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Lesson 1-1

Patterns and Inductive Reasoning

Lesson Objective	NAEP 2005 Strand: Geometry
1 Use inductive reasoning to make conjectures	Topic: Mathematical Reasoning
	Local Standards: ____________

Vocabulary

Inductive reasoning is reasoning based on patterns you observe.

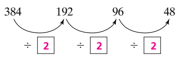
A conjecture is a conclusion you reach using inductive reasoning.

A counterexample is an example for which the conjecture is incorrect.

Example

1 **Finding and Using a Pattern** Find a pattern for the sequence. Use the pattern to find the next two terms in the sequence.

384, 192, 96, 48, . . .



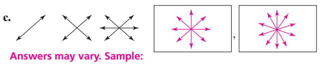
Each term is half the preceding term. The next two terms are 48 ÷ 2 = 24 and 24 ÷ 2 = 12.

Quick Check

1. Find the next two terms in each sequence.

a. 1, 2, 4, 7, 11, 16, 22, 29, 37, . . .

b. Monday, Tuesday, Wednesday, Thursday, Friday, . . .

c.  , . . .

Answers may vary. Sample:

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Example

2 **Using Inductive Reasoning** Make a conjecture about the sum of the cubes of the first 25 counting numbers.

Find the first few sums. Notice that each sum is a perfect square and that the perfect squares form a pattern.

$1^3 = 1 = 1^2 = 1^2$

$1^3 + 2^3 = 9 = 3^2 = (1 + 2)^2$

$1^3 + 2^3 + 3^3 = 36 = 6^2 = (1 + 2 + 3)^2$

$1^3 + 2^3 + 3^3 + 4^3 = 100 = 10^2 = (1 + 2 + 3 + 4)^2$

$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225 = 15^2 = (1 + 2 + 3 + 4 + 5)^2$

The sum of the first two cubes equals the square of the sum of the first two counting numbers. The sum of the first three cubes equals the square of the sum of the first three counting numbers. This pattern continues for the fourth and fifth rows. So a conjecture might be that the sum of the cubes of the first 25 counting numbers equals the square of the sum of the first 25 counting numbers, or $(1 + 2 + 3 + \ldots + 25)^2$.

Quick Check

2. Make a conjecture about the sum of the first 35 odd numbers. Use your calculator to verify your conjecture.

$1 = 1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

$1 + 3 + 5 + 7 + 9 = 25 = 5^2$

The sum of the first 35 odd numbers is 35^2, or 1225.

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Lesson 1-2

Drawings, Nets, and Other Models

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Make isometric and orthographic drawings	Topic: Dimension and Shape
2 Draw nets for three-dimensional figures	Local Standards: ____________

Vocabulary

An isometric drawing of a three-dimensional object shows a corner view of the figure drawn on isometric dot paper.

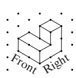
An orthographic drawing is the top view, front view, and right-side view of a three-dimensional figure.

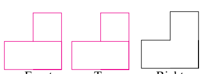
A net is a two-dimensional pattern you can fold to form a three-dimensional figure.

Example

1 **Orthographic Drawing** Make an orthographic drawing of the isometric drawing at right.

Orthographic drawings flatten the depth of a figure. An orthographic drawing shows three views. Because no edge of the isometric drawing is hidden in the top, front, and right views, all lines are solid.

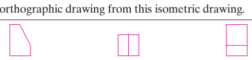




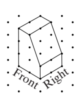
Front Top Right

Quick Check

1. Make an orthographic drawing from this isometric drawing.



Front Top Right



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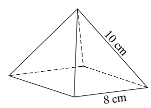
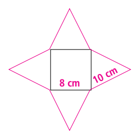
Example

2 **Drawing a Net** Draw a net for the figure with a square base and four isosceles triangle faces. Label the net with its dimensions.

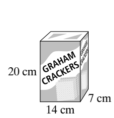


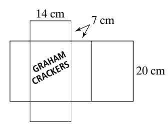
Think of the sides of the square base as hinges, and "unfold" the figure at these edges to form a net. The base of each of the four isosceles triangle faces is a side of the square. Write in the known dimensions.



Quick Check

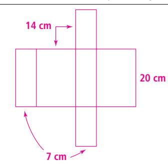
2. The drawing shows one possible net for the Graham Crackers box.





Draw a different net for this box. Show the dimensions in your diagram.

Answers may vary. Example:



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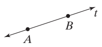
Lesson 1-3
Points, Lines, and Planes

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Understand basic terms of geometry	Topic: Dimension and Shape
2 Understand basic postulates of geometry	Local Standards: ____________

Vocabulary and Key Concepts

Postulate 1-1

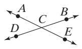
Through any two points there is exactly one line.



Line *t* is the only line that passes through points *A* and B.

Postulate 1-2

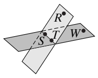
If two lines intersect, then they intersect in exactly one point.



$\overleftrightarrow{AE}$ and $\overleftrightarrow{BD}$ intersect at C.

Postulate 1-3

If two planes intersect, then they intersect in exactly one line.



Plane *RST* and plane *STW* intersect in $\overleftrightarrow{ST}$.

Postulate 1-4

Through any three noncollinear points there is exactly one plane.

A point is a location.

Space is the set of all points.

A line is a series of points that extends in two opposite directions without end.

Collinear points are points that lie on the same line.

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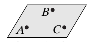
A plane is a flat surface that has no thickness.



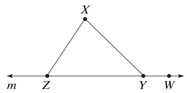
Plane *ABC*

Two points or lines are coplanar if they lie on the same plane.

A postulate or axiom is an accepted statement of fact.

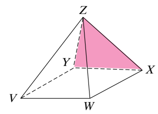
Examples

1. **Identifying Collinear Points** In the figure at right, name three points that are collinear and three points that are not collinear.



Points Y, Z, and W lie on a line, so they are collinear.

2. **Using Postulate 1-4** Shade the plane that contains *X*, *Y*, and *Z*.



Points *X*, *Y*, and *Z* are the vertices of one of the four triangular faces of the pyramid. To shade the plane, shade the interior of the triangle formed by X, Y, and Z.

Quick Check

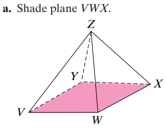
1. Use the figure in Example 1.
   a. Are points *W*, *Y*, and *X* collinear?

   no

   b. Name line *m* in three different ways.

   Answers may vary. Sample: $\overleftrightarrow{ZW}$, $\overleftrightarrow{WY}$, $\overleftrightarrow{YZ}$.

2. a. Shade plane *VWX*.



   b. Name a point that is coplanar with points *V*, *W*, and *X*.

   Y

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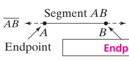
Lesson 1-4
Segments, Rays, Parallel Lines and Planes

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Identify segments and rays	Topic: Relationships Among Geometric Figures
2 Recognize parallel lines	Local Standards: ____________

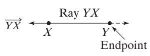
Vocabulary

A segment is the part of a line consisting of two endpoints and all points between them.

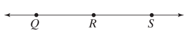


Endpoint

A ray is the part of a line consisting of one endpoint and all the points of the line on one side of the endpoint.



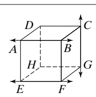
Opposite rays are two collinear rays with the same endpoint.



$\overrightarrow{RQ}$ and $\overrightarrow{RS}$ are opposite rays.

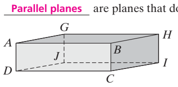
Parallel lines are coplanar lines that do not intersect.

Skew lines are noncoplanar; therefore, they are not parallel and do not intersect.



$\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{EF}$.
$\overleftrightarrow{AB}$ and $\overleftrightarrow{CG}$ are skew lines.

Parallel planes are planes that do not intersect.



Plane *ABCD* is parallel to plane *GHIJ*.

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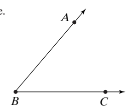
Examples

1. **Naming Segments and Rays** Name the segments and rays in the figure.



The labeled points in the figure are *A*, *B*, and *C*.

A segment is a part of a line consisting of two endpoints and all points between them. A segment is named by its two endpoints. So the segments are $\overline{BA}$ (or $\overline{AB}$) and $\overline{BC}$ (or $\overline{CB}$).

A ray is a part of a line consisting of one endpoint and all the points of the line on one side of that endpoint. A ray is named by its endpoint first, followed by any other point on the ray. So the rays are $\overrightarrow{BA}$ and $\overrightarrow{BC}$.

2. **Identifying Parallel Planes** Identify a pair of parallel planes in your classroom.

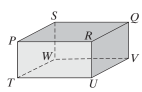
Planes are parallel if they do not intersect. If the walls of your classroom are vertical, opposite walls are parts of parallel planes. If the ceiling and floor of the classroom are level, they are parts of parallel planes.

Quick Check

1. **Critical Thinking** Use the figure in Example 1. $\overrightarrow{CB}$ and $\overrightarrow{BC}$ form a line. Are they opposite rays? Explain.

   No; they do not have the same endpoint.

2. Use the diagram to the right.



   a. Name three pairs of parallel planes.

   PSWT ∥ RQVU, PRUT ∥ SQVW, PSQR ∥ TWVU

   b. Name a line that is parallel to $\overleftrightarrow{PQ}$.

   $\overleftrightarrow{TV}$

   c. Name a line that is parallel to plane *QRUV*.

   Answers may vary. Sample: $\overleftrightarrow{PS}$

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Lesson 1-5

Measuring Segments

Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find the lengths of segments	Topic: Measuring Physical Attributes
	Local Standards: ____________

Vocabulary and Key Concepts

Postulate 1-5: Ruler Postulate

The points of a line can be put into one-to-one correspondence with the real numbers so that the distance between any two points is the absolute value of the difference of the corresponding numbers.

Postulate 1-6: Segment Addition Postulate

If three points A, B, and C are collinear and B is between A and C, then $AB + BC = AC$.

A coordinate is a point's distance and direction from zero on a number line.

the length of $\overline{AB}$

$AB = |\ a - b\ |$

coordinate of A coordinate of B

Congruent (≅) segments are segments with the same length.

2 cm, 2 cm, $AB = CD \rightarrow \overline{AB} \cong \overline{CD}$

A midpoint is a point that divides a segment into two congruent segments.

midpoint

$\overline{AB} \cong \overline{BC}$

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Examples

1 Using the Segment Addition Postulate If $AB = 25$, find the value of x. Then find AN and NB.

$2x - 6$, $x + 7$; A, N, B

Use the Segment Addition Postulate (Postulate 1-6) to write an equation.

$AN + NB = AB$ Segment Addition Postulate

$(2x - 6) + (x + 7) = 25$ Substitute.

$3x + 1 = 25$ Simplify the left side.

$3x = 24$ Subtract 1 from each side.

$x = 8$ Divide each side by 3.

$AN = 2x - 6 = 2(8) - 6 = 10$

$NB = x + 7 = (8) + 7 = 15$ Substitute 8 for x.

$AN = 10$ and $NB = 15$, which checks because the sum equals 25.

2 Finding Lengths M is the midpoint of $\overline{RT}$. Find RM, MT, and RT.

$5x + 9$, $8x - 36$; R, M, T

Use the definition of midpoint to write an equation.

$RM = MT$ Definition of midpoint

$5x + 9 = 8x - 36$ Substitute.

$5x + 45 = 8x$ Add 36 to each side.

$45 = 3x$ Subtract 5x from each side.

$15 = x$ Divide each side by 3.

$RM = 5x + 9 = 5(15) + 9 = 84$

$MT = 8x - 36 = 8(15) - 36 = 84$ Substitute 15 for x.

$RT = RM + MT = 168$ Segment Addition Postulate

RM and MT are each 84, which is half of 168, the length of $\overline{RT}$.

Quick Check

1. $EG = 100$. Find the value of x. Then find EF and FG.
   $4x - 20$, $2x + 30$; E, F, G
   $x = 15$, $EF = 40$; $FG = 60$
2. Z is the midpoint of $\overline{XY}$, and $XY = 27$. Find XZ.
   13.5

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Lesson 1-6

Measuring Angles

Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find the measures of angles	Topic: Measuring Physical Attributes
2 Identify special angle pairs	Local Standards: ____________

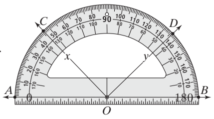
Vocabulary and Key Concepts

Postulate 1-7: Protractor Postulate

Let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be opposite rays in a plane. $\overrightarrow{OA}$, $\overrightarrow{OB}$, and all the rays with endpoint O that can be drawn on one side of $\overleftrightarrow{AB}$ can be paired with the real numbers from 0 to 180 so that

a. $\overrightarrow{OA}$ is paired with 0 and $\overrightarrow{OB}$ is paired with 180.

b. If $\overrightarrow{OC}$ is paired with x and $\overrightarrow{OD}$ is paired with y, then $m\angle COD = |x - y|$.



Postulate 1-8: Angle Addition Postulate

If point B is in the interior of $\angle AOC$, then $m\angle AOB + m\angle BOC = m\angle AOC$.

If $\angle AOC$ is a straight angle, then $m\angle AOB + m\angle BOC = 180$.

An angle (∠) is formed by two rays with the same endpoint. The rays are the sides of the angle and the endpoint is the vertex of the angle.

$\angle TBQ$

acute angle $0 < x < 90$

right angle $x = 90$

obtuse angle $90 < x < 180$

straight angle $x = 180$

An acute angle has measurement between 0° and 90°.

A right angle has a measurement of exactly 90°.

An obtuse angle has measurement between 90° and 180°.

A straight angle has a measurement of exactly 180°.

Congruent angles are two angles with the same measure.

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Examples

1 Naming Angles Name the angle at right in four ways.

The name can be the number between the sides of the angle: ∠3.

The name can be the vertex of the angle: ∠G.

Finally, the name can be a point on one side, the vertex, and a point on the other side of the angle: ∠AGC or ∠CGA.

2 Using the Angle Addition Postulate Suppose that $m\angle 1 = 42$ and $m\angle ABC = 88$. Find $m\angle 2$.

Use the Angle Addition Postulate (Postulate 1-8) to solve.

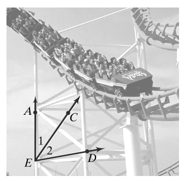
$m\angle 1 + m\angle 2 = m\angle ABC$ Angle Addition Postulate

$42 + m\angle 2 = 88$ Substitute 42 for $m\angle 1$ and 88 for $m\angle ABC$.

$m\angle 2 = 46$ Subtract 42 from each side.

Quick Check

1. a. Name ∠CED two other ways.
   ∠2, ∠DEC
   b. **Critical Thinking** Would it be correct to name any of the angles ∠E? Explain.
   No, 3 angles have E for a vertex, so you need more information in the name to distinguish them from one another.



2. If $m\angle DEG = 145$, find $m\angle GEF$.
   35

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Lesson 1-7

Basic Constructions

Lesson Objectives	NAEP 2005 Strand: Geometry
Use a compass and a straightedge to construct congruent segments and congruent angles Use a compass and a straightedge to bisect segments and angles	Topic: Relationships Among Geometric Figures Local Standards: ____________________

Vocabulary

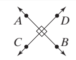
Construction is using a straightedge and a compass to draw a geometric figure.

A straightedge is a ruler with no markings on it.

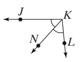
A compass is a geometric tool used to draw circles and parts of circles called arcs.

Perpendicular lines are two lines that intersect to form right angles.

A perpendicular bisector of a segment is a line, segment, or ray that is perpendicular to the segment at its midpoint, thereby bisecting the segment into two congruent segments.



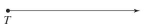
An angle bisector is a ray that divides an angle into two congruent coplanar angles.



Examples

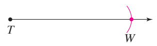
1 Constructing Congruent Segments Construct $\overline{TW}$ congruent to $\overline{KM}$.

Step 1 Draw a ray with endpoint T.



Step 2 Open the compass the length of $\overline{KM}$.

Step 3 With the same compass setting, put the compass point on point T. Draw an arc that intersects the ray. Label the point of intersection W.



$\overline{KM} \cong \overline{TW}$

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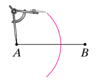
2 Constructing the Perpendicular Bisector

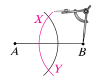
Given: $\overline{AB}$.

Construct: $\overleftrightarrow{XY}$ so that $\overleftrightarrow{XY} \perp \overline{AB}$ at the midpoint M of $\overline{AB}$.

Step 1 Put the compass point on point A and draw a long arc. Be sure that the opening is greater than $\frac{1}{2}AB$.

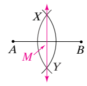


Step 2 With the same compass setting, put the compass point on point B and draw another long arc. Label the points where the two arcs intersect as X and Y.



Step 3 Draw $\overleftrightarrow{XY}$. The point of intersection of $\overline{AB}$ and $\overleftrightarrow{XY}$ is M, the midpoint of $\overline{AB}$.

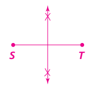
$\overleftrightarrow{XY} \perp \overline{AB}$ at the midpoint of $\overline{AB}$, so $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$.



Quick Check

1. Use a straightedge to draw $\overline{XY}$. Then construct $\overline{RS}$ so that $RS = 2XY$.

2. Draw $\overline{ST}$. Construct its perpendicular bisector.



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Lesson 1-8

The Coordinate Plane

Lesson Objectives	NAEP 2005 Strand: Measurement
Find the distance between two points in the coordinate plane Find the coordinates of the midpoint of a segment in the coordinate plane	Topic: Measuring Physical Attributes Local Standards: ____________________

Key Concepts

Formula: The Distance Formula

The distance d between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Formula: The Midpoint Formula

The coordinates of the midpoint M of $\overline{AB}$ with endpoints $A(x_1, y_1)$ and $B(x_2, y_2)$ are the following:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

Examples

1 Finding the Midpoint $\overline{AB}$ has endpoints $(8, 9)$ and $(-6, -3)$. Find the coordinates of its midpoint M.

Use the Midpoint Formula. Let (x_1, y_1) be $(8, 9)$ and (x_2, y_2) be $(-6, -3)$.

The midpoint has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$ Midpoint Formula

The x-coordinate is $\frac{8 + (-6)}{2} = \frac{2}{2} = 1$ Substitute 8 for x_1 and -6 for x_2. Simplify.

The y-coordinate is $\frac{9 + (-3)}{2} = \frac{6}{2} = 3$ Substitute 9 for y_1 and -3 for y_2. Simplify.

The coordinates of the midpoint M are $(1, 3)$.

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2 Finding an Endpoint The midpoint of $\overline{DG}$ is $M(-1, 5)$. One endpoint is $D(1, 4)$. Find the coordinates of the other endpoint G.

Use the Midpoint Formula. Let (x_1, y_1) be $(1, 4)$ and the midpoint $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ be $(-1, 5)$. Solve for x_2 and y_2, the coordinates of G.

Find the x-coordinate of G.

$-1 = \frac{1 + x_2}{2}$	← Use the Midpoint Formula. →	$5 = \frac{4 + y_2}{2}$
$-2 = 1 + x_2$	← Multiply each side by 2. →	$10 = 4 + y_2$
$-3 = x_2$	← Simplify. →	$6 = y_2$

The coordinates of G are $(-3, 6)$.

Quick Check

1. Find the coordinates of the midpoint of $\overline{XY}$ with endpoints $X(2, -5)$ and $Y(6, 13)$.

(4, 4)

2. The midpoint of $\overline{XY}$ has coordinates $(4, -6)$. X has coordinates $(2, -3)$. Find the coordinates of Y.

(6, −9)

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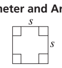
Lesson 1-9

Perimeter, Circumference, and Area

Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find perimeters of rectangles and squares, and circumferences of circles	Topic: Measuring Physical Attributes
2 Find areas of rectangles, squares, and circles	Local Standards: ________

Key Concepts

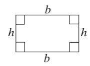
Perimeter and Area



Square with side length s.

Perimeter P = 4s

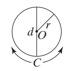
Area A = s^2



Rectangle with base b and height h.

Perimeter P = 2b + 2h

Area A = bh



Circle with radius r and diameter d.

Circumference C = πd or C = $2\pi r$

Area = πr^2

Postulate 1-9

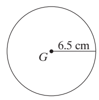
If two figures are congruent, then their areas are equal.

Postulate 1-10

The area of a region is the sum of the areas of its non-overlapping parts.

Examples

1 Finding Circumference $\odot G$ has a radius of 6.5 cm. Find the circumference of $\odot G$ in terms of π. Then find the circumference to the nearest tenth.



$C = 2\,\boxed{\pi}\,\boxed{r}$ — Formula for circumference of a circle

$C = 2\pi(\boxed{6.5})$ — Substitute 6.5 for r.

$C = \boxed{13}\,\pi$ — Exact answer

C = 13 × π = 40.840704 — Use a calculator.

The circumference of $\odot G$ is 13π, or about 40.8 cm.

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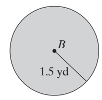
2 Finding Area of a Circle Find the area of $\odot B$ in terms of π.



In $\odot B$, r = 1.5 yd.

$A = \pi\,\boxed{r^2}$ — Formula for the area of a circle

$A = \pi(\boxed{1.5})^2$ — Substitute 1.5 for r.

$A = \boxed{2.25}\,\pi$

The area of $\odot B$ is 2.25π yd^2.

Quick Check

1. a. Find the circumference of a circle with a radius of 18 m in terms of π.

   36π m

   b. Find the circumference of a circle with a diameter of 18 m to the nearest tenth.

   56.5 m

2. You are designing a rectangular banner for the front of a museum. The banner will be 4 ft wide and 7 yd high. How much material do you need in square yards?

   $9\frac{1}{3}$ yd^2

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Lesson 2-1

Conditional Statements

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Recognize conditional statements	Topic: Mathematical Reasoning
2 Write converses of conditional statements	Local Standards: ________

Vocabulary and Key Concepts

Conditional Statements and Converses

Statement	Example	Symbolic Form	You read it
Conditional	If an angle is a straight angle, then its measure is 180°.	$p \rightarrow q$	If p, then q.
Converse	If the measure of an angle is 180°, then it is a straight angle.	$q \rightarrow p$	If q, then p.

A conditional is an *if-then* statement.

The hypothesis is the part that follows *if* in an *if-then* statement.

The conclusion is the part of an *if-then* statement (conditional) that follows *then*.

The truth value of a statement is "true" or "false" according to whether the statement is true or false, respectively.

The converse of the conditional "if p, then q" is the conditional "if q, then p."

Examples

1 Identifying the Hypothesis and the Conclusion Identify the hypothesis and conclusion: If two lines are parallel, then the lines are coplanar.

In a conditional statement, the clause after *if* is the hypothesis and the clause after *then* is the conclusion.

Hypothesis: Two lines are parallel.

Conclusion: The lines are coplanar.

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2 Writing the Converse of a Conditional Write the converse of the following conditional.

If $x = 9$, then $x + 3 = 12$.

The converse of a conditional exchanges the hypothesis and the conclusion.

Conditional		Converse	
Hypothesis	Conclusion	Hypothesis	Conclusion
$x = 9$	$x + 3 = 12$	$x + 3 = 12$	$x = 9$

So the converse is: If $x + 3 = 12$, then $x = 9$.

Quick Check

1. Identify the hypothesis and the conclusion of this conditional statement: If $y - 3 = 5$, then $y = 8$.

   Hypothesis: $y - 3 = 5$

   Conclusion: $y = 8$

2. Write the converse of the following conditional: If two lines are not parallel and do not intersect, then they are skew.

   If two lines are skew, then they are not parallel and do not intersect.

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Lesson 2-2

Biconditionals and Definitions

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Write biconditionals	Topics: Dimension and Shape; Mathematical Reasoning
2 Recognize good definitions	Local Standards: ____

Vocabulary and Key Concepts

Biconditional Statements

A biconditional combines $p \to q$ and $q \to p$ as $p \leftrightarrow q$.

Statement	Example	Symbolic Form	You read it
Biconditional	An angle is a straight angle if and only if its measure is 180°.	$p \leftrightarrow q$	p if and only if q.

A biconditional statement is the combination of a conditional statement and its converse.

A biconditional contains the words "if and only if."

Examples

1 **Writing a Biconditional** Consider the true conditional statement. Write its converse. If the converse is also true, combine the statements as a biconditional.

Conditional: If $x = 5$, then $x + 15 = 20$.

To write the converse, exchange the hypothesis and conclusion.

Converse: If $x + 15 = 20$, then $x = 5$.

When you subtract 15 from each side to solve the equation, you get $x = 5$. Because both the conditional and its converse are true, you can combine them in a biconditional using the phrase *if and only if*.

Biconditional: $x = 5$ if and only if $x + 15 = 20$.

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2 **Identifying a Good Definition** Is the following statement a good definition? Explain.

An apple is a fruit that contains seeds.

The statement is true as a description of an apple.

Exchange "An apple" and "a fruit that contains seeds." The converse reads: A fruit that contains seeds is an apple.

There are many fruits that contain seeds but are not apples, such as lemons and peaches. These are counterexamples, so the converse of the statement is false.

The original statement is not a good definition because the statement is not reversible.

Quick Check

1. Consider the true conditional statement. Write its converse. If the converse is also true, combine the statements as a biconditional.

Conditional: If three points are collinear, then they lie on the same line.

Converse:

If three points lie on the same line, then they are collinear.

The converse is true.

Biconditional:

Three points are collinear if and only if they lie on the same line.

2. Is the following statement a good definition? Explain.

A square is a figure with four right angles.

It is not a good definition because a rectangle has four right angles and is not necessarily a square.

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Lesson 2-3

Deductive Reasoning

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use the Law of Detachment	Topic: Mathematical Reasoning
2 Use the Law of Syllogism	Local Standards: ____

Vocabulary and Key Concepts

Law of Detachment

If a conditional is true and its hypothesis is true, then its conclusion is true.

In symbolic form:

If $p \to q$ is a true statement and p is true, then q is true.

Law of Syllogism

If $p \to q$ and $q \to r$ are true statements, then $p \to r$ is a true statement.

Deductive reasoning is a process of reasoning logically from given facts to a conclusion.

Examples

1 **Using the Law of Detachment** A gardener knows that if it rains, the garden will be watered. It is raining. What conclusion can he make?

The first sentence contains a conditional statement. The hypothesis is it rains.

Because the hypothesis is true, the gardener can conclude that the garden will be watered.

2 **Using the Law of Detachment** For the given statements, what can you conclude?

Given: If $\angle A$ is acute, then $m\angle A < 90°$. $\angle A$ is acute.

A conditional and its hypothesis are both given as true.

By the Law of Detachment, you can conclude that the conclusion of the conditional, $m\angle A < 90°$, is true.

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3 **Using the Law of Syllogism** Use the Law of Syllogism to draw a conclusion from the following true statements:

If a quadrilateral is a square, then it contains four right angles.

If a quadrilateral contains four right angles, then it is a rectangle.

The conclusion of the first conditional is the hypothesis of the second conditional. This means that you can apply the Law of Syllogism.

The Law of Syllogism: If $p \to q$ and $q \to r$ are true statements, then $p \to r$ is a true statement.

So you can conclude:

If a quadrilateral is a square, then it is a rectangle.

Quick Check

1. Suppose that a mechanic begins work on a car and finds that the car will not start. Can the mechanic conclude that the car has a dead battery? Explain.

No, there could be other things wrong with the car, such as a faulty starter.

2. If a baseball player is a pitcher, then that player should not pitch a complete game two days in a row. Vladimir Nuñez is a pitcher. On Monday, he pitches a complete game. What can you conclude?

Answers may vary. Sample: Vladimir Nuñez should not pitch a complete game on Tuesday.

3. If possible, state a conclusion using the Law of Syllogism. If it is not possible to use this law, explain why.

a. If a number ends in 0, then it is divisible by 10.
If a number is divisible by 10, then it is divisible by 5.

If a number ends in 0, then it is divisible by 5.

b. If a number ends in 6, then it is divisible by 2.
If a number ends in 4, then it is divisible by 2.

Not possible; the conclusion of one statement is not the hypothesis of the other statement.

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Lesson 2-4 — Reasoning in Algebra

Lesson Objective	NAEP 2005 Strand: Algebra and Geometry
Connect reasoning in algebra and geometry	Topics: Algebraic Representations; Mathematical Reasoning
	Local Standards: ____________

Key Concepts

Properties of Equality

- **Addition Property** If $a = b$, then $a + c = b + c$.
- **Subtraction Property** If $a = b$, then $a - c = b - c$.
- **Multiplication Property** If $a = b$, then $a \times c = b \times c$.
- **Division Property** If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
- **Reflexive Property** $a = a$.
- **Symmetric Property** If $a = b$, then $b = a$.
- **Transitive Property** If $a = b$ and $b = c$, then $a = c$.
- **Substitution Property** If $a = b$, then b can replace a in any expression.
- **Distributive Property** $a(b + c) = ab + ac$.

Properties of Congruence

- **Reflexive Property** $\overline{AB} \cong \overline{AB}$; $\angle A \cong \angle A$
- **Symmetric Property** If $\overline{AB} \cong \overline{CD}$, then $\overline{CD} \cong \overline{AB}$. If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
- **Transitive Property** If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$. If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

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Examples

1. **Justifying Steps in Solving an Equation** Justify each step used to solve $5x - 12 = 32 + x$ for x.

   Given: $5x - 12 = 32 + x$
   - $5x = 44 + x$ — Addition Property of Equality
   - $4x = 44$ — Subtraction Property of Equality
   - $x = 11$ — Division Property of Equality

2. **Using Properties of Equality and Congruence** Name the property that justifies each statement.

   If $\angle P \cong \angle Q$, $\angle Q \cong \angle R$, and $\angle R \cong \angle S$, then $\angle P \cong \angle S$.

   Use the Transitive Property of Congruence for the first two parts of the hypothesis:

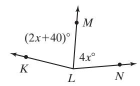
   If $\angle P \cong \angle Q$ and $\angle Q \cong \angle R$, then $\angle P \cong \angle R$.

   Use the Transitive Property of Congruence for $\angle P \cong \angle R$ and the third part of the hypothesis:

   If $\angle P \cong \angle R$ and $\angle R \cong \angle S$, then $\angle P \cong \angle S$.

Quick Check

1. Fill in each missing reason.



   Given: $\overrightarrow{LM}$ bisects $\angle KLN$.
   - $\overrightarrow{LM}$ bisects $\angle KLN$ — Given
   - $m\angle MLN = m\angle KLM$ — Definition of Angle Bisector
   - $4x = 2x + 40$ — Substitution Property of Equality
   - $2x = 40$ — Subtraction Property of Equality
   - $x = 20$ — Division Property of Equality

2. Name the property of equality or congruence illustrated.

   a. $\overline{XY} \cong \overline{XY}$ — Reflexive Property of Congruence

   b. If $m\angle A = 45$ and $45 = m\angle B$, then $m\angle A = m\angle B$ — Transitive or Substitution Property of Equality

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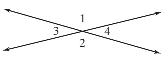
Lesson 2-5 — Proving Angles Congruent

Lesson Objectives	NAEP 2005 Strand: Geometry
Prove and apply theorems about angles	Topic: Relationships Among Geometric Figures
	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 2-1: Vertical Angles Theorem
Vertical angles are congruent.
$\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$



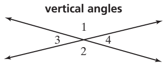
Theorem 2-2: Congruent Supplements Theorem
If two angles are supplements of the same angle (or of congruent angles), then the two angles are congruent.

Theorem 2-3: Congruent Complements Theorem
If two angles are complements of the same angle (or of congruent angles), then the two angles are congruent.

Theorem 2-4
All right angles are congruent.

Theorem 2-5
If two angles are congruent and supplementary, then each is a right angle.

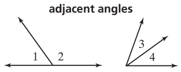
vertical angles



$\angle 1$ and $\angle 2$ are vertical angles, as are $\angle 3$ and $\angle 4$.

Vertical angles are two angles whose sides form two pairs of opposite rays.

adjacent angles



$\angle 1$ and $\angle 2$ are adjacent angles, as are $\angle 3$ and $\angle 4$.

Adjacent angles are two coplanar angles that have a common side and a common vertex but no common interior points.

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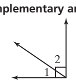
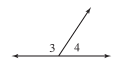
complementary angles



$\angle 1$ and $\angle 2$ are complementary angles.

Two angles are complementary angles if the sum of their measures is 90°.

supplementary angles



$\angle 3$ and $\angle 4$ are supplementary angles.

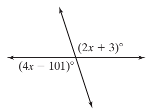
Two angles are supplementary angles if the sum of their measures is 180°.

A theorem is a conjecture that is proven.

A paragraph proof is a convincing argument that uses deductive reasoning in which statements and reasons are connected in sentences.

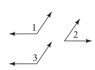
Examples

1. **Using the Vertical Angles Theorem** Find the value of x.



   The angles with labeled measures are vertical angles. Apply the Vertical Angles Theorem to find x.
   - $4x - 101 = 2x + 3$ — Vertical Angles Theorem
   - $4x = 2x + 104$ — Addition Property of Equality
   - $2x = 104$ — Subtraction Property of Equality
   - $x = 52$ — Division Property of Equality

2. **Proving Theorem 2-2** Write a paragraph proof of Theorem 2-2 using the diagram at the right.



   Start with the given: $\angle 1$ and $\angle 2$ are supplementary, $\angle 3$ and $\angle 2$ are supplementary. By the definition of supplementary angles, $m\angle 1 + m\angle 2 = 180$ and $m\angle 3 + m\angle 2 = 180$. By substitution, $m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$. Using the Subtraction Property of Equality, subtract $m\angle 2$ from each side. You get $m\angle 1 = m\angle 3$, or $\angle 1 \cong \angle 3$.

Quick Check

1. Refer to the diagram for Example 1.

   a. Find the measures of the labeled pair of vertical angles. 107°

   b. Find the measures of the other pair of vertical angles. 73°

   c. Check to see that adjacent angles are supplementary. 107° + 73° = 180°

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Lesson 3-1

Properties of Parallel Lines

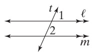
Lesson Objectives	
Identify angles formed by two lines and a transversal Prove and use properties of parallel lines	**NAEP 2005 Strand:** Measurement **Topic:** Measuring Physical Attributes **Local Standards:** ____________

Vocabulary and Key Concepts

Postulate 3-1: Corresponding Angles Postulate

If a transversal intersects two parallel lines, then corresponding angles are congruent.

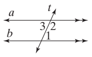
∠1 ≅ ∠2



Theorem 3-1: Alternate Interior Angles Theorem

If a transversal intersects two parallel lines, then alternate interior angles are congruent.

∠1 ≅ ∠3

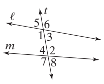


Theorem 3-2: Same-Side Interior Angles Theorem

If a transversal intersects two parallel lines, then same-side interior angles are supplementary.

m∠1 + m∠2 = 180

A transversal is a line that intersects two coplanar lines at two distinct points.



Alternate interior angles are nonadjacent interior angles that lie on opposite sides of the transversal.

∠1 and ∠2 are alternate interior angles.

Same-side interior angles are interior angles that lie on the same side of the transversal.

