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Geometrical and material nonlinear analysis of structures under static and dynamic loading based on quadratic path

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Abstract. The Newton-Raphson method, which is based on the Taylor series expansion, and which uses the tangent stiffness matrix, has been extensively used to solve nonlinear problems. This traditional method, especially for the large-scale, is time consuming. Consequently, iterative algorithms cannot be effective for analyzing the process. In the incremental-iterative analysis of elastic nonlinear structures, great saving in computation can be achieved if distinction is made between the predictor and corrector phases. This paper shows how a simple assumption can improve the computational efficiency of the nonlinear analysis of structures. It is shown that very high computational efficiency may be obtained by assuming the pursuit of each Degree Of Freedom (DOF) by a quadratic curve. Through examples, it is demonstrated how this efficiency significantly decreases the computing time of analysis compared with time taken to deploy the Newton-Raphson, modified Newton-Raphson and Conjugate Gradient (CG) methods.

1. Introduction

The stability analysis of nonlinear structures includes the solution of large systems of nonlinear algebraic equations for varying values of a control parameter, which is, in most cases, associated with load amplitude. In structural mechanics, this problem is usually referred to as that of tracing the equilibrium path of the system. The numerical method constitutes one of the most important aspects in the nonlinear analysis of structures. For deployment of the Newton-Raphson method, the displacement control methods, the perturbation method, the self-correcting incremental procedure and the incremental stiffness procedure, the initial value approach is the commonly used solution for nonlinear problems. Papadrakakis and Gantes [1] investigated a method to shorten the time taken to deploy Newton methods in nonlinear problems.

For three decades now, nonlinear elastic and inelastic analysis of frame structures has been a subject of considerable research. Kassimali [2] presented a numerical procedure for large deformation analysis of elastic-plastic plane frames. Tabatabaei and Saffari [3] studied large strain analysis of planar frames using a normal flow algorithm. Oran and Kassimali [4] investigated the large deformations for the nonlinear analysis of plane frames under static and dynamic loads. Tabatabaei et al. [5] applied the Newton-Raphson iterative algorithm along the normal flow path in nonlinear static analysis of frames. Hsiao and Hou [6] suggested a simple formulation for nonlinear analysis of elastic frames. Wen and Rahimzadeh [7] used a finite element method in nonlinear analysis of elastic frames.

Saffari et al. [8] introduced a new algorithm for nonlinear analysis of space trusses that can reduce the number of iterations and computing time. Thai and

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Kim [9] presented the large-deflection inelastic analysis of space truss structures, including consideration of both geometric and material non-linearities. Greco et al. [10] proposed a new geometric non-linear formulation for space truss analysis that uses nodal positions rather than nodal displacements. Kwasniewski [11] suggested the complete equilibrium paths for several Mises trusses. Saffari and Mansouri [12] applied a new algorithm for nonlinear analysis of trusses. Pourazarm et al. [13] proposed a numerical algorithm for nonlinear analysis of frames, using the unit displacement method in generating a reduced stiffness matrix of the structure. Recently, Saffari et al. [14] proposed a fast methodology for elasto-plastic analysis of frames using the homotopy perturbation concept. A method for large deformation elastic-plastic analysis of space frames was undertaken by Abbasnia and Kassimali [15,16], and inelastic post-buckling analysis of truss structures by the dynamic relaxation method has been investigated by Ramesh and Krishnamoorthy [17].

In this study, the effect of large displacements is considered. A quadratic function between forces and deformations is applied in order to enhance speed of convergence and to minimize the cycles required for calculating equilibrium of nodes. Hence, at the beginning of each load step, using force and given deformations, a parabolic curve is passed from three points. The deformations for the next step are approximated using this curve. This process, using the Newton-Raphson method, can be continued until the convergence criterion is satisfied.

In this paper, two categories of structure are considered: planar frames and space trusses. This approach is designed to emphasize computational load control, rather than to offer improvements (e.g. it ignores the inability of the Newton-Raphson method in passing limit points) to the Newton-Raphson approach per se. However, these problems can be overcome using a form of other methods [8].

2. System equilibrium equation

Consider an arbitrarily framed structure loaded at the nodes only, and let x denote symbolically the corresponding deformed configuration. The equations of equilibrium of the system can be written as:

$$\{f(x)\} = \{P\}, \tag{1}$$

in which $\{f(x)\}$ is the resultant of the nodal internal forces and $\{P\}$ represents the external nodal forces. The member force deformation relationships denote that $\{f\}$ is a highly nonlinear function of $\{x\}$.

In the special case of a plane frame, the end forces in the global coordinates are represented by a set of relations as follows [2]:

$$\{F\} = [B]\{S\}, \tag{2}$$

where $[B]$ is transformation matrix:

$$[B] = \frac{1}{L'}\begin{bmatrix} -n & -n & mL' \\ m & m & nL' \\ L' & 0 & 0 \\ n & n & -mL' \\ -m & -m & -nL' \\ 0 & L' & 0 \end{bmatrix}, \tag{3}$$

with:

$$m = \cos\alpha, \qquad n = \sin\alpha. \tag{4}$$

In Eqs. (3) and (4), L' and α refer to the length and orientation, respectively, of the chord of the element in its deformed configurations, as shown in Figure 1. A procedure for obtaining L', m, n has been published in [2] and is not repeated herein. In Eq. (2):

$$\{S\} = \begin{Bmatrix} M_1 \\ M_2 \\ Q \end{Bmatrix}, \tag{5}$$

denote the local member end forces as shown in Figure 2 [2].

