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Lesson 1-1 — Patterns and Inductive Reasoning

Lesson Objective	NAEP 2005 Strand: Geometry
Use inductive reasoning to make conjectures	Topic: Mathematical Reasoning
	Local Standards: ____________

Vocabulary

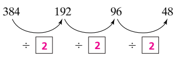
Inductive reasoning is reasoning based on patterns you observe.

A conjecture is a conclusion you reach using inductive reasoning.

A counterexample is an example for which the conjecture is incorrect.

Examples

1 Finding and Using a Pattern Find a pattern for the sequence. Use the pattern to find the next two terms in the sequence.
384, 192, 96, 48, . . .



Each term is half the preceding term. The next two terms are 48 ÷ 2 = 24 and 24 ÷ 2 = 12.

2 Using Inductive Reasoning Make a conjecture about the sum of the cubes of the first 25 counting numbers.
Find the first few sums. Notice that each sum is a perfect square and that the perfect squares form a pattern.

$1^3 = 1 = 1^2 = 1^2$
$1^3 + 2^3 = 9 = 3^2 = (1 + 2)^2$
$1^3 + 2^3 + 3^3 = 36 = 6^2 = (1 + 2 + 3)^2$
$1^3 + 2^3 + 3^3 + 4^3 = 100 = 10^2 = (1 + 2 + 3 + 4)^2$
$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225 = 15^2 = (1 + 2 + 3 + 4 + 5)^2$

The sum of the first two cubes equals the square of the sum of the first two counting numbers. The sum of the first three cubes equals the square of the sum of the first three counting numbers. This pattern continues for the fourth and fifth rows. So a conjecture might be that the sum of the cubes of the first 25 counting numbers equals the square of the sum of the first 25 counting numbers, or $(1 + 2 + 3 + \ldots + 25)^2$.

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3 Finding a Counterexample Find a counterexample for the conjecture.
Since $3^2 + 4^2 = 5^2$, the sum of the squares of two consecutive numbers is the square of the next consecutive number.

Begin with 4. The next consecutive number is 4 + 1 = 5.

The sum of the squares of these two consecutive numbers is $4^2 + 5^2 = 16 + 25 = 41$.

The conjecture is that 41 is the square of the next consecutive number.
The square of the next consecutive number is $(5 + 1)^2 = 6^2 = 36$.
So, since $4^2 + 5^2$ and 6^2 are not the same, this conjecture is false.

4 Applying Conjectures to Business The price of overnight shipping was $8.00 in 2000, $9.50 in 2001, and $11.00 in 2002. Make a conjecture about the price in 2003.

Write the data in a table. Find a pattern.

2000	2001	2002
$8.00	$9.50	$11.00

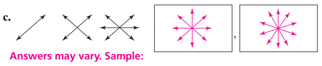
Each year the price increased by $1.50. A possible conjecture is that the price in 2003 will increase by $1.50. If so, the price in 2003 would be $11.00 + $1.50 = $12.50.

Quick Check

1. Find the next two terms in each sequence.

a. 1, 2, 4, 7, 11, 16, 22, 29, 37, . . .

b. Monday, Tuesday, Wednesday, Thursday, Friday, . . .

c.  , . . .

Answers may vary. Sample:

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2. Make a conjecture about the sum of the first 35 odd numbers. Use your calculator to verify your conjecture.

$1 = 1 = 1^2$
$1 + 3 = 4 = 2^2$
$1 + 3 + 5 = 9 = 3^2$
$1 + 3 + 5 + 7 = 16 = 4^2$
$1 + 3 + 5 + 7 + 9 = 25 = 5^2$

The sum of the first 35 odd numbers is 35^2, or 1225.

3. **Finding a Counterexample** Find a counterexample for the conjecture.
Some products of 5 and other numbers are shown in the table.

5 × 7 = 35	5 × 13 = 65
5 × 3 = 15	5 × 9 = 45
5 × 11 = 55	5 × 25 = 125

Therefore, the product of 5 and any positive integer ends in 5.

Since 5 × 2 = 10, not all products of 5 end in 5. (But they all either end in 5 or 0.)

4. Suppose the price of two-day shipping was $6.00 in 2000, $7.00 in 2001, and $8.00 in 2002. Make a conjecture about the price in 2003.

Each year the price increased by $1.00. A possible conjecture is that the price in 2003 will increase by $1.00. If so, the price in 2003 would be $8.00 + $1.00 = $9.00.

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Lesson 1-2 — Drawings, Nets, and Other Models

Lesson Objectives	NAEP 2005 Strand: Geometry
Make isometric and orthographic drawings	Topic: Dimension and Shape
Draw nets for three-dimensional figures	Local Standards: ____________

Vocabulary

An isometric drawing of a three-dimensional object shows a corner view of the figure drawn on isometric dot paper.

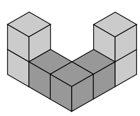
An orthographic drawing is the top view, front view, and right-side view of a three-dimensional figure.

A foundation drawing shows the base of a structure and the height of each part.

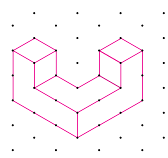
A net is a two-dimensional pattern you can fold to form a three-dimensional figure.

Examples

1 Isometric Drawing Make an isometric drawing of the cube structure at right.



An isometric drawing shows three sides of a figure from a corner view. Hidden edges are not shown, so all edges are solid lines.



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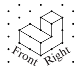
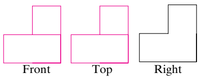
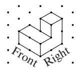
❷ **Orthographic Drawing** Make an orthographic drawing of the isometric drawing at right.

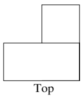


Orthographic drawings flatten the depth of a figure. An orthographic drawing shows [three] views. Because no edge of the isometric drawing is hidden in the top, front, and right views, all lines are solid.

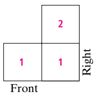


❸ **Foundation Drawing** Make a foundation drawing for the isometric drawing.
To make a foundation drawing, use the top view of the orthographic drawing.





Because the top view is formed from [3] squares, show [3] squares in the foundation drawing.
Identify the square that represents the tallest part. Write the number 2 in the back square to indicate that the back section is [2] cubes high.
Write the number 1 in each of the two front squares to indicate that each front section is [1] cube high.



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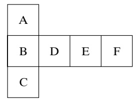
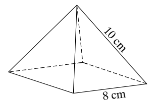
❹ **Identifying Solids from Nets** Is the pattern a net for a cube? If so, name two letters that will be opposite faces.



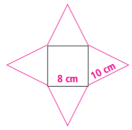
The pattern [is] a net because you [can] fold it to form a cube. Fold squares A and C up to form the back and front of the cube. Fold D up to form a side. Fold E over to form the top. Fold F down to form another side.

After the net is folded, the following pairs of letters are on opposite faces:
[A] and [C] are the back and front faces.
[B] and [E] are the [bottom] and [top] faces.
[D] and [F] are the right and left side faces.

❺ **Drawing a Net** Draw a net for the figure with a square base and four isosceles triangle faces. Label the net with its dimensions.

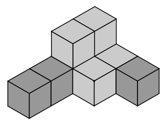


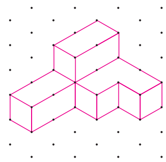
Think of the sides of the square base as hinges, and "unfold" the figure at these edges to form a net. The base of each of the four isosceles triangle faces is a side of the [square]. Write in the known dimensions.



Quick Check

1. Make an isometric drawing of the cube structure below.





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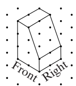
2. Make an orthographic drawing from this isometric drawing.



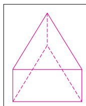
3. a. How many cubes would you use to make the structure in Example 3?

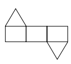
4

b. **Critical Thinking** Which drawing did you use to answer part (a), the foundation drawing or the isometric drawing? Explain.

Answers may vary. Sample: the foundation drawing; you can just add the three numbers.

4. Sketch the three-dimensional figure that corresponds to the net.





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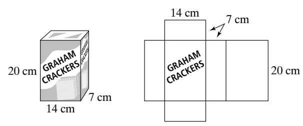
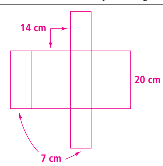
5. The drawing shows one possible net for the Graham Crackers box.



Draw a different net for this box. Show the dimensions in your diagram.

Answers may vary. Example:



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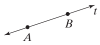
Lesson 1-3 — Points, Lines, and Planes

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Understand basic terms of geometry 2 Understand basic postulates of geometry	Topic: Dimension and Shape Local Standards: ____

Vocabulary and Key Concepts

Postulate 1-1

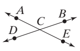
Through any two points there is **exactly one line.**



Line *t* is the only line that passes through points *A* and **B**.

Postulate 1-2

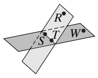
If two lines intersect, then they intersect in **exactly one point.**



$\overleftrightarrow{AE}$ and $\overleftrightarrow{BD}$ intersect at **C**.

Postulate 1-3

If two planes intersect, then they intersect in **exactly one line.**



Plane *RST* and plane *STW* intersect in $\overleftrightarrow{ST}$.

Postulate 1-4

Through any three noncollinear points there is **exactly one plane.**

A point is **a location.**

Space is **the set of all points.**

A line is **a series of points that extends in two opposite directions without end.**

Collinear points are **points that lie on the same line.**

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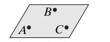
A plane is **a flat surface that has no thickness.**



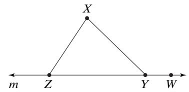
Plane *ABC*

Two points or lines are coplanar if **they lie on the same plane.**

A postulate or axiom is **an accepted statement of fact.**

Examples

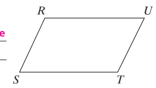
1 Identifying Collinear Points In the figure at right, name three points that are collinear and three points that are not collinear.



Points **Y**, **Z**, and **W** lie on a line, so they are collinear.

Any other set of three points in the figure do not lie on a line, so no other set of three points is collinear. For example, *X*, *Y*, and *Z* form a **triangle** and are not collinear.

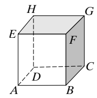
2 Naming a Plane Name the plane shown in two different ways.



You can name a plane using **any three or more points on that plane that are not collinear.**

Some possible names for the plane shown are the following: plane **RST**, plane **RSU**, plane **RTU**, plane **STU**, and plane **RSTU**.

3 Finding the Intersection of Two Planes Use the diagram at right. What is the intersection of plane *HGC* and plane *AED*?



As you look at the cube, the front face is on plane *AEFB*, the back face is on plane *HGC*, and the left face is on plane *AED*. The back and left faces of the cube intersect at **HD**. Planes *HGC* and *AED* intersect vertically at $\overleftrightarrow{HD}$.

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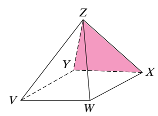
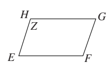
4 Using Postulate 1-4 Shade the plane that contains *X*, *Y*, and *Z*.

Points *X*, *Y*, and *Z* are the vertices of one of the four triangular faces of the pyramid. To shade the plane, shade the interior of the triangle formed by **X**, **Y**, and **Z**.

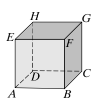


Quick Check

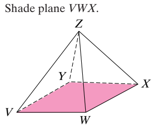
1. Use the figure in Example 1.
   a. Are points *W*, *Y*, and *X* collinear?
      no
   b. Name line *m* in three different ways.
      Answers may vary. Sample: $\overleftrightarrow{ZW}$, $\overleftrightarrow{WY}$, $\overleftrightarrow{YZ}$.
   c. **Critical Thinking** Why do you think arrowheads are used when drawing a line or naming a line such as $\overleftrightarrow{ZW}$?
      Arrowheads are used to show that the line extends in opposite directions without end.
2. List three different names for plane *Z*.
   Answers may vary. Sample: *HEF*, *HEFG*, *FGH*.



3. Name two planes that intersect in $\overleftrightarrow{BF}$.
   ABF* and *CBF



4. a. Shade plane *VWX*.



   b. Name a point that is coplanar with points *V*, *W*, and *X*.
      Y

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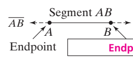
Lesson 1-4 — Segments, Rays, Parallel Lines and Planes

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Identify segments and rays 2 Recognize parallel lines	Topic: Relationships Among Geometric Figures Local Standards: ____

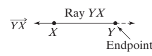
Vocabulary

A segment is **the part of a line consisting of two endpoints and all points between them.**

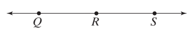


Endpoint **Endpoint**

A ray is **the part of a line consisting of one endpoint and all the points of the line on one side of the endpoint.**



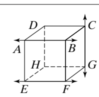
Opposite rays are **two collinear rays with the same endpoint.**



$\overrightarrow{RQ}$ and $\overrightarrow{RS}$ are opposite rays.

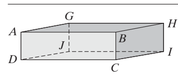
Parallel lines are **coplanar lines that do not intersect.**

Skew lines are **noncoplanar; therefore, they are not parallel and do not intersect.**



$\overleftrightarrow{AB}$ is **parallel** to $\overleftrightarrow{EF}$.
$\overleftrightarrow{AB}$ and $\overleftrightarrow{CG}$ are **skew** lines.

Parallel planes are **planes that do not intersect.**



Plane *ABCD* is **parallel** to plane *GHIJ*.

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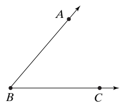
Examples

1 Naming Segments and Rays Name the segments and rays in the figure.

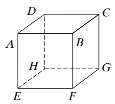


The labeled points in the figure are A, B, and C.

A segment is a part of a line consisting of two endpoints and all points between them. A segment is named by its two endpoints. So the segments are **$\overline{BA}$ (or $\overline{AB}$)** and **$\overline{BC}$ (or $\overline{CB}$)**.

A ray is a part of a line consisting of one endpoint and all the points of the line on one side of that endpoint. A ray is named by its endpoint first, followed by any other point on the ray. So the rays are **$\overrightarrow{BA}$** and **$\overrightarrow{BC}$**.

2 Identifying Parallel and Skew Segments Use the figure at right. Name all segments that are parallel to $\overline{AE}$. Name all segments that are skew to $\overline{AE}$.



Parallel segments lie in the same plane, and the lines that contain them do not intersect. The three segments in the figure that are parallel to $\overline{AE}$ are **$\overline{BF}$**, **$\overline{CG}$**, and **$\overline{DH}$**.

Skew segments are segments that do not lie in the same plane. The four segments in the figure that do not lie in the same plane as $\overline{AE}$ are **$\overline{BC}$**, **$\overline{CD}$**, **$\overline{FG}$**, and **$\overline{GH}$**.

3 Identifying Parallel Planes Identify a pair of parallel planes in your classroom.

Planes are parallel if they **do not intersect**. If the walls of your classroom are vertical, **opposite** walls are parts of parallel planes. If the ceiling and floor of the classroom are level, they are parts of parallel planes.

Quick Check

1. **Critical Thinking** Use the figure in Example 1. $\overrightarrow{CB}$ and $\overrightarrow{BC}$ form a line. Are they opposite rays? Explain.

No; they do not have the same endpoint.

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2. Use the diagram in Example 2.

   a. Name all labeled segments that are parallel to $\overline{GF}$.

   $\overline{HE}$, $\overline{CB}$, $\overline{DA}$

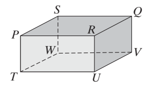
   b. Name all labeled segments that are skew to $\overline{GF}$.

   $\overline{AB}$, $\overline{DC}$, $\overline{AE}$, $\overline{DH}$

   c. Name another pair of parallel segments and another pair of skew segments.

   Answers may vary. Sample: $\overline{CG}$, $\overline{BF}$; $\overline{EF}$, $\overline{DH}$

3. Use the diagram to the right.



   a. Name three pairs of parallel planes.

   PSWT∥RQVU, PRUT∥SQVW, PSQR∥TWVU

   b. Name a line that is parallel to $\overleftrightarrow{PQ}$.

   $\overleftrightarrow{TV}$

   c. Name a line that is parallel to plane $QRUV$.

   Answers may vary. Sample: $\overleftrightarrow{PS}$

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Lesson 1-5 Measuring Segments

Lesson Objectives ▼ Find the lengths of segments	**NAEP 2005 Strand:** Measurement **Topic:** Measuring Physical Attributes **Local Standards:** ________

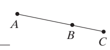
Vocabulary and Key Concepts

Postulate 1-5: Ruler Postulate

The points of a line can be put into one-to-one correspondence with the real numbers so that the distance between any two points is the **absolute value of the difference of the corresponding numbers.**

Postulate 1-6: Segment Addition Postulate

If three points A, B, and C are collinear and B is between A and C, then **$AB + BC = AC$.**

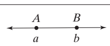


A coordinate is **a point's distance and direction from zero on a number line.**

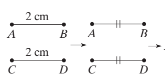
the length of $\overline{AB}$

$AB = |$ **a** $-$ **b** $|$

coordinate of A coordinate of B

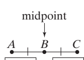


Congruent (≅) segments are **segments with the same length.**



$\rightarrow AB = CD \rightarrow$ **$\overline{AB}$** ≅ **$\overline{CD}$**

A midpoint is **a point that divides a segment into two congruent segments.**



$\overline{AB}$ ≅ **$\overline{BC}$**

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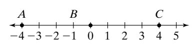
Examples

1 Comparing Segment Lengths Find AB and BC. Are $\overline{AB}$ and $\overline{AC}$ congruent?



$AB = |-5 -$ **(−1)** $| = |-4| =$ **4**

$BC = |$ **−1** $- 4| =$ **$|-5|$** $= 5$

$AB \neq BC$ so $\overline{AB}$ and $\overline{AC}$ are **not congruent.**

2 Using the Segment Addition Postulate If $AB = 25$, find the value of x. Then find AN and NB.

Use the Segment Addition Postulate (Postulate 1-6) to write an equation.

AN + **NB** $= AB$ — **Segment Addition Postulate**

(**2x − 6**) + (**x + 7**) $= 25$ — **Substitute.**

3x + **1** $= 25$ — **Simplify the left side.**

$3x =$ **24** — **Subtract 1 from each side.**

$x =$ **8** — **Divide each side by 3.**

$AN = 2x - 6 = 2($ **8** $) - 6 =$ **10**

$NB = x + 7 = ($ **8** $) + 7 =$ **15** — **Substitute 8 for x.**

$AN =$ **10** and $NB =$ **15**, which checks because the sum of the segment lengths equals 25.

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3 Using the Midpoint M is the midpoint of $\overline{RT}$. Find RM, MT, and RT.

$5x + 9$ $\quad$ $8x - 36$

R M T

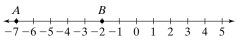
Use the definition of midpoint to write an equation.

RM = MT	Definition of midpoint	
5x + 9 = 8x − 36	Substitute.	
5x + 45 = 8x	Add 36 to each side.	
45 = 3x	Subtract 5x from each side.	
15 = x	Divide each side by 3.	
$RM = 5x + 9 = 5(15) + 9 = 84$	Substitute 15 for x.	
$MT = 8x - 36 = 8(15) - 36 = 84$		
$RT = RM + MT = 168$	Segment Addition Postulate	

RM and MT are each 84, which is half of 168, the length of $\overline{RT}$.

Quick Check

1. Find AB. Find C, different from A, such that $\overline{AB}$ and $\overline{AC}$ are congruent.



$AB = |-7 - (-2)| = |-5| = 5$

We also want $BC = 5$. That means that C must lie 5 units on the other side of B from A. So C must lie at $-2 + 5 = 3$.

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2. $EG = 100$. Find the value of x. Then find EF and FG.

$4x - 20$ $\quad$ $2x + 30$

E F G

$x = 15$, $EF = 40$; $FG = 60$

3. Z is the midpoint of $\overline{XY}$, and $XY = 27$. Find XZ.

13.5

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Lesson 1-6 Measuring Angles

Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find the measures of angles	Topic: Measuring Physical Attributes
2 Identify special angle pairs	Local Standards: ____________

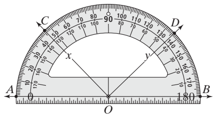
Vocabulary and Key Concepts

Postulate 1-7: Protractor Postulate

Let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be opposite rays in a plane. $\overrightarrow{OA}$, $\overrightarrow{OB}$, and all the rays with endpoint O that can be drawn on one side of $\overleftrightarrow{AB}$ can be paired with the real numbers from 0 to 180 so that

a. $\overrightarrow{OA}$ is paired with 0 and $\overrightarrow{OB}$ is paired with 180.

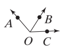
b. If $\overrightarrow{OC}$ is paired with x and $\overrightarrow{OD}$ is paired with y, then $m\angle COD = |x - y|$.



Postulate 1-8: Angle Addition Postulate

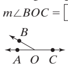
If point B is in the interior of $\angle AOC$, then

$m\angle AOB + m\angle BOC = m\angle AOC$.



If $\angle AOC$ is a straight angle, then

$m\angle AOB + m\angle BOC = 180$.



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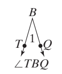
An angle (∠) is formed by two rays with the same endpoint. The rays are the sides of the angle and the endpoint is the vertex of the angle.



acute angle	right angle	obtuse angle	straight angle
$0 < x < 90$	$x = 90$	$90 < x < 180$	$x = 180$

An acute angle has a measurement between 0° and 90°.

A right angle has a measurement of exactly 90°.

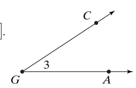
An obtuse angle has a measurement between 90° and 180°.

A straight angle has a measurement of exactly 180°.

Congruent angles are two angles with the same measure.

Examples

1 Naming Angles Name the angle at right in four ways.



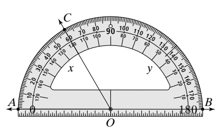
The name can be the number between the sides of the angle: ∠3.

The name can be the vertex of the angle: ∠G.

Finally, the name can be a point on one side, the vertex, and a point on the other side of the angle: ∠AGC or ∠CGA.

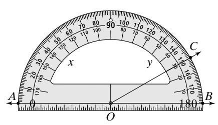
2 Measuring and Classifying Angles Find the measure of each ∠AOC. Classify each as *acute*, *right*, *obtuse*, or *straight*.

a.



60, acute

b.



150, obtuse

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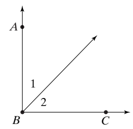
❸ **Using the Angle Addition Postulate** Suppose that $m\angle 1 = 42$ and $m\angle ABC = 88$. Find $m\angle 2$.



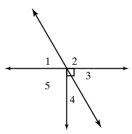
Use the Angle Addition Postulate (Postulate 1-8) to solve.

$m\angle$ **1** $+ m\angle$ **2** $= m\angle ABC$ — **Angle Addition** Postulate

42 $+ m\angle 2 =$ **88** — Substitute **42** for $m\angle 1$ and **88** for $m\angle ABC$.

$m\angle 2 =$ **46** — Subtract **42** from each side.

❹ **Identifying Angle Pairs** In the diagram identify pairs of numbered angles that are related as follows:



a. complementary

∠3 and ∠4

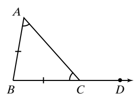
b. supplementary

∠1 and ∠2; ∠2 and ∠3

c. vertical angles

∠1 and ∠3

❺ **Making Conclusions From a Diagram** Can you make each conclusion from the diagram?



a. $\angle A \cong \angle C$

Yes; the markings show they are congruent.

b. $\angle B$ and $\angle ACD$ are supplementary

No; there are no markings.

c. $m(\angle BCA) + m(\angle DCA) = 180$

Yes; you can conclude that the angles are supplementary from the diagram.

d. $\overline{AB} \cong \overline{BC}$

Yes; the markings show they are congruent.

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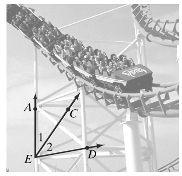
Quick Check



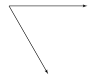
1. a. Name $\angle CED$ two other ways.

∠2, ∠DEC

b. **Critical Thinking** Would it be correct to name any of the angles $\angle E$? Explain.

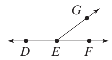
No, 3 angles have *E* for a vertex, so you need more information in the name to distinguish them from one another.

2. Find the measure of the angle. Classify it as *acute*, *right*, *obtuse*, or *straight*.



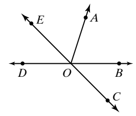
60; acute

3. If $m\angle DEG = 145$, find $m\angle GEF$.



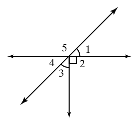
35

4. Name an angle or angles in the diagram supplementary to each of the following:
   a. $\angle DOA$
   b. $\angle EOB$



a. ∠AOB
b. ∠BOC or ∠DOE

5. Can you make each conclusion from the information in the diagram? Explain.
   a. $\angle 1 \cong \angle 3$
   b. $\angle 4$ and $\angle 5$ are supplementary
   c. $m(\angle 1) + m(\angle 5) = 180$



a. Yes; the markings show they are congruent.
b. Yes; you can conclude the angles are adjacent and supplementary from the diagram.
c. Yes; you can conclude that the angles are supplementary from the diagram and their sum is 180 by the Angle Addition Postulate.

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Lesson 1-7 Basic Constructions

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use a compass and a straightedge to construct congruent segments and congruent angles	Topic: Relationships Among Geometric Figures
2 Use a compass and a straightedge to bisect segments and angles	Local Standards: ____________________

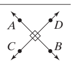
Vocabulary

Construction is **using a straightedge and a compass to draw a geometric figure.**

A straightedge is **a ruler with no markings on it.**

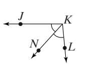
A compass is **a geometric tool used to draw circles and parts of circles called arcs.**

Perpendicular lines are **two lines that intersect to form right angles.**



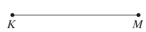
A perpendicular bisector of a segment is **a line, segment, or ray that is perpendicular to the segment at its midpoint, thereby bisecting the segment into two congruent segments.**

An angle bisector is **a ray that divides an angle into two congruent coplanar angles.**

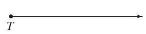


Examples

❶ **Constructing Congruent Segments** Construct $\overline{TW}$ congruent to $\overline{KM}$.



Step 1 Draw a ray with endpoint T.



Step 2 Open the compass the length of $\overline{KM}$.

Step 3 With the same compass setting, put the compass point on point T. Draw an arc that intersects the ray. Label the point of intersection W.

$\overline{KM} \cong \overline{TW}$

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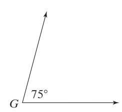
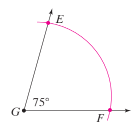
❷ **Constructing Congruent Angles** Construct $\angle Y$ so that $\angle Y \cong \angle G$.

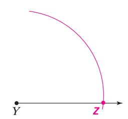


Step 1 Draw a ray with endpoint Y.

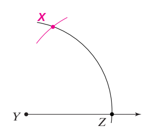
Step 2 With the compass point on point G, draw an arc that intersects both sides of $\angle G$. Label the points of intersection E and F.



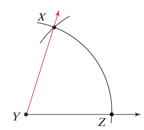
Step 3 With the same compass setting, put the compass point on point Y. Draw an arc that intersects the ray. Label the point of intersection Z.



Step 4 Open the compass to the length EF. Keeping the same compass setting, put the compass point on Z. Draw an arc that intersects with the arc you drew in Step 3. Label the point of intersection X.



Step 5 Draw $\overrightarrow{YX}$ to complete $\angle Y$.



$\angle Y \cong \angle G$

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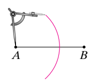
3 Constructing the Perpendicular Bisector

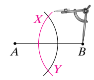
Given: $\overline{AB}$.

Construct: $\overleftrightarrow{XY}$ so that $\overleftrightarrow{XY} \perp \overline{AB}$ at the midpoint M of $\overline{AB}$.

Step 1 Put the compass point on point A and draw a long arc. Be sure that the opening is **greater** than $\frac{1}{2}AB$.

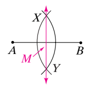


Step 2 With the **same** compass setting, put the compass point on point B and draw another long arc. Label the points where the two arcs **intersect** as X and Y.

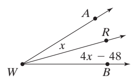


Step 3 Draw $\overleftrightarrow{XY}$. The point of intersection of $\overline{AB}$ and $\overleftrightarrow{XY}$ is M, the **midpoint** of $\overline{AB}$.

$\overleftrightarrow{XY} \perp \overline{AB}$ at the midpoint of $\overline{AB}$, so $\overleftrightarrow{XY}$ is the **perpendicular bisector** of $\overline{AB}$.



4 Finding Angle Measures $\overrightarrow{WR}$ bisects $\angle AWB$ so that $m\angle AWR = x$ and $m\angle BWR = 4x - 48$. Find $m\angle AWB$.



$m\angle$ **AWR** $= m\angle$ **BWR**	Definition of **angle bisector**
x $=$ **4x − 48**	Substitute **x** for $m\angle AWR$ and **4x − 48** for $m\angle BWR$.
−3x $= -48$	Subtract **4x** from each side.
$x =$ **16**	Divide each side by **−3**.
$m\angle AWR =$ **16**	Substitute **16** for x.
$m\angle BWR = 4($**16**$) - 48 =$ **16**	
$m\angle AWB = m\angle$ **AWR** $+ m\angle$ **BWR**	**Angle Addition** Postulate
$m\angle AWB =$ **16** $+$ **16** $=$ **32**	Substitute **16** for $m\angle AWR$ and for $m\angle BWR$.

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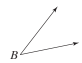
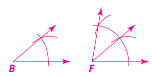
Quick Check

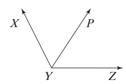
1. Use a straightedge to draw $\overline{XY}$. Then construct $\overline{RS}$ so that $RS = 2XY$.

2. a. Construct $\angle F$ with $m\angle F = 2m\angle B$.



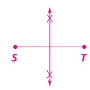


b. Explain how you can use your protractor to check that $\overrightarrow{YP}$ is the angle bisector of $\angle XYZ$.

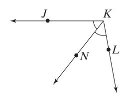


Measure ∠XYP and ∠PYZ to see that they are congruent.

3. Draw $\overline{ST}$. Construct its perpendicular bisector.



4. If $m\angle JKN = 50°$, then find $m\angle NKL$ and $m\angle JKL$. $\overrightarrow{KN}$ bisects $\angle JKL$.



$m\angle NKL =$ **50°**, $m\angle JKL =$ **100°**

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Lesson 1-8

The Coordinate Plane

Lesson Objectives	
1 Find the distance between two points in the coordinate plane 2 Find the coordinates of the midpoint of a segment in the coordinate plane	**NAEP 2005 Strand:** Measurement **Topic:** Measuring Physical Attributes **Local Standards:** ____________

Key Concepts

Formula: The Distance Formula

The distance d between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

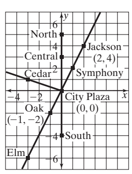
Formula: The Midpoint Formula

The coordinates of the midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ are the following:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Examples

1 Applying the Distance Formula How far is the subway ride from Oak to Symphony? Round to the nearest tenth. Each unit represents 1 mile.



Oak has coordinates **(−1, −2)**. Let **(x_1, y_1)** represent Oak. Symphony has coordinates **(1, 2)**. Let **(x_2, y_2)** represent Symphony.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Use the Distance Formula.
$d = \sqrt{(1 - (-1))^2 + (2 - (-2))^2}$	Substitute.
$d = \sqrt{2^2 + 4^2}$	Simplify within parentheses.
$d = \sqrt{4 + 16} = \sqrt{20}$	Simplify.
20 [√] = **4.472135955**	Use a calculator.

To the nearest tenth, the subway ride from Oak to Symphony is **4.5** miles.

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2 Finding the Midpoint $\overline{AB}$ has endpoints $(8, 9)$ and $(-6, -3)$. Find the coordinates of its midpoint M.

Use the Midpoint Formula. Let (x_1, y_1) be **(8, 9)** and (x_2, y_2) be **(−6, −3)**.

The midpoint has coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$. **Midpoint Formula**

The x-coordinate is $\frac{8 + (-6)}{2} = \frac{2}{2} = 1$ **Substitute 8 for x_1 and −6 for x_2. Simplify.**

The y-coordinate is $\frac{9 + (-3)}{2} = \frac{6}{2} = 3$ **Substitute 9 for y_1 and −3 for y_2. Simplify.**

The coordinates of the midpoint M are **(1, 3)**.

3 Finding an Endpoint The midpoint of $\overline{DG}$ is $M(-1, 5)$. One endpoint is $D(1, 4)$. Find the coordinates of the other endpoint G.

Use the Midpoint Formula. Let (x_1, y_1) be **(1, 4)** and the midpoint $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ be **(−1, 5)**. Solve for **x_2** and **y_2**, the coordinates of G.

Find the x-coordinate of G.

$-1 = \frac{1 + x_2}{2}$	← **Use the Midpoint Formula.** →	$5 = \frac{4 + y_2}{2}$
$-2 = 1 + x_2$	← **Multiply each side by 2.** →	$10 = 4 + y_2$
$-3 = x_2$	← **Simplify.** →	$6 = x_2$

The coordinates of G are **(−3, 6)**.

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Quick Check

1. a. Using the map in Example 1, find the distance between Elm and Symphony.

   about 8.9 miles

   b. Maple is located 6 miles west and 2 miles north of City Plaza. Find the distance between Cedar and Maple.

   about 3.2 miles

2. Find the coordinates of the midpoint of $\overline{XY}$ with endpoints $X(2, -5)$ and $Y(6, 13)$.

   (4, 4)

3. The midpoint of $\overline{XY}$ has coordinates $(4, -6)$. X has coordinates $(2, -3)$. Find the coordinates of Y.

   (6, −9)

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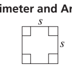
Lesson 1-9

Perimeter, Circumference, and Area

Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find perimeters of rectangles and squares, and circumferences of circles	**Topic:** Measuring Physical Attributes
2 Find areas of rectangles, squares, and circles	**Local Standards:** ____________________

Key Concepts

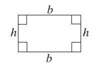
Perimeter and Area



Square with side length s.

Perimeter P = 4s

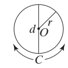
Area A = s^2



Rectangle with base b and height h.

Perimeter P = 2b + 2h

Area A = bh



Circle with radius r and diameter d.

Circumference C = πd

or C = $2\pi r$

Area = πr^2

Postulate 1-9

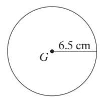
If two figures are congruent, then their areas are equal.

Postulate 1-10

The area of a region is the sum of the areas of its non-overlapping parts.

Examples

1 **Finding Circumference** $\odot G$ has a radius of 6.5 cm. Find the circumference of $\odot G$ in terms of π. Then find the circumference to the nearest tenth.



$C = 2\,\pi\,r$ Formula for circumference of a circle

$C = 2\pi(6.5)$ Substitute 6.5 for r.

$C = 13\pi$ Exact answer

$C = 13$ × π = 40.840704 Use a calculator.

The circumference of $\odot G$ is 13π, or about 40.8 cm.

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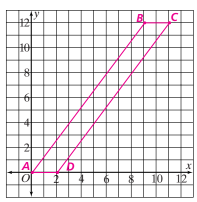
2 **Finding Perimeter in the Coordinate Plane** Quadrilateral $ABCD$ has vertices $A(0, 0)$, $B(9, 12)$, $C(11, 12)$, and $D(2, 0)$. Find the perimeter.

Draw and label $ABCD$ on a coordinate plane.



Find the length of each side. Add the lengths to find the perimeter.

$AB = \sqrt{(9 - 0)^2 + (12 - 0)^2} = \sqrt{9^2 + 12^2}$ Use the Distance Formula.

$= \sqrt{81 + 144} = \sqrt{225} = 15$

$BC = |11 - 9| = |2| = 2$ Ruler Postulate

$CD = \sqrt{(2 - 11)^2 + (0 - 12)^2}$ Use the Distance Formula.

$= \sqrt{(-9)^2 + (-12)^2}$

$= \sqrt{81 + 144} = \sqrt{225} = 15$

$DA = |2 - 0| = |2| = 2$ Ruler Postulate

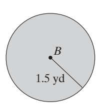
Perimeter = $AB + BC + CD + DA$

$= 15 + 2 + 15 + 2$

$= 34$

The perimeter of quadrilateral $ABCD$ is 34 units.

3 **Finding Area of a Circle** Find the area of $\odot B$ in terms of π.



In $\odot B$, r = 1.5 yd.

$A = \pi r^2$ Formula for the area of a circle

$A = \pi(1.5)^2$ Substitute 1.5 for r.

$A = 2.25\pi$

The area of $\odot B$ is 2.25π yd^2.

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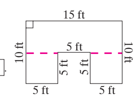
4 **Finding Area of an Irregular Shape** Find the area of the figure to the right.



Draw a horizontal line to separate the figure into three non-overlapping figures: one rectangle and two squares.

Find each area. Then add the areas.

$A_R = bh$ ← Use the formulas. → $A_S = s^2$

$= (15)(5)$ ← Substitute. → $= (5)^2$

$= 75$ ← Simplify. → $= 25$

$A = 75 + 25 + 25$ Add the areas.

$A = 125$ Simplify.

The area of the figure is 125 ft^2.

Quick Check

1. a. Find the circumference of a circle with a radius of 18 m in terms of π.

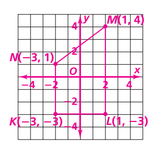
   36π m

   b. Find the circumference of a circle with a diameter of 18 m to the nearest tenth.

   56.5 m

2. Graph quadrilateral $KLMN$ with vertices $K(-3, -3)$, $L(1, -3)$, $M(1, 4)$, and $N(-3, 1)$. Find the perimeter of $KLMN$.

   20 units



3. You are designing a rectangular banner for the front of a museum. The banner will be 4 ft wide and 7 yd high. How much material do you need in square yards?

   $9\frac{1}{3}$ yd^2

4. Copy the figure in Example 4. Separate it in a different way. Find the area.

   125 ft^2

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Lesson 2-1

Conditional Statements

Lesson Objectives	NAEP 2005 Strand: Geometry
Recognize conditional statements	Topic: Mathematical Reasoning
Write converses of conditional statements	Local Standards: ____________

Vocabulary and Key Concepts

Conditional Statements and Converses

Statement	Example	Symbolic Form	You read it
Conditional	If an angle is a straight angle, then its measure is 180°.	$p \to q$	If **p**, then **q**.
Converse	If the measure of an angle is 180°, then it is a straight angle.	$q \to p$	If **q**, then **p**.

A conditional is **an *if-then* statement.**

The hypothesis is **the part that follows *if* in an *if-then* statement.**

The conclusion is **the part of an *if-then* statement (conditional) that follows *then*.**

The truth value of a statement is **"true" or "false" according to whether the statement is true or false, respectively.**

The converse of the conditional "if p, then q" **is the conditional "if q, then p."**

Examples

1 Identifying the Hypothesis and the Conclusion Identify the hypothesis and conclusion: If two lines are parallel, then the lines are coplanar.

In a conditional statement, the clause after *if* is the hypothesis and the clause after *then* is the conclusion.

Hypothesis: **Two lines are parallel.**

Conclusion: **The lines are coplanar.**

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2 Finding a Counterexample Find a counterexample to show that this conditional is false: If $x^2 \geq 0$, then $x \geq 0$.

A counterexample is a case in which the hypothesis is **true** and the conclusion is **false**. This counterexample must be an example in which $x^2 \geq 0$ (hypothesis true) and $x \ngeq 0$ or $x < 0$ (conclusion false).

Because any negative number has a positive square, one possible counterexample is -1.

Because $(-1)^2 = 1$, which is greater than 0, the hypothesis is **true**.

Because $-1 < 0$, the conclusion is **false**.

The counterexample shows that the conditional is **false**.

3 Writing the Converse of a Conditional Write the converse of the following conditional.

If $x = 9$, then $x + 3 = 12$.

The converse of a conditional exchanges the hypothesis and the conclusion.

