10-4 Perimeters and Areas of Similar Figures

What You'll Learn

- To find the perimeters and areas of similar figures

. . . And Why

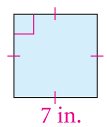
To find the expected yield of a garden, as in Example 3

Check Skills You'll Need

GO for Help Lesson 1-9

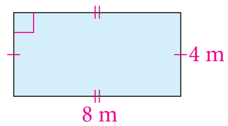
Find the perimeter and area of each figure.

1.



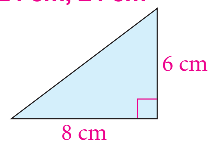
28 in.; 49 in.^{2}

2.



24 m; 32 m^2

3. 24 cm; 24 cm^2



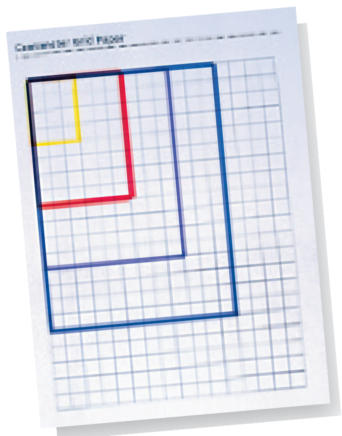
Find the perimeter and area of each rectangle with the given base and height.

4. $b = 1$ cm, $h = 3$ cm 8 cm; 3 cm^2

5. $b = 2$ cm, $h = 6$ cm 16 cm; 12 cm^2

6. $b = 3$ cm, $h = 9$ cm 24 cm; 27 cm^2

1 Finding Perimeters and Areas of Similar Figures



Hands-On Activity: Perimeters and Areas of Similar Rectangles

- On a piece of grid paper, draw a 3-unit by 4-unit rectangle.
- Draw three different rectangles, each similar to the original rectangle. Label them I, II, and III.

1. Use your drawings to complete a chart like this. **Check students' work.**

Rectangle	Perimeter	Area
Original		
I		
II		
III		

2. Use the information from the first chart to complete a chart like this. **Check students' work.**

Rectangle	Similarity Ratio	Ratio of Perimeters	Ratio of Areas
I to Original			
II to Original			
III to Original			

3. How do the ratios of perimeters and the ratios of areas compare with the similarity ratios? **See left.**

3. The ratio for perimeters is the same, but the ratio for areas is the similarity ratio squared.

Differentiated Instruction Solutions for All Learners

Special Needs L1

For the Hands-On Activity, have students use geoboards to create the similar rectangles. Have students start with a 2-unit by 3-unit rectangle, and restrict the choice of scale factors to fit on a geoboard.

learning style: tactile

Below Level L2

Before you go over Theorem 10-7, have students draw a triangle and three midsegments. Discuss how the four congruent triangles relate to Theorem 10-7.

learning style: visual

10-4

1. Plan

Objectives

1 To find the perimeters and areas of similar figures

Examples

1 Finding Ratios in Similar Figures
2 Finding Areas Using Similar Figures
3 Finding Similarity and Perimeter Ratios

Professional Development

Math Background

The Distributive Property readily proves that the ratio of the perimeters of two similar figures with the similarity ratio $a : b$ is also $a : b$. To prove that the ratio of the areas of two similar triangles with the similarity ratio $a : b$ is $a^2 : b^2$, draw altitudes to corresponding sides and prove that the right triangles thus formed are similar. The Transitive Property allows the proportional relationship of the triangles' sides to be extended to their altitudes.

More Math Background: p. 530C

Lesson Planning and Resources

See p. 530E for a list of the resources that support this lesson.

PowerPoint

Bell Ringer Practice

Check Skills You'll Need
For intervention, direct students to:

Finding Perimeter
Lesson 1-9: Examples 1 and 2
Extra Skills, Word Problems, Proof Practice, Ch. 1

Finding Area
Lesson 1-9: Example 4
Extra Skills, Word Problems, Proof Practice, Ch. 1

2. Teach

Guided Activity

Hands-On Activity
Because all rectangles have four right angles, remind students that all rectangles with a 3 : 4 ratio of sides are similar.