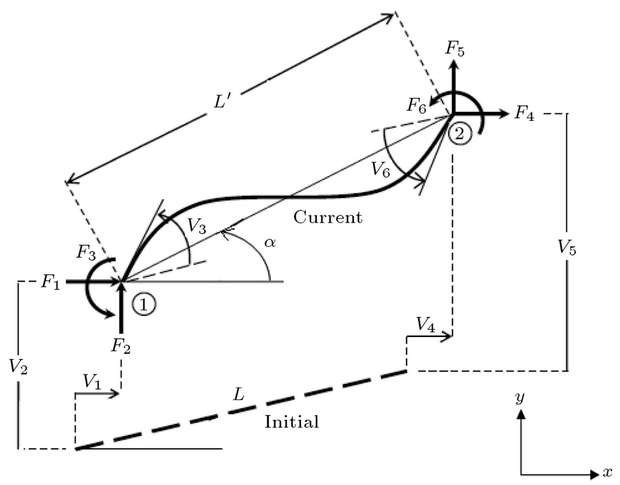

Figure 1. Member forces and deformations in global coordinates.

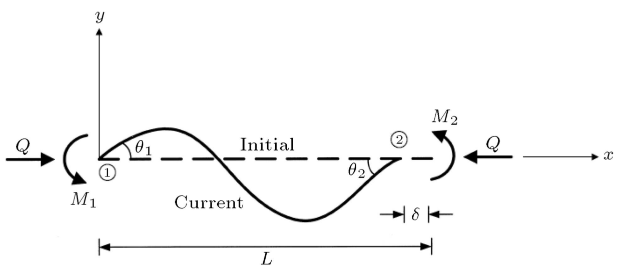

Figure 2. Member forces and deformations in local coordinates.

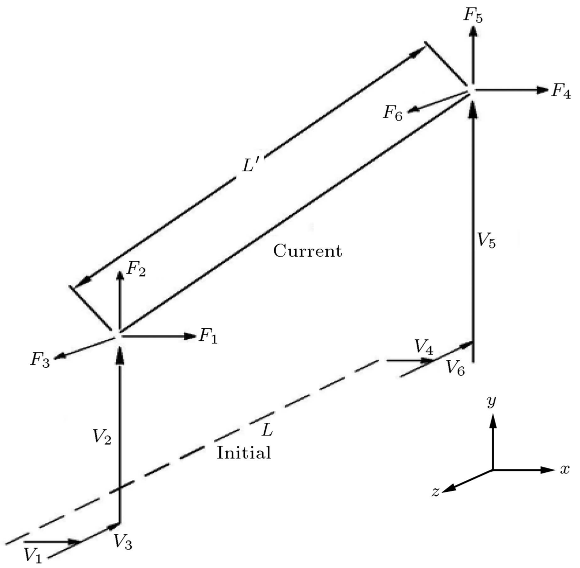

Figure 3. Initial and current configuration for typical truss member.

For a space truss element (Figure 3) the relationship between the end forces of the member in global and local coordinates is [18]:

$$\{F\} = \{B\}Q. \tag{6}$$

Here, Q is the local internal axial force of the truss member, and $\{B\}$ is the transformation matrix:

$$\{B\} = \begin{Bmatrix} l \\ m \\ n \\ -l \\ -m \\ -n \end{Bmatrix}, \tag{7}$$

in which l, m and n are the cosine directors of the element axis.

A detailed discussion for obtaining l, m, n and Q is provided in [18].

2.1. Material nonlinearity analysis

2.1.1. Truss element

The accuracy of the inelastic response of structures depends on the accuracy of the member's load-displacement relationship used in the analysis. A number of models have been introduced in the literature to predict the nonlinear behavior of space trusses. In this study, a stress-strain relationship proposed by Hill et al. [19] is adopted to predict the inelastic post-buckling behavior of trusses. The force-strain curve $(Q - u/L)$ is assumed applicable for steel material both in tension and compression states and is shown in Figure 4.

The curve can be expressed by the following relations:

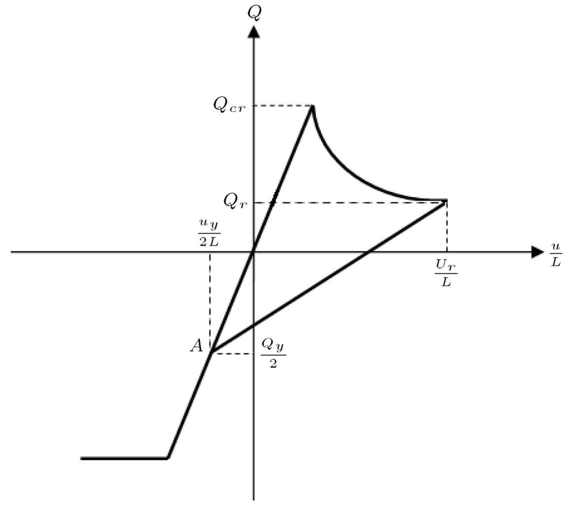

Figure 4. Proposed solution scheme.