∠1 and ∠4 are same-side interior angles.

Corresponding angles are angles that lie on the same side of the transversal and in corresponding positions relative to the coplanar lines.

∠1 and ∠7 are corresponding angles.

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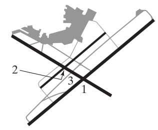
Examples

1 Applying Properties of Parallel Lines In the diagram of Lafayette Regional Airport, the black segments are runways and the gray areas are taxiways and terminal buildings.

Compare ∠2 and the angle vertical to ∠1. Classify the angles as alternate interior angles, same-side interior angles, or corresponding angles.

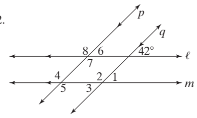


The angle vertical to ∠1 is between the runway segments.

∠2 is between the runway segments and on the opposite side of the transversal runway. Because alternate interior angles are not adjacent and lie between the lines on opposite sides of the transversal, ∠2 and the angle vertical to ∠1 are alternate interior angles.

2 Finding Measures of Angles

In the diagram at right, ℓ‖m and p‖q. Find m∠1 and m∠2.



∠1 and the 42° angle are corresponding angles.

Because ℓ‖m, m∠1 = 42 by the Corresponding Angles Postulate.

Because ∠1 and ∠2 are adjacent angles that form a straight angle, m∠1 + m∠2 = 180 by the Angle Addition Postulate. If you substitute 42 for m∠1, the equation becomes 42 + m∠2 = 180. Subtract 42 from each side to find m∠2 = 138.

Quick Check

1. Use the diagram in Example 1. Classify ∠2 and ∠3 as alternate interior angles, same-side interior angles, or corresponding angles.

   same-side interior angles

2. Using the diagram in Example 2 find the measure of each angle. Justify each answer.

   a. ∠3 42; Vertical angles are congruent

   b. ∠4 138; Corresponding angles are congruent

   c. ∠5 138; Same-side interior angles are supplementary

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Lesson 3-2

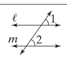
Proving Lines Parallel

Lesson Objectives	
Use a transversal in proving lines parallel	**NAEP 2005 Strand:** Measurement **Topic:** Measuring Physical Attributes **Local Standards:** ____________

Vocabulary and Key Concepts

Postulate 3-2: Converse of the Corresponding Angles Postulate

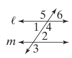
If two lines and a transversal form corresponding angles that are congruent, then the two lines are parallel.



ℓ ‖ m

Theorem 3-5: Converse of the Alternate Interior Angles Theorem

If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.



If ∠1 ≅ ∠2, then ℓ ‖ m.

Theorem 3-6: Converse of the Same-Side Interior Angles Theorem

If two lines and a transversal form same-side interior angles that are supplementary, then the two lines are parallel.

If ∠2 and ∠4 are supplementary, then ℓ ‖ m.

Theorem 3-7: Converse of the Alternate Exterior Angles Theorem

If two lines and a transversal form alternate exterior angles that are congruent, then the two lines are parallel.

If ∠3 ≅ ∠5, then ℓ ‖ m.

Theorem 3-8: Converse of the Same-Side Exterior Angles Theorem

If two lines and a transversal form same-side exterior angles that are supplementary, then the two lines are parallel.

If ∠3 and ∠6 are supplementary, then ℓ ‖ m.

A flow proof uses arrows to show the logical connections between the statements.

Reasons are written below the statements.

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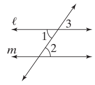
Examples

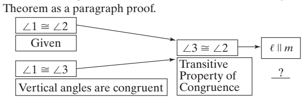
1 Proving Theorem 3-5 *If two lines and a transversal form alternate interior angles that are congruent, then the two lines are parallel.*

Given: ∠1 ≅ ∠2

Prove: ℓ ‖ m



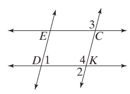
Write the flow proof below of the Alternate Interior Angles Theorem as a paragraph proof.



By the Vertical Angles Theorem, ∠3 ≅ ∠1. ∠1 ≅ ∠2, so ∠3 ≅ ∠2 by the Transitive Property of Congruence. Because ∠3 and ∠2 are corresponding angles, ℓ ‖ m by the Converse of the Corresponding Angles Postulate.

2 Using Postulate 3-2

Use the diagram at the right. Which lines, if any, must be parallel if ∠3 and ∠2 are supplementary? Justify your answer with a theorem or postulate.



It is given that ∠3 and ∠2 are supplementary. The diagram shows that ∠4 and ∠2 are supplementary. Because supplements of the same angle are congruent (Congruent Supplements Theorem), ∠3 ≅ ∠4. Because ∠3 and ∠4 are congruent corresponding angles, $\overleftrightarrow{EC}$ ‖ $\overleftrightarrow{DK}$ by the Converse of the Corresponding Angles Postulate.

Quick Check

1. Supply the missing reason in the flow proof from Example 1.

   If corresponding angles are congruent, then the lines are parallel.

2. Use the diagram from Example 1. Which lines, if any, must be parallel if ∠3 ≅ ∠4? Explain.

   $\overleftrightarrow{EC}$ ‖ $\overleftrightarrow{DK}$; Converse of Corresponding Angles Postulate

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Lesson 3-3

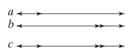
Parallel and Perpendicular Lines

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Relate parallel and perpendicular lines	Topic: Relationships Among Geometric Figures Local Standards: ____________

Key Concepts

Theorem 3-9

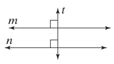
If two lines are [parallel to the same line], then they are [parallel] to each other.



[a] ∥ [b].

Theorem 3-10

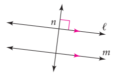
In a plane, if two lines are [perpendicular to the same line], then they are [parallel] to each other.



[m] ∥ [n].

Theorem 3-11

In a plane, if a line is perpendicular to one of two parallel lines, then it is also perpendicular to the other.



[n] ⊥ [m]

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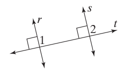
Examples

1 Proof of Theorem 3-10

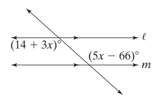
Use the diagram at the right. Which angle would you use with ∠1 to prove the theorem *In a plane, if two lines are perpendicular to the same line, then they are parallel to each other* (Theorem 3-10) using the Converse of the Alternate Interior Angles Theorem instead of the Converse of the Corresponding Angles Postulate?



By the Vertical Angles Theorem, ∠1 is [congruent] to its vertical angle.
Because ∠1 ≅ [∠2], ∠2 is congruent to the vertical angle of [∠1] by the [Transitive] Property of Congruence. Because alternate interior angles are congruent, you can use the vertical angle of [∠1] and the [Converse of the Alternate Interior Angles] Theorem to prove that the lines are parallel.

2 Using Algebra

Find the value of x for which $\ell \parallel m$.



The labeled angles are [alternate interior angles].

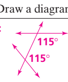
If $\ell \parallel m$, the alternate interior angles are [congruent], and their measures are [equal]. Write and solve the equation [5x] − 66 = [14] + 3x.

[5x] − 66 = [14] + 3x
[5x] = [80] + 3x — Add 66 to each side.
[2x] = [80] — Subtract 3x from each side.
x = [40] — Divide each side by 2.

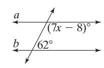
Quick Check

1. **Critical Thinking** In a plane, if two lines form congruent angles with a third line, must the lines be parallel? Draw a diagram to support your answer.

No; answers may vary. Sample:



2. Find the value of x for which $a \parallel b$. Explain how you can check your answer.



$x = 18$; $7(18) - 8 = 118°$, and $62° + 118° = 180°$

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Lesson 3-4

Parallel Lines and the Triangle Angle-Sum Theorem

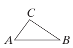
Lesson Objectives	NAEP 2005 Strand: Geometry
1 Classify triangles and find the measures of their angles 2 Use exterior angles of triangles	Topic: Relationships Among Geometric Figures Local Standards: ____________

Vocabulary and Key Concepts

Theorem 3-12: Triangle Angle-Sum Theorem

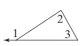
The sum of the measures of the angles of a triangle is [180].

$m\angle A + m\angle B + m\angle C = 180$

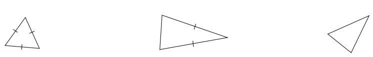


Theorem 3-13: Triangle Exterior Angle Theorem

The measure of each exterior angle of a triangle equals the [sum of the measures of its two remote interior angles].



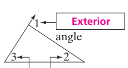
An acute triangle has [three acute angles.]
A right triangle has [one right angle.]
An obtuse triangle has [one obtuse angle.]
An equiangular triangle has [three congruent angles.]



An equilateral triangle has [three congruent sides.]
An isosceles triangle has [at least two congruent sides.]
A scalene triangle has [no congruent sides.]

An exterior angle of a polygon is [an angle formed by a side and an extension of an adjacent side.]

[Remote interior angles] are the two nonadjacent interior angles corresponding to each exterior angle of a triangle.



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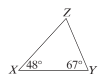
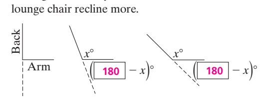
Examples

1 Applying the Triangle Angle-Sum Theorem Find $m\angle Z$.



[48] + [67] + $m\angle Z$ = [180] — Triangle Angle-Sum Theorem
[115] + $m\angle Z$ = 180 — Simplify.
$m\angle Z$ = [65] — Subtract 115 from each side.

2 Applying the Triangle Exterior Angle Theorem Explain what happens to the angle formed by the back of the chair and the armrest as you make a lounge chair recline more.



The exterior angle and the angle formed by the back of the chair and the armrest are [adjacent angles], which together form a [straight angle]. As one measure increases, the other measure [decreases]. The angle formed by the back of the chair and the armrest [increases] as you make a lounge chair recline more.

Quick Check

1. a. △MNP is a right triangle. ∠M is a right angle and $m\angle N$ is 58. Find $m\angle P$.

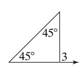
32

b. **Reasoning** Explain why this statement must be true:
If a triangle is a right triangle, its acute angles are complementary.

The sum of the measures of the angles of a triangle is 180. If you subtract the measure of the right angle from 180, you get 90. The sum of the measures of the other two angles is 90, so they are complementary.

2. a. Find $m\angle 3$.

90



b. **Critical Thinking** Is it true that if two acute angles of a triangle are complementary, then the triangle must be a right triangle? Explain.

True, because the measures of the two complementary angles add to 90, leaving 90 for the third angle.

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Lesson 3-5

The Polygon Angle-Sum Theorems

Lesson Objectives	**NAEP 2005 Strand:** Geometry
1 Classify polygons 2 Find the sums of the measures of the interior and exterior angles of polygons	**Topic:** Relationships Among Geometric Figures **Local Standards:** ____________

Vocabulary and Key Concepts

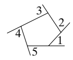
Theorem 3-14: Polygon Angle-Sum Theorem

The sum of the measures of the angles of an *n*-gon is **(n − 2)180**.

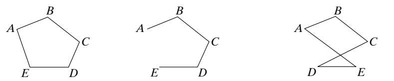
Theorem 3-15: Polygon Exterior Angle-Sum Theorem

The sum of the measures of the exterior angles of a polygon, one at each vertex, is **360**.

For the pentagon, $m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 =$ **360**.



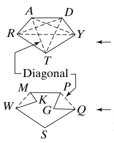
A polygon is **a closed plane figure with at least three sides that are segments. The sides intersect only at their endpoints, and no two adjacent sides are collinear.**



A polygon — Not a polygon; not a **closed** figure — Not a polygon; two sides **intersect** between endpoints.

A **convex polygon** does not have diagonal points outside of the polygon.

A concave polygon has **at least one diagonal with points outside of the polygon.**



← A **convex** polygon

← A **concave** polygon

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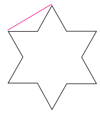
An **equilateral polygon** has all sides congruent.

An equiangular polygon has **all angles congruent**.

A **regular polygon** is both equilateral and equiangular.

Examples

1 Classifying Polygons Classify the polygon at the right by its sides. Identify it as convex or concave.



Starting with any side, count the number of sides clockwise around the figure. Because the polygon has **12** sides, it is a dodecagon. Think of the polygon as a star. Draw a diagonal connecting two adjacent points of the star. That diagonal lies **outside** the polygon, so the dodecagon is **concave**.

2 Finding a Polygon Angle Sum Find the sum of the measures of the angles of a decagon.

A decagon has **10** sides, so $n =$ **10**.

Sum $= (n - 2)(180)$ — **Polygon Angle-Sum** Theorem

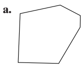
$= (\mathbf{10} - 2)(180)$ — Substitute **10** for *n*.

$= \mathbf{8} \cdot 180$ — Subtract.

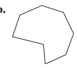
$= \mathbf{1440}$ — Simplify.

Quick Check

1. Classify each polygon by its sides. Identify each as convex or concave.

a. 

hexagon; convex

b. 

octagon; concave

2. Find the sum of the measures of the angles of a 13-gon.

1980

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Lesson 3-6

Lines in the Coordinate Plane

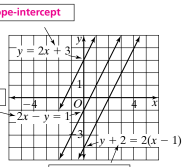
Lesson Objectives	**NAEP 2005 Strand:** Algebra
1 Graph lines given their equations 2 Write equations of lines	**Topics:** Patterns, Relations, and Functions; Algebraic Representations **Local Standards:** ____________

Vocabulary

The slope-intercept form of a linear equation is $y = mx + b$.

The **standard** form of a linear equation is $Ax + By = C$.

The point-slope form for a nonvertical line is $y - y_1 = m(x - x_1)$.



Examples

1 Graphing Lines Using Intercepts Use the *x*-intercept and *y*-intercept to graph $5x - 6y = 30$.

To find the *x*-intercept, substitute 0 for *y* and solve for *x*.

$5x - 6y = 30$

$5x - 6(\mathbf{0}) = 30$

$5x - \mathbf{0} = 30$

$5x = \mathbf{30}$

$x = \mathbf{6}$

The *x*-intercept is **6**.

A point on the line is (**6**, 0).

To find the *y*-intercept, substitute 0 for *x* and solve for *y*.

$5x - 6y = 30$

$5(\mathbf{0}) - 6y = 30$

$\mathbf{0} - 6y = 30$

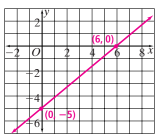
$-6y = \mathbf{30}$

$y = \mathbf{-5}$

The *y*-intercept is **−5**.

A point on the line is (0, **−5**).

Plot (6, 0) and (0, −5). Draw the line containing the two points.



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2 Using Point-Slope Form Write an equation in point-slope form of the line with slope −8 that contains $P(3, -6)$.

$y - \mathbf{y_1} = \mathbf{m}(x - \mathbf{x_1})$ — Use point-slope form.

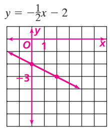
$y - (\mathbf{-6}) = \mathbf{-8}(x - \mathbf{3})$ — Substitute −8 for *m* and (3, −6) for (x_1, y_1).

$y + \mathbf{6} = \mathbf{-8}(x - \mathbf{3})$ — Simplify.

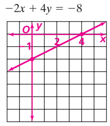
Quick Check

1. Graph each equation.

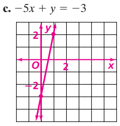
a. $y = -\frac{1}{2}x - 2$



b. $-2x + 4y = -8$



c. $-5x + y = -3$



2. Write an equation of the line that contains the points $P(5, 0)$ and $Q(7, -3)$.

$y - 0 = -\frac{3}{2}(x - 5)$ or $y + 3 = -\frac{3}{2}(x - 7)$

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Lesson 3-7

Slopes of Parallel and Perpendicular Lines

Lesson Objectives	NAEP 2005 Strand: Measurement
1 Relate slope and parallel lines 2 Relate slope and perpendicular lines	**Topic:** Measuring Physical Attributes **Local Standards:** ____________

Key Concepts

Slopes of Parallel Lines

If two nonvertical lines are parallel, their slopes are equal.

If the slopes of two distinct nonvertical lines are equal, the lines are parallel.

Any two vertical lines are parallel.

Slopes of Perpendicular Lines

If two nonvertical lines are perpendicular, the product of their slopes is −1.

If the slopes of two lines have a product of −1, the lines are perpendicular.

Any horizontal line and vertical line are perpendicular.

Examples

1 **Determining Whether Lines are Parallel** Are the lines $y = -5x + 4$ and $x = -5y + 4$ parallel? Explain.

The equation $y = -5x + 4$ is in slope-intercept form. Write the equation $x = -5y + 4$ in slope-intercept form.

$x = -5y + 4$

$x - 4 = -5y$ — Subtract 4 from each side.

$-\frac{1}{5}x + \frac{4}{5} = y$ — Divide each side by −5.

$y = -\frac{1}{5}x + \frac{4}{5}$

The line $x = -5y + 4$ has slope $-\frac{1}{5}$.

The line $y = -5x + 4$ has slope −5.

The lines are not parallel because their slopes are not equal.

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2 **Finding Slopes for Perpendicular Lines** Find the slope of a line perpendicular to $5x + 2y = 1$.

To find the slope of the given line, rewrite the equation in slope-intercept form.

$5x + 2y = 1$

$2y = -5x + 1$ — Subtract 5x from each side.

$y = -\frac{5}{2}x + \frac{1}{2}$ — Divide each side by 2.

The line $5x + 2y = 1$ has slope $-\frac{5}{2}$.

Find the slope of a line perpendicular to $5x + 2y = 1$. Let m be the slope of the perpendicular line.

$-\frac{5}{2}m = -1$ — The product of the slopes of perpendicular lines is −1.

$m = -1 \cdot \left(-\frac{2}{5}\right)$ — Multiply each side by $-\frac{2}{5}$.

$m = \frac{2}{5}$ — Simplify.

Quick Check

1. Are the lines $y = -\frac{1}{2}x + 5$ and $2x + 4y = 9$ parallel? Explain.

   Yes; Each line has slope $-\frac{1}{2}$ and the y-intercepts are different.

2. Find the slope of a line perpendicular to $5y - x = 10$.

   $m = -5$

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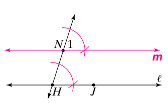
Lesson 3-8

Constructing Parallel and Perpendicular Lines

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Construct parallel lines 2 Construct perpendicular lines	**Topic:** Relationships Among Geometric Figures **Local Standards:** ____________

Example

1 **Constructing $\ell \parallel m$** Examine the diagram at right. Explain how to construct ∠1 congruent to ∠H. Construct the angle.



Use the method learned for constructing congruent angles.

Step 1 With the compass point on point H, draw an arc that intersects the sides of ∠H.

Step 2 With the same compass setting, put the compass point on point N. Draw an arc.

Step 3 Put the compass point below point N where the arc intersects $\overline{HN}$. Open the compass to the length where the arc intersects line ℓ. Keeping the same compass setting, put the compass point above point N where the arc intersects $\overline{HN}$. Draw an arc to locate a point.

Step 4 Use a straightedge to draw line m through the point you located and point N.

Quick Check

1. Use Example 1. Explain why lines ℓ and m must be parallel.

   If corresponding angles are congruent, the lines are parallel by the Converse of Corresponding Angles Postulate.

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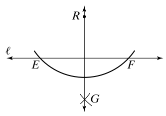
Example

2 **Perpendicular From a Point to a Line** Examine the construction. At what special point does $\overleftrightarrow{RG}$ meet line ℓ?



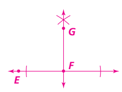
Point R is the same distance from point E as it is from point F because the arc was made with one compass opening.

Point G is the same distance from point E as it is from point F because both arcs were made with the same compass opening.

This means that $\overleftrightarrow{RG}$ intersects line ℓ at the midpoint of $\overline{EF}$, and that $\overleftrightarrow{RG}$ is the perpendicular bisector of $\overline{EF}$.

Quick Check

2. Use a straightedge to draw $\overleftrightarrow{EF}$. Construct $\overleftrightarrow{FG}$ so that $\overleftrightarrow{FG} \perp \overleftrightarrow{EF}$ at point F.



Name______________________ Class______________ Date__________

Lesson 4-1

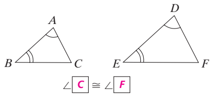
Congruent Figures

Lesson Objective	NAEP 2005 Strand: Geometry
▼ Recognize congruent figures and their corresponding parts	Topic: Transformation of Shapes and Preservation of Properties
	Local Standards: ______

Vocabulary and Key Concepts

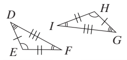
Theorem 4–1

If two angles of one triangle are congruent to two angles of another triangle, then **the third angles are congruent.**



$\angle$ **C** $\cong \angle$ **F**

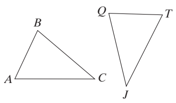
Congruent polygons are **polygons that have corresponding sides congruent and corresponding angles congruent.**



$\triangle$ **DEF** $\cong \triangle$ **GHI**

Examples

1 **Naming Congruent Parts** $\triangle ABC \cong \triangle QTJ$. List the congruent corresponding parts.



List the corresponding sides and angles in the same order.

Angles: $\angle A \cong \angle Q$ $\quad \angle B \cong \angle$ **T** $\quad \angle$ **C** $\cong \angle J$

Sides: $\overline{AB} \cong \overline{QT}$ $\quad$ **BC** $\cong \overline{TJ}$ $\quad \overline{AC} \cong$ **QJ**

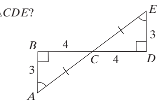
2 **Using Congruency** $\triangle XYZ \cong \triangle KLM$, $m\angle Y = 67$, and $m\angle M = 48$. Find $m\angle X$.

Use the Triangle Angle-Sum Theorem and the definition of congruent polygons to find $m\angle X$.

$m\angle X + m\angle Y + m\angle Z =$ **180**	Triangle Angle-Sum Theorem
$m\angle Z =$ **$m\angle M$**	Corresponding angles of congruent triangles are congruent.
$m\angle Z =$ **48**	Substitute 48 for $m\angle M$.
$m\angle X +$ **67** $+$ **48** $= 180$	Substitute.
$m\angle X +$ **115** $= 180$	Simplify.
$m\angle X =$ **65**	Subtract 115 from each side.

Name______________________ Class______________ Date__________

3 **Finding Congruent Triangles** Can you conclude that $\triangle ABC \cong \triangle CDE$?



List corresponding vertices in the same order.

If $\triangle ABC \cong \triangle CDE$, then $\angle BAC \cong$ **$\angle DCE$**.

The diagram above shows $\angle BAC \cong$ **$\angle DEC$**, not $\angle DCE$.

The statement $\triangle ABC \cong \triangle CDE$ **is not** true.

Notice that $\overline{BC} \cong$ **DC**, $\overline{BA} \cong$ **DE**, and $\overline{AC} \cong$ **EC**.

Also, $\angle CBA \cong$ **$\angle CDE$** and $\angle BAC \cong$ **$\angle DEC$**.

Using Theorem 4-1, you can conclude that $\angle ECD \cong$ **$\angle ACB$**.

Since all of the corresponding sides and angles are congruent, the triangles are congruent. The correct way to state this is $\triangle ABC \cong$ **$\triangle EDC$**.

Quick Check

1. $\triangle WYS \cong \triangle MKV$. List the congruent corresponding parts. Use three letters for each angle.

Sides: $WY \cong MK$ **WS ≅ MV** **YS ≅ KV**

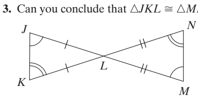
Angles: $\angle WSY \cong \angle MVK$ **∠SWY ≅ ∠VMK** **∠WYS ≅ ∠MKV**

2. It is given that $\triangle WYS \cong \triangle MKV$. If $m\angle Y = 35$, what is $m\angle K$? Explain.

$m\angle K =$ **35**

Corresponding angles of congruent triangles are congruent and have the same measure.

3. Can you conclude that $\triangle JKL \cong \triangle MNL$? Justify your answer.



No. The corresponding sides are not necessarily equal.

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Lesson 4-2

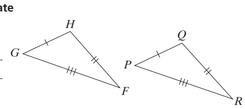
Triangle Congruence by SSS and SAS

Lesson Objective	NAEP 2005 Strand: Geometry
▼ Prove two triangles congruent using the SSS and SAS Postulates	Topic: Transformation of Shapes and Preservation of Properties
	Local Standards: ______

Key Concepts

Postulate 4-1: Side-Side-Side (SSS) Postulate

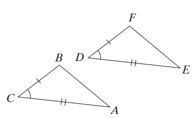
If the three sides of one triangle are congruent to the three sides of another triangle, then **the two triangles are congruent.**



$\triangle$ **GHF** $\cong \triangle$ **PQR**

Postulate 4-2: Side-Angle-Side (SAS) Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then **the two triangles are congruent.**



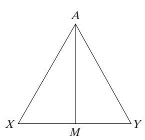
$\triangle$ **BCA** $\cong \triangle$ **FDE**

Examples

1 **Proving Triangles Congruent**

Given: M is the midpoint of $\overline{XY}$, $\overline{AX} \cong \overline{AY}$

Prove: $\triangle AMX \cong \triangle AMY$



Write a paragraph proof.

You are given that M is the midpoint of **XY**, and $\overline{AX} \cong$ **AY**.

Midpoint M implies that $\overline{MX} \cong$ **MY**. $\overline{AM} \cong \overline{AM}$ by the **Reflexive Property of Congruence**, so $\triangle AMX \cong \triangle AMY$ by the **SSS Postulate**.

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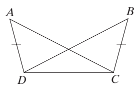
2 **Using SAS** $\overline{AD} \cong \overline{BC}$. What other information do you need to prove $\triangle ADC \cong \triangle BCD$ by SAS?



It is given that $\overline{AD} \cong \overline{BC}$. Also, $\overline{DC} \cong \overline{CD}$ by the **Reflexive Property of Congruence**.

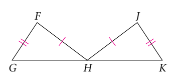
You now have two pairs of corresponding congruent sides.

Therefore, if you know **∠ADC ≅ ∠BCD**, you can prove $\triangle ADC \cong \triangle BCD$ by **SAS**.

Quick Check

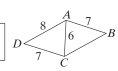
1. *Given:* $\overline{HF} \cong \overline{HJ}$, $\overline{FG} \cong \overline{JK}$, H is the midpoint of $\overline{GK}$.

Prove: $\triangle FGH \cong \triangle JKH$



Statements	Reasons
1. $\overline{HF} \cong \overline{HJ}$, $\overline{FG} \cong \overline{JK}$	1. **Given**
2. H is the midpoint of $\overline{GK}$.	2. **Given**
3. $\overline{GH} \cong \overline{HK}$	3. **Definition of Midpoint**
4. **△FGH ≅ △JKH**	4. **SSS Postulate**

2. What other information do you need to prove $\triangle ABC \cong \triangle CDA$ by SAS?



∠DCA ≅ ∠BAC

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Lesson 4-3

Triangle Congruence by ASA and AAS

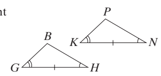
Lesson Objective	NAEP 2005 Strand: Geometry
Prove two triangles congruent using the ASA Postulate and the AAS Theorem	Topic: Transformation of Shapes and Preservation of Properties Local Standards: ____________

Key Concepts

Postulate 4-3: Angle-Side-Angle (ASA) Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent.

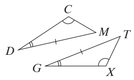
$\triangle$ HGB $\cong \triangle$ NKP



Theorem 4-2: Angle-Angle-Side (AAS) Theorem

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of another triangle, then the triangles are congruent.

$\triangle$ CDM $\cong \triangle$ XGT



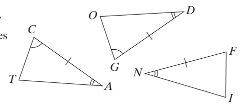
Example

1 Using ASA Suppose that $\angle F$ is congruent to $\angle C$ and $\angle I$ is *not* congruent to $\angle C$. Name the triangles that are congruent by the ASA Postulate.

The diagram shows $\angle N \cong \angle A \cong \angle D$ and $\overline{FN} \cong \overline{CA} \cong \overline{GD}$.

If $\angle F \cong \angle C$, then $\angle F \cong \angle C \cong \angle$ G.

Therefore, $\triangle FNI \cong \triangle CAT \cong \triangle GDO$ by ASA.



Quick Check

1. Using only the information in the diagram, can you conclude that $\triangle INF$ is congruent to either of the other two triangles? Explain.

No; Only one angle and one side are shown to be congruent. At least one more congruent side or angle is necessary to prove congruence with SAS, ASA, or AAS.

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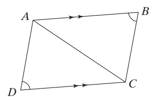
Example

2 Writing a Proof Write a two-column proof that uses AAS.

Given: $\angle B \cong \angle D$, $\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ABC \cong \triangle CDA$



Statements	Reasons
1. $\angle B \cong \angle D$, $\overline{AB} \parallel \overline{CD}$	1. Given
2. $\angle BAC \cong \angle DCA$	2. If lines are parallel, then alternate interior angles are congruent.
3. $\overline{AC} \cong \overline{AC}$	3. Reflexive Property of Congruence
4. $\triangle ABC \cong \triangle CDA$	4. AAS Theorem

Quick Check

2. In Example 2, explain how you could prove $\triangle ABC \cong \triangle CDA$ using ASA.

It is given that $\angle B \cong \angle D$. You know that $\angle BAC \cong \angle DCA$ by the Alternate Interior Angles Theorem. By Theorem 4-1, $\angle BCA \cong \angle DAC$. By the Reflexive Property of Congruence, $\overline{AC} \cong \overline{AC}$. Therefore, $\triangle ABC \cong \triangle CDA$ by ASA.

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Lesson 4-4

Using Congruent Triangles: CPCTC

Lesson Objective	NAEP 2005 Strand: Geometry
Use triangle congruence and CPCTC to prove that parts of two triangles are congruent	Topic: Transformation of Shapes and Preservation of Properties Local Standards: ____________

Vocabulary

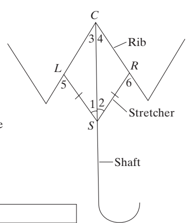
CPCTC stands for:

C orresponding P arts of C ongruent T riangles are C ongruent.

Examples

1 Congruence Statements The diagram shows the frame of an umbrella. What congruence statements besides $\angle 3 \cong \angle 4$ can you prove from the diagram, in which $\overline{SL} \cong \overline{SR}$ and $\angle 1 \cong \angle 2$ are given?

$\overline{SC} \cong \overline{SC}$ by the Reflexive Property of Congruence, and $\triangle LSC \cong \triangle RSC$ by SAS. $\angle 3 \cong \angle 4$ because corresponding parts of congruent triangles are congruent.



When two triangles are congruent, you can form congruence statements about three pairs of corresponding angles and three pairs of corresponding sides. List the congruence statements.

Sides:

$\overline{SL} \cong \overline{SR}$	Given
$\overline{SC} \cong \overline{SC}$	Reflexive Property of Congruence
$\overline{CL} \cong \overline{CR}$	Other congruence statement

Angles:

$\angle 1 \cong \angle 2$	Given
$\angle 3 \cong \angle 4$	Corresponding Parts of Congruent Triangles
$\angle CLS \cong \angle CRS$	Other congruence statement

The congruence statements that remain to be proved are $\angle CLS \cong \angle CRS$ and $\overline{CL} \cong \overline{CR}$.

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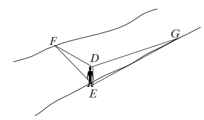
2 Using Right Triangles According to legend, one of Napoleon's followers used congruent triangles to estimate the width of a river. On the riverbank, the officer stood up straight and lowered the visor of his cap until the farthest thing he could see was the edge of the opposite bank. He then turned and noted the spot on his side of the river that was in line with his eye and the tip of his visor.



Given: $\angle DEG$ and $\angle DEF$ are right angles; $\angle EDG \cong \angle EDF$.

The officer then paced off the distance to this spot and declared that distance to be the width of the river!

The given states that $\angle DEG$ and $\angle DEF$ are right angles. What conditions must hold for that to be true?

$\angle DEG$ and $\angle DEF$ are the angles that the officer makes with the ground. So the officer must stand perpendicular to the ground, and the ground must be level or flat.

Quick Check

1. In Example 1, what can you say about $\angle 5$ and $\angle 6$? Explain.

They are congruent, because supplements of congruent angles are congruent.

2. Recall Example 2. About how wide was the river if the officer paced off 20 paces and each pace was about $2\frac{1}{2}$ feet long?

50 feet

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Lesson 4-5

Isosceles and Equilateral Triangles

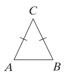
Lesson Objective	NAEP 2005 Strand: Geometry
Use and apply properties of isosceles triangles	Topic: Relationships Among Geometric Figures Local Standards: ____

Vocabulary and Key Concepts

Theorem 4-3: Isosceles Triangle Theorem

If two sides of a triangle are congruent, then **the angles opposite those sides are congruent.**

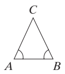
$\angle$ **A** $\cong \angle$ **B**



Theorem 4-4: Converse of Isosceles Triangle Theorem

If two angles of a triangle are congruent, then **the sides opposite those sides are congruent.**

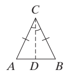
$\overline{AC} \cong \overline{BC}$



Theorem 4-5

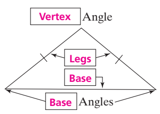
The bisector of the vertex angle of an isosceles triangle is the **perpendicular bisector** of the base.

$\overline{CD} \perp \overline{AB}$ and $\overline{CD}$ bisects $\overline{AB}$.



The legs of an isosceles triangle **are the congruent sides.**

The **base** of an isosceles triangle is the third, or non-congruent, side.



The **vertex angle** of an isosceles triangle is formed by the two congruent sides (legs). The base angles of an isosceles triangle **are formed by the base and the legs.**

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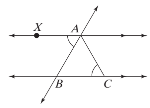
Example

Using the Isosceles Triangle Theorems Explain why $\triangle ABC$ is isosceles.



$\angle ABC$ and $\angle XAB$ are **alternate interior** angles formed by $\overleftrightarrow{XA}$, $\overleftrightarrow{BC}$, and the transversal $\overline{AB}$. Because $\overleftrightarrow{XA} \parallel \overleftrightarrow{BC}$, $\angle ABC \cong \angle$ **XAB**.

The diagram shows that $\angle XAB \cong \angle ACB$. By **the Transitive Property of Congruence**, $\angle ABC \cong \angle$ **ACB**.

You can use **the Converse of the Isosceles Triangle Theorem** to conclude that $\overline{AB} \cong$ **$\overline{AC}$**.

By the definition of an isosceles triangle, $\triangle ABC$ is isosceles.

Quick Check

a. In the figure, suppose $m\angle ACB = 55$. Find $m\angle CBA$ and $m\angle BAC$.

$m\angle CBA = 55$; $m\angle BAC = 70$

b. In the figure, can you deduce that $\triangle ABX$ is isosceles?

No; neither $\angle AXB$ nor $\angle XBA$ can be shown to be congruent to $\angle A$.

