Geometric Series Test.

Basic Divergence Test.

p-Series Test.

Integral Test.

Basic Comparison Test.

Limit Comparison Test.

Root Test

Ratio Test

Alternating Series Test for Convergence:

If the alternating series $\sum_{n=1}^{\infty}(-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \ldots \quad b_n > 0$
satisfies

(a) $b_{n+1} \le b_n$ for all n (b) $\lim_{n\to\infty} b_n = 0$

then the series is convergent.

Note: If $\lim_{n\to\infty} b_n \neq 0$, the series diverges by the Basic Test for Divergence, NOT by the Alternating Series Test.

Ex. $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$

Absolute and Conditional Convergence

If the series $\sum_{n=1}^{\infty} |a_n|$ converges, then the series $\sum_{n=1}^{\infty} a_n$ also converges.

$\sum_{n=1}^{\infty} a_n$ is **absolutely convergent** if $\sum_{n=1}^{\infty} |a_n|$ converges.

$\sum_{n=1}^{\infty} a_n$ is **conditionally convergent** if $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges

Note (rearrangement invariance):

If a series converges absolutely, then it will converge to the same value regardless of the order in which the terms are summed.

If the series converges conditionally, then the terms of the series can be rearranged to sum to any desired value.

Popper

1. $$\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$$

a. converges absolutely

b. converges conditionally

c. diverges

2. $$\sum_{n=1}^{\infty} \frac{2n+1}{5n^3+2n}$$

a. converges

b. diverges

Popper

3. $$\sum_{n=1}^{\infty} \frac{3n+1}{5n^2+2n}$$

a. converges

b. diverges

4. $$\sum_{n=1}^{\infty} \frac{n}{3^n}$$

a. converges

b. diverges

Popper

5. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

a. converges absolutely

b. converges conditionally

c. diverges

Popper

6. $$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} \arctan(n)}{n^2}$$

a. converges absolutely

b. converges conditionally

c. diverges

7. $$\sum_{n=1}^{\infty} \frac{n\,cos(n\pi)}{2^n}$$

a. converges absolutely

b. converges conditionally

c. diverges

8. $$\sum_{n=1}^{\infty} \frac{n\,cos(n\pi)}{n^2+1}$$

a. converges absolutely

b. converges conditionally

c. diverges

Popper

9. $$\sum_{n=1}^{\infty} \frac{\ln n}{n}$$

a. converges

b. diverges

Alternating Series Remainder

If a convergent alternating series satisfies the condition $0 < a_{n+1} \le a_n$, then the remainder R_N involved in approximating the sum S by S_N is less in magnitude than the first neglected (truncated) term. That is, $|R_N| = |S - S_N| \le a_{N+1}$.

The alternating Series Remainder is called remainder, error or $|S - S_N|$.

Ex: Approximate the sum of $\sum_{n=1}^{\infty}(-1)^{n-1}\left(\frac{1}{n!}\right)$ by its first six terms, and find the error.

Ex: Approximate the sum of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4}$ with an error of less than 0.001.

Section 11.5
Taylor Polynomials in x

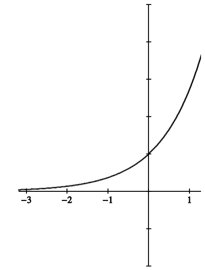
There are many functions that we only know at one point, or a handful of isolated points. Such as the trigonometric functions, e^x, ln x, etc.