- For tensile members:

$$Q = \begin{cases} \frac{A.E}{L}u & \text{for } |u| < u_y \\ A.F_y & \text{for } |u| \geq u_y \end{cases} \tag{8}$$

where L is length of element, A is cross-sectional area, E is modulus of elasticity, F_y denotes yield stress and u_y is $F_y.L/E$.

- For compressive members:

$$Q = \begin{cases} \frac{A.E}{L}u \\ \quad \text{for } u < u_{cr} \\ Q_l + (Q_{cr} - Q_l).e^{[-(X_1+X_2\sqrt{u'/L})u'/L]} \\ \quad \text{for } u \geq u_{cr} \end{cases} \tag{9}$$

Here:

$$Q_{cr} = \pi^2 EI/L^2.$$

I is weak axis moment of inertia. Q_l is the asymptotic lower stress limit and is defined as $Q_l = r.Q_{cr}$. The corresponding critical buckling displacement is $u_{cr} = Q_{cr}.L/(A.E)$, while u' is defined as $u' = u - u_{cr}$. Parameters X_1 and X_2 are constants, depending on the slenderness ratio of the compressive members.

It should be noted that when a member is in a compression state and $u \geq u_{cr}$, the tangent modulus, E_t, has to be used instead of E. The tangent modulus is obtained as:

$$E_t = -\frac{1}{A}(Q_{cr} - Q_l).e^{[-(X_1+X_2\sqrt{u'/L})u'/L]} \\ .(X_1 + \frac{3}{2}X_2\sqrt{u'/L}). \tag{10}$$

It can be seen that if the loading path reaches point A, the member behavior follows the relations in Eq. (8).

2.1.2. Frame element

A-perfectly plastic material associated with the plastic hinge concept is used in this study to consider the material non-linearity effect. In an elastic perfectly-plastic material, the effects of strain hardening are disregarded. It further implies that once the yield moment M_{pc} is reached, the material yields and cannot withstand further stress.

It is of note that yield moment is commonly defined by a yield criterion. A variety of yield criteria definitions have been introduced in structural engineering. In this paper, the AISC-LRFD 2005 [20] criterion, considering bending moment and axial force interaction, is used for steel elements. This criterion and its corresponding descriptive relation are shown below:

$$\begin{cases} \frac{|Q|}{2Q_c} + \left(\frac{M_{cx}}{M_{pcx}} + \frac{M_{cy}}{M_{pcy}}\right) = 1 \\ \quad \text{for } \frac{|Q|}{Q_c} < 0.2 \\ \\ \frac{|Q|}{Q_c} + \frac{8}{9}\left(\frac{M_{cx}}{M_{pcx}} + \frac{M_{cy}}{M_{pcy}}\right) = 1 \\ \quad \text{for } \frac{|Q|}{Q_c} \geq 0.2, \end{cases} \tag{11}$$

in which $M_c = \phi_b Z F_y$, M_{pc} represents reduced plastic moment capacity in the presence of axial force ($Q_c = \phi_c Q_n$); where ϕ_b and ϕ_c are bending and axial resistance factors, respectively; F_y denotes yield stress, and Z stands for plastic modulus.

3. Nonlinear dynamic analysis

The method of analysis used in the present study is briefly reviewed herein. Detailed derivation of the method can be found in [18].

3.1. Equations of motion

Among time integration methods, the Newmark method is the most extensively use in nonlinear dynamic analysis of structures because of its accuracy and stability. Therefore, the Newmark method is adopted here to solve the nonlinear equation of motions of structures. The Newton-Raphson iteration is performed at each time step to dissipate any unbalanced forces.

The incremental equation of motion of a structure can be expressed as:

$$[M]\{\Delta\ddot{x}\} + [C]\{\Delta\dot{x}\} + [\tau]\{\Delta x\} = \{\Delta P\}, \tag{12}$$

in which $\{\Delta P\}$ indicates load increments, $\{\Delta\ddot{x}\}$, $\{\Delta\dot{x}\}$ and $\{\Delta x\}$ represent increments of accelerations, velocities and displacements, respectively, and $[M]$, $[C]$ and $[\tau]$ are the mass, damping, and tangent stiffness matrices, respectively.

With the adoption of the average acceleration method of the Newmark family ($\beta = 0.25, \gamma = 0.5$), incremental acceleration and velocity at the first iteration of each time step can be expressed as:

$$\{\Delta\ddot{x}\} = \frac{4}{\Delta t^2}\{\Delta x\} - \frac{4}{\Delta t}\{\dot{x}_i\} - 2\{\ddot{x}_i\}, \tag{13}$$

$$\{\Delta\dot{x}\} = \frac{2}{\Delta t}\{\Delta x\} - 2\{\ddot{x}_i\}. \tag{14}$$

Substituting Eqs. (13) and (14) into Eq. (12), the incremental displacement can be given by:

$$[\bar{\tau}]\{\Delta x\} = \{\Delta\bar{P}\}, \tag{15}$$

in which $[\bar{\tau}]$ and $\{\Delta\bar{P}\}$ are the effective stiffness matrix and incrementally effective load vector, respectively, given by:

$$[\bar{\tau}] = \frac{4}{\Delta t^2}[M] + \frac{2}{\Delta t}[C] + [\tau], \tag{16}$$

$$\{\Delta\bar{P}\} = \{\Delta P\} + \left(\frac{4}{\Delta t}[M] + 2[C]\right)\{\dot{x}_i\} + 2[M]\{\ddot{x}_i\}. \tag{17}$$