Conditional		Converse	
Hypothesis	Conclusion	Hypothesis	Conclusion
$x = 9$	$x + 3 = 12$	**$x + 3 = 12$**	**$x = 9$**

So the converse is: **If $x + 3 = 12$, then $x = 9$.**

Quick Check

1. Identify the hypothesis and the conclusion of this conditional statement:
If $y - 3 = 5$, then $y = 8$.

Hypothesis: **$y - 3 = 5$**

Conclusion: **$y = 8$**

2. Show that this conditional is false by finding a counterexample:
If the name of a state includes the word "New," then the state borders an ocean.

Counterexample:

New Mexico does not border an ocean. It is a counterexample, so the conditional is false.

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Example

4 Finding the Truth Value of a Converse Write the converse of the conditional. Then determine the truth value of each.

If $a^2 = 25$, then $a = 5$.

Conditional: **If $a^2 = 25$, then $a = 5$.**

The converse exchanges the **hypothesis** and the **conclusion**.

Converse: **If $a = 5$, then $a^2 = 25$.**

The conditional is **false**. A counterexample is $a = -5$: $(-5)^2 = 25$, and $-5 \neq 5$.

Because $5^2 = 25$, the converse is **true**.

Quick Check

3. Write the converse of the following conditional:
If two lines are not parallel and do not intersect, then they are skew.

If two lines are skew, then they are not parallel and do not intersect.

4. Write the converse of each conditional. Determine the truth value of the conditional and its converse. (*Hint:* One of these conditionals is *not* true.)

a. If two lines do not intersect, then they are parallel.
Converse:
If two lines are parallel, then they do not intersect.
The conditional is **false** and the converse is **true**.

b. If $x = 2$, then $|x| = 2$.
Converse:
If $|x| = 2$, then $x = 2$.
The conditional is **true** and the converse is **false**.

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Lesson 2-2

Biconditionals and Definitions

Lesson Objectives	NAEP 2005 Strand: Geometry
Write biconditionals	Topics: Dimension and Shape; Mathematical Reasoning
Recognize good definitions	Local Standards: ____________

Vocabulary and Key Concepts

Biconditional Statements

A biconditional combines $p \to q$ and $q \to p$ as $p \leftrightarrow q$.

Statement	Example	Symbolic Form	You read it
Biconditional	An angle is a straight angle if and only if its measure is 180°.	$p \leftrightarrow q$	**p** if and only if **q**.

A biconditional statement is **the combination of a conditional statement and its converse.**

A biconditional contains the words **"if and only if."**

Examples

1 Writing a Biconditional Consider the true conditional statement. Write its converse. If the converse is also true, combine the statements as a biconditional.

Conditional: If $x = 5$, then $x + 15 = 20$.

To write the converse, exchange the hypothesis and conclusion.

Converse: **If $x + 15 = 20$, then $x = 5$.**

When you subtract 15 from each side to solve the equation, you get $x = 5$. Because both the conditional and its converse are **true**, you can combine them in a **biconditional** using the phrase ***if and only if***.

Biconditional: **$x = 5$ if and only if $x + 15 = 20$.**

2 Separating a Biconditional into Parts Write the two statements that form this biconditional.

Biconditional: Lines are skew if and only if they are noncoplanar.

Write a biconditional as two conditionals that are converses of each other.

Conditional: **If lines are skew, then they are noncoplanar.**

Converse: **If lines are noncoplanar, then they are skew.**

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❸ **Writing a Definition as a Biconditional** Show that this definition of *triangle* is reversible. Then write it as a true biconditional.

Definition: A triangle is a polygon with exactly three sides.

The original conditional is true.

Conditional: If a polygon is a triangle, then it has exactly three sides.

The converse is also true.

Converse: If a polygon has exactly three sides, then it is a triangle.

Because both statements are true, they can be combined to form a biconditional.

Biconditional: A polygon is a triangle if and only if it has exactly three sides.

❹ **Identifying a Good Definition** Is the following statement a good definition? Explain.

An apple is a fruit that contains seeds.

The statement is true as a description of an apple.

Exchange "An apple" and "a fruit that contains seeds." The converse reads:

A fruit that contains seeds is an apple.

There are many fruits that contain seeds but are not apples, such as lemons and peaches. These are counterexamples, so the converse of the statement is false.

The original statement is not a good definition because the statement is not reversible.

Quick Check

1. Consider the true conditional statement. Write its converse. If the converse is also true, combine the statements as a biconditional.

Conditional: If three points are collinear, then they lie on the same line.

Converse:

If three points lie on the same line, then they are collinear.

The converse is true.

Biconditional:

Three points are collinear if and only if they lie on the same line.

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2. Write two statements that form this biconditional about integers greater than 1:

A number is prime if and only if it has only two distinct factors, 1 and itself.

Statement:

If a number is prime, then it has only two distinct factors, 1 and itself.

Statement:

If a number has only two distinct factors, 1 and itself, then it is prime.

3. Show that this definition of *right angle* is reversible. Then write it as a true biconditional.

Definition: A right angle is an angle whose measure is 90°.

Conditional:

If an angle is a right angle, then its measure is 90°.

Converse:

If an angle has measure 90°, then it is a right angle.

The two statements are true.

Biconditional:

An angle is a right angle if and only if its measure is 90°.

4. Is the following statement a good definition? Explain.

A square is a figure with four right angles.

It is not a good definition because a rectangle has four right angles and is not necessarily a square.

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Lesson 2-3 Deductive Reasoning

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use the Law of Detachment	Topic: Mathematical Reasoning
2 Use the Law of Syllogism	Local Standards: ____________

Vocabulary and Key Concepts

Law of Detachment

If a conditional is true and its hypothesis is true, then its conclusion is true.

In symbolic form:

If $p \to q$ is a true statement and p is true, then q is true.

Law of Syllogism

If $p \to q$ and $q \to r$ are true statements, then $p \to r$ is a true statement.

Deductive reasoning is a process of reasoning logically from given facts to a conclusion.

Examples

❶ **Using the Law of Detachment** A gardener knows that if it rains, the garden will be watered. It is raining. What conclusion can he make?

The first sentence contains a conditional statement. The hypothesis is it rains.

Because the hypothesis is true, the gardener can conclude that the garden will be watered.

❷ **Using the Law of Detachment** For the given statements, what can you conclude?

Given: If $\angle A$ is acute, then $m\angle A < 90°$. $\angle A$ is acute.

A conditional and its hypothesis are both given as true.

By the Law of Detachment, you can conclude that the conclusion of the conditional, $m\angle A < 90°$, is true.

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❸ **Using the Law of Syllogism** Use the Law of Syllogism to draw a conclusion from the following true statements:

If a quadrilateral is a square, then it contains four right angles.
If a quadrilateral contains four right angles, then it is a rectangle.

The conclusion of the first conditional is the hypothesis of the second conditional. This means that you can apply the Law of Syllogism.

The Law of Syllogism: If $p \to q$ and $q \to r$ are true statements, then $p \to r$ is a true statement.

So you can conclude:

If a quadrilateral is a square, then it is a rectangle.

❹ **Drawing Conclusions** Use the Laws of Detachment and Syllogism to draw a possible conclusion.

If the circus is in town, then there are tents at the fairground. If there are tents at the fairground, then Paul is working as a night watchman. The circus is in town.

Because the conclusion of the first statement is the hypothesis of the second statement, you can apply the Law of Syllogism to write a new conditional:

If the circus is in town, then Paul is working as a night watchman.

The third statement means that the hypothesis of the new conditional is true. You can use the Law of Detachment to form the conclusion:

Paul is working as a night watchman.

Quick Check

1. Suppose that a mechanic begins work on a car and finds that the car will not start. Can the mechanic conclude that the car has a dead battery? Explain.

No, there could be other things wrong with the car, such as a faulty starter.

2. If a baseball player is a pitcher, then that player should not pitch a complete game two days in a row. Vladimir Nuñez is a pitcher. On Monday, he pitches a complete game. What can you conclude?

Answers may vary. Sample: Vladimir Nuñez should not pitch a complete game on Tuesday.

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3. If possible, state a conclusion using the Law of Syllogism. If it is not possible to use this law, explain why.
   a. If a number ends in 0, then it is divisible by 10.
   If a number is divisible by 10, then it is divisible by 5.
   If a number ends in 0, then it is divisible by 5.
   b. If a number ends in 6, then it is divisible by 2.
   If a number ends in 4, then it is divisible by 2.
   Not possible; the conclusion of one statement is not the hypothesis of the other statement.

4. Use the Law of Detachment and the Law of Syllogism to draw conclusions.
   The Volga River is in Europe.
   If a river is less than 2300 miles long, it is not one of the world's ten longest rivers.
   If a river is in Europe, then it is less than 2300 miles long.
   Conclusion:
   The Volga River is less than 2300 miles long.
   Conclusion:
   The Volga River is not one of the world's ten longest rivers.

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Lesson 2-4 Reasoning in Algebra

Lesson Objective	NAEP 2005 Strand: Algebra and Geometry
Connect reasoning in algebra and geometry	Topics: Algebraic Representations; Mathematical Reasoning Local Standards: ____________

Key Concepts

Properties of Equality

- **Addition Property** If $a = b$, then $a + c = b + c$.
- **Subtraction Property** If $a = b$, then $a - c = b - c$.
- **Multiplication Property** If $a = b$, then $a \times c = b \times c$.
- **Division Property** If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
- **Reflexive Property** $a = a$.
- **Symmetric Property** If $a = b$, then $b = a$.
- **Transitive Property** If $a = b$ and $b = c$, then $a = c$.
- **Substitution Property** If $a = b$, then b can replace a in any expression.
- **Distributive Property** $a(b + c) = ab + ac$.

Properties of Congruence

- **Reflexive Property** $\overline{AB} \cong \overline{AB}$; $\angle A \cong \angle A$
- **Symmetric Property** If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$. If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
- **Transitive Property** If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$. If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

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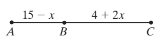
Examples

1. **Justifying Steps in Solving an Equation** Justify each step used to solve $5x - 12 = 32 + x$ for x.

Given: $5x - 12 = 32 + x$

$5x = 44 + x$	**Addition Property of Equality**
$4x = 44$	**Subtraction Property of Equality**
$x = 11$	**Division Property of Equality**

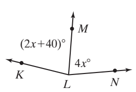
2. **Justifying Steps in Solving an Equation** Suppose that points A, B, and C are collinear with point B between points A and C. Solve for x if $AC = 21$, $AB = 15 - x$, and $BC = 4 + 2x$. Justify each step.



$AB + BC = AC$	**Segment Addition Postulate**
$(15 - x) + (4 + 2x) = 21$	**Substitution Property of Equality**
$19 + x = 21$	Simplify.
$x = 2$	**Subtraction Property of Equality**

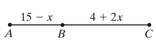
Quick Check

1. Fill in each missing reason.
   Given: $\overrightarrow{LM}$ bisects $\angle KLN$.



$\overrightarrow{LM}$ bisects $\angle KLN$	**Given**
$m\angle MLN = m\angle KLM$	Definition of **Angle** Bisector
$4x = 2x + 40$	**Substitution Property of Equality**
$2x = 40$	**Subtraction Property of Equality**
$x = 20$	**Division Property of Equality**

2. Recall that the length of line AC is 21.



   Find the lengths of $\overline{AB}$ and $\overline{BC}$ by substituting $x = 2$ into the expressions in the diagram. Check that $AB + BC = 21$.

   $AB =$ **13** $BC =$ **8** $AB + BC =$ **13** + **8** = **21**

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Example

3. **Using Properties of Equality and Congruence** Name the property that justifies each statement.
   a. If $x = y$ and $y + 4 = 3x$, then $x + 4 = 3x$.
   The conclusion of the conditional statement is the same as the equation $y + 4 = 3x$ (given) after x has been substituted for **y** (given). The property used is the **Substitution Property of Equality.**
   b. If $x + 4 = 3x$, then $4 = 2x$.
   The conclusion of the conditional statement shows the result after x is **subtracted** from each side of the equation in the hypothesis.
   The property used is the **Subtraction Property of Equality.**
   c. If $\angle P \cong \angle Q$, $\angle Q \cong \angle R$, and $\angle R \cong \angle S$, then $\angle P \cong \angle S$.
   Use the **Transitive Property of Congruence** for the first two parts of the hypothesis:
   If $\angle P \cong \angle Q$ and $\angle Q \cong \angle R$, then **$\angle P \cong \angle R$**.
   Use the **Transitive Property of Congruence** for $\angle P \cong \angle R$ and the third part of the hypothesis:
   If $\angle P \cong \angle R$ and $\angle R \cong \angle S$, then **$\angle P \cong \angle S$**.
   The property used is the **Transitive Property of Congruence.**

Quick Check

3. Name the property of equality or congruence illustrated.
   a. $\overline{XY} \cong \overline{XY}$
   Reflexive Property of Congruence
   b. If $m\angle A = 45$ and $45 = m\angle B$, then $m\angle A = m\angle B$
   Transitive or Substitution Property of Equality

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Lesson 2-5

Proving Angles Congruent

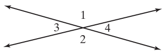
Lesson Objectives	NAEP 2005 Strand: Geometry
1 Prove and apply theorems about angles	Topic: Relationships Among Geometric Figures
	Local Standards: ____________________

Vocabulary and Key Concepts

Theorem 2-1: Vertical Angles Theorem

Vertical angles are **congruent**.

∠1 ≅ **∠2** and ∠3 ≅ **∠4**



Theorem 2-2: Congruent Supplements Theorem

If two angles are supplements of the same angle (or of congruent angles), then the two angles are **congruent**.

Theorem 2-3: Congruent Complements Theorem

If two angles are complements of the same angle (or of congruent angles), then the two angles are **congruent**.

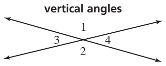
Theorem 2-4

All **right** angles are congruent.

Theorem 2-5

If two angles are congruent and supplementary, then each is a **right** angle.

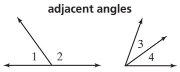
vertical angles



∠1 and **∠2** are vertical angles, as are ∠3 and **∠4**.

Vertical angles **are two angles whose sides form two pairs of opposite rays.**

adjacent angles



∠1 and **∠2** are adjacent angles, as are ∠3 and **∠4**.

Adjacent angles **are two coplanar angles that have a common side and a common vertex but no common interior points.**

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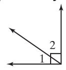
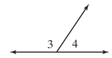
complementary angles



∠1 and **∠2** are complementary angles.

Two angles are complementary angles if **the sum of their measures is 90°.**

supplementary angles



∠3 and **∠4** are supplementary angles.

Two angles are supplementary angles if **the sum of their measures is 180°.**

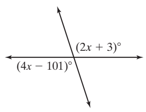
A theorem **is a conjecture that is proven.**

A paragraph proof **is a convincing argument that uses deductive reasoning in which statements and reasons are connected in sentences.**

Examples

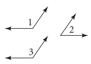
1. **Using the Vertical Angles Theorem** Find the value of x.

The angles with labeled measures are vertical angles. Apply the Vertical Angles Theorem to find x.



$4x - 101 = 2x + 3$	**Vertical Angles Theorem**
$4x = 2x + 104$	**Addition Property of Equality**
$2x = 104$	**Subtraction Property of Equality**
$x = 52$	**Division Property of Equality**

2. **Proving Theorem 2-2** Write a paragraph proof of Theorem 2-2 using the diagram at the right.



Start with the given: ∠1 and ∠2 are supplementary, ∠3 and ∠2 are supplementary. By the definition of **supplementary angles**, $m\angle 1 + m\angle 2 = 180$ and $m\angle 3 + m\angle 2 = 180$. By substitution, $m\angle 1 + m\angle 2 =$ **$m\angle 2$** + **$m\angle 3$**. Using the **Subtraction Property of Equality**, subtract **$m\angle 2$** from each side. You get $m\angle 1 =$ **$m\angle 3$**, or **∠1** ≅ **∠3**.

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Quick Check

1. Refer to the diagram for Example 1.
   a. Find the measures of the labeled pair of vertical angles.
   107°
   b. Find the measures of the other pair of vertical angles.
   73°
   c. Check to see that adjacent angles are supplementary.
   107° + 73° = 180°
2. Recall the proof of Theorem 2-2. Does the size of the angles in the diagram affect the proof? Would the proof change if ∠1 and ∠3 were acute rather than obtuse? Explain.
   No, the size of the angles does not affect the proof or the truth of the theorem.

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Lesson 3-1

Properties of Parallel Lines

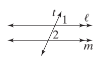
Lesson Objectives	NAEP 2005 Strand: Measurement
1 Identify angles formed by two lines and a transversal	Topic: Measuring Physical Attributes
2 Prove and use properties of parallel lines	Local Standards: ____________________

Vocabulary and Key Concepts

Postulate 3-1: Corresponding Angles Postulate

If a transversal intersects two parallel lines, then corresponding angles are **congruent**.

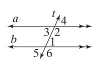
∠1 ≅ **∠2**



Theorem 3-1: Alternate Interior Angles Theorem

If a transversal intersects two parallel lines, then alternate interior angles are **congruent**.

∠1 ≅ **∠3**



Theorem 3-2: Same-Side Interior Angles Theorem

If a transversal intersects two parallel lines, then same-side interior angles are **supplementary**.

$m\angle 1 + m\angle 2 =$ **180**

Theorem 3-3: Alternate Exterior Angles Theorem

If a transversal intersects two parallel lines, then alternate exterior angles are **congruent**.

∠4 ≅ **∠5**

Theorem 3-4: Same-Side Exterior Angles Theorem

If a transversal intersects two parallel lines, then same-side exterior angles are **supplementary**.

$m\angle 4 + m\angle 6 =$ **180**

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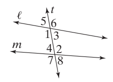
A transversal is a line that intersects two coplanar lines at two distinct points.



Alternate interior angles are nonadjacent interior angles that lie on opposite sides of the transversal.

∠1 and ∠2 are alternate interior angles.

Same-side interior angles are interior angles that lie on the same side of the transversal.

∠1 and ∠4 are same-side interior angles.

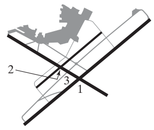
Corresponding angles are angles that lie on the same side of the transversal and in corresponding positions relative to the coplanar lines.

∠1 and ∠7 are corresponding angles.

A two-column proof is a display that shows the steps to prove a theorem. The first column shows the steps and the second column shows the reason for each step.

Examples

1 Applying Properties of Parallel Lines In the diagram of Lafayette Regional Airport, the black segments are runways and the gray areas are taxiways and terminal buildings.



Compare ∠2 and the angle vertical to ∠1. Classify the angles as alternate interior angles, same-side interior angles, or corresponding angles.

The angle vertical to ∠1 is between the runway segments.

∠2 is between the runway segments and on the opposite side of the transversal runway. Because alternate interior angles are not adjacent and lie between the lines on opposite sides of the transversal, ∠2 and the angle vertical to ∠1 are alternate interior angles.

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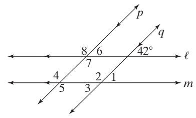
2 Finding Measures of Angles

In the diagram at right, $\ell \| m$ and $p \| q$. Find $m\angle 1$ and $m\angle 2$.



∠1 and the 42° angle are corresponding angles.

Because $\ell \| m$, $m\angle 1 =$ 42 by the Corresponding Angles Postulate.

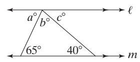
Because ∠1 and ∠2 are adjacent angles that form a straight angle, $m\angle 1 + m\angle 2 =$ 180 by the Angle Addition Postulate.

If you substitute 42 for $m\angle 1$, the equation becomes 42 + $m\angle 2$ = 180.

Subtract 42 from each side to find $m\angle 2 =$ 138.

3 Using Algebra to Find Angle Measures

In the diagram, $\ell \| m$. Find the values of a, b, and c.



$a =$ 65 Alternate Interior Angles Theorem

$c =$ 40 Alternate Interior Angles Theorem

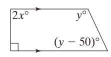
$a + b + c =$ 180 Angle Addition Postulate

65 + b + 40 = 180 Substitution Property of Equality

$b =$ 75 Subtraction Property of Equality

Quick Check

1. Use the diagram in Example 1. Classify ∠2 and ∠3 as alternate interior angles, same-side interior angles, or corresponding angles.
   same-side interior angles
2. Using the diagram in Example 2 find the measure of each angle. Justify each answer.
   a. ∠3 42; Vertical angles are congruent
   b. ∠4 138; Corresponding angles are congruent
   c. ∠5 138; Same-side interior angles are supplementary
   d. ∠6 42; Corresponding angles are congruent
   e. ∠7 138; Alternate interior angles are congruent
   f. ∠8 138; Vertical angles are congruent
3. Find the values of x and y. Then find the measures of the four angles in the trapezoid.
   $x = 45$, $y = 115$; 90, 90, 115, 65



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Lesson 3-2

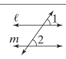
Proving Lines Parallel

Lesson Objectives	NAEP 2005 Strand: Measurement
1 Use a transversal in proving lines parallel	Topic: Measuring Physical Attributes
	Local Standards: ________

Vocabulary and Key Concepts

Postulate 3-2: Converse of the Corresponding Angles Postulate

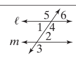
If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.



ℓ ∥ m

Theorem 3-5: Converse of the Alternate Interior Angles Theorem

If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.



If ∠1 ≅ ∠2, then ℓ ∥ m.

Theorem 3-6: Converse of the Same-Side Interior Angles Theorem

If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

If ∠2 and ∠4 are supplementary, then ℓ ∥ m.

Theorem 3-7: Converse of the Alternate Exterior Angles Theorem

If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.

If ∠3 ≅ ∠5, then ℓ ∥ m.

Theorem 3-8: Converse of the Same-Side Exterior Angles Theorem

If two lines and a transversal form same-side exterior angles that are supplementary, then the two lines are parallel.

If ∠3 and ∠6 are supplementary, then ℓ ∥ m.

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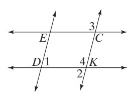
A flow proof uses arrows to show the logical connections between the statements.

Reasons are written below the statements.

Examples

1 Using Postulate 3-2

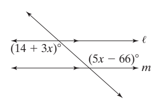
Use the diagram at the right. Which lines, if any, must be parallel if ∠3 and ∠2 are supplementary? Justify your answer with a theorem or postulate.



It is given that ∠3 and ∠2 are supplementary. The diagram shows that ∠4 and ∠2 are supplementary. Because supplements of the same angle are congruent (Congruent Supplements Theorem), ∠3 ≅ ∠4. Because ∠3 and ∠4 are congruent corresponding angles, $\overleftrightarrow{EC} \| \overleftrightarrow{DK}$ by the Converse of the Corresponding Angles Postulate.

2 Using Algebra

Find the value of x for which $\ell \| m$.



The labeled angles are alternate interior angles.

If $\ell \| m$, the alternate interior angles are congruent, and their measures are equal. Write and solve the equation $5x - 66 = 14 + 3x$.

$5x - 66 = 14 + 3x$

$5x = 80 + 3x$ Add 66 to each side.

$2x = 80$ Subtract $3x$ from each side.

$x = 40$ Divide each side by 2.

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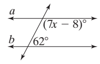
Quick Check

1. Use the diagram from Example 1. Which lines, if any, must be parallel if $\angle 3 \cong \angle 4$? Explain.

   $\overleftrightarrow{EC} \parallel \overleftrightarrow{DK}$; Converse of Corresponding Angles Postulate

2. Find the value of x for which $a \parallel b$. Explain how you can check your answer.

   $x = 18$; $7(18) - 8 = 118°$, and $62° + 118° = 180°$



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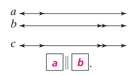
Lesson 3-3 Parallel and Perpendicular Lines

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Relate parallel and perpendicular lines	Topic: Relationships Among Geometric Figures
	Local Standards: ____________

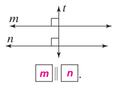
Key Concepts

Theorem 3-9
If two lines are parallel to the same line, then they are parallel to each other.



a ∥ b.

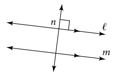
Theorem 3-10
In a plane, if two lines are perpendicular to the same line, then they are parallel to each other.



m ∥ n.

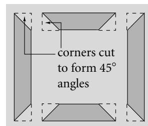
Theorem 3-11
In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other.

n ⊥ m



Examples

1 Real-World Connection
A picture frame is assembled as shown. Given the book's explanation for why the outer edges on opposite sides of the frame are parallel, why must the inner edges on opposite sides be parallel, too?



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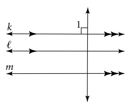
The sides are constructed so that the inner and outer edges of each side are parallel. And if an inner edge on one side is parallel to the outer edge on the same side, and at the same time the outer edge is parallel to the outer edge on the opposite side, then by Theorem 3-9 each inner edge is parallel to the outer edge on the opposite side. But that outer edge is parallel to the inner edge on the same side, so again by Theorem 3-9, inner edges on opposite sides are parallel.

2 Using Theorem 3-11
Write a paragraph proof.
Given: In a plane, $k \parallel \ell$ and $k \parallel m$. Also, $m\angle 1 = 90°$.
Prove: The transversal is perpendicular to line m.

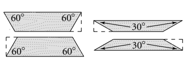


Since $m\angle 1 = 90°$, the transversal is perpendicular to line k. Since $k \parallel m$, by Theorem 3-11 the transversal is also perpendicular to line m.

Quick Check

1. Can you assemble the framing at the right into a frame with opposite sides parallel? Explain.

   Yes; $30° + 60° = 90°$



2. From what is given in Example 2, can you also conclude that the transversal is perpendicular to line l?

   Yes, by Theorem 3-11 and the fact that $k \parallel l$

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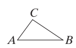
Lesson 3-4 Parallel Lines and the Triangle Angle-Sum Theorem

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Classify triangles and find the measures of their angles	Topic: Relationships Among Geometric Figures
2 Use exterior angles of triangles	Local Standards: ____________

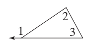
Vocabulary and Key Concepts

Theorem 3-12: Triangle Angle-Sum Theorem
The sum of the measures of the angles of a triangle is 180.

$m\angle A + m\angle B + m\angle C = 180$



Theorem 3-13: Triangle Exterior Angle Theorem
The measure of each exterior angle of a triangle equals the sum of the measures of its two remote interior angles.

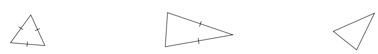


An acute triangle has three acute angles.

A right triangle has one right angle.

An obtuse triangle has one obtuse angle.

An equiangular triangle has three congruent angles.



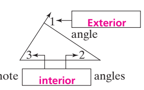
An equilateral triangle has three congruent sides.

An isosceles triangle has at least two congruent sides.

A scalene triangle has no congruent sides.

An exterior angle of a polygon is an angle formed by a side and an extension of an adjacent side.

Remote interior angles are the two nonadjacent interior angles corresponding to each exterior angle of a triangle.



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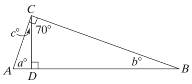
Examples

1 Applying the Triangle Angle-Sum Theorem In triangle ABC, $\angle ACB$ is a right angle, and $\overline{CD} \perp \overline{AB}$. Find the values of a and c.



Find c first, using the fact that $\angle ACB$ is a right angle.

$m\angle ACB =$ 90 — Definition of a right angle

$c +$ 70 $= 90$ — Angle Addition Postulate

$c =$ 20 — Subtract 70 from each side.

To find a, use $\triangle ADC$.

$a + m\angle ADC + c =$ 180 — Triangle Angle-Sum Theorem

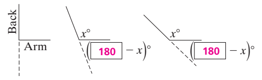
$m\angle ADC =$ 90 — Definition of perpendicular lines

$a +$ 90 $+$ 20 $=$ 180 — Substitute 90 for $m\angle ADC$ and 20 for c.

$a +$ 110 $=$ 180 — Simplify.

$a =$ 70 — Subtract 110 from each side.

2 Applying the Triangle Exterior Angle Theorem Explain what happens to the angle formed by the back of the chair and the armrest as you make a lounge chair recline more.



The exterior angle and the angle formed by the back of the chair and the armrest are adjacent angles, which together form a straight angle. As one measure increases, the other measure decreases. The angle formed by the back of the chair and the armrest increases as you make a lounge chair recline more.

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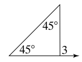
Quick Check

1. **Critical Thinking** Describe how you could use either $\triangle CAB$ or $\triangle CDB$ to find the value of b in Example 1.

   For $\triangle CAB$, $(70 + 20) + 70 + b = 180$. Then $160 + b = 180$ and $b = 20$.
   For $\triangle CDB$, $70 + 90 + b = 180$. Then $160 + b = 180$ and $b = 20$.

2. a. Find $m\angle 3$.

   90



   b. **Critical Thinking** Is it true that if two acute angles of a triangle are complementary, then the triangle must be a right triangle? Explain.

   True, because the measures of the two complementary angles add to 90, leaving 90 for the third angle.

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Lesson 3-5

The Polygon Angle-Sum Theorems

Lesson Objectives	
1 Classify polygons 2 Find the sums of the measures of the interior and exterior angles of polygons	**NAEP 2005 Strand:** Geometry **Topic:** Relationships Among Geometric Figures **Local Standards:** ____________________

Vocabulary and Key Concepts

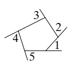
Theorem 3-14: Polygon Angle-Sum Theorem

The sum of the measures of the angles of an n-gon is $(n - 2)180$.

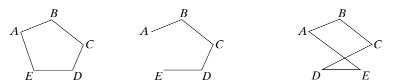
Theorem 3-15: Polygon Exterior Angle-Sum Theorem

The sum of the measures of the exterior angles of a polygon, one at each vertex, is 360.

For the pentagon, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 =$ 360.



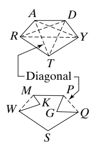
A polygon is a closed plane figure with at least three sides that are segments. The sides intersect only at their endpoints, and no two adjacent sides are collinear.



A polygon — Not a polygon; not a closed figure — Not a polygon; two sides intersect between endpoints.

A convex polygon does not have diagonal points outside of the polygon.

A concave polygon has at least one diagonal with points outside of the polygon.



← A convex polygon

← A concave polygon

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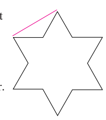
An equilateral polygon has all sides congruent.

An equiangular polygon has all angles congruent.

A regular polygon is both equilateral and equiangular.

Examples

1 Classifying Polygons Classify the polygon at the right by its sides. Identify it as convex or concave.



Starting with any side, count the number of sides clockwise around the figure. Because the polygon has 12 sides, it is a dodecagon. Think of the polygon as a star. Draw a diagonal connecting two adjacent points of the star. That diagonal lies outside the polygon, so the dodecagon is concave.

2 Finding a Polygon Angle Sum Find the sum of the measures of the angles of a decagon.

A decagon has 10 sides, so $n =$ 10.

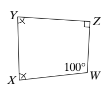
Sum $= (n - 2)(180)$ — Polygon Angle-Sum Theorem

$= ($10$ - 2)(180)$ — Substitute 10 for n.

$=$ 8 $\cdot 180$ — Subtract.

$=$ 1440 — Simplify.

3 Using the Polygon Angle-Sum Theorem Find $m\angle X$ in quadrilateral $XYZW$.



The figure has 4 sides, so $n =$ 4.

$m\angle X + m\angle Y + m\angle Z + m\angle W =$ $(4 - 2)(180)$ — Polygon Angle-Sum Theorem

$m\angle X + m\angle Y +$ 90 $+$ 100 $=$ 360 — Substitute.

$m\angle X + m\angle Y +$ 190 $=$ 360 — Simplify.

$m\angle X + m\angle Y =$ 170 — Subtract 190 from each side.

$m\angle X +$ $m\angle X$ $=$ 170 — Substitute $m\angle X$ for $m\angle Y$.

$2m\angle X$ $=$ 170 — Simplify.

$m\angle X =$ 85 — Divide each side by 2.

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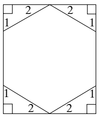
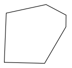
4 Applying Theorem 3-15 A regular hexagon is inscribed in a rectangle. Explain how you know that all the angles labeled ∠1 have equal measures.



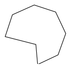
The hexagon is regular, so all its angles are **congruent**. An exterior angle is the **supplement** of a polygon's angle because they are adjacent angles that form a straight angle. Because **supplements** of congruent angles are **congruent**, all the angles marked ∠1 have equal measures.

Quick Check

1. Classify each polygon by its sides. Identify each as convex or concave.

a. 

hexagon; convex

b. 

octagon; concave

2. a. Find the sum of the measures of the angles of a 13-gon.

1980

b. **Critical Thinking** The sum of the measures of the angles of a given polygon is 720. How can you use Sum = $(n-2)180$ to find the number of sides in the polygon?

You can solve the equation $(n-2)180 = 720$.

3. Pentagon $ABCDE$ has 5 congruent angles. Find the measure of each angle.

108

4. a. In the figure from Example 4, find $m\angle 1$ by using the Polygon Exterior Angle-Sum Theorem.

60

b. Find $m\angle 2$. Is ∠2 an exterior angle? Explain.

30; no, it is not formed by extending one side of the polygon.

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Lesson 3-6

Lines in the Coordinate Plane

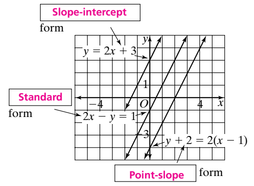
Lesson Objectives	NAEP 2005 Strand: Algebra
Graph lines given their equations Write equations of lines	**Topics:** Patterns, Relations, and Functions; Algebraic Representations **Local Standards:** ____________

Vocabulary

The slope-intercept form of a linear equation is **$y = mx + b$.**

The standard form of a linear equation is **$Ax + By = C$.**

The point-slope form for a nonvertical line is **$y - y_1 = m(x - x_1)$.**



Examples

1 Graphing Lines Using Intercepts Use the x-intercept and y-intercept to graph $5x - 6y = 30$.

To find the x-intercept, substitute 0 for y and solve for x.

$5x - 6y = 30$

$5x - 6(\boxed{0}) = 30$

$5x - \boxed{0} = 30$

$5x = \boxed{30}$

$x = \boxed{6}$

The x-intercept is $\boxed{6}$.

A point on the line is $(\boxed{6}, 0)$.

To find the y-intercept, substitute 0 for x and solve for y.

$5x - 6y = 30$

$5(\boxed{0}) - 6y = 30$

$\boxed{0} - 6y = 30$

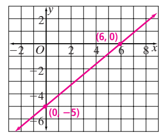
$-6y = \boxed{30}$

$y = \boxed{-5}$

The y-intercept is $\boxed{-5}$.

A point on the line is $(0, \boxed{-5})$.

Plot (6, 0) and (0, −5). Draw the line containing the two points.



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2 Transforming to Slope-Intercept Form Transform the equation $-6x + 3y = 12$ to slope-intercept form. Then graph the resulting equation.

Step 1 Transform the equation to slope-intercept form.

$-6x + 3y = 12$

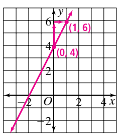
$3y = \boxed{6x} + 12$ — **Add $\boxed{6x}$ to each side.**

$\frac{3y}{\boxed{3}} = \frac{\boxed{6x}}{\boxed{3}} + \frac{12}{\boxed{3}}$ — **Divide each side by $\boxed{3}$.**

$y = \boxed{2x} + \boxed{4}$ — **Simplify.**

The y-intercept is $\boxed{4}$ and the slope is $\boxed{2}$.

Step 2 Use the y-intercept and the slope to plot two points and draw the line containing them.



3 Using Point-Slope Form Write an equation in point-slope form of the line with slope −8 that contains $P(3, -6)$.

$y - \boxed{y_1} = \boxed{m}(x - \boxed{x_1})$ — **Use point-slope form.**

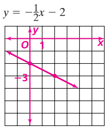
$y - (\boxed{-6}) = \boxed{-8}(x - \boxed{3})$ — **Substitute $\boxed{-8}$ for m and $\boxed{(3, -6)}$ for (x_1, y_1).**

$y + \boxed{6} = \boxed{-8}(x - \boxed{3})$ — **Simplify.**

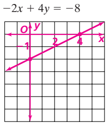
Quick Check

1. Graph each equation.

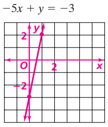
a. $y = -\frac{1}{2}x - 2$



b. $-2x + 4y = -8$



c. $-5x + y = -3$



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Example

4 Writing an Equation of a Line Given Two Points Write an equation in point-slope form of the line that contains the points $G(4, -9)$ and $H(-1, 1)$.

Step 1 Find the slope.

$m = \frac{\boxed{y_2} - \boxed{y_1}}{\boxed{x_2} - \boxed{x_1}}$ — **Use the formula for slope.**

$m = \frac{\boxed{1} - (\boxed{-9})}{\boxed{-1} - \boxed{4}}$ — **Substitute $\boxed{(4, -9)}$ for (x_1, y_1) and $\boxed{(-1, 1)}$ for (x_2, y_2).**

$= \frac{\boxed{10}}{\boxed{-5}}$ — **Simplify.**

$= \boxed{-2}$

Step 2 Select one of the points. Write the equation in point-slope form.

$y - \boxed{y_1} = \boxed{m}(x - \boxed{x_1})$ — **Point-slope form.**

$y - (\boxed{-9}) = \boxed{-2}(x - \boxed{4})$ — **Substitute $\boxed{-2}$ for m and $\boxed{(4, -9)}$ for (x_1, y_1).**

$y + \boxed{9} = \boxed{-2}(x - \boxed{4})$ — **Simplify.**

Quick Check

2. Write an equation of the line with slope −1 that contains the point $P(2, -4)$.

$y + 4 = -1(x - 2)$

3. Write an equation of the line that contains the points $P(5, 0)$ and $Q(7, -3)$.

$y - 0 = -\frac{3}{2}(x - 5)$ or $y + 3 = -\frac{3}{2}(x - 7)$

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Lesson 3-7

Slopes of Parallel and Perpendicular Lines

Lesson Objectives	NAEP 2005 Strand: Measurement
1 Relate slope and parallel lines	Topic: Measuring Physical Attributes
2 Relate slope and perpendicular lines	Local Standards: ____________

Key Concepts

Slopes of Parallel Lines

If two nonvertical lines are parallel, their **slopes** are **equal.**

If the **slopes** of two distinct nonvertical lines are equal, the lines are **parallel.**

Any two vertical lines are **parallel.**

Slopes of Perpendicular Lines

If two nonvertical lines are perpendicular, the product of their **slopes** is **−1.**

If the **slopes** of two lines have a product of −1, the lines are **perpendicular.**

Any horizontal line and vertical line are **perpendicular.**

Examples

1 **Determining Whether Lines are Parallel** Are the lines $y = -5x + 4$ and $x = -5y + 4$ parallel? Explain.

The equation $y = -5x + 4$ is in **slope-intercept** form. Write the equation $x = -5y + 4$ in slope-intercept form.

$x = -5y + 4$

$x - \boxed{4} = -5y$ Subtract $\boxed{4}$ from each side.

$\boxed{-}\frac{\boxed{1}}{\boxed{5}}x + \frac{\boxed{4}}{\boxed{5}} = y$ Divide each side by $\boxed{-5}$.

$y = \boxed{-}\frac{\boxed{1}}{\boxed{5}}x + \frac{\boxed{4}}{\boxed{5}}$

The line $x = -5y + 4$ has slope $\boxed{-\frac{1}{5}}$.

The line $y = -5x + 4$ has slope $\boxed{-5}$.

The lines **are not** parallel because their slopes are **not equal**.

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2 **Writing Equations of Parallel Lines** Write an equation in point-slope form for the line parallel to $6x - 3y = 9$ that contains $(-5, -8)$.

Step 1 To find the slope of the line, rewrite the equation in slope-intercept form.

$6x - 3y = 9$

$\boxed{-3}y = \boxed{-6x} + 9$ Subtract $\boxed{6x}$ from each side.