1 EXAMPLE Math Tip

Point out that the ratios of the perimeters and areas were found without calculating the perimeter or area of either trapezoid. In fact, those measurements cannot be found for the given figures because only one side length of each is known.

2 EXAMPLE Error Prevention

Remind students not to use the similarity ratio as the ratio of the areas. Point out that area is measured in *square* units, so the ratio of the areas is the *square* of the similarity ratio.

Teaching Tip
After students finish Example 2, ask: *How do you know that all regular pentagons are similar?* **All regular figures are equilateral and equiangular. So, all angles of regular pentagons are congruent, and the ratio of the sides of any two regular pentagons is constant.**

3 EXAMPLE Visual Learners

Have students draw a rectangle for each plot of land to help them visualize the descriptions.

4 EXAMPLE Connection to Algebra

Students are used to solving an equation for one variable but not for the ratio of two variables. Discuss why taking the square root of a ratio is like solving two equations.

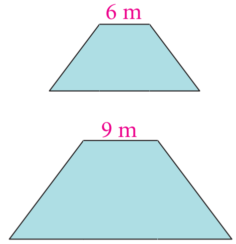
To compare areas of similar figures, you can square the similarity ratio.

Key Concepts

Theorem 10-7 Perimeters and Areas of Similar Figures

If the similarity ratio of two similar figures is $\frac{a}{b}$, then
(1) the ratio of their perimeters is $\frac{a}{b}$ and
(2) the ratio of their areas is $\frac{a^2}{b^2}$.

1 EXAMPLE Finding Ratios in Similar Figures



The trapezoids at the right are similar. The ratio of the lengths of corresponding sides is $\frac{6}{9}$, or $\frac{2}{3}$.

a. Find the ratio (smaller to larger) of the perimeters.

The ratio of the perimeters is the same as the ratio of corresponding sides, which is $\frac{2}{3}$.

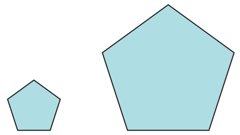
b. Find the ratio (smaller to larger) of the areas.

The ratio of the areas is the square of the ratio of corresponding sides, which is $\frac{2^2}{3^2}$, or $\frac{4}{9}$.

Quick Check 1 Two similar polygons have corresponding sides in the ratio 5 : 7.
a. Find the ratio of their perimeters. **5:7**
b. Find the ratio of their areas. **25:49**

When you know the area of one of two similar polygons, you can use a proportion to find the area of the other polygon.

2 EXAMPLE Finding Areas Using Similar Figures



Multiple Choice The area of the smaller regular pentagon is about 27.5 cm^2. What is the best approximation for the area of the larger regular pentagon?

Ⓐ 11 cm^2 Ⓑ 69 cm^2
Ⓒ 172 cm^2 Ⓓ 275 cm^2

Test-Taking Tip
You can eliminate choice A immediately. The area of the larger pentagon must be greater than 27.5 cm^2.

Regular pentagons are similar because all angles measure 108 and all sides in each are congruent. Here the ratio of corresponding-side lengths is $\frac{4}{10}$, or $\frac{2}{5}$. The ratio of the areas is $\frac{2^2}{5^2}$, or $\frac{4}{25}$.

$\frac{4}{25} = \frac{27.5}{A}$ **Write a proportion.**

$4A = 687.5$ **Cross-Product Property**

$A = \frac{687.5}{4} = 171.875$ **Solve for *A*.**

The area of the larger pentagon is about 172 cm^2. The answer is C.

Quick Check 2 The corresponding sides of two similar parallelograms are in the ratio $\frac{3}{4}$. The area of the larger parallelogram is 96 in.^{2}. Find the area of the smaller parallelogram. **54 in.^{2}**

Differentiated Instruction Solutions for All Learners

Advanced Learners L4
After Example 2, have students prove that the ratio of the areas of two similar regular polygons equals the square of the ratios of their sides.

learning style: verbal

English Language Learners ELL
Some students may confuse the *ratio of perimeters of similar figures* with the *ratio of areas of similar figures*. Point out that perimeter is a linear measure while area is measured in square units, and its similarity ratio is squared.

learning style: verbal

Real-World Connection

Many cities make city land available to the community for gardening.

