Geometry: A Complete Course

(with Trigonometry)

Module A – Instructor's Guide with Detailed Solutions for Progress Tests

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ERRATA
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VideoText Interactive

Quiz Form B

Name ______________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry
Part A - What is Geometry?
Lesson 1 - Origin
Lesson 2 - Structure

1. In our study of Algebra, the symbols used to name numbers were examples of the "things" of mathematics, or the objects around which our study revolves. How many new things did we discuss in Lesson 2? *4*
Name them.

Point   *Line*   *Plane*   *Space*

2. Tell what part of mathematical speech each of the following is:

a) $\div$ *Operation Symbol*

b) π *Number Symbol*

c) $||$ *Relation Symbol*

d) $\sqrt{17}$ *Number Symbol*

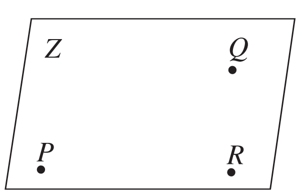
e) $+$ *Operation Symbol*

f) $\{\}$ *Grouping Symbol*

g) e *Number Symbol*

h) $\not\equiv$ *Relation Symbol*

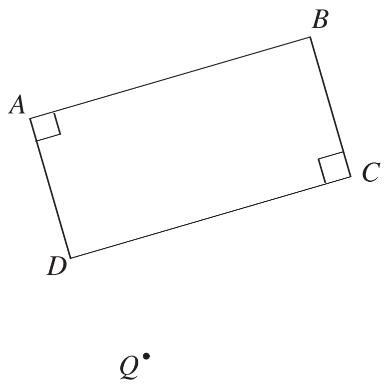
3. Name the plane shown in two ways

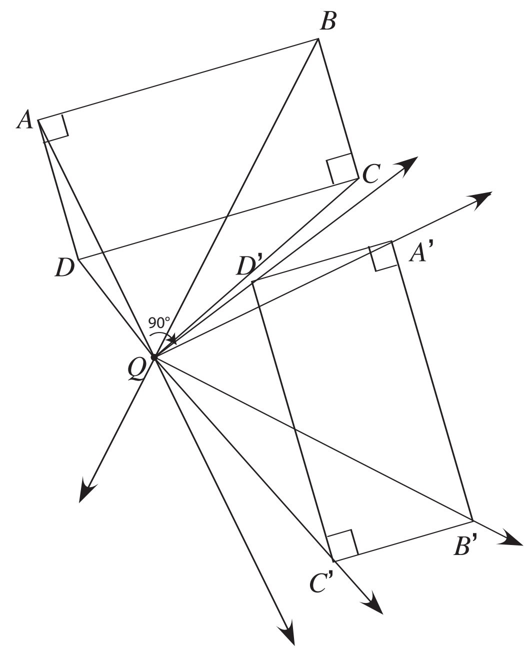


Plane Z   *Plane PQR*

Name ____________________________

3. Draw the image of the given rectangle after a rotation of 90° clockwise around the center of rotation Q.





1) Draw a segment from vertex A to the center of rotation point Q

2) Measure a 90° Angle clockwise at point Q. Draw the angle.

3) Use a ruler to locate A' on the ray of the angle forming angle AQA'.
The measure of segment QA' will equal the measure of segment QA.

4) Repeat steps 2 through 4 for each vertex. Connect the vertices to form rectangle A'B'C'D'.

Quiz Form A

Name ____________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry
Part A - What is Geometry?
Lesson 5 - More on Groupings

1. Write D = {1, 3, 5, ...} using set-builder notation. *D = {x | x is an odd natural number}*
or D = {x | x is an odd counting number}

2. Write $B = \{x \mid x = 3y + 1, y \in I, y \geq 0\}$ using the roster method. *B = {1, 4, 7, 10, 13, 16...}*

Rewrite the statements in exercises 3 and 4 using set notation. Use the roster method if possible.

3. The set made up of even counting numbers less than ten is an improper subset of the set made up of even counting numbers less than ten. $\{2, 4, 6, 8\} \subseteq \{2, 4, 6, 8\}$

4. The set whose only element is 0 is not a subset of the empty set. $\{0\} \not\subset \{\ \}$

Name ______________________________

Consider these sets for questions 5 through 10.

A = {a, b, c, d, e} B = {a, b, c, d, e, f, g}

C = {m, i, n, t} D = {d, e}

5. $A \cap B$ = $\{a, b, c, d, e\}$ *These are the elements that are common to both sets*

6. $A \cup C$ = $\{a, b, c, d, e, m, i, n, t\}$ *These are the elements in one, or the other, or both of the sets.*

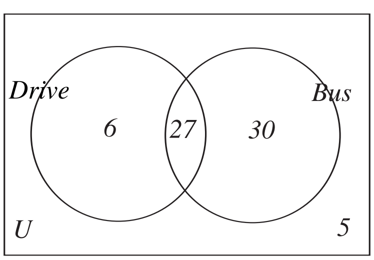
7. $A \cap D$ = $\{d, e\}$ *These are the elements that are common to both sets*

8. $C \cap D$ = { } *These are the elements that are common to both sets*

9. Is D ⊆ {d, e}? *Yes, all of the elements in set D are also in the set {d,e}*

10. Does A = C? *No, sets A and C do not contain exactly the same elements.*

11. Of 68 people surveyed, 33 most often drive to work, 57 usually take the bus to work, and 27 do both equally as often. How many of these surveyed did neither? *5*
Use a Venn Diagram to show your work.



Quiz Form A

Name ______________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry
Part B - The Scope of Our Geometry
Lesson 4 - Solids

Complete each sentence in exercises 1 through 5 with the appropriate geometric term(s).

1. The three basic three dimensional shapes in all the world are *Prisms*, *Pyramids*, and *Spheres*.

2. A prism has *Parallelograms* for sides.

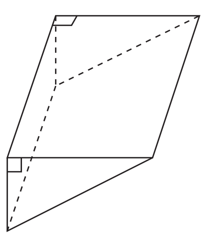
3. A pyramid has *Triangles* for sides.

4. Cones and cylinders have *Circles* for bases.

5. A sphere is a surface which is everywhere the same *Distance* from a fixed point.

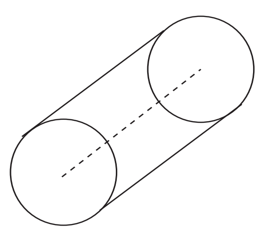
Identify the solids in exercises 6 through 11 as prisms, cylinders, pyramids, cones, or spheres. Note the shape of the base when naming a prism or pyramid and be as specific as possible.

6.



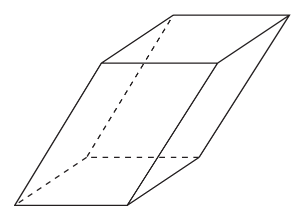
Triangular Prism

7.



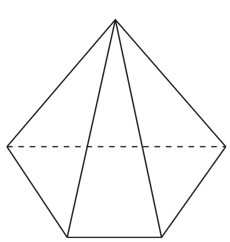
Cylinder
(or circular prism)

8.



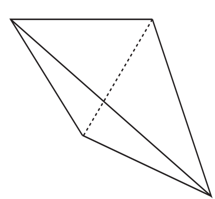
Square Prism

9.



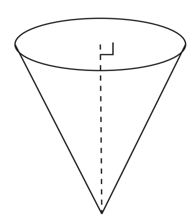
Trapezoidal Pyramid

10.



Triangular Pyramid

11.



Cone
(or circular pyramid)

Quiz Form B

Name ____________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry
Part B - The Scope of Our Geometry
Lesson 4 - Solids

Complete each sentence in exercises 1 through 5 with the appropriate geometric term(s).

1. A prisim is named after the shape of its ___*base*___.

2. A cone has a circular___*base*___.

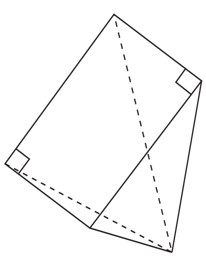
3. The sides of a prism are ___*parallelograms*___ .

4. The sides of a pyramid are ___*triangles*___.

5. A cylinder has two___*circular*___bases.

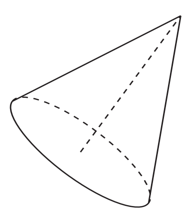
Identify the solids in exercises 6 through 11 as prisms, cylinders, pyramids, cones, or spheres. Note the shape of the base when naming a prism or pyramid, and be as specific as possible.

6.



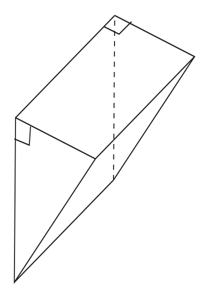
Rectangular Pyramid

7.



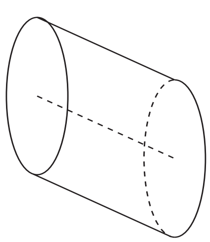
Cone (Circular Pyramid)

8.



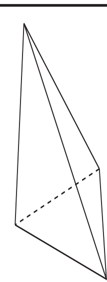
Triangular Prism

9.



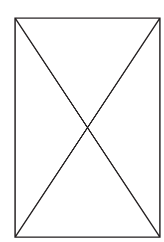
Elliptical Cylinder
(or Elliptical Prism)

10.



Triangular Pyramid

11.



Rectangular Pyramid
(top view)

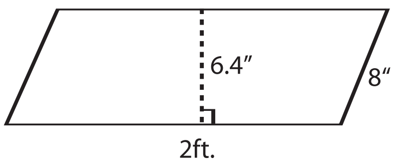
Quiz Form A

Name ______________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry
Part C - Measurement
Lesson 2 - Parallelograms

1. Find the area and perimeter of the given parallelogram.



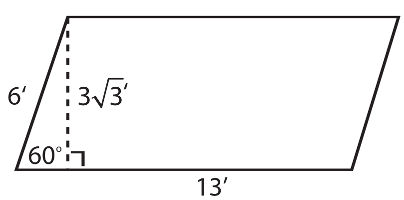
Area: *153.6 sq. inches*

Perimeter: *64 inches*

$$\begin{aligned} Area &= base \cdot height \\ 2\,ft &= 2 \cdot 12in = 24in \\ &= 24" \cdot 6.4" \\ &= 153.6 \text{ square inches} \\ &= 153.6 \text{ inches}^2 \end{aligned}$$

$$\begin{aligned} Perimeter &= \text{sum of lengths of the sides} \\ &= 24" + 8" + 24" + 8" \\ &= 32" + 32" \\ &= 64" \end{aligned}$$

2. Find the area and perimeter of the given parallelogram.



Area: *$39\sqrt{3}$ square feet*

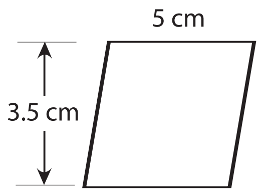
Perimeter: *38 feet*

$$\begin{aligned} Area &= base \cdot height \\ &= 13' \cdot 3\sqrt{3}' \\ &= (13)(3)\left(\sqrt{3}\right) \\ &= 39\sqrt{3} \text{ square feet} \end{aligned}$$

$$\begin{aligned} Perimeter &= \text{Sum of lengths of the sides} \\ &= 13' + 6' + 13' + 6' \\ &= 19' + 19' \\ &= 38 \text{ feet} \end{aligned}$$

Name ______________________________

3. Find the area and perimeter of the given rhombus.



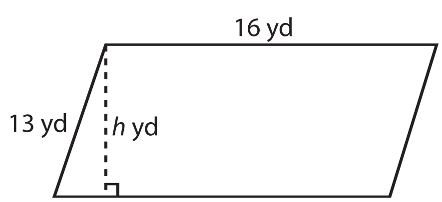
Area: *17.5 sq. cm.*

Perimeter: *20 cm.*

$$\begin{aligned} Area &= Base \cdot Height \\ &= 5 \cdot 3.5 \\ &= 17.5 \textit{ square cm} \end{aligned}$$

$$\begin{aligned} Perimeter &= Sum\ of\ lengths\ of\ the\ sides \\ &= 5+5+5+5 \\ &= 20cm \end{aligned}$$

4. Find the perimeter and the height of this parallelogram.



Area: 176 sq yds

Perimeter: *58 yds.*

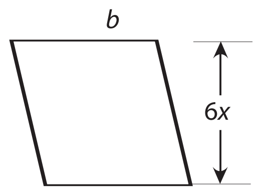
height: *11 yds.*

$$\begin{aligned} Area &= base \cdot height \\ 176 &= 16 \cdot h \\ \frac{1}{16} \cdot 176 &= \frac{1}{16} \cdot 16 \cdot h \\ \frac{176}{16} &= 1 \cdot h \\ 11 \textit{ yards} &= h \end{aligned}$$

$$\begin{aligned} Perimeter &= Sum\ of\ lengths\ of\ the\ sides \\ &= 16+13+16+13 \\ &= 29+29 \\ &= 58 \textit{yards} \end{aligned}$$

Name ______________________________

5. Find the perimeter and the base of the given rhombus.



Area: $48x^2$

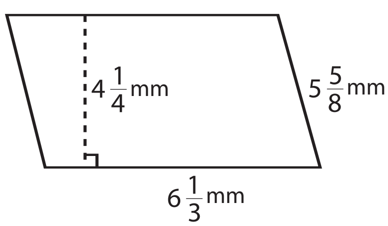
Perimeter: $32x$ units

base: $8x$ units

$$Area = Base \cdot Height$$
$$48x^2 = b \cdot 6x$$
$$\frac{1}{6x} \cdot \frac{48x^2}{1} = b \cdot \frac{6x}{1} \cdot \frac{1}{6x}$$
$$\frac{6 \cdot 8 \cdot x \cdot x}{6x} = b$$
$$8x \text{ units} = b$$

$$Perimeter = \text{Sum of lengths of the sides}$$
$$= 8x + 8x + 8x + 8x$$
$$= 32x \text{ units}$$

6. Find the area and perimeter of the given parallelogram.



Area: $26\frac{11}{12} mm^2$

Perimeter: $23\frac{11}{12} mm$

$$Area = base \cdot height$$
$$= 6\frac{1}{3} \cdot 4\frac{1}{4}$$
$$= \frac{19}{3} \cdot \frac{17}{4}$$
$$= \frac{19 \cdot 17}{3 \cdot 4}$$
$$= \frac{323}{12} \text{ square mm or } 26\frac{11}{12} mm^2$$

$$Perimeter = \text{Sum of lengths of the sides}$$
$$= 6\frac{1}{3} + 5\frac{5}{8} + 6\frac{1}{3} + 5\frac{5}{8}$$
$$= \frac{19}{3} + \frac{45}{8} + \frac{19}{3} + \frac{45}{8}$$
$$= \frac{19}{3} \cdot \frac{8}{8} + \frac{45}{8} \cdot \frac{3}{3} + \frac{19}{3} \cdot \frac{8}{8} + \frac{45}{8} \cdot \frac{3}{3}$$
$$= \frac{152 + 135 + 152 + 135}{24}$$
$$= \frac{574}{24} \text{ mm or } 23\frac{22}{24} \text{ mm or } 23\frac{11}{12} \text{ mm}$$

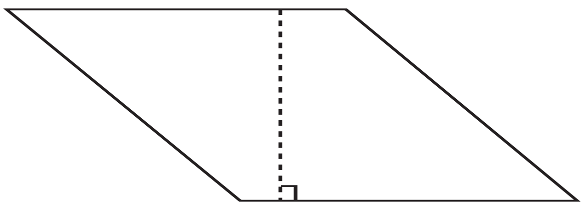
Quiz Form B

Name ______________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry
Part C - Measurement
Lesson 2 - Parallelograms

1. Find the height of the given parallelogram.

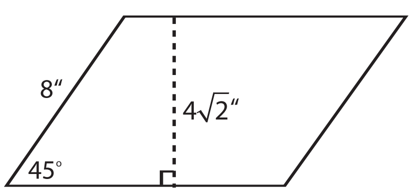


Area: 84 square inches

height: *7 inches*

$$\begin{aligned} Area &= base \cdot height \\ 84 &= 12 \cdot h \\ \frac{1}{12} \cdot \frac{84}{1} &= \frac{1}{12} \cdot \frac{12}{1} \cdot h \\ \frac{7 \cdot 12}{12} &= h \\ 7 &= h \end{aligned}$$

2. Find the area and perimeter of the given parallelogram.



Area: $48\sqrt{2}$ *sq. inches*

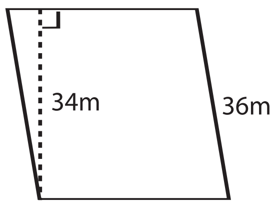
Perimeter: *40 inches*

$$\begin{aligned} Area &= base \cdot height \\ &= 12'' \cdot 4\sqrt{2}\,'' \\ &= (12)\cdot(4)\cdot\left(\sqrt{2}\right) \\ &= 48\sqrt{2} \text{ square inches} \end{aligned}$$

$$\begin{aligned} Perimeter &= Sum\ of\ lengths\ of\ the\ sides \\ &= 12 + 8 + 12 + 8 \\ &= 20 + 20 \\ &= 40 \text{ inches} \end{aligned}$$

Name ______________________________

3. Find the area and perimeter of the given rhombus.



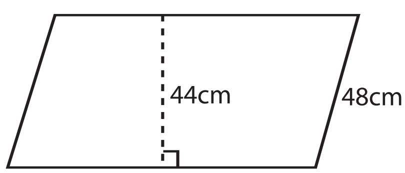
Area: *1224 sq. meters*

Perimeter: *144 meters*

$$\begin{aligned} Area &= base \cdot height \\ &= 36 \cdot 34 \\ &= 1224 \text{ } square\ meters \end{aligned}$$

$$\begin{aligned} Perimeter &= Sum\ of\ lengths\ of\ the\ sides \\ &= 36+36+36+36 \\ &= 144 \text{ } meters \end{aligned}$$

4. Find the perimeter of the given parallelogram.



Area: 2508 sq cm

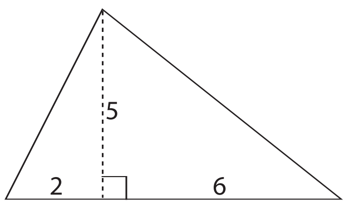
Perimeter: *210 cm*

$$Area = base \cdot height$$
$$2508 = b \cdot 44$$
$$\frac{1}{44} \cdot \frac{2508}{1} = b \cdot \frac{44}{1} \cdot \frac{1}{44}$$
$$\frac{1 \cdot 44 \cdot 57}{44} = b$$
$$57cm = b$$

$$\begin{aligned} Perimeter &= Sum\ of\ lengths\ of\ the\ sides \\ &= 57+48+57+48 \\ &= 210cm \end{aligned}$$

Name ____________________________________

3.