$y = \boxed{2x} - \boxed{3}$ Divide each side by $\boxed{-3}$.

The line $6x - 3y = 9$ has slope $\boxed{2}$.

Step 2 Use point-slope form to write an equation for the new line.

$y - \boxed{y_1} = \boxed{m}(x - \boxed{x_1})$

$y - (\boxed{-8}) = \boxed{2}(x - (\boxed{-5}))$ Substitute $\boxed{2}$ for m and $\boxed{(-5, -8)}$ for (x_1, y_1).

$y + \boxed{8} = \boxed{2}(x + \boxed{5})$ Simplify.

Quick Check

1. Are the lines parallel? Explain.

a. $y = -\frac{1}{2}x + 5$ and $2x + 4y = 9$

Yes; Each line has slope $-\frac{1}{2}$ and the *y*-intercepts are different.

b. $y = -\frac{1}{2}x + 5$ and $2x + 4y = 20$

No; the lines have the same slope and *y*-intercept, so they are the same line.

2. Write an equation for the line parallel to $y = -x + 4$ that contains $(-2, 5)$.

$y - 5 = -(x + 2)$

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Example

3 **Writing Equations for Perpendicular Lines** Write an equation for a line perpendicular to $5x + 2y = 1$ that contains $(10, 0)$.

Step 1 To find the slope of the given line, rewrite the equation in slope-intercept form.

$5x + 2y = 1$

$2y = \boxed{-5}x + 1$ Subtract $\boxed{5x}$ from each side.

$y = \boxed{-}\frac{\boxed{5}}{\boxed{2}}x + \frac{\boxed{1}}{\boxed{2}}$ Divide each side by $\boxed{2}$.

The line $5x + 2y = 1$ has slope $\boxed{-\frac{5}{2}}$.

Step 2 Find the slope of a line perpendicular to $5x + 2y = 1$. Let m be the slope of the perpendicular line.

$\boxed{-}\frac{\boxed{5}}{\boxed{2}}m = \boxed{-1}$ The product of the slopes of perpendicular lines is $\boxed{-1}$.

$m = \boxed{-1} \cdot \left(\boxed{-}\frac{\boxed{2}}{\boxed{5}}\right)$ Multiply each side by $\boxed{-}\frac{\boxed{2}}{\boxed{5}}$.

$m = \frac{\boxed{2}}{\boxed{5}}$ Simplify.

Step 3 Use point-slope form, $y - \boxed{y_1} = m(x - \boxed{x_1})$, to write an equation for the new line.

$y - \boxed{0} = \frac{\boxed{2}}{\boxed{5}}(x - \boxed{10})$ Substitute $\frac{\boxed{2}}{\boxed{5}}$ for m and $\boxed{(10, 0)}$ for (x_1, y_1).

$y = \frac{\boxed{2}}{\boxed{5}}(x - \boxed{10})$ Simplify.

Quick Check

3. Write an equation for the line perpendicular to $5y - x = 10$ that contains $(15, -4)$.

$y + 4 = -5(x - 15)$

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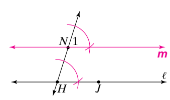
Lesson 3-8

Constructing Parallel and Perpendicular Lines

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Construct parallel lines	Topic: Relationships Among Geometric Figures
2 Construct perpendicular lines	Local Standards: ____________

Example

1 **Constructing $\ell \parallel m$** Examine the diagram at right. Explain how to construct ∠1 congruent to ∠*H*. Construct the angle.



Use the method learned for constructing congruent angles.

Step 1 With the compass point on point *H*, draw an arc that intersects the sides of $\boxed{\angle H}$.

Step 2 With the same compass setting, put the compass point on point $\boxed{N}$. Draw an arc.

Step 3 Put the compass point below point *N* where the arc intersects $\boxed{\overleftrightarrow{HN}}$. Open the compass to the length where the arc intersects line $\boxed{\ell}$. Keeping the same compass setting, put the compass point above point *N* where the arc intersects $\boxed{\overleftrightarrow{HN}}$. Draw an arc to locate a point.

Step 4 Use a straightedge to draw line *m* through the point you located and point *N*.

Quick Check

1. Use Example 1. Explain why lines ℓ and *m* must be parallel.

If corresponding angles are congruent, the lines are parallel by the Converse of Corresponding Angles Postulate.

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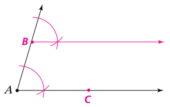
Example

2 Constructing a Special Quadrilateral Construct a quadrilateral with both pairs of sides parallel.

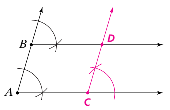
Step 1 Draw point A and two rays with endpoints at A. Label point B on one ray and point C on the other ray.

Step 2 Construct a ray parallel to $\overrightarrow{AC}$ through point B.



Step 3 Construct a ray parallel to $\overrightarrow{AB}$ through point C.

Step 4 Label point D where the ray parallel to $\overrightarrow{AC}$ intersects the ray parallel to $\overrightarrow{AB}$. Quadrilateral $ABDC$ has both pairs of opposite sides parallel.



Quick Check

2. Draw two segments. Label their lengths c and d. Construct a quadrilateral with one pair of parallel sides of lengths c and $2d$.

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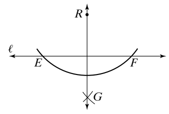
Example

3 Perpendicular From a Point to a Line Examine the construction. At what special point does $\overleftrightarrow{RG}$ meet line ℓ?



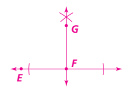
Point R is the same distance from point E as it is from point F because the arc was made with one compass opening.

Point G is the same distance from point E as it is from point F because both arcs were made with the same compass opening.

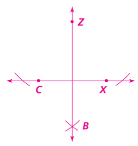
This means that $\overleftrightarrow{RG}$ intersects line ℓ at the midpoint of $\overline{EF}$, and that $\overleftrightarrow{RG}$ is the perpendicular bisector of $\overline{EF}$.

Quick Check

3. **a.** Use a straightedge to draw $\overleftrightarrow{EF}$. Construct $\overleftrightarrow{FG}$ so that $\overleftrightarrow{FG} \perp \overleftrightarrow{EF}$ at point F.



b. Draw a line $\overleftrightarrow{CX}$ and a point Z not on $\overleftrightarrow{CX}$. Construct $\overleftrightarrow{ZB}$ so that $\overleftrightarrow{ZB} \perp \overleftrightarrow{CX}$.



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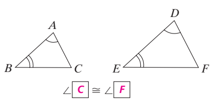
Lesson 4-1 Congruent Figures

Lesson Objective	NAEP 2005 Strand: Geometry
Recognize congruent figures and their corresponding parts	Topic: Transformation of Shapes and Preservation of Properties
	Local Standards: ____________________

Vocabulary and Key Concepts

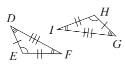
Theorem 4–1

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent.



$\angle$ C $\cong \angle$ F

Congruent polygons are polygons that have corresponding sides congruent and corresponding angles congruent.

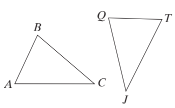


$\triangle$ DEF $\cong \triangle$ GHI

Examples

1 Naming Congruent Parts $\triangle ABC \cong \triangle QTJ$. List the congruent corresponding parts.

List the corresponding sides and angles in the same order.



Angles: $\angle A \cong \angle Q$ $\angle B \cong \angle$ T $\angle$ C $\cong \angle J$

Sides: $\overline{AB} \cong \overline{QT}$ $\overline{BC} \cong \overline{TJ}$ $\overline{AC} \cong \overline{QJ}$

2 Using Congruency $\triangle XYZ \cong \triangle KLM$, $m\angle Y = 67$, and $m\angle M = 48$. Find $m\angle X$.

Use the Triangle Angle-Sum Theorem and the definition of congruent polygons to find $m\angle X$.

$m\angle X + m\angle Y + m\angle Z = 180$	Triangle Angle-Sum Theorem
$m\angle Z = m\angle M$	Corresponding angles of congruent triangles are congruent.
$m\angle Z = 48$	Substitute 48 for $m\angle M$.
$m\angle X + 67 + 48 = 180$	Substitute.
$m\angle X + 115 = 180$	Simplify.
$m\angle X = 65$	Subtract 115 from each side.

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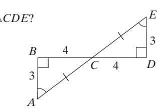
3 Finding Congruent Triangles Can you conclude that $\triangle ABC \cong \triangle CDE$? List corresponding vertices in the same order.



If $\triangle ABC \cong \triangle CDE$, then $\angle BAC \cong$ ∠DCE.

The diagram above shows $\angle BAC \cong$ ∠DEC, not $\angle DCE$.

The statement $\triangle ABC \cong \triangle CDE$ is not true.

Notice that $\overline{BC} \cong$ $\overline{DC}$, $\overline{BA} \cong$ $\overline{DE}$, and $\overline{AC} \cong$ $\overline{EC}$.

Also, $\angle CBA \cong$ ∠CDE and $\angle BAC \cong$ ∠DEC.

Using Theorem 4-1, you can conclude that $\angle ECD \cong$ ∠ACB.

Since all of the corresponding sides and angles are congruent, the triangles are congruent. The correct way to state this is △ABC $\cong$ △EDC.

Quick Check

1. $\triangle WYS \cong \triangle MKV$. List the congruent corresponding parts. Use three letters for each angle.

Sides: $\overline{WY} \cong \overline{MK}$ $\overline{WS} \cong \overline{MV}$ $\overline{YS} \cong \overline{KV}$

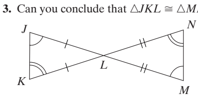
Angles: $\angle WSY \cong \angle MVK$ $\angle SWY \cong \angle VMK$ $\angle WYS \cong \angle MKV$

2. It is given that $\triangle WYS \cong \triangle MKV$. If $m\angle Y = 35$, what is $m\angle K$? Explain.

$m\angle K =$ 35

Corresponding angles of congruent triangles are congruent and have the same measure.

3. Can you conclude that $\triangle JKL \cong \triangle MNL$? Justify your answer.



No. The corresponding sides are not necessarily equal.

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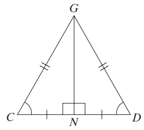
Example

4 Proving Triangles Congruent

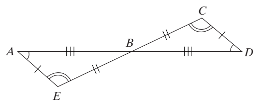
Given: $\overline{CG} \cong \overline{DG}$, $\overline{CN} \cong \overline{DN}$, $\angle C \cong \angle D$, $\overline{GN} \perp \overline{CD}$
Prove: $\triangle CNG \cong \triangle DNG$



Statements	Reasons
1. $\overline{CG} \cong \overline{DG}$	1. Given
2. $\overline{CN} \cong \overline{DN}$	2. Given
3. $\overline{GN} \cong \overline{GN}$	3. Reflexive Property of Congruence
4. $\angle C \cong \angle D$	4. Given
5. $\angle CNG \cong \angle DNG$	5. Right angles are congruent
6. $\angle CGN \cong \angle DGN$	6. If two angles of one triangle are congruent to two angles of another triangle, then the third angles are congruent (Theorem 4-1).
7. $\triangle CNG \cong \triangle DNG$	7. Definition of congruent triangles

Quick Check

4. *Given:* $\angle A \cong \angle D$, $\angle E \cong \angle C$, $\overline{AE} \cong \overline{DC}$, $\overline{EB} \cong \overline{CB}$, $\overline{BA} \cong \overline{BD}$
Prove: $\triangle AEB \cong \triangle DCB$



Statements	Reasons
1. $\angle A \cong \angle D$; $\angle E \cong \angle C$	1. Given
2. $\angle ABE \cong \angle DBC$	2. Vertical angles are congruent.
3. $\overline{AE} \cong \overline{CD}$; $\overline{AB} \cong \overline{BD}$; $\overline{EB} \cong \overline{BC}$	3. Given
4. $\triangle AEB \cong \triangle DCB$	4. Definition of Congruent Triangles

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Lesson 4-2

Triangle Congruence by SSS and SAS

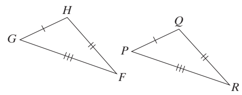
Lesson Objective	NAEP 2005 Strand: Geometry
Prove two triangles congruent using the SSS and SAS Postulates	Topic: Transformation of Shapes and Preservation of Properties Local Standards: ____________

Key Concepts

Postulate 4-1: Side-Side-Side (SSS) Postulate

If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent.

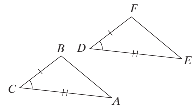
$\triangle GHF \cong \triangle PQR$



Postulate 4-2: Side-Angle-Side (SAS) Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.

$\triangle BCA \cong \triangle FDE$



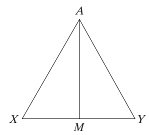
Examples

1 Proving Triangles Congruent

Given: M is the midpoint of $\overline{XY}$, $\overline{AX} \cong \overline{AY}$
Prove: $\triangle AMX \cong \triangle AMY$

Write a paragraph proof.

You are given that M is the midpoint of $\overline{XY}$, and $\overline{AX} \cong \overline{AY}$. Midpoint M implies that $\overline{MX} \cong \overline{MY}$. $\overline{AM} \cong \overline{AM}$ by the Reflexive Property of Congruence, so $\triangle AMX \cong \triangle AMY$ by the SSS Postulate.



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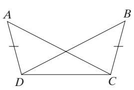
2 Using SAS $\overline{AD} \cong \overline{BC}$. What other information do you need to prove $\triangle ADC \cong \triangle BCD$ by SAS?

It is given that $\overline{AD} \cong \overline{BC}$. Also, $\overline{DC} \cong \overline{CD}$ by the Reflexive Property of Congruence.

You now have two pairs of corresponding congruent sides. Therefore, if you know $\angle ADC \cong \angle BCD$, you can prove $\triangle ADC \cong \triangle BCD$ by SAS.

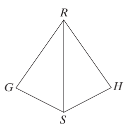


3 Are the Triangles Congruent? From the information given, can you prove $\triangle RSG \cong \triangle RSH$? Explain.

Given: $\angle RSG \cong \angle RSH$, $\overline{SG} \cong \overline{SH}$

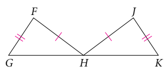
It is given that $\angle RSG \cong \angle RSH$ and $\overline{SG} \cong \overline{SH}$. $\overline{RS} \cong \overline{RS}$ by the Reflexive Property of Congruence.

Two pairs of corresponding sides and their included angles are congruent, so $\triangle RSG \cong \triangle RSH$ by the SAS Postulate.



Quick Check

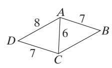
1. *Given:* $\overline{HF} \cong \overline{HJ}$, $\overline{FG} \cong \overline{JK}$, H is the midpoint of $\overline{GK}$.
Prove: $\triangle FGH \cong \triangle JKH$



Statements	Reasons
1. $\overline{HF} \cong \overline{HJ}$, $\overline{FG} \cong \overline{JK}$	1. Given
2. H is the midpoint of $\overline{GK}$.	2. Given
3. $\overline{GH} \cong \overline{HK}$	3. Definition of Midpoint
4. $\triangle FGH \cong \triangle JKH$	4. SSS Postulate

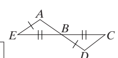
2. What other information do you need to prove $\triangle ABC \cong \triangle CDA$ by SAS?

$\angle DCA \cong \angle BAC$



3. From the information given, can you prove $\triangle AEB \cong \triangle DBC$? Explain.
Given: $\overline{EB} \cong \overline{CB}$, $\overline{AE} \cong \overline{DB}$

No; you don't know that $\angle AEB \cong \angle DBC$ or that $\overline{AB} \cong \overline{DC}$.



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Lesson 4-3

Triangle Congruence by ASA and AAS

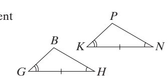
Lesson Objective	NAEP 2005 Strand: Geometry
Prove two triangles congruent using the ASA Postulate and the AAS Theorem	Topic: Transformations of Shapes and Preservation of Properties Local Standards: ____________

Key Concepts

Postulate 4-3: Angle-Side-Angle (ASA) Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

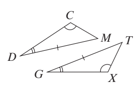
$\triangle HGB \cong \triangle NKP$



Theorem 4-2: Angle-Angle-Side (AAS) Theorem

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

$\triangle CDM \cong \triangle XGT$



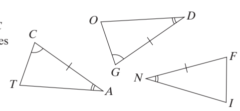
Example

1 Using ASA Suppose that $\angle F$ is congruent to $\angle C$ and $\angle I$ is *not* congruent to $\angle C$. Name the triangles that are congruent by the ASA Postulate.

The diagram shows $\angle N \cong \angle A \cong \angle D$ and $\overline{FN} \cong \overline{CA} \cong \overline{GD}$.

If $\angle F \cong \angle C$, then $\angle F \cong \angle C \cong \angle G$.

Therefore, $\triangle FNI \cong \triangle CAT \cong \triangle GDO$ by ASA.



Quick Check

1. Using only the information in the diagram, can you conclude that $\triangle INF$ is congruent to either of the other two triangles? Explain.

No; Only one angle and one side are shown to be congruent. At least one more congruent side or angle is necessary to prove congruence with SAS, ASA, or AAS.

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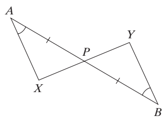
Examples

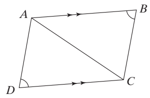
2 Writing a Proof Using ASA Write a paragraph proof.



Given: $\angle A \cong \angle B$, $\overline{AP} \cong \overline{BP}$
Prove: $\triangle APX \cong \triangle BPY$

It is given that $\angle A \cong \angle B$ and $\overline{AP} \cong \overline{BP}$. $\angle APX \cong \angle BPY$ by the **Vertical Angles** Theorem. Because two pairs of corresponding angles and their included sides are congruent, $\triangle APX \cong \triangle BPY$ by **ASA**.

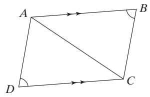
3 Planning a Proof Write a plan for a proof that uses AAS.



Given: $\angle B \cong \angle D$, $\overline{AB} \parallel \overline{CD}$
Prove: $\triangle ABC \cong \triangle CDA$

Because $\overline{AB} \parallel \overline{CD}$, $\angle BAC \cong \angle$ **DCA** by the Alternate Interior Angles Theorem. Then $\triangle ABC \cong \triangle$ **CDA** if a pair of corresponding sides are congruent. By the Reflexive Property, $\overline{AC} \cong$ **$\overline{AC}$**, so $\triangle ABC \cong \triangle CDA$ by **AAS**.

4 Writing a Proof Write a two-column proof that uses AAS.



Given: $\angle B \cong \angle D$, $\overline{AB} \parallel \overline{CD}$
Prove: $\triangle ABC \cong \triangle CDA$

Statements	Reasons
1. $\angle B \cong \angle D$, $\overline{AB} \parallel \overline{CD}$	1. **Given**
2. **$\angle BAC \cong \angle DCA$**	2. If lines are parallel, then **alternate interior** angles are congruent.
3. $\overline{AC} \cong \overline{AC}$	3. **Reflexive Property of Congruence**
4. $\triangle ABC \cong \triangle CDA$	4. **AAS Theorem**

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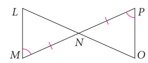
Quick Check

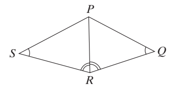
2. Write a proof.



Given: $\overline{NM} \cong \overline{NP}$, $\angle M \cong \angle P$
Prove: $\triangle NML \cong \triangle NPO$

It is given that $\overline{NM} \cong \overline{NP}$ and $\angle M \cong \angle P$. $\angle MNL \cong \angle PNO$ because vertical angles are congruent. $\triangle NML \cong \triangle NPO$ by ASA.

3. Write a flow proof.



Given: $\angle S \cong \angle Q$, $\overline{RP}$ bisects $\angle SRQ$.
Prove: $\triangle SRP \cong \triangle QRP$

- $\overline{RP}$ bisects $\angle SRQ$ — **Given**
  - → $\angle SRP \cong$ **$\angle QRP$** — **Definition of angle bisector**
- $\angle S \cong \angle Q$ — **Given**
- $\overline{RP} \cong$ **$\overline{RP}$** — **Reflexive Property of Congruence**
- → **$\triangle SRP \cong \triangle QRP$** — **AAS**

4. Recall Example 4. Explain how you could prove $\triangle ABC \cong \triangle CDA$ using ASA.

It is given that $\angle B \cong \angle D$. You know that $\angle BAC \cong \angle DCA$ by the Alternate Interior Angles Theorem. By Theorem 4-1, $\angle BCA \cong \angle DAC$. By the Reflexive Property of Congruence, $\overline{AC} \cong \overline{AC}$. Therefore, $\triangle ABC \cong \triangle CDA$ by ASA.

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Lesson 4-4

Using Congruent Triangles: CPCTC

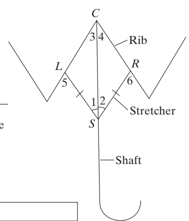
Lesson Objective	NAEP 2005 Strand: Geometry
Use triangle congruence and CPCTC to prove that parts of two triangles are congruent	Topic: Transformation of Shapes and Preservation of Properties Local Standards: ____________________

Vocabulary

CPCTC stands for:
C **orresponding** P **arts** of C **ongruent**
T **riangles** are C **ongruent**.

Examples

1 Congruence Statements The diagram shows the frame of an umbrella. What congruence statements besides $\angle 3 \cong \angle 4$ can you prove from the diagram, in which $\overline{SL} \cong \overline{SR}$ and $\angle 1 \cong \angle 2$ are given?



$\overline{SC} \cong$ **$\overline{SC}$** by the Reflexive Property of Congruence, and $\triangle LSC \cong \triangle RSC$ by **SAS**. $\angle 3 \cong \angle 4$ because **corresponding parts of congruent triangles are congruent.**

When two triangles are congruent, you can form congruence statements about three pairs of corresponding angles and three pairs of corresponding sides. List the congruence statements.

Sides:

$\overline{SL} \cong \overline{SR}$	Given
$\overline{SC} \cong$ **$\overline{SC}$**	**Reflexive Property of Congruence**
$\overline{CL} \cong$ **$\overline{CR}$**	**Other congruence statement**

Angles:

$\angle 1 \cong \angle 2$	Given
$\angle 3 \cong \angle$ **4**	**Corresponding Parts of Congruent Triangles**
$\angle CLS \cong \angle CRS$	**Other congruence statement**

The congruence statements that remain to be proved are **$\angle CLS \cong \angle CRS$ and $\overline{CL} \cong \overline{CR}$.**

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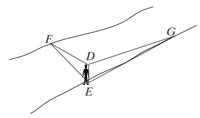
2 Using Right Triangles According to legend, one of Napoleon's followers used congruent triangles to estimate the width of a river. On the riverbank, the officer stood up straight and lowered the visor of his cap until the farthest thing he could see was the edge of the opposite bank. He then turned and noted the spot on his side of the river that was in line with his eye and the tip of his visor.



Given: $\angle DEG$ and $\angle DEF$ are right angles; $\angle EDG \cong \angle EDF$.

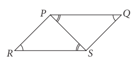
The officer then paced off the distance to this spot and declared that distance to be the width of the river!

The given states that $\angle DEG$ and $\angle DEF$ are **right angles**. What conditions must hold for that to be true?

$\angle DEG$ and $\angle DEF$ are the angles that the officer makes with the ground. So the officer must stand **perpendicular** to the ground, and the ground must be **level or flat**.

Quick Check

1. *Given:* $\angle Q \cong \angle R$, $\angle QPS \cong \angle RSP$
   Prove: $\overline{SQ} \cong \overline{PR}$



It is given that $\angle Q \cong \angle R$, $\angle QPS \cong \angle RSP$. $\overline{PS} \cong \overline{SP}$ by the Reflexive Property of $\cong$. $\triangle QPS \cong \triangle RSP$ by ASA. $\overline{SQ} \cong \overline{PR}$ by CPCTC.

2. Recall Example 2. About how wide was the river if the officer paced off 20 paces and each pace was about $2\frac{1}{2}$ feet long?

50 feet

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Lesson 4-5 — Isosceles and Equilateral Triangles

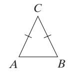
Lesson Objective	NAEP 2005 Strand: Geometry
Use and apply properties of isosceles triangles	Topic: Relationships Among Geometric Figures Local Standards: ____________

Vocabulary and Key Concepts

Theorem 4-3: Isosceles Triangle Theorem

If two sides of a triangle are congruent, then the angles opposite those sides are congruent.

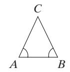
$\angle A \cong \angle B$



Theorem 4-4: Converse of Isosceles Triangle Theorem

If two angles of a triangle are congruent, then the sides opposite those sides are congruent.

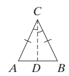
$\overline{AC} \cong \overline{BC}$



Theorem 4-5

The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

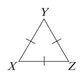
$\overline{CD} \perp \overline{AB}$ and $\overline{CD}$ bisects $\overline{AB}$.



Corollary to Theorem 4-3

If a triangle is equilateral, then the triangle is equiangular.

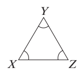
$\angle X \cong \angle Y \cong \angle Z$



Corollary to Theorem 4-4

If a triangle is equiangular, then the triangle is equilateral.

$\overline{XY} \cong \overline{YZ} \cong \overline{ZX}$



A corollary is a statement that follows immediately from a theorem.

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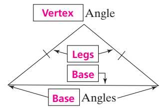
The legs of an isosceles triangle are the congruent sides.

The base of an isosceles triangle is the third, or non-congruent, side.

The vertex angle of an isosceles triangle is formed by the two congruent sides (legs).

The base angles of an isosceles triangle are formed by the base and the legs.



Example

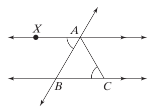
1 Using the Isosceles Triangle Theorems Explain why $\triangle ABC$ is isosceles.

$\angle ABC$ and $\angle XAB$ are alternate interior angles formed by $\overrightarrow{XA}$, $\overrightarrow{BC}$, and the transversal $\overline{AB}$. Because $\overrightarrow{XA} \parallel \overrightarrow{BC}$, $\angle ABC \cong \angle XAB$.

The diagram shows that $\angle XAB \cong \angle ACB$. By the Transitive Property of Congruence, $\angle ABC \cong \angle ACB$.

You can use the Converse of the Isosceles Triangle Theorem to conclude that $\overline{AB} \cong \overline{AC}$.

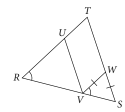
By the definition of an isosceles triangle, $\triangle ABC$ is isosceles.



Quick Check

1. Can you deduce that $\triangle RUV$ is isosceles? Explain.

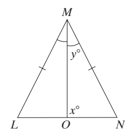
No; neither $\angle RUV$ nor $\angle RVU$ can be shown to be congruent to $\angle R$.



Examples

2 Using Algebra Suppose that $m\angle L = y$. Find the values of x and y.

$\overline{MO} \perp \overline{LN}$	The bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.
$x = 90$	Definition of perpendicular
$m\angle N = m\angle L$	Isosceles Triangle Theorem



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$m\angle L = y$	Given
$m\angle N = y$	Transitive Property of Equality
$m\angle N + m\angle NMO + m\angle MON = 180$	Triangle-Sum Theorem
$y + y + 90 = 180$	Substitute.
$2y + 90 = 180$	Simplify.
$2y = 90$	Subtract 90 from each side.
$y = 45$	Divide each side by 2.

Therefore, $x = 90$ and $y = 45$.

3 Using Isosceles Triangles The garden shown at the right is in the shape of a regular hexagon. Suppose that a segment is drawn between the endpoints of the angle marked x. Find the angle measures of the triangle that is formed. The measure of $\angle x$ is 120.

Because the garden is a regular hexagon, the sides have equal length, so the triangle is isosceles.

By the Isosceles Triangle Theorem, the unknown angles are congruent.

The measure of the angle marked x is 120, and the sum of the angle measures of a triangle is 180.

Label each unknown angle y.

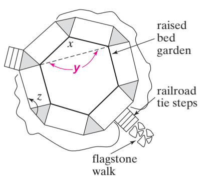
$x + y + y = 180$

$120 + 2y = 180$

$2y = 60$

$y = 30$

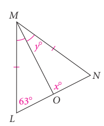
So the angle measures of the triangle are 120, 30, and 30.



Quick Check

2. Suppose $m\angle L = 43$. Find the values of x and y.

$x = 90$, $y = 47$



3. Use the diagram from Example 3. The path around the garden is made up of rectangles and equilateral triangles. What is the measure of $\angle z$, the angle at the outside corner of the path?

150

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Lesson 4-6 — Congruence in Right Triangles

Lesson Objective	NAEP 2005 Strand: Geometry
Prove triangles congruent using the HL Theorem	Topic: Transformation of Shapes and Preservation of Properties Local Standards: ____________

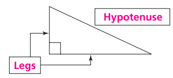
Vocabulary and Key Concepts

Theorem 4-6: Hypotenuse-Leg (HL) Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent.

The hypotenuse of a right triangle is its longest side, or the side opposite the right angle.

The legs of a right triangle are the two shortest sides, or the sides that are *not* opposite the right angle.



Examples

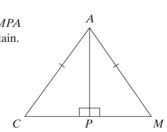
1 Proving Triangles Congruent One student wrote "$\triangle CPA \cong \triangle MPA$ by the HL Theorem" for the diagram. Is the student correct? Explain.

The diagram shows the following congruent parts.

$\overline{CA} \cong \overline{MA}$

$\angle CPA \cong \angle MPA$

$\overline{PA} \cong \overline{PA}$



Since $\overline{AC}$ is the hypotenuse and $\overline{PA}$ is a leg of right triangle CPA, and $\overline{MA}$ is the hypotenuse and $\overline{PA}$ is a leg of right triangle MPA, the triangles are congruent by the HL Theorem.

The student is correct.

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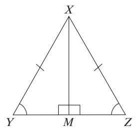
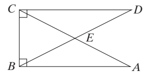
2 Flow Proof—Using the HL Theorem △XYZ is isosceles. From vertex X, a perpendicular is drawn to $\overline{YZ}$, intersecting $\overline{YZ}$ at point M. Explain why △XMY ≅ △XMZ.



- $\overline{XM} \perp \overline{YZ}$ — **Given** → ∠XMY and ∠XMZ are **right angles** — Definition of perpendicular lines → △XMY and △XMZ are **right triangles** — Definition of right triangle → △XMY ≅ △XMZ — **HL Theorem**
- $\overline{XY} \cong$ **$\overline{XZ}$** — Definition of **isosceles** → △XMY ≅ △XMZ
- $\overline{XM} \cong \overline{XM}$ — **Reflexive Property of Congruence** → △XMY ≅ △XMZ

3 Two-Column Proof—Using the HL Theorem

Given: ∠ABC and ∠DCB are right angles, $\overline{AC} \cong \overline{DB}$
Prove: △ABC ≅ △DCB



Statements	Reasons
1. ∠ABC and ∠DCB are **right angles**	1. Given
2. △ABC and △**DCB** are right triangles	2. **Definition of Right Triangle**
3. $\overline{AC} \cong$ **$\overline{DB}$**	3. **Given**
4. **$\overline{BC} \cong \overline{CB}$**	4. Reflexive Property of Congruence
5. △ABC ≅ △**DCB**	5. **If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then the triangles are congruent (HL Theorem).**

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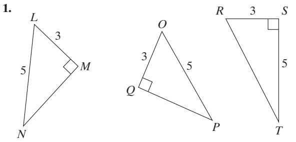
Quick Check

1.



Which two triangles are congruent by the HL Theorem? Write a correct congruence statement.

△LMN ≅ △OQP

2. Write a paragraph proof of Example 2.

It is given that $\overline{XM}$ is perpendicular to $\overline{YZ}$. This means that ∠YMX and ∠ZMX are right angles, since that is the definition of perpendicular lines. It follows, then, that △YMX and △ZMX are right triangles, since they each contain a right angle. It is given that △XYZ is isosceles, so $\overline{XY} \cong \overline{XZ}$ by the definition of isosceles. $\overline{XM} \cong \overline{XM}$ by the Reflexive Property of Congruence. Therefore, since the hypotenuses are congruent and one leg is congruent, △XMY ≅ △XMZ by the HL Theorem.

3. You know that two legs of one right triangle are congruent to two legs of another right triangle. Explain how to prove the triangles are congruent.

Since all right angles are congruent, the triangles are congruent by SAS.

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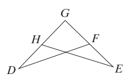
Lesson 4-7

Using Corresponding Parts of Congruent Triangles

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Identify congruent overlapping triangles	Topic: Relationships Among Geometric Figures
2 Prove two triangles congruent by first proving two other triangles congruent	Local Standards: ____________________

Examples

1 Identifying Common Parts Name the parts of the sides that △DFG and △EHG share.

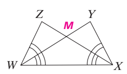


Identify the overlapping triangles.

Parts of sides $\overline{DG}$ and **$\overline{EG}$** are shared by △DFG and △EHG.

These parts are **$\overline{HG}$** and **$\overline{FG}$**, respectively.

2 Using Common Parts Write a plan for a proof that does not use overlapping triangles.



Given: ∠ZXW ≅ ∠YWX, ∠ZWX ≅ ∠YXW
Prove: $\overline{ZW} \cong \overline{YX}$

Label point M where $\overline{ZX}$ intersects $\overline{WY}$.

$\overline{ZW} \cong \overline{YX}$ by CPCTC if **△ZWM ≅ △YXM**.

You can prove these triangles congruent using ASA as follows:

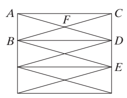
Look at △MWX. $\overline{MW} \cong \overline{MX}$ by the **Converse of the Isosceles Triangle Theorem**.

∠ZMW ≅ ∠YMX because **vertical angles** are congruent. By the Angle Addition Postulate, ∠ZWM ≅ ∠YXM. So △ZWM ≅ △YXM by **ASA**.

So $\overline{ZW} \cong \overline{YX}$ by **CPCTC**.

Quick Check

1. The diagram shows triangles from the scaffolding that workers used when they repaired and cleaned the Statue of Liberty.



a. Name the common side in △ACD and △BCD.

$\overline{CD}$

b. Name another pair of triangles that share a common side. Name the common side.

Answers may vary. Sample: △ABD and △CBD; $\overline{BD}$

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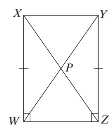
Examples

3 Using Two Pairs of Triangles Write a paragraph proof.

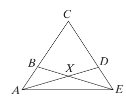


Given: $\overline{XW} \cong \overline{YZ}$, ∠XWZ and ∠YZW are right angles.
Prove: △XPW ≅ △YPZ

Plan: △XPW ≅ △YPZ by AAS if ∠WXZ ≅ **∠ZYW**. These angles are congruent by **CPCTC** if △XWZ ≅ △YZW. These triangles are congruent by **SAS**.

Proof: You are given $\overline{XW} \cong \overline{YZ}$. Because ∠XWZ and ∠YZW are **right angles**, ∠XWZ ≅ ∠YZW. $\overline{WZ} \cong \overline{ZW}$ by the **Reflexive Property of Congruence**. Therefore, △XWZ ≅ △YZW by SAS. ∠WXZ ≅ ∠**ZYW** by CPCTC, and ∠XPW ≅ ∠**YPZ** because **vertical angles** are congruent. Therefore, △XPW ≅ △YPZ by **AAS**.

4 Separating Overlapping Triangles



Given: $\overline{CA} \cong \overline{CE}$, $\overline{BA} \cong \overline{DE}$

Write a two-column proof to show that ∠CBE ≅ ∠CDA.

Plan: ∠CBE ≅ ∠CDA by **CPCTC** if △CBE ≅ △CDA. This congruence holds by **SAS** if $\overline{CB} \cong \overline{CD}$.

Proof:

Statement	Reason
1. ∠BCE ≅ ∠DCA	1. **Reflexive Property of Congruence**
2. $\overline{CA} \cong$ **$\overline{CE}$**, $\overline{BA} \cong$ **$\overline{DE}$**	2. Given
3. CA = **CE**, BA = **DE**	3. Congruent sides have equal measure.
4. CA − BA = CE − DE	4. **Subtraction Property of Equality**
5. CA − BA = CB CE − DE = CD	5. **Segment Addition Postulate**
6. CB = CD	6. Substitution
7. $\overline{CB} \cong \overline{CD}$	7. **Definition of Congruence**
8. △CBE ≅ △CDA	8. **SAS**
9. ∠CBE ≅ ∠CDA	9. **CPCTC**

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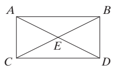
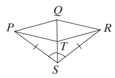
Quick Check

2. Write a paragraph proof.
Given: $\triangle ACD \cong \triangle BDC$
Prove: $\overline{CE} \cong \overline{DE}$



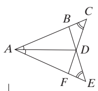
It is given that $\triangle ACD \cong \triangle BDC$, so $\angle ADC \cong \angle BCD$ by CPCTC. This means that $\triangle CED$ is isosceles by the Converse of Isosceles Triangle Theorem. Then $\overline{CE} \cong \overline{DE}$ by the definition of isosceles triangle.

3. Write a two-column proof.
Given: $\overline{PS} \cong \overline{RS}$, $\angle PSQ \cong \angle RSQ$
Prove: $\triangle QPT \cong \triangle QRT$



	Statement		Reason
1.	$\overline{PS} \cong \overline{RS}$, $\angle PSQ \cong \angle RSQ$	1.	Given
2.	$\overline{QS} \cong \overline{QS}$	2.	Reflexive Property of Congruence
3.	$\triangle PSQ \cong \triangle RSQ$	3.	SAS
4.	$\overline{PQ} \cong \overline{RQ}$	4.	CPCTC
5.	$\angle PQT \cong \angle RQT$	5.	CPCTC
6.	$\overline{QT} \cong \overline{QT}$	6.	Reflexive Property of Congruence
7.	$\triangle QPT \cong \triangle QRT$	7.	SAS

4. Write a two-column proof.
Given: $\angle CAD \cong \angle EAD$, $\angle C \cong \angle E$
Prove: $\overline{BD} \cong \overline{FD}$



	Statement		Reason
1.	$\angle CAD \cong \angle EAD$, $\angle C \cong \angle E$	1.	Given
2.	$\overline{AD} \cong \overline{AD}$	2.	Reflexive Property of Congruence
3.	$\triangle ACD \cong \triangle AED$	3.	AAS
4.	$\overline{CD} \cong \overline{ED}$	4.	CPCTC
5.	$\angle BDC \cong \angle FDE$	5.	Vertical Angles are Congruent
6.	$\triangle BDC \cong \triangle FDE$	6.	ASA
7.	$\overline{BD} \cong \overline{FD}$	7.	CPCTC

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Lesson 5-1

Midsegments of Triangles

Lesson Objective	NAEP 2005 Strand: Geometry
Use properties of midsegments to solve problems	Topics: Relationships Among Geometric Figures Local Standards: ____________

Vocabulary and Key Concepts

Theorem 5-1: Triangle Midsegment Theorem

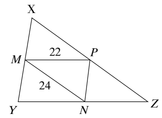
If a segment joins the midpoints of two sides of a triangle, then the segment is parallel to the third side, and is half its length.

A midsegment of a triangle is a segment connecting the midpoints of two sides.

A coordinate proof is a form of proof in which coordinate geometry and algebra are used to prove a theorem.

Examples

1. **Finding Lengths** In $\triangle XYZ$, M, N, and P are midpoints. The perimeter of $\triangle MNP$ is 60. Find NP and YZ.



Because the perimeter of $\triangle MNP$ is 60, you can find NP.

$NP + MN + MP = 60$ Definition of perimeter

$NP + 24 + 22 = 60$ Substitute 24 for MN and 22 for MP.

$NP + 46 = 60$ Simplify.

$NP = 14$ Subtract 46 from each side.

Use the Triangle Midsegment Theorem to find YZ.