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Lesson 4-6

Congruence in Right Triangles

Lesson Objective	NAEP 2005 Strand: Geometry
Prove triangles congruent using the HL Theorem	Topic: Transformation of Shapes and Preservation of Properties Local Standards: ____

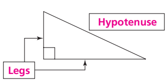
Vocabulary and Key Concepts

Theorem 4-6: Hypotenuse-Leg (HL) Theorem

If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and leg of another right triangle, then **the triangles are congruent.**

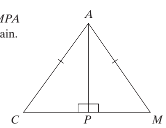
The hypotenuse of a right triangle is **its longest side, or the side opposite the right angle.**

The **legs** of a right triangle are the two shortest sides, or the sides that are *not* opposite the right angle.



Examples

1. **Proving Triangles Congruent** One student wrote "$\triangle CPA \cong \triangle MPA$ by the HL Theorem" for the diagram. Is the student correct? Explain.



The diagram shows the following congruent parts.

$\overline{CA} \cong$ **$\overline{MA}$**

$\angle CPA \cong$ **$\angle MPA$**

$\overline{PA} \cong$ **$\overline{PA}$**

Since $\overline{AC}$ is the **hypotenuse** and $\overline{PA}$ is a **leg** of right triangle CPA, and $\overline{MA}$ is the **hypotenuse** and $\overline{PA}$ is a **leg** of right triangle MPA, the triangles are congruent by the **HL Theorem**.

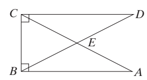
The student **is** correct.

Name______________________ Class______________ Date__________

2. **Two-Column Proof—Using the HL Theorem**

Given: $\angle ABC$ and $\angle DCB$ are right angles, $\overline{AC} \cong \overline{DB}$

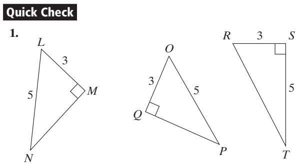
Prove: $\triangle ABC \cong \triangle DCB$



Statements	Reasons
1. $\angle ABC$ and $\angle DCB$ are **right angles**	1. Given
2. $\triangle ABC$ and $\triangle$ **DCB** are right triangles	2. Definition of Right Triangle
3. $\overline{AC} \cong$ **$\overline{DB}$**	3. Given
4. **$\overline{BC} \cong \overline{CB}$**	4. Reflexive Property of Congruence
5. $\triangle ABC \cong \triangle$ **DCB**	5. **HL**

Quick Check

1.



Which two triangles are congruent by the HL Theorem? Write a correct congruence statement.

$\triangle LMN \cong \triangle OQP$

2. You know that two legs of one right triangle are congruent to two legs of another right triangle. Explain how to prove the triangles are congruent.

Since all right angles are congruent, the triangles are congruent by SAS.

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Lesson 4-7

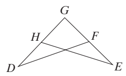
Using Corresponding Parts of Congruent Triangles

Lesson Objectives	NAEP 2005 Strand: Geometry
Identify congruent overlapping triangles Prove two triangles congruent by first proving two other triangles congruent	Topic: Relationships Among Geometric Figures Local Standards: ____________

Examples

1 Identifying Common Parts Name the parts of the sides that $\triangle DFG$ and $\triangle EHG$ share.

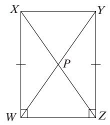
Identify the overlapping triangles.



Parts of sides $\overline{DG}$ and **$\overline{EG}$** are shared by $\triangle DFG$ and $\triangle EHG$.

These parts are **$\overline{HG}$** and **$\overline{FG}$**, respectively.

2 Using Two Pairs of Triangles Write a paragraph proof.



Given: $\overline{XW} \cong \overline{YZ}$, $\angle XWZ$ and $\angle YZW$ are right angles.

Prove: $\triangle XPW \cong \triangle YPZ$

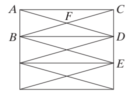
Plan: $\triangle XPW \cong \triangle YPZ$ by AAS if $\angle WXZ \cong$ **$\angle ZYW$**. These angles are congruent by **CPCTC** if $\triangle XWZ \cong \triangle YZW$. These triangles are congruent by **SAS**.

Proof: You are given $\overline{XW} \cong \overline{YZ}$. Because $\angle XWZ$ and $\angle YZW$ are **right angles**, $\angle XWZ \cong \angle YZW$. $\overline{WZ} \cong \overline{ZW}$ by the **Reflexive Property of Congruence**. Therefore, $\triangle XWZ \cong \triangle YZW$ by SAS. $\angle WXZ \cong \angle$ **ZYW** by CPCTC, and $\angle XPW \cong \angle$ **YPZ** because **vertical angles** are congruent. Therefore, $\triangle XPW \cong \triangle YPZ$ by **AAS**.

Name______________________ Class______________________ Date______________

Quick Check

1. The diagram shows triangles from the scaffolding that workers used when they repaired and cleaned the Statue of Liberty.



a. Name the common side in $\triangle ACD$ and $\triangle BCD$.

$\overline{CD}$

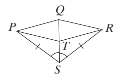
b. Name another pair of triangles that share a common side. Name the common side.

Answers may vary. Sample: $\triangle ABD$ and $\triangle CBD$; $\overline{BD}$

2. Write a two-column proof.

Given: $\overline{PS} \cong \overline{RS}$, $\angle PSQ \cong \angle RSQ$

Prove: $\triangle QPT \cong \triangle QRT$



Statements	Reasons
1. **$\overline{PS} \cong \overline{RS}$, $\angle PSQ \cong \angle RSQ$**	1. Given
2. $\overline{QS} \cong$ **$\overline{QS}$**	2. Reflexive Property of Congruence
3. $\triangle PSQ \cong$ **$\triangle RSQ$**	3. SAS
4. $\overline{PQ} \cong$ **$\overline{RQ}$**	4. CPCTC
5. $\angle PQT \cong$ **$\angle RQT$**	5. CPCTC
6. $\overline{QT} \cong$ **$\overline{QT}$**	6. Reflexive Property of Congruence
7. $\triangle QPT \cong$ **$\triangle QRT$**	7. SAS

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Lesson 5-1

Midsegments of Triangles

Lesson Objective	NAEP 2005 Strand: Geometry
Use properties of midsegments to solve problems	Topics: Relationships Among Geometric Figures Local Standards: ____________

Vocabulary and Key Concepts

Theorem 5-1: Triangle Midsegment Theorem

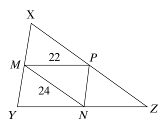
If a segment joins the midpoints of two sides of a triangle, then the segment is **parallel** to the third side, and is **half** its length.

A midsegment of a triangle is **a segment connecting the midpoints of two sides.**

A **coordinate proof** is a form of proof in which coordinate geometry and algebra are used to prove a theorem.

Examples

1 Finding Lengths In $\triangle XYZ$, M, N, and P are midpoints. The perimeter of $\triangle MNP$ is 60. Find NP and YZ.



Because the perimeter of $\triangle MNP$ is 60, you can find NP.

$NP + MN + MP = 60$ Definition of perimeter

$NP +$ **24** $+$ **22** $= 60$ Substitute **24** for *MN* and **22** for *MP*.

$NP +$ **46** $= 60$ Simplify.

$NP =$ **14** Subtract **46** from each side.

Use the Triangle Midsegment Theorem to find YZ.

$MP = \frac{\mathbf{1}}{\mathbf{2}} YZ$ Triangle Midsegment Theorem

22 $= \frac{1}{2} YZ$ Substitute **22** for *MP*.

44 $= YZ$ Multiply each side by **2**.

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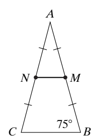
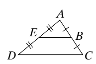
2 Identifying Parallel Segments Find $m\angle AMN$ and $m\angle ANM$. $\overline{MN}$ and $\overline{BC}$ are cut by transversal $\overline{AB}$, so $\angle AMN$ and $\angle B$ are **corresponding** angles. $\overline{MN} \parallel \overline{BC}$ by the **Triangle Midsegment** Theorem, so $\angle AMN \cong \angle B$ by the **Corresponding Angles** Postulate. $m\angle AMN =$ **75** because congruent angles have the same measure. In $\triangle AMN$, $AM =$ **AN**, so $m\angle ANM =$ **$m\angle AMN$** by the **Isosceles Triangle** Theorem. $m\angle ANM =$ **75** by substitution.



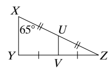
Quick Check

1. $AB = 10$ and $CD = 18$. Find EB, BC, and AC.



$EB = 9$; $BC = 10$; $AC = 20$

2. **Critical Thinking** Find $m\angle VUZ$. Justify your answer.



65; $\overline{UV} \parallel \overline{XY}$ so $\angle VUZ$ and $\angle YXZ$ are corresponding and congruent.

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Lesson 5-2

Bisectors in Triangles

Lesson Objective	NAEP 2005 Strand: Geometry
1 Use properties of perpendicular bisectors and angle bisectors	Topics: Relationships Among Geometric Figures
	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 5-2: Perpendicular Bisector Theorem

If a point is on the perpendicular bisector of a segment, then it is equidistant from the endpoints of the segment.

Theorem 5-3: Converse of the Perpendicular Bisector Theorem

If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

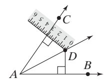
Theorem 5-4: Angle Bisector Theorem

If a point is on the bisector of an angle, then it is equidistant from the sides of the angle.

Theorem 5-5: Converse of the Angle Bisector Theorem

If a point in the interior of an angle is equidistant from the sides of the angle, then it is on the angle bisector.

The distance from a point to a line is the length of the perpendicular segment from the point to the line.



D is 3 in. from $\overleftrightarrow{AB}$ and $\overleftrightarrow{AC}$.

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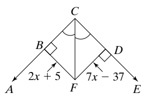
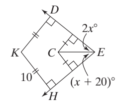
Example

Using the Angle Bisector Theorem
Find x, FB, and FD in the diagram at right.



$FD = FB$	Angle Bisector Theorem
$7x - 37 = 2x + 5$	Substitute.
$7x = 2x + 42$	Add 37 to each side.
$5x = 42$	Subtract 2x from each side.
$x = 8.4$	Divide each side by 5.
$FB = 2(8.4) + 5 = 21.8$	Substitute.
$FD = 7(8.4) - 37 = 21.8$	Substitute.

Quick Check



a. According to the diagram, how far is K from $\overrightarrow{EH}$? From $\overrightarrow{ED}$?
10; 10

b. What can you conclude about $\overrightarrow{EK}$?
$\overrightarrow{EK}$ is the angle bisector of $\angle DEH$.

c. Find the value of x.
20

d. Find $m\angle DEH$.
80

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Lesson 5-3

Concurrent Lines, Medians, and Altitudes

Lesson Objective	NAEP 2005 Strand: Geometry
1 Identify properties of perpendicular bisectors and angle bisectors	Topics: Relationships Among Geometric Figures
2 Identify properties of medians and altitudes of a triangle	Local Standards: ____________

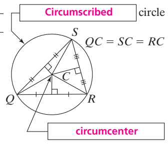
Vocabulary

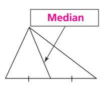
Concurrent lines are three or more lines that meet in one point.

The point of concurrency is the point at which concurrent lines intersect.

A circle is circumscribed about a polygon when the vertices of the polygon are on the circle.

The circumcenter of a triangle is the point of concurrency of the perpendicular bisectors of a triangle.

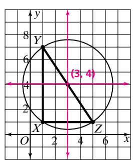




A median of a triangle is a segment that has as its endpoints a vertex of the triangle and the midpoint of the opposite side.

Examples

1 Finding the Circumcenter Find the center of the circle that circumscribes $\triangle XYZ$.



Because X has coordinates (1, 1) and Y has coordinates (1, 7), $\overline{XY}$ lies on the vertical line $x = 1$. The perpendicular bisector of $\overline{XY}$ is the horizontal line that passes through $(1, \frac{1+7}{2})$ or (1, 4), so the equation of the perpendicular bisector of $\overline{XY}$ is $y = 4$.

Because X has coordinates (1, 1) and Z has coordinates (5, 1), $\overline{XZ}$ lies on the horizontal line $y = 1$. The perpendicular bisector of $\overline{XZ}$ is the vertical line that passes through $(\frac{1+5}{2}, 1)$ or (3, 1), so the equation of the perpendicular bisector of $\overline{XZ}$ is $x = 3$.

Draw the lines $y = 4$ and $x = 3$. They intersect at the point (3, 4). This point is the center of the circle that circumscribes $\triangle XYZ$.

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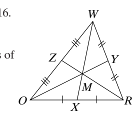
2 Finding Lengths of Medians M is the centroid of $\triangle WOR$, and $WM = 16$. Find WX.



The centroid is the point of concurrency of the medians of a triangle.

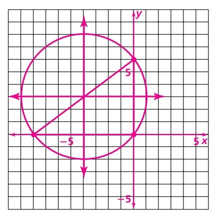
The medians of a triangle are concurrent at a point that is two-thirds the distance from each vertex to the midpoint of the opposite side. (Theorem 5-8)

Because M is the centroid of $\triangle WOR$, $WM = \frac{2}{3}WX$.

$WM = \frac{2}{3}WX$	Theorem 5-8
$16 = \frac{2}{3}WX$	Substitute 16 for WM.
$24 = WX$	Multiply each side by $\frac{3}{2}$.

Quick Check

1. a. Find the center of the circle that you can circumscribe about the triangle with vertices (0, 0), (−8, 0), and (0, 6).
(−4, 3)



b. **Critical Thinking** In Example 1, explain why it is not necessary to find the third perpendicular bisector.
Theorem 5-6: All the perpendicular bisectors of the sides of a triangle are concurrent.

2. Using the diagram in Example 2, find MX. Check that $WM + MX = WX$.
8

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Lesson 5-4

Inverses, Contrapositives, and Indirect Reasoning

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Write the negation of a statement and the inverse and contrapositive of a conditional statement	Topics: Mathematical Reasoning
2 Use indirect reasoning	Local Standards: ____________

Vocabulary and Key Concepts

Negation, Inverse, and Contrapositive Statements

Statement	Example	Symbolic Form	You Read It
Conditional	If an angle is a straight angle, then its measure is 180.	$p \to q$	If p, then q.
Negation (of p)	An angle is not a straight angle.	$\sim p$	Not p.
Inverse	If an angle is not a straight angle, then its measure is not 180.	$\sim p \to \sim q$	If not p, then not q.
Contrapositive	If an angle's measure is not 180, then it is not a straight angle.	$\sim q \to \sim p$	If not q, then not p.

The negation of a statement has the opposite truth value from that of the original statement.

The inverse of a conditional negates both the hypothesis and the conclusion.

The contrapositive of a conditional switches the hypothesis and the conclusion and negates both.

A conditional and its contrapositive always have the same truth value.

Equivalent statements have the same truth value.

Examples

1 Writing the Negation of a Statement Write the negation of "*ABCD* is not a convex polygon."

The negation of a statement has the opposite truth value. The negation of *is not* in the original statement removes the word not.

The negation of "*ABCD* is not a convex polygon" is "*ABCD* is a convex polygon."

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2 Writing the Inverse and Contrapositive Write the inverse and contrapositive of the conditional statement "If △ABC is equilateral, then it is isosceles."

To write the inverse of a conditional, negate both the hypothesis and the conclusion.

Hypothesis ↓ Conclusion ↓

Conditional: If $\triangle ABC$ is equilateral, then it is isosceles.

↓ Negate both. ↓

Inverse: If $\triangle ABC$ is not equilateral, then it is not isosceles.

To write the contrapositive of a conditional, switch the hypothesis and conclusion, then negate both.

Hypothesis ↓ Conclusion ↓

Conditional: If $\triangle ABC$ is equilateral, then it is isosceles.

Switch and negate both.

Contrapositive: If $\triangle ABC$ is not isosceles, then it is not equilateral.

Quick Check

1. Write the negation of each statement.
   a. $m\angle XYZ > 70$.
   The measure of $\angle XYZ$ is not more than 70.
   b. Today is not Tuesday.
   Today is Tuesday.
2. Write (a) the inverse and (b) the contrapositive of Maya Angelou's statement "If you don't stand for something, you'll fall for anything."
   a. If you stand for something, you won't fall for anything.
   b. If you won't fall for anything, then you stand for something.

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Lesson 5-5

Inequalities in Triangles

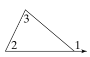
Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use inequalities involving angles of triangles	Topics: Relationships Among Geometric Figures
2 Use inequalities involving sides of triangles	Local Standards: ____________

Key Concepts

Corollary to the Triangle Exterior Angle Theorem

The measure of an exterior angle of a triangle is greater than the measure of each of its remote interior angles.

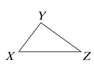
$m\angle 1 > m\angle 2$ and $m\angle 1 > m\angle 3$



Theorem 5-10

If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.

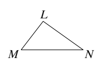
If $XZ > XY$, then $m\angle Y > m\angle Z$.



Theorem 5-12: Triangle Inequality Theorem

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$LM + MN > LN$ $MN + LN > LM$ $LN + LM > MN$

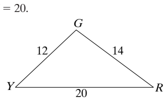


Examples

1 Applying Theorem 5-10 In $\triangle RGY$, $RG = 14$, $GY = 12$, and $RY = 20$. List the angles from largest to smallest.

No two sides of $\triangle RGY$ are congruent, so the largest angle lies opposite the longest side.

Find the angle opposite each side.



The longest side is 20. The opposite angle, $\angle G$, is the largest. The shortest side is 12. The opposite angle, $\angle R$, is the smallest. From largest to smallest, the angles are $\angle G$, $\angle Y$, $\angle R$.

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2 Using the Triangle Inequality Theorem Can a triangle have sides with the given lengths? Explain.

a. 2 cm, 2 cm, 4 cm

According to the Triangle Inequality Theorem, the sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$2 + 2 \not> 4$ The sum of 2 and 2 is not greater than 4.

No, a triangle cannot have sides with these lengths.

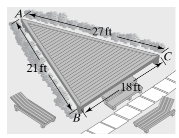
b. 8 in., 15 in., 12 in.

$8 + 15 > 12$ $8 + 12 > 15$ $15 + 12 > 8$

Yes, a triangle can have sides with these lengths.

Quick Check

1. List the angles of $\triangle ABC$ in order from smallest to largest.
   $\angle A$, $\angle C$, $\angle B$



2. Can a triangle have sides with the given lengths? Explain.
   a. 2 m, 7 m, and 9 m
   no; 2 + 7 is not greater than 9
   b. 4 yd, 6 yd, and 9 yd
   yes; 4 + 6 > 9, 6 + 9 > 4, and 4 + 9 > 6

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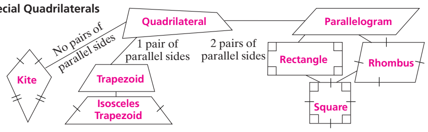
Lesson 6-1

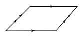
Classifying Quadrilaterals

Lesson Objective	NAEP 2005 Strand: Geometry
1 Define and classify special types of quadrilaterals	**Topic:** Relationships Among Geometric Figures **Local Standards:** __________

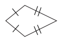
Key Concepts

Special Quadrilaterals

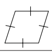




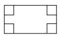
A **parallelogram** is a quadrilateral with both pairs of opposite sides **parallel**.



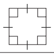
A **kite** is a quadrilateral with two pairs of **adjacent sides** congruent and no **opposite sides** congruent.



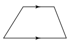
A **rhombus** is a parallelogram with four **congruent sides**.



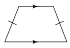
A **rectangle** is a parallelogram with four **right angles**.



A **square** is a parallelogram with four **congruent sides** and four **right angles**.



A **trapezoid** is a quadrilateral with **exactly one pair of parallel sides**.



An **isosceles trapezoid** is a trapezoid whose nonparallel sides are congruent.

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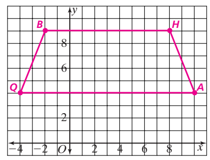
Example

Classifying by Coordinate Methods Determine the most precise name for the quadrilateral with vertices $Q(-4, 4)$, $B(-2, 9)$, $H(8, 9)$, and $A(10, 4)$. Graph quadrilateral $QBHA$.



Find the slope of each side.

Slope of $\overline{QB} = \frac{9-4}{-2-(-4)} = \frac{5}{2}$

Slope of $\overline{BH} = \frac{9-9}{8-(-2)} = 0$

Slope of $\overline{HA} = \frac{4-9}{10-8} = -\frac{5}{2}$

Slope of $\overline{QA} = \frac{4-4}{-4-10} = 0$

$\overline{BH}$ is parallel to $\overline{QA}$ because their slopes are **equal**.
$\overline{HA}$ is not parallel to $\overline{QB}$ because their slopes are **not equal**.
One pair of opposite sides is parallel, so $QBHA$ is a **trapezoid**.
Next, use the distance formula to see whether any pairs of sides are congruent.

$QB = \sqrt{(-2-(-4))^2 + (9-4)^2} = \sqrt{4 + 25} = \sqrt{29}$

$HA = \sqrt{(10-8)^2 + (4-9)^2} = \sqrt{4 + 25} = \sqrt{29}$

$BH = \sqrt{(8-(-2))^2 + (9-9)^2} = \sqrt{100 + 0} = 10$

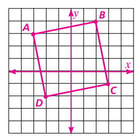
$QA = \sqrt{(-4-10)^2 + (4-4)^2} = \sqrt{196 + 0} = 14$

Because $QB =$ **HA**, $QBHA$ is **an isosceles trapezoid**.

Quick Check

a. Graph quadrilateral $ABCD$ with vertices $A(-3, 3)$, $B(2, 4)$, $C(3, -1)$, and $D(-2, -2)$.
b. Classify $ABCD$ in as many ways as possible.

ABCD is a quadrilateral because it has four sides. It is a parallelogram because both pairs of opposite sides are parallel. It is a rhombus because all four sides are congruent. It is a rectangle because it has four right angles and its opposite sides are congruent. It is a square because it has four right angles and its sides are all congruent.



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Lesson 6-2

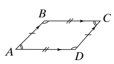
Properties of Parallelograms

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use relationships among sides and among angles of parallelograms 2 Use relationships involving diagonals of parallelograms and transversals	**Topic:** Relationships Among Geometric Figures **Local Standards:** __________

Vocabulary and Key Concepts

Theorem 6-1

Opposite sides of a parallelogram are congruent.

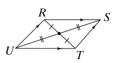


Theorem 6-2

Opposite angles of a parallelogram are **congruent**.

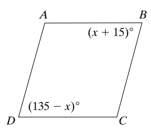
Theorem 6-3

The diagonals of a parallelogram **bisect** each other.



Examples

1 **Using Algebra** Find the value of x in ▱$ABCD$. Then find $m\angle A$.



$x + 15 = 135 - x$	Opposite angles of a parallelogram are **congruent**.
$2x + 15 = 135$	Add x to each side.
$2x = 120$	Subtract 15 from each side.
$x = 60$	Divide each side by 2.
$m\angle B = 60 + 15 = 75$	Substitute **60** for x.
$m\angle A + m\angle B = 180$	Consecutive angles of a parallelogram are **supplementary**.
$m\angle A + 75 = 180$	Substitute **75** for $m\angle B$.
$m\angle A = 105$	Subtract **75** from each side.

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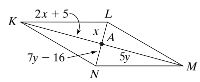
2 **Using Algebra** Find the values of x and y in ▱$KLMN$.



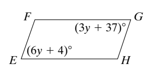
$x = 7y - 16$	The diagonals of a parallelogram **bisect** each other.
$2x + 5 = 5y$	
$2(7y - 16) + 5 = 5y$	Substitute $7y - 16$ for x in the second equation to solve for y.
$14y - 32 + 5 = 5y$	Distribute.
$14y - 27 = 5y$	Simplify.
$-27 = -9y$	Subtract **14y** from each side.
$3 = y$	Divide each side by **−9**.
$x = 7(3) - 16$	Substitute **3** for y in the first equation to solve for x.
$x = 5$	Simplify.

So $x = 5$ and $y = 3$.

Quick Check

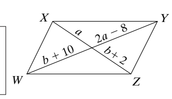
1. Find the value of y in ▱$EFGH$. Then find $m\angle E$, $m\angle F$, $m\angle G$, and $m\angle H$.

y = 11; m∠E = 70, m∠F = 110, m∠G = 70, and m∠H = 110



2. Find the values of a and b.

a = 16, b = 14



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Lesson 6-3

Proving That a Quadrilateral Is a Parallelogram

Lesson Objective	NAEP 2005 Strand: Geometry
Determine whether a quadrilateral is a parallelogram	Topic: Geometry Local Standards: ____________

Key Concepts

Theorem 6-5
If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

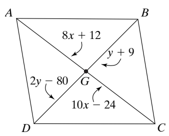
Theorem 6-6
If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Theorem 6-7
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

Theorem 6-8
If one pair of opposite sides of a quadrilateral is both parallel and congruent, then the quadrilateral is a parallelogram.

Examples

1. **Finding Values for Parallelograms** Find values for x and y for which $ABCD$ must be a parallelogram.



If the diagonals of quadrilateral $ABCD$ bisect each other, then $ABCD$ is a parallelogram by Theorem 6-7. Write and solve two equations to find values of x and y for which the diagonals bisect each other.

$10x - 24 = 8x + 12$	Diagonals of parallelograms bisect each other.	$2y - 80 = y + 9$
$2x - 24 = 12$	Collect the variables on one side.	$y - 80 = 9$
$2x = 36$	Solve.	$y = 89$
$x = 18$		

If $x = 18$ and $y = 89$, then $ABCD$ is a parallelogram.

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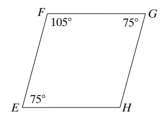
2. **Is the Quadrilateral a Parallelogram?** Can you prove the quadrilateral is a parallelogram from what is given? Explain.

Given: $m\angle E = m\angle G = 75°, m\angle F = 105°$
Prove: $EFGH$ is a parallelogram.



The sum of the measures of the angles of a polygon is $(n - 2)180$ where n represents the number of sides, so the sum of the measures of the angles of a quadrilateral is $(4 - 2)180 = 360$.

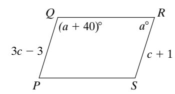
If x represents the measure of the unmarked angle, $x + 75 + 105 + 75 = 360$, so $x = 105$.

Theorem 6-6 states If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Because both pairs of opposite angles are congruent, the quadrilateral is a parallelogram by Theorem 6-6.

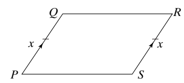
Quick Check

1. Find the values of a and c for which $PQRS$ must be a parallelogram.



$a = 70, c = 2$

2. Can you prove the quadrilateral is a parallelogram? Explain.
Given: $\overline{PQ} \cong \overline{SR}, \overline{PQ} \parallel \overline{SR}$
Prove: $PQRS$ is a parallelogram.



Yes; a pair of opposite sides are parallel and congruent (Theorem 6-8).

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Lesson 6-4

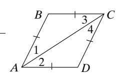
Special Parallelograms

Lesson Objectives	NAEP 2005 Strand: Geometry
1. Use properties of diagonals of rhombuses and rectangles 2. Determine whether a parallelogram is a rhombus or a rectangle	Topic: Geometry Local Standards: ____________

Key Concepts

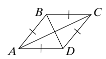
Rhombuses

Theorem 6-9
Each diagonal of a rhombus bisects two angles of the rhombus.



$\overline{AC}$ bisects $\angle BAD$, so $\angle 1 \cong \angle 2$.
$\overline{AC}$ bisects $\angle BCD$, so $\angle 3 \cong \angle 4$.

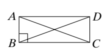
Theorem 6-10
The diagonals of a rhombus are perpendicular.



$\overline{AC} \perp \overline{BD}$

Rectangles

Theorem 6-11
The diagonals of a rectangle are congruent.



$\overline{AC} \cong \overline{BD}$

Parallelograms

Theorem 6-12
If one diagonal of a parallelogram bisects two angles of the parallelogram, the parallelogram is a rhombus.

Theorem 6-13
If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

Theorem 6-14
If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.

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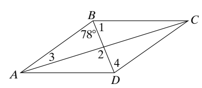
Examples

1. **Finding Angle Measures** Find the measures of the numbered angles in the rhombus.



Theorem 6-9 states that each diagonal of a rhombus bisects two angles of the rhombus, so $m\angle 1 = 78$.

Theorem 6-10 states that the diagonals of the rhombus are perpendicular, so $m\angle 2 = 90$. Because the four angles formed by the diagonals all must have measure 90, $\angle 3$ and $\angle ABD$ must be complementary. Because $m\angle ABD = 78, m\angle 3 = 90 - 78 = 12$. Finally, because $BC = DC$, the Isosceles Triangle Theorem allows you to conclude that $\angle 1 \cong \angle 4$. So $m\angle 4 = 78$.

2. **Finding Diagonal Length** One diagonal of a rectangle has length $8x + 2$. The other diagonal has length $5x + 11$. Find the length of each diagonal.

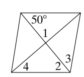
By Theorem 6-11, the diagonals of a rectangle are congruent.

$5x + 11 = 8x + 2$	Diagonals of a rectangle are congruent.
$11 = 3x + 2$	Subtract $5x$ from each side.
$9 = 3x$	Subtract 2 from each side.
$3 = x$	Divide each side by 3.
$8x + 2 = 8(3) + 2 = 26$	Substitute.
$5x + 11 = 5(3) + 11 = 26$	Substitute.

The length of each diagonal is 26.

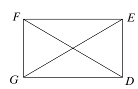
Quick Check

1. Find the measures of the numbered angles in the rhombus.



$m\angle 1 = 90, m\angle 2 = 50, m\angle 3 = 50, m\angle 4 = 40$

2. Find the length of the diagonals of rectangle $GFED$ if $FD = 5y - 9$ and $GE = y + 5$.



$8\frac{1}{2}$

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Lesson 6-5

Trapezoids and Kites

Lesson Objective	NAEP 2005 Strand: Geometry
Verify and use properties of trapezoids and kites	Topic: Relationships Among Geometric Figures
	Local Standards: ______

Vocabulary and Key Concepts

Trapezoids

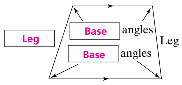
Theorem 6-15

The base angles of an isosceles trapezoid are **congruent**.

Theorem 6-16

The **diagonals** of an isosceles trapezoid are congruent.

The base angles of a trapezoid are **two angles that share a base of the trapezoid.**



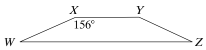
Kites

Theorem 6-17

The diagonals of a kite are **perpendicular**.

Examples

1 **Finding Angle Measures in Trapezoids** $WXYZ$ is an isosceles trapezoid, and $m\angle X = 156$. Find $m\angle Y$, $m\angle Z$, and $m\angle W$.



$m\angle X + m\angle W =$ **180** — Two angles that share a leg of a trapezoid are **supplementary**.

156 $+ m\angle W =$ **180** — Substitute.

$m\angle W =$ **24** — Subtract **156** from each side.

Because the base angles of an isosceles trapezoid are **congruent**, $m\angle Y = m\angle X = 156$ and $m\angle Z =$ **$m\angle W$** = **24**.

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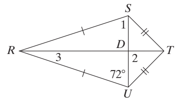
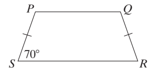
2 **Finding Angle Measures in Kites** Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ in the kite.



$m\angle 2 = 90$	Diagonals of a kite are **perpendicular**.
$RU =$ **RS**	Definition of a kite
$m\angle 1 =$ **72**	**Isosceles** Triangle Theorem
$m\angle 3 + m\angle RDU + 72 =$ **180**	**Triangle Angle-Sum Theorem**
$m\angle RDU =$ **90**	Diagonals of a kite are perpendicular.
$m\angle 3 +$ **90** $+ 72 =$ **180**	Substitute.
$m\angle 3 +$ **162** $=$ **180**	Simplify.
$m\angle 3 =$ **18**	Subtract **162** from each side.

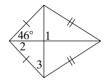
Quick Check

1. In the isosceles trapezoid, $m\angle S = 70$. Find $m\angle P$, $m\angle Q$, and $m\angle R$.



110, 110, 70

2. Find $m\angle 1$, $m\angle 2$, and $m\angle 3$ in the kite.



$m\angle 1 = 90$, $m\angle 2 = 46$, and $m\angle 3 = 44$

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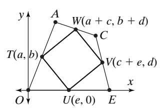
Lesson 6-6

Placing Figures in the Coordinate Plane

Lesson Objective	NAEP 2005 Strand: Geometry
Name coordinates of special figures by using their properties	Topic: Position and Direction
	Local Standards: ______

Example

1 **Proving Congruency** Show that $TWVU$ is a parallelogram by proving pairs of opposite sides congruent.



If both pairs of opposite sides of a quadrilateral are **congruent**, then the quadrilateral is a parallelogram by **Theorem 6-6**.

You can prove that $TWVU$ is a parallelogram by showing that $TW = VU$ and **WV** = **TU**. Use the distance formula.

Use the coordinates $T(a, b)$, $W(\mathbf{a + c}, \mathbf{b + d})$, $V(\mathbf{c + e}, \mathbf{d})$, and $U(e, 0)$.

$$TW = \sqrt{((a + c) - a)^2 + ((b + d) - b)^2} = \sqrt{c^2 + d^2}$$

$$VU = \sqrt{((c + e) - e)^2 + (d - 0)^2} = \sqrt{c^2 + d^2}$$

$$WV = \sqrt{((a + c) - (c + e))^2 + ((b + d) - d)^2} = \sqrt{(a - e)^2 + b^2}$$

$$TU = \sqrt{(a - e)^2 + (b - 0)^2} = \sqrt{(a - e)^2 + b^2}$$

Because $TW = VU$ and $WV = TU$, $TWVU$ is a **parallelogram**.

Quick Check

1. Use the diagram above. Use a different method: Show that $TWVU$ is a parallelogram by finding the midpoints of the diagonals.

Midpoint of $\overline{TV} = \left(\frac{a + c + e}{2}, \frac{b + d}{2}\right)$ = midpoint of $\overline{UW}$. Thus, the diagonals bisect each other, and $TWVU$ is a parallelogram.

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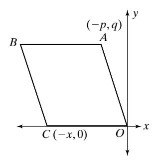
Example

2 **Naming Coordinates** Use the properties of parallelogram $OCBA$ to find the missing coordinates. Do not use any new variables.



The vertex O is the origin with coordinates O **(0, 0)**.

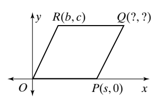
Because point A is p units to the left of point O, point B is also **p** units to the left of point **C**, because $OCBA$ is a parallelogram. So the first coordinate of point B is **$-p - x$**.

Because $\overline{AB} \parallel \overline{CO}$ and $\overline{CO}$ is horizontal, $\overline{AB}$ is also **horizontal**. So point B has the same second coordinate, **q**, as point A.

The missing coordinates are O **(0, 0)** and B **$(-p - x, q)$**.

Quick Check

2. Use the properties of parallelogram $OPQR$ to find the missing coordinates. Do not use any new variables.



$Q(s + b, c)$

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Lesson 6-7

Proofs Using Coordinate Geometry

Lesson Objective	NAEP 2005 Strand: Geometry
1 Prove theorems using figures in the coordinate plane	**Topics:** Position and Direction; Mathematical Reasoning
	Local Standards: ____________

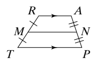
Vocabulary and Key Concepts

Theorem 6-18: Trapezoid Midsegment Theorem

(1) The midsegment of a trapezoid is **parallel** to the bases.