Area: *20 sq. units*

Perimeter: $\left(8+\sqrt{29}+\sqrt{61}\right)$ *units*

$$\begin{aligned} Area &= \frac{1}{2}\cdot base\cdot height \\ &= \frac{1}{2}\cdot 8\cdot 5 \\ &= \frac{8\cdot 5}{2} \\ &= \frac{2\cdot 4\cdot 5}{2} \\ &= 4\cdot 5 \\ &= 20 \text{ square units} \end{aligned}$$

Pythagorean Theorem

$$a^2+b^2=c^2$$

$$\begin{aligned} 2^2+5^2 &= x^2 & 6^2+5^2 &= y^2 \\ 4+25 &= x^2 & 36+25 &= y^2 \\ 29 &= x^2 & 61 &= y^2 \\ \sqrt{29} &= x & \sqrt{61} &= y \end{aligned}$$

Perimeter = Sum of Lengths of the Sides

$$Perimeter = \left(8+\sqrt{29}+\sqrt{61}\right) \text{ units}$$

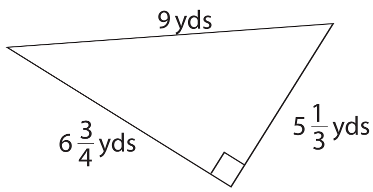
4. Find the area of a triangle with base ($2x$ + 3) units and height ($4x$ - 2) units.

Area: $\left(4x^2+4x-3\right)$ *square units*

$$\begin{aligned} Area &= \frac{1}{2}\cdot base\cdot height \\ &= \frac{1}{2}\cdot(2x+3)\cdot(4x-2) \\ &= \frac{1\cdot(2x+3)(4x-2)}{2} \\ &= \frac{2x\cdot 4x+3\cdot 4x+2x\cdot(-2)+3\cdot(-2)}{2} \\ &= \frac{8x^2+12x+{}^-4x+{}^-6}{2} \\ &= \frac{8x^2+8x-6}{2} \\ &= \frac{2\left(4x^2+4x-3\right)}{2} \\ &= \left(4x^2+4x-3\right) \text{ square units} \end{aligned}$$

Name ______________________

5. Find the area and perimeter of the given triangle.



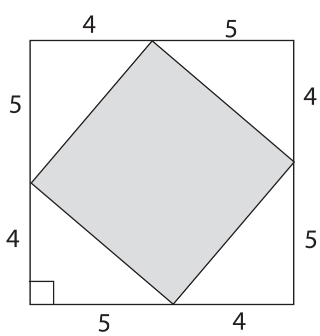
Area: *18 sq. yards*

Perimeter: $21\frac{1}{12}$ *yards*

$$\begin{aligned} Area &= \frac{1}{2}\cdot base\cdot height \\ &= \frac{1}{2}\cdot 6\frac{3}{4}\cdot 5\frac{1}{3} \\ &= \frac{1}{2}\cdot\frac{27}{4}\cdot\frac{16}{3} \\ &= \frac{27\cdot 16}{2\cdot 4\cdot 3} \\ &= \frac{3\cdot 9\cdot 4\cdot 2\cdot 2}{2\cdot 4\cdot 3} \\ &= 9\cdot 2 \\ &= 18\ square\ yards \end{aligned}$$

$$\begin{aligned} Perimeter &= Sum\ of\ lengths\ of\ the\ sides \\ &= 6\frac{3}{4}+5\frac{1}{3}+9 \\ &= \frac{27}{4}+\frac{16}{3}+9 \\ &= \frac{27\cdot 3}{4\cdot 3}+\frac{16\cdot 4}{3\cdot 4}+\frac{9\cdot 12}{1\cdot 12} \\ &= \frac{81}{12}+\frac{64}{12}+\frac{108}{12} \\ &= \frac{81+64+108}{12} \\ &= \frac{253}{12}\ or\ 21\frac{1}{12}\ yards \end{aligned}$$

6. Find the area and perimeter of the shaded square in the given figure.



Area: *41 sq. units*

Perimeter: $4\sqrt{41}$ *units*

Area of Larger Square:

$$\begin{aligned} Area &= (5+4)\cdot(5+4) \\ &= 9\cdot 9 \\ &= 81\ square\ units \end{aligned}$$

Pythagorean Theorem helps us find c.

$$\begin{aligned} a^2+b^2 &= c^2 \\ 4^2+5^2 &= c^2 \\ 16+25 &= c^2 \\ 41 &= c^2 \\ \sqrt{41} &= c \end{aligned}$$

c) Area of Shaded Square

$$\begin{aligned} Area &= \sqrt{41}\cdot\sqrt{41} \\ &= \sqrt{41\cdot 41} \\ &= \sqrt{1681} \\ &= 41\ square\ units \end{aligned}$$

d) Perimeter is the sum of the lengths of the sides.

$$\begin{aligned} Perimeter &= \sqrt{41}+\sqrt{41}+\sqrt{41}+\sqrt{41} \\ &= (1+1+1+1)\sqrt{41} \\ &= 4\sqrt{41}\ units \end{aligned}$$

Quiz Form B

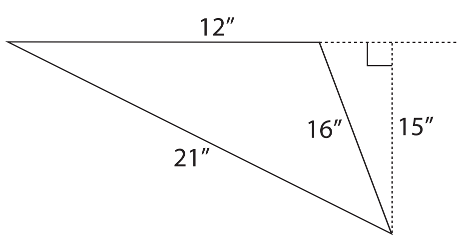
Name ______________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry
Part C - Measurement
Lesson 3 - Triangles

Find the area and perimeter of the given triangles in exercises 1 through 3. Note: You may first have to use the Pythagorean Theorem ($a^2 + b^2 = c^2$) to find some missing parts.

1.



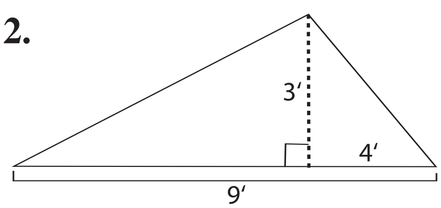
Area: *90 sq. inches*

Perimeter: *49 inches*

$$\begin{aligned} Area &= \frac{1}{2} \cdot base \cdot height \\ &= \frac{1}{2} \cdot 12 \cdot 15 \\ &= \frac{1 \cdot 12 \cdot 15}{2} \\ &= \frac{1 \cdot 2 \cdot 6 \cdot 15}{2} \\ &= 6 \cdot 15 \\ &= 90 \textit{ square inches} \end{aligned}$$

$$\begin{aligned} Perimeter &= Sum\ of\ Lengths\ of\ the\ Sides \\ &= 21 + 16 + 12 \\ &= 21 + 28 \\ &= 49 \textit{ inches} \end{aligned}$$

2.



Area: $13\frac{1}{2}$ *sq. feet*

Perimeter: $\left(14 + \sqrt{34}\right)$ *feet*

$$\begin{aligned} Area &= \frac{1}{2} \cdot base \cdot height \\ &= \frac{1}{2} \cdot 9 \cdot 3 \\ &= \frac{1 \cdot 9 \cdot 3}{2 \cdot 1 \cdot 1} \\ &= \frac{27}{2} \text{ or } 13\frac{1}{2} \textit{ square feet} \end{aligned}$$

Pythagorean Theorem

$$a^2 + b^2 = c^2$$

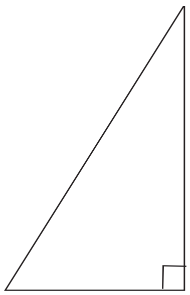
Two Missing Pieces:

$$\begin{aligned} 3^2 + 4^2 &= x^2 \\ 9 + 16 &= x^2 \\ 25 &= x^2 \\ \sqrt{25} &= x \\ 5 &= x \end{aligned} \qquad \begin{aligned} 3^2 + 5^2 &= y^2 \\ 9 + 25 &= y^2 \\ 34 &= y^2 \\ \sqrt{34} &= y \end{aligned}$$

$$\begin{aligned} Perimeter &= Sum\ of\ Lengths\ of\ the\ Sides \\ &= \left(9 + 5 + \sqrt{34}\right) \textit{ feet} \\ &= \left(14 + \sqrt{34}\right) \textit{ feet} \end{aligned}$$

Name ______________________________

3.



Area: *27 sq. cm*

Perimeter: $(15+3\sqrt{13})cm$

$$Area = \frac{1}{2}\cdot base\cdot height$$
$$= \frac{1}{2}\cdot 6\cdot 9$$
$$= \frac{1\cdot 6\cdot 9}{2}$$
$$= \frac{1\cdot 2\cdot 3\cdot 9}{2}$$
$$= 27 \textit{ square cm}$$

Pythagorean Theorem

$$a^2+b^2=c^2$$
$$6^2+9^2=c^2$$
$$36+81=c^2$$
$$117=c^2$$
$$\sqrt{117}=c$$
$$\sqrt{3^2\cdot 13}=c$$
$$3\sqrt{13}=c$$

$$Perimeter = Sum\ of\ Lengths\ of\ the\ Sides$$
$$= 6+9+3\sqrt{13}$$
$$= (15+3\sqrt{13})cm$$

4. Find the area of a triangle with base (2x - 4) units and height (x - 2) units

Area: (x^2-4x+4) *square units*

$$Area = \frac{1}{2}\cdot base\cdot height$$
$$= \frac{1}{2}(2x-4)(x-2)$$
$$= \frac{1}{2}(2x^2-8x+8)$$
$$= (x^2-4x+4) \textit{ square units}$$

Quiz Form A

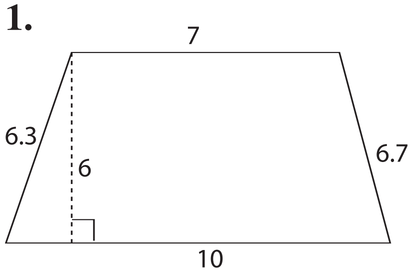
Name ______________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry
Part C - Measurement
Lesson 4 - Trapezoids

Find the area and perimeter of each of the given trapezoids in exercises 1 through 3. Assume all measures are in inches.

1.



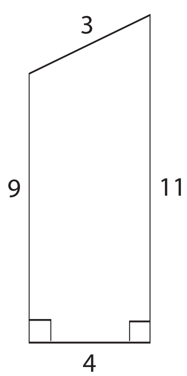
Area: *51 sq. inches*

Perimeter: *30 inches*

$$\begin{aligned} Area &= \frac{1}{2}\cdot height \cdot sum\ of\ the\ bases \\ &= \frac{1}{2}\cdot 6\cdot(10+7) \\ &= \frac{6\cdot 17}{2} \\ &= \frac{2\cdot 3\cdot 17}{2} \\ &= 51\ square\ inches \end{aligned}$$

$$\begin{aligned} Perimeter &= sum\ of\ lengths\ of\ the\ sides \\ &= 10+6.7+7+6.3 \\ &= 30\ inches \end{aligned}$$

2.



Area: *40 sq. inches*

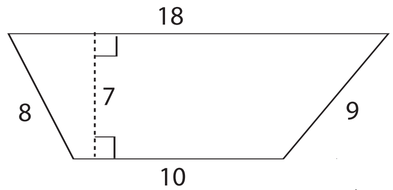
Perimeter: *27 inches*

$$\begin{aligned} Area &= \frac{1}{2}\cdot height \cdot sum\ of\ the\ bases \\ &= \frac{1}{2}\cdot 4\cdot(11+9) \\ &= \frac{4\cdot 20}{2} \\ &= \frac{2\cdot 2\cdot 20}{2} \\ &= 40\ square\ inches \end{aligned}$$

$$\begin{aligned} Perimeter &= sum\ of\ lengths\ of\ the\ sides \\ &= 4+11+3+9 \\ &= 27\ inches \end{aligned}$$

Name ______________________________

3.



Area: *98 sq. inches*

Perimeter: *45 inches*

$$\begin{aligned} Area &= \frac{1}{2} \cdot height \cdot sum\ of\ the\ bases \\ &= \frac{1}{2} \cdot 7 \cdot (10+18) \\ &= \frac{7 \cdot 28}{2} \\ &= \frac{7 \cdot 2 \cdot 14}{2} \\ &= 98\ square\ inches \end{aligned}$$

$$\begin{aligned} Perimeter &= sum\ of\ lengths\ of\ the\ sides \\ &= 8+10+9+18 \\ &= 45\ inches \end{aligned}$$

4. The area of a trapezoid is 100 square centimeters. The sum of the lengths of the bases is 50 centimeters. Find the height.

height: *4 cm*

$$\begin{aligned} Area &= \frac{1}{2} \cdot height \cdot sum\ of\ the\ bases \\ 100 &= \frac{1}{2} \cdot h \cdot (50) \\ \frac{2}{1} \cdot 100 &= \frac{2}{1} \cdot \frac{1}{2} \cdot h \cdot 50 \\ 200 &= 1 \cdot h \cdot 50 \\ \frac{1}{50} \cdot \frac{200}{1} &= h \cdot 50 \cdot \frac{1}{50} \\ \frac{50 \cdot 4}{50} &= h \cdot \frac{50}{50} \\ 4cm &= h \end{aligned}$$

Name ______________________________

5. The area of a trapezoid is 420m². The height is 12m. One base is 20m. Find the length b_2 of the other base.

$$Area = \frac{1}{2} \cdot height \cdot sum\ of\ the\ bases$$

$$420 = \frac{1}{2} \cdot 12 \cdot \left(20 + b_2\right)$$

$$420 = \frac{1}{2} \cdot \frac{12}{1} \cdot (20 + b_2)$$

$$420 = \frac{1 \cdot 2 \cdot 6}{2 \cdot 1} \cdot \left(20 + b_2\right)$$

$$420 = 6 \cdot 20 + 6 \cdot b_2$$

$$420 = 120 + 6 \cdot b_2$$

$$420 - 120 = 120 - 120 + 6b_2$$

$$300 = 0 + 6b_2$$

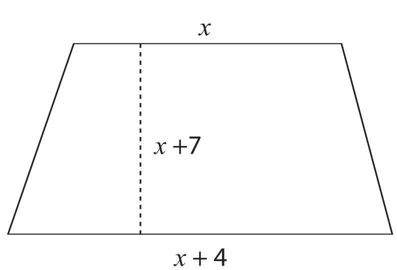
$$\frac{1}{6} \cdot \frac{300}{1} = \frac{1}{6} \cdot 6 \cdot b_2$$

$$\frac{1 \cdot 6 \cdot 50}{6 \cdot 1} = \frac{6}{6} \cdot b_2$$

$$50m = b_2$$

other base: *50 meters*

6. Application: The area of a trapezoid is 66 square units. The length of its longer base is 4 units longer than the length of its shorter base, and its height is 7 units longer than the length of its shorter base. Find the length of each base and the height of the trapezoid. (draw a diagram and label the necessary parts)



length of short base: *4 units*

length of long base: *8 units*

height: *11 units*

$$Area = \frac{1}{2} \cdot height \cdot sum\ of\ the\ bases$$

$$66 = \frac{1}{2} \cdot (x+7) \cdot \left[(x) + (x+4)\right]$$

$$66 = \frac{1}{2} \cdot (x+7) \cdot (2x+4)$$

$$2 \cdot 66 = \frac{2}{1} \cdot \frac{1}{2} \cdot (x+7)(2x+4)$$

$$132 = (x+7)(2x+4)$$

$$132 = x \cdot 2x + 7 \cdot 2x + x \cdot 4 + 7 \cdot 4$$

$$132 = 2x^2 + 18x + 28$$

$$132 - 132 = 2x^2 + 18x + 28 - 132$$

$$0 = 2x^2 + 18x - 104$$

$$0 = 2\left(x^2 + 9x - 52\right)$$

$$0 = 2(x-4)(x+13)$$

$$0 = 2\ (false)$$

$$0 = x - 4$$

$$0 + 4 = x - 4 + 4$$

$$4 = x + 0$$

$$4 = x$$

$$0 = x + 13$$

$$0 - 13 = x + 13 - 13$$

$$^-13 = x + 0$$

$$^-13 = x\ (can't\ be\ negative)$$

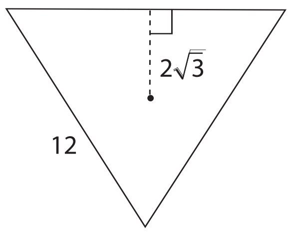
$$x = 4\ (base)$$

$$x + 7 = 11\ (height)$$

$$x + 4 = 8\ (base)$$

Name ______________________________

3.