$MP = \frac{1}{2} YZ$ Triangle Midsegment Theorem

$22 = \frac{1}{2} YZ$ Substitute 22 for MP.

$44 = YZ$ Multiply each side by 2.

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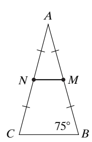
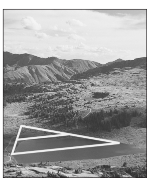
2. **Identifying Parallel Segments** Find $m\angle AMN$ and $m\angle ANM$. $\overline{MN}$ and $\overline{BC}$ are cut by transversal $\overline{AB}$, so $\angle AMN$ and $\angle B$ are corresponding angles. $\overline{MN} \parallel \overline{BC}$ by the Triangle Midsegment Theorem, so $\angle AMN \cong \angle B$ by the Corresponding Angles Postulate. $m\angle AMN = 75$ because congruent angles have the same measure. In $\triangle AMN$, $AM = AN$, so $m\angle ANM = m\angle AMN$ by the Isosceles Triangle Theorem. $m\angle ANM = 75$ by substitution.

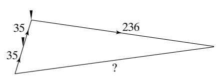


3. **Applying the Triangle Midsegment Theorem** Dean plans to swim the length of the lake, as shown in the photo. Explain how Dean could use the Triangle Midsegment Theorem to measure the length of the lake.

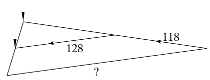


To find the length of the lake, Dean starts at the middle of the left edge of the lake and paces straight along that end of the lake. He counts the number of strides (35). Where the lake ends, he sets a stake. He paces the same number of strides (35) in the same direction and sets another stake. The first stake marks the midpoint of one side of a triangle. Dean paces from the second stake straight to the middle of the other end of the lake. He counts the number of his strides (236).

Dean finds the midsegment of the second side by pacing exactly half the number of strides back toward the second stake. He paces 118 strides. From this midpoint of the second side of the triangle, he returns to the midpoint of the first side, counting the number of strides (128). Dean has paced a triangle. He has also formed a midsegment of a triangle whose third side is the length of the lake.



By the Triangle Midsegment Theorem, the segment connecting the two midpoints is half the distance across the lake. So, Dean multiplies the length of the midsegment by 2 to find the length of the lake.



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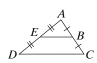
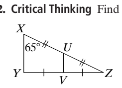
Quick Check

1. $AB = 10$ and $CD = 18$. Find EB, BC, and AC.



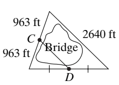
$EB = 9$; $BC = 10$; $AC = 20$

2. **Critical Thinking** Find $m\angle VUZ$. Justify your answer.



65; $\overline{UV} \parallel \overline{XY}$ so $\angle VUZ$ and $\angle YXZ$ are corresponding and congruent.

3. $\overline{CD}$ is a new bridge being built over a lake as shown. Find the length of the bridge.



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Lesson 5-2

Bisectors in Triangles

Lesson Objective	NAEP 2005 Strand: Geometry
Use properties of perpendicular bisectors and angle bisectors	Topics: Relationships Among Geometric Figures
	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 5-2: Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Theorem 5-3: Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

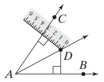
Theorem 5-4: Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

Theorem 5-5: Converse of the Angle Bisector Theorem

If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the angle bisector.

The distance from a point to a line is the length of the perpendicular segment from the point to the line.



D is 3 in. from $\overleftrightarrow{AB}$ and $\overrightarrow{AC}$.

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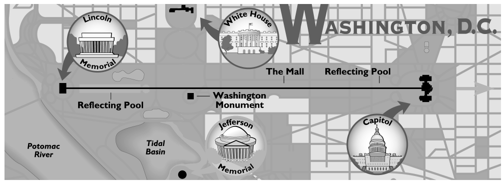
Examples

1 Applying the Perpendicular Bisector Theorem Use the map of Washington, D.C. Describe the set of points that are equidistant from the Lincoln Memorial and the Capitol.

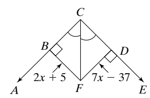


The Converse of the Perpendicular Bisector Theorem states: *If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.*

A point that is equidistant from the Lincoln Memorial and the Capitol must be on the perpendicular bisector of the segment whose endpoints are the Lincoln Memorial and the Capitol.

2 Using the Angle Bisector Theorem

Find x, FB, and FD in the diagram at right.



$FD = FB$	Angle Bisector Theorem
$7x - 37 = 2x + 5$	Substitute.
$7x = 2x + 42$	Add 37 to each side.
$5x = 42$	Subtract $2x$ from each side.
$x = 8.4$	Divide each side by 5.
$FB = 2(8.4) + 5 = 21.8$	Substitute.
$FD = 7(8.4) - 37 = 21.8$	Substitute.

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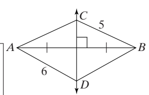
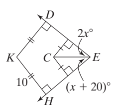
Quick Check

1. Use the information given in the diagram. $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$. Find CA and DB. Explain your reasoning.



$CA = 5$; $DB = 6$; $\overleftrightarrow{CD}$ is the perpendicular bisector of $\overline{AB}$; therefore $CA = CB$ and $DA = DB$.

2. a. According to the diagram, how far is K from $\overrightarrow{EH}$? From $\overrightarrow{ED}$?



10; 10

b. What can you conclude about $\overrightarrow{EK}$?

$\overrightarrow{EK}$ is the angle bisector of $\angle DEH$.

c. Find the value of x.

20

d. Find $m\angle DEH$.

80

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Lesson 5-3

Concurrent Lines, Medians, and Altitudes

Lesson Objective	NAEP 2005 Strand: Geometry
1 Identify properties of perpendicular bisectors and angle bisectors	Topics: Relationships Among Geometric Figures
2 Identify properties of medians and altitudes of a triangle	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 5-6

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices.

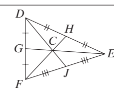
Theorem 5-7

The bisectors of the angles of a triangle are concurrent at a point equidistant from the sides.

Theorem 5-8

The medians of a triangle are concurrent at a point that is two-thirds the distance from each vertex to the midpoint of the opposite side.

$DC = \frac{2}{3}DJ$ $\quad EC = \frac{2}{3}EG$ $\quad FC = \frac{2}{3}FH$



Theorem 5-9

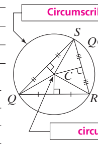
The lines that contain the altitudes of a triangle are concurrent.

Concurrent lines are three or more lines that meet in one point.

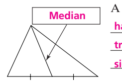
The point of concurrency is the point at which concurrent lines intersect.

A circle is circumscribed about a polygon when the vertices of the polygon are on the circle.

The circumcenter of a triangle is the point of concurrency of the perpendicular bisectors of a triangle.



A median of a triangle is a segment that has as its endpoints a vertex of the triangle and the midpoint of the opposite side.



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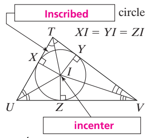
A circle is inscribed in a polygon if the sides of the polygon are tangent to the circle.

The incenter of a triangle is the point of currency of the angle bisectors of the triangle.

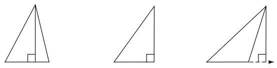
An altitude of a triangle is the perpendicular segment from a vertex to the line containing the opposite side.

Inscribed circle

$XI = YI = ZI$



incenter



Acute Triangle Altitude is inside.

Right Triangle Altitude is a side.

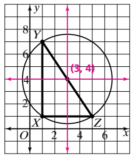
Obtuse Triangle Altitude is outside.

The centroid of a triangle is the point of intersection of the medians of the triangle.

The orthocenter of a triangle is the point of intersection of the lines containing the altitudes of the triangle.

Examples

1 Finding the Circumcenter Find the center of the circle that circumscribes $\triangle XYZ$.



Because X has coordinates (1, 1) and Y has coordinates (1, 7), $\overline{XY}$ lies on the vertical line $x =$ 1. The perpendicular bisector of $\overline{XY}$ is the horizontal line that passes through $(1, \frac{1+7}{2})$ or (1, 4), so the equation of the perpendicular bisector of $\overline{XY}$ is $y =$ 4.

Because X has coordinates (1, 1) and Z has coordinates (5, 1), $\overline{XZ}$ lies on the horizontal line $y =$ 1. The perpendicular bisector of $\overline{XZ}$ is the vertical line that passes through $(\frac{1+5}{2}, 1)$ or (3, 1), so the equation of the perpendicular bisector of $\overline{XZ}$ is $x =$ 3.

Draw the lines $y =$ 4 and $x =$ 3. They intersect at the point (3, 4). This point is the center of the circle that circumscribes $\triangle XYZ$.

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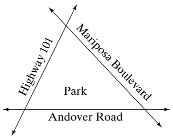
2 Applying Theorem 5-7 City planners want to locate a fountain equidistant from three straight roads that enclose a park. Explain how they can find the location.

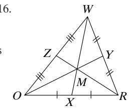


The roads form a triangle around the park.

Theorem 5-7 states that the bisectors of the angles of a triangle are concurrent at a point equidistant from the sides.

The city planners should find the point of concurrency of the angle bisectors of the triangle formed by the three roads and locate the fountain there.

3 Finding Lengths of Medians M is the centroid of $\triangle WOR$, and $WM = 16$. Find WX.



The centroid is the point of concurrency of the medians of a triangle.

The medians of a triangle are concurrent at a point that is two-thirds the distance from each vertex to the midpoint of the opposite side. (Theorem 5-8)

Because M is the centroid of $\triangle WOR$, $WM =$ $\frac{2}{3}WX$.

$WM = \frac{2}{3}WX$ Theorem 5-8

$16 = \frac{2}{3}WX$ Substitute 16 for WM.

$24 = WX$ Multiply each side by $\frac{3}{2}$.

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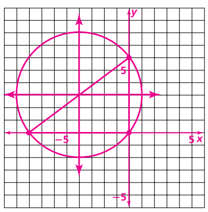
Quick Check

1. a. Find the center of the circle that you can circumscribe about the triangle with vertices (0, 0), (−8, 0), and (0, 6).

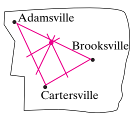
(−4, 3)



b. **Critical Thinking** In Example 1, explain why it is not necessary to find the third perpendicular bisector.

Theorem 5-6: All the perpendicular bisectors of the sides of a triangle are concurrent.

2. a. The towns of Adamsville, Brooksville, and Cartersville want to build a library that is equidistant from the three towns. Show on the diagram where they should build the library. Explain.



Draw segments connecting the towns. Build the library at the intersection point of the perpendicular bisectors of the segments.

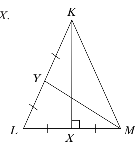
b. What theorem did you use to find the location?

The perpendicular bisectors of the sides of a triangle are concurrent at a point equidistant from the vertices (Theorem 5-6).

3. Using the diagram in Example 3, find MX. Check that $WM + MX = WX$.

8

4. Is $\overline{MY}$ a median, an altitude, or neither? Explain.



median; $\overline{MY}$ is a segment drawn from vertex M to the midpoint of the opposite side.

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Lesson 5-4

Inverses, Contrapositives, and Indirect Reasoning

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Write the negation of a statement and the inverse and contrapositive of a conditional statement	Topics: Mathematical Reasoning
2 Use indirect reasoning	Local Standards: ____________

Vocabulary and Key Concepts

Negation, Inverse, and Contrapositive Statements

Statement	Example	Symbolic Form	You Read It
Conditional	If an angle is a straight angle, then its measure is 180.	$p \rightarrow q$	If p, then q.
Negation (of p)	An angle is not a straight angle.	$\sim p$	Not p.
Inverse	If an angle is not a straight angle, then its measure is not 180.	$\sim p \rightarrow \sim q$	If not p, then not q.
Contrapositive	If an angle's measure is not 180, then it is not a straight angle.	$\sim q \rightarrow \sim p$	If not q, then not p.

Writing an Indirect Proof

Step 1 State as an assumption the opposite (negation) of what you want to prove.

Step 2 Show that this assumption leads to a contradiction.

Step 3 Conclude that the assumption must be false and that what you want to prove must be true.

The negation of a statement has the opposite truth value from that of the original statement.

The inverse of a conditional statement negates both the hypothesis and the conclusion.

The contrapositive of a conditional switches the hypothesis and the conclusion and negates both.

A conditional and its contrapositive always have the same truth value.

Equivalent statements have the same truth value.

In indirect reasoning all possibilities are considered and then all but one are proved false. The remaining possibility must be true.

An indirect proof is a proof involving indirect reasoning.

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Examples

❶ **Writing the Negation of a Statement** Write the negation of "*ABCD* is not a convex polygon."

The negation of a statement has the **opposite** truth value. The negation of *is not* in the original statement removes the word ***not***.

The negation of "*ABCD* is not a convex polygon" is

"ABCD is a convex polygon."

❷ **Writing the Inverse and Contrapositive** Write the inverse and contrapositive of the conditional statement "If △ABC is equilateral, then it is isosceles."

To write the inverse of a conditional, negate both the hypothesis and the conclusion.

Hypothesis ↓ Conclusion ↓

Conditional: If △*ABC* is equilateral, then it is isosceles.

↓ Negate both. ↓

Inverse: If △*ABC* is **not equilateral**, then it is **not isosceles**.

To write the contrapositive of a conditional, switch the hypothesis and conclusion, then negate both.

Hypothesis ↓ Conclusion ↓

Conditional: If △*ABC* is equilateral, then it is isosceles.

Switch and negate both.

Contrapositive: If △*ABC* is **not isosceles**, then it is **not equilateral**.

❸ **The First Step of an Indirect Proof** Write the first step of an indirect proof.

Prove: A triangle cannot contain two right angles.

In the first step of an indirect proof, you assume as true the **negation** of what you want to prove.

Because you want to prove that a triangle cannot contain two right angles, you assume that a triangle **can** contain two right angles.

The first step is **"Assume that a triangle contains two right angles."**

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❹ **Identifying Contradictions** Identify the two statements that contradict each other.

I. *P*, *Q*, and *R* are coplanar.
II. *P*, *Q*, and *R* are collinear.
III. $m\angle PQR = 60$

Two statements **contradict** each other when they cannot both be true at the same time. Examine each pair of statements to see whether they contradict each other.

I and II	I and III	II and III
P, *Q*, and *R* are coplanar and collinear.	*P*, *Q*, and *R* are coplanar, and $m\angle PQR = 60$.	*P*, *Q*, and *R* are collinear, and $m\angle PQR = 60$.
Three points that lie on the same line are both coplanar and collinear, so these two statements **do not** contradict each other.	Three points that lie on an angle are coplanar, so these two statements **do not** contradict each other.	If three distinct points are collinear, they form a straight angle, so $m\angle PQR$ cannot equal 60. Statements II and III **do** contradict each other.

❺ **Indirect Proof** Write an indirect proof.

Prove: △*ABC* cannot contain two obtuse angles.

Step 1 Assume as true the **opposite** of what you want to prove.
That is, assume that △*ABC* contains two **obtuse** angles.
Let **∠A** and **∠B** be **obtuse**.

Step 2 If ∠*A* and ∠*B* are **obtuse**, $m\angle A + m\angle B + m\angle C >$ **180**.
This contradicts the **Triangle Angle-Sum** Theorem, which states that $m\angle A + m\angle B + m\angle C =$ **180**.

Step 3 The assumption in Step 1 must be **false**. △*ABC* cannot contain two **obtuse** angles.

Quick Check

1. Write the negation of each statement.
 a. $m\angle XYZ > 70$.
 The measure of ∠XYZ is not more than 70.
 b. Today is not Tuesday.
 Today is Tuesday.

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2. Write (a) the inverse and (b) the contrapositive of Maya Angelou's statement "If you don't stand for something, you'll fall for anything."
 a. **If you stand for something, you won't fall for anything.**
 b. **If you won't fall for anything, then you stand for something.**

3. You want to prove each statement true. Write the first step of an indirect proof.
 a. The shoes cost no more than $20.
 The shoes cost more than $20.
 b. $m\angle A > m\angle B$
 $m\angle A \leq m\angle B$

4. Identify the two statements that contradict each other. Explain.
 I. $\overline{FG} \parallel \overline{KL}$ II. $\overline{FG} \perp \overline{KL}$ III. $\overline{FG} \cong \overline{KL}$
 I and II;
 Two segments cannot be parallel as well as perpendicular. I and II contradict each other.
 Two segments can be parallel and congruent. I and III do not contradict each other.
 Two segments can be perpendicular and congruent. II and III do not contradict each other.

5. **Critical Thinking** You plan to write an indirect proof showing that ∠*X* is an obtuse angle. In the first step you assume that ∠*X* is an acute angle. What have you overlooked?
 ∠X could be a right angle.

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Lesson 5-5 Inequalities in Triangles

Lesson Objectives	
1 Use inequalities involving angles of triangles 2 Use inequalities involving sides of triangles	**NAEP 2005 Strand:** Geometry **Topics:** Relationships Among Geometric Figures **Local Standards:** ____________________

Key Concepts

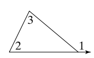
Comparison Property of Inequality

If $a = b + c$ and $c > 0$, then **a** > **b**.

Corollary to the Triangle Exterior Angle Theorem

The measure of an **exterior** angle of a triangle is greater than the measure of each of its remote **interior** angles.

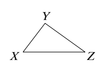
m∠1 > $m\angle 2$ and **m∠1** > $m\angle 3$



Theorem 5-10

If two sides of a triangle are not congruent, then **the larger angle lies opposite the longer side.**

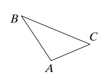
If $XZ > XY$, then $m\angle Y$ **>** $m\angle Z$.



Theorem 5-11

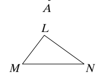
If two angles of a triangle are not congruent, then **the longer side lies opposite the larger angle.**

If $m\angle A > m\angle B$, then *BC* **>** *AC*.



Theorem 5-12: Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is **greater than the length of the third side.**



$LM + MN >$ **LN**
$MN + LN >$ **LM**
$LN + LM >$ **MN**

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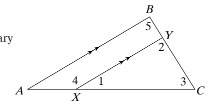
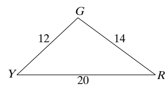
Examples

1 Applying the Corollary Explain why $m\angle 4 > m\angle 5$.



$\angle 4$ is an **exterior** angle of $\triangle XYC$. The Corollary to the Exterior Angle Theorem states that the measure of an exterior angle of a triangle is **greater** than the measure of each of its remote interior angles. The remote interior angles are **$\angle 2$** and **$\angle 3$**, so $m\angle 4 > m\angle 2$. $\angle 2$ and $\angle 5$ are **corresponding** angles formed by transversal **$\overline{BC}$**. Because $\overline{AB} \parallel \overline{XY}$, you can conclude that $\angle 2 \cong$ **$\angle 5$** by the Corresponding Angles Postulate. Thus, $m\angle 2 =$ **$m\angle 5$**. Substituting **$m\angle 5$** for $m\angle 2$ in the inequality $m\angle 4 > m\angle 2$ produces the inequality **$m\angle 4$** $>$ **$m\angle 5$**.

2 Applying Theorem 5-10 In $\triangle RGY$, $RG = 14$, $GY = 12$, and $RY = 20$. List the angles from largest to smallest.



Theorem 5-10: *If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.*

No two sides of $\triangle RGY$ are congruent, so the largest angle lies opposite the longest side.

Find the angle opposite each side.

The longest side is **20**. The opposite angle, **$\angle G$**, is the largest. The shortest side is **12**. The opposite angle, **$\angle R$**, is the smallest. From largest to smallest, the angles are **$\angle G$**, **$\angle Y$**, **$\angle R$**.

3 Using the Triangle Inequality Theorem Can a triangle have sides with the given lengths? Explain.

a. 2 cm, 2 cm, 4 cm

According to the **Triangle Inequality** Theorem, the sum of the lengths of any two sides of a triangle is **greater** than the length of the third side.

$2 + 2$ **$\not>$** 4 The sum of 2 and 2 **is not** greater than 4.

No, a triangle **cannot** have sides with these lengths.

b. 8 in., 15 in., 12 in.

$8 + 15$ **$>$** 12 $8 + 12$ **$>$** 15 $15 + 12$ **$>$** 8

Yes, a triangle **can** have sides with these lengths.

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4 Finding Possible Side Lengths In $\triangle FGH$, $FG = 9$ m and $GH = 17$ m. Describe the possible lengths of $\overline{FH}$.

The Triangle Inequality Theorem: *The sum of the lengths of any two sides of a triangle is* **greater** *than the length of the third side.*

Solve three inequalities.

$FH + 9 >$ **17** $FH + 17 >$ **9** $9 +$ **17** $> FH$

$FH >$ **8** $FH >$ **−8** **26** $> FH$

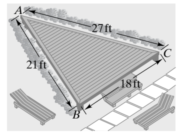
FH must be longer than **8** m and shorter than **26** m.

Quick Check

1. Use the diagram in Example 1. If $\overline{CY} \cong \overline{XY}$, explain why $m\angle 4 > m\angle 1$.

$m\angle 1 = m\angle 3$ by the Isosceles Triangle Theorem
$m\angle 4 > m\angle 3$ by the Corollary to the Triangle Exterior Angle Theorem
$m\angle 4 > m\angle 1$ by Substitution

2. List the angles of $\triangle ABC$ in order from smallest to largest.



$\angle A$, $\angle C$, $\angle B$

3. Can a triangle have sides with the given lengths? Explain.

a. 2 m, 7 m, and 9 m

no; 2 + 7 is not greater than 9

b. 4 yd, 6 yd, and 9 yd

yes; 4 + 6 > 9, 6 + 9 > 4, and 4 + 9 > 6

4. A triangle has sides of lengths 3 in. and 12 in. Describe the possible lengths of the third side.

$9 < x < 15$

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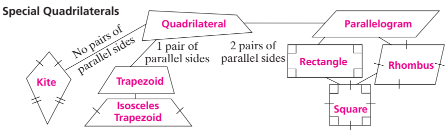
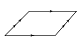
Lesson 6-1 Classifying Quadrilaterals

Lesson Objective	NAEP 2005 Strand: Geometry
1 Define and classify special types of quadrilaterals	Topic: Relationships Among Geometric Figures
	Local Standards: ____________

Key Concepts

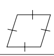
Special Quadrilaterals



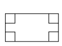


A **parallelogram** is a quadrilateral with both pairs of opposite sides **parallel**.

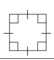
A **kite** is a quadrilateral with two pairs of **adjacent sides** congruent and no **opposite sides** congruent.



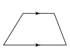
A **rhombus** is a parallelogram with four **congruent sides**.



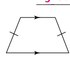
A **rectangle** is a parallelogram with four **right angles**.



A **square** is a parallelogram with four **congruent sides** and four **right angles**.



A **trapezoid** is a quadrilateral with **exactly one pair of parallel sides**.



An **isosceles trapezoid** is a trapezoid whose nonparallel sides are congruent.

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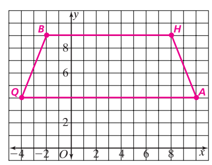
Examples

1 Classifying by Coordinate Methods Determine the most precise name for the quadrilateral with vertices $Q(-4, 4)$, $B(-2, 9)$, $H(8, 9)$, and $A(10, 4)$. Graph quadrilateral $QBHA$.



Find the slope of each side.

Slope of $\overline{QB} = \frac{9 - 4}{-2 - (-4)} =$ **$\frac{5}{2}$**

Slope of $\overline{BH} = \frac{9 - 9}{8 - (\textbf{−2})} =$ **0**

Slope of $\overline{HA} = \frac{4 - \textbf{9}}{10 - \textbf{8}} =$ **$-\frac{5}{2}$**

Slope of $\overline{QA} = \frac{\textbf{4} - \textbf{4}}{\textbf{−4} - \textbf{10}} =$ **0**

$\overline{BH}$ is parallel to $\overline{QA}$ because their slopes are **equal**.

$\overline{HA}$ is not parallel to $\overline{QB}$ because their slopes are **not equal**.

One pair of opposite sides is parallel, so $QBHA$ is a **trapezoid**.

Next, use the distance formula to see whether any pairs of sides are congruent.

$QB = \sqrt{(-2 - (-4))^2 + (9 - \textbf{4})^2} = \sqrt{4 + \textbf{25}} = \boldsymbol{\sqrt{29}}$

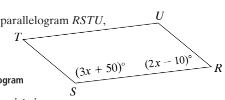
$HA = \sqrt{(10 - 8)^2 + (\textbf{4} - \textbf{9})^2} = \sqrt{\textbf{4} + \textbf{25}} = \boldsymbol{\sqrt{29}}$

$BH = \sqrt{(8 - (-2))^2 + (\textbf{9} - \textbf{9})^2} = \sqrt{\textbf{100} + \textbf{0}} = \textbf{10}$

$QA = \sqrt{(-4 - \textbf{10})^2 + (\textbf{4} - \textbf{4})^2} = \sqrt{\textbf{196} + \textbf{0}} = \textbf{14}$

Because $QB =$ **HA**, $QBHA$ is **an isosceles trapezoid**.

2 Using the Properties of Special Quadrilaterals In parallelogram $RSTU$, $m\angle R = 2x - 10$ and $m\angle S = 3x + 50$. Find x.



$RSTU$ is a parallelogram.	**Given**
$\overline{ST} \parallel$ **$\overline{RU}$**	**Definition of parallelogram**
$m\angle R + m\angle S =$ **180**	**If lines are parallel, then interior angles on the same side of a transversal are** **supplementary**.
$(2x - 10)$ $+$ **$(3x + 50)$** $= 180$	**Substitute** **$2x - 10$** **for $m\angle R$ and** **$3x + 50$** **for $m\angle S$.**
$5x$ $+$ **40** $= 180$	**Simplify.**
$5x$ $=$ **140**	**Subtract 40 from each side.**
$x =$ **28**	**Divide each side by 5.**

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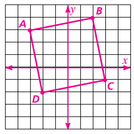
Quick Check

1. a. Graph quadrilateral $ABCD$ with vertices $A(-3, 3)$, $B(2, 4)$, $C(3, -1)$, and $D(-2, -2)$.



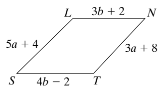
b. Classify $ABCD$ in as many ways as possible.

ABCD is a quadrilateral because it has four sides. It is a parallelogram because both pairs of opposite sides are parallel. It is a rhombus because all four sides are congruent. It is a rectangle because it has four right angles and its opposite sides are congruent. It is a square because it has four right angles and its sides are all congruent.

c. Which name gives the most information about $ABCD$? Explain.

Square; the name *square* means a figure is both a rectangle and a rhombus, since its sides are all congruent and it has four right angles.

2. Find the values of the variables in the rhombus. Then find the lengths of the sides.



$a = 2$, $b = 4$, $LN = ST = NT = SL = 14$

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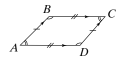
Lesson 6-2 Properties of Parallelograms

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use relationships among sides and among angles of parallelograms 2 Use relationships involving diagonals of parallelograms and transversals	**Topic:** Relationships Among Geometric Figures **Local Standards:** ________

Vocabulary and Key Concepts

Theorem 6-1

Opposite sides of a parallelogram are congruent.

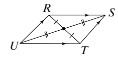


Theorem 6-2

Opposite angles of a parallelogram are congruent.

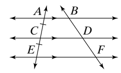
Theorem 6-3

The diagonals of a parallelogram bisect each other.



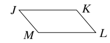
Theorem 6-4

If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.



$\overline{BD} \cong \overline{DF}$

Consecutive angles of a polygon share a common side.



In ▱$JKLM$, $\angle J$ and $\angle M$ are consecutive angles, as are $\angle J$ and $\angle K$. $\angle J$ and $\angle L$ are *not* consecutive.

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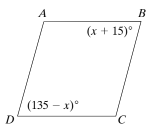
1 **Using Algebra** Find the value of x in ▱$ABCD$. Then find $m\angle A$.



$x + 15 = 135 - x$ — Opposite angles of a parallelogram are congruent.

$2x + 15 = 135$ — Add x to each side.

$2x = 120$ — Subtract 15 from each side.

$x = 60$ — Divide each side by 2.

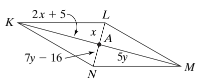
$m\angle B = 60 + 15 = 75$ — Substitute 60 for x.

$m\angle A + m\angle B = 180$ — Consecutive angles of a parallelogram are supplementary.

$m\angle A + 75 = 180$ — Substitute 75 for $m\angle B$.

$m\angle A = 105$ — Subtract 75 from each side.

2 **Using Algebra** Find the values of x and y in ▱$KLMN$.



$x = 7y - 16$ — The diagonals of a parallelogram bisect each other.

$2x + 5 = 5y$

$2(7y - 16) + 5 = 5y$ — Substitute $7y - 16$ for x in the second equation to solve for y.

$14y - 32 + 5 = 5y$ — Distribute.

$14y - 27 = 5y$ — Simplify.

$-27 = -9y$ — Subtract $14y$ from each side.

$3 = y$ — Divide each side by -9.

$x = 7(3) - 16$ — Substitute 3 for y in the first equation to solve for x.

$x = 5$ — Simplify.

So $x = 5$ and $y = 3$.

3 **Using Theorem 6-4** Theorem 6-4 states *If three (or more) parallel lines cut off congruent segments on one transversal, then* they cut off congruent segments on every transversal.

Explain how to divide a blank card into five equal rows using Theorem 6-4 and a sheet of lined paper.

Place a corner of the top of the card on the first line of the lined paper.
Place a corner that forms a consecutive angle with the first corner on the sixth line.
Mark the points where the lines intersect one side of the card.
Mark the points where the lines intersect the opposite side of the card.
Connect the marks on opposite sides using a straightedge.

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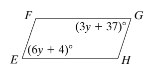
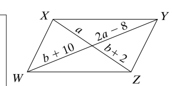
Quick Check

1. Find the value of y in ▱$EFGH$. Then find $m\angle E$, $m\angle F$, $m\angle G$, and $m\angle H$.



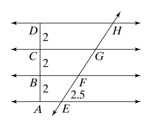
$y = 11$; $m\angle E = 70$, $m\angle F = 110$, $m\angle G = 70$, and $m\angle H = 110$

2. Find the values of a and b.



$a = 16$, $b = 14$

3. In the figure, $\overleftrightarrow{DH} \parallel \overleftrightarrow{CG} \parallel \overleftrightarrow{BF} \parallel \overleftrightarrow{AE}$, $AB = BC = CD = 2$, and $EF = 2.5$. Find EH.



7.5

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Lesson 6-3

Proving That a Quadrilateral Is a Parallelogram

Lesson Objective	
Determine whether a quadrilateral is a parallelogram	**NAEP 2005 Strand:** Geometry **Topic:** Geometry **Local Standards:** ________

Key Concepts

Theorem 6-5
If both pairs of opposite sides of a quadrilateral are congruent, then
the quadrilateral is a parallelogram.

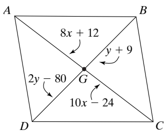
Theorem 6-6
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Theorem 6-7
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Theorem 6-8
If one pair of opposite sides of a quadrilateral is both congruent and parallel, then the quadrilateral is a parallelogram.

Examples

1 Finding Values for Parallelograms Find values for x and y for which $ABCD$ must be a parallelogram.



If the diagonals of quadrilateral $ABCD$ bisect each other, then $ABCD$ is a parallelogram by Theorem 6-7. Write and solve two equations to find values of x and y for which the diagonals bisect each other.

Diagonals of parallelograms bisect each other.

$10x - 24 = 8x + 12$ | $2y - 80 = y + 9$

Collect the variables on one side.

$2x - 24 = 12$ | $y - 80 = 9$

Solve.

$2x = 36$ | $y = 89$

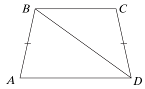
$x = 18$

If $x = 18$ and $y = 89$, then $ABCD$ is a parallelogram.

Name____________ Class____________ Date____________

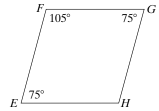
2 Is the Quadrilateral a Parallelogram? Can you prove the quadrilateral is a parallelogram from what is given? Explain.

a. Given: $\overline{AB} \cong \overline{CD}$
Prove: $ABCD$ is a parallelogram.



All you know about the quadrilateral is that only one pair of opposite sides is congruent. Therefore, you cannot conclude that the quadrilateral is a parallelogram.

b. Given: $m\angle E = m\angle G = 75°$, $m\angle F = 105°$
Prove: $EFGH$ is a parallelogram.



The sum of the measures of the angles of a polygon is $(n - 2)180$ where n represents the number of sides, so the sum of the measures of the angles of a quadrilateral is $(4 - 2)180 = 360$.

If x represents the measure of the unmarked angle, $x + 75 + 105 + 75 = 360$, so $x = 105$.

Theorem 6-6 states If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Because both pairs of opposite angles are congruent, the quadrilateral is a parallelogram by Theorem 6-6.

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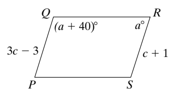
Quick Check

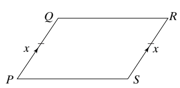
1. Find the values of a and c for which $PQRS$ must be a parallelogram.



$a = 70, c = 2$

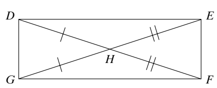
2. Can you prove the quadrilateral is a parallelogram? Explain.

a. Given: $\overline{PQ} \cong \overline{SR}$, $\overline{PQ} \parallel \overline{SR}$
Prove: $PQRS$ is a parallelogram.



Yes; a pair of opposite sides are parallel and congruent (Theorem 6-8).

b. Given: $\overline{DH} \cong \overline{GH}$, $\overline{EH} \cong \overline{FH}$
Prove: $DEFG$ is a parallelogram.



No; the diagonals do not necessarily bisect each other—the figure could be a trapezoid, with $\overline{DG}$ and $\overline{EF}$ being parallel but of unequal length.

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Lesson 6-4

Special Parallelograms

Lesson Objectives	
1 Use properties of diagonals of rhombuses and rectangles 2 Determine whether a parallelogram is a rhombus or a rectangle	**NAEP 2005 Strand:** Geometry **Topic:** Geometry **Local Standards:** ________

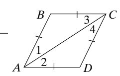
Key Concepts

Rhombuses

Theorem 6-9
Each diagonal of a rhombus bisects two angles of the rhombus.

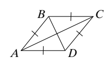
$\overline{AC}$ bisects $\angle BAD$, so $\angle 1 \cong \angle 2$

$\overline{AC}$ bisects $\angle BCD$, so $\angle 3 \cong \angle 4$



Theorem 6-10
The diagonals of a rhombus are perpendicular.

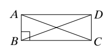
$\overline{AC} \perp \overline{BD}$



Rectangles

Theorem 6-11
The diagonals of a rectangle are congruent.

$\overline{AC} \cong \overline{BD}$



Parallelograms

Theorem 6-12
If one diagonal of a parallelogram bisects two angles of the parallelogram, then
the parallelogram is a rhombus.

Theorem 6-13
If the diagonals of a parallelogram are perpendicular, then
the parallelogram is a rhombus.

Theorem 6-14
If the diagonals of a parallelogram are congruent, then
the parallelogram is a rectangle.

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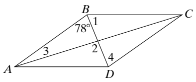
Examples

❶ **Finding Angle Measures** Find the measures of the numbered angles in the rhombus.



Theorem 6-9 states that each diagonal of a rhombus bisects two angles of the rhombus, so $m\angle 1 = 78$.

Theorem 6-10 states that [the diagonals of the rhombus are perpendicular],

so $m\angle 2 =$ [90].

Because the four angles formed by the diagonals all must have measure 90, $\angle 3$ and $\angle ABD$ must be [complementary]. Because $m\angle ABD = 78$, $m\angle 3 = 90 - 78 =$ [12].

Finally, because $BC = DC$, the [Isosceles Triangle Theorem] allows you to conclude that $\angle$[1] $\cong \angle 4$. So $m\angle 4 =$ [78].

❷ **Finding Diagonal Length** One diagonal of a rectangle has length $8x + 2$. The other diagonal has length $5x + 11$. Find the length of each diagonal.

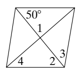
By Theorem 6-11, the diagonals of a rectangle are [congruent].

[5x + 11] = [8x + 2]	Diagonals of a rectangle are congruent.
11 = [3x] + 2	Subtract [5x] from each side.
[9] = [3x]	Subtract [2] from each side.
[3] = x	Divide each side by [3].
$8x + 2 = 8($[3]$) + 2 =$ [26]	Substitute.
$5x + 11 = 5($[3]$) + 11 =$ [26]	Substitute.

The length of each diagonal is [26].

Quick Check

1. Find the measures of the numbered angles in the rhombus.



$m\angle 1 = 90$, $m\angle 2 = 50$, $m\angle 3 = 50$, $m\angle 4 = 40$

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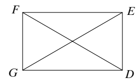
Example

❸ **Identifying Special Parallelograms** The diagonals of $ABCD$ are such that $AC = 16$ cm and $BD = 8$ cm. Can you conclude that $ABCD$ is a rhombus or a rectangle? Explain.

$ABCD$ [cannot] be a rectangle, because $\overline{AC} \not\cong \overline{BD}$ and the diagonals of a rectangle are [congruent] (Theorem 6-11). $ABCD$ may be a rhombus, but it may also not be one, depending on whether [$\overline{AC} \perp \overline{BD}$] (Theorem 6-10).

Quick Check

2. Find the length of the diagonals of rectangle $GFED$ if $FD = 5y - 9$ and $GE = y + 5$.



$8\frac{1}{2}$

3. A parallelogram has angles of 30°, 150°, 30°, and 150°. Can you conclude that it is a rhombus or a rectangle? Explain.

No; there is not enough information to conclude that the parallelogram is a rhombus. It cannot be a rectangle because it has no right angles.

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Lesson 6-5 Trapezoids and Kites

Lesson Objective	NAEP 2005 Strand: Geometry
▼ Verify and use properties of trapezoids and kites	Topic: Relationships Among Geometric Figures
	Local Standards: ____________________

Vocabulary and Key Concepts

Trapezoids

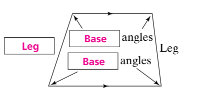
Theorem 6-15

The base angles of an isosceles trapezoid are [congruent].

Theorem 6-16

The [diagonals] of an isosceles trapezoid are congruent.

The base angles of a trapezoid are two angles that share a base of the trapezoid.



Kites

Theorem 6-17

The diagonals of a kite are [perpendicular].

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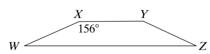
Examples

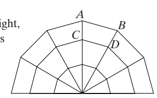
❶ **Finding Angle Measures in Trapezoids** $WXYZ$ is an isosceles trapezoid, and $m\angle X = 156$. Find $m\angle Y$, $m\angle Z$, and $m\angle W$.



$m\angle X + m\angle W =$ [180]	Two angles that share a leg of a trapezoid are [supplementary].
[156] $+ m\angle W =$ [180]	Substitute.
$m\angle W =$ [24]	Subtract [156] from each side.

Because the base angles of an isosceles trapezoid are [congruent], $m\angle Y = m\angle X = 156$ and $m\angle Z =$ [$m\angle W$] = [24].

❷ **Using Isosceles Trapezoids** Half of a spider's web is shown at the right, formed by layers of congruent isosceles trapezoids. Find the measures of the angles in $ABDC$.



Trapezoid $ABDC$ is part of an isosceles triangle whose vertex is at the center of the web. Because there are 6 adjacent congruent vertex angles at the center of the web, together forming a [straight] angle, each vertex angle measures $\frac{180}{6}$, or [30].

By the Triangle Angle-Sum Theorem, $m\angle A + m\angle B + 30 =$ [180], so $m\angle A + m\angle B =$ [150].