(2) The length of the midsegment of a trapezoid is half the sum of the **lengths of the bases**.

The midsegment of a trapezoid **is the segment that joins the midpoints of the nonparallel opposite sides of the trapezoid.**

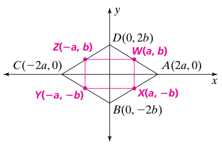


$\overline{MN} \parallel$ **$\overline{TP}$**, $\overline{MN} \parallel$ **$\overline{RA}$**, and $MN = \frac{1}{2}($**TP** $+$ **RA**$)$

Example

Using Coordinate Geometry Use coordinate geometry to prove that the quadrilateral formed by connecting the midpoints of rhombus $ABCD$ is a rectangle.

Draw quadrilateral $XYZW$ by connecting the midpoints of $ABCD$.



From Lesson 6-6, you know that $XYZW$ is a parallelogram.

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If the diagonals of a parallelogram are **congruent**, then the parallelogram is a **rectangle** from Theorem 6–14.

To show that $XYZW$ is a rectangle, find the lengths of its diagonals, and then compare them to show that they are **equal**.

$$XZ = \sqrt{(-a - \boxed{a})^2 + (b - (\boxed{-b}))^2}$$
$$= \sqrt{(\boxed{-2a})^2 + (\boxed{2b})^2}$$
$$= \sqrt{\boxed{4a^2 + 4b^2}}$$

$$YW = \sqrt{(-a - \boxed{a})^2 + (-b - (\boxed{b}))^2}$$
$$= \sqrt{(\boxed{-2a})^2 + (\boxed{-2b})^2}$$
$$= \sqrt{\boxed{4a^2 + 4b^2}}$$

XZ **=** YW

Because the diagonals are congruent, parallelogram $XYZW$ is a **rectangle**.

Quick Check

Use the diagram from the Example on page 82. Explain why the proof using $A(2a, 0)$, $B(0, -2b)$, $C(-2a, 0)$, and $D(0, 2b)$ is easier than the proof using $A(a, 0)$, $B(0, -b)$, $C(-a, 0)$, and $D(0, b)$.

Using multiples of 2 in the coordinates for *A*, *B*, *C*, and *D* eliminates the use of fractions when finding midpoints, since finding midpoints requires division by 2.

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Lesson 7-1

Ratios and Proportions

Lesson Objective	NAEP 2005 Strand: Geometry
1 Write ratios and solve proportions	**Topic:** Position and Direction
	Local Standards: ____________

Vocabulary and Key Concepts

Properties of Proportions

$\frac{a}{b} = \frac{c}{d}$ is equivalent to

(1) $ad =$ **bc** (2) $\frac{b}{a} =$ **$\frac{d}{c}$** (3) $\frac{a}{c} =$ **$\frac{b}{d}$** (4) $\frac{a+b}{b} =$ **$\frac{c+d}{d}$**

A proportion is **a statement that two ratios are equal.**

$\frac{a}{b} = \frac{c}{d}$ and $a : b = c : d$ are examples of proportions.

An **extended proportion** is a statement that three or more ratios are equal.

$\frac{6}{24} = \frac{4}{16} = \frac{1}{4}$ is an example of an extended proportion.

The Cross-Product Property states that **the product of the extremes of a proportion is equal to the product of the means.**

means

$a : b = c : d$

extremes

$\frac{a}{b} = \frac{c}{d}$

a **d** $= b$ **c**

A **scale drawing** is a drawing in which all lengths are proportional to corresponding actual lengths.

A scale is **the ratio of any length in a scale drawing to the corresponding actual length. The lengths may be in different units.**

Examples

1 **Finding Ratios** A scale model of a car is 4 in. long. The actual car is 15 ft long. What is the ratio of the length of the model to the length of the car?

Write both measurements in the same units.

15 ft = 15 × 12 in. = **180** in.

$\frac{\text{length of model}}{\text{length of car}} = \frac{4 \text{ in.}}{15 \text{ ft}} = \frac{4 \text{ in.}}{180 \text{ in.}} = \frac{4}{180} =$ **$\frac{1}{45}$**

The ratio of the length of the scale model to the length of the car is **1** : **45**.

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2 **Solving for a Variable** Solve each proportion.

a. $\frac{2}{5} = \frac{n}{35}$

$5n =$ **2**(**35**) Cross-Product Property

$5n =$ **70** Simplify.

$n =$ **14** Divide each side by **5**.

b. $\frac{x+1}{3} = \frac{x}{2}$

$3x =$ **2**(**x** + **1**) Cross-Product Property

$3x =$ **2x** + **2** Distributive Property

$x =$ **2** Subtract **2x** from each side.

Quick Check

1. A photo that is 8 in. wide and $5\frac{1}{3}$ in. high is enlarged to a poster that is 2 ft wide and $1\frac{1}{3}$ ft high. What is the ratio of the height of the photo to the height of the poster?

1 : 3

2. Solve each proportion.

a. $\frac{5}{z} = \frac{20}{3}$

$z = 0.75$

b. $\frac{18}{n+6} = \frac{6}{n}$

$n = 3$

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Lesson 7-2

Similar Polygons

Lesson Objectives	NAEP 2005 Strand: Geometry and Measurement
1. Identify similar polygons 2. Apply similar polygons	**Topics:** Transformation of Shapes and Preservation of Properties; Measuring Physical Attributes **Local Standards:** ____________

Vocabulary

Similar figures have the same shape but not necessarily the same size. Two polygons are similar if corresponding angles are congruent and corresponding sides are proportional.

The mathematical symbol for similarity is ~.

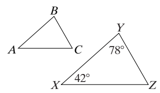
The similarity ratio is the ratio of the lengths of corresponding sides of similar figures.

A golden rectangle is a rectangle that can be divided into a square and a rectangle that is similar to the original rectangle.

The golden ratio is the ratio of the length to the width of any golden rectangle, about 1.618 : 1.

Example

1 Understanding Similarity $\triangle ABC \sim \triangle XYZ$.
Complete each statement.



a. $m\angle B = \blacksquare$

$\angle B \cong \angle$ Y and $m\angle Y = 78$, so $m\angle B =$ 78 because congruent angles have the same measure.

b. $\frac{BC}{YZ} = \frac{\blacksquare}{XZ}$

AC corresponds to $\overline{XZ}$, so $\frac{BC}{YZ} = \frac{AC}{XZ}$.

Quick Check

1. Refer to the diagram for Example 1. Complete:

$m\angle A =$ 42 and $\frac{BC}{YZ} = \frac{AB}{XY}$

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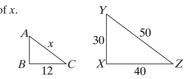
Example

2 Using Similar Figures If $\triangle ABC \sim \triangle YXZ$, find the value of x.
Because $\triangle ABC \sim \triangle YXZ$, you can write and solve a proportion.



$\frac{AC}{YZ} = \frac{BC}{XZ}$ Corresponding sides are proportional.

$\frac{x}{50} = \frac{12}{40}$ **Substitute.**

$x = \frac{12}{40} \cdot 50$ **Solve for x.**

$x = 15$ **Simplify.**

Quick Check

2. Refer to the diagram for Example 2. Find AB.

9

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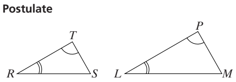
Lesson 7-3

Proving Triangles Similar

Lesson Objectives	NAEP 2005 Strand: Geometry
1. Use AA, SAS, and SSS similarity statements 2. Apply AA, SAS, and SSS similarity statements	**Topic:** Transformation of Shapes and Preservation of Properties **Local Standards:** ____________

Vocabulary and Key Concepts

Postulate 7-1: Angle-Angle Similarity (AA~) Postulate
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.



$\triangle TRS \sim \triangle$ PLM

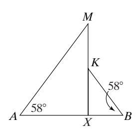
Theorem 7-1: Side-Angle-Side Similarity (SAS~) Theorem
If an angle of one triangle is congruent to an angle of a second triangle, and the sides including the two angles are proportional, then the triangles are similar.

Theorem 7-2: Side-Side-Side Similarity (SSS~) Theorem
If the corresponding sides of two triangles are proportional, then the triangles are similar.

Indirect measurement is a way of measuring things that are difficult to measure directly.

Examples

1 Using the AA~ Postulate $\overline{MX} \perp \overline{AB}$. Explain why the triangles are similar. Write a similarity statement.



Because $\overline{MX} \perp \overline{AB}$, $\angle AXM$ and $\angle BXK$ are right angles, so $\angle AXM \cong \angle$ BXK.

$\angle A \cong \angle$ B because their measures are equal.

$\triangle AMX \sim \triangle BKX$ by the Angle-Angle Similarity (AA~) Postulate.

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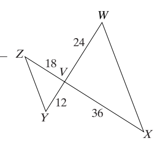
2 Using Similarity Theorems Explain why the triangles must be similar. Write a similarity statement.



$\angle YVZ \cong \angle WVX$ because they are vertical angles.

$\frac{VY}{VW} = \frac{12}{24} = \frac{1}{2}$ and $\frac{VZ}{VX} = \frac{18}{36} = \frac{1}{2}$,

so corresponding sides are proportional.

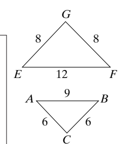
Therefore, $\triangle YVZ \sim \triangle WVX$ by the Side-Angle-Side Similarity (SAS~) Theorem.

Quick Check

1. In Example 1, you have enough information to write a similarity statement. Do you have enough information to find the similarity ratio? Explain.

No; none of the side lengths are given.

2. Explain why the triangles at the right must be similar. Write a similarity statement.



$\frac{AC}{EG} = \frac{CB}{GF} = \frac{AB}{EF} = \frac{3}{4}$, so the triangles are similar by the SSS~ Theorem; $\triangle ABC \sim \triangle EFG$

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Lesson 7-4

Similarity in Right Triangles

Lesson Objective	NAEP 2005 Strand: Geometry
Find and use relationships in similar right triangles	**Topic:** Transformation of Shapes and Preservation of Properties **Local Standards:** ____________

Vocabulary and Key Concepts

Theorem 7-3

The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are **similar** to the original triangle and **similar** to each other.

Corollary 1 to Theorem 7-3

The length of the altitude to the hypotenuse of a right triangle is the **geometric mean** of the lengths of the segments of the hypotenuse.

Corollary 2 to Theorem 7-3

The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the **adjacent hypotenuse segment** and the length of the **hypotenuse**.

The geometric mean of two positive numbers a and b **is the positive number x such that $\frac{a}{x} = \frac{x}{b}$.**

Examples

1 Finding the Geometric Mean Find the geometric mean of 3 and 12.

$\frac{3}{x} = \frac{x}{12}$ Write a proportion.

$x^2 = 36$ Cross-Product Property.

$x = \sqrt{36}$ Find the positive square root.

$x = 6$

The geometric mean of 3 and 12 is **6**.

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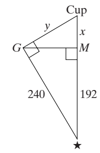
2 Finding Distance At a golf course, Maria drove her ball 192 yd straight toward the cup. Her brother Gabriel drove his ball straight 240 yd, but not toward the cup. The diagram shows the results. Find x and y, their remaining distances from the cup.



Use Corollary 2 of Theorem 7-3 to solve for x.

$\frac{x + 192}{240} = \frac{240}{192}$ Write a proportion.

$192(x + 192) = 240^2$ Cross-Product Property

$192x + 36{,}864 = 57{,}600$ Distributive Property

$192x = 20{,}736$ Solve for x.

$x = 108$

Use Corollary 2 of Theorem 7-3 to solve for y.

$\frac{x + 192}{y} = \frac{y}{x}$ Write a proportion.

$\frac{108 + 192}{y} = \frac{y}{108}$ Substitute **108** for x.

$\frac{300}{y} = \frac{y}{108}$ Simplify.

$y^2 = 32{,}400$ Cross-Product Property

$y = \sqrt{32{,}400}$ Find the positive square root.

$y = 180$

Maria's ball is **108** yd from the cup, and Gabriel's ball is **180** yd from the cup.

Quick Check

1. Find the geometric mean of 5 and 20.

   10

2. Recall Example 2. Find the distance between Maria's ball and Gabriel's ball.

   144 yd

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Lesson 7-5

Proportions in Triangles

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use the Side-Splitter Theorem 2 Use the Triangle-Angle-Bisector Theorem	**Topic:** Transformation of Shapes and Preservation of Properties **Local Standards:** ____________

Key Concepts

Theorem 7-4: Side-Splitter Theorem

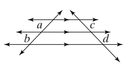
If a line is parallel to one side of a triangle and intersects the other two sides, then **it divides those sides proportionally.**

Theorem 7-5: Triangle-Angle-Bisector Theorem

If a ray bisects an angle of a triangle, then **it divides the opposite side into two segments that are proportional to the other two sides of the triangle.**

Corollary to Theorem 7-4

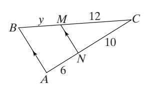
If three parallel lines intersect two transversals, then **the segments intercepted on the transversals are proportional.**



$\frac{a}{b} = \frac{c}{d}$

Examples

1 Using the Side-Splitter Theorem Find y.



$\frac{CM}{MB} = \frac{CN}{NA}$ Side-Spitter Theorem

$\frac{12}{y} = \frac{10}{6}$ Substitute.

$10y = 72$ Cross-Product Property

$y = 7.2$ Solve for y.

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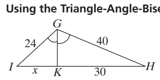
2 Using the Triangle-Angle-Bisector Theorem Find the value of x.



$\frac{IG}{GH} = \frac{IK}{HK}$ Triangle-Angle-Bisector Theorem.

$\frac{24}{40} = \frac{x}{30}$ Substitute.

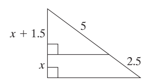
$\frac{24}{40}(30) = x$ Solve for x.

$x = 18$

Quick Check

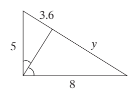
1. Use the Side-Splitter Theorem to find the value of x.

   1.5



2. Find the value of y.

   5.76



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Lesson 8-1 — The Pythagorean Theorem and Its Converse

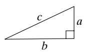
Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use the Pythagorean Theorem 2 Use the Converse of the Pythagorean Theorem	Topic: Relationships Among Geometric Figures Local Standards: ____________

Vocabulary and Key Concepts

Theorem 8-5: Pythagorean Theorem

In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

$a^2 + b^2 = c^2$



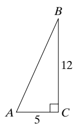
Theorem 8-6: Converse of the Pythagorean Theorem

If the square of the length of one side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.

A Pythagorean triple is a set of nonzero whole numbers a, b, and c that satisfy the equation $a^2 + b^2 = c^2$.

Examples

1 **Pythagorean Triples** Find the length of the hypotenuse of $\triangle ABC$. Do the lengths of the sides of $\triangle ABC$ form a Pythagorean triple?



$a^2 + b^2 = c^2$ — Use the Pythagorean Theorem.

$5^2 + 12^2 = c^2$ — Substitute 5 for a, and 12 for b.

$169 = c^2$ — Simplify.

$\sqrt{169} = c$ — Take the square root.

$13 = c$ — Simplify.

The length of the hypotenuse is 13. The lengths of the sides, 5, 12, and 13 form a Pythagorean triple because they are whole numbers that satisfy $a^2 + b^2 = c^2$.

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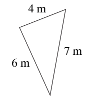
2 **Is It a Right Triangle?** Is this triangle a right triangle?



$a^2 + b^2 \stackrel{?}{=} c^2$

$4^2 + 6^2 \stackrel{?}{=} 7^2$ — Substitute 4 for a, 6 for b, and 7 for c.

$16 + 36 \stackrel{?}{=} 49$ — Simplify.

$52 \neq 49$

Because $a^2 + b^2 \neq c^2$, the triangle is not a right triangle.

Quick Check

1. A right triangle has a hypotenuse of length 25 and a leg of length 10. Find the length of the other leg. Do the lengths of the sides form a Pythagorean triple?

$5\sqrt{21}$; no

2. A triangle has sides of lengths 16, 48, and 50. Is the triangle a right triangle?

no

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Lesson 8-2 — Special Right Triangles

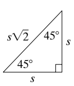
Lesson Objectives	NAEP 2005 Strand: Geometry
1 Use the properties of 45°-45°-90° triangles 2 Use the properties of 30°-60°-90° triangles	Topic: Relationships Among Geometric Figures Local Standards: ____________

Key Concepts

Theorem 8-5: 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, both legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

hypotenuse = $\sqrt{2}$ · leg

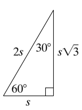


Theorem 8-6: 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

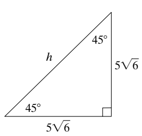
hypotenuse = 2 · shorter leg

longer leg = $\sqrt{3}$ · shorter leg



Examples

1 **Finding the Length of the Hypotenuse** Find the value of the variable. Use the 45°-45°-90° Triangle Theorem to find the hypotenuse.



$h = \sqrt{2} \cdot 5\sqrt{6}$ — hypotenuse = $\sqrt{2}$ · leg

$h = 5\sqrt{12}$ — Simplify.

$h = 5\sqrt{4(3)}$

$h = 5(2)\sqrt{3}$

$h = 10\sqrt{3}$

The length of the hypotenuse is $10\sqrt{3}$.

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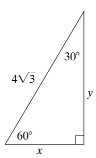
2 **Using the Length of One Side** Find the value of each variable. Use the 30°-60°-90° Triangle Theorem to find the lengths of the legs.



$4\sqrt{3} = 2 \cdot x$ — hypotenuse = 2 · shorter leg

$x = \frac{4\sqrt{3}}{2}$ — Divide each side by 2.

$x = 2\sqrt{3}$ — Simplify.

$y = \sqrt{3}$ · shorter leg — 30°-60°-90° Triangle Theorem

$y = \sqrt{3} \cdot 2\sqrt{3}$ — Substitute $2\sqrt{3}$ for shorter leg.

$y = 2 \cdot \sqrt{3} \cdot \sqrt{3}$ — Simplify.

$y = 6$

The length of the shorter leg is $2\sqrt{3}$, and the length of the longer leg is 6.

Quick Check

1. Find the length of the hypotenuse of a 45°-45°-90° triangle with legs of length $5\sqrt{3}$.

$5\sqrt{6}$

2. Find the lengths of the legs of a 30°-60°-90° triangle with hypotenuse of length 12.

6; $6\sqrt{3}$

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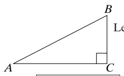
Lesson 8-3 — The Tangent Ratio

Lesson Objective	NAEP 2005 Strand: Measurement
Use tangent ratios to determine side lengths in triangles	Topic: Measuring Physical Attributes Local Standards: ____________

Vocabulary

The tangent of acute ∠A in a right triangle is the ratio of the length of the leg opposite ∠A to the length of the leg adjacent to ∠A.

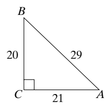


tangent of ∠A = $\frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$

You can abbreviate the equation as tan A = $\frac{\text{opposite}}{\text{adjacent}}$.

Examples

1 Writing Tangent Ratios Write the tangent ratios for ∠A and ∠B.

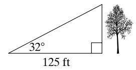


tan A = $\frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC} = \frac{20}{21}$

tan B = $\frac{\text{opposite}}{\text{adjacent}} = \frac{AC}{BC} = \frac{21}{20}$

Name_______________ Class_______________ Date_______________

2 Using a Tangent Ratio To measure the height of a tree, Alma walked 125 ft from the tree and measured a 32° angle from the ground to the top of the tree. Estimate the height of the tree.



The tree forms a right angle with the ground, so you can use the tangent ratio to estimate the height of the tree.

tan 32° = $\frac{\text{height}}{125}$ — Use the tangent ratio.

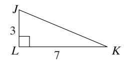
height = 125 (tan 32°) — Solve for height.

125 TAN 32 ENTER 78.108669 — Use a calculator.

The tree is about 78 feet tall.

Quick Check

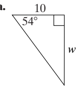
1. Write the tangent ratios for ∠K and ∠J.



$\frac{3}{7}, \frac{7}{3}$

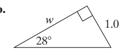
2. Find the value of w to the nearest tenth.

a.



13.8

b.



1.9

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Lesson 8-4 — Sine and Cosine Ratios

Lesson Objective	NAEP 2005 Strand: Measurement
Use sine and cosine to determine side lengths in triangles	Topic: Measuring Physical Attributes Local Standards: ____________

Vocabulary

The sine of ∠A is the ratio of the length of the leg opposite ∠A to the length of the hypotenuse.

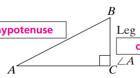
The cosine of ∠A is the ratio of the length of the leg adjacent to ∠A to the hypotenuse.

sine of ∠A = $\frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$

This can be abbreviated sin A = $\frac{\text{opposite}}{\text{hypotenuse}}$.

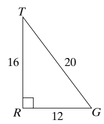
cosine of ∠A = $\frac{\text{leg adjacent to } \angle A}{\text{hypotenuse}}$

This can be abbreviated cos A = $\frac{\text{adjacent}}{\text{hypotenuse}}$.



Examples

1 Writing Sine and Cosine Ratios Use the triangle to find sin T, cos T, sin G, and cos G. Write your answers in simplest terms.



sin T = $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{20} = \frac{3}{5}$

cos T = $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{16}{20} = \frac{4}{5}$

sin G = $\frac{\text{opposite}}{\text{hypotenuse}} = \frac{16}{20} = \frac{4}{5}$

cos G = $\frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{20} = \frac{3}{5}$

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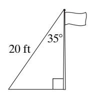
2 Using the Cosine Ratio A 20-ft wire supporting a flagpole forms a 35° angle with the flagpole. To the nearest foot, how high is the flagpole?



The flagpole, wire, and ground form a right triangle with the wire as the hypotenuse. Because you know an angle and the measure of its hypotenuse, you can use the cosine ratio to find the height of the flagpole.

cos 35° = $\frac{\text{height}}{20}$ — Use the cosine ratio.

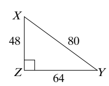
height = 20 · cos 35° — Solve for height.

20 COS 35 ENTER 16.383041 — Use a calculator.

The flagpole is about 16 feet tall.

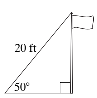
Quick Check

1. Write the sine and cosine ratios for ∠X and ∠Y.



sin X = $\frac{64}{80}$, cos X = $\frac{48}{80}$, sin Y = $\frac{48}{80}$, cos Y = $\frac{64}{80}$

2. In Example 2, suppose that the angle the wire makes with the ground is 50°. What is the height of the flagpole to the nearest foot?



15 feet

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Lesson 8-5

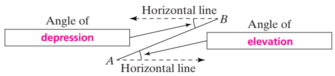
Angles of Elevation and Depression

Lesson Objective	**NAEP 2005 Strand:** Measurement
▼ Use angles of elevation and depression to solve problems	**Topic:** Measuring Physical Attributes
	Local Standards: ____________

Vocabulary

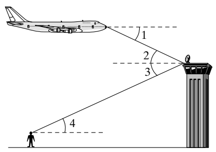
An angle of elevation **is the angle formed by a horizontal line and the line of sight to an object above the horizontal line.**

An **angle of depression** is the angle formed by a horizontal line and the line of sight to an object below the horizontal line.



Examples

1 **Identifying Angles of Elevation and Depression** Describe ∠1 and ∠2 as they relate to the situation shown.



One side of the angle of depression is a horizontal line. ∠1 is the angle of **depression** from the **airplane** to the **building**.

One side of the angle of elevation is a horizontal line. ∠2 is the angle of **elevation** from the **building** to the **airplane**.

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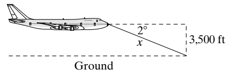
2 **Aviation** An airplane flying 3500 ft above the ground begins a 2° descent to land at the airport. How many miles from the airport is the airplane when it starts its descent? (Note: The angle is not drawn to scale.)



$\sin 2° = \frac{3{,}500}{x}$ Use the **sine** ratio.

$x = \frac{3500}{\sin 2°}$ Solve for x.

3500 ÷ SIN 2 ENTER 100287.9792 Use a calculator.

100287.9792 ÷ 5280 ENTER 18.993935 Divide by **5280** to convert feet to miles.

The airplane is about **19** miles from the airport when it starts its descent.

Quick Check

1. Describe each angle as it relates to the situation in Example 1.
   a. ∠3
   The angle of depression from the building to the person on the ground
   b. ∠4
   The angle of elevation from the person on the ground to the building
2. An airplane pilot sees a life raft at a 26° angle of depression. The airplane's altitude is 3 km. What is the airplane's surface distance d from the raft?
   about 6.8 km

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Lesson 8-6

Vectors

Lesson Objectives	**NAEP 2005 Strand:** Geometry
▼ Describe vectors	**Topic:** Position and Direction
▼ Solve problems that involve vector addition	**Local Standards:** ____________

Vocabulary and Key Concepts

Adding Vectors

For $\vec{a} = \langle x_1, y_1\rangle$ and $\vec{c} = \langle x_2, y_2\rangle$, $\vec{a} + \vec{c} =$ **$\langle x_1 + x_2, y_1 + y_2\rangle$**.

A vector is **any quantity with magnitude (size) and direction.**

A **vector** can be represented with an arrow.

The magnitude of a vector is **its size, or length.**

The **initial point** of a vector is the point at which it starts.

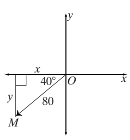
The terminal point of a vector is **the point at which it ends.**

A **resultant vector** is the sum of other vectors.

Examples

1 **Describing a Vector** Describe $\overrightarrow{OM}$ as an ordered pair.

Give coordinates to the nearest tenth.
Use the sine and cosine ratios to find the values of x and y.



$\cos 40° = \frac{x}{80}$ Use sine and cosine. $\sin 40° = \frac{y}{80}$

$x = 80(\cos 40°)$ Solve for the variable. $y = 80(\sin 40°)$

$x \approx 61.28355545$ Use a calculator. $y \approx 51.42300878$

Because point M is in the **third** quadrant, both coordinates are **negative**. To the nearest tenth, $\overrightarrow{OM} \approx$ **$\langle -61.3, -51.4\rangle$**.

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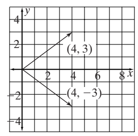
2 **Adding Vectors** Vectors $\vec{v}$ ⟨4, 3⟩ and $\vec{w}$ ⟨4, −3⟩ are shown at the right. Write the sum of the two vectors as an ordered pair. Then draw $\vec{s}$, the sum of $\vec{v}$ and $\vec{w}$.



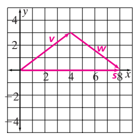
To find the first coordinate of $\vec{s}$, add the **first** coordinates of $\vec{v}$ and $\vec{w}$.

To find the second coordinate of $\vec{s}$, add the **second** coordinates of $\vec{v}$ and $\vec{w}$.

$\vec{s} = \langle 4, 3\rangle + \langle 4, -3\rangle$

$= \langle$ **4 + 4**, **3 + (−3)** $\rangle$ Add the coordinates.

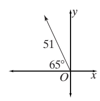
$= \langle$ **8**, **0** $\rangle$ Simplify.



Draw vector $\vec{v}$ using the origin as the initial point. Draw vector $\vec{w}$ using the terminal point of $\vec{v}$, (4, 3), as the initial point. Draw the resultant vector $\vec{s}$ from the **initial point** of $\vec{v}$ to the **terminal point** of $\vec{w}$.

Quick Check

1. Describe the vector at the right as an ordered pair. Give the coordinates to the nearest tenth.
   ⟨−21.6, 46.2⟩



2. Write the sum of the two vectors ⟨2, 3⟩ and ⟨−4, −2⟩ as an ordered pair.
   ⟨−2, 1⟩

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Lesson 9-1

Translations

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Identify isometries 2 Find translation images of figures	**Topic:** Transformation of Shapes and Preservation of Properties; Position and Direction **Local Standards:** ________

Vocabulary

A transformation of a geometric figure is **a change in its position, shape, or size.**

In a transformation, the **preimage** is the original image before changes are made.

In a transformation, the image is **the resulting figure after changes are made.**

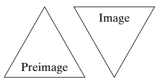
An **isometry** is a transformation in which the preimage and the image are congruent.

A translation (slide) is **a transformation that maps all points the same distance and in the same direction.**

A **composition of transformations** is a combination of two or more transformations.

Examples

1 Identifying Isometries Does the transformation appear to be an isometry?



The image appears to be the same as the preimage, but **turned**. Because the figures appear to be **congruent**, the transformation appears to be an isometry.

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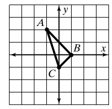
2 Finding a Translation Image Find the image of $\triangle ABC$ under the translation $(x, y) \to (x + 2, y - 3)$.



$A(-1, 2) \to A'($ **−1 + 2**, **2** $- 3)$ **Use the rule (x, y) → (x + 2, y − 3)**

$B(1, 0) \to B'($ **1** $+ 2,$ **0 − 3** $)$

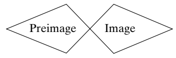
$C(0, -1) \to C'($ **0 + 2**, **−1 − 3** $)$

The image of $\triangle ABC$ is $\triangle A'B'C'$ with $A'($**1**, **−1**$)$, $B'($**3**, **−3**$)$, and $C'($**2**, **−4**$)$.

Quick Check

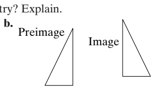
1. Does the transformation appear to be an isometry? Explain.

a.



Yes; the figures appear to be congruent by a flip.

b.



Yes; the figures appear to be congruent by a flip and a slide.

2. Find the image of $\triangle ABC$ for the translation $(x - 2, y + 1)$.

A′(−3, 3), B′(−1, 1), C′(−2, 0)

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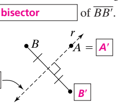
Lesson 9-2

Reflections

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Find reflection images of figures	**Topics:** Transformation of Shapes and Preservation of Properties **Local Standards:** ________

Vocabulary

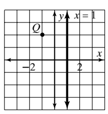
A reflection in line r is a transformation such that if a point A is on line r, then the image of A is **itself**, and if a point B is not on line r, then its image B' is the point such that r is the **perpendicular bisector** of $\overline{BB'}$.



Line of **reflection**

Example

1 Finding Reflection Images If point $Q(-1, 2)$ is reflected across line $x = 1$, what are the coordinates of its reflection image?



Q is **2** units to the **left** of the reflection line, so its image Q' is **2** units to the **right** of the reflection line. The reflection line is the perpendicular bisector of $\overline{QQ'}$ if Q' is at **(3, 2)**.

Quick Check

1. What are the coordinates of the image of Q if the reflection line is $y = -1$?

(−1, −4)

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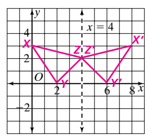
Examples

2 Drawing Reflection Images $\triangle XYZ$ has vertices $X(0, 3)$, $Y(2, 0)$, and $Z(4, 2)$. Draw $\triangle XYZ$ and its reflection image in the line $x = 4$.

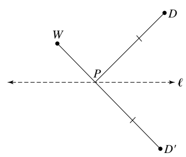


First locate vertices X, Y, and Z and draw $\triangle XYZ$ in a coordinate plane.

Locate points X', Y', and Z' such that the line of reflection $x = 4$ is the **perpendicular bisector** of $\overline{XX'}$, $\overline{YY'}$, and $\overline{ZZ'}$.

Draw the reflection image $X'Y'Z'$.

3 Congruent Angles D' is the reflection of D across ℓ. Show that $\overline{PD}$ and $\overline{PW}$ form congruent angles with line ℓ.



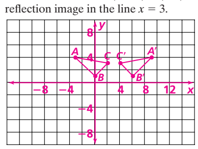
Because a reflection is an **isometry**, $\overline{PD}$ and $\overline{PD'}$ form congruent angles with line ℓ.

$\overline{PD'}$ and $\overline{PW}$ also form congruent angles with line ℓ because **vertical** angles are congruent.

Therefore, $\overline{PD}$ and $\overline{PW}$ form congruent angles with line ℓ by the **Transitive** Property.

Quick Check

2. $\triangle ABC$ has vertices $A(-3, 4)$, $B(0, 1)$, and $C(2, 3)$. Draw $\triangle ABC$ and its reflection image in the line $x = 3$.



3. In Example 3, what kind of triangle is $\triangle DPD'$? Imagine the image of point W reflected across line ℓ. What can you say about $\triangle WPW'$ and $\triangle DPD'$?

isosceles; they are similar by SAS

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Lesson 9-3

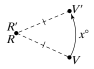
Rotations

Lesson Objective	NAEP 2005 Strand: Geometry
▼ Draw and identify rotation images of figures	**Topic:** Transformation of Shapes and Preservation of Properties
	Local Standards: ____________

Vocabulary

A rotation of $x°$ about a point R is a transformation for which the following are true:

- The image of R is **itself** (that is, $R' =$ **R**).
- For any point V, $RV' =$ **RV** and $m\angle VRV' =$ **x**.



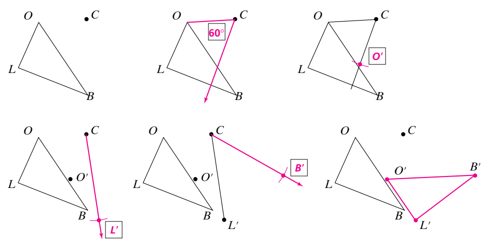
Examples

1 Drawing a Rotation Image Draw the image of $\triangle LOB$ under a 60° rotation about C.

Step 1 Use a protractor to draw a 60° angle at vertex C with one side $\overline{CO}$.

Step 2 Use a compass to construct $\overline{CO'} \cong \overline{CO}$.

Step 3 Locate L' and B' in a similar manner. Then draw $\triangle L'O'B'$.



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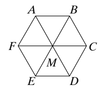
2 Identifying a Rotation Image Regular hexagon $ABCDEF$ is divided into six equilateral triangles.



a. Name the image of B for a 240° rotation about M.

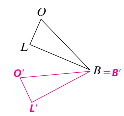
Because $360 \div 6 =$ **60**, each central angle of $ABCDEF$ measures **60°**. A 240° counterclockwise rotation about center M moves point B across **four** triangles. The image of point B is point **D**.

b. Name the image of M for a 60° rotation about F.

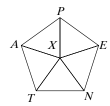
$\triangle AMF$ is equilateral, so $\angle AFM$ has measure $180 \div 3 =$ **60**. A 60° rotation of $\triangle AMF$ about point F would superimpose $\overline{FM}$ on $\overline{FA}$, so the image of M under a 60° rotation about point F is point **A**.

Quick Check

1. Draw the image of LOB for a 50° rotation about point B. Label the vertices of the image.



2. Regular pentagon $PENTA$ is divided into 5 congruent triangles. Name the image of T for a 144° rotation about point X.



E

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Lesson 9-4

Symmetry

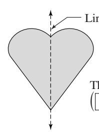
Lesson Objective	NAEP 2005 Strand: Geometry
▼ Identify the type of symmetry in a figure	**Topic:** Transformation of Shapes and Preservation of Properties
	Local Standards: ____________

Vocabulary

A figure has symmetry if **there is an isometry that maps the figure onto itself.**

A figure has **reflectional symmetry** if there is symmetry that maps the figure onto itself.