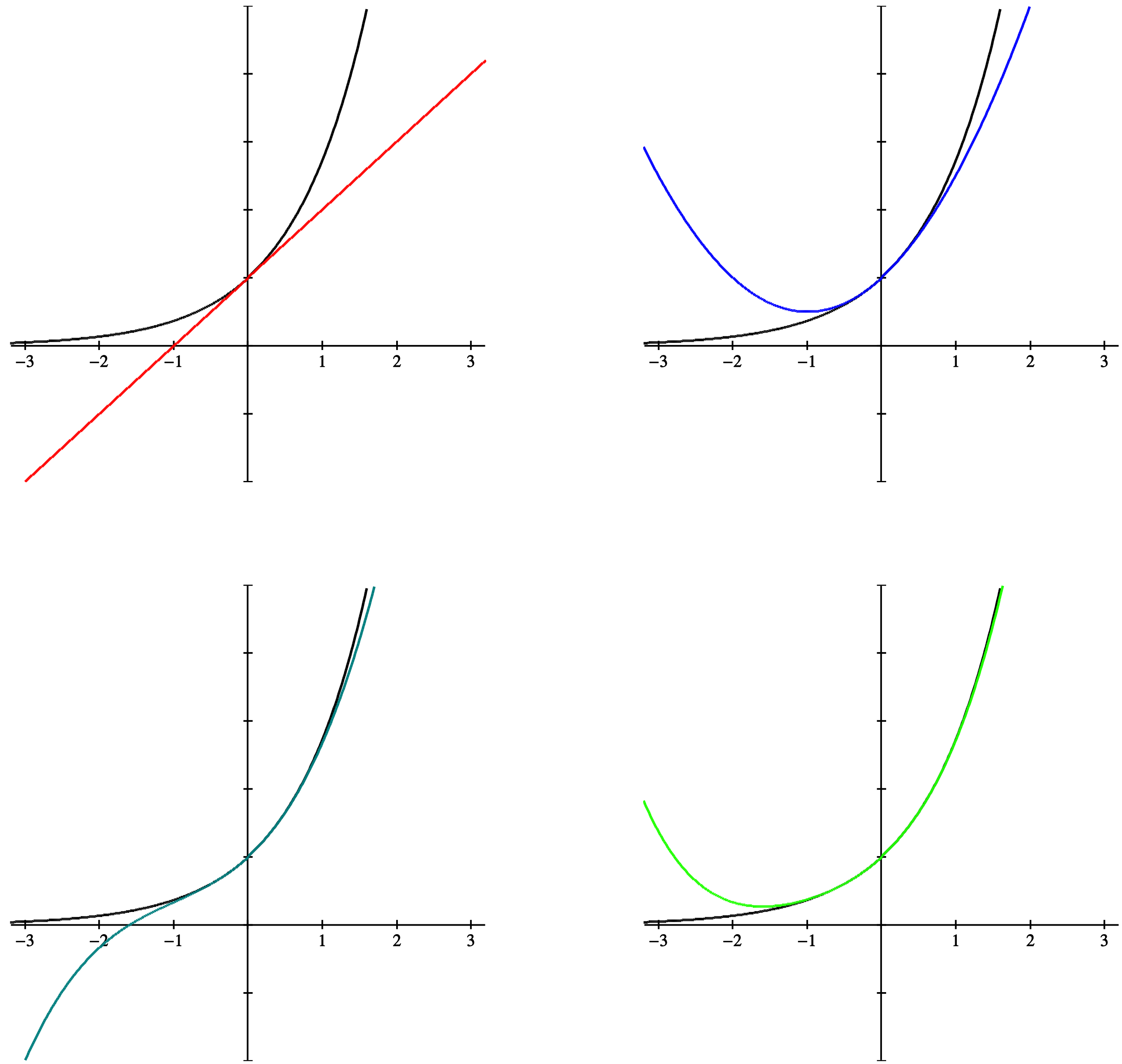
Let's create a polynomial P(x) that has the same properties as some function f(x) that we know very well at x = a, such as sin (x) or e^x around x = 0.

The properties that we need to consider are the function and derivative properties.

Why a polynomial?

1) Find a polynomial of degree n = 4 for $f(x) = e^x$.