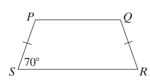
Because $ABDC$ is part of an isosceles triangle, $m\angle A = m\angle B$, so $2(m\angle A) =$ [150] and $m\angle A = m\angle B =$ [75].

Another way to find the measure of each acute angle is to divide the difference between 180 and the measure of the vertex angle by 2: $\frac{180 - 30}{2} =$ [75].

Because the bases of a trapezoid are parallel, the two angles that share a leg are [supplementary], so $m\angle C = m\angle D = 180 - 75 =$ [105].

Quick Check

1. In the isosceles trapezoid, $m\angle S = 70$. Find $m\angle P$, $m\angle Q$, and $m\angle R$.



110, 110, 70

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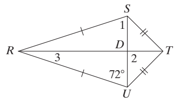
Example

3 Finding Angle Measures in Kites Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ in the kite.



$m\angle 2 = 90$ — Diagonals of a kite are **perpendicular**.

$RU =$ **RS** — Definition of a kite

$m\angle 1 =$ **72** — **Isosceles** Triangle Theorem

$m\angle 3 + m\angle RDU + 72 =$ **180** — **Triangle Angle-Sum Theorem**

$m\angle RDU =$ **90** — Diagonals of a kite are perpendicular.

$m\angle 3 +$ **90** $+ 72 =$ **180** — Substitute.

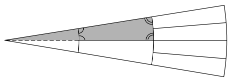
$m\angle 3 +$ **162** $=$ **180** — Simplify.

$m\angle 3 =$ **18** — Subtract **162** from each side.

Quick Check

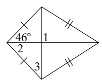
2. The middle ring of the piece of ceiling shown is made from congruent isosceles trapezoids. Imagine a circular glass ceiling made from the ceiling pieces with 18 angles meeting at the center. What are the measures of the two sets of base angles of the trapezoids in the middle ring?

The measure of both inner angles is 85.
The measure of both outer angles is 95.



3. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ in the kite.

$m\angle 1 = 90$, $m\angle 2 = 46$, and $m\angle 3 = 44$



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Lesson 6-6

Placing Figures in the Coordinate Plane

Lesson Objective	NAEP 2005 Strand: Geometry
Name coordinates of special figures by using their properties	Topic: Position and Direction
	Local Standards: ____________________

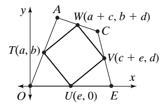
Example

1 Proving Congruency Show that *TWVU* is a parallelogram by proving pairs of opposite sides congruent.

If both pairs of opposite sides of a quadrilateral are **congruent**, then the quadrilateral is a parallelogram by **Theorem 6-5**.

You can prove that *TWVU* is a parallelogram by showing that $TW = VU$ and **WV** $=$ **TU**. Use the distance formula.

Use the coordinates $T(a, b)$, $W(a + c, b + d)$, $V(c + e, d)$, and $U(e, 0)$.



$TW = \sqrt{((a + c) - a)^2 + ((b + d) - b)^2} = \sqrt{c^2 + d^2}$

$VU = \sqrt{((c + e) - e)^2 + (d - 0)^2} = \sqrt{c^2 + d^2}$

$WV = \sqrt{((a + c) - (c + e))^2 + ((b + d) - d)^2} = \sqrt{(a - e)^2 + b^2}$

$TU = \sqrt{(a - e)^2 + (b - 0)^2} = \sqrt{(a - e)^2 + b^2}$

Because $TW = VU$ and $WV = TU$, *TWVU* is a **parallelogram**.

Quick Check

1. Use the diagram above. Use a different method: Show that *TWVU* is a parallelogram by finding the midpoints of the diagonals.

Midpoint of $\overline{TV} = \left(\frac{a + c + e}{2}, \frac{b + d}{2}\right)$ = midpoint of $\overline{UW}$. Thus, the diagonals bisect each other, and *TWVU* is a parallelogram.

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Example

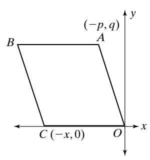
2 Naming Coordinates Use the properties of parallelogram *OCBA* to find the missing coordinates. Do not use any new variables.

The vertex *O* is the origin with coordinates O **(0, 0)**.

Because point *A* is *p* units to the left of point *O*, point *B* is also *p* units to the left of point **C**, because *OCBA* is a parallelogram. So the first coordinate of point *B* is **$-p - x$**.

Because $\overline{AB} \parallel \overline{CO}$ and $\overline{CO}$ is horizontal, $\overline{AB}$ is **also horizontal**. So point *B* has the same second coordinate, **q**, as point *A*.

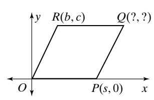
The missing coordinates are O **(0, 0)** and B **$(-p - x, q)$**.



Quick Check

2. Use the properties of parallelogram *OPQR* to find the missing coordinates. Do not use any new variables.

$Q(s + b, c)$



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Lesson 6-7

Proofs Using Coordinate Geometry

Lesson Objective	NAEP 2005 Strand: Geometry
Prove theorems using figures in the coordinate plane	Topics: Position and Direction; Mathematical Reasoning
	Local Standards: ____________________

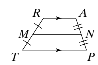
Vocabulary and Key Concepts

Theorem 6-18: Trapezoid Midsegment Theorem

(1) The midsegment of a trapezoid is **parallel** to the bases.

(2) The length of the midsegment of a trapezoid is half the sum of the **lengths of the bases**.

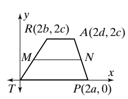
The midsegment of a trapezoid **is the segment that joins the midpoints of the nonparallel opposite sides of the trapezoid.**



$\overline{MN} \parallel$ **$\overline{TP}$**, $\overline{MN} \parallel$ **$\overline{RA}$**, and $MN = \frac{1}{2}($**TP** $+$ **RA**$)$

Examples

1 Planning a Coordinate Geometry Proof Examine trapezoid *TRAP*. Explain why you can assign the same *y*-coordinate to points *R* and *A*.



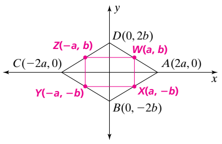
In a trapezoid, only one pair of sides is parallel. In *TRAP*, $\overline{TP} \parallel$ **$\overline{RA}$**. Because $\overline{TP}$ lies on the horizontal *x*-axis, $\overline{RA}$ also must be **horizontal**.

The *y*-coordinates of all points on a horizontal line are the same, so points *R* and **A** have the same *y*-coordinates.

Name______________________ Class______________________ Date____________

❷ **Using Coordinate Geometry** Use coordinate geometry to prove that the quadrilateral formed by connecting the midpoints of rhombus $ABCD$ is a rectangle.

Draw quadrilateral $XYZW$ by connecting the midpoints of $ABCD$.



From Lesson 6-6, you know that $XYZW$ is a parallelogram.

If the diagonals of a parallelogram are **congruent**, then the parallelogram is a rectangle from Theorem 6–14.

To show that $XYZW$ is a rectangle, find the lengths of its diagonals, and then compare them to show that they are **equal**.

$$XZ = \sqrt{(-a-a)^2 + (b-(-b))^2} = \sqrt{(\boxed{-2a})^2 + (\boxed{2b})^2} = \sqrt{\boxed{4a^2 + 4b^2}}$$

$$YW = \sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(\boxed{-2a})^2 + (\boxed{-2b})^2} = \sqrt{\boxed{4a^2 + 4b^2}}$$

$XZ \ \boxed{=} \ YW$

Because the diagonals **are** congruent, parallelogram $XYZW$ is a **rectangle**.

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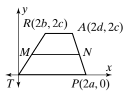
Quick Check

1. Complete the proof of Theorem 6-18.

Given: $\overline{MN}$ is the midsegment of trapezoid $TRAP$.

Prove: $\overline{MN} \parallel \overline{TP}$, $\overline{MN} \parallel \overline{RA}$, and $MN = \frac{1}{2}(TP + RA)$



a. Find the coordinates of midpoints M and N. How do the multiples of 2 help?

M(b, c) and N(a + d, c); starting with multiples of 2 eliminates fractions when using the midpoint formula.

b. Find and compare the slopes of $\overline{MN}$, $\overline{TP}$, and $\overline{RA}$.

0, 0, 0; they are all parallel.

c. Find and compare the lengths MN, TP, and RA.

$MN = d + a - b$, $TP = 2a$, $RA = 2d - 2b$. So $RA + TP = 2d + 2a - 2b$ which is twice MN. So the midsegment is half the sum of the bases.

d. In parts (b) and (c), how does placing a base along the x-axis help?

The base along the x-axis allows calculating length by subtracting x-values.

2. Use the diagram from Example 2. Explain why the proof using $A(2a, 0)$, $B(0, -2b)$, $C(-2a, 0)$, and $D(0, 2b)$ is easier than the proof using $A(a, 0)$, $B(0, -b)$, $C(-a, 0)$, and $D(0, b)$.

Using multiples of 2 in the coordinates for A, B, C, and D eliminates the use of fractions when finding midpoints, since finding midpoints requires division by 2.

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Lesson 7-1 Ratios and Proportions

Lesson Objective	NAEP 2005 Strand: Geometry
1 Write ratios and solve proportions	Topic: Position and Direction
	Local Standards: ____________

Vocabulary and Key Concepts

Properties of Proportions

$\frac{a}{b} = \frac{c}{d}$ is equivalent to

(1) $ad = \boxed{bc}$ (2) $\frac{b}{a} = \boxed{\frac{d}{c}}$ (3) $\frac{a}{c} = \boxed{\frac{b}{d}}$ (4) $\frac{a+b}{b} = \boxed{\frac{c+d}{d}}$

A proportion is **a statement that two ratios are equal.**

$\frac{a}{b} = \frac{c}{d}$ and $a : b = c : d$ are examples of proportions.

An extended proportion is **a statement that three or more ratios are equal.**

$\frac{6}{24} = \frac{4}{16} = \frac{1}{4}$ is an example of an extended proportion.

The Cross-Product Property states that **the product of the extremes of a proportion is equal to the product of the means.**

means (b and c)
$a : b = c : d$
extremes (a and d)

$\frac{a}{b} = \frac{c}{d}$

$a\boxed{d} = b\boxed{c}$

A scale drawing is **a drawing in which all lengths are proportional to corresponding actual lengths.**

A scale is **the ratio of any length in a scale drawing to the corresponding actual length. The lengths may be in different units.**

Examples

❶ **Finding Ratios** A scale model of a car is 4 in. long. The actual car is 15 ft long. What is the ratio of the length of the model to the length of the car?

Write both measurements in the same units.

15 ft = 15 × 12 in. = **180** in.

$$\frac{\text{length of model}}{\text{length of car}} = \frac{4 \text{ in.}}{15 \text{ ft}} = \frac{4 \text{ in.}}{180 \text{ in.}} = \frac{4}{180} = \boxed{\frac{1}{45}}$$

The ratio of the length of the scale model to the length of the car is **1** : **45**.

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❷ **Properties of Proportions**

Complete: If $\frac{a}{4} = \frac{12}{b}$, then $\frac{b}{12} = \frac{\blacksquare}{\blacksquare}$.

$ab = \boxed{48}$ Cross-Product Property

$\frac{ab}{\boxed{12a}} = \frac{48}{\boxed{12a}}$ Divide each side by **12a**.

$\frac{b}{12} = \frac{\boxed{4}}{\boxed{a}}$ Simplify.

❸ **Solving for a Variable** Solve each proportion.

a. $\frac{2}{5} = \frac{n}{35}$

$5n = \boxed{2}(\boxed{35})$ **Cross-Product** Property

$5n = \boxed{70}$ Simplify.

$n = \boxed{14}$ Divide each side by **5**.

b. $\frac{x+1}{3} = \frac{x}{2}$

$3x = \boxed{2}(\boxed{x} + \boxed{1})$ **Cross-Product** Property

$3x = \boxed{2x} + \boxed{2}$ **Distributive** Property

$x = \boxed{2}$ Subtract **2x** from each side.

Quick Check

1. A photo that is 8 in. wide and $5\frac{1}{3}$ in. high is enlarged to a poster that is 2 ft wide and $1\frac{1}{3}$ ft high. What is the ratio of the height of the photo to the height of the poster?

1 : 3

2. Write two expressions that are equivalent to $\frac{m}{4} = \frac{n}{11}$.

Answers may vary. Sample: $\frac{4}{m} = \frac{11}{n}$, $\frac{m+4}{4} = \frac{n+11}{11}$

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Example

4 Using Proportions Two cities are $3\frac{1}{2}$ in. apart on a map with the scale 1 in. = 50 mi. Find the actual distance.

Let d represent the actual distance.

$\frac{\text{map distance (in.)}}{\text{actual distance (mi.)}} = \frac{1 \text{ in.}}{50 \text{ mi}}$

$\frac{3\frac{1}{2}}{d} = \frac{1}{50}$ Substitute.

$d = 50(3\frac{1}{2})$ Cross-Product Property

$d = 175$ Simplify.

The cities are actually 175 miles apart.

Quick Check

3. Solve each proportion.

a. $\frac{5}{z} = \frac{20}{3}$

$z = 0.75$

b. $\frac{18}{n+6} = \frac{6}{n}$

$n = 3$

4. Recall Example 4. You want to make a new map with a scale of 1 in. = 35 mi. Two cities that are actually 175 miles apart are to be represented on your map. What would be the distance in inches between the cities on the new map?

5 inches

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Lesson 7-2 Similar Polygons

Lesson Objectives	NAEP 2005 Strand: Geometry and Measurement
1 Identify similar polygons 2 Apply similar polygons	Topics: Transformation of Shapes and Preservation of Properties; Measuring Physical Attributes Local Standards: ____________

Vocabulary

Similar figures have the same shape but not necessarily the same size. Two polygons are similar if corresponding angles are congruent and corresponding sides are proportional.

The mathematical symbol for similarity is ~.

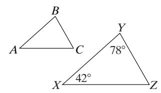
The similarity ratio is the ratio of the lengths of corresponding sides of similar figures.

A golden rectangle is a rectangle that can be divided into a square and a rectangle that is similar to the original rectangle.

The golden ratio is the ratio of the length to the width of any golden rectangle, about 1.618 : 1.

Example

1 Understanding Similarity $\triangle ABC \sim \triangle XYZ$. Complete each statement.



a. $m\angle B = \blacksquare$

$\angle B \cong \angle Y$ and $m\angle Y = 78$, so $m\angle B = 78$ because congruent angles have the same measure.

b. $\frac{BC}{YZ} = \frac{\blacksquare}{XZ}$

$\overline{AC}$ corresponds to $\overline{XZ}$, so $\frac{BC}{YZ} = \frac{AC}{XZ}$.

Quick Check

1. Refer to the diagram for Example 1. Complete:

$m\angle A = 42$ and $\frac{BC}{YZ} = \frac{AB}{XY}$

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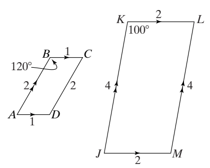
Examples

2 Determining Similarity Determine whether the parallelograms are similar. Explain.



Check that corresponding sides are proportional.

$\frac{AB}{JK} = \frac{2}{4}$ $\frac{BC}{KL} = \frac{1}{2}$

$\frac{CD}{LM} = \frac{2}{4}$ $\frac{DA}{MJ} = \frac{1}{2}$

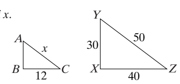
Corresponding sides of the two parallelograms are proportional.

Check that corresponding angles are congruent.

$\angle B$ corresponds to $\angle K$, but $m\angle B \neq m\angle K$, so corresponding angles are not congruent.

Although corresponding sides are proportional, the parallelograms are not similar because the corresponding angles are not congruent.

3 Using Similar Figures If $\triangle ABC \sim \triangle YXZ$, find the value of x.



Because $\triangle ABC \sim \triangle YXZ$, you can write and solve a proportion.

$\frac{AC}{YZ} = \frac{BC}{XZ}$ Corresponding sides are proportional.

$\frac{x}{50} = \frac{12}{40}$ Substitute.

$x = \frac{12}{40} \cdot 50$ Solve for x.

$x = 15$ Simplify.

4 Using the Golden Ratio The dimensions of a rectangular tabletop are in the Golden Ratio. The shorter side is 40 in. Find the longer side.

Let ℓ represent the longer side of the tabletop.

$\frac{\ell}{40} = \frac{1.618}{1}$ Write a proportion using the Golden Ratio.

$\ell = 64.72$ Cross-Product Property

The table is about 65 in. long.

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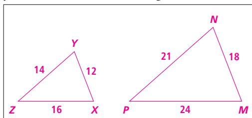
Quick Check

2. Sketch $\triangle XYZ$ and $\triangle MNP$ with $\angle X \cong \angle M$, $\angle Y \cong \angle N$, and $\angle Z \cong \angle P$. Also, $XY = 12$, $YZ = 14$, $ZX = 16$, $MN = 18$, $NP = 21$, and $PM = 24$. Can you conclude that the two triangles are similar?



Yes; corresponding angles are congruent and corresponding sides are proportional.

3. Refer to the diagram for Example 3. Find AB.

9

4. A golden rectangle has shorter sides of length 20 cm. Find the length of the longer sides.

about 32.4 cm

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Lesson 7-3

Proving Triangles Similar

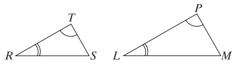
Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use AA, SAS, and SSS similarity statements 2 Apply AA, SAS, and SSS similarity statements	**Topic:** Transformation of Shapes and Preservation of Properties **Local Standards:** ____________

Vocabulary and Key Concepts

Postulate 7-1: Angle-Angle Similarity (AA~) Postulate

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

$\triangle TRS \sim \triangle$ **PLM**



Theorem 7-1: Side-Angle-Side Similarity (SAS~) Theorem

If an angle of one triangle is congruent to an angle of a second triangle, and the sides including the two angles are proportional, then **the triangles are similar.**

Theorem 7-2: Side-Side-Side Similarity (SSS~) Theorem

If the corresponding sides of two triangles are proportional, then **the triangles are similar.**

Indirect measurement is **a way of measuring things that are difficult to measure directly.**

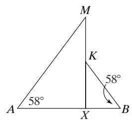
Examples

1 Using the AA~ Postulate $\overline{MX} \perp \overline{AB}$. Explain why the triangles are similar. Write a similarity statement.

Because $\overline{MX} \perp \overline{AB}$, $\angle AXM$ and $\angle BXK$ are **right angles**, so $\angle AXM \cong \angle$**BXK**.

$\angle A \cong \angle$**B** because their measures are equal.

$\triangle AMX \sim \triangle BKX$ by the **Angle-Angle Similarity (AA~)** Postulate.



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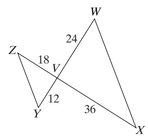
2 Using Similarity Theorems Explain why the triangles must be similar. Write a similarity statement.

$\angle YVZ \cong \angle WVX$ because **they are vertical angles.**

$\frac{VY}{VW} = \frac{12}{24} = \frac{\mathbf{1}}{\mathbf{2}}$ and $\frac{VZ}{VX} = \frac{\mathbf{18}}{\mathbf{36}} = \frac{\mathbf{1}}{\mathbf{2}}$,

so corresponding sides are proportional.

Therefore, $\triangle YVZ \sim \triangle WVX$ by the **Side-Angle-Side Similarity (SAS~)** Theorem.

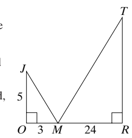


3 Using Similar Triangles Joan places a mirror 24 ft from the base of a tree. When she stands 3 ft from the mirror, she can see the top of the tree reflected in it. If her eyes are 5 ft above the ground, how tall is the tree?

TR represents the height of the tree, Point M represents the mirror, and point J represents Joan's eyes.

Both Joan and the tree are **perpendicular** to the ground, so $m\angle JOM = m\angle TRM$, and therefore **$\angle JOM \cong \angle TRM$**.

The light reflects off the mirror at the same angle at which it hits the mirror, so $\angle JMO \cong \angle$**TMR**.



Use similar triangles to find the height of the tree.

$\triangle JOM \sim \triangle TRM$ **AA~** Postulate

$\frac{TR}{JO} = \frac{\mathbf{RM}}{OM}$ Corresponding **sides of similar triangles are proportional.**

$\frac{TR}{\mathbf{5}} = \frac{\mathbf{24}}{\mathbf{3}}$ Substitute.

$TR = \frac{\mathbf{24}}{\mathbf{3}} \cdot \mathbf{5}$ Solve for TR.

$TR = \mathbf{40}$

The tree is **40** feet tall.

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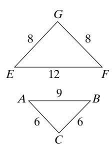
Quick Check

1. In Example 1, you have enough information to write a similarity statement. Do you have enough information to find the similarity ratio? Explain.

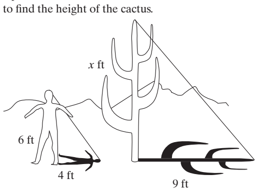
No; none of the side lengths are given.

2. Explain why the triangles at the right must be similar. Write a similarity statement.

$\frac{AC}{EG} = \frac{CB}{GF} = \frac{AB}{EF} = \frac{3}{4}$, so the triangles are similar by the SSS~ Theorem; $\triangle ABC \sim \triangle EFG$



3. In sunlight, a cactus casts a 9-ft shadow. At the same time, a person 6 ft tall casts a 4-ft shadow. Use similar triangles to find the height of the cactus.



13.5 ft

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Lesson 7-4

Similarity in Right Triangles

Lesson Objective	NAEP 2005 Strand: Geometry
1 Find and use relationships in similar right triangles	**Topic:** Transformation of Shapes and Preservation of Properties **Local Standards:** ____________

Vocabulary and Key Concepts

Theorem 7-3

The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are **similar** to the original triangle and **similar** to each other.

Corollary 1 to Theorem 7-3

The length of the altitude to the hypotenuse of a right triangle is the **geometric mean** of the lengths of the segments of the hypotenuse.

Corollary 2 to Theorem 7-3

The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the **adjacent hypotenuse segment** and the length of the **hypotenuse**.

The geometric mean of two positive numbers a and b **is the positive number x such that $\frac{a}{x} = \frac{x}{b}$.**

Examples

1 Finding the Geometric Mean Find the geometric mean of 3 and 12.

$\frac{3}{x} = \frac{\mathbf{x}}{\mathbf{12}}$ Write a proportion.

$x^2 = \mathbf{36}$ Cross-Product Property.

$x = \mathbf{\sqrt{36}}$ Find the positive square root.

$x = \mathbf{6}$

The geometric mean of 3 and 12 is **6**.

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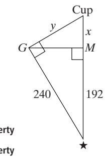
❷ Finding Distance At a golf course, Maria drove her ball 192 yd straight toward the cup. Her brother Gabriel drove his ball straight 240 yd, but not toward the cup. The diagram shows the results. Find x and y, their remaining distances from the cup.



Use Corollary 2 of Theorem 7-3 to solve for x.

$\frac{x + 192}{240} = \frac{240}{192}$ Write a proportion.

$192(x + 192) = 240^2$ Cross-Product Property

$192x + 36{,}864 = 57{,}600$ Distributive Property

$192x = 20{,}736$ Solve for x.

$x = 108$

Use Corollary 2 of Theorem 7-3 to solve for y.

$\frac{x + 192}{y} = \frac{y}{x}$ Write a proportion.

$\frac{108 + 192}{y} = \frac{y}{108}$ Substitute 108 for x.

$\frac{300}{y} = \frac{y}{108}$ Simplify.

$y^2 = 32{,}400$ Cross-Product Property

$y = \sqrt{32{,}400}$ Find the positive square root.

$y = 180$

Maria's ball is 108 yd from the cup, and Gabriel's ball is 180 yd from the cup.

Quick Check

1. Find the geometric mean of 15 and 20.

   $10\sqrt{3}$

2. Recall Example 2. Find the distance between Maria's ball and Gabriel's ball.

   144 yd

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Lesson 7-5 Proportions in Triangles

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use the Side-Splitter Theorem 2 Use the Triangle-Angle-Bisector Theorem	Topic: Transformation of Shapes and Preservation of Properties Local Standards: ____________

Key Concepts

Theorem 7-4: Side-Splitter Theorem

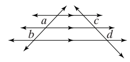
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.

Theorem 7-5: Triangle-Angle-Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle.

Corollary to Theorem 7-4

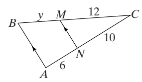
If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.



$\frac{a}{b} = \frac{c}{d}$

Examples

❶ Using the Side-Splitter Theorem Find y.



$\frac{CM}{MB} = \frac{CN}{NA}$ Side-Splitter Theorem

$\frac{12}{y} = \frac{10}{6}$ Substitute.

$10y = 72$ Cross-Product Property

$y = 7.2$ Solve for y.

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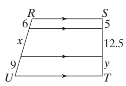
❷ Using the Corollary to the Side-Splitter Theorem The segments joining the sides of trapezoid $RSTU$ are parallel to its bases. Find x and y.



Because $RSTU$ is a trapezoid, $\overline{RS} \parallel \overline{UT}$.

$\frac{6}{x} = \frac{5}{12.5}$ Corollary to the Side-Splitter Theorem

$5x = 75$ Cross-Product Property

$x = 15$ Solve for x.

$\frac{x}{9} = \frac{12.5}{y}$ Corollary to the Side-Splitter Theorem

$\frac{15}{9} = \frac{12.5}{y}$ Substitute 15 for x.

$15y = 112.5$ Cross-Products Property

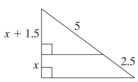
$y = 7.5$ Solve for y.

$x = 15$ and $y = 7.5$

Quick Check

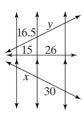
1. Use the Side-Splitter Theorem to find the value of x.

   1.5



2. Solve for x and y.

   $x = \frac{225}{13}$; $y = 28.6$



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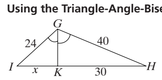
Example

❸ Using the Triangle-Angle-Bisector Theorem Find the value of x.



$\frac{IG}{GH} = \frac{IK}{HK}$ Triangle-Angle-Bisector Theorem.

$\frac{24}{40} = \frac{x}{30}$ Substitute.

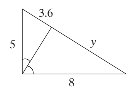
$\frac{24}{40}(30) = x$ Solve for x.

$x = 18$

Quick Check

3. Find the value of y.

   5.76



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Lesson 8-1

The Pythagorean Theorem and Its Converse

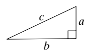
Lesson Objectives	NAEP 2005 Strand: Geometry
Use the Pythagorean Theorem	Topic: Relationships Among Geometric Figures
Use the Converse of the Pythagorean Theorem	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 8-1: Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the **legs** is equal to the square of the length of the **hypotenuse**.

$a^2 + b^2 = \boxed{c}^2$

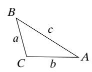


Theorem 8-2: Converse of the Pythagorean Theorem

If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a **right** triangle.

Theorem 8-3

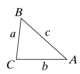
If the square of the length of the longest side of a triangle is greater than the sum of the squares of the lengths of the other two sides, the triangle is **obtuse**.



If $c^2 > a^2 + b^2$, the triangle is **obtuse**.

Theorem 8-4

If the square of the length of the longest side of a triangle is less than the sum of the squares of the lengths of the other two sides, the triangle is **acute**.



If $c^2 < a^2 + b^2$, the triangle is **acute**.

A Pythagorean triple is **a set of nonzero whole numbers *a*, *b*, and *c* that satisfy the equation $a^2 + b^2 = c^2$.**

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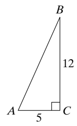
Example

1 Pythagorean Triples Find the length of the hypotenuse of $\triangle ABC$. Do the lengths of the sides of $\triangle ABC$ form a Pythagorean triple?



$a^2 + b^2 = c^2$ — Use the **Pythagorean** Theorem.

$\boxed{5^2} + \boxed{12^2} = c^2$ — Substitute **5** for *a*, and **12** for *b*.

$\boxed{169} = c^2$ — Simplify.

$\sqrt{\boxed{169}} = c$ — Take the square root.

$\boxed{13} = c$ — Simplify.

The length of the hypotenuse is **13**. The lengths of the sides, 5, 12, and **13**, form a **Pythagorean triple** because they are **whole** numbers that satisfy $a^2 + b^2 = c^2$.

Quick Check

1. A right triangle has a hypotenuse of length 25 and a leg of length 10. Find the length of the other leg. Do the lengths of the sides form a Pythagorean triple?

$5\sqrt{21}$; no

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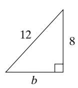
Example

2 Using Simplest Radical Form Find the value of *b*. Leave your answer in simplest radical form



$a^2 + b^2 = c^2$ — Use the **Pythagorean** Theorem.

$\boxed{8^2} + b^2 = \boxed{12^2}$ — Substitute **8** for *a*, **12** for *c*.

$\boxed{64} + b^2 = \boxed{144}$ — Simplify.

$b^2 = \boxed{80}$ — Subract **64** from both sides.

$b = \sqrt{\boxed{80}}$ — Take the square root.

$b = \sqrt{\boxed{16}(\boxed{5})}$ — Simplify.

$b = \boxed{4\sqrt{5}}$

Quick Check

2. The hypotenuse of a right triangle has length 12. One leg has length 6. Find the length of the other leg. Leave your answer in simplest radical form.

$6\sqrt{3}$

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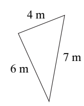
Examples

3 Is It a Right Triangle? Is this triangle a right triangle?



$a^2 + b^2 \stackrel{?}{=} c^2$

$\boxed{4^2} + \boxed{6^2} \stackrel{?}{=} \boxed{7^2}$ — Substitute **4** for *a*, **6** for *b*, and **7** for *c*.

$\boxed{16} + \boxed{36} \stackrel{?}{=} \boxed{49}$ — Simplify.

$\boxed{52} \neq \boxed{49}$

Because $a^2 + b^2 \boxed{\neq} c^2$, the triangle **is not** a right triangle.

4 Classifying Triangles as Acute, Obtuse, or Right The numbers 10, 15, and 20 represent the lengths of the sides of a triangle. Classify the triangle as acute, obtuse, or right.

$c^2 \stackrel{?}{=} a^2 + b^2$ — Compare c^2 with $a^2 + b^2$.

$20^2 \stackrel{?}{=} \boxed{10^2} + \boxed{15^2}$ — Substitute the greatest length for **c**.

$\boxed{400} \stackrel{?}{=} \boxed{100} + \boxed{225}$ — Simplify.

$\boxed{400} > \boxed{325}$

Because $c^2 \boxed{>} a^2 + b^2$, the triangle is a(n) **obtuse** triangle.

Quick Check

3. A triangle has sides of lengths 16, 48, and 50. Is the triangle a right triangle?

no

4. A triangle has sides of lengths 7, 8, and 9. Classify the triangle by its angles.

acute

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Lesson 8-2

Special Right Triangles

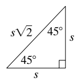
Lesson Objectives	NAEP 2005 Strand: Geometry
1. Use the properties of 45°-45°-90° triangles	Topic: Relationships Among Geometric Figures
2. Use the properties of 30°-60°-90° triangles	Local Standards: ____________

Key Concepts

Theorem 8-5: 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, both legs are **congruent** and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

hypotenuse = $\sqrt{2}$ · **leg**

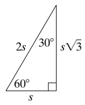


Theorem 8-6: 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse is **twice** the length of the **shorter leg**. The length of the longer leg is $\sqrt{3}$ times the length of the **shorter leg**.

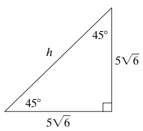
hypotenuse = **2** · **shorter leg**

longer leg = $\sqrt{3}$ · **shorter leg**



Examples

1 Finding the Length of the Hypotenuse Find the value of the variable. Use the 45°-45°-90° Triangle Theorem to find the hypotenuse.



$h = \sqrt{2} \cdot 5\sqrt{6}$ — hypotenuse = $\sqrt{2}$ · leg

$h = 5\sqrt{12}$ — Simplify.

$h = 5\sqrt{(4)(3)}$

$h = 5(2)\sqrt{3}$

$h = 10\sqrt{3}$

The length of the hypotenuse is $10\sqrt{3}$.

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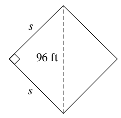
2 Applying the 45°-45°-90° Triangle Theorem The distance from one corner to the opposite corner of a square playground is 96 ft. To the nearest foot, how long is each side of the playground?

The distance from one corner to the opposite corner, 96 ft, is the length of the hypotenuse of a 45°-45°-90° triangle.



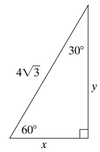
$96 = \sqrt{2} \cdot \text{leg}$ — hypotenuse = $\sqrt{2}$ · leg

$\text{leg} = \frac{96}{\sqrt{2}}$ — Divide each side by $\sqrt{2}$.

leg = 67.882251 — Use a calculator.

Each side of the playground is about **68** ft.

3 Using the Length of One Side Find the value of each variable. Use the 30°-60°-90° Triangle Theorem to find the lengths of the legs.



$4\sqrt{3} = 2 \cdot x$ — hypotenuse = 2 · shorter leg

$x = \frac{4\sqrt{3}}{2}$ — Divide each side by 2.

$x = 2\sqrt{3}$ — Simplify.

$y = \sqrt{3}$ · shorter leg — **30°-60°-90°** Triangle Theorem

$y = \sqrt{3} \cdot 2\sqrt{3}$ — Substitute $2\sqrt{3}$ for shorter leg.

$y = 2 \cdot \sqrt{3} \cdot \sqrt{3}$ — Simplify.

$y = 6$

The length of the shorter leg is $2\sqrt{3}$, and the length of the longer leg is **6**.

Quick Check

1. Find the length of the hypotenuse of a 45°-45°-90° triangle with legs of length $5\sqrt{3}$.

   $5\sqrt{6}$

2. A square garden has sides of 100 ft long. You want to build a brick path along a diagonal of the square. How long will the path be? Round your answer to the nearest foot.

   141 ft

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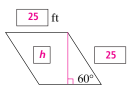
Example

4 Applying the 30°-60°-90° Triangle Theorem A rhombus-shaped garden has a perimeter of 100 ft and a 60° angle. Find the area of the garden to the nearest square foot.



Because a rhombus has four sides of equal length, each side is **25** ft.

Because a rhombus is also a parallelogram, you can use the area formula $A = bh$. Draw the altitude h, and then solve for h.

The height h is the longer leg of the right triangle. To find the height h, you can use the properties of 30°-60°-90° triangles. Then apply the area formula.

$25 = 2 \cdot \text{shorter leg}$ — hypotenuse = 2 · shorter leg

$\text{shorter leg} = \frac{25}{2} = 12.5$ — Divide each side by 2.

$h = 12.5\sqrt{3}$ — longer leg = $\sqrt{3}$ · shorter leg

$A = bh$ — Use the formula for the area of a **parallelogram**.

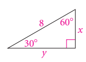
$A = (25)(12.5\sqrt{3})$ — Substitute 25 for b and $12.5\sqrt{3}$ for h.

$A = 541.26588$ — Use a calculator.

To the nearest square foot, the area is **541** ft^2.

Quick Check

3. Find the value of each variable.



   $x = 4, y = 4\sqrt{3}$

4. A rhombus has 10-in. sides, two of which meet to form the indicated angle. Find the area of each rhombus. (*Hint:* Use a special right triangle to find height.)

   a. a 30° angle — $50\ \text{in.}^2$

   b. a 60° angle — $50\sqrt{3}\ \text{in.}^2$

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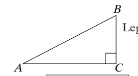
Lesson 8-3

The Tangent Ratio

Lesson Objective	NAEP 2005 Strand: Measurement
1. Use tangent ratios to determine side lengths in triangles	Topic: Measuring Physical Attributes
	Local Standards: ____________

Vocabulary

The tangent of acute ∠A in a right triangle is **the ratio of the length of the leg opposite ∠A to the length of the leg adjacent to ∠A.**



Leg **opposite** ∠A

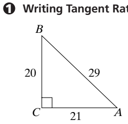
Leg **adjacent to** ∠A

tangent of ∠A = $\frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$

You can abbreviate the equation as $\tan A = \frac{\text{opposite}}{\text{adjacent}}$.

Examples

1 Writing Tangent Ratios Write the tangent ratios for ∠A and ∠B.



$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC} = \frac{20}{21}$

$\tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{AC}{BC} = \frac{21}{20}$

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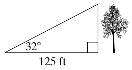
❷ Using a Tangent Ratio To measure the height of a tree, Alma walked 125 ft from the tree and measured a 32° angle from the ground to the top of the tree. Estimate the height of the tree.



The tree forms a **right** angle with the ground, so you can use the tangent ratio to estimate the height of the tree.

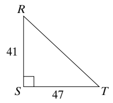
$\tan 32° = \frac{\text{height}}{125}$ — Use the tangent ratio.

height = **125** (**tan 32°**) — Solve for height.

125 TAN 32 ENTER 78.108669 — Use a calculator.

The tree is about **78** feet tall.

❸ Using the Inverse of Tangent Find $m\angle R$ to the nearest degree.



$\tan R = \frac{47}{41}$

$m\angle R \approx \tan^{-1}\left(\frac{47}{41}\right)$

47 ÷ 41 ENTER 1.14634146

TAN⁻¹ 1.14634146 ENTER 48.900494

So $m\angle R \approx$ **49**.

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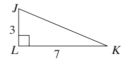
Quick Check

1. a. Write the tangent ratios for ∠K and ∠J.

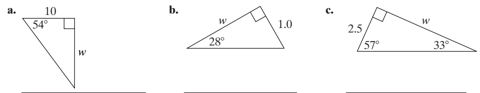


$\frac{3}{7}, \frac{7}{3}$

b. How is tan K related to tan J?

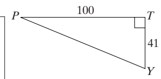
They are reciprocals.

2. Find the value of w to the nearest tenth.



a. **13.8** b. **1.9** c. **3.8**

3. Find $m\angle Y$ to the nearest degree.



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Lesson 8-4 Sine and Cosine Ratios

Lesson Objective	NAEP 2005 Strand: Measurement
▼ Use sine and cosine to determine side lengths in triangles	Topic: Measuring Physical Attributes
	Local Standards: ____________

Vocabulary

The sine of ∠A is **the ratio of the length of the leg opposite ∠A to the length of the hypotenuse.**

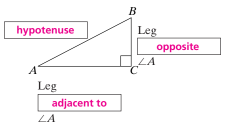
The cosine of ∠A is **the ratio of the length of the leg adjacent to ∠A to the hypotenuse.**

sine of ∠A = $\frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$

This can be abbreviated sin A = $\frac{\text{opposite}}{\text{hypotenuse}}$

cosine of ∠A = $\frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$

This can be abbreviated cos A = $\frac{\text{adjacent}}{\text{hypotenuse}}$

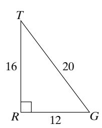


An identity is **an equation that is true for all allowed values of the variable.**

An example of a trigonometric identity is sin x° = cos **(90 − x)°**.

Examples

❶ Writing Sine and Cosine Ratios Use the triangle to find sin T, cos T, sin G, and cos G. Write your answers in simplest terms.



$\sin T = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{20} = \frac{3}{5}$

$\cos T = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{16}{20} = \frac{4}{5}$

sin $G = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{16}{20} = \frac{4}{5}$

cos $G = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{20} = \frac{3}{5}$

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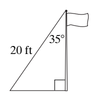
❷ Using the Cosine Ratio A 20-ft wire supporting a flagpole forms a 35° angle with the flagpole. To the nearest foot, how high is the flagpole?



The flagpole, wire, and ground form a **right triangle** with the wire as the **hypotenuse**. Because you know an angle and the measure of its hypotenuse, you can use the **cosine** ratio to find the height of the flagpole.

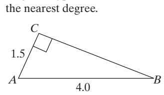
$\cos 35° = \frac{\text{height}}{20}$ — Use the cosine ratio.

height = **20** · **cos 35°** — Solve for height.