Line symmetry is the same as **reflectional symmetry.**

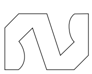


Line of **Symmetry**

The heart-shaped figure has **reflectional** (**line**) symmetry.

A figure has **rotational symmetry** if it is its own image for some rotation of 180° or less.

A figure has point symmetry if **it has 180° rotational symmetry.**



The figure is its own image after one half-turn, so it has rotational symmetry with a **180**° angle of rotation. The figure also has **point** symmetry.

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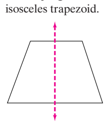
Examples

1 Identifying Lines of Symmetry Draw all lines of symmetry for the isosceles trapezoid.



Draw any lines that divide the isosceles trapezoid so that half of the figure is a mirror image of the other half.

There is **one** line of symmetry.

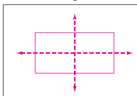
2 Identifying Rotational Symmetry Judging from appearance, do the letters V and H have rotational symmetry? If so, give an angle of rotation.

The letter V does not have rotational symmetry because it must be rotated **360**° before it is its own image.

The letter H is its own image after one half-turn, so it has rotational symmetry with a **180**° angle of rotation.

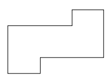
Quick Check

1. Draw a rectangle and all of its lines of symmetry.



2. **a.** Judging from appearance, tell whether the figure at the right has rotational symmetry. If so, give the angle of rotation.

yes; 180°



b. Does the figure have point symmetry?

yes

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Lesson 9-5

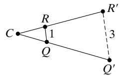
Dilations

Lesson Objective	**NAEP 2005 Strand:** Geometry
Locate dilation images of figures	**Topic:** Geometry
	Local Standards: ____________

Vocabulary

A dilation is a transformation with center C and scale factor n for which the following are true:

- The image of C is itself (that is, $C' =$ C).
- For any point R, R' is on $\overrightarrow{CR}$ and $CR' =$ n $\times$ CR.



$\overline{R'Q'}$ is the image of $\overline{RQ}$ under a dilation with center C and scale factor 3.

An enlargement is a dilation with a scale factor greater than 1.

A reduction is a dilation with a scale factor less than 1.

Examples

1 **Finding a Scale Factor** Circle A with 3-cm diameter and center C is a dilation of concentric circle B with 8-cm diameter. Describe the dilation.

The circles are concentric, so the dilation has center C.

Because the diameter of the dilation image is smaller, the dilation is a reduction.

Scale factor: $\frac{\text{diameter of dilation image}}{\text{diameter of preimage}} = \frac{3}{8}$

The dilation is a reduction with center C and scale factor $\frac{3}{8}$.

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2 **Scale Drawings** The scale factor on a museum's floor plan is 1 : 200. The length and width of one wing on the drawing are 8 in. and 6 in. Find the actual dimensions of the wing in feet and inches.

The floor plan is a reduction of the actual dimensions by a scale factor of $\frac{1}{200}$.

Multiply each dimension on the drawing by 200 to find the actual dimensions. Then write the dimensions in feet and inches.

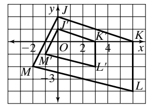
8 in. × 200 = 1600 in. = 133 ft, 4 in.

6 in. × 200 = 1200 in. = 100 ft

The museum wing measures 133 ft, 4 in. by 100 ft.

Quick Check

1. Quadrilateral $J'K'L'M'$ is a dilation image of quadrilateral $JKLM$. Describe the dilation.



The dilation is a reduction with center (0, 0) and scale factor $\frac{1}{2}$.

2. The height of a tractor-trailer truck is 4.2 m. The scale factor for a model truck is $\frac{1}{54}$. Find the height of the model to the nearest centimeter.

8 cm

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Lesson 9-6

Compositions of Reflections

Lesson Objectives	**NAEP 2005 Strand:** Geometry
Use a composition of reflections	**Topic:** Transformation of Shapes and Preservation of Properties
Identify glide reflections	**Local Standards:** ____________

Vocabulary and Key Concepts

Theorem 9-1

A translation or rotation is a composition of two reflections.

Theorem 9-2

A composition of reflections across two parallel lines is a translation.

Theorem 9-3

A composition of reflections across two intersecting lines is a rotation.

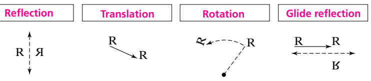
Theorem 9-4: Fundamental Theorem of Isometries

In a plane, one of two congruent figures can be mapped onto the other by a composition of at most three reflections.

Theorem 9-5: Isometry Classification Theorem

There are only four isometries. They are the following:

Reflection | Translation | Rotation | Glide reflection



A glide reflection is the composition of a glide (translation) and a reflection across a line parallel to the direction of translation.

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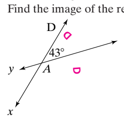
Example

Composition of Reflections in Intersecting Lines The letter D is reflected in line x and then in line y. Describe the resulting rotation.

Find the image of D through a reflection across line x.

Find the image of the reflection through another reflection across line y.



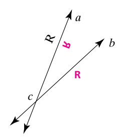
The composition of two reflections across intersecting lines is a rotation.

The center of rotation is the point where the lines intersect, and the angle is twice the angle formed by the intersecting lines. So, the letter D is rotated 86° clockwise, or 274° counterclockwise, with the center of rotation at point A.

Quick Check

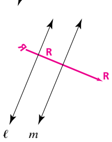
a. Reflect the letter R across a and then b. Describe the resulting rotation.

Answers may vary. The result is a clockwise rotation about point c through an angle of 2$m\angle acb$.



b. Use parallel lines ℓ and m. Draw R between ℓ and m. Find the image of R for a reflection across line ℓ and then across line M. Describe the resulting translation.

Answers may vary. The result is a translation twice the distance between ℓ and m.



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Lesson 9-7 Tessellations

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Identify transformation in tessellations, and figures that will tessellate	Topic: Geometry
2 Identify symmetries in tessellations	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 9-6
Every triangle tessellates.

Theorem 9-7
Every quadrilateral tessellates.

A tessellation, or tiling, is **a repeating pattern of figures that completely covers a plane, without gaps or overlaps.**

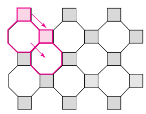
Translational symmetry is the type of symmetry for which there is a translation that maps a figure onto itself.

Glide reflectional symmetry is **the type of symmetry for which there is a glide reflection that maps a figure onto itself.**

Examples

1 **Identifying the Transformation in a Tessellation** Identify the repeating figures and a transformation in the tessellation.

A repeated combination of an **octagon** and one adjoining **square** will completely cover the plane without gaps or overlaps. Use arrows to show a translation.



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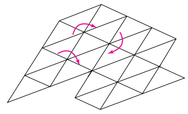
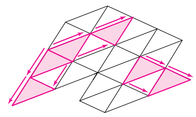
2 **Identifying Symmetries in Tessellations** List the symmetries in the tessellation.

Starting at any vertex, the tessellation can be mapped onto itself using a **180°** rotation, so the tessellation has **point** symmetry centered at any vertex.

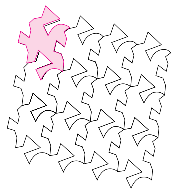


The tessellation also has **translational** symmetry, as can be seen by sliding any triangle onto a copy of itself along any of the lines.



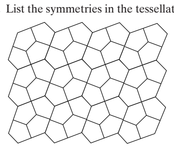
Quick Check

1. Identify a transformation and outline the smallest repeating figure in the tessellation below.



translation

2. List the symmetries in the tessellation.



line symmetry, rotational symmetry, glide reflectional symmetry, translational symmetry

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Lesson 10-1 Areas of Parallelograms and Triangles

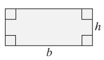
Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find the area of a parallelogram	Topic: Measuring Physical Attributes
2 Find the area of a triangle	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 10-1: Area of a Rectangle

The area of a rectangle is the product of its **base** and **height**.

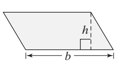
$A =$ **bh**



Theorem 10-2: Area of a Parallelogram

The area of a parallelogram is the product of a **base** and the corresponding **height**.

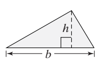
$A =$ **bh**



Theorem 10-3: Area of a Triangle

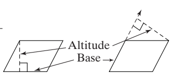
The area of a triangle is **half** the product of a **base** and the corresponding **height**.

$A =$ **$\frac{1}{2}bh$**



A base of a parallelogram is **any of its sides.**

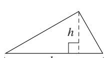
The **altitude** of a parallelogram corresponding to a given base is the segment perpendicular to the line containing that base drawn from the side opposite the base.



The height of a parallelogram is **the length of its altitude.**

A **base** of a triangle is any of its sides.

The height of a triangle is **the length of the altitude to the line containing that base.**



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Examples

1 **Finding the Area of a Parallelogram** A parallelogram has 9-in. and 18-in. sides. The height corresponding to the 9-in. base is 15 in. Find the area of the parallelogram.

$A = bh$ Area of a parallelogram

$A =$ **9**(**15**) Substitute **9** for *b* and **15** for *h*.

$A =$ **135** Simplify.

The area of the parallelogram is **135** in.^{2}

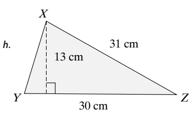
2 **Finding the Area of a Triangle** Find the area of $\triangle XYZ$.

$A = \frac{1}{2}bh$ Area of a **triangle**

$A = \frac{1}{2}$(**30**)(**13**) Substitute **30** for *b* and **13** for *h*.

$A =$ **195** Simplify.

$\triangle XYZ$ has area **195** cm^2.

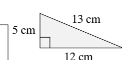


Quick Check

1. A parallelogram has sides 15 cm and 18 cm. The height corresponding to a 15-cm base is 9 cm. Find the area of the parallelogram.

135 cm^2

2. Find the area of the triangle.



30 cm^2

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Lesson 10-2
Areas of Trapezoids, Rhombuses, and Kites

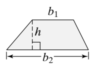
Lesson Objectives	
Find the area of a trapezoid Find the area of a rhombus or a kite	**NAEP 2005 Strand:** Measurement **Topic:** Measuring Physical Attributes **Local Standards:** ____

Vocabulary and Key Concepts

Theorem 10-4: Area of a Trapezoid

The area of a trapezoid is half the product of the height and the sum of the bases.

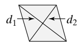
$A = \frac{1}{2}h(b_1 + b_2)$



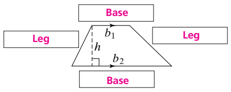
Theorem 10-5: Area of a Rhombus or a Kite

The area of a rhombus or a kite is half the product of the lengths of its diagonals.

$A = \frac{1}{2}d_1d_2$

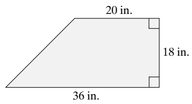


The height of a trapezoid is the perpendicular distance *h* between the bases.



Examples

1 Applying the Area of a Trapezoid A car window is shaped like the trapezoid shown. Find the area of the window.



$A = \frac{1}{2}h(b_1 + b_2)$ Area of a trapezoid

$A = \frac{1}{2}(18)(20 + 36)$ Substitute 18 for h, 20 for b_1, and 36 for b_2.

$A = 504$ Simplify.

The area of the car window is 504 $in.^2$.

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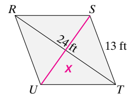
2 Finding the Area of a Rhombus Find the area of rhombus *RSTU*. To find the area, you need to know the lengths of both diagonals. Draw diagonal $\overline{SU}$, and label the intersection of the diagonals point *X*.



△*SXT* is a right triangle because the diagonals of a rhombus are perpendicular.

The diagonals of a rhombus bisect each other, so $TX = 12$ ft.

You can use the Pythagorean triple 5, 12, 13 or the Pythagorean Theorem to conclude that $SX = 5$ ft.

$SU = 10$ ft because the diagonals of a rhombus bisect each other.

$A = \frac{1}{2}d_1d_2$ Area of a rhombus

$A = \frac{1}{2}(24)(10)$ Substitute 24 for d_1 and 10 for d_2.

$A = 120$ Simplify.

The area of rhombus *RSTU* is 120 ft^2.

Quick Check

1. Find the area of a trapezoid with height 7 cm and bases 12 cm and 15 cm.

   94.5 cm^2

2. **Critical Thinking** In Example 2, explain how you can use a Pythagorean triple to conclude that $XU = 5$ ft.

   $5^2 + 12^2 = 13^2$

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Lesson 10-3
Areas of Regular Polygons

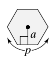
Lesson Objective	
Find the area of a regular polygon	**NAEP 2005 Strand:** Measurement **Topic:** Measuring Physical Attributes **Local Standards:** ____

Vocabulary and Key Concepts

Theorem 10-6: Area of a Regular Polygon

The area of a regular polygon is half the product of the apothem and the perimeter.

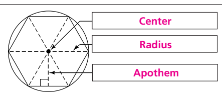
$A = \frac{1}{2}ap$



The center of a regular polygon is the center of the circumscribed circle.

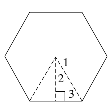
The radius of a regular polygon is the distance from the center to a vertex.

The apothem of a regular polygon is the perpendicular distance from the center to a side.



Examples

1 Finding Angle Measures This regular hexagon has an apothem and radii drawn. Find the measure of each numbered angle.



$m\angle 1 = \frac{360}{6} = 60$ Divide 360 by the number of sides.

$m\angle 2 = \frac{1}{2}m\angle 1$ The apothem bisects the vertex angle of the isosceles triangle formed by the radii.

$m\angle 2 = \frac{1}{2}(60) = 30$ Substitute 60 for $m\angle 1$.

$m\angle 3 = 180 - (90 + 30) = 60$ The sum of the measures of the angles of a triangle is 180.

$m\angle 1 = 60$, $m\angle 2 = 30$, and $m\angle 3 = 60$

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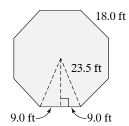
2 Finding the Area of a Regular Polygon A library is in the shape of a regular octagon. Each side is 18.0 ft. The radius of the octagon is 23.5 ft. Find the area of the library to the nearest 10 ft^2.



Consecutive radii form an isosceles triangle, so an apothem bisects the side of the octagon.

To apply the area formula $A = \frac{1}{2}ap$, you need to find *a* and *p*.

Step 1 Find the apothem *a*.

$a^2 + (9.0)^2 = (23.5)^2$ Pythagorean Theorem

$a^2 + 81 = 552.25$ Solve for *a*.

$a^2 = 471.25$

$a \approx 21.7$

Step 2 Find the perimeter *p*.

$p = ns$ Find the perimeter, where n = the number of sides of a regular polygon.

$p = (8)(18.0) = 144$ Substitute 8 for *n* and 18.0 for *s*, and simplify.

Step 3 Find the area *A*.

$A = \frac{1}{2}ap$ Area of a regular polygon

$A \approx \frac{1}{2}(21.7)(144)$ Substitute 21.7 for *a* and 144 for *p*.

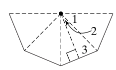
$A \approx 1562.4$ Simplify.

To the nearest 10 ft^2, the area is 1560 ft^2.

Quick Check

1. At the right, a portion of a regular octagon has radii and an apothem drawn. Find the measure of each numbered angle.

   $m\angle 1 = 45;\ m\angle 2 = 22.5;\ m\angle 3 = 67.5$



2. Find the area of a regular pentagon with 11.6-cm sides and an 8-cm apothem.

   232 cm^2

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Lesson 10-4

Perimeters and Areas of Similar Figures

Lesson Objective	NAEP 2005 Strand
▼ Find the perimeters and areas of similar figures	**NAEP 2005 Strand:** Measurement and Number Properties and Operations **Topics:** Systems of Measurement; Ratios and Proportional Reasoning **Local Standards:** ____________

Key Concepts

Theorem 10-7: Perimeters and Areas of Similar Figures

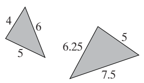
If the similarity ratio of two similar figures is $\frac{a}{b}$, then

(1) the ratio of their perimeters is $\frac{a}{b}$ and

(2) the ratio of their areas is $\frac{a^2}{b^2}$.

Examples

1 Finding Ratios in Similar Figures The triangles at the right are similar. Find the ratio (larger to smaller) of their perimeters and of their areas.



The shortest side of the left-hand triangle has length 4, and the shortest side of the right-hand triangle has length 5.

From larger to smaller, the similarity ratio is $\frac{5}{4}$.

By the Perimeters and Areas of Similar Figures Theorem, the ratio of the perimeters is $\frac{5}{4}$, and the ratio of the areas is $\frac{5^2}{4^2}$, or $\frac{25}{16}$.

2 Finding Areas Using Similar Figures The ratio of the length of the corresponding sides of two regular octagons is $\frac{8}{3}$. The area of the larger octagon is 320 ft^2. Find the area of the smaller octagon.

All regular octagons are similar.

Because the ratio of the lengths of the corresponding sides of the regular octagons is $\frac{8}{3}$, the ratio of their areas is $\frac{8^2}{3^2}$, or $\frac{64}{9}$.

$\frac{64}{9} = \frac{320}{A}$ Write a proportion.

$64A = 2880$ Use the Cross-Product Property.

$A = 45$ Divide each side by 64.

The area of the smaller octagon is 45 ft^2.

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3 Using Similarity Ratios Benita plants the same crop in two rectangular fields. Each dimension of the larger field is $3\frac{1}{2}$ times the dimension of the smaller field. Seeding the smaller field costs $8. How much money does seeding the larger field cost?

The similarity ratio of the fields is 3.5 : 1, so the ratio of the areas of the fields is $(3.5)^2 : (1)^2$, or 12.25 to 1.

Because seeding the smaller field costs $8, seeding 12.25 times as much land costs 12.25 × $8.

Seeding the larger field costs $98.

Quick Check

1. Two similar polygons have corresponding sides in the ratio 5 : 7.
   a. Find the ratio of their perimeters. 5 : 7
   b. Find the ratio of their areas. 25 : 49
2. The corresponding sides of two similar parallelograms are in the ratio $\frac{3}{4}$. The area of the smaller parallelogram is 54 in.^{2}. Find the area of the larger parallelogram.
   96 in.^{2}
3. The similarity ratio of the dimensions of two similar pieces of window glass is 3 : 5. The smaller piece costs $2.50. What should be the cost of the larger piece?
   $6.94

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Lesson 10-5

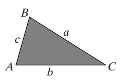
Trigonometry and Area

Lesson Objectives	NAEP 2005 Strand
▼ Find the area of a regular polygon using trigonometry ▼ Find the area of a triangle using trigonometry	**NAEP 2005 Strand:** Measurement **Topic:** Measuring Physical Attributes **Local Standards:** ____________

Key Concepts

Theorem 10-8: Area of a Triangle Given SAS

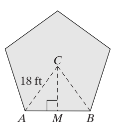
The area of a triangle is one half the product of the lengths of two sides and the sine of the included angle.



Area of $\triangle ABC = \frac{1}{2}bc(\sin A)$

Examples

1 Finding Area The radius of a garden in the shape of a regular pentagon is 18 feet. Find the area of the garden.



Find the perimeter p and apothem a, and then find the area using the formula $A = \frac{1}{2}ap$.

Because a pentagon has five sides, $m\angle ACB = \frac{360}{5} = 72$.

$\overline{CA}$ and $\overline{CB}$ are radii, so $CA = CB$. Therefore, $\triangle ACM \cong \triangle BCM$ by the HL Theorem, so $m\angle ACM = \frac{1}{2}m\angle ACB = 36$.

Use the cosine ratio to find a.

$\cos 36° = \frac{a}{18}$ Use the ratio.

$a = 18(\cos 36°)$ Solve.

Use the sine ratio to find AM.

$\sin 36° = \frac{AM}{18}$

$AM = 18(\sin 36°)$

Use AM to find p. Because $\triangle ACM \cong \triangle BCM$, $AB = 2 \cdot AM$. Because the pentagon is regular, $p = 5 \cdot AB$.

So $p = 5(2 \cdot AM) = 10 \cdot AM = 10 \cdot 18(\sin 36°) = 180(\sin 36°)$.

Finally, substitute into the area formula $A = \frac{1}{2}ap$.

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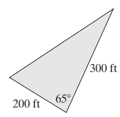
$A = \frac{1}{2} \cdot 18(\cos 36°) \cdot 180(\sin 36°)$ Substitute for *a* and *p*.

$A = 1620(\cos 36°) \cdot (\sin 36°)$ Simplify.

$A \approx 770.355778$ Use a calculator.

The area is about 770 ft^2.

2 Surveying A triangular park has two sides that measure 200 ft and 300 ft and form a 65° angle. Find the area of the park to the nearest hundred square feet.



Use Theorem 10-8: The area of a triangle is one half the product of the lengths of two sides and the sine of the included angle.

Area $= \frac{1}{2} \cdot$ side length $\cdot$ side length $\cdot$ sine of included angle Theorem 10-8

Area $= \frac{1}{2} \cdot 200 \cdot 300 \cdot \sin 65°$ Substitute.

Area $= 30{,}000 \sin 65°$ Substitute.

≈ 27189.23361 Use a calculator.

The area of the park is approximately 27,200 ft^2.

Quick Check

1. Find the area of a regular octagon with a perimeter of 80 in. Give the area to the nearest tenth.
   482.8 in.^{2}
2. Two sides of a triangular building plot are 120 ft and 85 ft long. They include an angle of 85°. Find the area of the building plot to the nearest square foot.
   5081 ft^2

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Lesson 10-6 — Circles and Arcs

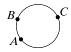
Lesson Objectives	NAEP 2005 Strand: Measurement
Find the measures of central angles and arcs	Topic: Measuring Physical Attributes
Find circumference and arc length	Local Standards: ____________

Vocabulary and Key Concepts

Postulate 10-1: Arc Addition Postulate

The measure of the arc formed by two adjacent arcs is the sum of the measures of the two arcs.

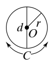
$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$



Theorem 10-9: Circumference of a Circle

The circumference of a circle is π times the diameter.

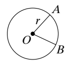
$C = \pi d$ or $C = 2\pi r$



Theorem 10-10: Arc Length

The length of an arc of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the circumference of the circle.

length of $\overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360} \cdot 2\pi r$



A circle is the set of all points equidistant from a given point called the center.

A center of a circle is the point from which all points are equidistant.

A radius is a segment that has one endpoint at the center and the other endpoint on the circle.

A central angle is an angle whose vertex is the center of the circle.

Circumference of a circle is the distance around the circle.

Pi (π) is the ratio of the circumference of a circle to its diameter.

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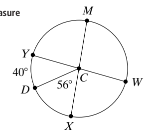
Examples

1 Finding the Measures of Arcs Find $m\overset{\frown}{XY}$ and $m\overset{\frown}{DXM}$ in circle C.

$m\overset{\frown}{XY} = m\overset{\frown}{XD} + m\overset{\frown}{DY}$	Arc Addition Postulate
$m\overset{\frown}{XY} = m\angle XCD + m\overset{\frown}{DY}$	The measure of a minor arc is the measure of its corresponding central angle.
$m\overset{\frown}{XY} = 56 + 40$	Substitute.
$m\overset{\frown}{XY} = 96$	Simplify.
$m\overset{\frown}{DXM} = m\overset{\frown}{DX} + m\overset{\frown}{XWM}$	Arc Addition Postulate
$m\overset{\frown}{DXM} = 56 + 180$	Substitute.
$m\overset{\frown}{DXM} = 236$	Simplify.



2 Finding Arc Length Find the length of $\overset{\frown}{ADB}$ in circle M in terms of π.

Because $m\overset{\frown}{AB} = 150$,

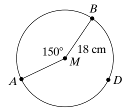
$m\overset{\frown}{ADB} = 360 - 150 = 210$ — Arc Addition Postulate

length of $\overset{\frown}{ADB} = \frac{m\overset{\frown}{ADB}}{360} \cdot 2\pi r$ — Arc Length Postulate

$= \frac{210}{360} \cdot 2\pi(18)$ — Substitute.

$= 21\pi$

The length of $\overset{\frown}{ADB}$ is 21π cm.



Quick Check

1. Use the diagram in Example 1. Find $m\angle YCD$, $m\overset{\frown}{YMW}$, $m\overset{\frown}{MW}$, and $m\overset{\frown}{XMY}$.

   40; 180; 96; 264

2. Find the length of a semicircle with radius 1.3 m in terms of π.

   1.3π m

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Lesson 10-7 — Areas of Circles and Sectors

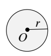
Lesson Objective	NAEP 2005 Strand: Measurement
Find the areas of circles, sectors, and segments of circles	Topic: Measuring Physical Attributes
	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 10-11: Area of a Circle

The area of a circle is the product of π and the square of the radius.

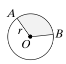
$A = \pi r^2$



Theorem 10-12: Area of a Sector of a Circle

The area of a sector of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the area of the circle.

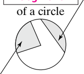
Area of sector $AOB = \frac{m\overset{\frown}{AB}}{360} \cdot \pi r^2$



A sector of a circle is a region bounded by two radii and their intercepted arc.

A segment of a circle is the part bounded by an arc and the segment joining its endpoints.

Segment of a circle



Sector of a circle

Examples

1 Applying the Area of a Circle A circular archery target has a 2-ft diameter. It is yellow except for a red bull's-eye at the center with a 6-in. diameter. Find the area of the yellow region to the nearest whole number.

First find the areas of the archery target and the red bull's-eye.

The radius of the archery target is $\frac{1}{2}(2 \text{ ft}) = 1$ ft $= 12$ in.

The area of the archery target is $\pi r^2 = \pi(12)^2 = 144\pi$ in.^{2}

The radius of the red bull's-eye region is $\frac{1}{2}(6 \text{ in.}) = 3$ in.

The area of the red region is $\pi r^2 = \pi(3)^2 = 9\pi$ in.^{2}

area of archery target	−	area of red region	=	area of yellow region
144π	−	9π	=	135π
			=	424.11501 — Simplify.

The area of the yellow region is about 424 in.^{2}

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2 Finding the Area of a Sector of a Circle Find the area of sector ACB. Leave your answer in terms of π.

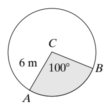
area of sector $ACB = \frac{m\overset{\frown}{AB}}{360} \cdot \pi r^2$

$= \frac{100}{360} \cdot \pi(6)^2$ — Substitute.

$= \frac{5}{18} \cdot 36\pi$ — Simplify.

$= 10\pi$ — Simplify.

The area of sector ACB is 10π m^2.



Quick Check

1. How much more pizza is in a 14-in.-diameter pizza than in a 12-in. pizza?

   about 41 in.^{2}

2. **Critical Thinking** A circle has a diameter of 20 cm. What is the area of a sector bounded by a 208° major arc? Round your answer to the nearest tenth.

   181.5 cm^2

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Lesson 10-8

Geometric Probability

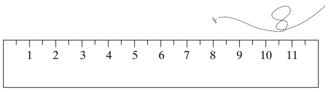
Lesson Objective	NAEP 2005 Strand: Data Analysis and Probability
Use segment and area models to find the probabilities of events	Topic: Probability
	Local Standards: ____________

Vocabulary

Geometric probability is a model in which you let points represent outcomes.

Example

1 Finding Probability Using Segments A gnat lands at random on the edge of the ruler below. Find the probability that the gnat lands on a point between 2 and 10.



The length of the segment between 2 and 10 is 10 − 2 = 8.

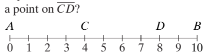
The length of the ruler is 12.

P(landing between 2 and 10)

$= \dfrac{\text{length of favorable segment}}{\text{length of entire segment}} = \dfrac{8}{12} = \dfrac{2}{3}$

Quick Check

1. A point on $\overline{AB}$ is selected at random. What is the probability that it is a point on $\overline{CD}$?

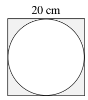


$\frac{2}{5}$

Name_______________ Class_______________ Date_______________

Example

2 Finding Probability Using Area A circle is inscribed in a square target with 20-cm sides. Find the probability that a dart landing randomly within the square does not land within the circle.



Find the area of the square.

$A = s^2 = 20^2 = 400\ \text{cm}^2$

Find the area of the circle. Because the square has sides of length 20 cm, the circle's diameter is 20 cm, so its radius is 10 cm.

$A = \pi r^2 = \pi(10)^2 = 100\pi\ \text{cm}^2$

Find the area of the region between the square and the circle.

$A = (400 - 100\pi)\ \text{cm}^2$

Use areas to calculate the probability that a dart landing randomly in the square does not land within the circle. Use a calculator. Round to the nearest thousandth.

$P(\text{between square and circle}) = \dfrac{\text{area between square and circle}}{\text{area of square}}$

$= \dfrac{400 - 100\pi}{400}$

$= 1 - \dfrac{\pi}{4} \approx 0.215$

The probability that a dart landing randomly in the square does not land within the circle is about 21.5%.

Quick Check

2. Use the diagram in Example 2. If you change the radius of the circle as indicated, what then is the probability of hitting outside the circle?

a. Divide the radius by 2.

about 80.4%

b. Divide the radius by 5.

about 96.9%

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Lesson 11-1

Space Figures and Cross Sections

Lesson Objective	NAEP 2005 Strand: Geometry
Recognize polyhedra and their parts	Topic: Dimension and Shape
Visualize cross sections of space figures	Local Standards: ____________

Vocabulary and Key Concepts

Euler's Formula

The numbers of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula $F + V = E + 2$.

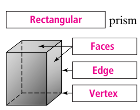
A polyhedron is a three-dimensional figure whose surfaces are polygons.

A face is a flat surface of a polyhedron in the shape of a polygon.

An edge is a segment that is formed by the intersection of two faces.

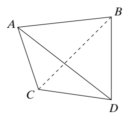
A vertex is a point where three or more edges intersect.

A cross section is the intersection of a solid and a plane.



Examples

1 Identifying Vertices, Edges, and Faces How many vertices, edges, and faces are there in the polyhedron shown? Give a list of each.



There are four vertices: A, B, C, and D.

There are six edges: $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, $\overline{AC}$, and $\overline{BD}$.

There are four faces: △ABC, △ABD, △ACD, and △BCD.

Name_______________ Class_______________ Date_______________

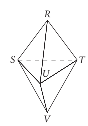
2 Using Euler's Formula Use Euler's Formula to find the number of edges on a solid with 6 faces and 8 vertices.

$F + V = E + 2$	Euler's Formula
$6 + 8 = E + 2$	Substitute the number of faces and vertices.
$12 = E$	Simplify.

A solid with 6 faces and 8 vertices has 12 edges.

Quick Check

1. List the vertices, edges, and faces of the polyhedron.



R, S, T, U, V; $\overline{RS}$, $\overline{RU}$, $\overline{RT}$, $\overline{VS}$, $\overline{VU}$, $\overline{VT}$, $\overline{SU}$, $\overline{UT}$, $\overline{TS}$; △RSU, △RUT, △RTS, △VSU, △VUT, △VTS

2. Use Euler's Formula to find the number of edges on a polyhedron with eight triangular faces.

12 edges

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Lesson 11-2

Surface Areas of Prisms and Cylinders

Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find the surface area of a prism 2 Find the surface area of a cylinder	**Topic:** Measuring Physical Attributes **Local Standards:** ________

Vocabulary and Key Concepts

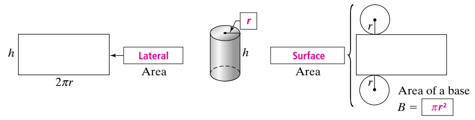
Theorem 11-1: Lateral and Surface Area of a Cylinder

The lateral area of a right cylinder is the product of the circumference of the base and the height of the cylinder.

L.A. $= 2\pi rh$, or L.A. $= \pi dh$

The surface area of a right cylinder is the sum of the lateral area and the areas of the two circular bases.

S.A. $=$ L.A. $+ 2B$, or S.A. $= 2\pi rh + 2\pi r^2$



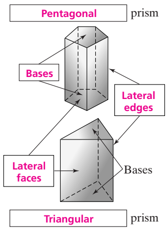
A prism is a polyhedron with exactly two congruent, parallel faces.

The bases of a polyhedron are the parallel faces.

Lateral faces are the faces on a prism that are not bases.

The lateral area of a prism is the sum of the areas of the lateral faces.

The surface area of a prism or a cylinder is the sum of the lateral area and the areas of the two bases.

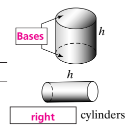


Name__________________ Class______________ Date __________

A cylinder is a three-dimensional figure with two congruent circular bases that lie in parallel planes.

In a cylinder, the bases are parallel circles.

The lateral area of a cylinder is the area of the curved surface.



Example

Finding Surface Area of Cylinders A company sells cornmeal and barley in cylindrical containers. The diameter of the base of the 6-in.-high cornmeal container is 4 in. The diameter of the base of the 4-in.-high barley container is 6 in. Which container has the greater surface area?

Find the surface area of each container. Remember that $r = \frac{d}{2}$.

Cornmeal Container		Barley Container
S.A. $=$ L.A. $+ 2B$	Use the formula for surface area.	S.A. $=$ L.A. $+ 2B$
$= 2\pi rh + 2\pi r^2$	Substitute the formulas for lateral area of a cylinder and area of a circle.	$= 2\pi rh + 2\pi r^2$
$= 2\pi(2)(6) + 2\pi(2^2)$	Substitute for r and h.	$= 2\pi(3)(4) + 2\pi(3^2)$
$= 24\pi + 8\pi$	Simplify.	$= 24\pi + 18\pi$
$= 32\pi$	Combine like terms.	$= 42\pi$

Because 42π in.^{2} $>$ 32π in.^{2}, the barley container has the greater surface area.

Quick Check

a. Find the surface area of a cylinder with height 10 cm and radius 10 cm in terms of π.

400π cm^2

b. The company in the Example wants to make a label to cover the cornmeal container. The label will cover the container all the way around, but will not cover any part of the top or bottom. What is the area of the label to the nearest tenth of a square inch?

75.4 in.^{2}

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Lesson 11-3

Surface Areas of Pyramids and Cones

Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find the surface area of a pyramid 2 Find the surface area of a cone	**Topic:** Measuring Physical Attributes **Local Standards:** ________

Vocabulary and Key Concepts

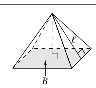
Theorem 11-3: Lateral and Surface Area of a Regular Pyramid

The lateral area of a regular pyramid is half the product of the perimeter of the base and the slant height.

L.A. $= \frac{1}{2}p\ell$

The surface area of a regular pyramid is the sum of the lateral area and the area of the base.

S.A. $=$ L.A. $+ B$



A regular pyramid is a pyramid whose base is a regular polygon and whose lateral faces are congruent isosceles triangles.

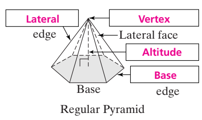
The altitude of a pyramid or cone is the perpendicular segment from the vertex to the plane of the base.

The height of a pyramid or a cone is the length of the altitude.

The slant height of a regular pyramid is the length of the altitude of a lateral face.

