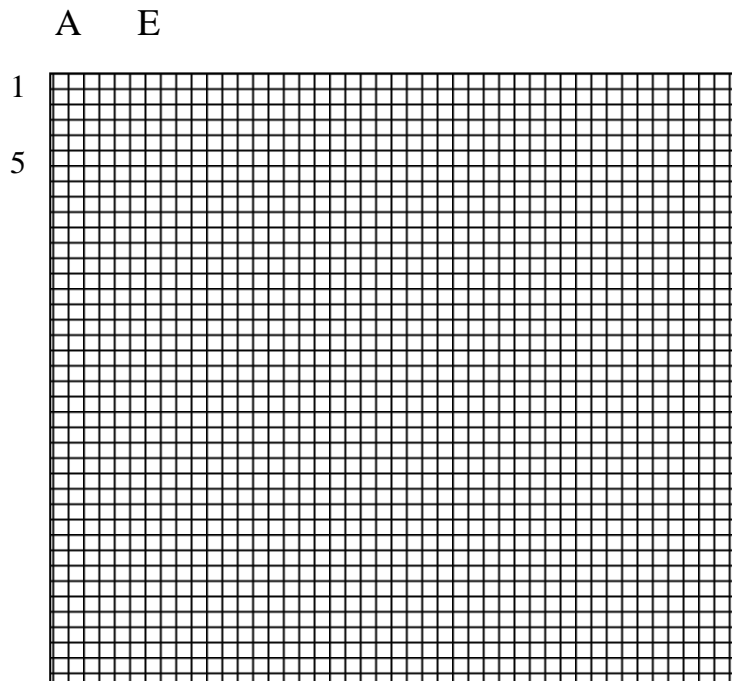
UTM

Lat/Long

# Reference Systems

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## ✦ Rectangular grids



Simple system  
used in atlases  
and on road maps;

May be added to  
ArcMap layouts,  
along with grids using  
real (LL, UTM, SP)  
coordinates

# *Coordinate System Codes*

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- ✦ FIPSZONES for state plane
- ✦ EPSG codes are commonly used to identify a coordinate system
  - European Petroleum Survey Group
  - <http://www.epsg.org>
  - <http://spatialreference.org/ref/epsg/>
- ✦ Also WKIDs (well-known IDs) for GCS
  - <https://developers.arcgis.com/javascript/jshelp/gcs.html>.

**NAD83 Mass. State Plane Mainland:**  
FIPSZONE = 2001  
EPSG = 26986

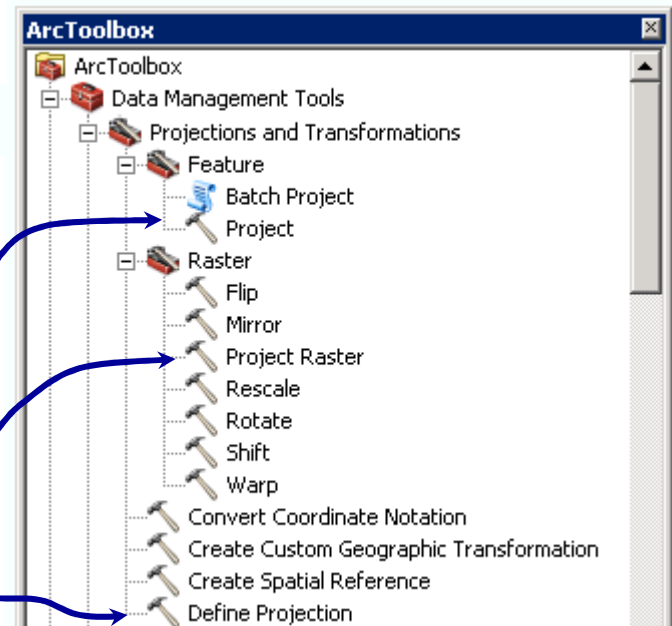
# Projections in ArcGIS

- ✦ All datasets have some coordinate system (aka “spatial reference”) although it may not be known (defined)
- ✦ Data in different coordinate systems are projected, or converted from one to another, on the fly, **ONLY IF** spatial references are properly defined.
  - Use the **DEFINE PROJECTION** tool if spatial reference is unknown or incorrect
  - If datasets with different spatial references are not properly defined, they will not align properly on the map.
- ✦ The **PROJECT** tool will create a new dataset with a converted (transformed) coordinate system.

PROJECT (vector)

PROJECT (raster)

DEFINE PROJECTION



# *Vertical Control*

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- ✦ Measurements of elevation that may serve as a reference to topographic mapping (i.e. of contour lines or digital elevation models).
- ✦ 2 networks have been devised:
  - National Geodetic Vertical Datum of 1929
  - North American Vertical Datum of 1988
- ✦ GPS now determines elevations based on the WGS 84 Ellipsoid latest revision



# Vertical Control

- ✦ The relationship between the reference ellipsoid height ( $h$ ) and the difference ( $N$ ) between the geoid and the ellipsoid determines vertical height ( $H$ ).

$$H = h - N$$

$$\text{Ex. } h = 20$$

$$N = -30$$

$$H = h - (-30) = 50$$