3 EXAMPLE Real-World Connection

Community Service During the summer, a group of high school students used a plot of city land and harvested 13 bushels of vegetables that they gave to a food pantry. Their project was so successful that next summer the city will let them use a larger, similar plot of land.

In the new plot, each dimension is 2.5 times the corresponding dimension of the original plot. How many bushels can they expect to harvest next year?

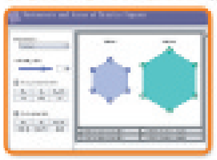
The ratio of the dimensions is $2.5 : 1$. So, the ratio of the areas is $(2.5)^2 : 1^2$, or $6.25 : 1$. With 6.25 times as much land next year, the students can expect to harvest 6.25(13), or about 81 bushels.

Quick Check 3 The similarity ratio of the dimensions of two similar pieces of window glass is $3 : 5$. The smaller piece costs \$2.50. What should be the cost of the larger piece? **\$6.94**

When you know the ratio of the areas of two similar figures, you can work backward to find the ratio of their perimeters.

4 EXAMPLE Finding Similarity and Perimeter Ratios

Online active math



For: Perimeter and Area Activity
Use: Interactive Textbook, 10-4

The areas of two similar triangles are 50 cm^2 and 98 cm^2. What is the similarity ratio? What is the ratio of their perimeters?

Find the similarity ratio $a : b$.

$\frac{a^2}{b^2} = \frac{50}{98}$ **The ratio of the areas is $a^2 : b^2$.**

$\frac{a^2}{b^2} = \frac{25}{49}$ **Simplify.**

$\frac{a}{b} = \frac{5}{7}$ **Take square roots.**

The ratio of the perimeters equals the similarity ratio $5 : 7$.

Quick Check 4 The areas of two similar rectangles are 1875 ft^2 and 135 ft^2. Find the ratio of their perimeters. **$5\sqrt{5} : 3$**

EXERCISES

For more exercises, see *Extra Skill, Word Problem, and Proof Practice.*

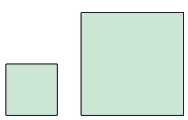
Practice and Problem Solving

A Practice by Example

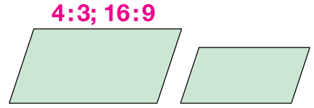
Example 1 (page 554)

GO for Help

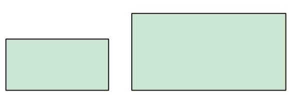
The figures in each pair are similar. Compare the first figure to the second. Give the ratio of the perimeters and the ratio of the areas.

1. 

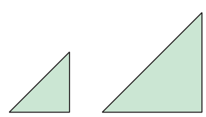
1:2; 1:4

2. 

4:3; 16:9

3. 

2:3; 4:9

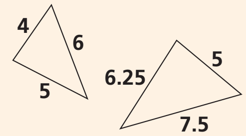
4. 

3:5; 9:25

PowerPoint

Additional Examples

1 The triangles below are similar. Find the ratio (larger to smaller) of their perimeters and of their areas.



perimeters: $\frac{5}{4}$; areas: $\frac{25}{16}$

2 The ratio of the lengths of the corresponding sides of two regular octagons is $\frac{8}{3}$. The area of the larger octagon is 320 ft^2. Find the area of the smaller octagon. **45 ft^2**

3 Benita plants the same crop in two rectangular fields, each with side lengths in a ratio of 2 : 3. Each dimension of the larger field is $3\frac{1}{2}$ times the dimension of the smaller field. Seeding the smaller field costs \$8. How much money does seeding the larger field cost? **\$98**

4 The areas of two similar pentagons are 32 in.^{2} and 72 in.^{2} What is their similarity ratio? What is the ratio of their perimeters? **2 : 3; 2 : 3**