20 COS 35 ENTER 16.383041 — Use a calculator.

The flagpole is about **16** feet tall.

❸ Using the Inverse of Cosine and Sine A right triangle has a leg 1.5 units long and hypotenuse 4.0 units long. Find the measures of its acute angles to the nearest degree.



Use the inverse of the cosine function to find $m\angle A$.

$\cos A = \frac{1.5}{4.0} = 0.375$ — Use the cosine ratio.

m∠A = $\cos^{-1}$(**0.375**) — Use the inverse of the cosine.

COS⁻¹ 0.375 ENTER 67.975687 — Use a calculator.

$m\angle A \approx$ **68** — Round to the nearest degree.

To find $m\angle B$, use the fact that the acute angles of a right triangle are **complementary**.

$m\angle A + m\angle B =$ **90** — Definition of complementary angles

68 + $m\angle B \approx$ **90** — Substitute.

$m\angle B \approx$ **22** — Simplify.

The acute angles, rounded to the nearest degree, measure **68** and **22**.

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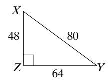
Quick Check

1. a. Write the sine and cosine ratios for ∠X and ∠Y.

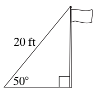


$\sin X = \frac{64}{80}$, $\cos X = \frac{48}{80}$, $\sin Y = \frac{48}{80}$, $\cos Y = \frac{64}{80}$

b. In general, how are sin X and cos Y related? Explain.

sin X = cos Y when ∠X and ∠Y are complementary.

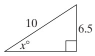
2. In Example 2, suppose that the angle the wire makes with the ground is 50°. What is the height of the flagpole to the nearest foot?



15 feet

3. Find the value of x. Round your answer to the nearest degree.

a.



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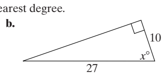
b.



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Lesson 8-5

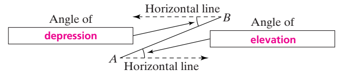
Angles of Elevation and Depression

Lesson Objective	NAEP 2005 Strand: Measurement
Use angles of elevation and depression to solve problems	Topic: Measuring Physical Attributes Local Standards: ____________

Vocabulary

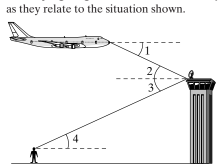
An angle of elevation is the angle formed by a horizontal line and the line of sight to an object above the horizontal line.

An angle of depression is the angle formed by a horizontal line and the line of sight to an object below the horizontal line.



Examples

1. **Identifying Angles of Elevation and Depression** Describe ∠1 and ∠2 as they relate to the situation shown.



One side of the angle of depression is a horizontal line. ∠1 is the angle of depression from the airplane to the building.

One side of the angle of elevation is a horizontal line. ∠2 is the angle of elevation from the building to the airplane.

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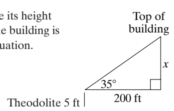
2. **Surveying** A surveyor stands 200 ft from a building to measure its height with a 5-ft tall theodolite. The angle of elevation to the top of the building is 35°. How tall is the building? Use a diagram to represent the situation.



$\tan 35° = \frac{x}{200}$ Use the tangent ratio.

$x = 200 \cdot \tan 35°$ Solve for x.

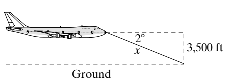
200 TAN 35 ENTER 140.041508 Use a calculator.

$x \approx 140$

To find the height of the building, add the height of the theodolite, which is 5 feet tall.

The building is about 140 ft + 5 ft, or 145 ft tall.

3. **Aviation** An airplane flying 3500 ft above the ground begins a 2° descent to land at the airport. How many miles from the airport is the airplane when it starts its descent? (Note: The angle is not drawn to scale.)



$\sin 2° = \frac{3{,}500}{x}$ Use the sine ratio.

$x = \frac{3500}{\sin 2°}$ Solve for x.

3500 ÷ SIN 2 ENTER 100287.9792 Use a calculator.

100287.9792 ÷ 5280 ENTER 18.993935 Divide by 5280 to convert feet to miles.

The airplane is about 19 miles from the airport when it starts its descent.

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Quick Check

1. Describe each angle as it relates to the situation in Example 1.

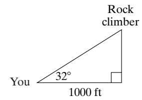
a. ∠3

The angle of depression from the building to the person on the ground

b. ∠4

The angle of elevation from the person on the ground to the building

2. You sight a rock climber on a cliff at a 32° angle of elevation. The horizontal ground distance to the cliff is 1000 ft. Find the line-of-sight distance to the rock climber.



about 1179 ft

3. An airplane pilot sees a life raft at a 26° angle of depression. The airplane's altitude is 3 km. What is the airplane's surface distance d from the raft?

about 6.8 km

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Lesson 8-6 — Vectors

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Describe vectors 2 Solve problems that involve vector addition	Topic: Position and Direction Local Standards: ____________

Vocabulary and Key Concepts

Adding Vectors

For $\vec{a} = \langle x_1, y_1 \rangle$ and $\vec{c} = \langle x_2, y_2 \rangle$, $\vec{a} + \vec{c} =$ $\langle x_1 + x_2, y_1 + y_2 \rangle$.

A vector is any quantity with magnitude (size) and direction.

A vector can be represented with an arrow.

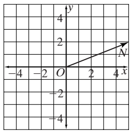
The magnitude of a vector is its size, or length.

The initial point of a vector is the point at which it starts.

The terminal point of a vector is the point at which it ends.

A resultant vector is the sum of other vectors.

The magnitude of vector $\overrightarrow{ON}$ is the distance from its initial point O to its terminal point N. The ordered pair notation for the vector is $\langle 5, 2 \rangle$.



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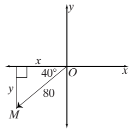
Examples

1 Describing a Vector Describe $\overrightarrow{OM}$ as an ordered pair.

Give coordinates to the nearest tenth.

Use the sine and cosine ratios to find the values of x and y.



$\cos 40° = \dfrac{x}{80}$ — Use sine and cosine. — $\sin 40° = \dfrac{y}{80}$

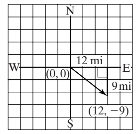
$x = 80(\cos 40°)$ — Solve for the variable. — $y = 80(\sin 40°)$

$x \approx 61.28355545$ — Use a calculator. — $y \approx 51.42300878$

Because point M is in the third quadrant, both coordinates are negative. To the nearest tenth, $\overrightarrow{OM} \approx \langle -61.3, -51.4 \rangle$.

2 Describing a Vector Direction A boat sailed 12 mi east and 9 mi south. The trip can be described by the vector $\langle 12, -9 \rangle$. Use distance and direction to describe this vector a second way.

Draw a diagram for the situation.



To find the distance sailed, use the Distance Formula.

$d = \sqrt{(12 - 0)^2 + (-9 - 0)^2}$ — Distance Formula

$d = \sqrt{144 + 81}$ — Simplify.

$d = \sqrt{225}$ — Simplify.

$d = 15$ — Take the square root.

To find the direction that the boat sails, find the angle that the vector forms with the x-axis.

$\tan x° = \dfrac{9}{12} = 0.75$ — Use the tangent ratio.

$x = \tan^{-1}(0.75)$ — Find the angle whose tangent is 0.75.

TAN⁻¹ 0.75 ENTER 36.869898 — Use a calculator.

The boat sailed 15 miles at about 37° south of east.

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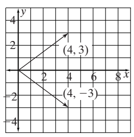
3 Adding Vectors Vectors $\vec{v}\,\langle 4, 3 \rangle$ and $\vec{w}\,\langle 4, -3 \rangle$ are shown at the right. Write the sum of the two vectors as an ordered pair. Then draw $\vec{s}$, the sum of $\vec{v}$ and $\vec{w}$.



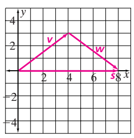
To find the first coordinate of $\vec{s}$, add the first coordinates of $\vec{v}$ and $\vec{w}$.

To find the second coordinate of $\vec{s}$, add the second coordinates of $\vec{v}$ and $\vec{w}$.

$\vec{s} = \langle 4, 3 \rangle + \langle 4, -3 \rangle$

$= \langle 4 + 4, 3 + (-3) \rangle$ — Add the coordinates.

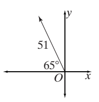
$= \langle 8, 0 \rangle$ — Simplify.



Draw vector $\vec{v}$ using the origin as the initial point. Draw vector $\vec{w}$ using the terminal point of $\vec{v}$, $(4, 3)$, as the initial point. Draw the resultant vector $\vec{s}$ from the initial point of $\vec{v}$ to the terminal point of $\vec{w}$.

Quick Check

1. Describe the vector at the right as an ordered pair. Give the coordinates to the nearest tenth.



$\langle -21.6, 46.2 \rangle$

2. A small airplane lands 246 mi east and 76 mi north of the point from which it took off. Describe the magnitude and the direction of its flight vector.

about 257 mi at 17° N of E

3. Write the sum of the two vectors $\langle 2, 3 \rangle$ and $\langle -4, -2 \rangle$ as an ordered pair.

$\langle -2, 1 \rangle$

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Lesson 9-1 — Translations

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Identify isometries 2 Find translation images of figures	Topic: Transformation of Shapes and Preservation of Properties; Position and Direction Local Standards: ____________

Vocabulary

A transformation of a geometric figure is a change in its position, shape, or size.

In a transformation, the preimage is the original image before changes are made.

In a transformation, the image is the resulting figure after changes are made.

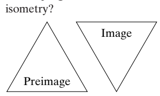
An isometry is a transformation in which the preimage and the image are congruent.

A translation (slide) is a transformation that maps all points the same distance and in the same direction.

A composition of transformations is a combination of two or more transformations.

Examples

1 Identifying Isometries Does the transformation appear to be an isometry?



The image appears to be the same as the preimage, but turned. Because the figures appear to be congruent, the transformation appears to be an isometry.

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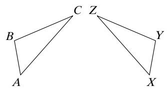
❷ **Naming Images and Corresponding Parts** In the diagram, $\triangle XYZ$ is an image of $\triangle ABC$.



a. Name the images of $\angle B$ and $\angle C$.

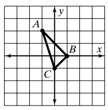
Because corresponding vertices of the preimage and the image are listed in the same order, the image of $\angle B$ is $\angle$ [Y], and the image of $\angle C$ is $\angle$ [Z].

b. List all pairs of corresponding sides.

Because corresponding sides of the preimage and the image are listed in the same order, the following pairs are corresponding sides:

$\overline{AB}$ and [$\overline{XY}$], $\overline{AC}$ and [$\overline{XZ}$], $\overline{BC}$ and [$\overline{YZ}$].

❸ **Finding a Translation Image** Find the image of $\triangle ABC$ under the translation $(x, y) \to (x + 2, y - 3)$.



$A(-1, 2) \to A'($[$-1 + 2$], [2] $- 3)$ **Use the rule $(x, y) \to (x + 2, y - 3)$**

$B(1, 0) \to B'($[1] $+ 2$, [$0 - 3$]$)$

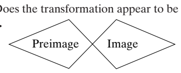
$C(0, -1) \to C'($[$0 + 2$], [$-1 - 3$]$)$

The image of $\triangle ABC$ is $\triangle A'B'C'$ with $A'($[1], [-1]$)$, $B'($[3], [-3]$)$, and $C'($[2], [-4]$)$.

Quick Check

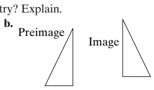
1. Does the transformation appear to be an isometry? Explain.

a.



[Yes; the figures appear to be congruent by a flip.]

b.

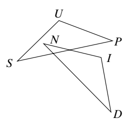


[Yes; the figures appear to be congruent by a flip and a slide.]

2. In the diagram, $NID \to SUP$.

a. Name the images of $\angle I$ and point D.

[$\angle U$, P]



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b. List all pairs of corresponding sides.

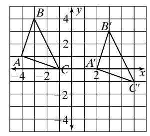
[$\overline{NI}$ and $\overline{SU}$; $\overline{ID}$ and $\overline{UP}$; $\overline{ND}$ and $\overline{SP}$]

3. Find the image of $\triangle ABC$ for the translation $(x - 2, y + 1)$.

[$A'(-3, 3)$, $B'(-1, 1)$, $C'(-2, 0)$]

Examples

❹ **Writing a Rule to Describe a Translation** Write a rule to describe the translation $\triangle ABC \to \triangle A'B'C'$.



You can use any point on $\triangle ABC$ and its image on $\triangle A'B'C'$ to describe the translation. Using $A(-4, 1)$ and its image [$A'(2, 0)$], the horizontal change is $2 -$ ([-4]), or [6], and the vertical change is $0 -$ [1], or [-1].

The rule is $(x, y) \to$ [$(x + 6, y - 1)$].

❺ **Adding Translations** Tritt rides his bicycle 3 blocks north and 5 blocks east of a pharmacy to deliver a prescription. Then he rides 4 blocks south and 8 blocks west to make a second delivery. How many blocks is he now from the pharmacy?

The rule [$(x, y) \to (x + 5, y + 3)$] represents a ride of 3 blocks north and 5 blocks east.

The rule [$(x, y) \to (x - 8, y - 4)$] represents a ride of 4 blocks south and 8 blocks west.

Tritt's position after the second delivery is the [composition] of the two translations.

([0], [0]) translates to ([0] + [5], [0] + [3]) or ([5], [3]). Then, ([5], [3]) translates to ([5] + [-8], [3] + [-4]) or ([-3], [-1]).

Tritt is [1] block [south] and [3] blocks [west] of the pharmacy.

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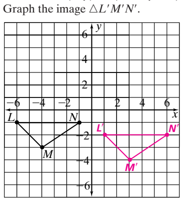
Quick Check

4. Use the rule $(x, y) \to (x + 7, y - 1)$ to find the translation image of $\triangle LMN$. Graph the image $\triangle L'M'N'$.



5. Refer to Example 5. Tritt now makes a third delivery. Starting from where the second delivery was, he goes 5 blocks north and 1 block west. How many blocks is Tritt from the pharmacy?

[He is 4 blocks north and 4 blocks west of the pharmacy.]

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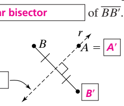
Lesson 9-2 Reflections

Lesson Objectives	NAEP 2005 Strand: Geometry
▼ Find reflection images of figures	**Topics:** Transformation of Shapes and Preservation of Properties
	Local Standards: ____________

Vocabulary

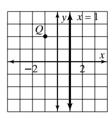
A reflection in line r is a transformation such that if a point A is on line r, then the image of A is [itself], and if a point B is not on line r, then its image B' is the point such that r is the [perpendicular bisector] of $\overline{BB'}$.



Line of [reflection]

Example

❶ **Finding Reflection Images** If point $Q(-1, 2)$ is reflected across line $x = 1$, what are the coordinates of its reflection image?



Q is [2] units to the [left] of the reflection line, so its image Q' is [2] units to the [right] of the reflection line. The reflection line is the perpendicular bisector of $\overline{QQ'}$ if Q' is at [(3, 2)].

Quick Check

1. What are the coordinates of the image of Q if the reflection line is $y = -1$?

[$(-1, -4)$]

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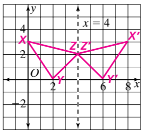
Examples

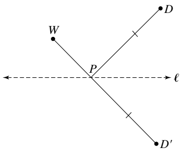
2 Drawing Reflection Images $\triangle XYZ$ has vertices $X(0, 3)$, $Y(2, 0)$, and $Z(4, 2)$. Draw $\triangle XYZ$ and its reflection image in the line $x = 4$. First locate vertices X, Y, and Z and draw $\triangle XYZ$ in a coordinate plane.



Locate points X', Y', and Z' such that the line of reflection $x = 4$ is the [perpendicular bisector] of $\overline{XX'}$, $\overline{YY'}$, and $\overline{ZZ'}$.

Draw the reflection image $X'Y'Z'$.

3 Congruent Angles D' is the reflection of D across ℓ. Show that $\overline{PD}$ and $\overline{PW}$ form congruent angles with line ℓ.



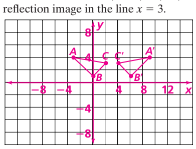
Because a reflection is an [isometry], $\overline{PD}$ and $\overline{PD'}$ form congruent angles with line ℓ.

$\overline{PD'}$ and $\overline{PW}$ also form congruent angles with line ℓ because [vertical] angles are congruent.

Therefore, $\overline{PD}$ and $\overline{PW}$ form congruent angles with line ℓ by the [Transitive] Property.

Quick Check

2. $\triangle ABC$ has vertices $A(-3, 4)$, $B(0, 1)$, and $C(2, 3)$. Draw $\triangle ABC$ and its reflection image in the line $x = 3$.



3. In Example 3, what kind of triangle is $\triangle DPD'$? Imagine the image of point W reflected across line ℓ. What can you say about $\triangle WPW'$ and $\triangle DPD'$?

isosceles; they are similar by SAS

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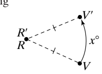
Lesson 9-3 Rotations

Lesson Objective	NAEP 2005 Strand: Geometry
Draw and identify rotation images of figures	Topic: Transformation of Shapes and Preservation of Properties
	Local Standards: ____________________

Vocabulary

A rotation of $x°$ about a point R is a transformation for which the following are true:



- The image of R is [itself] (that is, $R' =$ [R]).
- For any point V, $RV' = RV$ and $m\angle VRV' =$ [x].

The center of a regular polygon is a point that is [equidistant] from the polygon's [vertices].

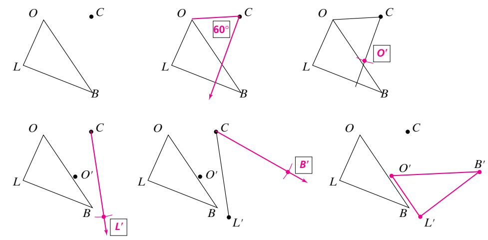
Examples

1 Drawing a Rotation Image Draw the image of $\triangle LOB$ under a 60° rotation about C.

Step 1 Use a protractor to draw a 60° angle at vertex C with one side $\overline{CO}$.

Step 2 Use a compass to construct $\overline{CO'} \cong \overline{CO}$.

Step 3 Locate L' and B' in a similar manner. Then draw $\triangle L'O'B'$.



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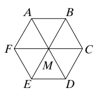
2 Identifying a Rotation Image Regular hexagon $ABCDEF$ is divided into six equilateral triangles.



a. Name the image of B for a 240° rotation about M.

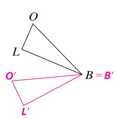
Because $360 \div 6 =$ [60], each central angle of $ABCDEF$ measures [60°]. A 240° counterclockwise rotation about center M moves point B across [four] triangles. The image of point B is point [D].

b. Name the image of M for a 60° rotation about F.

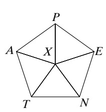
$\triangle AMF$ is equilateral, so $\angle AFM$ has measure $180 \div 3 =$ [60]. A 60° rotation of $\triangle AMF$ about point F would superimpose $\overline{FM}$ on $\overline{FA}$, so the image of M under a 60° rotation about point F is point [A].

Quick Check

1. Draw the image of LOB for a 50° rotation about point B. Label the vertices of the image.



2. Regular pentagon $PENTA$ is divided into 5 congruent triangles. Name the image of T for a 144° rotation about point X.



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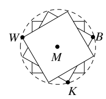
Examples

3 Finding an Angle of Rotation A regular 12-sided polygon can be formed by stacking congruent square sheets of paper rotated about the same center on top of each other. Find the angle of rotation about M that maps W to B.



Consecutive vertices of the three squares form the outline of a regular 12-sided polygon.

$360 \div 12 =$ [30], so each vertex of the polygon is a [30°] rotation about point M.

You must rotate counterclockwise through 7 vertices to map point W to point B, so the angle of rotation is $7 \cdot$ [30]° = [210]°.

4 Compositions of Rotations Describe the image of quadrilateral $XYZW$ for a composition of a 145° rotation and then a 215° rotation, both about point X.

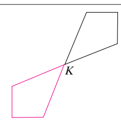
Two rotations of 145° and 215° about the same point make a total rotation of 145° + 215°, or [360]°. Because this forms a [complete rotation] about point X, the image is [the preimage $XYZW$].

Quick Check

3. In the figure from Example 3, find the angle of rotation about M that maps B to K.

270°

4. Draw the image of the kite for a composition of two 90° rotations about point K.



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Lesson 9-4

Symmetry

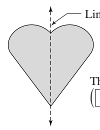
Lesson Objective	NAEP 2005 Strand: Geometry
Identify the type of symmetry in a figure	Topic: Transformation of Shapes and Preservation of Properties Local Standards: ____________

Vocabulary

A figure has symmetry if there is an isometry that maps the figure onto itself.

A figure has reflectional symmetry if there is a reflection that maps the figure onto itself.

Line symmetry is the same as reflectional symmetry.

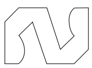


Line of Symmetry

The heart-shaped figure has reflectional (line) symmetry.

A figure has rotational symmetry if it is its own image for some rotation of 180° or less.

A figure has point symmetry if it has 180° rotational symmetry.



The figure is its own image after one half-turn, so it has rotational symmetry with a 180° angle of rotation.

The figure also has point symmetry.

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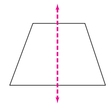
Examples

1. **Identifying Lines of Symmetry** Draw all lines of symmetry for the isosceles trapezoid.



Draw any lines that divide the isosceles trapezoid so that half of the figure is a mirror image of the other half.

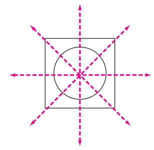
There is one line of symmetry.

2. **Identifying Rotational Symmetry** Judging from appearance, do the letters V and H have rotational symmetry? If so, give an angle of rotation.

The letter V does not have rotational symmetry because it must be rotated 360° before it is its own image.

The letter H is its own image after one half-turn, so it has rotational symmetry with a 180° angle of rotation.

3. **Finding Symmetry** A nut holds a bolt in place. Some nuts have square faces, like the top view shown below. Tell whether the nut has rotational symmetry and/or reflectional symmetry. Draw all lines of symmetry.



The nut has a square outline with a circular opening. The square and circle are concentric.

The nut is its own image after one quarter-turn, so it has 90° rotational symmetry.

The nut has 4 lines of symmetry.

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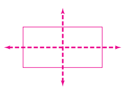
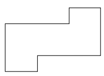
Quick Check

1. Draw a rectangle and all of its lines of symmetry.



2. a. Judging from appearance, tell whether the figure at the right has rotational symmetry. If so, give the angle of rotation.

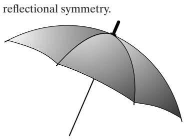


yes; 180°

b. Does the figure have point symmetry?

yes

3. Tell whether the umbrella has rotational symmetry about a line and/or reflectional symmetry.



The umbrella has both rotational and reflectional symmetry.

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Lesson 9-5

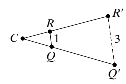
Dilations

Lesson Objective	NAEP 2005 Strand: Geometry
Locate dilation images of figures	Topic: Transformation of Shapes and Preservation of Properties Local Standards: ____________

Vocabulary

A dilation is a transformation with center C and scale factor n for which the following are true:

- The image of C is itself (that is, $C' = C$).
- For any point R, R' is on $\overrightarrow{CR}$ and $CR' = n \times CR$.



$\overline{R'Q'}$ is the image of $\overline{RQ}$ under a dilation with center C and scale factor 3.

An enlargement is a dilation with a scale factor greater than 1.

A reduction is a dilation with a scale factor less than 1.

Examples

1. **Finding a Scale Factor** Circle A with 3-cm diameter and center C is a dilation of concentric circle B with 8-cm diameter. Describe the dilation.

The circles are concentric, so the dilation has center C.

Because the diameter of the dilation image is smaller, the dilation is a reduction.

Scale factor: $\frac{\text{diameter of dilation image}}{\text{diameter of preimage}} = \frac{3}{8}$

The dilation is a reduction with center C and scale factor $\frac{3}{8}$.

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❷ Scale Drawings The scale factor on a museum's floor plan is 1 : 200. The length and width of one wing on the drawing are 8 in. and 6 in. Find the actual dimensions of the wing in feet and inches.

The floor plan is a reduction of the actual dimensions by a scale factor of $\frac{1}{200}$.

Multiply each dimension on the drawing by 200 to find the actual dimensions. Then write the dimensions in feet and inches.

8 in. × 200 = **1600** in. = **133** ft, **4** in.

6 in. × 200 = **1200** in. = **100** ft

The museum wing measures **133 ft, 4 in.** by **100 ft**.

❸ Graphing Dilation Images $\triangle ABC$ has vertices $A(-2, -3)$, $B(0, 4)$, and $C(6, -12)$. What are the coordinates of the image of $\triangle ABC$ for a dilation with center $(0, 0)$ and scale factor 0.75?

$A(-2, -3) \rightarrow A'(0.75 \cdot$ **(−2)**, **0.75 · (−3)**) The scale factor is **0.75**, so use

$B(0, 4) \rightarrow B'($**0.75 · 0**, $0.75 \cdot$ **4**) the rule $(x, y) \rightarrow$ **(0.75x, 0.75y)**.

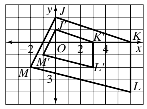
$C(6, -12) \rightarrow C'($**0.75 · 6**, **0.75 · (−12)**)

The vertices of the reduction image of $\triangle ABC$ are A' **(−1.5, −2.25)**, B' **(0, 3)**, and C' **(4.5, −9)**.

Name______________________ Class__________________ Date____________

Quick Check

1. Quadrilateral $J'K'L'M'$ is a dilation image of quadrilateral $JKLM$. Describe the dilation.

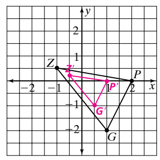


The dilation is a reduction with center (0, 0) and scale factor $\frac{1}{2}$.

2. The height of a tractor-trailer truck is 4.2 m. The scale factor for a model truck is $\frac{1}{54}$. Find the height of the model to the nearest centimeter.

8 cm

3. Find the image of $\triangle PZG$ for a dilation with center $(0, 0)$ and scale factor $\frac{1}{2}$. Draw the reduction on the grid and give the coordinates of the image's vertices.



Vertices: P' **(1, 0)**, Z' **$\left(-\frac{1}{2}, \frac{1}{4}\right)$**, and G' **$\left(\frac{1}{2}, -1\right)$**

Name______________________ Class__________________ Date____________

Lesson 9-6

Compositions of Reflections

Lesson Objectives	NAEP 2005 Strand: Geometry
▼ Use a composition of reflections	Topic: Transformation of Shapes and Preservation of Properties
▼ Identify glide reflections	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 9-1

A translation or rotation is a composition of two **reflections**.

Theorem 9-2

A composition of reflections across two parallel lines is a **translation**.

Theorem 9-3

A composition of reflections across two intersecting lines is a **rotation**.

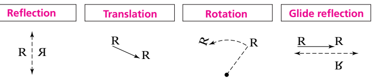
Theorem 9-4: Fundamental Theorem of Isometries

In a plane, one of two congruent figures can be mapped onto the other by a composition of at most three **reflections**.

Theorem 9-5: Isometry Classification Theorem

There are only four isometries. They are the following:

Reflection **Translation** **Rotation** **Glide reflection**



A glide reflection **is the composition of a glide (translation) and a reflection across a line parallel to the direction of translation.**

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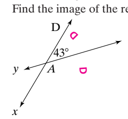
Examples

❶ Composition of Reflections in Intersecting Lines The letter D is reflected in line x and then in line y. Describe the resulting rotation.

Find the image of D through a reflection across line x.

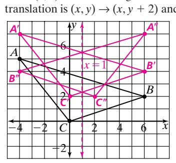
Find the image of the reflection through another reflection across line y.



The composition of two reflections across intersecting lines is a **rotation**.

The center of rotation is **the point where the lines intersect**, and the angle is **twice the angle formed by the intersecting lines**. So, the letter D is rotated **86**° clockwise, or **274**° counterclockwise, with the center of rotation at point **A**.

❷ Finding a Glide Reflection Image $\triangle ABC$ has vertices $A(-4, 5)$, $B(6, 2)$, and $C(0, 0)$. Find the image of $\triangle ABC$ for a glide reflection where the translation is $(x, y) \rightarrow (x, y + 2)$ and the reflection line is $x = 1$.



First, translate $\triangle ABC$ by $(x, y) \rightarrow (x, y + 2)$.

$(-4, 5) \rightarrow (-4 +$ **0**, $5 +$ **2**), or (**−4**, **7**)

$(6, 2) \rightarrow (6 +$ **0**, $2 +$ **2**), or (**6**, **4**)

$(0, 0) \rightarrow (0 +$ **0**, $0 +$ **2**), or (**0**, **2**)

Then, reflect the translated image across the line $x =$ **1**.

The glide reflection image $\triangle A''B''C''$ has vertices A'' **(6, 7)**, B'' **(−4, 4)**, and C'' **(2, 2)**.

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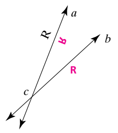
Quick Check

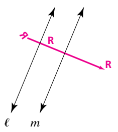
1. a. Reflect the letter R across *a* and then *b*. Describe the resulting rotation.

Answers may vary. The result is a clockwise rotation about point *c* through an angle of $2m\angle acb$.



b. Use parallel lines ℓ and *m*. Draw R between ℓ and *m*. Find the image of R for a reflection across line ℓ and then across line *M*. Describe the resulting translation.

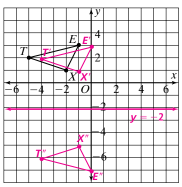
Answers may vary. The result is a translation twice the distance between ℓ and *m*.



2. a. Find the image of $\triangle TEX$ under a glide reflection where the translation is $(x, y) \rightarrow (x + 1, y)$ and the reflection line is $y = -2$. Draw the translation first, then the reflection.

b. Would the result of part (a) be the same if you reflected $\triangle TEX$ first, and then translated it? Explain.

yes; Order of steps does not matter in a glide reflection.



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Lesson 9-7

Tessellations

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Identify transformation in tessellations, and figures that will tessellate	Topic: Geometry
2 Identify symmetries in tessellations	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 9-6
Every triangle tessellates.

Theorem 9-7
Every quadrilateral tessellates.

A tessellation, or tiling, is a repeating pattern of figures that completely covers a plane, without gaps or overlaps.

Translational symmetry is the type of symmetry for which there is a translation that maps a figure onto itself.

Glide reflectional symmetry is the type of symmetry for which there is a glide reflection that maps a figure onto itself.

Examples

1 **Determining Figures That Will Tessellate** Determine whether a regular 15-gon tessellates a plane. Explain.

Because the figures in a tessellation do not overlap or leave gaps, the sum of the measures of the angles around any vertex must be 360°. Check to see whether the measure of an angle of a regular 15-gon is a factor of 360.

$a = \dfrac{180(n - 2)}{n}$ Use the formula for the measure of an angle of a regular polygon.

$a = \dfrac{180(15 - 2)}{15}$ Substitute 15 for *n*.

$a = 156$ Simplify.

Because 156 is not a factor of 360, a regular 15-gon will not tessellate a plane.

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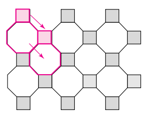
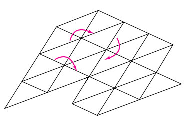
2 **Identifying the Transformation in a Tessellation** Identify the repeating figures and a transformation in the tessellation.

A repeated combination of an octagon and one adjoining square will completely cover the plane without gaps or overlaps. Use arrows to show a translation.

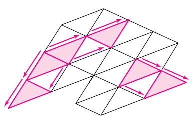


3 **Identifying Symmetries in Tessellations** List the symmetries in the tessellation.

Starting at any vertex, the tessellation can be mapped onto itself using a 180° rotation, so the tessellation has point symmetry centered at any vertex.



The tessellation also has translational symmetry, as can be seen by sliding any triangle onto a copy of itself along any of the lines.



Quick Check

1. Explain why you can tessellate a plane with an equilateral triangle.

The interior angles of an equilateral triangle measure 60°. Because 60 divides evenly into 360, it will tessellate.

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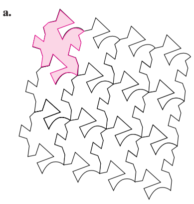
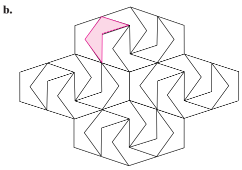
2. Identify a transformation and outline the smallest repeating figure in each tessellation below.

a.



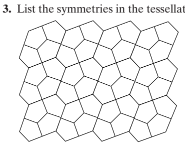
translation

b.



Answers may vary. Sample: rotation

3. List the symmetries in the tessellation.



line symmetry, rotational symmetry, glide reflectional symmetry, translational symmetry

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Lesson 10-1

Areas of Parallelograms and Triangles

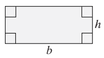
Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find the area of a parallelogram	Topic: Measuring Physical Attributes
2 Find the area of a triangle	Local Standards: ____________________

Vocabulary and Key Concepts

Theorem 10-1: Area of a Rectangle

The area of a rectangle is the product of its **base** and **height**.

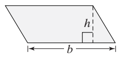
$A =$ **bh**



Theorem 10-2: Area of a Parallelogram

The area of a parallelogram is the product of a **base** and the corresponding **height**.

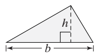
$A =$ **bh**



Theorem 10-3: Area of a Triangle

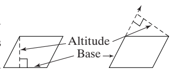
The area of a triangle is **half** the product of a **base** and the corresponding **height**.

$A =$ **$\frac{1}{2}bh$**



A base of a parallelogram is **any of its sides.**

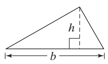
The altitude of a parallelogram corresponding to a given base is **the segment perpendicular to the line containing that base drawn from the side opposite the base.**



The height of a parallelogram is **the length of its altitude.**

A base of a triangle is **any of its sides.**

The height of a triangle is **the length of the altitude to the line containing that base.**



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Examples

1 Finding a Missing Dimension A parallelogram has 9-in. and 18-in. sides. The height corresponding to the 9-in. base is 15 in. Find the height corresponding to the 18-in. base.

First find the area of the parallelogram using the 9-in. base and its corresponding 15-in height.

$A = bh$ — Area of a parallelogram

$A = (\mathbf{9})(\mathbf{15})$ — Substitute **9** for b and **15** for h.

$A = \mathbf{135}$ — Simplify.

The area of the parallelogram is **135** in.^{2}

Use the area to find the height corresponding to the 18-in. base.

$A = \mathbf{bh}$ — Area of a parallelogram

$\mathbf{135} = \mathbf{18}\,h$ — Substitute **135** for A and **18** for b.

$\frac{\mathbf{135}}{\mathbf{18}} = h$ — Divide each side by **18**.

$\mathbf{7.5} = h$ — Simplify.

The height corresponding to the 18-in base is **7.5** in.

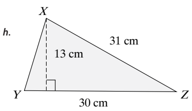
2 Finding the Area of a Triangle Find the area of $\triangle XYZ$.

$A = \frac{1}{2}bh$ — Area of a **triangle**

$A = \frac{1}{2}(\mathbf{30})(\mathbf{13})$ — Substitute **30** for b and **13** for h.

$A = \mathbf{195}$ — Simplify.

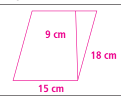
$\triangle XYZ$ has area **195** cm^2.



Quick Check

1. A parallelogram has sides 15 cm and 18 cm. The height corresponding to a 15-cm base is 9 cm. Find the height corresponding to an 18-cm base.

7.5 cm



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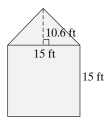
Example

3 Structural Design The front of a garage is a square 15 ft on each side with a triangular roof above the square. The height of the triangular roof is 10.6 ft. To the nearest hundred, how much force is exerted by an 80 mi/h wind blowing directly against the front of the garage? Use the formula $F = 0.004Av^2$, where F is the force in pounds, A is the area of the surface in square feet, and v is the velocity of the wind in miles per hour.

Use the area formulas for rectangles and triangles to find the area of the front of the garage.



Area of the square: $\mathbf{bh} = \mathbf{15}^2 = \mathbf{225}$ ft^2

Area of the triangle: $\mathbf{\frac{1}{2}bh} = \frac{1}{2}(\mathbf{15})(\mathbf{10.6}) = \mathbf{79.5}$ ft^2

The total area of the front of the garage is $\mathbf{225} + \mathbf{79.5} = \mathbf{304.5}$ ft^2.

Find the force of the wind against the front of the garage.

$F = 0.004Av^2$ — Use the formula for force.

$F = 0.004(\mathbf{304.5})(\mathbf{80})^2$ — Substitute **304.5** for A and **80** for v.

$F = \mathbf{7795.2}$ — Simplify.

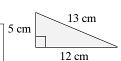
$F = \mathbf{7800}$ — Round to the nearest hundred.

An 80 mi/h wind exerts a force of about **7800** lb against the front of the garage.

Quick Check

2. Find the area of the triangle.

30 cm^2



3. **Critical Thinking** Suppose the bases of the square and triangle in Example 3 are doubled to 30 ft, but the height of each figure stays the same. How is the force of the wind against the building affected?

The force is doubled.

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Lesson 10-2

Areas of Trapezoids, Rhombuses, and Kites

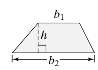
Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find the area of a trapezoid	Topic: Measuring Physical Attributes
2 Find the area of a rhombus or a kite	Local Standards: ____________________

Vocabulary and Key Concepts

Theorem 10-4: Area of a Trapezoid

The area of a trapezoid is **half the product of the height and the sum of the bases.**

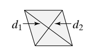
$A =$ **$\frac{1}{2}h(b_1 + b_2)$**



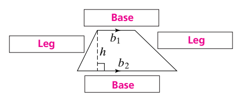
Theorem 10-5: Area of a Rhombus or a Kite

The area of a rhombus or a kite is **half the product of the lengths of its diagonals.**

$A =$ **$\frac{1}{2}d_1d_2$**

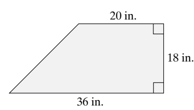


The height of a trapezoid is **the perpendicular distance h between the bases.**



Examples

1 Applying the Area of a Trapezoid A car window is shaped like the trapezoid shown. Find the area of the window.



$A = \frac{1}{2}h(b_1 + b_2)$ — Area of a **trapezoid**

$A = \frac{1}{2}(\mathbf{18})(\mathbf{20} + \mathbf{36})$ — Substitute **18** for h, **20** for b_1, and **36** for b_2.

$A = \mathbf{504}$ — Simplify.

The area of the car window is **504** in.^{2}.

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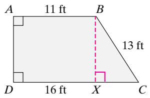
❷ **Finding Area Using a Right Triangle** Find the area of trapezoid *ABCD*. First draw an altitude from vertex *B* to $\overline{DC}$ that divides trapezoid *ABCD* into a rectangle and a right triangle. The altitude will meet $\overline{DC}$ at point *X*.



Because opposite sides of rectangle *ABXD* are congruent, $DX =$ 11 ft and $XC =$ 16 ft − 11 ft = 5 ft.

By the Pythagorean Theorem, $BX^2 + XC^2 = BC^2$, so $BX^2 = 13^2 - 5^2 = 144$.

Taking the square root, $BX =$ 12 ft. You may remember that 5, 12, 13 is a Pythagorean triple.

$A = \frac{1}{2}h(b_1 + b_2)$ Use the trapezoid area formula.

$A = \frac{1}{2}(12)(11 + 16)$ Substitute 12 for *h*, 11 for b_1, and 16 for b_2.

$A = 162$ Simplify.

The area of trapezoid *ABCD* is 162 ft^2.