At the first iteration of each time step, the total displacement, velocity, and acceleration at time $t + \Delta t$ is updated according to the incremental displacement vector $\{\Delta x\}$ as:

$$\{x_{i+1}\} = \{x_i\} + \{\Delta x\}, \tag{18}$$

$$\{\dot{x}_{i+1}\} = -\{\dot{x}_i\} + \frac{2}{\Delta t}\{\Delta x\}, \tag{19}$$

$$\{\ddot{x}_{i+1}\} = -\{\ddot{x}_i\} - \frac{4}{\Delta t}\{\dot{x}_i\} + \frac{4}{\Delta t^2}\{\Delta x\}. \tag{20}$$

For the second and subsequent iterations of each time step, the structural system is solved, subject to the effect of the unbalanced load $\{\Delta Q\}$, as:

$$[\bar{\tau}]\{\Delta\Delta x\} = \{\Delta Q\}, \tag{21}$$

where the unbalanced load $\{\Delta Q\}$ is determined based on the total external load $\{P\}$, inertial force, damping force, and updated internal force $\{f\}$, as follows:

$$\{\Delta Q\} = \{P_{i+1}\} - [M]\{\ddot{x}_{i+1}\} - [C]\{\dot{x}_{i+1}\} - \{f\}. \tag{22}$$

When the convergence criterion is satisfied, the structural response is updated for the next time step as:

$$\{\Delta x^{k+1}\} = \{\Delta x^k\} + \{\Delta\Delta x\}, \tag{23}$$

$$\{x_{i+1}\} = \{x_i\} + \{\Delta x^{k+1}\}, \tag{24}$$

$$\{\dot{x}_{i+1}\} = -\{\dot{x}_i\} + \frac{2}{\Delta t}\{\Delta x^{k+1}\}, \tag{25}$$

$$\{\ddot{x}_{i+1}\} = -\{\dot{x}_i\} - \frac{4}{\Delta t}\{\dot{x}_i\} + \frac{4}{\Delta t^2}\{\Delta x^{k+1}\}. \tag{26}$$

The details of the method for the application of the Newmark method and the Newton-Raphson iteration are as follows [21]:

Step 1. Predictor phase:

I. Form the effective stiffness matrix $[\bar{\tau}]$;

II. Form the effective force vector $\{\Delta\bar{P}\}$;

III. Solve for $\{\Delta x\}$ using Eq. (15).

Step 2. Corrector phase (force recovery):

I. Update structural configuration and motion;

II. Update the member force.

Step 3. Convergence:

I. Compute the unbalanced load;

II. Check convergence: If the convergence exists, update structural configuration and motion at time $t + \Delta t$ and go to the next time step. Otherwise, apply the unbalanced load on the structure system and go to Step 1.

4. Nonlinear analysis algorithms

In this section, three methods and a new approach for nonlinear analysis of structures are described.

4.1. Newton-Raphson method

The Newton-Raphson method offers one of the popular iterative methods for solving nonlinear equations. Via this method, an approximate solution is estimated, and then an unknown value is added as a corrector value to improve the initial solution. Using Taylor series, the system of nonlinear equations can be changed to linear form, and by solving this linear system, it is possible to achieve the corrector value and an improved solution. This process is continued until an acceptable approximation is obtained [22].

4.2. Modified Newton-Raphson method

For the conventional Newton-Raphson method, the tangent stiffness is reformed every iteration and, for the modified Newton-Raphson method, it is only reformed in the first iteration. The conventional Newton-Raphson method normally takes fewer iterations for convergence, but the computing time for each iteration in the modified Newton-Raphson method is shorter. This is significant because the solution and formation of the tangent stiffness matrix is a time-consuming process.

4.3. Conjugate Gradient (CG) method

The problem of solving nonlinear equations by the CG method may be viewed as a problem of minimizing a twice continuously differentiable nonquadratic function. Papadrakakis and Gantes [1] presented some procedures to truncate the Newton-Raphson method. The truncated Newton method is defined by the preconditioned CG to compute the search direction, and the details of this method can be found in the aforementioned reference.

4.4. Quadratic path as an initial guess

The incremental-iterative methods may be the most popular solution methods used in nonlinear analysis. In the linear incremental method, load-deflection behavior is approximated as piecewise linear, which produces unbalanced forces between externally applied loads and internal nodal loads. The presence of these unbalanced forces violates the equilibrium of the structure. If the unbalanced forces are not eliminated or reduced to a certain acceptable level, the calculated load-displacement relation will drift away from the true behavior of the structure. An iteration procedure may be used to eliminate these unbalanced forces in each incremental step. It is, therefore, necessary to use an incremental-iterative solution method to obtain more accurate results.

In the incremental-iterative analysis of elastic nonlinear structures, great saving in computation can be achieved if distinction is made between the predictor and corrector phases. The predictor relates to the solution of structural displacements for given load increments, which affects only the number of iterations. For the sake of the iterations, the equations used in the predictor need not be exact, but should be accurate enough not to mislead the direction of iterations. To approximate a curve, one naturally thinks of using a series of segments. The smaller the size for these segments, the closer the approximation will be, but the heavier the computational effort required.