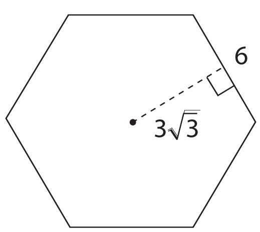
Area: $36\sqrt{3}$ *square cm*

Perimeter: *36 cm*

$$\begin{aligned} Perimeter &= Sum\ of\ Lengths\ of\ the\ sides \\ &= Number\ of\ sides \cdot \\ &\quad length\ of\ each\ side \\ &= n \cdot s \\ &= 3 \cdot 12 \\ &= 36\ cm \end{aligned}$$

$$\begin{aligned} Area &= \frac{1}{2}\ measure\ of\ a\ side \cdot Apothem \cdot \\ &\quad number\ of\ sides \\ &= \frac{1}{2} \cdot s \cdot a \cdot n \\ &\quad or \\ &= \frac{1}{2} \cdot a \cdot Perimeter \\ &= \frac{1}{2} \cdot \left(2\sqrt{3}\right) \cdot 36 \\ &= \sqrt{3} \cdot 36 \\ &= 36\sqrt{3}\ square\ cm \end{aligned}$$

4.



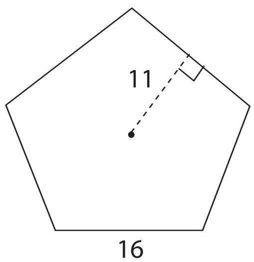
Area: $54\sqrt{3}$ *square cm*

Perimeter: *36 cm*

$$\begin{aligned} Perimeter &= Sum\ of\ Lengths\ of\ the\ sides \\ &= Number\ of\ sides \cdot \\ &\quad length\ of\ each\ side \\ &= n \cdot s \\ &= 6 \cdot 6 \\ &= 36\ cm \end{aligned}$$

$$\begin{aligned} Area &= \frac{1}{2}\ measure\ of\ a\ side \cdot Apothem \cdot \\ &\quad the\ number\ of\ sides \\ &= \frac{1}{2} \cdot s \cdot a \cdot n \\ &= \frac{1}{2} \cdot a \cdot Perimeter \\ &= \frac{1}{2} \cdot \left(3\sqrt{3}\right) \cdot 36 \\ &= \frac{3 \cdot \sqrt{3} \cdot 2 \cdot 18}{2} \\ &= 54\sqrt{3}\ square\ cm \end{aligned}$$

5.



Area: *440 square cm*

Perimeter: *80 cm*

$$\begin{aligned}Perimeter &= Sum\ of\ Lengths\ of\ the\ sides\\ &= Number\ of\ sides \cdot\\ &\quad length\ of\ each\ side\\ &= n \cdot s\\ &= 5 \cdot 16\\ &= 80\ cm\end{aligned}$$

$$\begin{aligned}Area &= \frac{1}{2}\ measure\ of\ a\ side \cdot Apothem \cdot\\ &\quad number\ of\ sides\\ &= \frac{1}{2} \cdot s \cdot a \cdot n\\ &or\\ &= \frac{1}{2} \cdot a \cdot Perimeter\\ &= \frac{1}{2} \cdot 11 \cdot 80\\ &= \frac{11 \cdot 2 \cdot 40}{2}\\ &= 440\ square\ cm\end{aligned}$$

6. Find the area and perimeter of a regular decagon with apothem 6.8cm and side length 4.4cm.

Area: *149.6 square cm*

Perimeter: *44 cm*

$$\begin{aligned}Perimeter &= Sum\ of\ Lengths\ of\ the\ sides\\ &= Number\ of\ sides \cdot\\ &\quad length\ of\ each\ side\\ &= n \cdot s\\ &= 10 \cdot (4.4)\\ &= 44\ cm\end{aligned}$$

$$\begin{aligned}Area &= \frac{1}{2}\ measure\ of\ a\ side \cdot Apothem \cdot\\ &\quad number\ of\ sides\\ &= \frac{1}{2} \cdot s \cdot a \cdot n\\ &= \frac{1}{2} \cdot a \cdot Perimeter\\ &= \frac{1}{2} \cdot (6.8) \cdot 44\\ &= \frac{2 \cdot (3.4) \cdot 44}{2}\\ &= 149.6\ square\ cm\end{aligned}$$

Name ______________________________

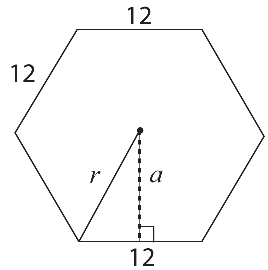
7. Application: Find the indicated measures in the given regular hexagon.

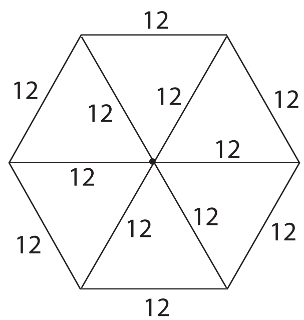
a = $6\sqrt{3}$ *units*

r = *12*

Area = $216\sqrt{3}$ *square units*

Perimeter = *72 units*

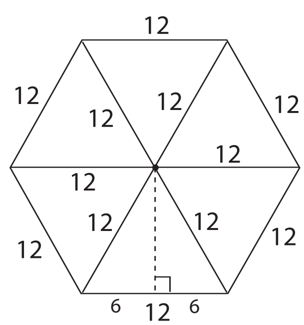




$$
\begin{aligned}
\textit{Perimeter} &= \textit{Sum of Lengths of the sides}\\
&= \textit{Number of sides} \cdot\\
&\quad \textit{length of each side}\\
&= n \cdot s\\
&= 6 \cdot 12\\
&= 72 \textit{ units}
\end{aligned}
$$

Hint #1: Complete the inside of the hexagon with line segments drawn from the center to all 6 vertices. The small triangles appear to be (and actually are) equilateral. Therefore, r = 12 units

Hint #2: To find "a", we can reason this way. Each triangle is an equilateral triangle and therefore an isosceles triangle. The Apothem, "a", is an altitude in the triangle and will bisect the base of 12. So, our hexagon could look like this: Observe what the apothem does to the side of length 12.



Pythagorean Theorem.

$$
\begin{aligned}
a^2 + b^2 &= c^2\\
a^2 + 6^2 &= 12^2\\
a^2 + 36 &= 144\\
a^2 + 36 + {}^{-}36 &= 144 + {}^{-}36\\
a^2 &= 108\\
a &= \sqrt{108}\\
a &= \sqrt{36 \cdot 3}\\
a &= 6\sqrt{3} \textit{ units}
\end{aligned}
$$

$$
\begin{aligned}
\textit{Area} &= \frac{1}{2} \textit{ measure of a side} \cdot \textit{Apothem} \cdot \textit{number of sides}\\
&= \frac{1}{2} \cdot s \cdot a \cdot n\\
&= \frac{1}{2} \cdot a \cdot \textit{Perimeter}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2} \cdot 6\sqrt{3} \cdot 72\\
&= \frac{2 \cdot 3 \cdot \sqrt{3} \cdot 72}{2}\\
&= 216\sqrt{3} \textit{ square units}
\end{aligned}
$$

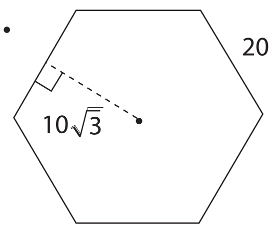
Name ______________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry
Part C - Measurement
Lesson 5 - Regular Polygons

Find the perimeter and area of each regular polygon in exercises 1 through 6. Assume all dimensions are in centimeters.

1.



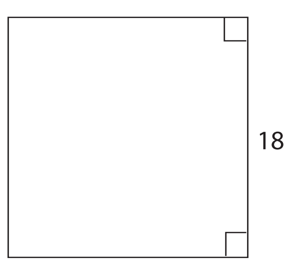
Area: *$600\sqrt{3}$ square cm*

Perimeter: *120 cm*

$$\begin{aligned} Perimeter &= Sum\ of\ Lengths\ of\ the\ sides \\ &= Number\ of\ sides \cdot \\ &\quad length\ of\ each\ side \\ &= n \cdot s \\ &= 6 \cdot 20 \\ &= 120\ cm \end{aligned}$$

$$\begin{aligned} Area &= \frac{1}{2}\ measure\ of\ a\ side \cdot Apothem \cdot \\ &\quad number\ of\ sides \\ &= \frac{1}{2} \cdot s \cdot a \cdot n \\ &\ or \\ &= \frac{1}{2} \cdot a \cdot Perimeter \\ &= \frac{1}{2} \cdot 10\sqrt{3} \cdot 120 \\ &= \frac{2 \cdot 5\sqrt{3} \cdot 120}{2} \\ &= 600\sqrt{3}\ square\ cm \end{aligned}$$

2.



Area: *324 square cm*

Perimeter: *72cm*

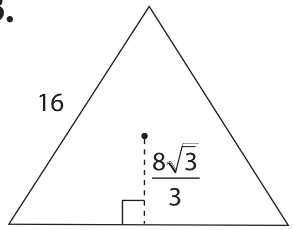
$$\begin{aligned} Perimeter &= Sum\ of\ Lengths\ of\ the\ sides \\ &= Number\ of\ sides \cdot \\ &\quad length\ of\ each\ side \\ &= n \cdot s \\ &= 4 \cdot 18 \\ &= 72\ cm \end{aligned}$$

$$\begin{aligned} Area &= \frac{1}{2}\ measure\ of\ a\ side \cdot Apothem \cdot \\ &\quad number\ of\ sides \\ &= \frac{1}{2} \cdot s \cdot a \cdot n \\ &= \frac{1}{2} \cdot a \cdot Perimeter \\ &= \frac{1}{2} \cdot 9 \cdot 72 \\ &= \frac{9 \cdot 2 \cdot 36}{2} \\ &= 324\ square\ cm \end{aligned}$$

Note: a = 18/2 or 9

Name ______________________________

3.



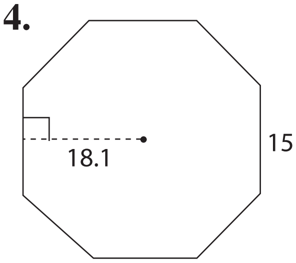
Area: $64\sqrt{3}$ *square cm*

Perimeter: *48 cm*

$$
\begin{aligned}
Perimeter &= Sum\ of\ Lengths\ of\ the\ sides \\
&= Number\ of\ sides \cdot \\
&\quad length\ of\ each\ side \\
&= n \cdot s \\
&= 3 \cdot 16 \\
&= 48\ cm
\end{aligned}
$$

$$
\begin{aligned}
Area &= \frac{1}{2}\ measure\ of\ a\ side \cdot Apothem \cdot \\
&\quad number\ of\ sides \\
&= \frac{1}{2} \cdot s \cdot a \cdot n \\
&= \frac{1}{2} \cdot a \cdot Perimeter \\
&= \frac{1}{2} \cdot \frac{8\sqrt{3}}{3} \cdot 48 \\
&= \frac{8\sqrt{3} \cdot 2 \cdot 3 \cdot 8}{2 \cdot 3} \\
&= 64\sqrt{3}\ square\ cm
\end{aligned}
$$

4.



Area: *1086 square cm*

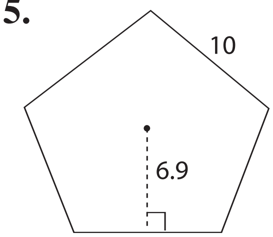
Perimeter: *120 cm*

$$
\begin{aligned}
Perimeter &= Sum\ of\ Lengths\ of\ the\ sides \\
&= Number\ of\ sides \cdot \\
&\quad length\ of\ each\ side \\
&= n \cdot s \\
&= 8 \cdot 15 \\
&= 120\ cm
\end{aligned}
$$

$$
\begin{aligned}
Area &= \frac{1}{2}\ measure\ of\ a\ side \cdot Apothem \cdot \\
&\quad number\ of\ sides \\
Area &= \frac{1}{2} \cdot s \cdot a \cdot n \\
Area &= \frac{1}{2} \cdot a \cdot Perimeter \\
Area &= \frac{1}{2} \cdot (18.1) \cdot 120 \\
&= \frac{(18.1) \cdot 2 \cdot 60}{2} \\
&= (18.1) \cdot 60 \\
&= 1086\ square\ cm
\end{aligned}
$$

Name ______________________________

5.



Area: *172.5 square cm*

Perimeter: *50 cm*

$$\begin{aligned} \textit{Perimeter} &= \textit{Sum of Lengths of the sides} \\ &= \textit{Number of sides} \cdot \textit{length of each side} \\ &= n \cdot s \\ &= 5 \cdot 10 \\ &= 50\ cm \end{aligned}$$

$$\begin{aligned} \textit{Area} &= \frac{1}{2}\ \textit{measure of a side} \cdot \textit{Apothem} \cdot \textit{number of sides} \\ &= \frac{1}{2} \cdot s \cdot a \cdot n \\ &= \frac{1}{2} \cdot a \cdot \textit{Perimeter} \\ &= \frac{1}{2} \cdot (6.9) \cdot 50 \\ &= \frac{(6.9) \cdot 2 \cdot 25}{2} \\ &= (6.9) \cdot 25 \\ &= 172.5\ \textit{square cm} \end{aligned}$$

6. A regular dodecagon has a side of length 2 in. and an approximate area of 44.78 in^2. Find the length of the apothem to the nearest tenth of an inch.

apothem: *3.7 inches*

$$\begin{aligned} \textit{Area} &= \frac{1}{2}\ \textit{measure of a side} \cdot \textit{Apothem} \cdot \textit{number of sides} \\ 44.78 &= \frac{1}{2} \cdot s \cdot a \cdot n \\ 44.78 &= \frac{1}{2} \cdot a \cdot 2 \cdot 12 \\ 44.78 &= \frac{2 \cdot a \cdot 12}{2} \\ 44.78 &= a \cdot 12 \\ \frac{1}{12} \cdot 44.78 &= a \cdot 12 \cdot \frac{1}{12} \\ 3.731 &= 1a \\ 3.731\ \textit{inches} &= a \end{aligned}$$

Name ______________________________

In exercises 7 and 8 find the approximation, correct to the nearest hundredth, of the circumference of a circle with the given radius. Use 3.142 to approximate π.