Resources

- Daily Notetaking Guide 10-4 L3
- Daily Notetaking Guide 10-4—Adapted Instruction L1

Closure

The similarity ratio of two similar triangles is 5 : 3. The perimeter of the smaller triangle is 36 cm, and its area is 18 cm^2. Find the perimeter and area of the larger triangle. **perimeter: 60 cm; area: 50 cm^2**

3. Practice

Assignment Guide

1 **A B** 1-40
C Challenge 41-44

Test Prep 45-49
Mixed Review 50-61

Homework Quick Check

To check students' understanding of key skills and concepts, go over Exercises 4, 12, 35, 38, 39.

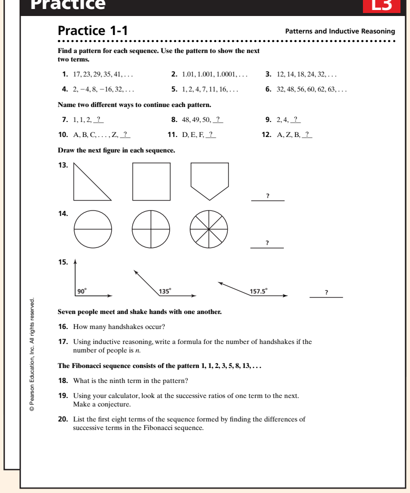
Connection to Statistics

Exercise 23 Misleading graphs often are found in magazines and newspapers, so everyone needs to know how to analyze graphs critically. Have students suggest how they would draw a more appropriate graph.

Exercise 22 Watch for students who use a ratio of $\frac{s^2}{4s^2}$ instead of $\frac{s^2}{(4s)^2} = \frac{s^2}{16s^2}$. Ask: If a side is four times larger, how much larger would its area be? $4^2 = 16$ **times larger**

Differentiated Instruction Resources

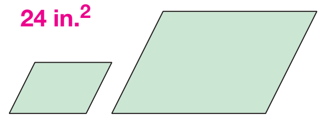
- GPS Guided Problem Solving L3
- Enrichment L4
- Reteaching L2
- Adapted Practice L1
- Practice L3


Practice 1-1 Patterns and Inductive Reasoning

Example 2 (page 554)

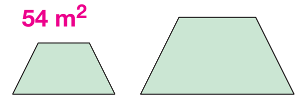
The figures in each pair are similar. The area of one figure is given. Find the area of the other figure to the nearest whole number.

5. **24 in.^{2}**



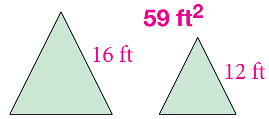
Area of smaller parallelogram $= 6$ in.^{2}

6. **54 m^2**



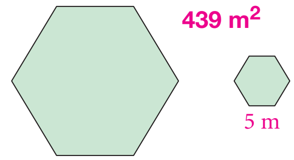
Area of larger trapezoid $= 121$ m^2

7. **59 ft^2**



Area of larger triangle $= 105$ ft^2

8. **439 m^2**



Area of smaller hexagon $= 65$ m^2

Example 3 (page 555)

9. Remodeling It costs a family \$216 to have a 9 ft-by-12 ft wooden floor refinished. At that rate, how much would it cost them to have a 12 ft-by-16 ft wooden floor refinished? **\$384**

10. Decorating An embroidered placemat costs \$2.95. An embroidered tablecloth is similar to the placemat, but four times as long and four times as wide. How much would you expect to pay for the tablecloth? **\$47.20**

Example 4 (page 555)

Find the similarity ratio and the ratio of perimeters for each pair of similar figures.