In this paper, it is assumed that each DOF of displacements follows from a quadratic function, independently. As an initial approximation, this assumption is used at the beginning of load stepping. Therefore, there is no need to construct the tangent stiffness matrix at the beginning of each load step and so, the analytical computing time will be reduced. Furthermore, a more accurate approximation may be obtained by using a quadratic function as a rapidly converging iterative method. In the proposed technique, and like the modified Newton-Raphson method, the tangent stiffness is reformed at only the first iteration. The procedure can be summarized as follows.

The total external nodal load on a structure, at each loading step $\{P_i\}$, can be calculated by production of a total load ratio at each step, λ_i, and a given reference load $\{P_{\text{ref}}\}$ (in the case of proportional loading, the total external load vector can be obtained by simple scaling of a reference load vector) is applied through a series of load increments $\{\Delta P\}$. Mathematically, this is stated as:

$$\{P_i\} = \lambda_i\{P_{\text{ref}}\} = \sum \Delta P. \tag{27}$$

The approximate displacements vector at the $(i+1)$th step, $\{d^{i+1}\}$, can be estimated as $(i \geq 3)$:

$$\{d^{i+1}\} = \mu\{d^i\}, \tag{28}$$

where:

$$\mu = \frac{d_{kj}^{i+1}}{d_j^i}. \tag{29}$$

Here, d_j^i is a scalar which represents the displacement of the jth DOF of the structures at converged step i, and d_{kj}^{i+1} is the estimated displacement of that DOF at the $(i+1)$th step and kth iteration, which can be calculated as follows:

$$d_{kj}^{i+1} = \left([A]^{-1}\{d\}\right)^T \begin{Bmatrix} \left(\lambda^{i+1}\right)^2 \\ \lambda^{i+1} \\ 1.0 \end{Bmatrix}, \tag{30}$$

in which matrix $[A]$ is:

$$[A] = \begin{bmatrix} \left(\lambda^{i-2}\right)^2 & \lambda^{i-2} & 1.0 \\ \left(\lambda^{i-1}\right)^2 & \lambda^{i-1} & 1.0 \\ \left(\lambda^{i}\right)^2 & \lambda^{i} & 1.0 \end{bmatrix}, \tag{31}$$

and:

$$\{d\} = \begin{Bmatrix} d_j^{i-2} \\ d_j^{i-1} \\ d_j^{i} \end{Bmatrix}, \tag{32}$$

and the superscript T represents transposition.

A program implementing the two-point algorithm has been written in MATLAB and representative results are provided. Material nonlinearity is not presently included in the algorithm. The graphical representation of this process is shown in Figure 5.

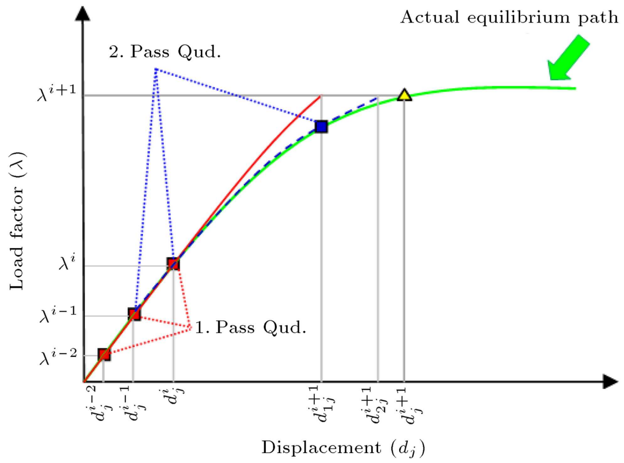

Figure 5. Proposed solution scheme.

5. Numerical examples

Four numerical examples were solved in a microcomputer environment so that the efficiency of the proposed procedure used together with the Newton-Raphson and modified Newton-Raphson methods in nonlinear behavior of planar frames could be compared. The computer program was developed based on the procedure described in this paper. All examples have been solved with a 32 bit Pentium 2.00 GHz processor (2 CPUs). For the solution of nonlinear equations, a new iterative method is adopted and the iterative process will stop when the convergence criteria are satisfied. The convergence criterion, based on displacement, used herein, is given by:

$$\sqrt{\frac{\sum_i (\Delta x_i)^2}{\sum_i (x_i)^2}} \leq e, \tag{33}$$

where e is the error tolerance. All the numerical examples presented here use a tolerance of 10^{-3}. It should be noted that the computing time is for twenty analyses in all examples.