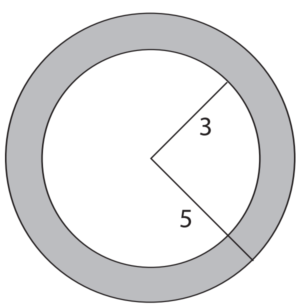
7. radius = $\frac{7}{3}$cm   Circumference $\doteq$ *14.66 cm.*

$$\begin{aligned} Circumference &= 2\pi r \\ &\doteq 2(3.142)\left(\frac{7}{3}\,cm\right) \\ &\doteq (6.284)\left(\frac{7}{3}\,cm\right) \\ &\doteq 14.66\ cm \end{aligned}$$

8. radius = 8.6 in.   Circumference $\doteq$ *54.04 in.*

$$\begin{aligned} Circumference &= 2\pi r \\ &\doteq 2(3.142)(8.6\ in) \\ &\doteq (6.284)(8.6\ in) \\ &\doteq 54.04\ in \end{aligned}$$

9. Find the area of the shaded region. Use 3.142 for π, and approximate the answer to the nearest tenth.



Area $\doteq$ *50.3 units²*

Area of shaded region = area of large circle - area of small circle.

$$\begin{aligned} Area &= \pi r^2 - \pi r^2 \\ &\doteq (3.142)(5)(5) - (3.142)(3)(3) \\ &\doteq (3.142)25 - (3.142)9 \\ &\doteq (3.142)(25-9) \\ &\doteq (3.142)(16) \\ &\doteq 50.272 \\ &\doteq 50.27 \end{aligned}$$

Name ______________________________

In exercises 7 and 8 find the approximation, correct to the nearest hundredth, of the circumference of a circle with the given radius. Use 3.142 for π.

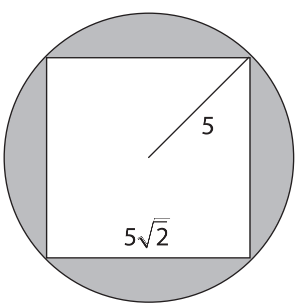
7. r = 2.1 km **Circumference =** 13.20 *km*

$$\begin{aligned} \textit{Circumference} &= 2\pi r \\ &= 2(3.142)(2.1) \\ &= (6.284)(2.1) \\ &= 13.1964 \\ &= 13.20\ km \end{aligned}$$

8. r = $\frac{7}{5}$ cm. **Circumference =** *8.80 cm*

$$\begin{aligned} \textit{Circumference} &= 2\pi r \\ &\doteq 2(3.142)\left(\frac{7}{5}\right) \\ &\doteq 2(3.142)(1.4) \\ &\doteq (6.284)(1.4) \\ &\doteq 8.7976 \\ &\doteq 8.80\ cm \end{aligned}$$

9. Find the area of the shaded region. Use 3.142 for π. Approximate the answer to the nearest tenth

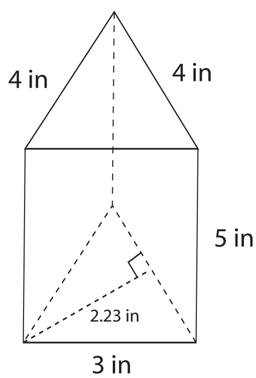


Area = *28.6 square units*

$$\begin{aligned} \textit{Area of shaded region} &= \textit{Area of circle} - \textit{Area of square} \\ &= \pi r^2 - s^2 \\ &= \pi \cdot 5 \cdot 5 - \left(5\sqrt{2}\right)\left(5\sqrt{2}\right) \\ &= \pi \cdot 5 \cdot 5 - \left(5\sqrt{2}\right)\left(5\sqrt{2}\right) \\ &= \pi \cdot 25 - \left(5 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{2}\right) \\ &= 25\pi - (25 \cdot 2) \\ &= 25\pi - 50 \\ &\doteq 25(3.142) - 50 \\ &\doteq 78.55 - 50 \\ &\doteq 28.55\ \textit{square units} \end{aligned}$$

Name ______________________________

4.



Lateral Area = $55in^2$

Total Area = $63.92in^2$

Volume = $22.3in^3$

Lateral Area of a Prism (L.A.) = Perimeter of the Base Multiplied by the height of the prism

$$L.A. = P \cdot h$$

$$L.A. = (4 + 4 + 3) \cdot 5$$

$$L.A. = 11 \cdot 5$$

$$L.A. = 55\ in^2$$

Total Area of a Prism (T.A.) = The sum of the lateral area plus the area of the bases

$$T.A. = L.A. + \frac{1}{2} \cdot b \cdot h + \frac{1}{2} \cdot b \cdot h$$

$$Area = 55 + \frac{1}{2} \cdot 4 \cdot 2.23 + \frac{1}{2} \cdot 4 \cdot 2.23$$

$$Area = 55 + 2 \cdot 2.23 + 2 \cdot 2.23$$

$$Area = 55 + 4.46 + 4.46$$

$$Area = 63.92\ in^2$$

Volume of Prism (V) = Area of one base multiplied by the height of the prism

$$= \frac{1}{2} \cdot b \cdot h \cdot 5$$

$$= \frac{1}{2} \cdot 4 \cdot 2.23 \cdot 5$$

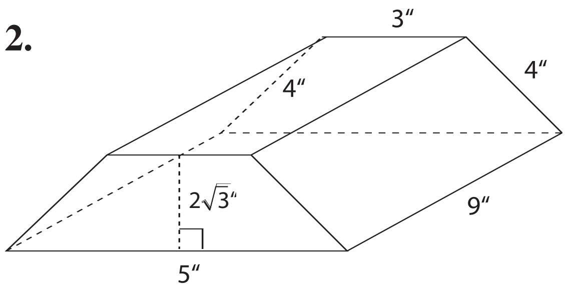
$$= 2 \cdot 2.23 \cdot 5$$

$$= 4.46 \cdot 5$$

$$= 22.3\ in^3$$

Name

2.



Lateral Area = $144\ in^2$

Total Area = $(144 + 16\sqrt{3})\ in^2$

Volume = $72\sqrt{3}\ in^3$

Lateral Area of a Prism (L.A.) = Perimeter of the Base Multiplied by the height of the prism.

Perimeter = Sum of the lengths of the sides.

Perimeter of Base = 3 + 4 + 5 + 4= 16 in.

$$L.A. = P \cdot h$$
$$= 16 \cdot 9$$
$$= 144\ in^2$$

Total Area of a Prism (T.A.)= The sum of the L.A. and the area of the bases

$$\text{Area of one base} = \frac{1}{2} \cdot h \cdot (b_1 + b_2)$$

$$= \frac{1}{2} \cdot \frac{2\sqrt{3}}{1} \cdot (3 \cdot 5)$$

$$= \sqrt{3} \cdot 8 \text{ sq. in. or } 8\sqrt{3}\ in^2$$

$$\text{Area of the two bases} = 8\sqrt{3}\ in^2 + 8\sqrt{3}\ in^2$$
$$= 16\sqrt{3}\ in^2$$

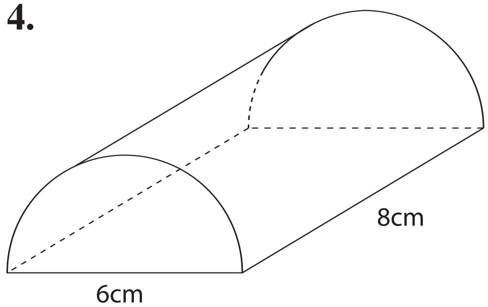
$$\text{Total area} = 144 + 16\sqrt{3}\ in^2$$
$$= (144 + 16\sqrt{3})in^2$$

Volume of a Prism (V) = Area of the Base x height

$$Volume = B \cdot h = 8\sqrt{3}\ in^2 \cdot 9\ in$$
$$= 72\sqrt{3}\ in^3 \text{ or } 72\sqrt{3} \text{ cubic inches}$$

Name ______________________________

4.



Lateral Area = $(48 + 24\pi)cm^2$

Total Area = $(33\pi + 48)\ cm^2$

Volume = $36\pi\ cm^3$

Lateral Area of a Cylinder (L.A.) = Circumference of the Base multiplied by height

$$\text{Diameter} = 2 \cdot \text{Radius}$$
$$d = 2 \cdot r$$

$$\text{Circumference} = \pi \cdot d$$

$$\begin{aligned} L.A. &= \pi \cdot d \cdot h \\ &= \pi \cdot 6\ cm \cdot 8\ cm \\ &= \pi \cdot 48\ cm^2 \\ * \quad &= 48\pi\ cm^2 \end{aligned}$$

*(*full cylinder) Since this figure has a semi-circle for a base, it is $\frac{1}{2}$ of a cylinder. The Lateral Area will be $\frac{1}{2}$ of the Lateral Area of the full cylinder plus the Area of the rectangular side created by cutting the full cylinder in half.*

$$\frac{1}{2} \cdot \text{Lateral Area of full cylinder} = \frac{1}{2} \cdot 48\pi\ cm^2 = 24\pi\ cm^2$$

$$\text{Area of Rectangle} = \ell \cdot w = 8\ cm \cdot 6\ cm = 48\ cm^2$$

$$\therefore \text{Lateral Area of } \frac{1}{2} \text{ cylinder} = 24\pi\ cm^2 + 48\ cm^2 = (48 + 24\pi)\ cm^2$$

Total Area of a Cylinder (T.A.)= The sum of the L.A. and the area of the bases

$$\begin{aligned} \text{Area of one base} &= \pi \cdot \text{radius} \cdot \text{radius} \\ &= \pi \cdot r^2 \\ &= \pi \cdot 3\ cm \cdot 3\ cm \\ &= \pi \cdot 9\ cm^2 \\ * \quad &= 9\pi\ cm^2 \\ &= 2 \cdot r \\ 6\ cm &= 2 \cdot r \\ \frac{1}{2} \cdot 6\ cm &= \frac{1}{2} \cdot 2 \cdot r \\ 3\ cm &= r \end{aligned}$$

$$\therefore \text{Area of one base of } \frac{1}{2} \text{ cylinder} = \frac{1}{2} \cdot 9\pi\ cm^2 = \frac{9}{2}\pi\ cm^2$$

$$\begin{aligned} \text{Area of two bases} &= 2 \cdot \frac{9}{2}\pi\ cm^2 \\ &= \frac{2}{1} \cdot \frac{9}{2} \cdot \pi\ cm^2 \\ &= 9\pi\ cm^2 \end{aligned}$$

$$\begin{aligned} \text{Total Area} &= 24\pi\ cm^2 + 48\ cm^2 + 9\pi cm^2 \\ &= (24\pi + 48 + 9\pi)\ cm^2 \\ &= (24\pi + 9\pi + 48)\ cm^2 \\ &= (33\pi + 48)\ cm^2 \end{aligned}$$

Volume of a Cylinder (V) = Area of the Base x height

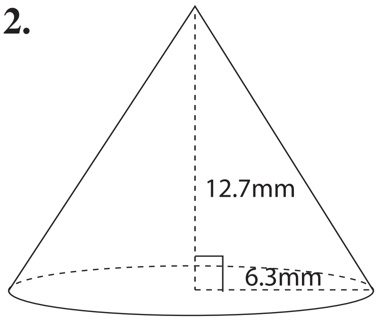
$$\text{Volume of a full cylinder} = 9\pi\ cm^2 \bullet 8\ cm = 72\pi\ cm^3$$

$$\begin{aligned} \text{Volume of a half cylinder} &= \frac{1}{2} \bullet 72\pi\ cm^3 \\ &= 36\pi\ cm^3 \text{ or } 36\pi \text{ cubic centimeters} \end{aligned}$$

Name ______________________________

Find the lateral area, total area, and volume of the right circular cone shown below.

2.



Lateral Area = *89.32π sq mm*

Total Area = *129.01π mm²*

Volume = *168.02π mm³*

Lateral Area of a Cone (L.A.) = $\frac{1}{2}$ times the circumference of the base times the slant of the height.

$$C = \pi \cdot d$$

$$d = 2 \cdot r$$

Use Pythagorean Theorem to find slant height.

$$\begin{aligned} L.A. &= \frac{1}{2} \cdot c \cdot l \\ &= \frac{1}{2} \cdot \pi \cdot d \cdot l \\ &= \frac{1}{2} \cdot \pi \cdot 2 \cdot r \cdot l \\ &= \frac{1 \cdot \pi \cdot \cancel{2} \cdot 6.3\text{ mm} \cdot 14.177\text{ mm}}{\cancel{2} \cdot 1 \cdot 1 \cdot 1 \cdot 1} \\ &= \pi \cdot 89.32\text{ mm}^2 \\ &= 89.32\pi \text{ sq mm} \end{aligned}$$

Total Area of a Cone (T.A.) = the sum of the Base Area and the Lateral Area.

$$\begin{aligned} T.A. &= B.A. + L.A. \\ &= (\text{Area of a circle}) + L.A. \\ &= \pi \cdot r^2 + 89.32 \text{ sq. mm} \\ &= \pi \cdot (6.3\text{ mm})^2 + 89.32\pi\text{ mm}^2 \\ &= \pi \cdot 39.69\text{ mm}^2 + 89.32\pi\text{ mm}^2 \\ &= (39.69\pi + 89.32\pi)\text{ mm}^2 \\ &= 129.01\pi\text{ mm}^2 \end{aligned}$$

Volume of a Cone (V) = $\frac{1}{3}$ times the Area of the Base times the height of the cone.

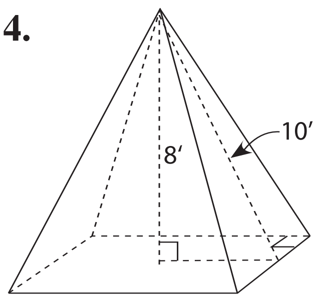
Use Pythagorean Theorem to find h.

$$\begin{aligned} V &= \frac{1}{3} \cdot B \cdot h \\ &= \frac{1}{3} \cdot (\text{Area of a circle}) \cdot h \\ &= \frac{1}{3}\pi \cdot r^2 \cdot h \\ &= \frac{1}{3} \cdot \pi \cdot (6.3\text{mm}) \cdot (6.3\text{mm}) \cdot (12.7\text{mm}) \\ &= \frac{1 \cdot \pi \cdot 504.063}{3 \cdot 1 \cdot 1} \\ &= \frac{504.063\pi}{3} \\ &\approx 168.02\pi\text{ mm}^3 \end{aligned}$$

Name ______________________________

Find the lateral area, total area, and volume of the right pyramid shown below.

4.



Lateral Area = $240\ ft^2$

Total Area = $384\ ft^2$

Volume = $384\ ft^3$

Lateral Area of Pyramid (L.A.) = $\frac{1}{2}$ *times the perimeter of the Base multiplied by the slant height.*

Use Pythagorean Theorem to find $\frac{1}{2}$ *the length of each side of the square.*

$$L.A. = \frac{1}{2} \cdot P \cdot \ell$$
$$= \frac{1}{2} \cdot (12 + 12 + 12 + 12) \cdot 10$$
$$= \frac{1}{2} \cdot 48\ ft \cdot 10\ ft$$
$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2} \cdot 24 \cdot 10}{1 \cdot 1}\ ft^2$$
$$= 240\ ft^2$$

Total Area of Pyramid (T.A.) = the sum of the Base Area and the Lateral Area.

$$T.A. = B.A. + L.A.$$
$$= 144\ ft^2 + 240\ ft^2$$
$$= 384\ ft^2$$

Volume of a Pyramid (V) = $\frac{1}{3}$ *times the Area of the Base times the height of the pyramid.*

Use Pythagorean Theorem to find h.

$$V = \frac{1}{3} \cdot B \cdot h$$
$$= \frac{1}{3} \cdot 144\ ft^2 \cdot 8\ ft$$
$$= \frac{1}{\cancel{3}} \cdot \frac{\cancel{3} \cdot 48\ ft^2 \cdot 8\ ft}{1 \cdot 1}\ ft^3$$
$$= 384\ ft^3$$

Quiz Form B

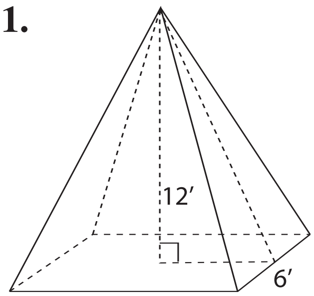
Name ______________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry
Part C - Measurement
Lesson 8 - Pyramids

Find the lateral area, total area, and volume of the right square pyramid shown below.

1.