The lateral area of a pyramid is the sum of the areas of the congruent lateral faces.

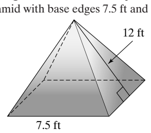
The surface area of a pyramid is the sum of the lateral area and the area of the base.



Regular Pyramid

Example

Finding Surface Area of a Pyramid Find the surface area of a square pyramid with base edges 7.5 ft and slant height 12 ft.



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The perimeter p of the square base is 4×7.5 ft, or 30 ft.

You are given $\ell = 12$ ft and you found that $p = 30$ ft, so you can find the lateral area.

L.A. $= \frac{1}{2}p\ell$ — Use the formula for lateral area of a pyramid.

$= \frac{1}{2}(30)(12)$ — Substitute.

$= 180$ — Simplify.

Find the area of the square base.

Because the base is a square with side length 7.5 ft,

$B = s^2 = 7.5^2 = 56.25$.

S.A. $=$ L.A. $+ B$ — Use the formula for surface area of a pyramid.

$= 180 + 56.25$ — Substitute.

$= 236.25$ — Simplify.

The surface area of the square pyramid is 236.25 ft^2.

Quick Check

Find the surface area of a square pyramid with base edges 5 m and slant height 3 m.

55 m^2

Name______________________ Class______________ Date__________

Lesson 11-4

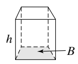
Volumes of Prisms and Cylinders

Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find the volume of a prism 2 Find the volume of a cylinder	Topic: Measuring Physical Attributes Local Standards: ____________

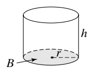
Vocabulary and Key Concepts

Theorem 11-5: Cavalieri's Principle
If two space figures have the same height and the same cross sectional area at every level, then they have the same volume.

Theorem 11-6: Volume of a Prism
The volume of a prism is the product of the area of a base and the height of the prism.
$V = Bh$



Theorem 11-7: Volume of a Cylinder
The volume of a cylinder is the product of the area of a base and the height of the cylinder.
$V = Bh$, or $V = \pi r^2 h$

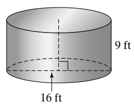


Volume is the space that a figure occupies.

A composite space figure is a three-dimensional figure that is the combination of two or more simpler figures.

Examples

1 Finding Volume of a Cylinder Find the volume of the cylinder. Leave your answer in terms of π.



The formula for the volume of a cylinder is $V = \pi r^2 h$.
The diagram shows h and d, but you must find r.

$r = \frac{1}{2}d = 8$

$V = \pi \cdot r^2 \cdot h$ — Use the formula for the volume of a cylinder.

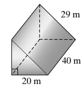
$= \pi \cdot 8^2 \cdot 9$ — Substitute.

$= 576\pi$ — Simplify.

The volume of the cylinder is 576π ft^3.

Name______________________ Class______________ Date__________

2 Finding Volume of a Triangular Prism Find the volume of the prism.



The prism is a right triangular prism with triangular bases. The base of the triangular prism is a right triangle where one leg is the base and the other leg is the altitude. Use the Pythagorean Theorem to calculate the length of the other leg.

$\sqrt{29^2 - 20^2} = \sqrt{841 - 400} = \sqrt{441} = 21$

The area B of the base is $\frac{1}{2}bh = \frac{1}{2}(20)(21) = 210$. Use the area of the base to find the volume of the prism.

$V = B \cdot h$ — Use the formula for the volume of a prism.

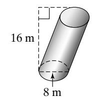
$= 210 \cdot 40$ — Substitute.

$= 8400$ — Simplify.

The volume of the triangular prism is 8400 m^3.

Quick Check

1. The cylinder shown is oblique.



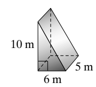
a. Find its volume in terms of π.

256π m^3

b. Find its volume to the nearest tenth of a cubic meter.

804.2 m^3

2. Find the volume of the triangular prism.



150 m^3

Name______________________ Class______________ Date__________

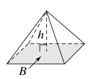
Lesson 11-5

Volumes of Pyramids and Cones

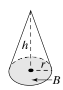
Lesson Objectives	NAEP 2005 Strand: Measurement
1 Find the volume of a pyramid 2 Find the volume of the cone	Topic: Measuring Physical Attributes Local Standards: ____________

Key Concepts

Theorem 11-8: Volume of a Pyramid
The volume of a pyramid is one third the product of the area of the base and the height of the pyramid.
$V = \frac{1}{3}Bh$



Theorem 11-9: Volume of a Cone
The volume of a cone is one third the product of the area of the base and the height of the cone.
$V = \frac{1}{3}Bh$, or $V = \frac{1}{3}\pi r^2 h$



Examples

1 Finding Volume of a Pyramid Find the volume of a square pyramid with base edges 15 cm and height 22 cm.

Because the base is a square, $B = 15 \cdot 15 = 225$.

$V = \frac{1}{3}Bh$ — Use the formula for volume of a pyramid.

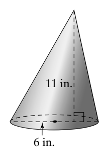
$= \frac{1}{3}(225)(22)$ — Substitute 225 for B and 22 for h.

$= 1650$ — Simplify.

The volume of the square pyramid is 1650 cm^3.

Name______________________ Class______________ Date__________

2 Finding Volume of an Oblique Cone Find the volume of the oblique cone in terms of π.



$r = \frac{1}{2}d = 3$ in.

$V = \frac{1}{3}\pi r^2 h$ — Use the formula for the volume of a cone.

$= \frac{1}{3}\pi(3)^2(11)$ — Substitute 3 for r and 11 for h.

$= 33\pi$ — Simplify.

The volume of the cone is 33π in.^{3}.

Quick Check

1. Find the volume of a square pyramid with base edges 24 m and slant height 13 m.

960 m^3

2. A small child's teepee is in the shape of a cone 6 ft tall and 7 ft in diameter. Find the volume of the teepee to the nearest cubic foot.

77 ft^3

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Lesson 11-6

Surface Areas and Volumes of Spheres

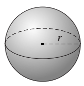
Lesson Objective	NAEP 2005 Strand: Measurement
Find the surface area and volume of a sphere	Topic: Measuring Physical Attributes
	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 11-10: Surface Area of a Sphere

The surface area of a sphere is four times the product of [π] and the [square] of the [radius] of the sphere.

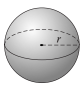
S.A. = [$4\pi r^2$]



Theorem 11-11: Volume of a Sphere

The volume of a sphere is four thirds the product of [π] and the [cube] of the [radius] of the sphere.

S.A. = [$\frac{4}{3}\pi r^3$]

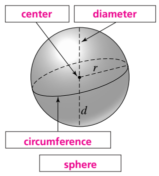


A sphere is the set of all points in space equidistant from a given point.

The center of a sphere is the given point from which all points on the sphere are equidistant.

The radius of a sphere is a segment that has one endpoint at the center and the other endpoint on the sphere.

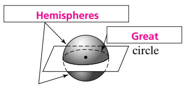
The diameter of a sphere is a segment passing through the center with endpoints on the sphere.



A great circle is the intersection of a sphere and a plane containing the center of the sphere.

The great circle divides a sphere into two congruent hemispheres.

The circumference of a sphere is the circumference of its great circle.



Name_______________ Class_______________ Date_______________

Examples

1 Finding Surface Area The circumference of a rubber ball is 13 cm. Calculate its surface area to the nearest whole number.

Step 1 First, find the radius.

$C = 2\pi r$ Use the formula for circumference.

$[13] = 2\pi r$ Substitute [13] for *C*.

$\frac{[13]}{2\pi} = r$ Solve for *r*.

Step 2 Use the radius to find the surface area.

S.A. $= 4\pi r^2$ Use the formula for surface area of a sphere.

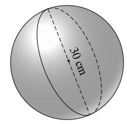
$= 4\pi \cdot \left(\frac{[13]}{2\pi}\right)^2$ Substitute $\frac{[13]}{2\pi}$ for *r*.

$= \frac{[169]}{\pi}$ Simplify.

$\approx [53.794371]$ Use a calculator.

To the nearest whole number, the surface area of the rubber ball is [54] cm^2.

2 Finding Volume Find the volume of the sphere. Leave your answer in terms of π.



$V = \frac{4}{3}[\pi][r^3]$ Use the formula for volume of a sphere.

$= \frac{4}{3}\pi \cdot [15]^3$ Substitute $r = \frac{30}{2} = [15]$.

$= [4500]\pi$ Simplify.

The volume of the sphere is [4500π] cm^3.

Quick Check

1. Find the surface area of a sphere with $d = 14$ in. Give your answer in two ways, in terms of π and rounded to the nearest square inch.

   196π in.^{2}, 616 in.^{2}

2. Find the volume to the nearest cubic inch of a sphere with diameter 60 in.

   113,097 in.^{3}

Name_______________ Class_______________ Date_______________

Lesson 11-7

Areas and Volumes of Similar Solids

Lesson Objective	NAEP 2005 Strand: Measurement
Find relationships between the ratios of the areas and volumes of similar solids	Topic: Systems of Measurement
	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 11-12: Areas and Volumes of Similar Solids

If the similarity ratio of two similar solids is $a : b$, then

(1) the ratio of their corresponding areas is [a^2] : [b^2], and

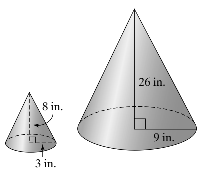
(2) the ratio of their volumes is [a^3] : [b^3].

Similar solids have the same shape and all of their corresponding parts are proportional.

The similarity ratio of two similar objects is the ratio of their corresponding linear dimensions.

Examples

1 Identifying Similar Solids Are the two solids similar? If so, give the similarity ratio.



Both figures have the same shape. Check that the ratios of the corresponding dimensions are equal.

The ratio of the radii is [$\frac{3}{9}$], and the ratio of the heights is [$\frac{8}{26}$].

The cones are [not similar] because [$\frac{3}{9} \neq \frac{8}{26}$].

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2 Finding the Similarity Ratio Find the similarity ratio of two similar cylinders with surface areas of 98π ft^2 and 2π ft^2.

Use the ratio of the surface areas to find the similarity ratios.

$\frac{a^2}{b^2} = \frac{[98\pi]}{[2\pi]}$ The ratio of the surface areas is [a^2] : [b^2].

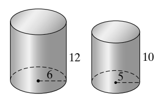
$\frac{a^2}{b^2} = \frac{[49]}{[1]}$ Simplify.

$\frac{a}{b} = \frac{[7]}{[1]}$ Take the [square root] of each side.

The similarity ratio is [7] : [1].

Quick Check

1. Are the two cylinders similar? If so, give the similarity ratio.



   yes; 6 : 5

2. Find the similarity ratio of two similar prisms with surface areas 144 m^2 and 324 m^2.

   2 : 3

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Lesson 12-1

Tangent Lines

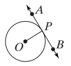
Lesson Objectives	NAEP 2005 Strand: Geometry
1. Use the relationship between a radius and a tangent	Topic: Relationships Among Geometric Figures
2. Use the relationship between two tangents from one point	Local Standards: ________

Vocabulary and Key Concepts

Theorem 12-1

If a line is tangent to a circle, then the line is perpendicular to the radius drawn to the point of tangency.

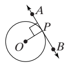
$\overleftrightarrow{AB} \perp \overline{OP}$



Theorem 12-2

If a line in the plane of a circle is perpendicular to a radius at its endpoint on the circle, then the line is tangent to the circle.

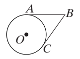
$\overleftrightarrow{AB}$ is tangent to ⊙ O.



Theorem 12-3

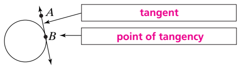
The two segments tangent to a circle from a point outside the circle are congruent.

$\overline{AB} \cong \overline{CB}$



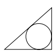
A tangent to a circle is a line, segment, or ray in the plane of the circle that intersects the circle in exactly one point.

The point of tangency is the point where a circle and a tangent intersect.



A triangle is inscribed in a circle if all vertices of the triangle lie on the circle.

A triangle is circumscribed about a circle if each side of the triangle is tangent to the circle.



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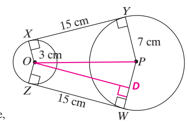
Example

Applying Tangent Lines A belt fits tightly around two circular pulleys, as shown. Find the distance between the centers of the pulleys. Round your answer to the nearest tenth.



Draw $\overline{OP}$. Then draw $\overline{OD}$ parallel to $\overline{ZW}$ to form rectangle $ODWZ$. Because $\overline{OZ}$ is a radius of ⊙ O, OZ = 3 cm.

Because opposite sides of a rectangle have the same measure, DW = 3 cm and OD = 15 cm.

Because ∠ODP is the supplement of a right angle, ∠ODP is also a right angle, and △OPD is a right triangle.

Because the radius of ⊙ P is 7 cm, PD = 7 − 3 = 4 cm.

$OD^2 + PD^2 = OP^2$ Pythagorean Theorem

$15^2 + 4^2 = OP^2$ Substitute.

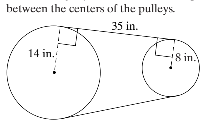
$241 = OP^2$ Simplify.

$OP \approx$ 15.524175 Use a calculator to find the square root.

The distance between the centers of the pulleys is about 15.5 cm.

Quick Check

A belt fits tightly around two circular pulleys, as shown. Find the distance between the centers of the pulleys.



about 35.5 in.

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Lesson 12-2

Chords and Arcs

Lesson Objectives	NAEP 2005 Strand: Geometry
1. Use congruent chords, arcs, and central angles	Topic: Relationships Among Geometric Figures
2. Recognize properties of lines through the center of a circle	Local Standards: ________

Vocabulary and Key Concepts

Theorem 12-4

Within a circle or in congruent circles

(1) Congruent central angles have congruent chords.

(2) Congruent chords have congruent arcs.

(3) Congruent arcs have congruent central angles.

Theorem 12-5

Within a circle or in congruent circles

(1) Chords equidistant from the center are congruent.

(2) Congruent chords are equidistant from the center.

Theorem 12-6

In a circle, a diameter that is perpendicular to a chord bisects the chord and its arcs.

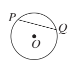
Theorem 12-7

In a circle, a diameter that bisects a chord (that is not a diameter) is perpendicular to the chord.

Theorem 12-8

In a circle, the perpendicular bisector of a chord contains the center of the circle.

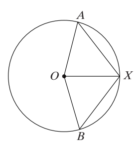
A chord is a segment whose endpoints are on a circle.



Name_______________ Class_______________ Date_______________

Examples

1. **Using Theorem 12-4** In the diagram, radius $\overline{OX}$ bisects ∠AOB. What can you conclude?

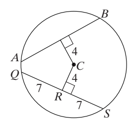


∠$AOX \cong$ ∠BOX by the definition of an angle bisector.

$\overline{AX} \cong \overline{BX}$ because congruent central angles have congruent chords.

$\overset{\frown}{AX} \cong \overset{\frown}{BX}$ because congruent chords have congruent arcs.

2. **Using Theorem 12-5** Find AB.



$QS = QR + RS$ Segment Addition Postulate

QS = 7 + 7 Substitute.

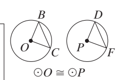
QS = 14 Simplify.

AB = QS Chords that are equidistant from the center of a circle are congruent.

AB = 14 Substitute 14 for QS.

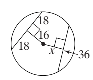
Quick Check

1. If you are given that $\overline{BC} \cong \overline{DF}$ in the circles, what can you conclude?



∠$O \cong$ ∠P; $\overset{\frown}{BC} \cong \overset{\frown}{DF}$

2. Find the value of x in the circle at the right.



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Lesson 12-3

Inscribed Angles

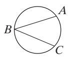
Lesson Objectives	NAEP 2005 Strand: Geometry
1 Find the measure of an inscribed angle	Topic: Relationships Among Geometric Figures
2 Find the measure of an angle formed by a tangent and a chord	Local Standards: ____________

Vocabulary and Key Concepts

Theorem 12-9: Inscribed Angle Theorem

The measure of an inscribed angle is half the measure of its intercepted arc.

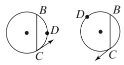
$m\angle B = \frac{1}{2}m\widehat{AC}$



Theorem 12-10

The measure of an angle formed by a tangent and a chord is half the measure of the intercepted arc.

$m\angle C = \frac{1}{2}m\widehat{BDC}$



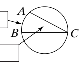
Corollaries to the Inscribed Angle Theorem

1. Two inscribed angles that intercept the same arc are congruent.
2. An angle inscribed in a semicircle is a right angle.
3. The opposite angles of a quadrilateral inscribed in a circle are supplementary.

An inscribed angle has a vertex that is on a circle and sides that are chords of the circle.

Intercepted arc

Inscribed angle



An intercepted arc is an arc with endpoints on the sides of an inscribed angle and its other points in the interior of the angle.

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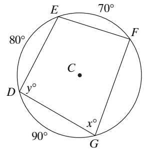
Example

Using the Inscribed Angle Theorem Find the values of x and y.



$x = \frac{1}{2}m\widehat{DEF}$ — Inscribed Angle Theorem

$x = \frac{1}{2}(m\widehat{DE} + m\widehat{EF})$ — Arc Addition Postulate

$x = \frac{1}{2}(80 + 70)$ — Substitute.

$x = 75$ — Simplify.

Because $\widehat{EFG}$ is the intercepted arc of $\angle D$, you need to find $m\widehat{FG}$ in order to find $m\widehat{EFG}$. The arc measure of a circle is 360°, so

$m\widehat{FG} = 360 - 70 - 80 - 90 = 120$.

$y = \frac{1}{2}m\widehat{EFG}$ — Inscribed Angle Theorem

$y = \frac{1}{2}(m\widehat{EF} + m\widehat{FG})$ — Arc Addition Postulate

$y = \frac{1}{2}(70 + 120)$ — Substitute.

$y = 95$ — Simplify.

Quick Check

In the Example, find $m\angle DEF$.

105

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Lesson 12-4

Angle Measures and Segment Lengths

Lesson Objectives	NAEP 2005 Strand: Geometry
1 Find the measures of angles formed by chords, secants, and tangents	Topic: Relationships Among Geometric Figures
2 Find the lengths of segments associated with circles	Local Standards: ____________

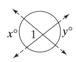
Key Concepts

Theorem 12-11

The measure of an angle formed by two lines that

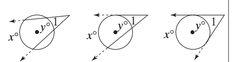
(1) intersect inside a circle is half the sum of the measures of the intercepted arcs.

$m\angle 1 = \frac{1}{2}(x + y)$



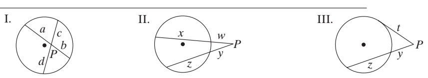
(2) intersect outside a circle is half the difference of the measures of the intercepted arcs.

$m\angle 1 = \frac{1}{2}(x - y)$



Theorem 12-12

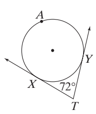
For a given point and circle, the product of the lengths of the two segments from the point to the circle is constant along any line through the point and circle.



I. $a \cdot b = c \cdot d$ II. $(w + x)w = (y + z)y$ III. $(y + z)y = t^2$

Examples

1 **Finding Arc Measures** An advertising agency wants a frontal photo of a "flying saucer" ride at an amusement park. The photographer stands at the vertex of the angle formed by tangents to the "flying saucer." What is the measure of the arc that will be in the photograph? In the diagram, the photographer stands at point T. $\overrightarrow{TX}$ and $\overrightarrow{TY}$ intercept minor arc $\widehat{XY}$ and major arc $\widehat{XAY}$.



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Let $m\widehat{XY} = x$.
Then $m\widehat{XAY} = 360 - x$.

$m\angle T = \frac{1}{2}(m\widehat{XAY} - m\widehat{XY})$ — Theorem 12-11(2)

$72 = \frac{1}{2}[(360 - x) - x]$ — Substitute.

$72 = \frac{1}{2}(360 - 2x)$ — Simplify.

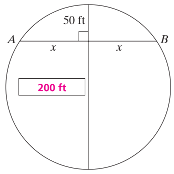
$72 = 180 - x$ — Distributive Property.

$x + 72 = 180$ — Add x to both sides.

$x = 108$ — Solve for x.

A 108° arc will be in the advertising agency's photo.

2 **Finding Segment Lengths** A tram travels from point A to point B along the arc of a circle with a radius of 125 ft. Find the shortest distance from point A to point B.



The perpendicular bisector of the chord $\overline{AB}$ contains the center of the circle. Because the radius is 125 ft, the diameter is $2 \cdot 125 = 250$ ft. The length of the other segment along the diameter is 250 ft − 50 ft, or 200 ft.

$x \cdot x = 50 \cdot 200$ — Theorem 12-12(1)

$x^2 = 10{,}000$ — Multiply.

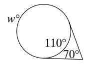
$x = 100$ — Solve for x.

The shortest distance from point A to point B is $2x$ or 200 ft.

Quick Check

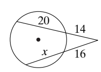
1. Find the value of w.

250



2. Find the value of x to the nearest tenth.

13.8



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Lesson 12-5

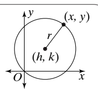
Circles in the Coordinate Plane

Lesson Objectives	NAEP 2005 Strand: Geometry
Write an equation of a circle	Topic: Position and Direction
Find the center and radius of a circle	Local Standards: ________

Key Concepts

Theorem 12-13

The standard form of an equation of a circle with center **(h, k)** and radius **r** is $(x - h)^2 + (y - k)^2 = r^2$.



Examples

1 Writing the Equation of a Circle Write the standard equation of a circle with center $(-8, 0)$ and radius $\sqrt{5}$.

$(x - \boxed{h})^2 + (y - \boxed{k})^2 = \boxed{r}^2$ Standard form

$[x - (\boxed{-8})]^2 + (y - \boxed{0})^2 = (\boxed{\sqrt{5}})^2$ Substitute $(-8, 0)$ for (h, k) and $\sqrt{5}$ for r.

$(x + \boxed{8})^2 + y^2 = \boxed{5}$ Simplify.

2 Using the Center and a Point on a Circle Write the standard equation of a circle with center $(5, 8)$ that passes through the point $(-15, -13)$.

First find the radius.

$r = \sqrt{(x - \boxed{h})^2 + (y - \boxed{k})^2}$ Use the Distance Formula to find r.

$= \sqrt{(\boxed{-15} - \boxed{5})^2 + (\boxed{-13} - \boxed{8})^2}$ Substitute $(5, 8)$ for (h, k) and $(-15, -13)$ for (x, y).

$= \sqrt{(\boxed{-20})^2 + (\boxed{-21})^2}$ Simplify.

$= \sqrt{\boxed{400} + \boxed{441}}$

$= \sqrt{\boxed{841}} = \boxed{29}$

Then find the standard equation of the circle with center $(5, 8)$ and radius **29**.

$(x - \boxed{h})^2 + (y - \boxed{k})^2 = \boxed{r}^2$ Standard form

$(x - \boxed{5})^2 + (y - \boxed{8})^2 = (\boxed{29})^2$ Substitute $(5, 8)$ for (h, k) and **29** for r.

$(x - \boxed{5})^2 + (y - \boxed{8})^2 = \boxed{841}$ Simplify.

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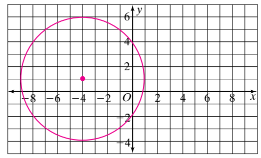
3 Graphing a Circle Given Its Equation Find the center and radius of the circle with equation $(x + 4)^2 + (y - 1)^2 = 25$. Then graph the circle.

$(x + 4)^2 + (y - 1)^2 = 25$

$(x - \boxed{-4})^2 + (y - \boxed{1})^2 = (\boxed{5})^2$ Relate the equation to the standard form $(x - h)^2 + (y - k)^2 = r^2$.

↑ h ↑ k ↑ r

The center is **(−4, 1)** and the radius is **5**.



Quick Check

1. Write the standard equation of the circle with center $(-2, -1)$ and radius $\sqrt{2}$.

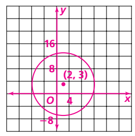
$(x + 2)^2 + (y + 1)^2 = 2$

2. Write the standard equation of the circle with center $(2, 3)$ that passes through the point $(-1, 1)$.

$(x - 2)^2 + (y - 3)^2 = 13$

3. Find the center and radius of the circle with equation $(x - 2)^2 + (y - 3)^2 = 100$. Then graph the circle.

center: (2, 3); radius: 10



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Lesson 12-6

Locus: A Set of Points

Lesson Objective	NAEP 2005 Strand: Geometry
Draw and describe a locus	Topic: Dimension and Shape
	Local Standards: ________

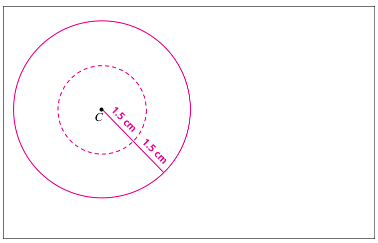
Vocabulary

A locus is **a set of points, all of which meet a stated condition.**

Examples

1 Describing a Locus in a Plane Draw and describe the locus of points in a plane that are 1.5 cm from ⊙ *C* with radius 1.5 cm.

Use a compass to draw ⊙ *C* with radius 1.5 cm.



The locus of points in the interior of ⊙ *C* that are **1.5** cm from ⊙ *C* is the center *C*.

The locus of points exterior to ⊙ *C* that are 1.5 cm from ⊙ *C* is a circle with center *C* and radius **3** cm.

The locus of points in a plane that are 1.5 cm from a point on ⊙ *C* with radius **1.5** cm is point *C* and a circle with radius **3** cm and center *C*.

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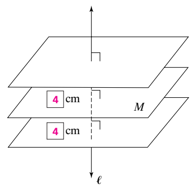
2 Describing a Locus in Space Describe the locus of points in space that are 4 cm from a plane *M*.

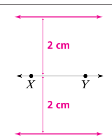


Imagine plane *M* as a horizontal plane. Because distance is measured along a perpendicular segment from a point to a plane, the locus of points in space that are 4 cm from a plane *M* are a plane **4** cm above and parallel to plane *M* and another plane **4** cm below and parallel to plane *M*.

Quick Check

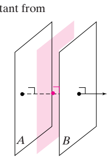
1. Draw and describe the locus: In a plane, the points 2 cm from line $\overleftrightarrow{XY}$.

Two lines parallel to $\overleftrightarrow{XY}$, each 2 cm from $\overleftrightarrow{XY}$.



2. Draw and describe the locus of points in space that are equidistant from two parallel planes.

A plane midway between the two parallel planes, parallel to and equidistant from each.



Chapter 1

Practice 1-1

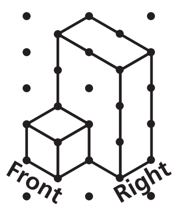
1. 47, 53 **2.** 42, 54 **3.** −64, 128 **4.** Sample: 2 or 3 **5.** 6 or 8 **6.** *Y* or *A* **7.** any hexagon **8.** a 168.75° angle **9.** 34 **10.** Sample: The farther out you go, the closer the ratio gets to a number that is approximately 0.618. **11.** 0, 1, 1, 2, 3, 5, 8, 13

Guided Problem Solving 1-1

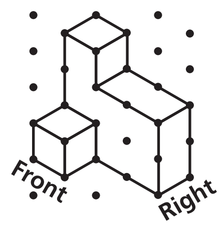
1. The pattern is easier to visualize. **2.** The graph will go up. **3.** Use Years for the horizontal axis. **4.** Use Number of Stations for the vertical axis. **5.** increasing **6.** greater **7.** Yes. Since the number of stations increases steadily from 1950 to 2000, we can be confident that the number of stations in 2010 will be greater than in 2000. **8.** Patterns are necessary to reach a conclusion through inductive reasoning. **9.** (any list of numbers without a pattern would apply) 2,435; 16,439; 16,454; 3,765; 210,564

Practice 1-2

1.



2.



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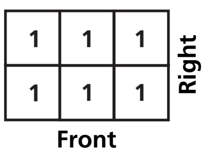

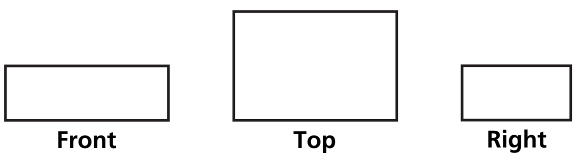

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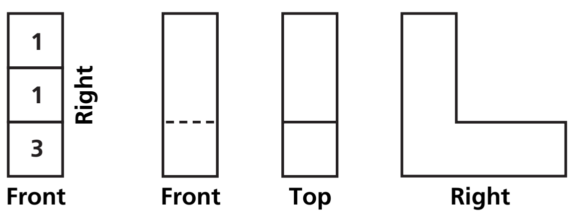
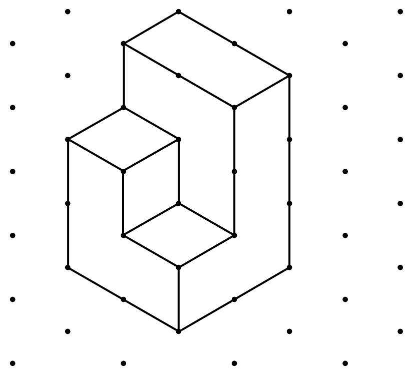


5. A, C, D **6.** C **7.** D **8.** B **9.** A

Guided Problem Solving 1-2

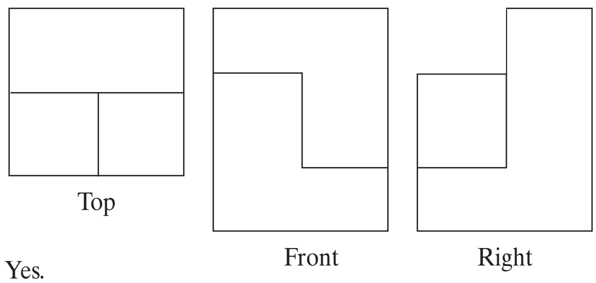
1. They represent three-dimensional objects on a two-dimensional surface. **2.** nine **3.** See the figure in 4, below.

4.



5. Yes. It is similar to the foundation drawing, except there are no numbers. **6.** no

7.



8. Yes.

9.

4	2
3	1

Practice 1-3

1. $\overleftrightarrow{AC}$ **2.** any two of the following: *ABD, DBC, CBE, ABE, ECD, ADE, ACE, ACD* **3.** yes **4.** no **5.** yes **6.** yes **7.** yes **8.** yes **9.** *G* **10.** $\overleftrightarrow{LM}$ **11.** the empty set **12.** $\overleftrightarrow{KP}$ **13.** Sample: plane *ABD* **14.** $\overleftrightarrow{AB}$ **15.** no **16.** yes **17.** the empty set **18.** no **19.** yes **20.** yes

Guided Problem Solving 1-3

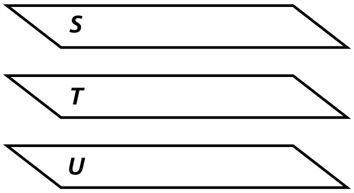
1. Collinear points lie on the same line. **2.** Answers may vary. (Some people might note that the *y*-coordinate of two of the points is the same so that the third point must have the same *y*-coordinate to be collinear. Since it does not, the points are not collinear.) **3.** horizontal **4.** No. **5.** No. **6.** All points must have the same *y*-coordinate, −3. **7.** No. **8.** $\left(1, -\frac{1}{2}\right)$

Practice 1-4

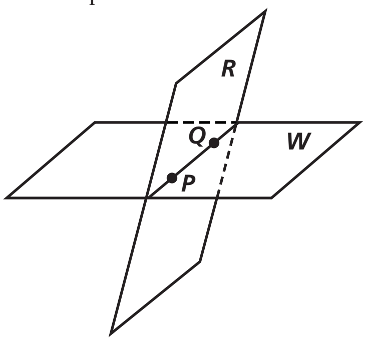
1. true **2.** false **3.** false **4.** false **5.** $\overline{JK}, \overline{HG}$ **6.** any three of the following pairs: $\overleftrightarrow{EF}$ and $\overleftrightarrow{JH}$; $\overleftrightarrow{EF}$ and $\overleftrightarrow{GK}$; $\overleftrightarrow{HG}$ and $\overleftrightarrow{JE}$; $\overleftrightarrow{HG}$ and $\overleftrightarrow{FK}$; $\overleftrightarrow{JK}$ and $\overleftrightarrow{EH}$; $\overleftrightarrow{JK}$ and $\overleftrightarrow{FG}$; $\overleftrightarrow{EJ}$ and $\overleftrightarrow{FG}$; $\overleftrightarrow{EH}$ and $\overleftrightarrow{FK}$; $\overleftrightarrow{JE}$ and $\overleftrightarrow{KG}$; $\overleftrightarrow{EH}$ and $\overleftrightarrow{KG}$; $\overleftrightarrow{JH}$ and $\overleftrightarrow{KF}$; $\overleftrightarrow{JH}$ and $\overleftrightarrow{GF}$ **7.** planes *A* and *B* **8.** planes *A* and *C*

9. Sample: $\overrightarrow{EG}$ **10.** $\overrightarrow{EF}$ and $\overrightarrow{ED}$ or $\overrightarrow{EG}$ and $\overrightarrow{ED}$
11. $\overrightarrow{FE}, \overrightarrow{FD}$ **12.** yes

13. Sample:

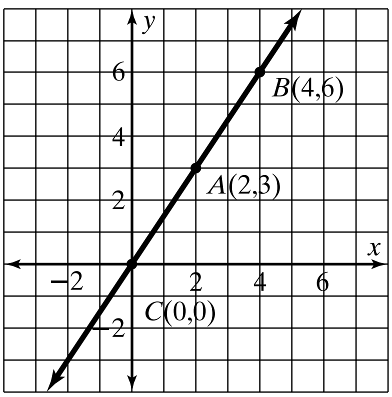


14. Sample:



Guided Problem Solving 1-4

1. Opposite rays are two collinear rays with the same endpoint. **2.** a line **3–4.** See graph in Exercise 5 answer. **5.** Answers may vary. Sample: (0,0) (Answers will be coordinates (x, y), where $y = \frac{3}{2}x$, $x < 2$.)