Quick Check

1. Find the area of a trapezoid with height 7 cm and bases 12 cm and 15 cm.

94.5 cm^2

2. In Example 2, suppose $\overline{BX}$ and $\overline{BC}$ change so that $m\angle C = 60$ while bases and angles *A* and *D* are unchanged. Find the area of trapezoid *ABCD*.

$67.5\sqrt{3}$ or 116.9 ft^2

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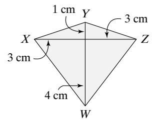
Examples

❸ **Finding the Area of a Kite** Find the area of kite *XYZW*.

Find the lengths of the diagonals of kite *XYZW*.



$XZ = d_1 = 3 + 3 = 6$ and $YW = d_2 = 1 + 4 = 5$

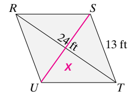
$A = \frac{1}{2}d_1 d_2$ Use the formula for the area of a kite.

$A = \frac{1}{2}(6)(5)$ Substitute 6 for d_1 and 5 for d_2.

$A = 15$ Simplify.

The area of kite *XYZW* is 15 cm^2.

❹ **Finding the Area of a Rhombus** Find the area of rhombus *RSTU*.



To find the area, you need to know the lengths of both diagonals. Draw diagonal $\overline{SU}$, and label the intersection of the diagonals point *X*.

△*SXT* is a right triangle because the diagonals of a rhombus are perpendicular.

The diagonals of a rhombus bisect each other, so *TX* = 12 ft.

You can use the Pythagorean triple 5, 12, 13 or the Pythagorean Theorem to conclude that *SX* = 5 ft.

SU = 10 ft because the diagonals of a rhombus bisect each other.

$A = \frac{1}{2}d_1 d_2$ Area of a rhombus

$A = \frac{1}{2}(24)(10)$ Substitute 24 for d_1 and 10 for d_2.

$A = 120$ Simplify.

The area of rhombus *RSTU* is 120 ft^2.

Quick Check

3. Find the area of a kite with diagonals that are 12 in. and 9 in. long.

54 in^2

4. **Critical Thinking** In Example 4, explain how you can use a Pythagorean triple to conclude that *XU* = 5 ft.

$5^2 + 12^2 = 13^2$

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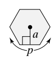
Lesson 10-3 Areas of Regular Polygons

Lesson Objective	NAEP 2005 Strand: Measurement
Find the area of a regular polygon	Topic: Measuring Physical Attributes
	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 10-6: Area of a Regular Polygon

The area of a regular polygon is half the product of the apothem and the perimeter.

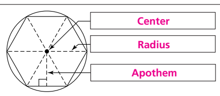


$A = \frac{1}{2}ap$

The center of a regular polygon is the center of the circumscribed circle.

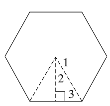
The radius of a regular polygon is the distance from the center to a vertex.

The apothem of a regular polygon is the perpendicular distance from the center to a side.



Examples

❶ **Finding Angle Measures** This regular hexagon has an apothem and radii drawn. Find the measure of each numbered angle.



$m\angle 1 = \frac{360}{6} = 60$ Divide 360 by the number of sides.

$m\angle 2 = \frac{1}{2}m\angle 1$ The apothem bisects the vertex angle of the isosceles triangle formed by the radii.

$m\angle 2 = \frac{1}{2}(60) = 30$ Substitute 60 for $m\angle 1$.

$m\angle 3 = 180 - (90 + 30) = 60$ The sum of the measures of the angles of a triangle is 180.

$m\angle 1 = 60$, $m\angle 2 = 30$, and $m\angle 3 = 60$

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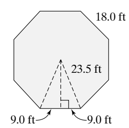
❷ **Finding the Area of a Regular Polygon** A library is in the shape of a regular octagon. Each side is 18.0 ft. The radius of the octagon is 23.5 ft. Find the area of the library to the nearest 10 ft^2.



Consecutive radii form an isosceles triangle, so an apothem bisects the side of the octagon.

To apply the area formula $A = \frac{1}{2}ap$, you need to find *a* and *p*.

Step 1 Find the apothem *a*.

$a^2 + (9.0)^2 = (23.5)^2$ Pythagorean Theorem

$a^2 + 81 = 552.25$ Solve for *a*.

$a^2 = 471.25$

$a \approx 21.7$

Step 2 Find the perimeter *p*.

$p = ns$ Find the perimeter, where *n* = the number of sides of a regular polygon.

$p = (8)(18.0) = 144$ Substitute 8 for *n* and 18.0 for *s*, and simplify.

Step 3 Find the area *A*.

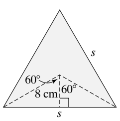
$A = \frac{1}{2}ap$ Area of a regular polygon

$A \approx \frac{1}{2}(21.7)(144)$ Substitute 21.7 for *a* and 144 for *p*.

$A \approx 1562.4$ Simplify.

To the nearest 10 ft^2, the area is 1560 ft^2.

❸ **Applying Theorem 10-6** Find the area of an equilateral triangle with apothem 8 cm. Leave your answer in simplest radical form.



This equilateral triangle shows two radii forming an angle that measures $\frac{360}{3} = 120$. Because the radii and a side form an isosceles triangle, the apothem bisects the 120° angle, forming two 60° angles. You can use a 30°-60°-90° triangle to find half the length of a side.

$\frac{1}{2}s = \sqrt{3} \cdot a$ longer leg = $\sqrt{3}$ · shorter leg

$\frac{1}{2}s = \sqrt{3} \cdot 8$ Substitute 8 for *a*.

$s = 16\sqrt{3}$ Multiply each side by 2.

$p = ns$ Find the perimeter.

$p = (3)(16\sqrt{3}) = 48\sqrt{3}$ Substitute 3 for *n* and $16\sqrt{3}$ for *s*, and simplify.

$A = \frac{1}{2}ap$ Area of a regular polygon

$A = \frac{1}{2}(8)(48\sqrt{3})$ Substitute 8 for *a* and $48\sqrt{3}$ for *p*.

$A = 192\sqrt{3}$ Simplify.

The area of the equilateral triangle is $192\sqrt{3}$ cm^2.

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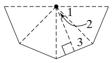
Quick Check

1. At the right, a portion of a regular octagon has radii and an apothem drawn. Find the measure of each numbered angle.



$m\angle 1 = 45$; $m\angle 2 = 22.5$; $m\angle 3 = 67.5$

2. Find the area of a regular pentagon with 11.6-cm sides and an 8-cm apothem.

$232\ cm^2$

3. The side of a regular hexagon is 16 ft. Find the area of the hexagon.

$384\sqrt{3}\ ft^2$

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Lesson 10-4

Perimeters and Areas of Similar Figures

Lesson Objective	NAEP 2005 Strand: Measurement and Number Properties and Operations
1 Find the perimeters and areas of similar figures	**Topics:** Systems of Measurement; Ratios and Proportional Reasoning **Local Standards:** ____________

Key Concepts

Theorem 10-7: Perimeters and Areas of Similar Figures

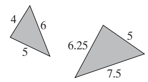
If the similarity ratio of two similar figures is $\frac{a}{b}$, then

(1) the ratio of their perimeters is $\frac{a}{b}$ and

(2) the ratio of their areas is $\frac{a^2}{b^2}$.

Examples

1. **Finding Ratios in Similar Figures** The triangles at the right are similar. Find the ratio (larger to smaller) of their perimeters and of their areas.



The shortest side of the left-hand triangle has length 4, and the shortest side of the right-hand triangle has length 5. From larger to smaller, the similarity ratio is $\frac{5}{4}$.

By the Perimeters and Areas of Similar Figures Theorem, the ratio of the perimeters is $\frac{5}{4}$, and the ratio of the areas is $\frac{5^2}{4^2}$, or $\frac{25}{16}$.

2. **Finding Areas Using Similar Figures** The ratio of the length of the corresponding sides of two regular octagons is $\frac{8}{3}$. The area of the larger octagon is 320 ft^2. Find the area of the smaller octagon.

All regular octagons are similar.

Because the ratio of the lengths of the corresponding sides of the regular octagons is $\frac{8}{3}$, the ratio of their areas is $\frac{8^2}{3^2}$, or $\frac{64}{9}$.

$\frac{64}{9} = \frac{320}{A}$ **Write a proportion.**

$64A = 2880$ **Use the Cross-Product Property.**

$A = 45$ **Divide each side by 64.**

The area of the smaller octagon is 45 ft^2.

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3. **Using Similarity Ratios** Benita plants the same crop in two rectangular fields. Each dimension of the larger field is $3\frac{1}{2}$ times the dimension of the smaller field. Seeding the smaller field costs \$8. How much money does seeding the larger field cost?

The similarity ratio of the fields is 3.5 : 1, so the ratio of the areas of the fields is $(3.5)^2 : (1)^2$, or 12.25 to 1.

Because seeding the smaller field costs \$8, seeding 12.25 times as much land costs 12.25 × \$8.

Seeding the larger field costs \$98.

Quick Check

1. Two similar polygons have corresponding sides in the ratio 5 : 7.
   a. Find the ratio of their perimeters. 5 : 7
   b. Find the ratio of their areas. 25 : 49

2. The corresponding sides of two similar parallelograms are in the ratio $\frac{3}{4}$. The area of the larger parallelogram is 96 $in.^2$. Find the area of the smaller parallelogram.

54 $in.^2$

3. The similarity ratio of the dimensions of two similar pieces of window glass is 3 : 5. The smaller piece costs \$2.50. What should be the cost of the larger piece?

\$6.94

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Example

4. **Finding Similarity and Perimeter Ratios** The areas of two similar pentagons are 32 $in.^2$ and 72 $in.^2$. What is their similarity ratio? What is the ratio of their perimeters?

Find the similarity ratio $a : b$.

$\frac{a^2}{b^2} = \frac{32}{72}$ **The ratio of the areas is $a^2 : b^2$.**

$\frac{a^2}{b^2} = \frac{16}{36}$ **Simplify.**

$\frac{a}{b} = \frac{4}{6} = \frac{2}{3}$ **Take the square root.**

The similarity ratio is 2 : 3.

By the Perimeters and Areas of Similar Figures Theorem, the ratio of the perimeters is also 2 : 3.

Quick Check

4. The areas of two similar rectangles are 1875 ft^2 and 135 ft^2. Find the ratio of their perimeters.

$5\sqrt{5} : 3$

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Lesson 10-5

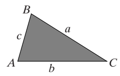
Trigonometry and Area

Lesson Objectives	
1 Find the area of a regular polygon using trigonometry 2 Find the area of a triangle using trigonometry	**NAEP 2005 Strand:** Measurement **Topic:** Measuring Physical Attributes **Local Standards:** ____________

Key Concepts

Theorem 10-8: Area of a Triangle Given SAS

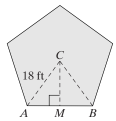
The area of a triangle is one half the product of the lengths of two sides and the sine of the included angle.



Area of $\triangle ABC =$ $\frac{1}{2}bc(\sin A)$

Examples

1 Finding Area The radius of a garden in the shape of a regular pentagon is 18 feet. Find the area of the garden.



Find the perimeter p and apothem a, and then find the area using the formula $A = \frac{1}{2}ap$.

Because a pentagon has five sides, $m\angle ACB = \frac{360}{5} = 72$.

$\overline{CA}$ and $\overline{CB}$ are radii, so $CA = CB$. Therefore, $\triangle ACM \cong \triangle BCM$ by the HL Theorem, so $m\angle ACM = \frac{1}{2}m\angle ACB = 36$.

Use the cosine ratio to find a. Use the sine ratio to find AM.

$\cos 36° = \frac{a}{18}$ Use the ratio. $\sin 36° = \frac{AM}{18}$

$a = 18(\cos 36°)$ Solve. $AM = 18(\sin 36°)$

Use AM to find p. Because $\triangle ACM \cong \triangle BCM$, $AB = 2 \cdot AM$. Because the pentagon is regular, $p = 5 \cdot AB$.

So $p = 5(2 \cdot AM) = 10 \cdot AM = 10 \cdot 18(\sin 36°) = 180(\sin 36°)$.

Finally, substitute into the area formula $A = \frac{1}{2}ap$.

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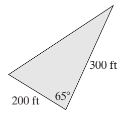
$A = \frac{1}{2} \cdot 18(\cos 36°) \cdot 180(\sin 36°)$ **Substitute for a and p.**

$A = 1620 (\cos 36°) \cdot (\sin 36°)$ **Simplify.**

$A \approx 770.355778$ **Use a calculator.**

The area is about 770 ft^2.

2 Surveying A triangular park has two sides that measure 200 ft and 300 ft and form a 65° angle. Find the area of the park to the nearest hundred square feet.



Use Theorem 10-8: The area of a triangle is one half the product of the lengths of two sides and the sine of the included angle.

Area $= \frac{1}{2} \cdot$ side length $\cdot$ side length $\cdot$ sine of included angle **Theorem 9-1**

Area $= \frac{1}{2} \cdot 200 \cdot 300 \cdot \sin 65°$ **Substitute.**

Area $= 30{,}000 \cdot \sin 65°$ **Substitute.**

≈ 27189.23361 **Use a calculator.**

The area of the park is approximately 27,200 ft^2.

Quick Check

1. Find the area of a regular octagon with a perimeter of 80 in. Give the area to the nearest tenth.

482.8 $in.^2$

2. Two sides of a triangular building plot are 120 ft and 85 ft long. They include an angle of 85°. Find the area of the building plot to the nearest square foot.

5081 ft^2

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Lesson 10-6

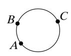
Circles and Arcs

Lesson Objectives	
1 Find the measures of central angles and arcs 2 Find circumference and arc length	**NAEP 2005 Strand:** Measurement **Topic:** Measuring Physical Attributes **Local Standards:** ____________

Vocabulary and Key Concepts

Postulate 10-1: Arc Addition Postulate

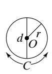
The measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs.



$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$

Theorem 10-9: Circumference of a Circle

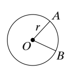
The circumference of a circle is π times the diameter.



$C = \pi d$ or $C = 2\pi r$

Theorem 10-10: Arc Length

The length of an arc of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the circumference of the circle.



length of $\widehat{AB} = \frac{m\widehat{AB}}{360} \cdot 2\pi r$

A circle is the set of all points equidistant from a given point called the center.

A center of a circle is the point from which all points are equidistant.

A radius is a segment that has one endpoint at the center and the other endpoint on the circle.

Congruent circles have congruent radii.

A diameter is a segment that contains the center of a circle and has both endpoints on the circle.

A central angle is an angle whose vertex is the center of the circle.

Circumference of a circle is the distance around the circle.

Pi (π) is the ratio of the circumference of a circle to its diameter.

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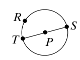
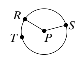
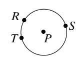


$\widehat{TRS}$ is a semicircle.
$m\widehat{TRS} = 180$
A semicircle is half of a circle.



$\widehat{RS}$ is a minor arc.
$m\widehat{RS} = m\angle RPS$
A minor arc is smaller than a semicircle.



$\widehat{RTS}$ is a major arc.
$m\widehat{RTS} = 360 - m\widehat{RS}$
A major arc is greater than a semicircle.

Adjacent arcs are arcs of the same circle that have exactly one point in common.

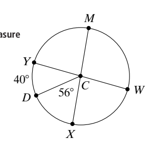
Concentric circles are circles that lie in the same plane and have the same center.

Arc length is a fraction of a circle's circumference.

Congruent arcs are arcs that have the same measure and are in the same circle or in congruent circles.

Example

1 Finding the Measures of Arcs Find $m\widehat{XY}$ and $m\widehat{DXM}$ in circle C.



$m\widehat{XY} = m\widehat{XD} + m\widehat{DY}$ **Arc Addition Postulate**

$m\widehat{XY} = m\angle XCD + m\widehat{DY}$ **The measure of a minor arc is the measure of its corresponding central angle.**

$m\widehat{XY} = 56 + 40$ **Substitute.**

$m\widehat{XY} = 96$ **Simplify.**

$m\widehat{DXM} = m\widehat{DX} + m\widehat{XWM}$ **Arc Addition Postulate**

$m\widehat{DXM} = 56 + 180$ **Substitute.**

$m\widehat{DXM} = 236$ **Simplify.**

Quick Check

1. Use the diagram in Example 1. Find $m\angle YCD$, $m\widehat{YMW}$, $m\widehat{MW}$, and $m\widehat{XMY}$.

40; 180; 96; 264

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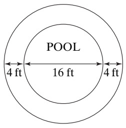
Examples

2 Applying Circumference A circular swimming pool 16 feet in diameter will be enclosed in a circular fence 4 ft from the pool. What length of fencing material is needed? Round your answer to the next whole number.



The pool and the fence are concentric circles. The diameter of the pool is 16 ft, so the diameter of the fence is 16 + [4] + [4] = [24] ft. Use the formula for the circumference of a circle to find the length of fencing material needed.

$C =$ [πd] Formula for the circumference of a circle

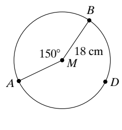
$C = \pi$ [24] Substitute.

$C \approx 3.14$([24]) Use 3.14 to approximate π.

$C \approx$ [75.36] Simplify.

About [76] ft of fencing material is needed.

3 Finding Arc Length Find the length of $\widehat{ADB}$ in circle M in terms of π.



Because $m\widehat{AB} = 150$,

$m\widehat{ADB} =$ [360] − [150] = [210] Arc Addition Postulate

length of $\widehat{ADB} = \frac{m\widehat{ADB}}{360} \cdot 2$ [πr] Arc Length Postulate

$= \frac{[210]}{360} \cdot 2\pi$([18]) Substitute.

$=$ [21] π

The length of $\widehat{ADB}$ is 21π cm.

Quick Check

2. The diameter of a bicycle wheel is 22 in. To the nearest whole number, how many revolutions does the wheel make when the bicycle travels 100 ft?

17 revolutions

3. Find the length of a semicircle with radius 1.3 m in terms of π.

1.3π m

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Lesson 10-7 Areas of Circles and Sectors

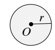
Lesson Objective	NAEP 2005 Strand: Measurement
Find the areas of circles, sectors, and segments of circles	Topic: Measuring Physical Attributes
	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 10-11: Area of a Circle

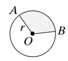
The area of a circle is [the product of π and the square of the radius].

$A =$ [πr^2]



Theorem 10-12: Area of a Sector of a Circle

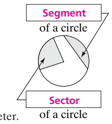
The area of a sector of a circle is the product of the ratio $\frac{\text{measure of the arc}}{[360]}$ and the [area of the circle.].



Area of sector $AOB =$ [$\frac{m\widehat{AB}}{360} \cdot \pi r^2$]

A sector of a circle is [a region bounded by two radii and their intercepted arc.]

A segment of a circle is [the part bounded by an arc and the segment joining its endpoints.]



Examples

1 Applying the Area of a Circle A circular archery target has a 2-ft diameter. It is yellow except for a red bull's-eye at the center with a 6-in. diameter. Find the area of the yellow region to the nearest whole number.

First find the areas of the archery target and the red bull's-eye.

The radius of the archery target is $\frac{1}{2}$([2] ft) = [1] ft = 12 in.

The area of the archery target is $\pi r^2 = \pi$([12])^2 = [144π] in.^{2}

The radius of the red bull's-eye region is $\frac{1}{2}$([6] in.) = [3] in.

The area of the red region is $\pi r^2 = \pi$([3])^2 = [9π] in.^{2}

area of archery target − area of red region = area of yellow region

[144][π] − [9][π] = [135][π]

= [424.11501] Simplify.

The area of the yellow region is about [424] in.^{2}

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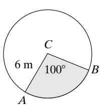
2 Finding the Area of a Sector of a Circle Find the area of sector ACB. Leave your answer in terms of π.



area of sector $ACB = \frac{m\widehat{AB}}{[360]} \cdot \pi r^2$

$= \frac{[100]}{[360]} \cdot \pi$([6])^2

$= \frac{[5]}{[18]} \cdot$ [36] π

$=$ [10] π

The area of sector ACB is [10π] m^2.

Quick Check

1. How much more pizza is in a 14-in.-diameter pizza than in a 12-in. pizza?

about 41 in.^{2}

2. **Critical Thinking** A circle has a diameter of 20 cm. What is the area of a sector bounded by a 208° major arc? Round your answer to the nearest tenth.

181.5 cm^2

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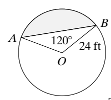
Example

3 Finding the Area of a Segment of a Circle Find the area of the shaded segment. Round your answer to the nearest tenth.



Step 1 Find the area of sector AOB.

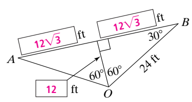
area of sector $AOB = \frac{[m\widehat{AB}]}{360} \cdot \pi r^2$ Use the formula for area of a [sector].

$= \frac{[120]}{360} \cdot \pi$([24])^2 Substitute.

$= \frac{[1]}{[3]} \cdot 576\pi =$ [192] π Simplify.

Step 2 Find the area of $\triangle AOB$.

You can use a 30°-60°-90° triangle to find the height h of $\triangle AOB$ and the length of $\overline{AB}$.



[24] = [2] h hypotenuse = [2] · shorter leg

[12] = h Divide each side by [2].

$\frac{AB}{2} = \sqrt{3} \cdot$ [12] = [$12\sqrt{3}$] longer leg = [$\sqrt{3}$] · shorter leg

$AB =$ [24][$\sqrt{3}$] Multiply each side by [2].

$\triangle AOB$ has base [$12\sqrt{3}$] ft + [$12\sqrt{3}$] ft, or [$24\sqrt{3}$] ft, and height [12] ft.

$A = \frac{1}{2}bh$ Area of a [triangle].

$A = \frac{1}{2}$([$24\sqrt{3}$])([12]) Substitute [$24\sqrt{3}$] for b and [12] for h.

$A =$ [144][$\sqrt{3}$] Simplify.

Step 3 Subtract the area of $\triangle AOB$ from the area of sector AOB to find the area of the segment of the circle.

area of segment = [192][π] − [144][$\sqrt{3}$]

$\approx$ [353.77047] Use a calculator.

To the nearest tenth, the area of the shaded segment is [353.8] ft^2.

Quick Check

3. A circle has a radius of 12 cm. Find the area of the smaller segment of the circle determined by a 60° arc. Round your answer to the nearest tenth.

13.0 cm^2

Name_______________ Class_______________ Date_______________

Lesson 10-8 — Geometric Probability

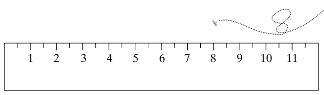
Lesson Objective	NAEP 2005 Strand: Data Analysis and Probability
Use segment and area models to find the probabilities of events	Topic: Probability
	Local Standards: ____________

Vocabulary

Geometric probability is a model in which you let points represent outcomes.

Example

1 Finding Probability Using Segments A gnat lands at random on the edge of the ruler below. Find the probability that the gnat lands on a point between 2 and 10.



The length of the segment between 2 and 10 is 10 − 2 = 8.

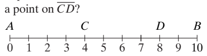
The length of the ruler is 12.

P(landing between 2 and 10)

$= \dfrac{\text{length of favorable segment}}{\text{length of entire segment}} = \dfrac{8}{12} = \dfrac{2}{3}$

Quick Check

1. A point on $\overline{AB}$ is selected at random. What is the probability that it is a point on $\overline{CD}$?

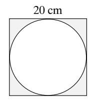


$\frac{2}{5}$

Name_______________ Class_______________ Date_______________

Example

2 Finding Probability Using Area A circle is inscribed in a square target with 20-cm sides. Find the probability that a dart landing randomly within the square does not land within the circle.



Find the area of the square.

$A = s^2 = 20^2 = 400$ cm^2

Find the area of the circle. Because the square has sides of length 20 cm, the circle's diameter is 20 cm, so its radius is 10 cm.

$A = \pi r^2 = \pi(10)^2 = 100\pi$ cm^2

Find the area of the region between the square and the circle.

$A = (400 - 100\pi)$ cm^2

Use areas to calculate the probability that a dart landing randomly in the square does not land within the circle. Use a calculator. Round to the nearest thousandth.

$P(\text{between square and circle}) = \dfrac{\text{area between square and circle}}{\text{area of square}}$

$= \dfrac{400 - 100\pi}{400}$

$= 1 - \dfrac{\pi}{4} \approx 0.215$

The probability that a dart landing randomly in the square does not land within the circle is about 21.5%.

Quick Check

2. Use the diagram in Example 2. If you change the radius of the circle as indicated, what then is the probability of hitting outside the circle?

a. Divide the radius by 2.

about 80.4%

b. Divide the radius by 5.

about 96.9%

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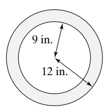
Example

3 Applying Geometric Probability To win a prize, you must toss a quarter so that it lands entirely within the outer region of the circle at right. Find the probability that this happens with a quarter of radius $\frac{15}{32}$ in. Assume that the quarter is equally likely to land anywhere completely inside the large circle.



The center of a quarter with a radius of $\frac{15}{32}$ in. must land at least $\frac{15}{32}$ in. beyond the boundary of the inner circle in order to lie entirely outside the inner circle. Because the inner circle has a radius of 9 in., the quarter must land outside the circle whose radius is 9 in. + $\frac{15}{32}$ in., or $9\frac{15}{32}$ in.

Find the area of the circle with a radius of $9\frac{15}{32}$ in.

$A = \pi r^2 = \pi\left(9\dfrac{15}{32}\right)^2 \approx 281.66648$ in.^{2}

Similarly, the center of a quarter with a radius of $\frac{15}{32}$ in. must land at least $\frac{15}{32}$ in. within the outer circle. Because the outer circle has a radius of 12 in., the quarter must land inside the circle whose radius is 12 in. − $\frac{15}{32}$ in., or $11\frac{17}{32}$ in.

Find the area of the circle with a radius of $11\frac{17}{32}$ in.

$A = \pi r^2 = \pi\left(11\dfrac{17}{32}\right)^2 \approx 417.73672$ in.^{2}

Use the area of the outer region to find the probability that the quarter lands entirely within the outer region of the circle.

$P(\text{outer region}) = \dfrac{\text{area of outer circle}}{\text{area of large circle}} \approx \dfrac{417.73672 - 281.66648}{417.73672}$

$= \dfrac{136.07024}{417.73672} \approx 0.32573$

The probability that the quarter lands entirely within the outer region of the circle is about 0.326 or 32.6 %.

Quick Check

3. **Critical Thinking** Use Example 3. Suppose you toss 100 quarters. Would you expect to win a prize? Explain.

Yes; theoretically you should win 32.6 times out of 100.

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Lesson 11-1 — Space Figures and Cross Sections

Lesson Objective	NAEP 2005 Strand: Geometry
Recognize polyhedra and their parts	Topic: Dimension and Shape
Visualize cross sections of space figures	Local Standards: ____________

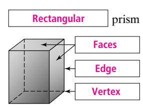
Vocabulary and Key Concepts

Euler's Formula

The numbers of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.

A polyhedron is a three-dimensional figure whose surfaces are polygons.

A face is a flat surface of a polyhedron in the shape of a polygon.



An edge is a segment that is formed by the intersection of two faces.

A vertex is a point where three or more edges intersect.

A cross section is the intersection of a solid and plane.

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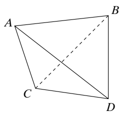
Examples

❶ **Identifying Vertices, Edges, and Faces** How many vertices, edges, and faces are there in the polyhedron shown? Give a list of each.



There are **four** vertices: **A, B, C, and D**.

There are **six** edges: **$\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, $\overline{AC}$, and $\overline{BD}$**.

There are **four** faces: **$\triangle ABC$, $\triangle ABD$, $\triangle ACD$, and $\triangle BCD$**.

❷ **Using Euler's Formula** Use Euler's Formula to find the number of edges on a solid with 6 faces and 8 vertices.

$F + V = E + 2$ Euler's Formula

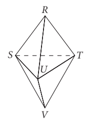
$6 + 8 = E + 2$ Substitute the number of faces and vertices.

$12 = E$ Simplify.

A solid with 6 faces and 8 vertices has **12** edges.

Quick Check

1. List the vertices, edges, and faces of the polyhedron.



R, S, T, U, V; $\overline{RS}$, $\overline{RU}$, $\overline{RT}$, $\overline{VS}$, $\overline{VU}$, $\overline{VT}$, $\overline{SU}$, $\overline{UT}$, $\overline{TS}$; $\triangle RSU$, $\triangle RUT$, $\triangle RTS$, $\triangle VSU$, $\triangle VUT$, $\triangle VTS$

2. Use Euler's Formula to find the number of edges on a polyhedron with eight triangular faces.

12 edges

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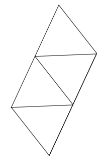
Example

❸ Verify Euler's Formula for the two-dimensional net of the solid in Example 1.



Count the regions: F = **4**

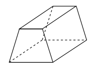
Count the vertices: V = **6**

Count the segments: E = **9**

4 + **6** = **9** + 1

Quick Check

3. The figure at the right is a trapezoidal prism.

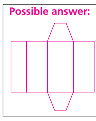


a. verify Euler's Formula $F + V = E + 2$ for the prism.

6 + 8 = 12 + 2

b. Draw a net for the prism.

Possible answer:



c. Verify Euler's Formula $F + V = E + 1$ for your two-dimensional net.

6 + 14 = 19 + 1

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Lesson 11-2 Surface Areas of Prisms and Cylinders

Lesson Objectives	NAEP 2005 Strand: Measurement
❶ Find the surface area of a prism	Topic: Measuring Physical Attributes
❷ Find the surface area of a cylinder	Local Standards: ____________

Vocabulary and Key Concepts

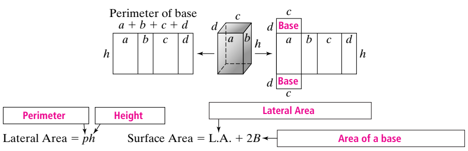
Theorem 11-1: Lateral and Surface Area of a Prism

The lateral area of a right prism is the product of the **perimeter of the base** and the **height**.

L.A. = ph

The surface area of a right prism is the sum of the **lateral area** and the **areas of the two bases**.

S.A. = L.A. + 2B



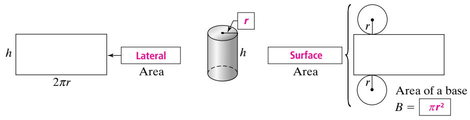
Theorem 11-2: Lateral and Surface Area of a Cylinder

The lateral area of a right cylinder is the product of the **circumference of the base** and the **height of the cylinder**.

L.A. = $2\pi rh$, or L.A. = πdh

The surface area of a right cylinder is the sum of the **lateral area** and the **areas of the two circular bases**.

S.A. = L.A. + 2B, or S.A. = $2\pi rh + 2\pi r^2$



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A prism is **a polyhedron with exactly two congruent, parallel faces.**

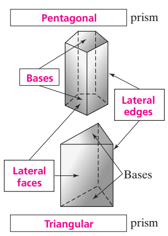
The bases of a polyhedron are **are the parallel faces.**

Lateral faces are **the faces on a prism that are not bases.**

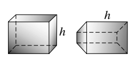
An altitude is **a perpendicular segment that joins the planes of the bases.**

The height is **the length of the altitude.**

The lateral area of a prism is **the sum of the areas of the lateral faces.**



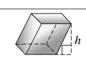
The surface area of a prism or a cylinder is **the sum of the lateral area and the areas of the two bases.**



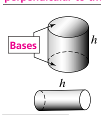
right prisms

A right prism is **a prism whose lateral faces are rectangular regions and a lateral edge is an altitude.**

An oblique prism is **a prism whose lateral faces are not perpendicular to the bases.**



oblique prism



right cylinders

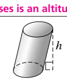
A cylinder is **a three-dimensional figure with two congruent circular bases that lie in parallel planes.**

In a cylinder, the bases are **parallel circles.**

The lateral area of a cylinder is **the area of the curved surface.**

A right cylinder is **one where the segment joining the centers of the bases is an altitude.**

An oblique cylinder is **one in which the segment joining the centers of the bases is not perpendicular to the bases.**



oblique cylinder

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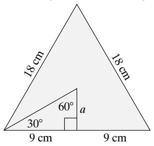
Examples

1 Using Formulas to Find Surface Area Find the surface area of a 10-cm-high right prism with triangular bases having 18-cm edges. Round to the nearest whole number.

Use the formula **L.A. = ph** to find the lateral area and the formula **S.A. = L.A. + 2B** to find the surface area of the prism. The area B of the base is $\frac{1}{2}ap$, where a is the apothem and p is the **perimeter**.

The triangle has sides of length 18 cm, so $p = 3 \cdot 18$ cm, or **54** cm.



Use the diagram of the base.

Use 30°-60°-90° triangles to find the **apothem**.

$\boxed{a} \cdot \boxed{\sqrt{3}} = 9$ — $\sqrt{3}$ · shorter leg = longer leg

$a = \frac{9}{\boxed{\sqrt{3}}}$ — Divide both sides by $\boxed{\sqrt{3}}$.

$a = \frac{9}{\boxed{\sqrt{3}}} \cdot \frac{\boxed{\sqrt{3}}}{\boxed{\sqrt{3}}} = \frac{\boxed{9\sqrt{3}}}{\boxed{3}} = \boxed{3}\boxed{\sqrt{3}}$ — Rationalize the denominator.

$B = \frac{1}{2}ap$

$B = \frac{1}{2} \cdot \boxed{3}\boxed{\sqrt{3}} \cdot \boxed{54}$ — Substitute $\boxed{81\sqrt{3}}$ for a and $\boxed{54}$ for p.

$B = \boxed{81}\boxed{\sqrt{3}}$ — Simplify.

The area of each base of the prism is $\boxed{81\sqrt{3}}$ cm².

S.A. = L.A. + 2B — Use the formula for surface area.

= $\boxed{ph}$ + 2B — Substitute $\boxed{ph}$ for L.A.

= $(\boxed{54})(\boxed{10}) + 2(\boxed{81}\boxed{\sqrt{3}})$ — Substitute $\boxed{54}$ for p, $\boxed{10}$ for h and $\boxed{81\sqrt{3}}$ for B.

= $\boxed{540} + \boxed{162\sqrt{3}}$ — Simplify.

≈ $\boxed{820.59223}$ — Use a calculator.

Rounded to the nearest whole number, the surface area is **821 cm²**.

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2 Finding Surface Area of Cylinders A company sells cornmeal and barley in cylindrical containers. The diameter of the base of the 6-in.-high cornmeal container is 4 in. The diameter of the base of the 4-in.-high barley container is 6 in. Which container has the greater surface area?

Find the surface area of each container. Remember that $r = \frac{d}{2}$.

Cornmeal Container

S.A. = $\boxed{L.A.} + \boxed{2}\boxed{B}$ — Use the formula for surface area.

= $2\pi rh + \boxed{2}\boxed{\pi}\boxed{r^2}$ — Substitute the formulas for lateral area of a cylinder and area of a circle.

= $2\pi(\boxed{2})(\boxed{6}) + 2\pi(\boxed{2}^2)$ — Substitute for r and h.

= $\boxed{24}\boxed{\pi} + \boxed{8}\boxed{\pi}$ — Simplify.

= $\boxed{32}\boxed{\pi}$ — Combine like terms.

Barley Container

S.A. = $\boxed{L.A.} + \boxed{2}\boxed{B}$

= $\boxed{2}\boxed{\pi}\boxed{r}\boxed{h} + 2\pi r^2$

= $2\pi(\boxed{3})(\boxed{4}) + 2\pi(\boxed{3}^2)$

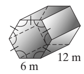
= $\boxed{24}\boxed{\pi} + \boxed{18}\boxed{\pi}$

= $\boxed{42}\boxed{\pi}$

Because 42π in.² $\boxed{>}$ 32π in.², the **barley** container has the greater surface area.

Quick Check

1. Use formulas to find the lateral area and surface area of the prism.



432 m²; about 619 m²

2. Find the surface area of a cylinder with height 10 cm and radius 10 cm in terms of π.

400π cm²

3. The company in Example 2 wants to make a label to cover the cornmeal container. The label will cover the container all the way around, but will not cover any part of the top or bottom. What is the area of the label to the nearest tenth of a square inch?

75.4 in.²

Name_____________ Class_____________ Date_____________

Lesson 11-3

Surface Areas of Pyramids and Cones

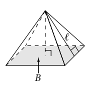
Lesson Objectives	
1 Find the surface area of a pyramid 2 Find the surface area of a cone	**NAEP 2005 Strand:** Measurement **Topic:** Measuring Physical Attributes **Local Standards:** ____________

Vocabulary and Key Concepts

Theorem 11-3: Lateral and Surface Area of a Regular Pyramid

The lateral area of a regular pyramid is half the product of the **perimeter of the base** and the **slant height**.

L.A. = $\frac{1}{2}p\ell$



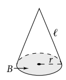
The surface area of a regular pyramid is the sum of the **lateral area** and the **area of the base**.

S.A. = $\boxed{L.A.} + \boxed{B}$

Theorem 11-4: Lateral and Surface Area of a Cone

The lateral area of a right cone is half the product of the **circumference of the base** and the **slant height**.

L.A. = $\frac{1}{2} \cdot 2\pi r \cdot \ell$, or L.A. = $\pi r\ell$



The surface area of a right cone is the sum of the lateral area and the area of the base.

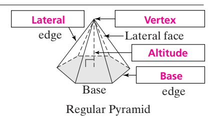
S.A. = $\boxed{L.A.} + \boxed{B}$

A regular pyramid is **a pyramid whose base is a regular polygon and whose lateral faces are congruent isosceles triangles.**

The altitude of a pyramid or a cone is **the perpendicular segment from the vertex to the plane of the base.**

The height of a pyramid or a cone is **the length of the altitude.**

The slant height of a regular pyramid is **the length of the altitude of a lateral face.**



Regular Pyramid

The lateral area of a pyramid is **is the sum of the areas of the congruent lateral faces.**

The surface area of a pyramid is **is the sum of the lateral area and the area of the base.**

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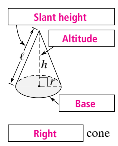
A right cone is **a cone in which the altitude is perpendicular to the plane of the base.**

The slant height of a cone is **the distance from the vertex to the edge of the base.**

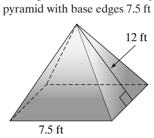
The lateral area of a cone is **the area of the curved lateral surface.**

The surface area of a cone is **the sum of the lateral area and the area of the base.**



Examples

1 Finding Surface Area of a Pyramid Find the surface area of a square pyramid with base edges 7.5 ft and slant height 12 ft.



The perimeter p of the square base is $4 \times \boxed{7.5}$ ft, or $\boxed{30}$ ft.

You are given $\ell = \boxed{12}$ ft and you found that $p = \boxed{30}$ ft, so you can find the lateral area.

L.A. = $\frac{\boxed{1}}{\boxed{2}}\boxed{p}\boxed{\ell}$ — Use the formula for lateral area of a pyramid.

= $\frac{1}{2}(\boxed{30})(\boxed{12})$ — Substitute.

= $\boxed{180}$ — Simplify.

Find the area of the square base.

Because the base is a square with side length 7.5 ft,

$B = s^2 = \boxed{7.5}^2 = \boxed{56.25}$.

S.A. = $\boxed{L.A.} + \boxed{B}$ — Use the formula for surface area of a pyramid.

= $\boxed{180} + \boxed{56.25}$ — Substitute.

= $\boxed{236.25}$ — Simplify.

The surface area of the square pyramid is **236.25** ft².

Name______________________ Class______________________ Date ______________

❷ Finding Lateral Area of a Cone Leandre uses paper cones to cover her plants in the early spring. The diameter of each cone is 1 ft, and its height is 1.5 ft. How much paper is in the cone? Round your answer to the nearest tenth.