Lateral Area = $36\sqrt{17}\ ft^2$

Total Area = $36(1+\sqrt{17})\ ft^2$

Volume = $144\ ft^3$

Lateral Area of Pyramid (L.A.) = $\frac{1}{2}$ *times the perimeter of the Base multiplied by the slant height.*

Use Pythagorean Theorem to find slant height.

$$\begin{aligned} L.A. &= \frac{1}{2}\cdot P\cdot \ell \\ &= \frac{1}{2}\cdot(6+6+6+6)\cdot\sqrt{153} \\ &= \frac{1}{2}\cdot(24\ ft)\cdot\sqrt{153}\,ft \\ &= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}\cdot 12\cdot\sqrt{153}}{1\cdot 1}\,ft^2 \\ &= 12\sqrt{153}\,ft^2 \\ &= 12\sqrt{9\cdot 17}\,ft^2 \\ &= 12\cdot 3\sqrt{17}\,ft^2 \\ &= 36\sqrt{17}\,ft^2 \end{aligned}$$

Total Area of Pyramid (T.A.) = the sum of the Base Area and the Lateral Area.

$$\begin{aligned} T.A. &= B.A. + L.A. \\ &= 36\ ft^2 + 36\sqrt{17}\ ft^2 \\ &= 36(1+\sqrt{17})\ ft^2 \end{aligned}$$

Volume of a Pyramid (V) = $\frac{1}{3}$ *times the Area of the Base times the height of the pyramid.*

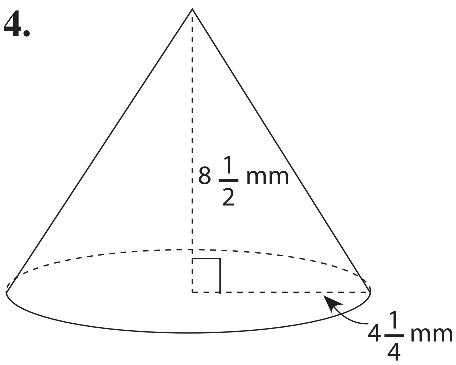
Use Pythagorean Theorem to find h.

$$\begin{aligned} V &= \frac{1}{3}\cdot B\cdot h \\ &= \frac{1}{3}\cdot 36\ ft^2\cdot 12\ ft \\ &= \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}\cdot 12\cdot 12}{1\cdot 1}\,ft^3 \\ &= 144\ ft^3 \end{aligned}$$

Name ______________________________

Find the lateral area, total area, and volume of the right circular cone shown below.

4.



Lateral Area = $\dfrac{289\pi\sqrt{5}}{16}\,mm^2$

Total Area = $\dfrac{289\pi\left(1+\sqrt{5}\right)}{16}\,mm^2$

Volume = $\dfrac{4913\pi}{96}\,mm^3$

Lateral Area of a Cone (L.A.) = $\frac{1}{2}$ *times the circumference of the base times the slant height of the cone.*

$$C = \pi \bullet d$$

$$d = 2 \bullet r$$

Use Pythagorean Theorem to find the slant height.

$$\begin{aligned} L.A. &= \frac{1}{2}\cdot c\cdot \ell \\ &= \frac{1}{2}\cdot \pi\cdot d\cdot \ell \\ &= \frac{1}{2}\cdot \pi\cdot 2r\cdot \ell \\ &= \frac{1}{2}\cdot \pi\cdot 2\cdot \frac{17}{4}\,mm\cdot \frac{17\sqrt{5}}{4}\,mm \\ &= \frac{1\cdot \pi\cdot \cancel{2}\cdot 17\cdot 17\sqrt{5}}{\cancel{2}\cdot 1\cdot 1\cdot 4\cdot 4}\,mm^2 \\ &= \frac{\pi\cdot 289\sqrt{5}}{16}\,mm^2 \\ &= \frac{289\pi\sqrt{5}}{16}\,mm^2 \end{aligned}$$

Total Area of a Cone (T.A.) = the sum of the Base Area and the Lateral Area.

$$\begin{aligned} T.A. &= B.A. + L.A. \\ &= (\text{Area of a circle}) + L.A. \\ &= \pi\cdot r^2 + \frac{289\pi\sqrt{5}}{16}\,mm^2 \\ &= \pi\cdot \frac{17}{4}\,mm\cdot \frac{17}{4}\,mm + \frac{289\pi\sqrt{5}}{16}\,mm^2 \\ &= \frac{\pi}{1}\cdot\frac{17}{4}\cdot\frac{17}{4}\,mm^2 + \frac{289\pi\sqrt{5}}{16}\,mm^2 \\ &= \frac{289\pi}{16}\,mm^2 + \frac{289\pi\sqrt{5}}{16}\,mm^2 \\ &= \frac{289\pi + 289\pi\sqrt{5}}{16}\,mm^2 \\ &= \frac{289\pi(1+\sqrt{5})}{16}\,mm^2 \end{aligned}$$

Volume of a Cone (V) = $\frac{1}{3}$ *times the Area of the Base times the height of the cone.*

$$\begin{aligned} V &= \frac{1}{3}\cdot B\cdot h \\ &= \frac{1}{3}\cdot(\text{Area of a circle})\cdot h \\ &= \frac{1}{3}\pi\cdot r^2\cdot h \\ &= \frac{1}{3}\pi\cdot\frac{17}{4}\,mm\cdot\frac{17}{4}\,mm\cdot\frac{17}{2}\,mm \\ &= \frac{1\cdot\pi\cdot 17\cdot 17\cdot 17}{3\cdot 1\cdot 4\cdot 4\cdot 2}\,mm^3 \\ &= \frac{4913\pi}{96}\,mm^3 \end{aligned}$$

Name ______________________________

3. If the volume of a sphere is 12π cubic units, find the radius and the surface area.

radius = $\sqrt[3]{9}$ *units*

Surface Area = $12\pi \cdot \sqrt[3]{3}$ *sq. units*

$$
\begin{aligned}
\textit{Volume of sphere} &= \frac{4}{3}\cdot\pi\cdot r^3 \\
12\pi &= \frac{4}{3}\cdot\pi r^3 \\
3\cdot 12\pi &= 3\cdot\frac{4}{3}\cdot\pi r^3 \\
36\pi &= 4\pi r^3 \\
\frac{1}{4\pi}\cdot\frac{36\pi}{1} &= \frac{1}{4\pi}\cdot\frac{4\pi}{1}\cdot r^3 \\
\frac{1}{4\pi}\cdot\frac{4\pi\cdot 9}{1} &= 1\cdot r^3 \\
9 &= r^3 \\
\sqrt[3]{9}\ \textit{units} &= r
\end{aligned}
$$

$$
\begin{aligned}
\textit{Surface Area of a sphere} &= 4\cdot\pi\cdot r^2 \\
&= 4\cdot\pi\cdot\sqrt[3]{9}\cdot\sqrt[3]{9} \\
&= 4\cdot\pi\cdot\sqrt[3]{9^2} \\
&= 4\cdot\pi\cdot\sqrt[3]{81} \\
&= 4\cdot\pi\cdot\sqrt[3]{27}\cdot\sqrt[3]{3} \\
&= 4\cdot\pi\cdot 3\cdot\sqrt[3]{3} \\
&= 12\pi\cdot\sqrt[3]{3}\ \textit{sq. units}
\end{aligned}
$$

Name ______________________________

5. The volume of a sphere is $\frac{9}{16}\pi$ cubic meters. Find the radius and surface area.

radius = $\underline{\frac{3}{4}\textit{ meter}}$

Surface Area = $\underline{\frac{9}{4}\pi\textit{ square meters}}$

$$\textit{Volume of sphere} = \frac{4}{3}\cdot\pi\cdot r^3$$
$$\frac{9}{16}\cdot\pi = \frac{4}{3}\cdot\pi\cdot r^3$$
$$\frac{48}{\pi}\cdot\frac{9}{16}\cdot\frac{\pi}{1} = \frac{48}{\pi}\cdot\frac{4}{3}\cdot\frac{\pi}{1}\cdot r^3$$
$$\frac{3\cdot\cancel{16}}{\cancel{\pi}}\cdot\frac{9}{\cancel{16}}\cdot\frac{\cancel{\pi}}{1} = \frac{\cancel{3}\cdot 16}{\cancel{\pi}}\cdot\frac{4}{\cancel{3}}\cdot\frac{\cancel{\pi}}{1}\cdot r^3$$
$$27 = 64r^3$$
$$\frac{1}{64}\cdot\frac{27}{1} = \frac{1}{\cancel{64}}\cdot\frac{\cancel{64}}{1}\cdot r^3$$
$$\frac{27}{64} = r^3$$
$$\sqrt[3]{\frac{27}{64}} = r$$
$$\frac{\sqrt[3]{27}}{\sqrt[3]{64}} = r$$
$$\frac{3}{4}\textit{ meter} = r$$

$$\textit{Surface Area of a sphere} = 4\cdot\pi\cdot r^2$$
$$= \frac{4}{1}\cdot\frac{\pi}{1}\cdot\frac{3}{4}\cdot\frac{3}{4}$$
$$= \frac{\cancel{4}}{1}\cdot\frac{\pi}{1}\cdot\frac{3}{\cancel{4}}\cdot\frac{3}{4}$$
$$= \frac{\pi}{1}\cdot\frac{9}{4}$$
$$= \frac{9}{4}\pi\textit{ square meters}$$

Quiz Form B

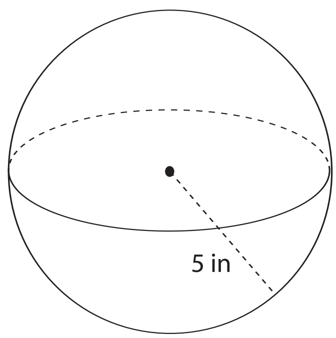
Name ____________________

Class ________ Date ________ Score ________

Unit I - The Structure of Geometry
Part C - Measurement
Lesson 9 - Spheres

Find the surface area and volume of the sphere illustrated below. Round all answers to the nearest tenth. Use 3.14 as an approximation for π.

1.



Surface Area = *314.0 sq inches*

Volume = *523.3 cubic inches*

Surface Area of a sphere $= 4 \cdot \pi \cdot r^2$

Total Area $= 4 \cdot \pi \cdot r \cdot r$

$= 4 \cdot 3.14 \cdot 5\ in \cdot 5\ in$

$= 4 \cdot 3.14 \cdot 5 \cdot 5\ in^2$

$= 314\ in^2$ *or 314.0 sq inches*

Volume of sphere $= \frac{4}{3} \cdot \pi \cdot r^3$

$= \frac{4}{3} \cdot \pi \cdot 5\ in \cdot 5\ in \cdot 5\ in$

$= \frac{4 \cdot 3.14 \cdot 5 \cdot 5 \cdot 5}{3}\ in^3$

$= \frac{1570}{3}\ in^3$

$= 523.\overline{3}$ *cubic inches*

Name ______________________________

3. The volume of a sphere is 36π mm^3. Find the radius and the surface area.

radius = *3 mm*

Surface Area = *36π sq mm*

$$\text{Volume of sphere} = \frac{4}{3}\cdot\pi\cdot r^3$$

$$36\pi = \frac{4}{3}\cdot\pi r^3$$

$$\frac{3}{4\pi}\cdot\frac{36\pi}{1} = \frac{3}{4\pi}\cdot\frac{4\pi}{3}\cdot r^3$$

$$\frac{3}{4\pi}\cdot\frac{4\pi\cdot 9}{1} = 1\cdot r^3$$

$$27 = r^3$$

$$\sqrt[3]{27} = r$$

$$3\text{ mm} = r$$

$$\text{Surface Area of a sphere} = 4\cdot\pi\cdot r^2$$

$$= 4\cdot\pi\cdot 3\text{ mm}\cdot 3\text{ mm}$$

$$= 4\cdot\pi\cdot 3\cdot 3\text{ mm}^2$$

$$= 36\pi\text{ sq mm}$$

Name ______________________________

5. The radius of a sphere is $8\sqrt{2}$ inches. Find the surface area and volume.

Surface Area = 512π *square in*

Volume = $\dfrac{4096\pi \cdot \sqrt{2}}{3}$ *cubic in*

$$\textit{Surface Area of a sphere} = 4 \cdot \pi \cdot r^2$$

$$\begin{aligned}
\textit{Total Area} &= 4 \cdot \pi \cdot 8\sqrt{2}\ in \cdot 8\sqrt{2}\ in \\
&= 4 \cdot \pi \cdot 8 \cdot 8 \cdot \sqrt{2} \cdot \sqrt{2}\ in^2 \\
&= 4 \cdot \pi \cdot 64 \cdot 2\ in^2 \\
&= 512\pi \textit{ square in}
\end{aligned}$$

$$\begin{aligned}
\textit{Volume of sphere} &= \frac{4}{3} \cdot \pi \cdot r^3 \\
&= \frac{4}{3} \cdot \pi \cdot 8\sqrt{2}\ in \cdot 8\sqrt{2}\ in \cdot 8\sqrt{2}\ in \\
&= \frac{4 \cdot \pi \cdot 8 \cdot \sqrt{2} \cdot 8 \cdot \sqrt{2} \cdot 8 \cdot \sqrt{2}}{3 \cdot 1 \cdot 1 \cdot 1 \cdot 1}\ in^3 \\
&= \frac{4 \cdot \pi \cdot 8 \cdot 8 \cdot 8 \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2}}{3}\ in^3 \\
&= \frac{4 \cdot \pi \cdot 512 \cdot 2 \cdot \sqrt{2}}{3}\ in^3 \\
&= \frac{4096\pi \cdot \sqrt{2}}{3} \textit{ cubic in}
\end{aligned}$$

Name ____________________

For each group listed in exercises 4 through 6, read the accompanying scenario illustrating a good use of inductive reasoning. Then write a scenario of your own. Check with another person for its validity.

4. Football Players

The opponent in the Cougar's next game throws a pass on first down 8 out of 10 times according to statistics from the first five games. The Cougars expect they will need to be prepared to use their pass defense the majority of the time on first down.

Answers will vary.

5. Employees

Employees of the Discount Mart Variety Store have a meeting every Friday morning one hour before the store opens. Their supervisor has been ten to fifteen minutes late to the meeting for the last 7 weeks. There has been a noticeable increase in the number of employees who are late since the meeting never seems to start on time anyway.

Answers will vary.

6. Police Officers

The intersection of 5th Street and Cumberland Avenue has been the scene of nine accidents in the last four weeks. Over the last three months the number of speeding citations issued on Cumberland Avenue has increased by 5% over the previous three month period. The Police Department has requested that a study of the daily traffic patterns be conducted to determine a remedy for the dangerous situation at this intersection.

Answers will vary.

 Name ______________________________

For each group listed, in exercises 4 through 6, read the accompanying scenario illustrating a good use of inductive reasoning. Then write a scenario of your own. Check with another person for its validity

4. Manufacturers

Each year, auto makers introduce new colors for their cars. One way auto makers choose colors for new cars is to find out which colors sold well in the past. Trends which are observed over a long period of time, say five years, help automakers to decide what color of automobile to produce, in an attempt to sell more cars.

Answers will vary.

5. Thieves

The owner of a small photography shop is observed leaving his business to go to the bank at approximately the same time every day. A thief would use such an observation to plan a confrontation and possible robbery. For the businessman, his responsibility is to avoid following the same routine everyday.

Answers will vary.

6. Explorers

Throughout history, there have been many explorers from Columbus to Lewis and Clark to Space Shuttle Astronauts. Early explorers had only limited knowledge about conditions they would encounter. More modern explorers can use technology to help them prepare for their travels. However, at all levels, explorers were required to make observations and record patterns of activity which might effect their success. For example, how did Columbus acquaint himself with the prevailing winds needed to push him across the ocean? How did Lewis and Clark know when the best time was to move? When is the best "window" for launching a space shuttle? Planning had to be based on observations such as weather patterns and moon phases, and using that information to make assumptions to be acted upon, to successfully complete the mission.

Answers will vary.

Name ______________________________

4. Use inductive reasoning to find the next two terms of the sequence given below. Describe how you found these terms.

A, B, D, G, K, <u>P</u>, <u>V</u>

Assign numbers 1 to 26 to the letters of the alphabet and numbers 1 to 7 to the position of the letter in the pattern A, B, C, G, K, P, V. The sum of the position of the letter in the alphabet and the positions of the letter in the pattern gives the numerical position in the alphabet of the next letter in the pattern.

Example:	$A \longrightarrow 1$	*1*	*1+1*	*=2*
	$B \longrightarrow 2$	*2*	*2+2*	*=4*
	$D \longrightarrow 4$	*3*	*4+3*	*=7*
	$G \longrightarrow 7$	*4*	*7+4*	*=11*
	$K \longrightarrow 11$	*5*	*11+5*	*=16*
	$P \longrightarrow 16$	*6*	*16+6*	*=22*
	$V \longrightarrow 22$			

↑

Position of letter in alphabet + Position of letter in pattern = Position of next letter in the alphabet.

Look for a pattern and predict the next two numbers in each sequence in exercises 5 through 8. Write a sentence describing how you found these numbers.

5. 1, 10, 100, 1000, <u>10,000</u>, <u>100,000</u>

Each term is multiplied by 10 to get the next term.

6. 180, 360, 540, 720, <u>900</u>, <u>1080</u>

Each term is found by adding 180 to the previous term.