5.1. Example 1

Figure 6 shows a two-bay, six-storey frame subjected to distributed gravity and lateral loads with its associated

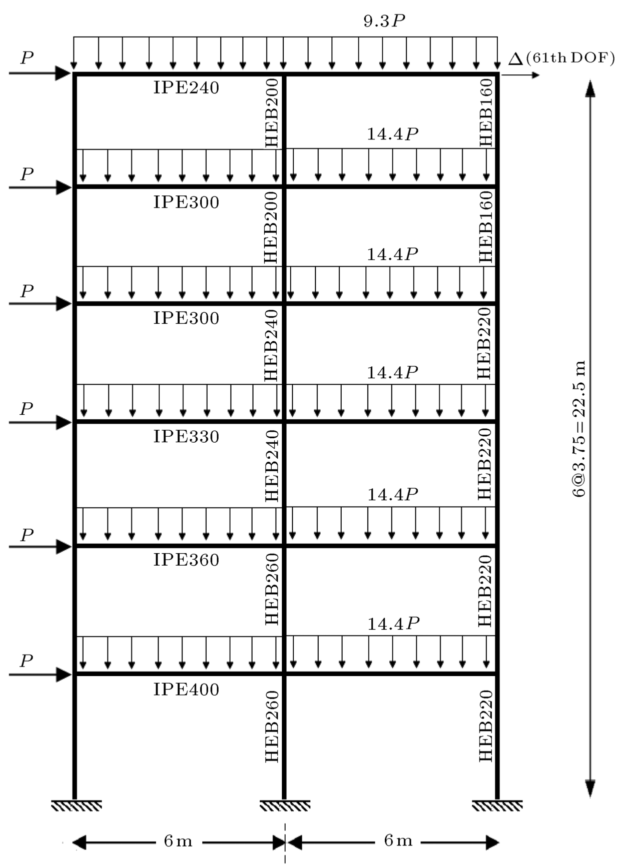

Figure 6. Two-bay six-storey frame.

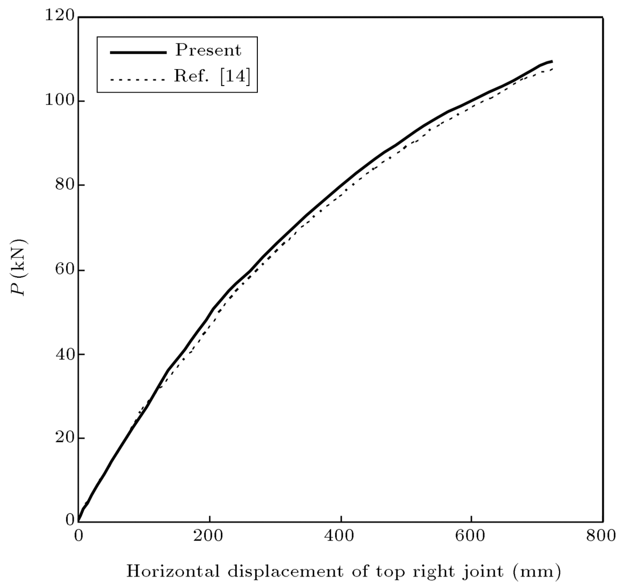

Figure 7. Load-displacement curve for two-bay six-storey frame.

data [22]. The elastic modulus for all members E is 20500 kN/cm^2. It is assumed that P = 102.2 kN and $\Delta\lambda$ = 0.02. In this example d_j is 61. The developed method here is applied to plane frames in which elastic-perfectly plastic behavior is assumed for structural material, while conventional plastic hinges of zero length are used to model the plasticity effect. The path of load-deformation curves is shown in Figure 7. To compare the performance of the proposed method, the results of analyses are summarized in Tables 1 and 2.

5.2. Example 2

The star truss, shown in Figure 8, having 24 elements and 13 nodes with pin supports at the outer nodes, is taken from [18], which provides a good opportunity to evaluate the efficiency of the method discussed here. The cross-section area, modulus of elasticity and mass per unit length for all members are A = 6.45 cm^2, E = 6.9 MPa and $\bar{m}$ = 690 N.S^2/cm^2, respectively. A dynamic forcing function considered for this structure is shown in Figure 9.

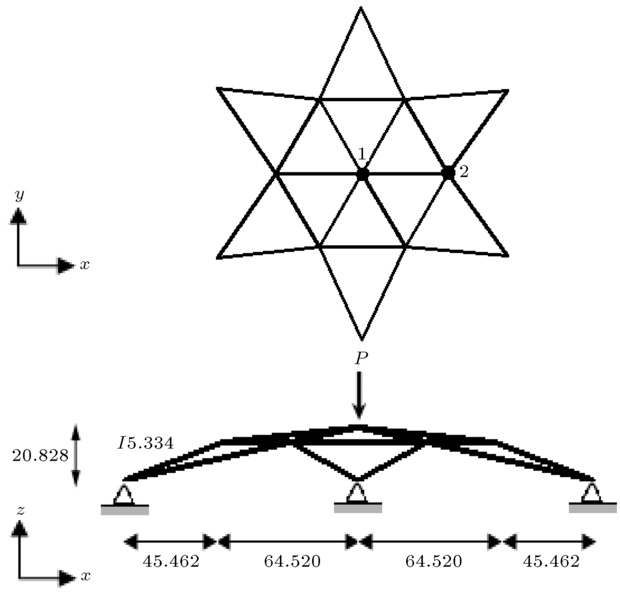

Figure 8. Star dome truss, dimensions are given in cm.