11. two regular octagons with areas 4 ft^2 and 16 ft^2 **1:2; 1:2**

12. two triangles with areas 75 m^2 and 12 m^2 **5:2; 5:2**

13. two trapezoids with areas 49 cm^2 and 9 cm^2 **7:3; 7:3**

14. two parallelograms with areas 18 in.^{2} and 32 in.^{2} **3:4; 3:4**

15. two equilateral triangles with areas $16\sqrt{3}$ ft^2 and $\sqrt{3}$ ft^2 **4:1; 4:1**

16. two circles with areas 2π cm^2 and 200π cm^2 **1:10; 1:10**

B Apply Your Skills

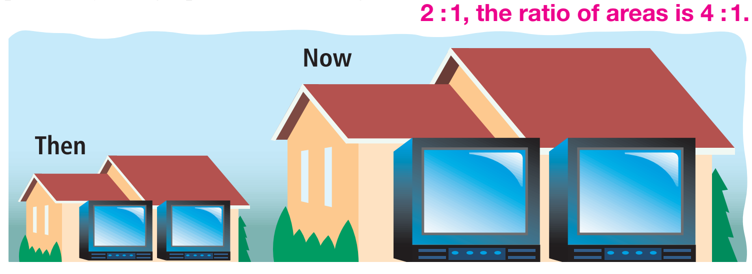
The similarity ratio of two similar polygons is given. Find the ratio of their perimeters and the ratio of their areas.

17. 3 : 1 **3:1; 9:1**
18. 2 : 5 **2:5; 4:25**
19. $\frac{2}{3}$ **2:3; 4:9**
20. $\frac{7}{4}$ **7:4; 49:16**
21. 6 : 1 **6:1; 36:1**

22. Multiple Choice The area of a regular decagon is 50 cm^2. What is the area of a regular decagon with sides four times the sides of the smaller decagon? **C**

(A) 200 cm^2 (B) 500 cm^2 (C) 800 cm^2 (D) 2000 cm^2

23. Error Analysis A reporter used the graphic below to show that the number of houses with more than two televisions had doubled in the past few years. Explain why this graphic is misleading. **While the ratio of lengths is 2 : 1, the ratio of areas is 4 : 1.**



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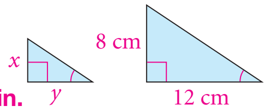
24. Medicine For some medical imaging, the scale of the image is 3 : 1. That means that if an image is 3 cm long, the corresponding length on the person's body is 1 cm. Find the actual area of a lesion if its image has area 2.7 cm^2. **0.3 cm^2**

25. The longer sides of a parallelogram are 5 m. The longer sides of a similar parallelogram are 15 m. The area of the smaller parallelogram is 28 m^2. What is the area of the larger parallelogram? **252 m^2**

Algebra **Find the values of x and y when the smaller triangle shown here has the given area.**

26. 3 cm^2 **27.** 6 cm^2 **28.** 12 cm^2
29. 16 cm^2 **30.** 24 cm^2 **31.** 48 cm^2 **26–31. See margin.**



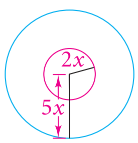
32. Two similar rectangles have areas 27 $in.^2$ and 48 $in.^2$. The length of one side of the larger rectangle is 16 in. What are the dimensions of both rectangles? **$2\frac{1}{4}$ in. by 12 in., 3 in. by 16 in.**

33. In $\triangle RST$, $RS = 20$ m, $ST = 25$ m, and $RT = 40$ m.
a. Open-Ended Choose a convenient scale. Then use a ruler and compass to draw $\triangle R'S'T' \sim \triangle RST$. **Check students' work.**
b. Constructions Construct an altitude of $\triangle R'S'T'$ and measure its length. Find the area of $\triangle R'S'T'$. **Check students' work.**
c. Estimation Estimate the area of $\triangle RST$. **Estimates may vary. Sample: 205 m^2**

34. Drawing Draw a square with an area of 8 $in.^2$. Draw a second square with an area that is four times as large. What is the ratio of their perimeters? **Ratio of small to large is 1:2.**

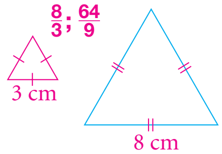
Compare the blue figure to the red figure. Find the ratios of (a) their perimeters and (b) their areas.

35.