7. $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$, $\frac{5}{6}$, $\frac{6}{6}$ *or 1*

Each new term is found by adding $\frac{1}{6}$ *to the previous term.*

Alternate solution: Each fraction divided by the next fraction in the sequence, gives the following fraction. This would give anwers of $\frac{3}{4}$ *and* $\frac{8}{9}$.

8. 2, 20, 10, 100, 50, <u>500</u>, <u>250</u>

The pattern is formed by multiplying the first term by 10 to get the second term. Then divide the second term by 2 to get third term. Repeat this process– multiply by 10, divide by 2.

Quiz Form B

Name ______________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry
Part D - Inductive Reasoning

Lesson 2 - Applications in Mathematics

1. In each of the past 8 months, Joe has received the rent payment for his rental house on the fourth or fifth day of the month. He makes a conjecture that he will receive next month's rent on the fourth or fifth of the month. Is this a good example of inductive reasoning? *Yes*

Explain your answer. *Yes, this is a good example of inductive reasoning. His tenants probably get paid on the first of each month and do not mail any bill payments until their pay check is securely in the bank. Their pattern has been established and will probably continue.*

2. Do you think there is a connection between inductive reasoning and the stock market investments made by investors in our country? *Yes*

Explain your answer. *Yes, there is a connection. We hear news reports mentioning "economic indicators" which are factors occurring within our economic system that have been related to good times for growth or bad times for growth. A stock broker will be aware of these indicators in deciding when and in what investments to place money.*

3. Anthony noted that $5 = 1^2 + 2^2$ and $13 = 2^2 + 3^2$. He concluded that every prime number may be expressed as the sum of the squares of two positive integers. Was he correct? *No* You might want to test some cases.

$2 = 1^2 + 1^2 = 1 + 1$ *Yes*

$3 = 1^2 + 1^2 = 1 + 1$ *or* $1^2 + 2^2 = 1 + 4$ *No*

$5 = 1^2 + 2^2 = 1 + 4 = 5$ *Yes*

$7 = 1^2 + 2^2 = 1 + 4$ *or* $1^2 + 3^2 = 1 + 9$ *No*

$11 = 1^2 + 3^2 = 1 + 9$ *or* $2^2 + 3^2 = 4 + 9$ *No*

$13 = 2^2 + 3^2 = 4 + 9 = 13$ *Yes*

$17 = 1^2 + 4^2 = 1 + 16 = 17$ *Yes*

$19 = 2^2 + 4^2 = 4 + 16 = 20$ *No*

$23 = 3^2 + 4^2 = 9 + 16 = 25$ *No*

$29 = 2^2 + 5^2 = 4 + 25 = 29$ *Yes*

Explain your answer *In the first 10 prime numbers, there are 5 which cannot be expressed as the sum of squares of two positive integers.*

Name ______________________________

4. Write a general formula for the sum of any number (n) of consecutive odd integers by examining the following cases and using inductive reasoning.

integers	number of integers	sum of integers	
1	1	1	*Let n = the number of integers added.*
1,3	2	4	$n = 2 \quad 2^2 = 4$
1,3,5	3	9	$n = 3 \quad 3^2 = 9$
1,3,5,7	4	16	$n = 4 \quad 4^2 = 16$
1,3,5,7,9	5	25	$n = 5 \quad 5^2 = 25$

For "*n*" consecutive odd integers, the sum will be *Sum can be found by squaring "n" or Sum of "n" odd integers is n^2.*

Look for a pattern and predict the next two numbers in each sequence in exercises 5 through 8. Write a sentence describing how you found these numbers.

5. 0, 10, 21, 33, 46, 60, *75*, *91*

Start at 0. The pattern of numbers is found by adding consecutive integers to each number beginning with 10.
0, 0+10=10, 10+11=21, 21+12=33

6. 1, 3, 4, 7, 11, 18, *29*, *47*

The pattern is formed by adding two adjacent numbers to get the next number, assuming that you start with 1 and 3.

7. 3, −12, 48, −192, 768, *−3072*, *12,288*

Each new number is found by multiplying the previous number by −4.

8. $\frac{1}{2}, 9, \frac{2}{3}, 10, \frac{5}{6}, 11, 1$, *12*, $1\frac{1}{6}$

Every number in an odd position in the pattern is found by adding $\frac{1}{6}$ to the previous number in an odd position. Every number in an even position in the pattern is found by adding 1 to the previous number in an even position.

Name ______________________________

3. General Statement - No one living in Oklahoma has a house on the beach.

Specific Statement - Jeremy has a beachfront house.

Conclusion - *We might want to conclude that Jeremy does not live in Oklahoma. However, this is not a valid conclusion. The condition of the general statement is that someone must live in Oklahoma. The specific statement does not satisfy that condition. It states the Jeremy has a beachfront house. Okay, but where does he have his house? For the reasoning to be valid, the specific statement must state whether or not Jeremy lives in Oklahoma.*

4. General Statement - The new fluoride toothpaste, Dent-Sure, prevents cavities.

Specific Statement - Carl had no cavities at his dental check-up today.

Conclusion - *We cannot arrive at a conclusion here. The condition of the general statement is that someone uses Dent-Sure toothpaste. The specific statement does not satisfy this condition. It states Carl had no cavities. We don't know what he does to prevent them.*

Name ______________________________

Suppose p stands for "Scientists are not uneducated" (a true statement) and q stands for "Geology involves the study of the earth" (a true statement). Write, in words, each of the statements in exercises 12 through 16. Then decide the truth of each compound statement.

12. $\sim(p \wedge q)$

It is false that scientists are not uneducated and geology involves the study of the earth. This is a false statement.

1) True and True —> True

2) Negated True —> False

13. $\sim p \vee \sim q$

It is not the case that scientists are not uneducated, or it is not the case that geology involves the study of the earth OR scientists are uneducated or geology does not involve the study of the earth. This is a false statement.

1) False or False —> False

14. $\sim(p \vee q)$

It is false that scientists are not uneducated or geology involves the study of the earth. This is a false statement.

1) True or True —> True

2) Negated True —> False

15. $(p \vee q)$

Scientists are not uneducated or geology involves the study of the earth. This is a true statement.

1) True or True —> True

16. $\sim p \wedge \sim q$

It is false that scientists are not uneducated <u>and</u> it is false that geology involves the study of the earth. Or, we could write it another way. Scientists are uneducated <u>and</u> geology does not involve the study of the earth. This is a false statement.

1) True and True —> True

2) ~True and ~True —> False

Name ______________________________

7. Suppose p stands for "Triangle ABC is right isosceles" and q stands for "Triangle ABC is a right triangle". Use these two statements to form a conjunction, a disjunction, and a negation of p.

Conjunction: *$\triangle ABC$ is right isosceles and $\triangle ABC$ is a right triangle.*

Disjunction: *$\triangle ABC$ is right isosceles or $\triangle ABC$ is a right triangle.*

Negation of p: *$\triangle ABC$ is not right isosceles.*

Given two statements as indicated in exercises 8 through 11, indicate whether the conjunction, disjunction, and negation of p are true or false.

8. both p and q are true.

Conjunction: *True and True* *True*

Disjunction: *True or True* *True*

Negation of q: *not True* *False*

9. p is true and q is false.

Conjunction: *True and False* *False*

Disjunction: *True or False* *True*

Negation of q:: *not False* *True*

10. both p and q are false.

Conjunction: *False and False* *False*

Disjunction: *False or False* *False*

Negation of q: *not False* *True*

11. p is false and q is true.

Conjunction: *False and True* *False*

Disjunction: *False or True* *True*

Negation of q: *not True* *False*

Name ____

Suppose p stands for "Algebra is a branch of mathematics" (a true statement) and q stands for "Geometry is not worthless" (a true statement). Write, in words, each of the statements in exercises 12 through 16. Then decide the truth of each compound statement.

12. $p \vee q$

Algebra is a branch of mathematics or geometry is not worthless. This is a true statement.

1) True or True —> True

13. $\sim(p \vee q)$

It is not the case that algebra is a branch of mathematics or geometry is not worthless. This is a false statement.

1) True or True —> True

2) Negated True —> False

14. $\sim p \wedge \sim q$

It is not the case that algebra is a branch of mathematics and it is not the case that geometry is not worthless. We could say this another way by saying algebra is not a branch of mathematics and geometry is worthless. This is a false statement.

1) ~True and ~True

False and False —> False

15. $\sim(p \wedge q)$

It is not the case that algebra is a branch of mathematics and geometry is not worthless. This is a false statement.

1) True and True —> True

2) Negated True —> False

16. $\sim p \vee \sim q$

It is not the case that algebra is a branch of mathematics or it is not the case that geometry is not worthless. This is a false statement.

1) ~True or ~True

False or False —> False

Consider each of the statements in Exercises 6-10 to be true. State the converse of each, and tell whether the converse is always, sometimes, or never true.

6. If a polygon is a hexagon, then it has exactly six sides.

Converse: *If a polygon has exactly six sides, then it is a hexagon. (always true)*

7. If $x = 3$, then $x^2 = 9$

Converse: *If $x^2 = 9$ then $x = 3$ (sometimes or partially true)*

8. If you are able to finish a triathlon, then you are in good shape.

Converse: *If you are in good shape, then you are able to finish a triathlon. (sometimes true)*

9. If a person is swimming, then that person is wet.

Converse: *If a person is wet, then that person is swimming. (sometimes true)*

10. If two lines have a common point, then the lines are intersecting lines.

Converse: *If two lines are intersecting, then they have a common point. (always true)*

When a statement and its converse are both always true, you can combine the two statements into a biconditional using the phrase "if and only if". For exercises 11 through 15, decide which of the statements from exercises 6 through 10 can be written in biconditional form, and if possible, write the biconditional. If not possible, explain why.

11. (using exercise 6) *This can be written as a biconditional. A polygon is a hexagon if and only if it has exactly six sides.*

12. (using exercise 7) *This statement and its converse cannot be put together as a biconditional because both are not always true. (the converse is sometimes false)*

13. (using exercise 8) *This statement and its converse cannot be put together as a biconditional because both are not always true. (the converse is sometimes false)*

14. (using exercise 9) *This statement and its converse cannot be put together as a biconditional because both are not always true. (the converse is sometimes false)*

15. (using exercise 10) *This can be written as a biconditional. Two lines have a common point if and only if the lines are intersecting lines.*

Name ______________________________

Consider each of the statements in Exercises 6-10 to be true. State the converse of each, and tell whether the converse is always, sometimes, or never true.

6. If a figure is a pentagon, then it is a polygon.

If a figure is a polygon, then the figure is a pentagon. (sometimes true)

7. If a whole number has exactly two whole number factors, then it is a prime number.

If a number is a prime whole number, then it has exactly two whole number factors. (always true)

8. If you are an elephant, then you do not know how to fly.

If you do not know how to fly, then you are an elephant. (sometimes true)

9. If you are at least 21 years old, then you can legally vote.

If you can legally vote, then you are at least 21 years old. (always true)

10. If an angle is acute, then it is smaller than an obtuse angle.

If an angle is smaller than an obtuse angle, then the angle is acute. (sometimes true)

When a statement and its converse are both always true, you can combine the two statements into a biconditional using the phrase "if and only if". For exercises 11 through 15, decide which of the statements from exercises 6 through 10 can be written in biconditional form, and if possible, write the biconditional. If not possible, explain why.

11. (using exercise 6) *This statement and its converse cannot be combined as a biconditional. The converse is not always true.*

12. (using exercise 7) *This statement and its converse can be combined as a biconditional because both the statement and its converse are always true. A whole number has exactly two whole number factors if and only if it is a prime whole number.*

13. (using exercise 8) *This statement and its converse cannot be combined as a biconditional because the statements are not both always true. The converse is not always true.*

14. (using exercise 9) *This statement and its converse can be combined as a biconditional because they are both always true. You can legally vote if and only if you are at least 21 years old.*

15. (using exercise 10) *This statement and its converse cannot be combined as a biconditional because both statements are not always true. The converse is false if the angle is a right angle.*

 Name ______________________________

6. A square is a rectangle.

Conditional: *If a figure is a square, then the figure is a rectangle. (true)*
Contrapositive: *If a figure is not a rectangle, then the figure is not a square. (true)*

7. A figure has three segments if it is a triangle.

Conditional: *If a figure is a triangle, then the figure has three segments. (true)*
Contrapositive: *If a figure does not have three segments, then the figure is not a triangle. (true)*

8. $x < 0$ implies $5x > 6x$

Conditional: *If $x < 0$, then $5x > 6x$ (true)*
Contrapositive: *If $5x \ngtr 6x$, then $x \nless 0$ (true)*

9. An integer is less than zero given that the integer is a negative integer.

Conditional: *If a number is a negative integer, then the number is less than zero. (true)*
Contrapositive: *If a number is not less than zero, then the number is not a negative integer. (true)*

In exercises 41 through 44, use the given information about a circle to find the missing measures. Use 3.14 as an approximation for π, and show your work. Give your answers to the nearest tenth, and label your results properly.

(C-6)

	radius	diameter	Circumference	Area
41.	*7.5"*	15"	$\doteq$ *47.1 inches*	$\doteq$ *176.6 sq. in.*

$$Diameter = 2 \bullet radius$$
$$15'' = 2 \bullet r$$
$$\frac{1}{2} \bullet \frac{15''}{1} = \frac{1}{2} \bullet 2 \bullet r$$
$$\frac{15''}{2} = r$$
$$7.5'' = r$$

$$Circumference = \pi \bullet d$$
$$= \pi \bullet 15''$$
$$\doteq (3.14) \bullet (15) \text{ inches}$$
$$\doteq 47.1 \text{ inches}$$

$$Area = \pi \bullet r^2$$
$$= \pi \bullet \frac{15''}{2} \bullet \frac{15''}{2}$$
$$\doteq (3.14)(7.5)(7.5) \text{ sq. in.}$$
$$\doteq 176.625 \text{ sq. in.}$$
$$\doteq 176.6 \text{ sq. in.}$$

(C-6)

	radius	diameter	Circumference	Area
42.	8 cm	*16 cm*	$\doteq$ *50.2 cm*	$\doteq 201.0\ cm^2$

$$Diameter = 2 \bullet radius$$
$$= 2 \bullet 8 \text{ cm}$$
$$= 16 \text{ cm}$$

$$Circumference = \pi \bullet d$$
$$= \pi \bullet 16 \text{ cm}$$
$$\doteq 3.14 \bullet 16 \text{ cm}$$
$$\doteq 50.24 \text{ cm}$$
$$\doteq 50.2 \text{ cm}$$

$$Area = \pi \bullet r^2$$
$$= \pi \bullet 8 \text{ cm} \bullet 8 \text{ cm}$$
$$= \pi \bullet 64 \text{ cm}^2$$
$$\doteq (3.14)(64) \text{ cm}^2$$
$$\doteq 200.96 \text{ cm}^2$$
$$\doteq 201.0 \text{ cm}^2$$

Quiz Form B

Name ______________________

Class ________ Date ________ Score ________

Unit I - The Structure of Geometry
Part F - Deductive Reasoning
Lesson 3 - Negations of Conditionals

Write the inverse, converse, and contrapositive of each conditional in exercises 1 through 3. Then determine which statements are true and which are false.

1. If $a > 1$, then $a^2 > a$.

Inverse: *If $a \ngtr 1$, the $a^2 \ngtr a$ (false)*

Converse: *If $a^2 > a$, then $a > 1$ (false)*

Contrapositive: *If $a^2 \ngtr a$, then $a \ngtr 1$ (true)*

2. If a triangle is an acute triangle, then it has no obtuse angles.

Inverse: *If a triangle is not an acute triangle, then the triangle has an obtuse angle. (false)*

Converse: *If a triangle has no obtuse angles, then the triangle is an acute triangle. (false)*

Contrapositive: *If a triangle has an obtuse angle, then the triangle is not an acute triangle. (true)*

3. If Sean lives in Pittsburgh, then he lives in Pennsylvania.

Inverse: *If Sean does not live in Pittsburgh, then he does not live in Pennsylvania. (false)*

Converse: *If Sean lives in Pennsylvania, then he lives in Pittsburgh. (false)*

Contrapositive: *If Sean does not live in Pennsylvania, then he does not live in Pittsburgh. (true)*

In exercises 4 through 9, write the given statement as a conditional in "if-then" form. Then form the contrapositive and decide the truth value of each statement.