6. yes **7.** $L(4, 2)$

Practice 1-5

1. 4 **2.** 12 **3.** 20 **4.** 6 **5.** 22 **6.** −3, 4 **7.** no **8.** −2 **9.** 11 **10.** 29 **11.** 29

Guided Problem Solving 1-5

1. $\overline{AD} \cong \overline{DC}$ **2.** $AD = DC$ **3.** Segment Addition Postulate.
4. Since $AD = DC$, $AC = 2(AD)$. **5.** $AC = 2(12) = 24$
6. $y = 15$ **7.** $DC = AD = 12$ **8.** Answers may vary.
9. $ED = 11, DB = 11, EB = 22$

Practice 1-6

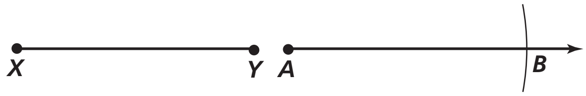
1. any three of the following: $\angle O, \angle MOP, \angle POM, \angle 1$
2. $\angle AOB$ **3.** $\angle EOC$ **4.** $\angle DOC$ **5.** 51 **6.** 90
7. 141 **8.** 68 **9.** $\angle ABD, \angle DBE, \angle EBF, \angle DBF, \angle FBC$
10. $\angle ABF, \angle DBC$ **11.** $\angle ABE, \angle EBC$

Guided Problem Solving 1-6

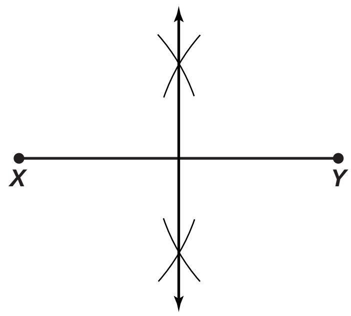
1. Angle Addition Postulate **2.** supplementary angles
3. $m\angle RQS + m\angle TQS = 180$ **4.** $(2x + 4) + (6x + 20) = 180$
5. $x = 19.5$ **6.** $m\angle RQS = 43; m\angle TQS = 137$ **7.** The sum of the angle measures should be 180; $m\angle RQS + m\angle TQS = 43 + 137 = 180$. **8a.** $x = 11$ **8b.** $m\angle AOB = 17$; $m\angle COB = 73$

Practice 1-7

1.



2.



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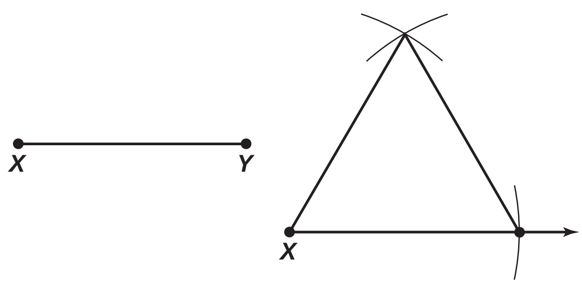

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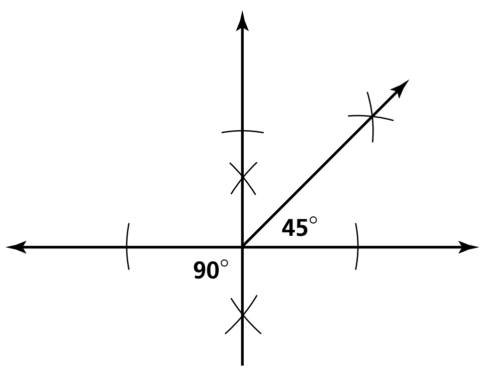

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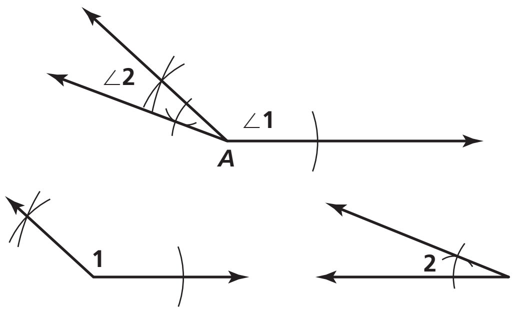

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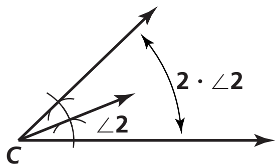

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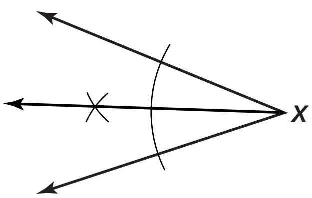
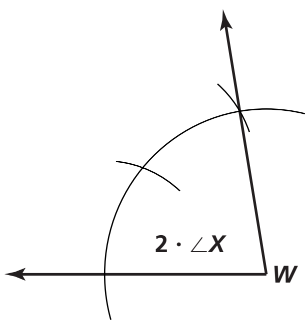


8.



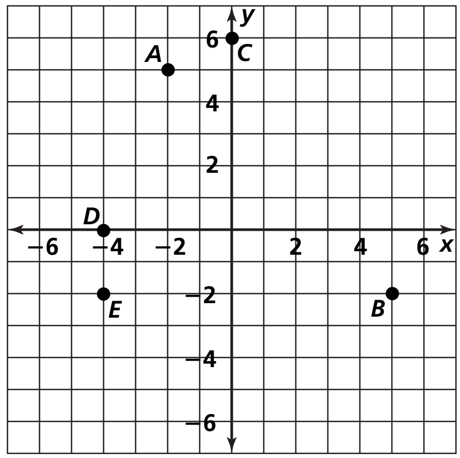
9. true **10.** false **11.** false **12.** true

Guided Problem Solving 1-7

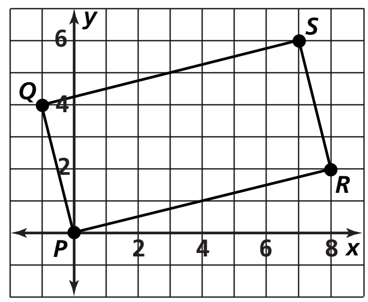
1. $\angle DBC \cong \angle ABC$ **2.** complementary angles
3. $\angle CBD$ **4.** $m\angle CBD = m\angle CBA = 41$
5. $m\angle ABD = m\angle CBA + m\angle CBD = 41 + 41 = 82$
6. $m\angle ABE + m\angle CBA = 90$
$m\angle ABE + 41 = 90$
$m\angle ABE = 49$
7. $m\angle DBF = m\angle ABE = 49$ **8.** Answers may vary. Sample: The sum of the measures of the complementary angles should be 90 and the sum of the measures of the supplementary angles should be 180. **9.** $m\angle CBD = 21$, $m\angle FBD = 69$, $m\angle CBA = 21$, and $m\angle EBA = 69$

Practice 1-8

1.–5.



6. 12 **7.** 13 **8.** (5, 5) **9.** $\left(-2\frac{1}{2}, 6\right)$ **10.** (−0.3, 3.4)
11. (5, −2) **12.** yes; $AB = BC = CD = DA = 6$
13. $\sqrt{401} \approx 20.025$
14.



15. ≈ 24.7 **16.** (3.5, 3)

Guided Problem Solving 1-8

1. Distance Formula **2.** The distance d between two points $A(x_1,y_1)$ to $B(x_2,y_2)$ is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
3. No; the differences are opposites but the squares of the differences are the same.
4. $XY = \sqrt{(5 - (-6))^2 + (-2 - 9)^2}$ **5.** To the nearest tenth, $XY = 15.6$ units. **6.** To the nearest tenth, $XZ = 12.0$ units. **7.** Z is closer to X. **8.** The results are the same; e.g., $XY = \sqrt{(-6 - 5)^2 + (9 - (-2))^2} = \sqrt{242}$, or about 15.6 units, as before.
9. $YZ = \sqrt{(17 - (-6))^2 + (-3 - 9)^2} = \sqrt{673}$; to the nearest tenth, $YZ = 25.9$ units. To the nearest tenth, $XY+YZ+XZ = 53.5$ units.

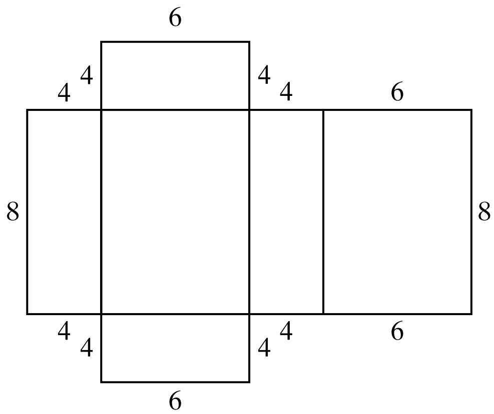
Practice 1-9

1. 792 in.^{2} **2.** 2.4 m^2 **3.** 16π **4.** 7.8π **5.** 26 cm; 42 cm^2 **6.** 29 in.; 42 in.^{2} **7.** 40 m; 99 m^2 **8.** 26; 22 **9.** 30; 44 **10.** 156.25π **11.** $10{,}000\pi$ **12.** 36 **13.** 26; 13

Guided Problem Solving 1-9

1. six **2.** It is a two-dimensional pattern you can fold to form a three-dimensional object. **3.** rectangles

4.



5. 208 in.^{2} **6.** They are equal. **7.** 208 in.^{2} **8.** Answers will vary. Sample: $2(4 \cdot 6) + 2(4 \cdot 8) + 2(6 \cdot 8)$; the results are the same, 208 in.^{2} **9.** $6(7^2) = 294$ in.^{2}

1A: Graphic Organizer

1. Tools of Geometry **2.** Answers may vary. Sample: patterns and inductive reasoning; measuring segments and angles; basic constructions; and the coordinate plane **3.** Check students' work.

1B: Reading Comprehension

1. Answer may vary. Sample: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $\overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$, $\overline{JK} \cong \overline{LM}$, $\overline{JL} \cong \overline{KM}$, $m\angle AJF + m\angle FJK = 180°$, $\angle HKB \cong \angle KMD$, $\overleftrightarrow{DN} \perp \overleftrightarrow{CD}$ **2.** Points *A, M,* and *S* are collinear. **3.** $\overleftrightarrow{AB}$, $\overleftrightarrow{HI}$, and $\overleftrightarrow{LN}$ intersect at point *M*. **4.** a

1C: Reading/Writing Math Symbols

1. Line *BC* is parallel to line *MN*. **2.** Line *CD* **3.** Line segment *GH* **4.** Ray *AB* **5.** The length of segment *XY* is greater than the length of segment *ST*. **6.** $MN = XY$ **7.** $GH = 2(KL)$ **8.** $\overline{ST} \perp \overline{UV}$ **9.** plane *ABC* ∥ plane *XYZ* **10.** $\overline{AB} \parallel \overline{DE}$

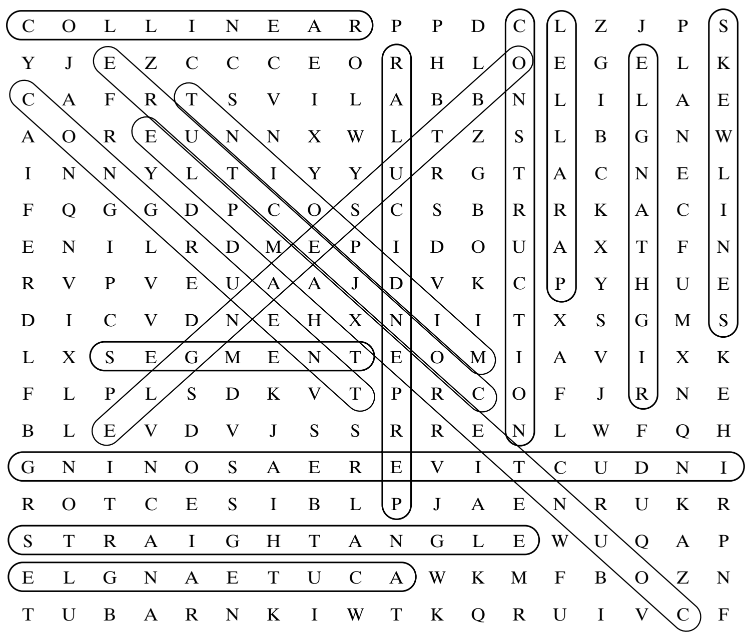
1D: Visual Vocabulary Practice

1. parallel planes **2.** Segment Addition Postulate **3.** supplementary angles **4.** opposite rays **5.** isometric drawing **6.** perpendicular lines **7.** foundation drawing **8.** right angle **9.** congruent sides

1E: Vocabulary Check

Net: A two-dimensional pattern that you can fold to form a three-dimensional figure.
Conjecture: A conclusion reached using inductive reasoning.
Collinear points: Points that lie on the same line.
Midpoint: A point that divides a line segment into two congruent segments.
Postulate: An accepted statement of fact.

1F: Vocabulary Review Puzzle



Chapter 2

Practice 2-1

1. Sample: It is 12:00 noon on a rainy day. **2.** Sample: 6 **3.** If you are strong, then you drink milk. **4.** If a rectangle is a square, then it has four sides the same length. **5.** If $x = 26$, then $x - 4 = 22$; true. **6.** If m is positive, then m^2 is positive; true. **7.** If lines are parallel, then their slopes are equal; true. **8.** Hypothesis: If you like to shop; conclusion: Visit Pigeon Forge outlets in Tennessee. **9.** If you visit Pigeon Forge outlets, then you like to shop. **10.** Drinking Sustain makes you train harder and run faster. **11.** If you drink Sustain, then you will train harder and run faster. **12.** If you train harder and run faster, then you drink Sustain.

Guided Problem Solving 2-1

1. Hypothesis: x is an integer divisible by 3. **2.** Conclusion: x^2 is an integer divisible by 3. **3.** Yes, it is true. Since 3 is a factor of x, it must be a factor of $x \cdot x = x^2$. **4.** If x^2 is an integer divisible by 3 then x is an integer divisible by 3. **5.** The converse is false. Counterexamples may vary. Let $x^2 = 3$. Then $x = \sqrt{3}$, which is not an integer and is not divisible by 3. **6.** No. The conditional is true, so there is no such counterexample. **7.** No. By definition, a general statement is false if a counterexample can be provided. **8.** If $5x + 3 = 23$, then $x = 4$. The original statement and the converse are both true.

Practice 2-2

1. Two angles have the same measure if and only if they are congruent. **2.** The converse, "If $|n| = 17$, then $n = 17$," is not true. **3.** If a whole number is a multiple of 5, then its last digit is either 0 or 5. If a whole number has a last digit of 0 or 5, then it is a multiple of 5. **4.** If two lines are perpendicular, then the lines form four right angles. If two lines form four right angles, then the lines are perpendicular. **5.** Sample: Other vehicles, such as trucks, fit this description. **6.** Sample: Baseball also fits this definition. **7.** Sample: *Pleasing, smooth,* and *rigid* all are too vague. **8.** yes **9.** no **10.** yes

Guided Problem Solving 2-2

1. A good definition is clearly understood, precise, and reversible. **2.** $\angle 3$ and $\angle 4$, $\angle 5$ and $\angle 6$ **3.** No **4.** They are not supplementary. **5.** A linear pair has a common vertex, shares a common side, and is supplementary. **6.** yes **7.** yes; yes **8.** linear pairs: $\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$; not linear pairs: $\angle 1$ and $\angle 3$, $\angle 1$ and $\angle 4$, $\angle 2$ and $\angle 3$, $\angle 2$ and $\angle 4$

Practice 2-3

1. Football practice is canceled for Monday. **2.** $\triangle DEF$ is a right triangle. **3.** If two lines are not parallel, then they intersect at a point. **4.** If you vacation at the beach, then you like Florida. **5.** Tamika lives in Nebraska. **6.** not possible **7.** It is not freezing outside. **8.** Shannon lives in the smallest state in the United States.

Guided Problem Solving 2-3

1. conditional; hypothesis **2.** Yes **3.** Beth will go. **4.** Anita, Beth, Aisha, Ramon **5.** No; only two students went. **6.** Beth, Aisha, Ramon; no—only two went. **7.** Aisha, Ramon **8.** The answer is reasonable. It is not possible for another pair to go to the concert. **9.** Ramon

Practice 2-4

1. $UT = MN$ **2.** $y = 51$ **3.** $\overline{JL}$ **4.** Addition; Subtraction Property of Equality; Multiplication Property of Equality; Division Property of Equality **5.** Substitution **6.** Substitution **7.** Symmetric Property of Congruence **8.** Definition of Complementary Angles; 90, Substitution; $3x$, Simplify; $3x$, 84, Subtraction Property of Equality; 28, Division Property of Equality

Guided Problem Solving 2-4

1. Angle Addition Postulate **2.** Substitution Property of Equality; Simplify; Addition Property of Equality; Division Property of Equality **3.** 40 **4.** yes; yes **5.** 13; 13

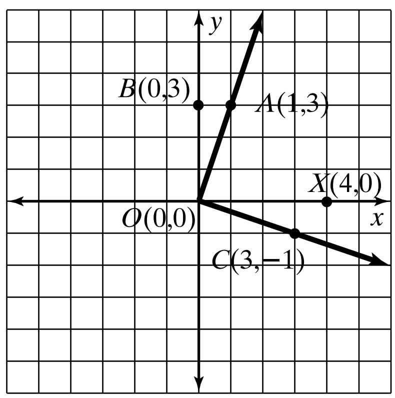
Practice 2-5

1. 30 **2.** 15 **3.** 6 **4.** $m\angle A = 135$; $m\angle B = 45$ **5.** $m\angle A = 10$; $m\angle B = 80$ **6.** $m\angle PMO = 55$; $m\angle PMQ = 125$; $m\angle QMN = 55$ **7.** $m\angle BWC = m\angle CWD$, $m\angle AWB + m\angle BWC = 180$; $m\angle CWD + m\angle DWA = 180$; $m\angle AWB = m\angle AWD$

Guided Problem Solving 2-5

1. 90 **2.** See graph in Exercise 5 answer. **3.** on the positive y-axis **4.** Answers may vary. B can be any point on the positive y-axis. Sample: $B(0, 3)$.

5.



6. They are adjacent complementary angles. **7.** Answers may vary. C can be any point on the line $y = -\frac{1}{3}x$, $x > 0$. Sample: $C(3, -1)$. **8.** a right angle **9.** Yes; their sum corresponds to the right angle formed by the positive x-axis and the positive y-axis. **10.** Answers may vary. D can be any point on the negative x-axis, sample: $D(-4, 0)$

2A: Graphic Organizer

1. Reasoning and Proof **2.** Answers may vary. Sample: conditional statements; writing biconditionals; converses; and using the Law of Detachment **3.** Check students' work.

2B: Reading Comprehension

1. 42 degrees **2.** 38 degrees **3.** b

2C: Reading/Writing Math Symbols

1. Segment MN is congruent to segment PQ. **2.** If p, then q.
3. The length of $\overline{MN}$ is equal to the length of $\overline{PQ}$.
4. Angle XQV is congruent to angle RDC. **5.** If q, then p.
6. The measure of angle XQV is equal to the measure of angle RDC. **7.** p if and only if q. **8.** $a \rightarrow b$ **9.** $AB = MN$
10. $m\angle XYZ = m\angle RPS$ **11.** $b \rightarrow a$ **12.** $\overline{AB} \cong \overline{MN}$
13. $a \leftrightarrow b$ **14.** $\angle XYZ \cong \angle RPS$

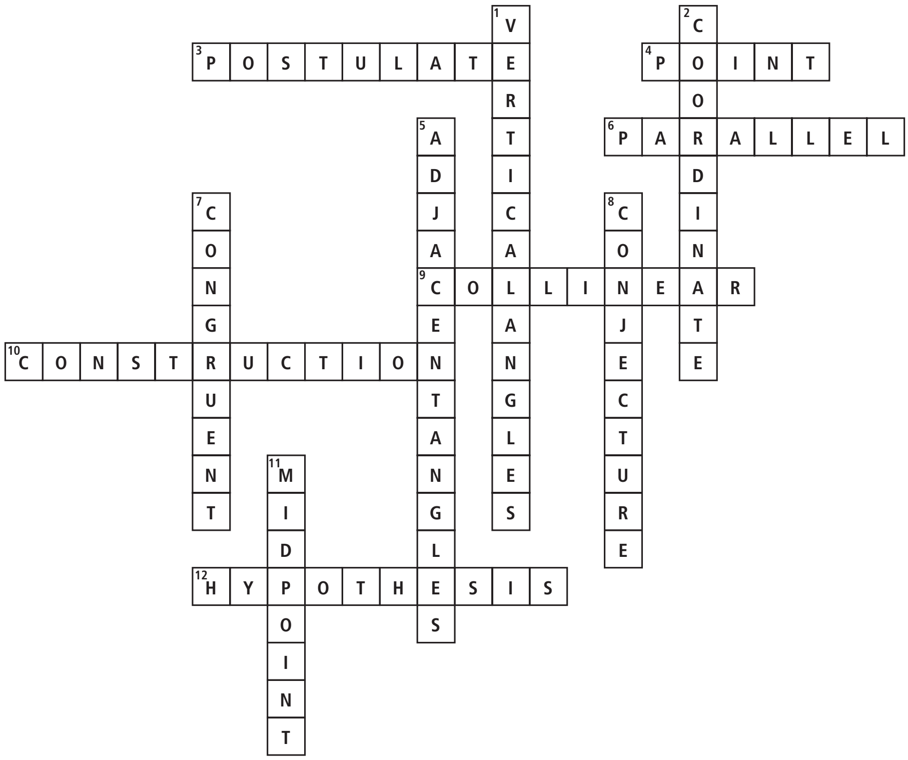
2D: Visual Vocabulary Practice

1. Law of Detachment **2.** hypothesis **3.** Distributive Property **4.** Reflexive Property **5.** Law of Syllogism
6. biconditional **7.** conclusion **8.** good definition
9. Symmetric Property

2E: Vocabulary Check

Truth value: "True" or "false" according to whether the statement is true or false, respectively
Hypothesis: The part that follows *if* in an *if-then* statement.
Biconditional: The combination of a conditional statement and its converse; it contains the words "if and only if."
Conclusion: The part of an *if-then* statement that follows *then*.
Conditional: An *if-then* statement.

2F: Vocabulary Review Puzzle



Chapter 3

Practice 3-1

1. corresponding angles **2.** alternate interior angles **3.** same-side interior angles **4.** $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 8$, $\angle 4$ and $\angle 7$ **5.** $\angle 4$ and $\angle 6$, $\angle 3$ and $\angle 5$ **6.** $\angle 4$ and $\angle 5$, $\angle 3$ and $\angle 6$ **7.** $m\angle 1 = 100$, alternate interior angles; $m\angle 2 = 100$, corresponding angles or vertical angles
8. $m\angle 1 = 135$, corresponding angles; $m\angle 2 = 135$, vertical angles **9.** $x = 103; 77°, 103°$ **10.** $x = 30; 85°, 85°$

Guided Problem Solving 3-1

1. The top and bottom sides are parallel, and the left and right sides are parallel. **2.** The two diagonals are transversals, and also each side of the parallelogram is a transversal for the two sides adjacent to it. **3.** Corresponding angles, interior and exterior angles are formed. **4.** v, w and x; By the Alternate Interior Angles Theorem, $v = 42$, $w = 25$ and $x = 76$.
5. Answers may vary. Possible answer: By the Same-Side Interior Angles Theorem, $(w + 42) + (y + 76) = 180$. Since $w = 25$, $y = 37$. (The two y's are equal by Theorem 3-1.)
6. $w = 25$, $y = 37$, $v = 42$, $x = 76$; yes **7.** $v = 42$, $w = 35$, $x = 57$, $y = 46$

Practice 3-2

1. *l* and *m*, Converse of Same-Side Interior Angles Theorem **2.** none **3.** $\overline{BC}$ and $\overline{AD}$, Converse of Same-Side Interior Angles Theorem **4.** $\overline{BH}$ and $\overline{CI}$, Converse of Corresponding Angles Postulate **5.** 43 **6.** 90 **7.** 38 **8.** 100

Guided Problem Solving 3-2

1. ℓ and m **2.** transversals **3.** x **4.** the angles measuring $19x°$ and $17x°$ **5.** 180° **6.** $17x°$ **7.** $180 - 19x = 17x$ or $19x + 17x = 180$ **8.** $x = 5$ **9.** With $x = 5$, $19x = 95$ and $17x = 85$. **10.** $x = 6$

Practice 3-3

1. True. Every avenue will be parallel to Founders Avenue, and therefore every avenue will be perpendicular to Center City Boulevard, and therefore every avenue will be perpendicular to any street that is parallel to Center City Boulevard.
2. True. The fact that one intersection is perpendicular, plus the fact that every street belongs to one of two groups of parallel streets, is enough to guarantee that all intersections are perpendicular. **3.** Not necessarily true. If there are more than three avenues and more than three boulevards, there will be some blocks bordered by neither Center City Boulevard nor Founders Avenue. **4.** $a \perp e$ **5.** $a \parallel e$ **6.** $a \parallel e$ **7.** $a \parallel e$
8. If the number of $\perp$statements is even, then $\ell_1 \parallel \ell_n$. If it is odd, then $\ell_1 \perp \ell_n$.

Guided Problem Solving 3-3

1. supplementary angles **2.** right angle **3.** Any one of the following: Postulate 3-1, or Theorem 3-1, 3-2, 3-3 or 3-4 **4.** 90 **5.** It is congruent; Postulate 3-1 **6.** 90 **7.** $a \perp c$ **8.** It is true for any line parallel to *b*. **9.** Yes. The point is that a transversal cannot be perpendicular to just one of two parallel lines. It has to be perpendicular to both, or else to neither.

Practice 3-4

1. 125 **2.** 143 **3.** 129 **4.** 136 **5.** $x = 35; y = 145; z = 25$ **6.** $v = 118; w = 37; t = 62$ **7.** 50 **8.** 88 **9.** $m\angle 1 = 33$; $m\angle 2 = 52$ **10.** right scalene **11.** obtuse isosceles **12.** equiangular equilateral

Guided Problem Solving 3-4

1. three **2.** 180 **3.** right triangle **4.** $z = 90$; Because it is given in the figure that $\overline{BD} \perp \overline{AC}$. **5.** Theorem 3-12, the Triangle Angle-Sum Theorem **6.** $x = 38$ **7.** $y = 36$
8. $\triangle ABD$ is a 36-54-90 right triangle. $\triangle BCD$ is a 38-52-90 right triangle. **9.** 74 **10.** $\triangle ABC$ is a 52-54-74 acute triangle.
11. Yes, all three are acute angles, with $\angle ABC$ visibly larger than $\angle A$ and $\angle C$. **12.** $\angle BCD$

Practice 3-5

1. $x = 120; y = 60$ **2.** $n = 51\frac{3}{7}$ **3.** $a = 108; b = 72$ **4.** 109 **5.** 133 **6.** 129 **7.** 30 **8.** 150 **9.** 6 **10.** 5 **11.** $BEDC$ **12.** $\angle FAE$ **13.** $\angle FAE$ and $\angle BAE$ **14.** $ABCDE$

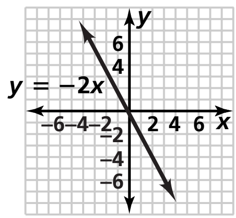
Guided Problem Solving 3-5

1. A theater stage, consisting of a large platform surrounding a smaller platform. The shapes in the bottom part of the figure may represent a ramp for actors to enter and exit. **2.** The measures of angles 1 and 2 **3.** 8; octagon **4.** $(8 - 2)180 = 1080$ degrees **5.** 135 **6.** 45 **7.** Yes, angle 1 is an obtuse angle and angle 2 is an acute angle. **8.** trapezoids **9.** 360

Practice 3-6

1. $y = \frac{1}{3}x - 7$ **2.** $y = -2x + 12$ **3.** $y = \frac{4}{5}x - 2$ **4.** $y = 4x - 13$

5.



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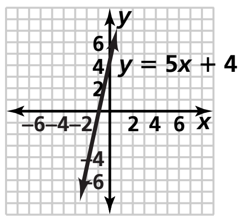

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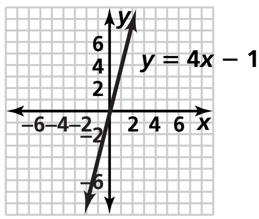
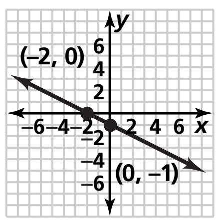


8. $y = x + 4$ **9.** $y = \frac{1}{2}x - 3$ **10.** $y = -\frac{1}{2}x - \frac{1}{2}$ **11.** $y = -6x + 45$ **12** $y = -11; x = 2$ **13.** $y = 2; x = 0$

14.



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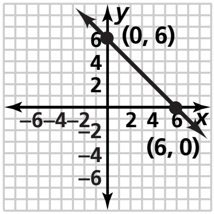

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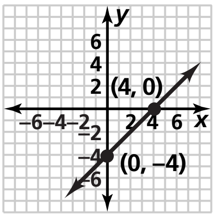
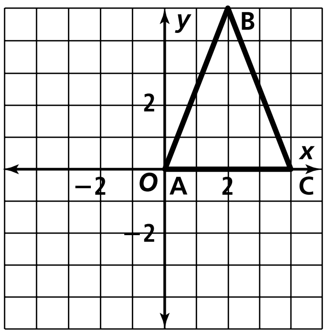


Guided Problem Solving 3-6

1.



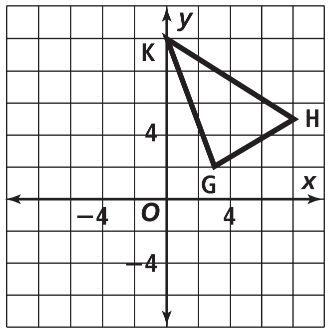
2. $m = \frac{y_2 - y_1}{x_2 - x_1}$ **3.** $y - y_1 = m(x - x_1)$ **4.** Slope of $\overleftrightarrow{AB} = \frac{5}{2}$; slope of $\overleftrightarrow{BC} = -\frac{5}{2}$. The absolute values of the slopes are the same, but one slope is positive and the other is negative. **5.** Point-slope form: $y - 0 = \frac{5}{2}(x - 0)$; slope-intercept form: $y = \frac{5}{2}x$ **6.** Point-slope form: $y - 5 = -\frac{5}{2}(x - 2)$ or $y - 0 = -\frac{5}{2}(x - 4)$; slope-intercept form: $y = -\frac{5}{2}x + 10$ **7.** Of line $\overleftrightarrow{AB}$: $(0, 0)$; of line $\overleftrightarrow{BC}$: $(0, 10)$ **8.** $\triangle ABC$ appears to be an isosceles triangle, which is consistent with a horizontal base and two remiaining sides having slopes of equal magnitude and opposite sign. **9.** Slope $= 0$; $y = 0$; y-intercept $= (0, 0)$ just as for line $\overleftrightarrow{AB}$ (they intersect on the y-axis).

Practice 3-7

1. neither; $3 \neq \frac{1}{3}, 3 \cdot \frac{1}{3} \neq -1$ **2.** perpendicular; $\frac{1}{2} \cdot -2 = -1$ **3.** parallel; $-\frac{2}{3} = -\frac{2}{3}$ **4.** perpendicular; $y = 2$ is a horizontal line, $x = 0$ is a vertical line
5. perpendicular; $-1 \cdot 1 = -1$ **6.** neither; $\frac{1}{2} \neq -\frac{5}{3}$, $\frac{1}{2} \cdot -\frac{5}{3} \neq -1$ **7.** neither; $\frac{9}{2} \neq 4, \frac{9}{2} \cdot 4 \neq -1$ **8.** parallel; $\frac{1}{2} = \frac{1}{2}$ **9.** $y = \frac{2}{3}x$ **10.** $y = 2x - 4$

Guided Problem Solving 3-7

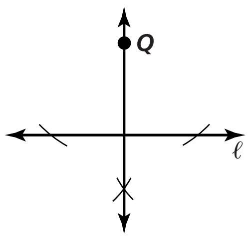
1.



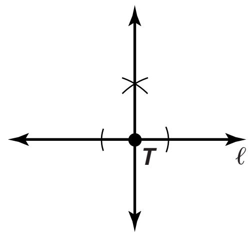
2. a right angle **3.** $m_1 \cdot m_2 = -1$ **4.** sides $\overline{GH}$ and $\overline{GK}$
5. Slope of $\overline{GH} = \frac{3}{5}$; slope of $\overline{GK} = \frac{8}{3}$
6. Product $= -\frac{8}{5} \neq -1$. Sides $\overline{GH}$ and $\overline{GK}$ are not perpendicular. **7.** $\triangle GHK$ has no pair of perpendicular sides. It is not a right triangle. **8.** No **9.** $\angle HGK$; approximately 80
10. Slope of $\overline{LM} = \frac{7}{2}$ and slope of $\overline{LN} = -\frac{2}{7}$. The product of the slopes is -1, so $\overline{LM}$ and $\overline{LN}$ are perpendicular.

Practice 3-8

1.



2.



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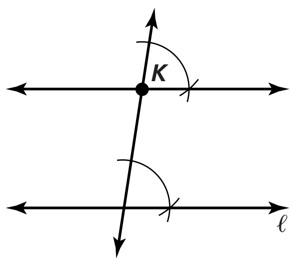
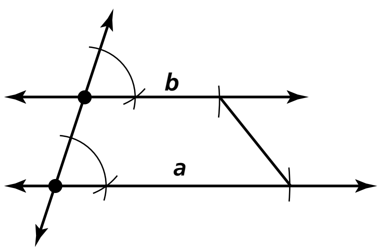
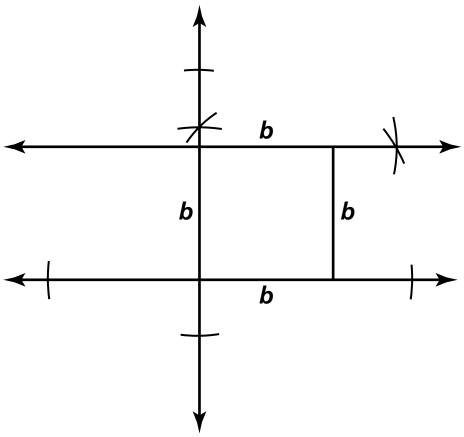


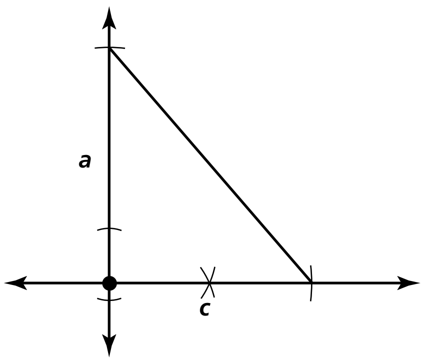
4. Sample:



5.

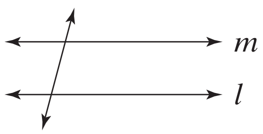


6.



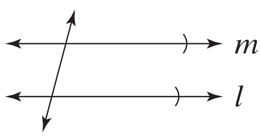
Guided Problem Solving 3-8

1. a line segment of length *c* **2.** Construct a quadrilateral with one pair of parallel sides of length *c*, and then examine the other pair. **3.** The procedure is given on p. 181 of the text.



4. Adjust the compass to exactly span line segment *c*, end to end. Then tighten down the compass adjustment as necessary.

5.



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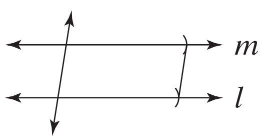


7. They appear to be both congruent and parallel. **8.** The answers to Step 7 are confirmed. **9.** yes; a parallelogram

3A: Graphic Organizer

1. Parallel and Perpendicular Lines **2.** Answers may vary. Sample: properties of parallel lines; finding the measures of angles in triangles; classifying polygons; and graphing lines **3.** Check students' work.