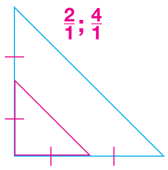


$\frac{5}{2}$; $\frac{25}{4}$

36. **$\frac{8}{3}$; $\frac{64}{9}$**

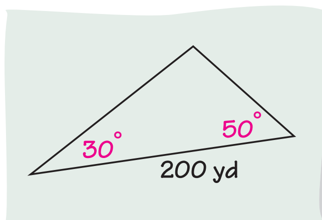


37. **$\frac{2}{1}$; $\frac{4}{1}$**

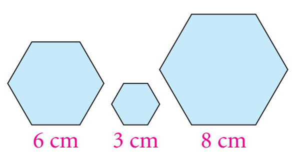


38. Writing The enrollment at an elementary school is going to increase from 200 students to 395 students. A parents' group is planning to increase the 100 ft-by-200 ft playground area to a larger area that is 200 ft by 400 ft. What would you tell the parents' group when they ask your opinion about whether the new playground will be large enough? **See left.**

39. a. Surveying A surveyor measured one side and two angles of a field as shown in the diagram. Use a ruler and a protractor to draw a similar triangle. **See margin.**
b. Measure the sides and altitude of your triangle and find its perimeter and area.
c. Estimation Estimate the perimeter and area of the field. **456 yd; 7600 yd^2**



40. a. Find the area of a regular hexagon with sides 2 cm long. Leave your answer in simplest radical form. **$6\sqrt{3}$ cm^2**
b. Use your answer to part (a) and Theorem 10-7 to find the areas of the regular polygons shown at the right.
$54\sqrt{3}$ cm^2; $13.5\sqrt{3}$ cm^2; $96\sqrt{3}$ cm^2



Real-World Connection

Careers Doctors use enlarged images to aid in certain medical procedures.

Problem Solving Hint

For Exercise 34, recall the length of a diagonal of a square with 2-in. sides.

38. Answers may vary. Sample: The proposed playground is more than adequate. The number of students has approximately doubled. The proposed playground would be four times larger than the original playground.

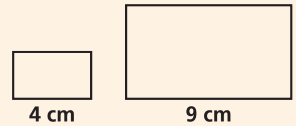
39b. 114 mm; 475 mm^2

4. Assess & Reteach

Lesson Quiz

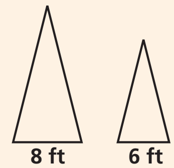
PowerPoint

1. For the similar rectangles, give the ratios (smaller to larger) of the perimeters and of the areas.



perimeters: $\frac{4}{9}$; areas: $\frac{16}{81}$

2. The triangles below are similar. The area of the larger triangle is 48 ft^2. Find the area of the smaller triangle.



27 ft^2

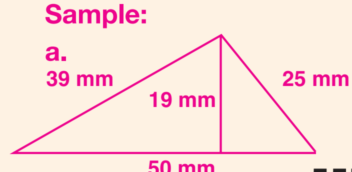
3. The similarity ratio of two regular octagons is 5 : 9. The area of the smaller octagon is 100 $in.^2$ Find the area of the larger octagon. **324 $in.^2$**
4. The areas of two equilateral triangles are 27 yd^2 and 75 yd^2. Find their similarity ratio and the ratio of their perimeters. **3 : 5; 3 : 5**
5. Mulch to cover an 8-ft by 16-ft rectangular garden costs $48. At the same rate, what would be the cost of mulch to cover a 12-ft by 24-ft rectangular garden? **$108**

Alternative Assessment

Have students work in pairs and use rulers and graph paper to estimate the area of a map of your state. Then have them use the map scale and Theorem 8-6 to estimate the actual area of the state.