4. All acute angles are congruent.

Conditional: *If angles are acute, then angles are congruent. (false)*

Contrapositive: *If angles are not congruent, then the angles are not acute. (false)*

5. A number is an integer, given that it is greater than zero.

Conditional: *If a number is greater than zero, then the number is an integer. (false)*

Contrapositive: *If a number is not an integer, then the number is not greater than zero. (false)*

For exercises 10 through 12, consider exercises 6 through 8 and decide which conditionals, if any, can be written as a biconditional. Then rewrite the biconditional form of the statement. If the conditional cannot be written as a biconditional, say, “not possible”.

A compound statement consisting of a given conditional and, its converse both of which are considered to be true, can be written as a biconditional.

10. Biconditional (if possible):

Exercise 6: *If $x = 4$, then $3x = 12$ (true)*

If $3x = 12$, then $x = 4$ (true)

$x = 4$ if and only if $3x = 12$

11. Biconditional (if possible):

Exercise 7: *If $a = b$ or $a = -b$, then $a^2 - b^2 = 0$ (true)*

If $a^2 - b^2 = 0$, then $a = b$ or $a = -b$ (true)

$a = b$ or $a = -b$ if and only if $a^2 - b^2 = 0$

12. Biconditional (if possible):

Exercise 8: *If A, B, and C are on one line, then A, B, and C are collinear. (true)*

If A, B, and C are collinear, then A, B, and C are on one line. (true)

A, B, and C are on one line if and only if A, B, and C are collinear.

13. (Bonus Problem) Arrange some or all of the given conditionals into an order that allows you to make the given conclusion.

Conditionals: If A, then B.
If X, then R.
If M, then Y.
If R, then M.
If Y, then A.

Conclusion: If X, then B.

If X, then R. If R, then M. If M, then Y. If Y, then A. If A, then B. Therefore, if X then B.

(F-1)
___*v*___**13.** disjunction

(C-9)
___*f*___**14.** sphere

(B-3)
___*h*___**15.** polygon

(A-5)
___*t*___**16.** $\cap$

(B-3)
___*y*___**17.** isosceles triangle

(B-3)
___*u*___**18.** rhombus

(C-7)
___*w*___**19.** right prism

(B-3)
___*x*___**20.** quadrilateral

(C-7)
___*i*___**21.** prism

(F-4)
___*k*___**22.** fallacy

(B-2)
___*e*___**23.** geometric figure

(A-1)
___*s*___**24.** [ ]

(B-3)
___*c*___**25.** hexagon

(C-6)
___*l*___**26.** radius

m) a prism whose lateral faces are at an angle other than 90^{o} with the base.

n) a three dimensional geometric figure created by "connecting" a polygon to a point not in the plane.

o) a system of reasoning, in an orderly fashion, which draws conclusions from specific premises.

p) two or more sets which have no members in common.

q) a polygon made with eight line segments

r) a triangle in which one of the angles is a right angle (90^{o})

s) grouping symbol; brackets

t) an operation on two or more sets which selects only those elements common to all of the original sets at the same time

u) a quadrilateral in which all four sides are of equal measure

v) an operation in logic which joins two simple statements using the word "or".

w) a prism whose lateral faces are at an angle of 90^{o} with the bases.

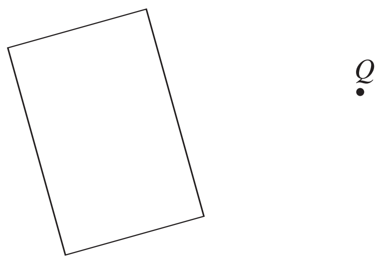
x) a polygon made with four line segments.

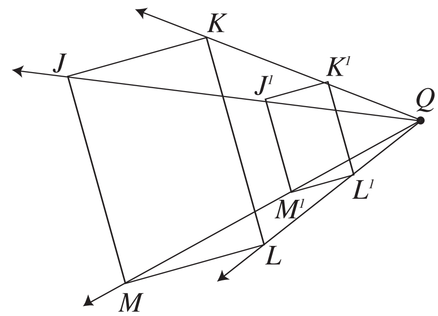
y) a triangle in which at least two of the three sides are of equal measure

z) the process of using a general statement which calls for a conclusion, based on certain conditions, and then applying a specific statement which satisfies those conditions, therefore establishing the validity of the conclusion.

(A-3)
29. Draw the image of the given rectangle, after a reduction with center Q, using the given scale factor.

Scale factor: .5





Draw rays QJ, QK, QL, and QM. Then measure line segments QJ^1, QK^1, QL^1, and QM^1 to lengths $\frac{1}{2}$ times the lengths of line segments QJ, QK, QL, and QM. Rectangle $J^1K^1L^1M^1$ is the reduction dilation image of rectangle JKLM.

(A-5)

30. For each of the following sets, list all of the members of the set. Use the set of whole numbers as the domain of *x*.

a) $\{x \mid 3x = 27\}$ *{9}*

b) $\{x \mid 3x = 27\}$ *{4, 5, 6}*

c) $\{x \mid x \le 4\}$ *{0, 1, 2, 3, 4}*

d) $\{x \mid x^2 - 5x + 6 = 0\}$ *{2, 3}*

e) $\{x \mid x^2 + 7x + 12 = 0\}$ *{ }*

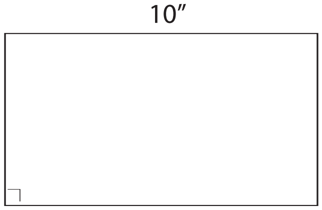
f) $\{x \mid x \le 4\} \cap \{x \mid x^2 - 5x + 6 = 0\}$ *{2, 3}*

g) $\{x \mid 3x = 27\} \cup \{x \mid x^2 + 7x + 12 = 0\}$ *{9}* *Note: -3 and -4 are not whole numbers, and are therefore not in the solution set.*

In exercises 33 through 40, identify the polygon, as specifically as possible, find the perimeter of the polygon, and find the area of the polygon. Show your work and label your answers properly.

(C-1)

33.



Polygon: *Rectangle*

Perimeter: *28 inches*

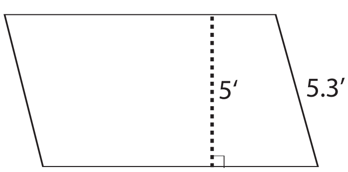
Area: *40 sq. in.*

Perimeter = sum of the lengths of the sides
= 10 in. + 4 in. + 10 in. + 4 in.
= 14 in. + 14 in.
= 28 inches

Area = length • width or base • height
= 10 in. • 4 in.
= 40 sq. in.

(C-2)

34.



Polygon: *Parallelogram*

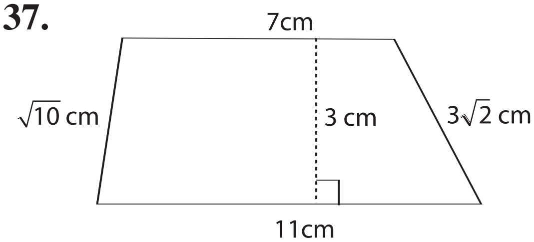
Perimeter: *26.6 ft.*

Area: *40 sq. ft.*

Perimeter = sum of the lengths of the sides
= 8 ft + 5.3 ft + 8 ft + 5.3 ft
= 13.3 ft + 13.3 ft
= 26.6 ft

Area = base • height
= 8 ft • 5 ft
= 40 ft^2 or 40 sq. ft.

(C-4)
37.



Polygon: *Trapezoid*

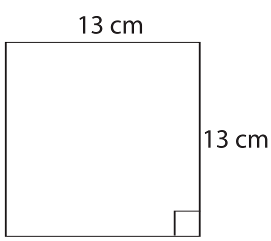
Perimeter: $(18 + 3\sqrt{2} + \sqrt{10})$ cm

Area: *27 sq. cm*

$$\begin{aligned} \text{Perimeter} &= \text{sum of the lengths of the sides} \\ &= \sqrt{10}\text{ cm} + 11\text{ cm} + 3\sqrt{2}\text{ cm} + 7\text{ cm} \\ &= (18 + 3\sqrt{2} + \sqrt{10})\text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2} \bullet \text{height} \bullet \text{sum of the bases} \\ &= \frac{1}{2} \bullet h \bullet (b_1 + b_2) \\ &= \frac{1}{2} \bullet 3\text{ cm} \bullet (7\text{ cm} + 11\text{ cm}) \\ &= \frac{1}{2} \bullet \frac{3\text{ cm}}{1} \bullet \frac{18\text{ cm}}{1} \\ &= \frac{1}{\cancel{2}} \bullet \frac{3}{1} \bullet \frac{\cancel{2} \bullet 9}{1}\text{ cm}^2 \\ &= 27\text{ cm}^2 \end{aligned}$$

(C-1)
38.



Polygon: *Square*

Perimeter: *52 cm*

Area: *169 sq. cm*

$$\begin{aligned} &\text{Perimeter} = \text{sum of the lengths of the sides or} \\ &P = 4 \bullet s \\ &= 13\text{ cm} + 13\text{ cm} + 13\text{ cm} + 13\text{ cm or} \\ &= 4 \bullet 13\text{ cm} \\ &= 52\text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Area} &= \text{length} \bullet \text{width or} \\ &= \text{side} \bullet \text{side} \\ &= 13\text{ cm} \bullet 13\text{ cm} \\ &= 13 \bullet 13\text{ cm}^2 \\ &= 169\text{ cm}^2 \end{aligned}$$

In exercises 41 through 44, use the given information about a circle to find the missing measures. Use 3.14 as an approximation for π, and show your work. Give your answers to the nearest tenth, and label your results properly.

	radius	diameter	Circumference	Area
(C-6) **41.**	*7.5"*	15"	$\doteq$ *47.1 inches*	$\doteq$ *176.6 sq. in.*

Diameter = 2 • radius

$$15'' = 2 \cdot r$$
$$\frac{1}{2} \cdot \frac{15''}{1} = \frac{1}{2} \cdot 2 \cdot r$$
$$\frac{15''}{2} = r$$
$$7.5'' = r$$

Circumference $= \pi \cdot d$

$$= \pi \cdot 15''$$
$$\doteq (3.14) \cdot (15) \text{ inches}$$
$$\doteq 47.1 \text{ inches}$$

Area $= \pi \cdot r^2$

$$= \pi \cdot \frac{15''}{2} \cdot \frac{15''}{2}$$
$$\doteq (3.14)(7.5)(7.5) \text{ sq. in.}$$
$$\doteq 176.625 \text{ sq. in.}$$
$$\doteq 176.6 \text{ sq. in.}$$

	radius	diameter	Circumference	Area
(C-6) **42.**	8 cm	*16 cm*	$\doteq$ *50.2 cm*	$\doteq 201.0\ cm^2$

Diameter = 2 • radius

$$= 2 \cdot 8 \text{ cm}$$
$$= 16 \text{ cm}$$

Circumference $= \pi \cdot d$

$$= \pi \cdot 16 \text{ cm}$$
$$\doteq 3.14 \cdot 16 \text{ cm}$$
$$\doteq 50.24 \text{ cm}$$
$$\doteq 50.2 \text{ cm}$$

Area $= \pi \cdot r^2$

$$= \pi \cdot 8 \text{ cm} \cdot 8 \text{ cm}$$
$$= \pi \cdot 64 \text{ cm}^2$$
$$\doteq (3.14)(64) \text{ cm}^2$$
$$\doteq 200.96 \text{ cm}^2$$
$$\doteq 201.0 \text{ cm}^2$$

(C-6)
43.

radius	diameter	Circumference	Area
$\doteq 1.3$ m	2.5 m	$\doteq 7.9$ meters	$\doteq 4.9$ square meters

Diameter = 2 • radius

$$2\frac{1}{2}m = 2 \cdot r$$
$$\frac{5}{2}m = 2 \cdot r$$
$$\frac{1}{2} \cdot \frac{5}{2}m = \frac{1}{2} \cdot 2 \cdot r$$
$$\frac{5}{4}m = r$$
$$1.25m = r$$
$$1.3m \doteq r$$

Circumference = π • d

$$= \pi \bullet 2\frac{1}{2} m$$
$$\doteq (3.14) \bullet (2.5)\ m$$
$$\doteq 7.9 \text{ meters}$$

Area = π • r²

$$= \pi \cdot \frac{5}{4}m \cdot \frac{5}{4}m$$
$$= \pi \cdot \frac{5}{4} \cdot \frac{5}{4} m^2$$
$$= \pi \cdot \frac{25}{16} m^2$$
$$\doteq (3.14)(1.25)(1.25)\ m^2$$
$$\doteq 4.90625\ m^2$$
$$\doteq 4.9 \text{ square meters}$$

(C-6)
44.

radius	diameter	Circumference	Area
4.2 ft	8.4 ft	$\doteq 26.4$ ft	$\doteq 55.4$ sq. ft

Diameter = 2 • radius

$$= 2 \bullet (4.2)\ ft$$
$$= 8.4\ ft$$

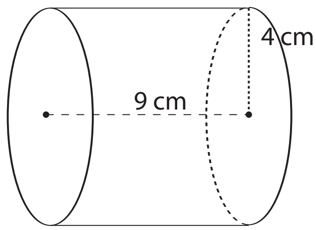
Circumference = π • d

$$= \pi \bullet 8.4\ ft$$
$$\doteq (3.14)(8.4)\ ft$$
$$\doteq 26.376\ ft$$
$$\doteq 26.4\ ft$$

Area = π • r²

$$= \pi \bullet (4.2\ ft)\ g\ (4.2\ ft)$$
$$\doteq (3.14)(4.2)(4.2)\ ft^2$$
$$\doteq 55.3896\ ft^2$$
$$\doteq 55.4 \text{ sq. ft}$$

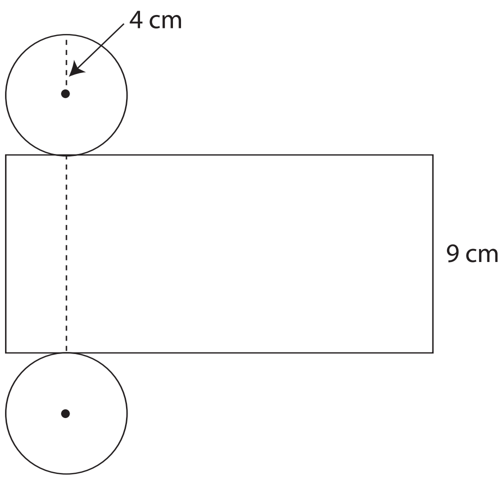
(C-7)
48.



Lateral Area = $72\pi\ cm^2$

Total Area = 104π *sq cm*

Volume = $144\pi\ cm^3$



Consider the cylinder, unfolded, and laid out, in a net. We can now see that the "lateral face" of the cylinder is a rectangle with length equal to the circumference of the circular base and width equal to the height of the cylinder.

Circumference $= \pi \bullet d\ (d = 2r)$
$= \pi \bullet 2 \bullet r$
$= \pi \bullet 2 \bullet 4\ cm$
$= 8\pi\ cm$

Lateral Area = length • width of rectangle or circumference • height
$= 8\pi\ cm \bullet 9\ cm$
$= 72\pi\ cm^2$ *or* 72π *sq. cm*

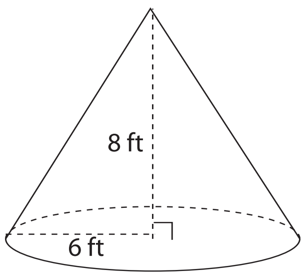
The bases of the cylinder are teo identical circles
Area $= \pi \bullet r^2$
$= \pi \bullet (4\ cm)^2$
$= \pi \bullet 4\ cm \bullet 4\ cm$
$= 16\pi\ cm^2$

Total Area is the sum of the areas of the two bases of the cylinder and the area of the lateral face.
T.A. = B.A. + L.A.
$= [(16\pi + 16\pi) + 72\pi]$ *sq cm or* 104π *sq cm*

The Volume of a cylinder is found by multiplying the area of one base by the height.
$V = B \bullet h$
$= (16\pi\ cm^2) \bullet (9\ cm)$
$= 144\pi\ cm^3$ *or* 144π *cubic centimeters*

For exercises 49 and 50, find the lateral area, total area, and volume of each right pyramid. Label your answers properly. Give exact answers where π is involved and label them properly.

(C-8)
49.



Lateral Area = $60\pi\ ft^2$

Total Area = 96π *sq ft*

Volume = $96\pi\ ft^3$

This figure is a right cone. Lateral Area of a cone is found by multiplying $\frac{1}{2}$ *times the perimeter of the base (i.e. circumference of the circular base) times the slant height of the cone.*

Find slant height. Use Pythagorean Theorem.