Use the formula L.A. = [$\pi r \ell$] to find the lateral area of the cone.

The cone's diameter is 1 ft, so its radius r is [0.5] ft.

The altitude of the cone, radius of the base, and slant height form a [right] triangle. Use the [Pythagorean Theorem] to find the slant height ℓ.

$\ell = \sqrt{[0.5]^2 + [1.5]^2} = \sqrt{[0.25] + [2.25]} = [\sqrt{2.5}]$

Find the lateral area.

L.A. = [π][r][ℓ] — Use the formula for lateral area of a cone.

= π([0.5])([$\sqrt{2.5}$]) — Substitute [0.5] for r and [$\sqrt{2.5}$] for ℓ.

≈ [2.4836471] — Use a calculator.

The lateral area of the cone is about [2.5] ft^2.

Quick Check

1. Find the surface area of a square pyramid with base edges 5 m and slant height 3 m.

55 m^2

2. Find the lateral area of a cone with radius 15 in. and height 20 in. to the nearest square inch.

1178 $in.^2$

Name______________________ Class______________________ Date ______________

Lesson 11-4

Volumes of Prisms and Cylinders

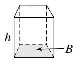
Lesson Objectives	NAEP 2005 Strand: Measurement
❶ Find the volume of a prism	Topic: Measuring Physical Attributes
❷ Find the volume of a cylinder	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 11-5: Cavalieri's Principle
If two space figures have the same height and the same cross sectional area at every level, then they have the same [volume].

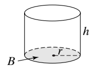
Theorem 11-6: Volume of a Prism
The volume of a prism is the product of the [area of a base] and the [height of the prism].

$V =$ [Bh]



Theorem 11-7: Volume of a Cylinder
The volume of a cylinder is the product of the [area of the base] and the [height of the cylinder].

$V = Bh$, or $V =$ [$\pi r^2 h$]

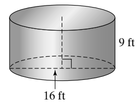


Volume is the space that a figure occupies.

A composite space figure is a three-dimensional figure that is the combination of two or more simpler figures.

Examples

❶ Finding Volume of a Cylinder Find the volume of the cylinder. Leave your answer in terms of π.



The formula for the volume of a cylinder is [$V = \pi r^2 h$].

The diagram shows h and d, but you must find r.

$r = \frac{1}{2}d =$ [8]

$V =$ [π][r^2][h] — Use the formula for the volume of a cylinder.

$= \pi \cdot$ [8^2] $\cdot$ [9] — Substitute.

$=$ [576][π] — Simplify.

The volume of the cylinder is [576π] ft^3.

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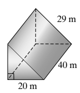
❷ Finding Volume of a Triangular Prism Find the volume of the prism.



The prism is a [right triangular prism] with triangular bases. The base of the triangular prism is a [right triangle] where one leg is the [base] and the other leg is the [altitude]. Use the Pythagorean Theorem to calculate the length of the other leg.

$\sqrt{[29]^2 - [20]^2} = \sqrt{[841] - [400]} = \sqrt{[441]} = [21]$

The area B of the base is $\frac{1}{2}bh = \frac{1}{2}$([20])([21]) = [210]. Use the area of the base to find the volume of the prism.

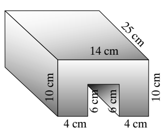
$V =$ [B][h] — Use the formula for the volume of a prism.

$=$ [210] $\cdot$ [40] — Substitute.

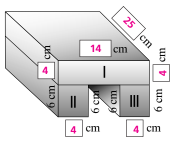
$=$ [8400] — Simplify.

The volume of the triangular prism is [8400] m^3.

❸ Finding Volume of a Composite Figure Find the volume of the composite space figure.



You can use three [rectangular prisms] to find the volume.



Each prism's volume can be found using the formula $V =$ [Bh].

Volume of Prism I = Bh = ([14] $\cdot$ [4]) $\cdot$ [25] = [1400]

Volume of Prism II = Bh = ([6] $\cdot$ [4]) $\cdot$ [25] = [600]

Volume of Prism III = Bh = ([6] $\cdot$ [4]) $\cdot$ [25] = [600]

Sum of the volumes = [1400] + [600] + [600] = [2600]

The volume of the composite space figure is [2600] cm^3.

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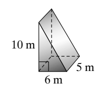
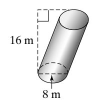
Quick Check

1. Find the volume of the triangular prism.



150 m^3

2. The cylinder shown is oblique.



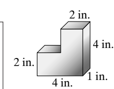
a. Find its volume in terms of π.

256π m^3

b. Find its volume to the nearest tenth of a cubic meter.

804.2 m^3

3. Find the volume of the composite space figure.



12 $in.^3$

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Lesson 11-5

Volumes of Pyramids and Cones

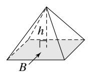
Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find the volume of a pyramid 2 Find the volume of the cone	Topic: Measuring Physical Attributes Local Standards: ____________________

Key Concepts

Theorem 11-8: Volume of a Pyramid

The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.

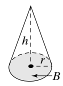
$V = \frac{1}{3}Bh$



Theorem 11-9: Volume of a Cone

The volume of a cone is one third the product of the area of the base and the height of the cone.

$V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2 h$



Examples

1 Finding Volume of a Pyramid Find the volume of a square pyramid with base edges 15 cm and height 22 cm.

Because the base is a square, $B = 15 \cdot 15 = 225$.

$V = \frac{1}{3}Bh$ Use the formula for volume of a pyramid.

$= \frac{1}{3}(225)(22)$ Substitute 225 for *B* and 22 for *h*.

$= 1650$ Simplify.

The volume of the square pyramid is 1650 cm^3.

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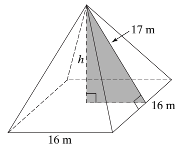
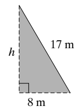
2 Finding Volume of a Pyramid Find the volume of a square pyramid with base edges 16 m and slant height 17 m.

The altitude of a right square pyramid intersects the base at the center of the square. Because each side of the square base is 16 m, the leg of the right triangle along the base is 8 m, as shown.





Step 1 Find the height of the pyramid.

$17^2 = 8^2 + h^2$ Use the Pythagorean Theorem.

$289 = 64 + h^2$ Simplify.

$225 = h^2$ Subtract 64 from each side.

$h = 15$ Find the square root of each side.

Step 2 Find the volume of the pyramid.

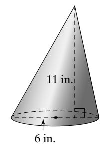
$V = \frac{1}{3}Bh$ Use the formula for the volume of a pyramid.

$= \frac{1}{3}(16 \cdot 16)15$ Substitute.

$= 1280$ Simplify.

The volume of the square pyramid is 1280 m^3.

3 Finding Volume of an Oblique Cone Find the volume of the oblique cone in terms of π.



$r = \frac{1}{2}d = 3$ in.

$V = \frac{1}{3}\pi r^2 h$ Use the formula for the volume of a cone.

$= \frac{1}{3}\pi(3^2)(11)$ Substitute 3 for *r* and 11 for *h*.

$= 33\pi$ Simplify.

The volume of the cone is 33π $in.^3$.

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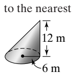
Quick Check

1. Find the volume of a square pyramid with base edges 24 m and slant height 13 m.

   960 m^3

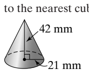
2. Find the volume of each cone in terms of π and also rounded as indicated.

   a. to the nearest cubic meter

   144π m^3; 452 m^3

   b. to the nearest cubic millimeter

   6174π mm^3; 19,396 mm^3

3. A small child's teepee is 6 ft tall and 7 ft in diameter. Find the volume of the teepee to the nearest cubic foot.

   77 ft^3

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Lesson 11-6

Surface Areas and Volumes of Spheres

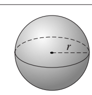
Lesson Objective	NAEP 2005 Strand: Measurement
Find the surface area and volume of a sphere	Topic: Measuring Physical Attributes Local Standards: ____________________

Vocabulary and Key Concepts

Theorem 11-10: Surface Area of a Sphere

The surface area of a sphere is four times the product of π and the square of the radius of the sphere.

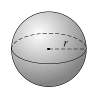
S.A. = $4\pi r^2$



Theorem 11-11: Volume of a Sphere

The volume of a sphere is four thirds the product of π and the cube of the radius of the sphere.

S.A. = $\frac{4}{3}\pi r^3$

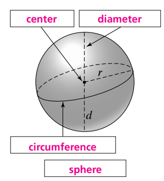


A sphere is the set of all points in space equidistant from a given point.

The center of a sphere is the given point from which all points on the sphere are equidistant.

The radius of a sphere is a segment that has one endpoint at the center and the other endpoint on the sphere.

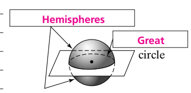
The diameter of a sphere is a segment passing through the center with endpoints on the sphere.



A great circle is the intersection of a plane and a sphere containing the center of the sphere.

A great circle divides a sphere into two congruent hemispheres.

The circumference of a sphere is the circumference of its great circle.



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Examples

1 Finding Surface Area The circumference of a rubber ball is 13 cm. Calculate its surface area to the nearest whole number.

Step 1 First, find the radius.

$C = 2\,\pi\,r$ Use the formula for circumference.

$13 = 2\pi r$ Substitute 13 for C.

$\frac{13}{2\pi} = r$ Solve for *r*.

Step 2 Use the radius to find the surface area.

S.A. $= 4\,\pi\,r^2$ Use the formula for surface area of a sphere.

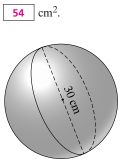
$= 4\pi \cdot \left(\frac{13}{2\pi}\right)^2$ Substitute $\frac{13}{2\pi}$ for *r*.

$= \frac{169}{\pi}$ Simplify.

≈ 53.794371 Use a calculator.

To the nearest whole number, the surface area of the rubber ball is 54 cm^2.

2 Finding Volume Find the volume of the sphere. Leave your answer in terms of π.



$V = \frac{4}{3}\,\pi\,r^3$ Use the formula for volume of a sphere.

$= \frac{4}{3}\pi \cdot 15^3$ Substitute $r = \frac{30}{2} = 15$.

$= 4500\,\pi$ Simplify.

The volume of the sphere is 4500π cm^3.

Quick Check

1. Find the surface area of a sphere with $d = 14$ in. Give your answer in two ways, in terms of π and rounded to the nearest square inch.

196π $in.^2$, 616 $in.^2$

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Example

3 Using Volume to Find Surface Area The volume of a sphere is 1 $in.^3$. Find its surface area to the nearest tenth.

$V = \frac{4}{3}\,\pi\,r^3$ Use the formula for volume of a sphere.

$1 = \frac{4}{3}\pi r^3$ Substitute.

$\frac{3}{4\pi} = r^3$ Solve for r^3.

$\sqrt[3]{\frac{3}{4\pi}} = r$ Find the cube root of each side.

S. A. ≈ 4.83597587 Use *r*, S.A. $= 4\pi r^2$, and a calculator.

To the nearest tenth, the surface area of the sphere is 4.8 $in.^2$.

Quick Check

2. Find the surface area of a spherical melon with circumference 18 in. Round your answer to the nearest ten square inches.

100 $in.^2$

3. Find the volume to the nearest cubic inch of a sphere with diameter 60 in.

113,097 $in.^3$

4. The volume of a sphere is 4200 ft^3. Find the surface area to the nearest tenth.

1258.9 ft^2

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Lesson 11-7 Areas and Volumes of Similar Solids

Lesson Objective	NAEP 2005 Strand: Measurement
Find relationships between the ratios of the areas and volumes of similar solids	Topic: Systems of Measurement Local Standards: ________

Vocabulary and Key Concepts

Theorem 11-12: Areas and Volumes of Similar Solids

If the similarity ratio of two similar solids is $a : b$, then

(1) the ratio of their corresponding areas is $a^2 : b^2$, and

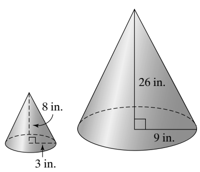
(2) the ratio of their volumes is $a^3 : b^3$.

Similar solids have the same shape and all of their corresponding parts are proportional.

The similarity ratio of two similar solids is the ratio of their corresponding linear dimensions.

Examples

1 Identifying Similar Solids Are the two solids similar? If so, give the similarity ratio.



Both figures have the same shape. Check that the ratios of the corresponding dimensions are equal.

The ratio of the radii is $\frac{3}{9}$, and the ratio of the heights is $\frac{8}{26}$.

The cones are not similar because $\frac{3}{9} \neq \frac{8}{26}$.

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2 Finding the Similarity Ratio Find the similarity ratio of two similar cylinders with surface areas of 98π ft^2 and 2π ft^2. Use the ratio of the surface areas to find the similarity ratios.

$\frac{a^2}{b^2} = \frac{98\pi}{2\pi}$ The ratio of the surface areas is $a^2 : b^2$.

$\frac{a^2}{b^2} = \frac{49}{1}$ Simplify.

$\frac{a}{b} = \frac{7}{1}$ Take the square root of each side.

The similarity ratio is 7 : 1.

3 Using a Similarity Ratio Two similar square pyramids have volumes of 48 cm^3 and 162 cm^3. The surface area of the larger pyramid is 135 cm^2. Find the surface area of the smaller pyramid.

Step 1 Find the similarity ratio.

$\frac{a^3}{b^3} = \frac{48}{162}$ The ratio of the volumes is $a^3 : b^3$.

$\frac{a^3}{b^3} = \frac{8}{27}$ Simplify.

$\frac{a}{b} = \frac{2}{3}$ Take the cube root of each side.

Step 2 Use the similarity ratio to find the surface area S_1 of the smaller pyramid.

$\frac{S_1}{S_2} = \frac{2^2}{3^2}$ The ratio of the surface areas is $a^2 : b^2$.

$\frac{S_1}{S_2} = \frac{4}{9}$ Simplify.

$\frac{S_1}{135} = \frac{4}{9}$ Substitute 135 for S_2, the surface area of the larger pyramid.

$S_1 = \frac{4}{9} \cdot 135$ Solve for S_1.

$S_1 = 60$ Simplify.

The surface area of the smaller pyramid is 60 cm^2.

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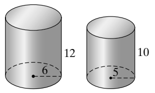
Quick Check

1. Are the two cylinders similar? If so, give the similarity ratio.

yes; 6 : 5



2. Find the similarity ratio of two similar prisms with surface areas 144 m^2 and 324 m^2.

2 : 3

3. The volumes of two similar solids are 128 m^3 and 250 m^3. The surface area of the larger solid is 250 m^2. What is the surface area of the smaller solid?

160 m^2

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Lesson 12-1

Tangent Lines

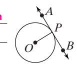
Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use the relationship between a radius and a tangent	Topic: Relationships Among Geometric Figures
2 Use the relationship between two tangents from one point	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 12-1

If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.

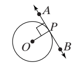
$\overleftrightarrow{AB} \perp \overline{OP}$



Theorem 12-2

If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

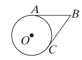
$\overleftrightarrow{AB}$ is tangent to $\odot O$.



Theorem 12-3

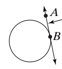
The two segments tangent to a circle from a point outside the circle are congruent.

$\overline{AB} \cong \overline{CB}$



A tangent to a circle is a line, segment, or ray in the plane of the circle that intersects the circle in exactly one point.

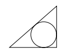
The point of tangency is the point where a circle and a tangent intersect.



A: tangent
B: point of tangency

A triangle is inscribed in a circle if all vertices of the triangle lie on the circle.

A triangle is circumscribed about a circle if each side of the triangle is tangent to the circle.



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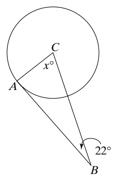
Examples

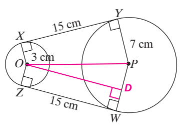
1. **Finding Angle Measures** $\overline{BA}$ is tangent to $\odot C$ at point A. Find the value of x.



Because $\overline{BA}$ is tangent to $\odot C$, $\angle A$ must be a right angle. Use the Triangle Angle-Sum Theorem to find x.

$m\angle A + m\angle B + m\angle C = 180$	Triangle Angle-Sum Theorem
$90 + 22 + x = 180$	Substitute.
$112 + x = 180$	Simplify.
$x = 68$	Solve.

2. **Applying Tangent Lines** A belt fits tightly around two circular pulleys, as shown. Find the distance between the centers of the pulleys. Round your answer to the nearest tenth.



Draw $\overline{OP}$. Then draw $\overline{OD}$ parallel to $\overline{ZW}$ to form rectangle $ODWZ$. Because $\overline{OZ}$ is a radius of $\odot O$, $OZ = 3$ cm.

Because opposite sides of a rectangle have the same measure, $DW = 3$ cm and $OD = 15$ cm.

Because $\angle ODP$ is the supplement of a right angle, $\angle ODP$ is also a right angle, and $\triangle OPD$ is a right triangle.

Because the radius of $\odot P$ is 7 cm, $PD = 7 - 3 = 4$ cm.

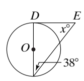
$OD^2 + PD^2 = OP^2$	Pythagorean Theorem
$15^2 + 4^2 = OP^2$	Substitute.
$241 = OP^2$	Simplify.
$OP \approx 15.524175$	Use a calculator to find the square root.

The distance between the centers of the pulleys is about 15.5 cm.

Quick Check

1. $\overline{ED}$ is tangent to $\odot O$. Find the value of x.

52



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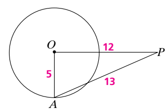
Examples

3. **Finding a Tangent** $\odot O$ has radius 5. Point P is outside $\odot O$ such that $PO = 12$, and point A is on $\odot O$ such that $PA = 13$. Is $\overline{PA}$ tangent to $\odot O$ at A? Explain.

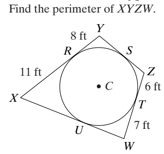


For $\overline{PA}$ to be tangent to $\odot O$ at A, $\angle A$ must be a right angle, $\triangle OAP$ must be a right triangle, and $PO^2 = PA^2 + OA^2$

$PO^2 \stackrel{?}{=} PA^2 + OA^2$	Is $\triangle OAP$ a right triangle?
$12^2 \stackrel{?}{=} 13^2 + 5^2$	Substitute.
$144 \neq 194$	Simplify.

Because $PO^2 \neq PA^2 + OA^2$, $\overline{PA}$ is not tangent to $\odot O$ at A.

4. **Circles Inscribed in Polygons** $\odot C$ is inscribed in quadrilateral $XYZW$. Find the perimeter of $XYZW$.



$UX = XR = 11$ ft
$YS = RY = 8$ ft
$SZ = ZT = 6$ ft
$WU = TW = 7$ ft

By Theorem 12-3, two segments tangent to a circle from a point outside the circle are congruent.

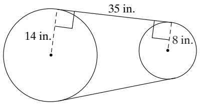
$p = XY + YZ + ZW + WX$	Definition of perimeter p
$= XR + RY + YS + SZ + ZT + TW + WU + UX$	Segment Addition Postulate
$= 11 + 8 + 8 + 6 + 6 + 7 + 7 + 11$	Substitute.
$= 64$	Simplify.

The perimeter is 64 ft.

Name______________________ Class______________________ Date______________

Quick Check

2. A belt fits tightly around two circular pulleys, as shown. Find the distance between the centers of the pulleys.

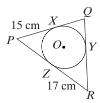


about 35.5 in.

3. In Example 3, if $OA = 4$, $AP = 7$, and $OP = 8$, is $\overline{PA}$ tangent to $\odot O$ at A? Explain.

No; $4^2 + 7^2 \neq 8^2$

4. $\odot O$ is inscribed in $\triangle PQR$. $\triangle PQR$ has a perimeter of 88 cm. Find QY.



12 cm

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Lesson 12-2

Chords and Arcs

Lesson Objectives	NAEP 2005 Strand: Geometry
Use congruent chords, arcs, and central angles	Topic: Relationships Among Geometric Figures
Recognize properties of lines through the center of a circle	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 12-4

Within a circle or in congruent circles

(1) Congruent central angles have congruent chords.

(2) Congruent chords have congruent arcs.

(3) Congruent arcs have congruent central angles.

Theorem 12-5

Within a circle or in congruent circles

(1) Chords equidistant from the center are congruent.

(2) Congruent chords are equidistant from the center.

Theorem 12-6

In a circle, a diameter that is perpendicular to a chord bisects the chord and its arcs.

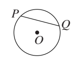
Theorem 12-7

In a circle, a diameter that bisects a chord (that is not a diameter) is perpendicular to the chord.

Theorem 12-8

In a circle, the perpendicular bisector of a chord contains the center of the circle.

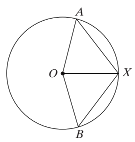
A chord is a segment whose endpoints are on a circle.



Name______________________ Class______________________ Date______________

Examples

1. **Using Theorem 12-4** In the diagram, radius $\overline{OX}$ bisects $\angle AOB$. What can you conclude?

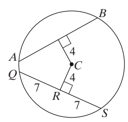


$\angle AOX \cong$ $\angle BOX$ by the definition of an angle bisector.

$\overline{AX} \cong$ $\overline{BX}$ because congruent central angles have congruent chords.

$\overset{\frown}{AX} \cong$ $\overset{\frown}{BX}$ because congruent chords have congruent arcs.

2. **Using Theorem 12-5** Find AB.



$QS = QR + RS$ — Segment Addition Postulate

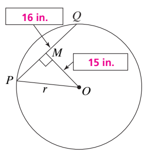
$QS = 7 + 7$ — Substitute.

$QS = 14$ — Simplify.

$AB = QS$ — Chords that are equidistant from the center of a circle are congruent.

$AB = 14$ — Substitute 14 for QS.

3. **Using Diameters and Chords** P and Q are points on $\odot O$. The distance from O to $\overline{PQ}$ is 15 in., and $PQ = 16$ in. Find the radius of $\odot O$.



The distance from the center of $\odot O$ to $\overline{PQ}$ is measured along a perpendicular line.

$PM = \frac{1}{2}PQ$ — A diameter that is perpendicular to a chord bisects the chord.

$PM = \frac{1}{2}(16) = 8$ — Substitute.

$OP^2 = PM^2 + OM^2$ — Use the Pythagorean Theorem.

$r^2 = 8^2 + 15^2$ — Substitute.

$r^2 = 289$ — Simplify.

$r = 17$ — Find the square root of each side.

The radius of $\odot O$ is 17 in.

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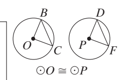
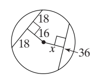
Quick Check

1. If you are given that $\overset{\frown}{BC} \cong \overset{\frown}{DF}$ in the circles, what can you conclude?



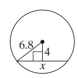
$\angle O \cong \angle P$; $\overline{BC} \cong \overline{DF}$

2. Find the value of x in the circle at the right.



16

3. Use the circle at the right.



a. Find the length of the chord to the nearest unit.

about 11

b. Find the distance from the midpoint of the chord to the midpoint of its minor arc.

2.8

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Lesson 12-3

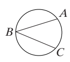
Inscribed Angles

Lesson Objectives	NAEP 2005 Strand: Geometry
Find the measure of an inscribed angle; Find the measure of an angle formed by a tangent and a chord	Topic: Relationships Among Geometric Figures; Local Standards: ____

Vocabulary and Key Concepts

Theorem 12-9: Inscribed Angle Theorem

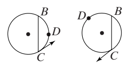
The measure of an inscribed angle is **half the measure of its intercepted arc.**



$m\angle B = \frac{1}{2}m\overarc{AC}$

Theorem 12-10

The measure of an angle formed by a tangent and a chord is **half the measure of the intercepted arc.**

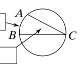


$m\angle C = \frac{1}{2}m\overarc{BDC}$

Corollaries to the Inscribed Angle Theorem

1. Two inscribed angles that intercept the same arc are **congruent**.
2. An angle inscribed in a semicircle is a **right** angle.
3. The opposite angles of a quadrilateral inscribed in a circle are **supplementary**.

An inscribed angle has **a vertex that is on a circle and sides that are chords of the circle.**



An intercepted arc is **an arc with endpoints on the sides of an inscribed angle and its other points in the interior of the angle.**

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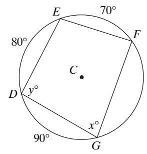
Examples

1 Using the Inscribed Angle Theorem Find the values of x and y.



$x = \frac{1}{2}m\overarc{DEF}$ Inscribed Angle Theorem

$x = \frac{1}{2}(m\overarc{DE} + m\overarc{EF})$ Arc Addition Postulate

$x = \frac{1}{2}(80 + 70)$ Substitute.

$x = 75$ Simplify.

Because $\overarc{EFG}$ is the intercepted arc of $\angle D$, you need to find $m\overarc{FG}$ in order to find $m\overarc{EFG}$. The arc measure of a circle is 360°, so

$m\overarc{FG} = 360 - 70 - 80 - 90 = 120$.

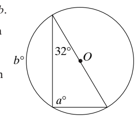
$y = \frac{1}{2}m\overarc{EFG}$ Inscribed Angle Theorem

$y = \frac{1}{2}(m\overarc{EF} + m\overarc{FG})$ Arc Addition Postulate

$y = \frac{1}{2}(70 + 120)$ Substitute.

$y = 95$ Simplify.

2 Using Corollaries to Find Angle Measures Find the values of a and b.



By Corollary 2 to the Inscribed Angle Theorem, an angle inscribed in a semicircle is a right angle, so $a = 90$.

The sum of the measures of the three angles of the triangle inscribed in $\odot O$ is **180**. Therefore, the angle whose intercepted arc has measure b must have measure $180 - 90 - 32$ or **58**.

Because the inscribed angle has **half** the measure of the intercepted arc, the intercepted arc has **twice** the measure of the inscribed angle, so $b = 2(58) = 116$.

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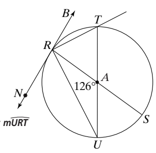
3 Using Theorem 12-10 $\overline{RS}$ and $\overline{TU}$ are diameters of circle A. $\overleftrightarrow{RB}$ is tangent to $\odot A$ at point R. Find $m\angle BRT$ and $m\angle TRS$.



$m\angle BRT = \frac{1}{2}m\overarc{RT}$ Theorem 12-10

$m\overarc{RT} = m\overarc{URT} - m\overarc{UR}$ Arc Addition Postulate

$m\angle BRT = \frac{1}{2}(180 - 126)$ Substitute 180 for $m\overarc{URT}$ and 126 for $m\overarc{UR}$.

$m\angle BRT = 27$ Simplify.

Use the properties of tangents to find $m\angle TRS$.

$m\angle BRS = 90$ A tangent is **perpendicular** to the radius of a circle at its point of tangency.

$m\angle BRS = m\angle BRT + m\angle TRS$ Angle Addition Postulate

$90 = 27 + m\angle TRS$ Substitute.

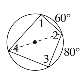
$63 = m\angle TRS$ Simplify.

Quick Check

1. In Example 1, find $m\angle DEF$.

   105

2. For the diagram at the right, find the measure of each numbered angle.

   $m\angle 1 = 90$, $m\angle 2 = 110$, $m\angle 3 = 90$, $m\angle 4 = 70$

3. In Example 3, describe two ways to find $m\angle NRS$ using Theorem 12-10.

   $m\angle NRS = m\angle NRU + m\angle URS = \frac{1}{2}m\overarc{RU} + 27 = 63 + 27 = 90$
   $m\angle NRS = \frac{1}{2}m\overarc{RUS} = \frac{1}{2}(180) = 90$

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Lesson 12-4

Angle Measures and Segment Lengths

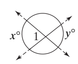
Lesson Objectives	NAEP 2005 Strand: Geometry
Find the measures of angles formed by chords, secants, and tangents; Find the lengths of segments associated with circles	Topic: Relationships Among Geometric Figures; Local Standards: ____

Vocabulary and Key Concepts

Theorem 12-11

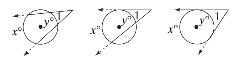
The measure of an angle formed by two lines that

(1) intersect inside a circle is half the **sum of the measures of the intercepted arcs.**



$m\angle 1 = \frac{1}{2}(x + y)$

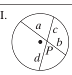
(2) intersect outside a circle is half the **difference of the measures of the intercepted arcs.**



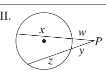
$m\angle 1 = \frac{1}{2}(x - y)$

Theorem 12-12

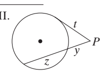
For a given point and circle, the product of the lengths of the two segments from the point to the circle is **constant along any line through the point and circle.**

I. 

$a \cdot b = c \cdot d$

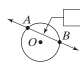
II. 

$(w + x)w = (y + z)y$

III. 

$(y + z)y = t^2$

A secant is **a line that intersects a circle at two points.**



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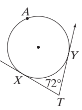
Examples

1 Finding Arc Measures An advertising agency wants a frontal photo of a "flying saucer" ride at an amusement park. The photographer stands at the vertex of the angle formed by tangents to the "flying saucer." What is the measure of the arc that will be in the photograph? In the diagram, the photographer stands at point T. $\overline{TX}$ and $\overline{TY}$ intercept minor arc $\overset{\frown}{XY}$ and major arc $\overset{\frown}{XAY}$.



Let $m\overset{\frown}{XY} = x$.

Then $m\overset{\frown}{XAY} = \boxed{360} - x$.

$m\angle T = \frac{\boxed{1}}{\boxed{2}}(m\overset{\frown}{XAY} - m\overset{\frown}{XY})$ — Theorem 12-11(2)

$\boxed{72} = \frac{1}{2}[(\boxed{360 - x}) - \boxed{x}]$ — Substitute.

$\boxed{72} = \frac{1}{2}(\boxed{360} - 2x)$ — Simplify.

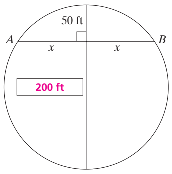
$\boxed{72} = \boxed{180} - x$ — Distributive Property.

$x + \boxed{72} = \boxed{180}$ — Add $\boxed{x}$ to both sides.

$x = \boxed{108}$ — Solve for x.

A $\boxed{108°}$ arc will be in the advertising agency's photo.

2 Finding Segment Lengths A tram travels from point A to point B along the arc of a circle with a radius of 125 ft. Find the shortest distance from point A to point B.



The perpendicular bisector of the chord $\overline{AB}$ contains the center of the circle. Because the radius is 125 ft, the diameter is $\boxed{2} \cdot \boxed{125} = \boxed{250}$ ft. The length of the other segment along the diameter is $\boxed{250}$ ft $-$ $\boxed{50}$ ft, or $\boxed{200}$ ft.

$x \cdot x = \boxed{50} \cdot \boxed{200}$ — Theorem 12-12(1)

$x^2 = \boxed{10{,}000}$ — Multiply.

$x = \boxed{100}$ — Solve for x.

The shortest distance from point A to point B is $2x$ or $\boxed{200}$ ft.

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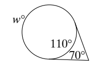
Quick Check

1. Find the value of w.

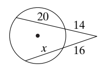


250

2. **Critical Thinking** To photograph a 160° arc of the "flying saucer" in Example 1, should you move toward or away from the object? What angle should the tangents form?

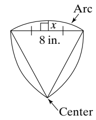
away; 20°

3. Find the value of x to the nearest tenth.



13.8

4. The basis of a design of a rotor for a Wankel engine is an equilateral triangle. Each side of the triangle is a chord to an arc of a circle. The opposite vertex of the triangle is the center of the arc. In the diagram at the right, each side of the equilateral triangle is 8 in. long.



   a. Use what you know about equilateral triangles and find the value of x.

   $(8 - 4\sqrt{3})$ in.

   b. **Critical Thinking** Complete the circle with the given center. Then use Theorem 12-12 to find the value of x. Show that your answers to parts (a) and (b) are equal.

   $\frac{16}{8 + 4\sqrt{3}}$ in.; $(8 - 4\sqrt{3}) = \frac{16}{8 + 4\sqrt{3}} \approx 1.07$

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Lesson 12-5

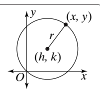
Circles in the Coordinate Plane

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Write an equation of a circle	Topic: Position and Direction
2 Find the center and radius of a circle	Local Standards: ____________

Key Concepts

Theorem 12-13

The standard form of an equation of a circle with center $\boxed{(h, k)}$ and radius $\boxed{r}$ is $(x - h)^2 + (y - k)^2 = r^2$.



Examples

1 Writing the Equation of a Circle Write the standard equation of a circle with center $(-8, 0)$ and radius $\sqrt{5}$.

$(x - \boxed{h})^2 + (y - \boxed{k})^2 = \boxed{r}^2$ — Standard form

$[x - (\boxed{-8})]^2 + (y - \boxed{0})^2 = (\boxed{\sqrt{5}})^2$ — Substitute $(\boxed{-8, 0})$ for (h, k) and $\boxed{\sqrt{5}}$ for r.

$(x + \boxed{8})^2 + y^2 = \boxed{5}$ — Simplify.

2 Using the Center and a Point on a Circle Write the standard equation of a circle with center $(5, 8)$ that passes through the point $(-15, -13)$.

First find the radius.

$r = \sqrt{(x - \boxed{h})^2 + (y - \boxed{k})^2}$ — Use the $\boxed{\text{Distance}}$ Formula to find r.

$= \sqrt{(\boxed{-15} - \boxed{5})^2 + (\boxed{-13} - \boxed{8})^2}$ — Substitute $(\boxed{5, 8})$ for (h, k) and $(\boxed{-15, -13})$ for (x, y).

$= \sqrt{(\boxed{-20})^2 + (\boxed{-21})^2}$ — Simplify.

$= \sqrt{\boxed{400} + \boxed{441}}$

$= \sqrt{\boxed{841}} = \boxed{29}$

Then find the standard equation of the circle with center $(5, 8)$ and radius $\boxed{29}$.

$(x - \boxed{h})^2 + (y - \boxed{k})^2 = \boxed{r}^2$ — Standard form

$(x - \boxed{5})^2 + (y - \boxed{8})^2 = (\boxed{29})^2$ — Substitute $(\boxed{5, 8})$ for (h, k) and $\boxed{29}$ for r.

$(x - \boxed{5})^2 + (y - \boxed{8})^2 = \boxed{841}$ — Simplify.

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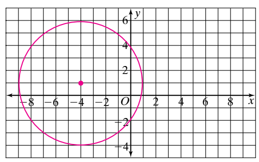
3 Graphing a Circle Given Its Equation Find the center and radius of the circle with equation $(x + 4)^2 + (y - 1)^2 = 25$. Then graph the circle.

$(x + \boxed{4})^2 + (y - \boxed{1})^2 = \boxed{25}$

$(x - \boxed{-4})^2 + (y - \boxed{1})^2 = (\boxed{5})^2$ — Relate the equation to the standard form $(x - h)^2 + (y - k)^2 = r^2$.

$\boxed{h}$ $\boxed{k}$ $\boxed{r}$

The center is $\boxed{(-4, 1)}$ and the radius is $\boxed{5}$.



4 Applying the Equation of a Circle A diagram locates a radio tower at $(6, -12)$ on a coordinate grid where each unit represents 1 mi. The radio signal's range is 80 mi. Find an equation that describes the position and range of the tower.

The center of a circular range is at $\boxed{(6, -12)}$, and the radius is $\boxed{80}$.

$(x - h)^2 + (y - k)^2 = r^2$ — Use standard form.

$(x - \boxed{6})^2 + [y - (\boxed{-12})]^2 = \boxed{80}^2$ — Substitute.

$(x - \boxed{6})^2 + (y + \boxed{12})^2 = \boxed{6400}$ — This is an equation for position and range of the tower.

Quick Check

1. Write the standard equation of each circle.

   a. center $(3, 5)$; radius 6

   $(x - 3)^2 + (y - 5)^2 = 36$

   b. center $(-2, -1)$; radius $\sqrt{2}$

   $(x + 2)^2 + (y + 1)^2 = 2$

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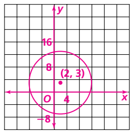
2. Write the standard equation of the circle with center (2, 3) that passes through the point (−1, 1).

$(x - 2)^2 + (y - 3)^2 = 13$

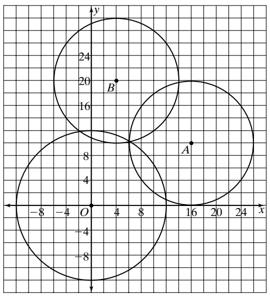
3. Find the center and radius of the circle with equation $(x - 2)^2 + (y - 3)^2 = 100$. Then graph the circle.

center: (2, 3); radius: 10



4. When you make a call on a cellular phone, a tower receives the call. In the diagram, the centers of circles O, A, and B are locations of cellular telephone towers. Write an equation that describes the position and range of Tower O.

$x^2 + y^2 = 144$



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Lesson 12-6 Locus: A Set of Points

Lesson Objective	NAEP 2005 Strand: Geometry
Draw and describe a locus	Topic: Dimension and Shape
	Local Standards: ____________

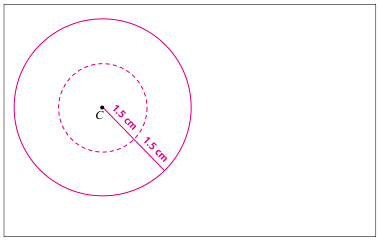
Vocabulary

A locus is a set of points, all of which meet a stated condition.

Examples

❶ **Describing a Locus in a Plane** Draw and describe the locus of points in a plane that are 1.5 cm from ⊙ C with radius 1.5 cm.

Use a compass to draw ⊙ C with radius 1.5 cm.



The locus of points in the interior of ⊙ C that are 1.5 cm from ⊙ C is the center C.

The locus of points exterior to ⊙ C that are 1.5 cm from ⊙ C is a circle with center C and radius 3 cm.

The locus of points in a plane that are 1.5 cm from a point on ⊙ C with radius 1.5 cm is point C and a circle with radius 3 cm and center C.

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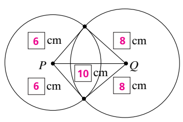
❷ **Drawing a Locus for Two Conditions** Point P is 10 cm from point Q. Describe the locus of points in a plane that are 6 cm from point P and 8 cm from point Q.

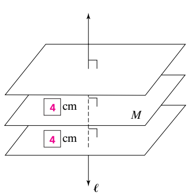


The set of points in a plane that are 6 cm from point P is a circle with center P and radius 6 cm.

The set of points in a plane that are 8 cm from point Q is a circle with center Q and radius 8 cm.

Because the sum (14 cm) of the radii is greater than 10 cm, the circles overlap by 4 cm on $\overline{PQ}$. The circles intersect at two points, so the locus is the two points where the circles intersect.

❸ **Describing a Locus in Space** Describe the locus of points in space that are 4 cm from a plane M.



Imagine plane M as a horizontal plane. Because distance is measured along a perpendicular segment from a point to a plane, the locus of points in space that are 4 cm from a plane M are a plane 4 cm above and parallel to plane M and another plane 4 cm below and parallel to plane M.

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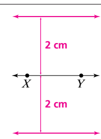
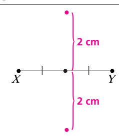
Quick Check

1. Draw and describe the locus: In a plane, the points 2 cm from line $\overleftrightarrow{XY}$.

Two lines parallel to $\overleftrightarrow{XY}$, each 2 cm from $\overleftrightarrow{XY}$.



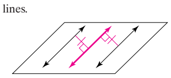
2. Draw the locus: In a plane, the points equidistant from two points X and Y and 2 cm from the midpoint of $\overline{XY}$.



3. Draw and describe each locus.

a. In a plane, the points that are equidistant from two parallel lines.

A line midway between the two parallel lines, parallel to and equidistant from each.



b. In space, the points that are equidistant from two parallel planes.

A plane midway between the two parallel planes, parallel to and equidistant from each.