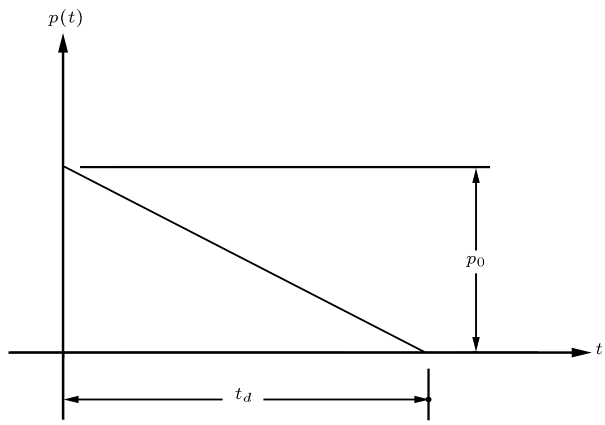

Figure 9. Dynamic forcing function (t_d = 0.01 Sec).

Figure 10 shows the load-displacement curve obtained by applying the method developed during the present study.

Table 1. Corresponding displacements (mm) obtained by current and other methods in Example 1.

Load (kN)	Newton-Raphson	Modified Newton-Raphson	CG method	Quad. path
16.35	61.84	61.84	61.83	61.84
34.75	145.13	145.13	144.28	146.21
77.26	399.09	399.09	399.41	400.00
105.16	681.85	681.85	682.07	681.77

Table 2. Computing time of 20 times analysis for two-bay six-storey frame (sec).

Newton-Raphson	Modified Newton-Raphson	CG method	Quad. path
292.22	198.72	99.01	77.55

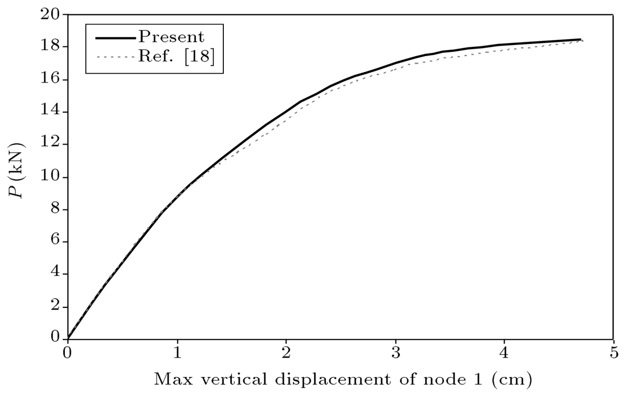

Figure 10. Load-deflection curve for Example 2.

The results show that the computing time used in deploying the classic Newton-Raphson approach is more than that used by applying our method. The results are presented in Tables 3 and 4.

5.3. Example 3

The circular dome truss taken from Thai and Kim [9] is shown in Figure 11. This structure is subjected to a vertical load at the apex and has 168 elements with 73 nodes, with a total of 147 degrees-of-freedom. The out-of-plane motion has been constrained with pin supports added to each end of the truss. The cross-sectional area, A, is equal to 50.431 cm^2 for all the members. Also, the following parameters are assumed for this dome: $E = 2.04 \times 10^4$ kN/cm^2, $P = 820$ kN and $\Delta\lambda = 0.024$.

For this truss, d_j can be given as 117. The proposed algorithm is applied to two cases of elastic and Inelastic Post-Buckling (IPB) analyses of this truss. Figure 12 shows the variation of vertical displacement with the load P. Tables 5 and 6 document the performance of the methods.

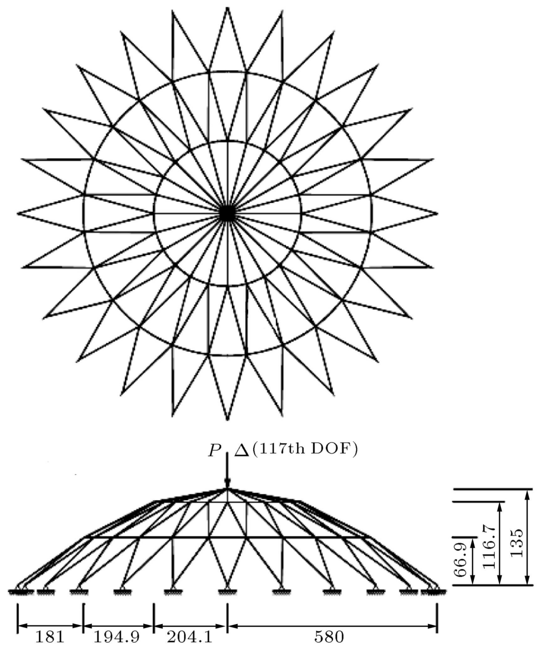

Figure 11. Circular dome truss; dimensions are given in cm.

5.4. Example 4

This truss, shown in Figure 13, with 264 elements and 97 nodes, with pin supports at the outer nodes, gives a possibility of comparison with results in the

Table 3. Corresponding displacements (cm) obtained by current and other methods in Example 2.

Load (kN)	Newton-Raphson	Modified Newton-Raphson	CG method	Quad. path
3.55	2.28	2.28	2.30	2.28
10.67	8.44	8.45	8.43	8.45
15.58	19.60	19.59	19.62	19.61

Table 4. Computing time of 20 times analysis for Example 2 (sec).

Newton-Raphson	Modified Newton-Raphson	CG method	Quad. path
44.03	26.48	19.37	15.92

Table 5. Corresponding displacements (cm) obtained by current and other methods in Example 3.