−3 −2 −1 1 2 3
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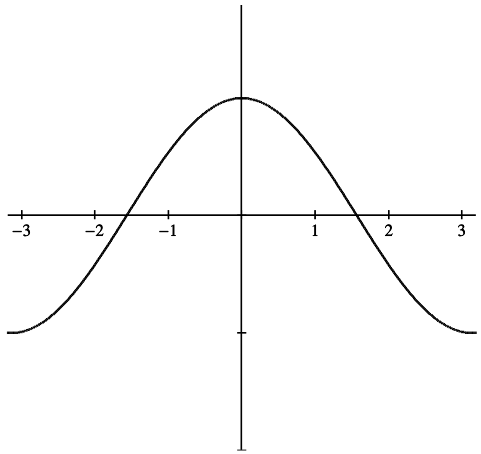
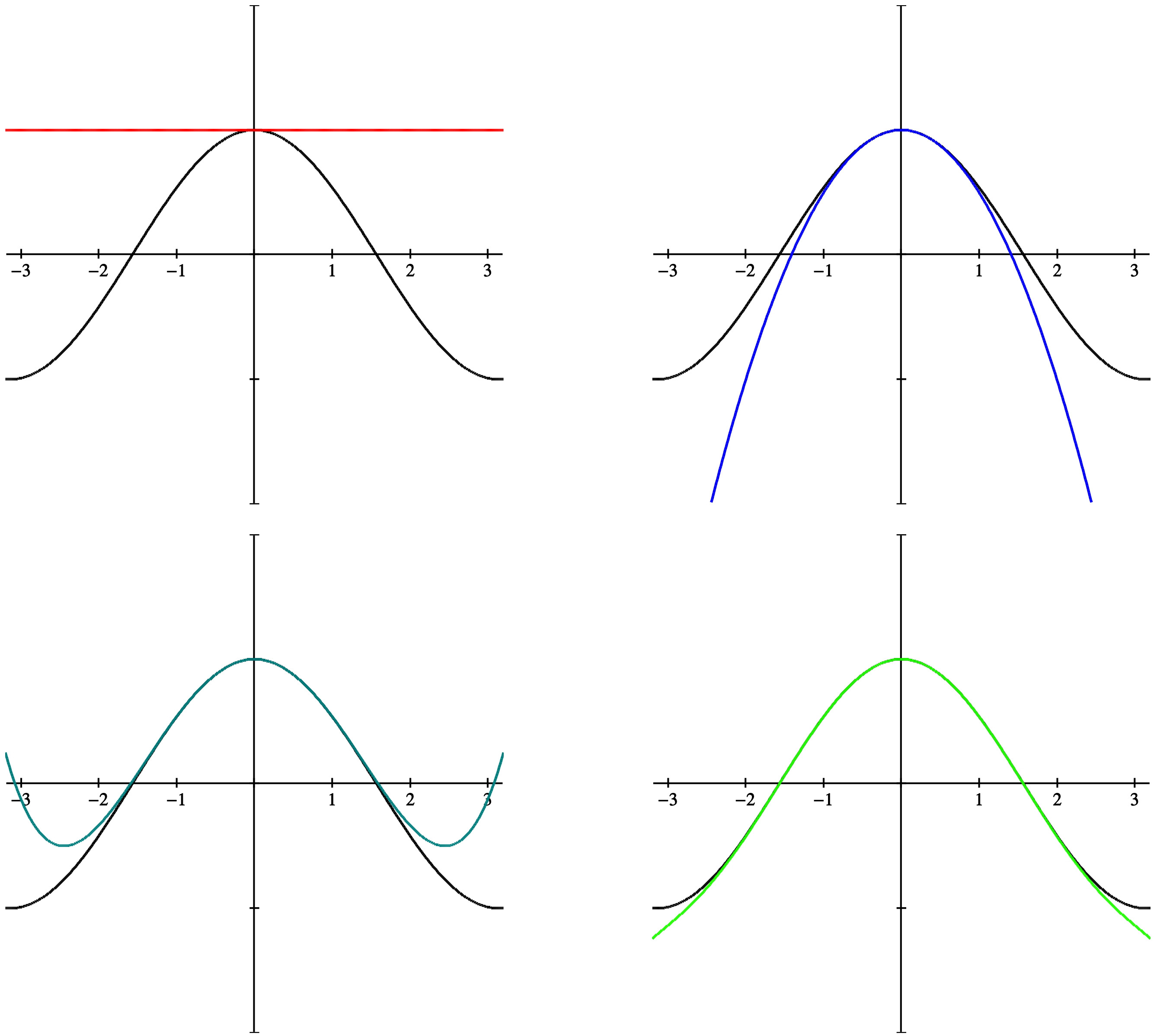
2) Find a polynomial of degree n = 6 for $f(x) = \cos x$.