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
8^2 + 6^2 &= c^2 \\
64 + 36 &= c^2 \\
100 &= c^2 \\
10 &= c
\end{aligned}
$$

$$
\begin{aligned}
\text{Lateral Area} &= \frac{1}{2} \bullet P \bullet \ell \\
&= \frac{1}{2} \bullet C \bullet \ell \\
&= \frac{1}{2} \bullet 2\pi r \bullet \ell \\
&= \frac{1}{2} \bullet \frac{2 \bullet \pi \bullet 6\ ft}{1} \bullet \frac{10\ ft}{1} \\
&= 60\pi\ ft^2 \text{ or } 60\pi\ sq\ ft
\end{aligned}
$$

The Total Area of a cone is found by adding the base area of the cone and the lateral area of the cone

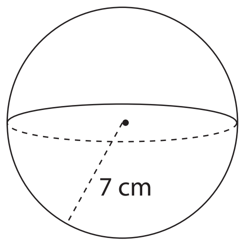
$$
\begin{aligned}
T.A. &= B.A. + L.A. \\
&= (\text{Area of circular base}) + L.A. \\
&= (\text{Area if circular base}) + L.A. \\
&= \pi \bullet r^2 + L.A. \\
&= 36\pi\ sq\ ft + 60\pi\ sq\ ft \\
&= 96\pi\ sq\ ft
\end{aligned}
$$

The Volume of a cone is found by multiplying $\frac{1}{3}$ *times the area of the circular base times the height of the cone.*

$$
\begin{aligned}
V &= \frac{1}{3} \bullet B \bullet h \\
&= \frac{1}{3} \bullet \frac{36\pi\ sq\ ft}{1} \bullet \frac{8\ sq\ ft}{1} \\
&= \frac{1}{3} \bullet \frac{3 \bullet 12\pi}{1} \bullet \frac{8}{1} \text{ cubic ft} \\
&= 96\pi\ ft^3 \text{ or } 96\pi \text{ cubic ft}
\end{aligned}
$$

Find the surface area and volume of this sphere. Substitute 3.14 for π and round your answers to the nearest tenth.

(C-9)
51.



Surface Area = $615.4\,cm^2$

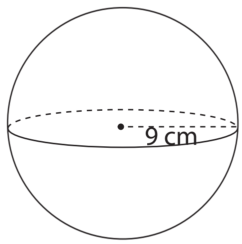
Volume = $1436.0\ cm^3$

$$\text{Surface area of a sphere} = 4 \bullet \pi \bullet r^2$$
$$T.A. = 4 \bullet \pi \bullet r^2$$
$$= 4 \bullet \pi \bullet 49\ cm^2$$
$$\doteq 615.4\,cm^2$$

$$\text{Volume of a sphere is } \frac{4}{3} \bullet \pi \bullet r^3$$
$$V = \frac{4}{3} \bullet \frac{\pi}{1} \bullet \frac{7\ cm}{1} \bullet \frac{7\ cm}{1} \bullet \frac{7\ cm}{1}$$
$$= \frac{4 \bullet 7 \bullet 7 \bullet 7 \bullet \pi}{3}\ \text{cubic cm}$$
$$\doteq 1436.0\ cm^3$$

Find the surface area and volume of this sphere. Give exact answers

(C-9)
52.



Surface Area = $324\pi\ cm^2$

Volume = $972\pi\ cm^3$

Surface area of a sphere $= 4 \cdot \pi \cdot r^2$

$$T.A. = 4 \cdot \pi \cdot 9\ cm \cdot 9\ cm$$
$$= 4 \cdot \pi \cdot 81\ cm^2$$
$$= 324\pi\ cm^2$$

Volume of a sphere is $\frac{4}{3} \cdot \pi \cdot r^3$

$$V = \frac{4}{3} \cdot \frac{\pi}{1} \cdot \frac{9\ cm}{1} \cdot \frac{9\ cm}{1} \cdot \frac{9\ cm}{1}$$
$$V = \frac{4}{\cancel{3}} \cdot \frac{\pi}{1} \cdot \frac{\cancel{3} \cdot 3}{1} cm \cdot \frac{9}{1} cm \cdot \frac{9}{1} cm$$
$$V = 12 \cdot 81 \cdot \pi\ cm^3$$
$$V = 972\ \pi\ cm^3$$

Unit Test Form B

Name ______________________

Class __________ Date __________ Score __________

Unit I - The Structure of Geometry

For each term in the left column, choose the letter for the expression in the right column which defines, or most clearly describes that term and place that letter in the blank.

(C-7) __v__ **1.** lateral area

(B-3) __o__ **2.** scalene triangle

(C-1) __y__ **3.** perimeter

(F-2) __n__ **4.** hypothesis

(C-5) __m__ **5.** circumscribed circle

(B-3) __w__ **6.** equilateral triangle

(C-7) __k__ **7.** net

(F-1) __q__ **8.** negation

(D-1) __b__ **9.** inductive reasoning

(B-3) __r__ **10.** parallelogram

(F-1) __u__ **11.** Venn Diagram

(B-3) __c__ **12.** dodecagon

a) a triangle with an angle greater than 90 degrees

b) the process of finding a general principle based upon the evidence of a finite number of specific cases.

c) a polygon made with twelve line segments

d) operation symbol; indicates taking the square root.

e) a polygon in which there are two distinct pairs of consecutive sides which are of equal measure.

f) a statement in logic which is made up of two or more simple statements.

g) a polygon made with five line segments.

h) a statement consisting of a hypothesis and a conclusion, generally in “if-then” form.

i) an operation in logic which joins two simple statements using the word “and”.

j) shortest distance from the center of a regular polygon to any one of the sides of the polygon.

k) the plane geometric figure obtained by “unfolding” a three-dimensional geometric figure, and laying it “flat” in a plane.

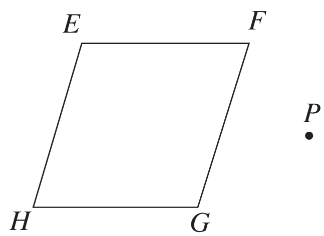
l) line CD

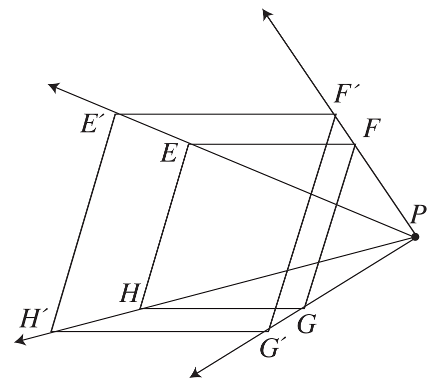
m) a circle which completely encloses a polygon

(A-3)

29. Draw the image of the given rhombus after a dilation with center P and the given scale factor.

Scale factor: 1.25





Draw rays PE, PF, PG and PH. Then measure and draw line segments PE´, PF´, PG´ and PH´ to lengths $1\frac{1}{4}$ times the lengths of line segments PE, PF, PG and PH. rhombus E´F´G´H´ is the enlargement dilation image of rhombus DFGH.

(A-5)

30. Suppose $X=\{2,4,6,...,20\}$ and $Y=\{4,8,12,...,20\}$

a) Name the elements in each set.

x= {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

y= {4, 8, 12, 16, 20}

b) Find $X \cap Y$

{4, 8, 12, 16, 20}

c) Find $X \cup Y$

{2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

d) Find $X \cap \{1, 2\}$

{2}

e) Find $Y \cap \{1, 2\}$

{ } (empty set)

(D-2)
31. Use inductive reasoning to determine the number of line segments determined by 12 collinear points. Explain your answer.

Number of Line Segments: *66*

• •	*2 points*	*1 line segment*
• • • *A B C*	*3 points*	*3 line segments (AB, BC, AC)*
• • • • *A B C D*	*4 points*	*6 line segments (AB, AC, AD, BC, BD, CD)*
• • • • • *A B C D E*	*5 points*	*10 line segments (AB, AC, AD, AE, BC, BD, BE, CD, CE, DE)*
• • • • • • *A B C D E F*	*6 points*	*15 line segments (AB, AC, AD, AE, AF, BC, BD, BE, BF, CD, CE, CF, DE, DF, EF)*

12 collinear points determine 66 line segments. Example: Number of points plus number of line segments determine the number of line segments when the number of points is increased by one. 5 + 10 = 15, 6 + 15 = 21, 7 + 21 = 28, etc.

(E-1)
32. Use the statements below to arrive at a conclusion, if possible. If no conclusion can be reached, say so.

General Statement - If students intend to go to college, they must take both Algebra and Geometry

Specific Statement - Jan intends to go to college.

Conclusion - *Jan must take both algebra and geometry.*

In exercises 41 through 44, use the given information about a circle to find the missing measures. Use 3.14 as an approximation for π, and show your work. Give your answers to the nearest tenth, and label your results properly.

	radius	diameter	Circumference	Area
(C-6) **41.**	$2\frac{5''}{16}$	$4\frac{5}{8}''$	$\doteq 14.5$ inches	$\doteq 16.8\ in^2$

Diameter = 2 • radius

$$4\frac{5}{8}'' = 2 \cdot r$$
$$\frac{37''}{8} = 2 \cdot r$$
$$\frac{1}{2} \cdot \frac{37''}{8} = \frac{1}{2} \cdot \frac{2}{1} \cdot r$$
$$\frac{37''}{16} = r$$
$$2\frac{5''}{16} = r$$

Circumference = π • d

$$\doteq 3.14 \cdot 4\frac{5}{8}''$$
$$\doteq 3.14 \cdot 4.625''$$
$$\doteq 14.5225''$$
$$\doteq 14.5 \text{ inches}$$

Area = π • r²

$$= \pi \cdot \frac{37''}{16} \cdot \frac{37''}{16}$$
$$\doteq (3.14)(2.3125)(2.3125)$$
$$\doteq 16.79\ in^2$$
$$\doteq 16.8\ in^2$$

	radius	diameter	Circumference	Area
(C-6) **42.**	2π cm	$\doteq 12.6$ cm	$\doteq 39.4$ cm	$\doteq 123.8\ cm^2$

Diameter = 2 • radius

$$= 2 \cdot 2\pi \text{ cm}$$
$$= 4\pi \text{ cm (exact answer)}$$
$$\doteq 4 \cdot 3.14 \text{ cm}$$
$$\doteq 12.56 \text{ cm}$$
$$\doteq 12.56 \text{ cm (approximate answer)}$$

Circumference = π • d

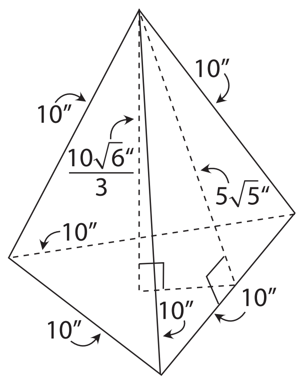
$$= \pi \cdot 4\pi \text{ cm}$$
$$= \pi \cdot \pi \cdot 4 \text{ cm}$$
$$= 4\pi^2 \text{ cm (exact answer)}$$
$$\doteq 4 \cdot 3.14 \cdot 3.14 \text{ cm}$$
$$\doteq 39.4384 \text{ cm}$$
$$\doteq 39.4 \text{ cm (approximate answer)}$$

Area = π • r²

$$= \pi \cdot 2\pi \text{ cm} \cdot 2\pi \text{ cm}$$
$$= \pi \cdot \pi \cdot \pi \cdot 2 \cdot 2$$
$$= \pi^3 \cdot 4 \text{ (exact answer)}$$
$$\doteq (3.14)(3.14)(3.14)(4)\ cm^2$$
$$\doteq 123.836576\ cm^2$$
$$\doteq 123.8\ cm^2$$

For exercises 49 and 50, find the lateral area, total area, and volume of each right pyramid or right circular cone. Label your answers properly. Give exact answers where π is involved in an answer.

(C-8)
49.



Lateral Area = $75\sqrt{5}\ in^2$

Total Area = $100\sqrt{5}\ in^2$

Volume = $\frac{250 \bullet \sqrt{30}\ in^3}{9}$

This is a regular right pyramid. The lateral area is found by multiplying $\frac{1}{2}$ times the perimeter of the base times the slant height

$$\text{Lateral Area} = \frac{1}{2} \bullet P \bullet \ell$$
$$= \frac{1}{2} \bullet (10'' + 10'' + 10'') \bullet 5\sqrt{5}''$$
$$= \frac{1}{2} \bullet \frac{30''}{1} \bullet \frac{5\sqrt{5}''}{1}$$
$$= \frac{1}{\cancel{2}} \bullet \frac{\cancel{2} \bullet 15''}{1} \bullet \frac{5\sqrt{5}''}{1}$$
$$= 75\sqrt{5}\text{ sq in. or } 75\sqrt{5}\text{ in}$$

The Total Area of a pyramid is the sum of the lateral area of the pyramid and the area of the base of the pyramid.

Area of the triangular base = $\frac{1}{2} \bullet$ base $\bullet$ height

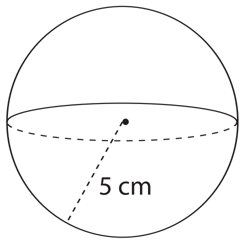
$$B.A. = \frac{1}{2} \bullet 10'' \bullet 5\sqrt{5}''$$
$$= \frac{1}{\cancel{2}} \bullet \frac{\cancel{2} \bullet 5''}{1} \bullet \frac{5\sqrt{5}''}{1}$$
$$= 25\sqrt{5}\ in^2 \text{ or } 25\sqrt{5}\text{ sq in.}$$

$$T.A. = B.A. + L.A.$$
$$= 75\sqrt{5}\ in^2 + 25\sqrt{5}\ in^2$$
$$= 100\sqrt{5}\ in^2$$

The Volume of a pyramid is found by multiplying $\frac{1}{3}$ times the area of the base times the height of the pyramid.

$$V = \frac{1}{3} \bullet B \bullet h$$
$$= \frac{1}{3} \bullet \frac{25\sqrt{5}\ in^2}{1} \bullet \frac{10\sqrt{6}\ in}{3}$$
$$= \frac{1 \bullet 25 \bullet \sqrt{5} \bullet 10 \bullet \sqrt{6}\ in^3}{3 \bullet 1 \bullet 3}$$
$$= \frac{1 \bullet 25 \bullet 10 \bullet \sqrt{5} \bullet \sqrt{6}\ in^3}{1 \bullet 3 \bullet 3}$$
$$= \frac{250 \bullet \sqrt{30}\ in^3}{9}$$

(C-9)
51. Find the surface area and volume of this sphere. Give exact answers.



Surface Area = $100\pi\ cm^2$

Volume = $\frac{500\pi}{3}\ cm^3$

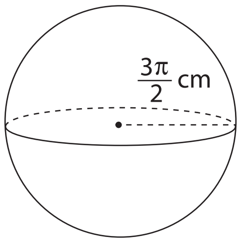
$$\text{Surface area of a sphere} = 4 \bullet \pi \bullet r^2$$

$$\begin{aligned} T.A. &= 4 \bullet \pi \bullet r^2 \\ &= 4 \bullet \pi \bullet 5\ cm \bullet 5\ cm \\ &= 4 \bullet \pi \bullet 25\ cm^2 \\ &= 100\pi\ cm^2 \end{aligned}$$

$$\text{Volume of a sphere is } \frac{4}{3} \bullet \pi \bullet r^3$$

$$\begin{aligned} V &= \frac{4}{3} \bullet \pi \bullet 5\ cm \bullet 5\ cm \bullet 5\ cm \\ &= \frac{4}{3} \bullet \pi \bullet 125\ cm^3 \\ &= \frac{4}{3} \bullet \frac{\pi}{1} \bullet \frac{125}{1}\ cm^3 \\ &= \frac{500\pi}{3}\ cm^3 \end{aligned}$$

(C-9)
52. Find the surface area and volume of this sphere. Substitute 3.14 for π and round your answers to the nearest tenth



Surface Area = $\doteq 278.6\ cm^2$

Volume = $437.5\ cm^3$

Surface area of a sphere $= 4 \cdot \pi \cdot r^2$

$$T.A. = 4 \cdot \pi \cdot \left(\frac{3\pi}{2}\right)^2$$
$$= \cancel{4} \cdot \pi \cdot \frac{3\pi}{\cancel{2}} \cdot \frac{3\pi}{\cancel{2}}$$
$$= 9\pi^3$$
$$\doteq 278.63 cm^2$$
$$\doteq 278.6\ cm^2$$

Volume of a sphere is $\frac{4}{3} \cdot \pi \cdot r^3$

$$V = \frac{\cancel{4}}{\cancel{3}} \cdot \pi \cdot \frac{\cancel{3}\pi}{\cancel{2}} \cdot \frac{3\pi}{\cancel{2}} \cdot \frac{3\pi}{2}$$
$$= \frac{9\pi^4}{2}$$
$$= 437.5\ cm^3$$