Load (kN)	Newton-Raphson	Modified Newton-Raphson	CG method	Quad. path
149.46	0.49	0.49	0.49	0.49
400.0	1.42	1.42	1.40	1.41
600.0	2.48	2.48	2.47	2.47
750.0	3.66	3.66	3.65	3.65

Table 6. Computing time of 20 times analysis for circular dome truss (sec).

Newton-Raphson	Modified Newton-Raphson	CG method	Quad. path
93.71	71.25	37.44	24.07

Table 7. Corresponding displacements (cm) obtained by current and other methods in Example 4.

Load (kN)	Newton-Raphson	Modified Newton-Raphson	CG method	Quad. path
5.46	0.89	0.89	0.90	0.89
17.65	3.41	3.42	3.42	3.43
27.11	7.96	7.96	7.97	7.93

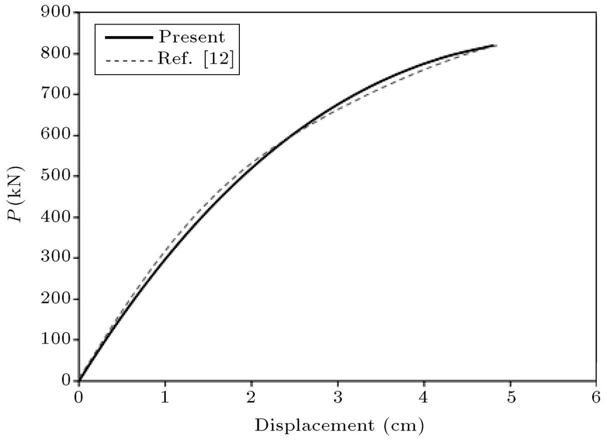

Figure 12. Load-displacement curve for circular dome truss.

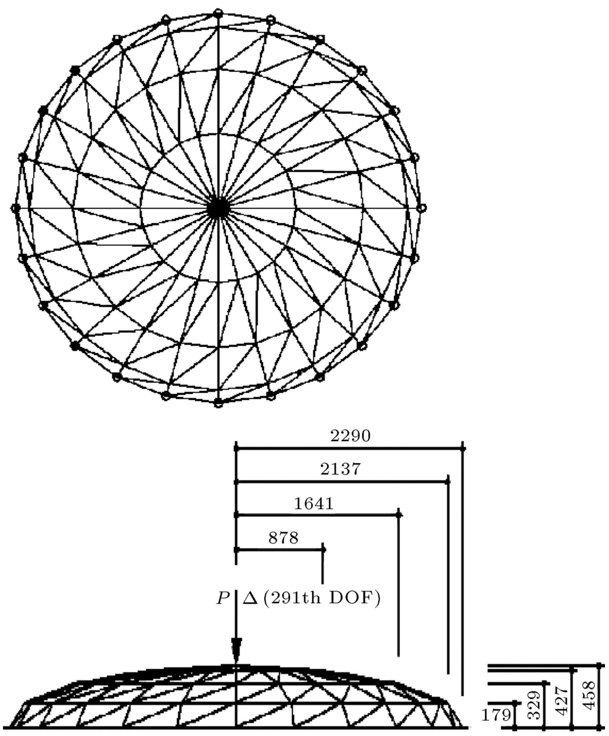

Figure 13. Schewdeler's dome truss; dimensions are given in cm.

Table 8. Computing time of 20 times analysis for Schewdeler's truss (sec).

Newton-Raphson	Modified Newton-Raphson	CG method	Quad. path
121.29	87.35	54.28	26.34

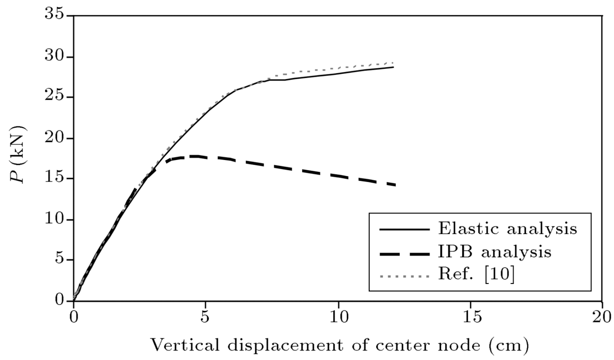

Figure 14. Load-displacement curve for Schewdeler's dome truss.

literature [10]. The axial stiffness for all members is $EA = 640 \times 10^3$ kN. The external loading was equipment loading, which consists of $P = 30$ kN at the crown node and $\Delta\lambda = 0.033$. For this structure, d_j is 291.

The load-displacement curve for this structure is shown in Figure 14. Tables 7 and 8 documents the comparison between results obtained using the Newton-Raphson and the newly introduced method.

6. Conclusions

The implementation of a simple and effective method has been discussed. The use of the quadratic function between load and deflections as the initial guess at the beginning of each load step allows easy implementation of large displacement analysis. In other words, in the proposed technique, the quadratic curve plays the predictor role. The deformations for the next step are approximated using this curve. In this method, there is no need to construct the tangent stiffness matrix at the beginning of each load step and, so, the computing time of analysis will be reduced. A numerical procedure was written into a computer program. The results reveal that deployment of the quadratic path offers high accuracy and can be an efficient technique for geometrical and material nonlinearity analysis of structures which are otherwise inconveniently time-consuming.

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