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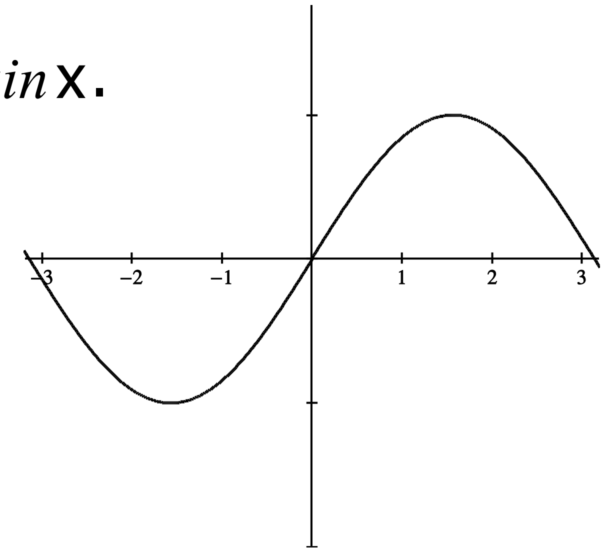
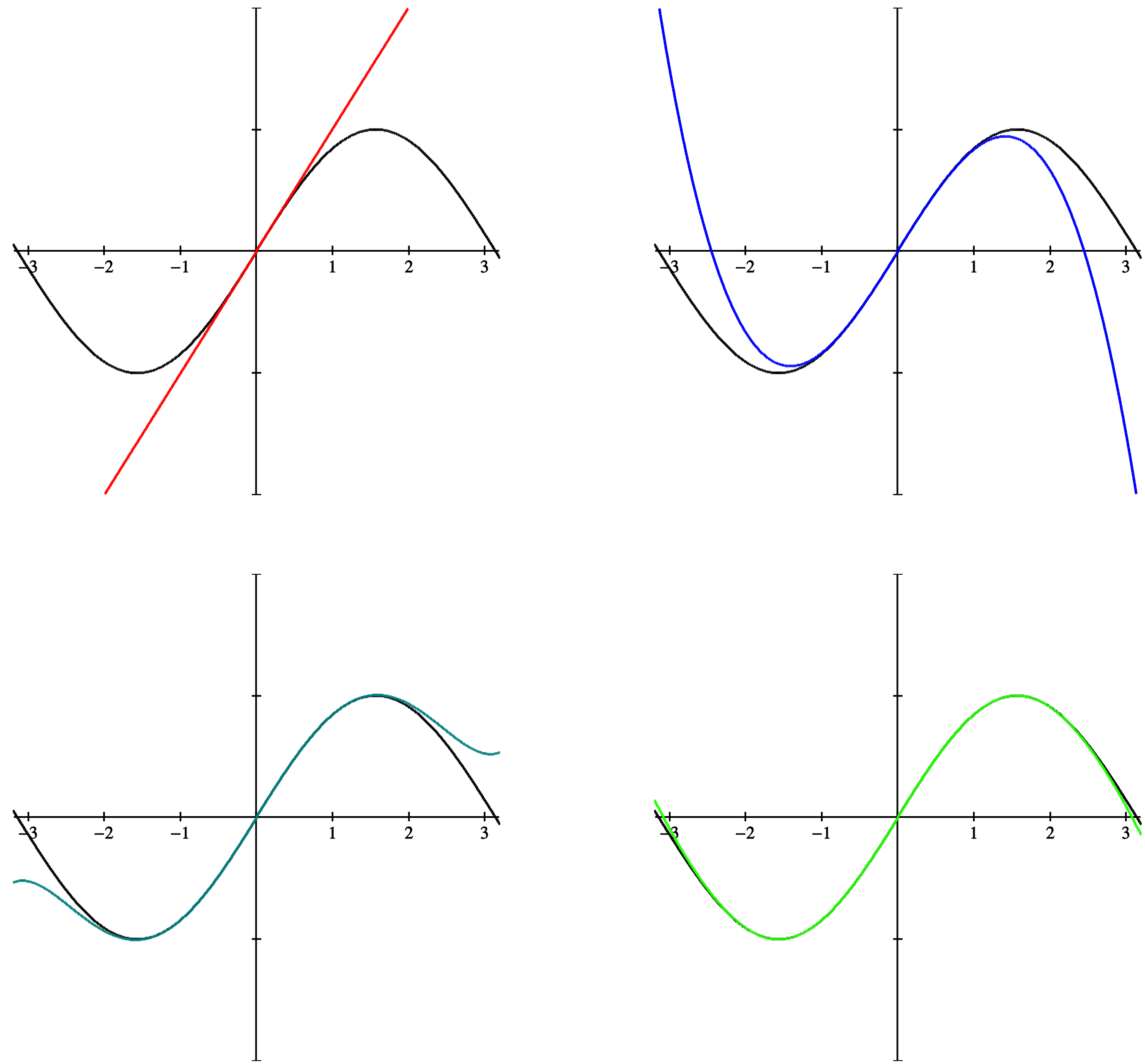
3) Find a polynomial of degree n = 5 for $f(x) = \sin x$.

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−3 −2 −1 1 2 3
−3 −2 −1 1 2 3

Definition of nth degree Taylor polynomial:

If f has n derivatives at c, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n$$

is called the nth degree Taylor polynomial for f at c.

If c = 0, then

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \ldots + \frac{f^{(n)}(0)}{n!}x^n$$

may be called the nth degree **Maclaurin** polynomial for f.

4) Use the Taylor approximation $e^x \approx 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ for x near 0 to find:

$$\lim_{x \to 0} \frac{e^x - 1}{2x}.$$