26. $x = 2$ cm, $y = 3$ cm
27. $x = 2\sqrt{2}$ cm, $y = 3\sqrt{2}$ cm
28. $x = 4$ cm, $y = 6$ cm
29. $x = \frac{8\sqrt{3}}{3}$ cm, $y = 4\sqrt{3}$ cm
30. $x = 4\sqrt{2}$ cm, $y = 6\sqrt{2}$ cm
31. $x = 8$ cm, $y = 12$ cm

39. Answers may vary. Sample:
a.



2. Teach

Guided Instruction

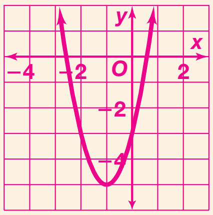
1 EXAMPLE Math Tip

Encourage students to enter $Y_1 = -3x^2 + 6x + 5$ into their graphing calculators. Have them press 2nd **TABLE** and identify pairs of points which are reflections across the axis of symmetry.

PowerPoint

Additional Examples

1 Graph the function $y = 2x^2 + 4x - 3$.



2 Suppose a particular star is projected from an aerial firework at a starting height of 610 ft with an initial upward velocity of 88 ft/s. How long will it take for the star to reach its maximum height? How far above the ground will it be? **2.75 s; 731 ft**

For: Quadratic Function Activity
Use: Interactive Textbook, 10-2

When you substitute $x = 0$ into the equation $y = ax^2 + bx + c$, $y = c$. So the y-intercept of a quadratic function is the value of c. You can use the axis of symmetry and the y-intercept to help you graph a quadratic function.

1 EXAMPLE Graphing $y = ax^2 + bx + c$

Graph the function $y = -3x^2 + 6x + 5$.

Step 1 Find the equation of the axis of symmetry and the coordinates of the vertex.

$x = \frac{-b}{2a} = \frac{-6}{2(-3)} = 1$ **Find the equation of the axis of symmetry.**

The axis of symmetry is $x = 1$.

$y = -3x^2 + 6x + 5$

$y = -3(1)^2 + 6(1) + 5$ **To find the y-coordinate of the vertex, substitute 1 for x.**

$= 8$

The vertex is $(1, 8)$.

Step 2 Find two other points on the graph.

Use the y-intercept.

For $x = 0$, $y = 5$, so one point is $(0, 5)$.

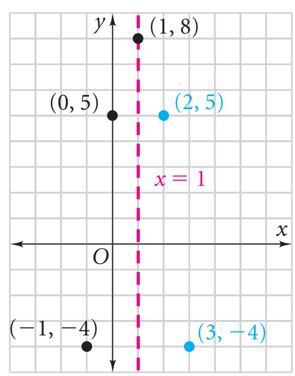
Choose a value for x on the same side of the vertex as the y-intercept. Let $x = -1$.

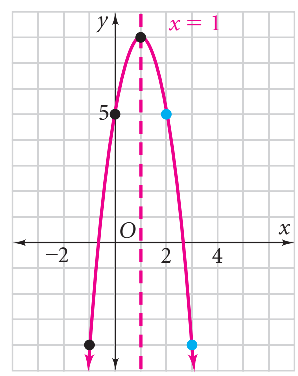
$y = -3(-1)^2 + 6(-1) + 5$ **Find the y-coordinate for $x = -1$.**

$= -4$

For $x = -1$, $y = -4$, so another point is $(-1, -4)$.

Step 3 Reflect $(0, 5)$ and $(-1, -4)$ across the axis of symmetry to get two more points. Then draw the parabola.





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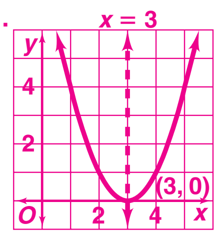

Quick Check

1 Graph $f(x) = x^2 - 6x + 9$. Label the axis of symmetry and the vertex.
See left.

You saw in the previous lesson that the formula $h = -16t^2 + c$ describes the height above the ground of an object falling from an initial height c, at time t. If an object is given an initial upward velocity v and continues with no additional force of its own, the formula $h = -16t^2 + vt + c$ describes its approximate height above the ground.

Differentiated Instruction Solutions for All Learners

Advanced Learners L4
Have students graph the quadratic function in Example 2.
learning style: verbal

English Language Learners ELL
Ask students if they have any ideas about how to find the area under a curve, as in Exercises 38 and 39. Explain that if no formula comes to mind, estimation is good problem-solving strategy for finding the answer to some questions.
learning style: verbal