3B: Reading Comprehension

1. 14 spaces **2.** 6 spaces **3.** 60° **4.** corresponding **5.** $7000; $480 **6.** the width of the stalls, 10 ft **7.** b

3C: Reading/Writing Math Symbols

1. $m \perp n$ **2.** $m\angle 1 + m\angle 2 = 180$ **3.** $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$
4. $m\angle MNP + m\angle MNQ = 90$ **5.** $\angle 3 \cong \angle EFD$
6. Line 1 is parallel to line 2. **7.** The measure of angle ABC is equal to the measure of angle XYZ.
8. Line AB is perpendicular to line DF. **9.** Angle ABC and angle ABD are complementary. **10.** Angle 2 is a right angle, or the measure of angle 2 is 90°. **11.** Sample answer: $\overleftrightarrow{CB} \parallel \overleftrightarrow{GD}$, $m\angle BAF = m\angle GFA$

3D: Visual Vocabulary Practice/High-Use Academic Words

1. property **2.** conclusion **3.** describe **4.** formula **5.** measure **6.** approximate **7.** compare **8.** contradiction **9.** pattern

3E: Vocabulary Check

Transversal: A line that intersects two coplanar lines in two points.
Alternate interior angles: Nonadjacent interior angles that lie on opposite sides of the transversal.
Same-side interior angles: Interior angles that lie on the same side of a transversal between two lines.
Corresponding angles: Angles that lie on the same side of a transversal between two lines, in corresponding positions.
Flow proof: A convincing argument that uses deductive reasoning, in which arrows show the logical connections between the statements.

3F: Vocabulary Review

1. C **2.** E **3.** D **4.** B **5.** A **6.** F **7.** K **8.** H **9.** L **10.** G **11.** I **12.** J

Chapter 4

Practice 4-1

1. $m\angle 1 = 110; m\angle 2 = 120$ **2.** $m\angle 3 = 90; m\angle 4 = 135$ **3.** $\overline{CA} \cong \overline{JS}, \overline{AT} \cong \overline{SD}, \overline{CT} \cong \overline{JD}$ **4.** $\angle C \cong \angle J$, $\angle A \cong \angle S, \angle T \cong \angle D$ **5.** Yes; $\angle GHJ \cong \angle IHJ$ by Theorem 4-1 and by the Reflexive Property of $\cong$. Therefore, $\triangle GHJ \cong \triangle IHJ$ by the definition of $\cong$ triangles. **6.** No; $\angle QSR \cong \angle TSV$ because vertical angles are congruent, and $\angle QRS \cong \angle TVS$ by Theorem 4-1, but none of the sides are necessarily congruent. **7a.** Given **7b.** Vertical angles are $\cong$. **7c.** Theorem 4-1 **7d.** Given **7e.** Definition of $\cong$ triangles

Guided Problem Solving 4-1

1. right triangles **2.** $m\angle A = 45, m\angle B = m\angle L = 90$, and $AB = 4$ in. **3.** $\triangle ABC$ is congruent to $\triangle KLM$ means corresponding sides and angles are congruent. **4.** x and t **5.** 45 **6.** $m\angle K = m\angle M = 45$ **7.** $3x = 45$ **8.** $x = 15$ **9.** 4 **10.** $2t = 4$ **11.** $t = 2$ **12.** The angle measures indicate that the two triangles are isosceles right triangles. This matches the appearance of the figure. **13.** $m\angle M = 60$

Practice 4-2

1. $\triangle ADB \cong \triangle CDB$ by SAS **2.** not possible **3.** $\triangle TUS \cong \triangle XWV$ by SSS **4.** not possible **5.** $\triangle DEC \cong \triangle GHF$ by SAS **6.** $\triangle PRN \cong \triangle PRQ$ by SSS **7.** $\angle C$ **8.** $\overline{AB}$ and $\overline{BC}$ **9.** $\angle A$ and $\angle B$ **10.** $\overline{AC}$ **11a.** Given **11b.** Reflexive Property of Congruence **11c.** SAS Postulate

Guided Problem Solving 4-2

1. $\overline{ISOS}$ and $\overline{SP}$ bisects $\angle ISO$. **2.** Prove whatever additional facts can be proven about $\triangle ISP$ and $\triangle OSP$, based on the given information. **3.** $\overline{IS} \cong \overline{SO}$. **4.** $\angle ISP \cong \angle PSO$ **5.** $\overline{SP}$ **6.** $\triangle ISP \cong \triangle OSP$ by Postulate 4-2, the Side-Angle-Side(SAS)Postulate **7.** It does not matter. The Side-Angle-Side Postulate applies whether or not they are collinear. **8.** It does follow, because $\triangle ISP \cong \triangle OSP$ and because $\overline{IP}$ and $\overline{PQ}$ are corresponding parts.

Practice 4-3

1. not possible **2.** ASA Postulate **3.** AAS Theorem **4.** ASA Postulate **5.** not possible **6.** AAS Theorem

7.

Statements	***Reasons***
1. $\angle K \cong \angle M, \overline{KL} \cong \overline{ML}$	**1.** Given
2. $\angle JLK \cong \angle PLM$	**2.** Vertical $\angle$s are $\cong$.
3. $\triangle JKL \cong \triangle PML$	**3.** ASA Postulate

8. $\overline{BC} \cong \overline{EF}$ **9.** $\angle KHJ \cong \angle HKG$ or $\angle KJH \cong \angle HGK$

Guided Problem Solving 4-3

1. Corresponding angles and alternate interior angles. **2.** $\angle EAB$ and $\angle DBC$. **3.** $\angle EBA$ and $\angle DCB$. **4.** $\angle EAB$ and $\angle DBC$ **5.** $\angle EAB \cong \angle DBC, \overline{AE} \cong \overline{BD}$, and $\angle E \cong \angle D$. **6.** $\triangle AEB \cong \triangle BDC$ by Postulate 4-3, the Angle-Side-Angle (ASA) Postulate **7.** Yes; Theorem 4-2, the Angle-Angle-Side (AAS) Theorem; $\angle EBA \cong \angle DCB$. **8.** No, because now there is no way to demonstrate a second pair of congruent sides, nor a second pair of congruent angles.

Practice 4-4

1. $\overline{BD}$ is a common side, so $\triangle ADB \cong \triangle CDB$ by SAS, and $\angle A \cong \angle C$ by CPCTC. **2.** $\overline{FH}$ is a common side, so $\triangle FHE \cong \triangle HFG$ by ASA, and $\overline{HE} \cong \overline{FG}$ by CPCTC. **3.** $\overline{QS}$ is a common side, so $\triangle QTS \cong \triangle SRQ$ by AAS. $\angle QST \cong \angle SQR$ by CPCTC. **4.** $\angle ZAY$ and $\angle CAB$ are vertical angles, so $\triangle ABC \cong \triangle AYZ$ by ASA. $\overline{ZA} \cong \overline{AC}$ by CPCTC. **5.** $\angle JKH$ and $\angle LKM$ are vertical angles, so $\triangle HJK \cong \triangle MLK$ by AAS, and $\overline{JK} \cong \overline{KL}$ by CPCTC. **6.** $\overline{PR}$ is a common side, so $\triangle PNR \cong \triangle RQP$ by SSS, and $\angle N \cong \angle Q$ by CPCTC. **7.** First, show that $\angle ACB$ and $\angle ECD$ are vertical angles. Then, show $\triangle ABC \cong \triangle EDC$ by ASA. Last, show $\angle A \cong \angle E$ by CPCTC.

Guided Problem Solving 4-4

1. A compass with a fixed setting was used to draw two circular arcs, both centered at point P but crossing ℓ in different locations, which were labeled A and B. The compass was used again, with a wider setting, to draw two intersecting circular arcs, one centered at A and one at B. The point at which the new arcs intersected was labeled C. Finally, line $\overleftrightarrow{CP}$ was drawn. **2.** Find equal lengths or distances and explain why $\overleftrightarrow{CP}$ is perpendicular to ℓ. **3.** $\triangle ACP$ and $\triangle BCP$ **4.** $AP = PB$ and $AC = BC$. **5.** $\triangle APC \cong \triangle BPC$, by Postulate 4-1, the Side-Side-Side (SSS) Postulate **6.** $\angle APC \cong \angle BPC$ by CPCTC **7.** Since $\angle APC \cong \angle BPC$, $m\angle APC = m\angle BPC$ and $m\angle APC + m\angle BPC = 180$, it follows that $m\angle APC = m\angle BPC = 90$. **8.** from the definition of perpendicular and the fact that $m\angle APC = m\angle BPC = 90$ **9.** The distances do not matter, so long as $AP = BP$ and $AC = BC$. That is what is required in order that $\triangle APC \cong \triangle BPC$. **10.** Draw a line, and use the construction technique of the problem to construct a second line perpendicular to the first. Then do the same thing again to construct a third line perpendicular to the second line. The first and third lines will be parallel, by Theorem 3-10.

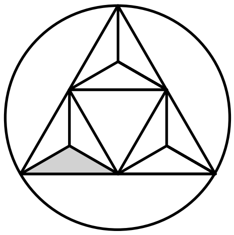
Practice 4-5

1. $x = 35; y = 35$ **2.** $x = 80; y = 90$ **3.** $t = 150$ **4.** $x = 55; y = 70; z = 125$ **5.** $x = 6$ **6.** $z = 120$ **7.** $\overline{AD}; \angle D \cong \angle F$ **8.** $\overline{KJ}; \angle KIJ \cong \angle KJI$ **9.** $\overline{BA}$; $\angle ABJ \cong \angle AJB$ **10.** 130 **11.** 130 **12.** $x = 70; y = 55$

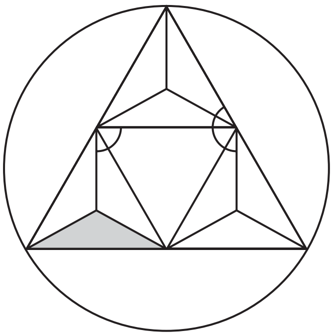
Guided Problem Solving 4-5

1. One angle is obtuse. The other two angles are acute and congruent. **2.** Highlight an obtuse isosceles angle and find its angle measures, then find all the other angle measures represented in the figure.

3. Possible answer:



4. 60 **5.** 30, because the measure of each base angle is half the measure of an angle of the equilateral triangle. **6.** 120° because the sum of the angles of the highlighted triangle must equal 180°.
7. The other measures are 90° and 150°. Examples:



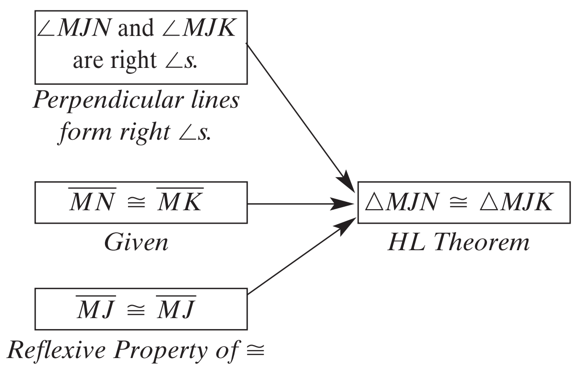
8. Yes; $3 \times 120 = 360$. **9.** Answers may vary.

Practice 4-6

1.

Statements	***Reasons***
1. $\overline{AB} \perp \overline{BC}$, $\overline{ED} \perp \overline{FE}$	**1.** Given
2. $\angle B$, $\angle E$ are right $\angle$s.	**2.** Perpendicular lines form right $\angle$s.
3. $\overline{AC} \cong \overline{FD}$, $\overline{AB} \cong \overline{ED}$	**3.** Given
4. $\triangle ABC \cong \triangle DEF$	**4.** HL Theorem

2.



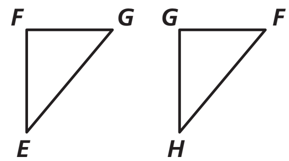
3. $\overline{RS} \cong \overline{VW}$ **4.** none **5.** $m\angle C$ and $m\angle F = 90$
6. $\overline{ST} \cong \overline{UV}$ or $\overline{SV} \cong \overline{UT}$ **7.** $m\angle A$ and $m\angle X = 90$
8. $\overline{GI} \perp \overline{JH}$

Guided Problem Solving 4-6

1. Two congruent right triangles. Each one has a leg and a hypotenuse labeled with a variable expression. **2.** the values of x and y for which the triangles are congruent by HL
3. The two shorter legs are congruent. **4.** $x = y + 1$ **5.** The hypotenuses are congruent. **6.** $x + 3 = 3y$ **7.** $x = 3; y = 2$
8. $\frac{\text{hypotenuse}}{\text{shorter leg}} = 2$; yes, this matches the figure. **9.** The solution remains the same: $x = 3$ and $y = 2$. The reason is that one is still solving the same two equations, $x = y + 1$ and $x + 3 = 3y$.

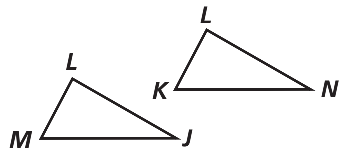
Practice 4-7

1. $\triangle ZWX \cong \triangle YXW$; SAS **2.** $\triangle LNP \cong \triangle LMO$; SAS
3. $\triangle ADF \cong \triangle BGE$; SAS
4.



common side: $\overline{FG}$

5.



common angle: $\angle L$

6. Sample:

Statements	***Reasons***
1. $\overline{AX} \cong \overline{AY}$	**1.** Given
2. $\overline{CX} \perp \overline{AB}$, $\overline{BY} \perp \overline{AC}$	**2.** Given
3. $m\angle CXA$ and $m\angle BYA = 90$	**3.** Perpendicular lines form right $\angle$s.
4. $\angle A \cong \angle A$	**4.** Reflexive Property of $\cong$
5. $\triangle BYA \cong \triangle CXA$	**5.** ASA Postulate

Guided Problem Solving 4-7

1. The figure, a list of parallel and perpendicular pairs of sides, and one known angle measure, namely $m\angle A = 56$
2. nine **3.** They are congruent and have equal measures.
4. $m\angle A = m\angle 1 = m\angle 2 = 56$ **5.** $m\angle 4 = 90$
6. $m\angle 3 = 34$ **7.** $m\angle DCE = 56$ **8.** $m\angle 5 = 22$
9. $m\angle FCG = 90$ **10.** $m\angle 6 = 34$ **11.** $m\angle 7 = 34$, $m\angle 8 = 68$, and $m\angle 9 = 112$ **12.** $m\angle 9 = 56 + 56 = 112$
13. $m\angle FIC = 180 - (m\angle 2 + m\angle 3) = 90$; $m\angle DHC = m\angle 4 = 90$; $m\angle FJC = 180 - m\angle 9 = 68$; $m\angle BIG = m\angle FIC = 90$

4A: Graphic Organizer

1. Congruent Triangles **2.** Answers may vary. Sample: congruent figures; triangle congruence by SSS, SAS, ASA, and AAS; proving parts of triangles congruent; the Isosceles Triangle Theorem **3.** Check students' work.

4B: Reading Comprehension

1. Yes. Using the Isosceles Triangle Theorem, $\angle W \cong \angle Y$. It is given that $\overline{WX} \cong \overline{YX}$ and $\overline{WU} \cong \overline{YV}$. Therefore $\triangle WUX \cong \triangle YVX$ by SAS. **2.** There is not enough information. You need to know if $\overline{AC} \cong \overline{EC}$, if $\angle A \cong \angle E$, or if $\angle B \cong \angle D$. **3.** a

4C: Reading/Writing Math Symbols

1. Angle-Angle-Side **2.** triangle XYZ **3.** angle PQR **4.** line segment BD **5.** line ST **6.** ray WX **7.** hypotenuse-leg **8.** line 3 **9.** angle 6 **10.** Angle-Side-Angle

4D: Visual Vocabulary Practice

1. theorem **2.** congruent polygons **3.** base angle of an isosceles triangle **4.** CPCTC **5.** postulate **6.** vertex angle of an isosceles triangle **7.** corollary **8.** base of an isosceles triangle **9.** legs of an isosceles triangle

4E: Vocabulary Check

Angle: Formed by two rays with the same endpoint.
Congruent angles: Angles that have the same measure.
Congruent segments: Segments that have the same length.
Corresponding polygons: Polygons that have corresponding sides congruent and corresponding angles congruent.
CPCTC: An abbreviation for "corresponding parts of congruent triangles are congruent."

4F: Vocabulary Review Puzzle

1. postulate **2.** hypotenuse **3.** angle **4.** vertex **5.** side **6.** leg **7.** perpendicular **8.** polygon **9.** supplementary **10.** parallel **11.** corresponding

Chapter 5

Practice 5-1

1a. 8 cm **1b.** 16 cm **1c.** 14 cm **2a.** 9.5 cm **2b.** 17.5 cm **2c.** 14.5 cm **3.** 17 **4.** 7 **5.** 42 **6.** 16.5 **7a.** 18 **7b.** 61 **8.** $\overline{PR} \parallel \overline{YZ}$, $\overline{PQ} \parallel \overline{XZ}$, $\overline{XY} \parallel \overline{RQ}$

Guided Problem Solving 5-1

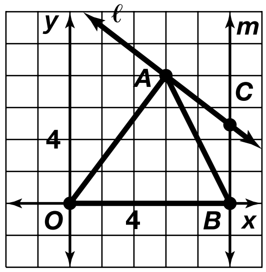
1. 30 units **2.** The three sides of the large triangle are each bisected by intersections with the two line segments lying in the interior of the large triangle. **3.** the value of x **4.** They are called midsegments. **5.** They are parallel, and the side labeled 30 is half the length of the side labeled x. **6.** $x = 60$ **7.** Yes; the side labeled x appears to be about twice as long as the side labeled 30. **8.** No. Those lengths are not fixed by the given information. (The triangle could be vertically stretched or shrunk without changing the lengths of the labeled sides.) All one can say is that the midsegment is half as long as the side it is parallel to.

Practice 5-2

1. $\overline{WY}$ is the perpendicular bisector of $\overline{XZ}$. **2.** 4 **3.** 9 **4.** right triangle **5.** 5 **6.** 17 **7.** isosceles triangle **8.** 3.5 **9.** 21 **10.** right triangle **11.** $\overrightarrow{JP}$ is the bisector of $\angle LJN$. **12.** 9 **13.** 45 **14.** 14 **15.** right isosceles triangle

Guided Problem Solving 5-2

1.



2. See answer to Step 1, above. **3.** See answer to Step 1, above. **4.** Plot a point and explain why it lies on the bisector of the angle at the origin. **5.** line ℓ: $y = -\frac{3}{4}x + \frac{25}{2}$; line m: $x = 10$ **6.** $C(10, 5)$ **7.** $CA = CB = 5$; yes **8.** Theorem 5-5, the Converse of the Angle Bisector Theorem **9.** $m\angle AOC = m\angle BOC \approx 27$ **10.** Draw ℓ, m, and C, then draw $\overline{OC}$. Since $OA = OB = 10$, it follows that $\triangle OAC \cong \triangle OBC$, by HL. Then $\overline{CA} \cong \overline{CB}$ and $\angle AOC \cong \angle BOC$ by CPCTC.

Practice 5-3

1. $(-2, 2)$ **2.** $(4, 0)$ **3.** altitude **4.** median **5.** perpendicular bisector **6.** angle bisector **7a.** $(2, 0)$ **7b.** $(-2, -2)$ **8a.** $(0, 0)$ **8b.** $(3, -4)$

Guided Problem Solving 5-3

1. the figure and a proof with some parts left blank **2.** Fill in the blanks. **3.** $\overline{AB}$ **4.** Theorem 5-2, the Perpendicular Bisector Theorem **5.** $\overline{BC}$; XC **6.** the Transitive Property of Equality **7.** Perpendicular Bisector. (This converse is Theorem 5-3.) **8.** The point of the proof is to *demonstrate* that n runs through point X. It would not be appropriate to show that fact as already given in the figure. **9.** Nothing essential would change. Point X would lie outside $\triangle ABC$ (below $\overleftrightarrow{BC}$), but the proof woud run just the same.

Practice 5-4

1. I and III **2.** I and II **3.** The angle measure is not 65. **4.** Tina does not have her driver's license. **5.** The figure does not have eight sides. **6.** $\triangle ABC$ is congruent to $\triangle XYZ$. **7a.** If you do not live in Toronto, then you do not live in Canada; false. **7b.** If you do not live in Canada, then you do not live in Toronto; true. **8.** Assume that $m\angle A \neq m\angle B$. **9.** Assume that $\overline{LM}$ does not intersect $\overline{NO}$. **10.** Assume that it is not sunny outside. **11.** Assume that $m\angle A \geq 90$. This means that $m\angle A + m\angle C \geq 180$. This, in turn, means that the sum of the angles of $\triangle ABC$ exceeds 180, which contradicts the Triangle Angle-Sum Theorem. So the assumption that $m\angle A \geq 90$ must be incorrect. Therefore, $m\angle A < 90$.

Guided Problem Solving 5-4

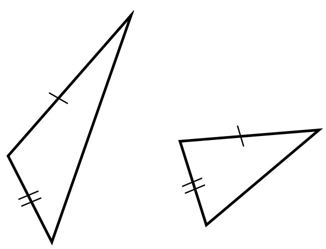
1. Ice is forming on the sidewalk in front of Toni's house. **2.** Use indirect reasoning to show that the temperature of the sidewalk surface must be 32°F or lower. **3.** The temperature of the sidewalk in front of Toni's house is greater than 32°F. **4.** Water is liquid (ice does not form) above 32°F. **5.** There is no ice forming on the sidewalk in front of Toni's house. **6.** The result from step 5 contradicts the information identified as given in step 1. **7.** The temperature of the sidewalk in front of Toni's house is less than or equal to 32°F. **8.** If the temperature is above 32°F, water remains liquid. This is reliably true. Converse: If water remains liquid, the temperature is above 32°F. This is not reliably true. Adding salt will cause water to remain liquid even below 32°F. **9.** Suppose two people are each the world's tallest person. Call them person A and person B. Then person A would be taller than everyone else, including B, but by the same token B would be taller than A. It is a contradiction for two people each to be taller than the other. So it is impossible for two people each to be the World's Tallest Person.

Practice 5-5

1. $\angle M, \angle N$ **2.** $\angle C, \angle D$ **3.** $\angle R, \angle P$ **4.** $\angle A, \angle T$ **5.** yes; $4 + 7 > 8, 7 + 8 > 4, 8 + 4 > 7$ **6.** no; $6 + 10 \ngtr 17$ **7.** yes; $4 + 4 > 4$ **8.** yes; $11 + 12 > 13, 12 + 13 > 11, 13 + 11 > 12$ **9.** no; $18 + 20 \ngtr 40$ **10.** no; $1.2 + 2.6 \ngtr 4.9$ **11.** $\overline{BO}, \overline{BL}, \overline{LO}$ **12.** $\overline{RS}, \overline{ST}, \overline{RT}$ **13.** $\angle D, \angle S, \angle A$ **14.** $\angle N, \angle S, \angle J$ **15.** $3 < x < 11$ **16.** $8 < x < 26$ **17.** $0 < x < 10$ **18.** $9 < x < 31$

Guided Problem Solving 5-5

1.



2. The side opposite the larger included angle is greater than the side opposite the smaller included angle. **3.** The angle opposite the larger side is greater than the angle opposite the smaller side **4.** The opposite sides each have a length of nearly the sum of the other two side lengths. **5.** The opposite sides are the same length. They are corresponding parts of triangles that are congruent by SAS.

5A: Graphic Organizer

1. Relationships Within Triangles **2.** Answers may vary. Sample: midsegments of triangles; bisectors in triangles; concurrent lines, medians, and altitudes; and inverses, contrapositives, and indirect reasoning **3.** Check students' work.

5B: Reading Comprehension

1. The width of the tar pit is 10 meters. **2.** b

5C: Reading/Writing Math Symbols

1. L **2.** F **3.** O **4.** G **5.** A **6.** I **7.** M **8.** H **9.** E **10.** K **11.** B **12.** D **13.** N **14.** J **15.** C

5D: Visual Vocabulary Practice

1. median **2.** negation **3.** circumcenter **4.** contrapositive **5.** centroid **6.** equivalent statements **7.** incenter **8.** inverse **9.** altitude

5E: Vocabulary Check

Midpoint: A point that divides a line segment into two congruent segments.
Midsegment of a triangle: The segment that joins the midpoints of two sides of a triangle.
Proof: A convincing argument that uses deductive reasoning.
Coordinate proof: A proof in which a figure is drawn on a coordinate plane and the formulas for slope, midpoint, and distance are used to prove properties of the figure.
Distance from a point to a line: The length of the perpendicular segment from the point to the line.

5F: Vocabulary Review

1. altitude **2.** line **3.** median **4.** negation **5.** contrapositive **6.** incenter **7.** orthocenter **8.** slope-intercept **9.** exterior **10.** obtuse **11.** alternate interior **12.** centroid **13.** equivalent **14.** right **15.** parallel

Chapter 6

Practice 6-1

1. parallelogram **2.** rectangle **3.** quadrilateral **4.** kite, quadrilateral **5.** trapezoid, isosceles trapezoid, quadrilateral **6.** square, rectangle, parallelogram, rhombus, quadrilateral **7.** $x = 7; AB = BD = DC = CA = 11$ **8.** $f = 5; g = 11; FG = GH = HI = IF = 17$ **9.** parallelogram **10.** kite

Guided Problem Solving 6-1

1. a labeled figure, which shows an isosceles trapezoid **2.** The nonparallel sides are congruent. **3.** the measures of the angles and the lengths of the sides **4.** $m\angle G = c$ **5.** $c + (4c - 20) = 180$ **6.** 40 **7.** $m\angle D = m\angle G = 40, m\angle E = m\angle F = 140$ **8.** $a - 4 = 11$ **9.** 15 **10.** $DE = FG = 11, EF = 15, DG = 32$ **11.** $40 + 40 + 140 + 140 = 360 = (4 - 2)180$ **12.** $m\angle D = m\angle G = 39, m\angle E = m\angle F = 141$

Practice 6-2

1. 15 **2.** 32 **3.** 7 **4.** 12 **5.** 9 **6.** 8 **7.** 100 **8.** 40; 140; 40 **9.** 113; 45; 22 **10.** 115; 15; 50 **11.** 55; 105; 55 **12.** 32; 98; 50 **13.** 16 **14.** 35 **15.** 28

Guided Problem Solving 6-2

1. the ratio of two different angle measures in a parallelogram
2. The consecutive angles are supplementary.
3.

4. the measures of the angles **5.** The angles are supplementary angles, because they are consecutive. **6.** $x + 9x = 180$ **7.** 18 and 162 **8.** No; the lengths of the sides are irrelevant in this problem. **9.** 30 and 150

Practice 6-3

1. no **2.** yes **3.** yes **4.** yes **5.** $x = 2; y = 3$ **6.** $x = 64; y = 10$ **7.** $x = 8$; the figure is a ▱ because both pairs of opposite sides are congruent. **8.** $x = 25$; the figure is a ▱ because the congruent opposite sides are $\parallel$ by the Converse of the Alternate Interior Angles Theorem. **9.** No; the congruent opposite sides do not have to be $\parallel$. **10.** No; the figure could be a trapezoid. **11.** Yes; both pairs of opposite sides are congruent. **12.** Yes; both pairs of opposite sides are $\parallel$ by the converse of the Alternate Interior Angles Theorem. **13.** No; only one pair of opposite angles is congruent. **14.** Yes; one pair of opposite sides is both congruent and $\parallel$.

Guided Problem Solving 6-3

1. a labeled figure, which shows a quadrilateral that appears to be a parallelogram **2.** The consecutive angles are supplementary. **3.** find values for x and y which make the quadrilateral a parallelogram **4.** $m\angle A + m\angle D = 180$, so that $\angle A$ and $\angle D$ meet the requirements for same-side interior angles on the transversal of two parallel lines (Theorems 3-2 and 3-6). **5.** $\angle B \cong \angle D$, by Theorem 6-2. **6.** $3x + 10 + 5y = 180; 8x + 5 = 5y$ **7.** $x = 15, y = 25$ **8.** $m\angle A = m\angle C = 55$ and $m\angle B = m\angle D = 125$, which matches the appearance of the figure. **9.** $(3x + 10) + (8x + 5) = 180$; yes

Practice 6-4

1a. rhombus **1b.** 72; 54; 54; 72 **2a.** rectangle **2b.** 37; 53; 106; 74 **3a.** rectangle **3b.** 60; 30; 60; 30 **4a.** rhombus **4b.** 22; 68; 68; 90 **5.** Possible; opposite angles are congruent in a parallelogram. **6.** Impossible; if the diagonals are perpendicular, then the parallelogram should be a rhombus, but the sides are not of equal length. **7.** $x = 7; HJ = 7; IK = 7$ **8.** $x = 6; HJ = 25; IK = 25$ **9a.** 90; 90; 29; 29 **9b.** 288 cm^2 **10a.** 38; 90; 90; 38 **10b.** 260 m^2

Guided Problem Solving 6-4

1. A labeled figure, which shows a parallelogram. One angle is a right angle, and two adjacent sides are congruent. Algebraic expressions are given for the lengths of three line segments. **2.** diagonals **3.** Find the values of x and y. **4.** It is a square. Theorem 6-1 and the fact that $\overline{AB} \cong \overline{AD}$ imply that all four sides are congruent. Theorems 3-11 and 6-2 plus the fact that $m\angle B = 90$ imply that all four angles are right angles. **5.** congruent; bisect **6.** $4x - y + 1 = (2x - 1) + (3y + 5)$; $2x - 1 = 3y + 5$ **7.** $x = 7\frac{1}{2}; y = 3$ **8.** It was not necessary to know $\overline{AB} \cong \overline{AD}$, but it was necessary to know $m\angle B = 90$. The key fact, which enables the use of Theorem 6-11 in addition to Theorem 6-3, is that $ABCD$ is a rectangle. It does not matter whether all four sides are congruent. **9.** 40

Practice 6-5

1. 118; 62 **2.** 59; 121 **3.** 96; 84 **4.** 101; 79 **5.** $x = 4$ **6.** $x = 1$ **7.** 105.5; 105.5 **8.** 118; 118 **9.** 90; 63; 63 **10.** 107; 107 **11.** $x = 8$ **12.** $x = 7$

Guided Problem Solving 6-5

1. Isosceles trapezoid $ABCD$ with $\overline{AB} \cong \overline{DC}$ **2.** $\angle B \cong \angle C$ and $\angle BAD \cong \angle D$ **3.** $\overline{AB} \cong \overline{DC}$ is Given. $\overline{DC} \cong \overline{AE}$ because opposite sides of a parallelogram are congruent (Theorem 6-1). $\overline{AB} \cong \overline{AE}$ is from the Transitive Property of Congruency. **4.** Isosceles; $\cong$; because base angles of an isosceles triangle are congruent. **5.** $\angle 1 \cong \angle C$ because corresponding angles on a transversal of two parallel lines are congruent. **6.** $\angle B \cong \angle C$ by the Transitive Property of Congruency. **7.** $\angle BAD$ is a same-side interior angle with $\angle B$, and $\angle D$ is a same-side interior angle with $\angle C$. **8.** This is not a problem, because for $AD > BC$ there is a similar proof with a line segment drawn from B to a point E lying on $\overline{AD}$.
9. The two drawn segments can be shown to be congruent, and then one has two congruent right triangles by the HL Theorem. $\angle B \cong \angle C$ follows by CPCTC and $\angle BAD \cong \angle D$ because they are supplements of congruent angles.

Practice 6-6

1. $(1.5a, 2b); a$ **2.** $(0.5a, 0); a$ **3.** $(0.5a, b); \sqrt{a^2 + 4b^2}$ **4.** 0 **5.** 1 **6.** $-\frac{1}{2}$ **7.** $\frac{2b}{3a}$ **8.** $-\frac{2b}{3a}$ **9.** $E(a, 3b); I(4a, 0)$ **10.** $D(4a, b); I(3a, 0)$ **11.** $(-4a, b)$ **12.** $(-b, 0)$

Guided Problem Solving 6-6

1. a rhombus with coordinates given for two vertices
2. Arhombus is a parallelogram with four congruent sides.
3. the coordinates of the other two vertices **4.** They are the diagonals. **5.** They bisect each other. **6.** $W(-2r, 0)$, $Z(0, -2t)$ **7.** No; neither Theorem 6-3 nor any other theorem or result would apply. **8.** Slope of $\overline{WX} = 0$ and slope of $\overline{YZ}$ is undefined. This confirms Theorem 6-10, which says that the diagonals of a rhombus are perpendicular.

Practice 6-7

1a. $\frac{p}{q}$ **1b.** $y = mx + b; q = \frac{p}{q}(p) + b; b = q - \frac{p^2}{q}$; $y = \frac{p}{q}x + q - \frac{p^2}{q}$ **1c.** $x = r + p$ **1d.** $y = \frac{p}{q}(r + p) + q - \frac{p^2}{q}; y = \frac{rp}{q} + \frac{p^2}{q} + q - \frac{p^2}{q}; y = \frac{rp}{q} + q$; intersection at $(r + p, \frac{rp}{q} + q)$ **1e.** $\frac{r}{q}$ **1f.** (r, q) **1g.** $y = mx + b;$ $q = \frac{r}{q}(r) + b; b = q - \frac{r^2}{q}; y = \frac{r}{q}x + q - \frac{r^2}{q}$
1h. $y = \frac{r}{q}(r + p) + q - \frac{r^2}{q}; y = \frac{r^2}{q} + \frac{rp}{q} + q - \frac{r^2}{q};$

$y = \frac{rp}{q} + q$; intersection at $(r + p, \frac{rp}{q} + q)$
1i. $(r + p, \frac{rp}{q} + q)$ **2a.** $(-2a, 0)$ **2b.** $(-a, b)$
2c. $(-\frac{3a}{2}, \frac{b}{2})$ **2d.** $\frac{b}{a}$

Guided Problem Solving 6-7

1. kite $DEFG$ with $DE = EF$ with the midpoint of each side identified **2.** A kite is a quadrilateral with two pairs of adjacent sides congruent and no opposite sides congruent. **3.** The midpoints are the vertices of a rectangle. **4.** $D(-2b, 2c), G(0, 0)$ **5.** $L(b, a + c), M(b, c), N(-b, c), K(-b, a + c)$ **6.** Slope of $\overline{KL}$ = slope of $\overline{NM} = 0$, slopes of $\overline{KN}$ and $\overline{LM}$ are undefined **7.** Opposite sides are parallel; it is a rectangle. **8.** Adjacent sides are perpendicular. **9.** right angles **10.** Answers will vary. Example: $a = 3, b = 2, c = 2$ yields the points $D(-4, 4), E(0, 6), F(4, 4), G(0, 0)$ with midpoints at $(-2, 2), (-2, 5), (2, 5)$, and $(2, 2)$. Connecting these midpoints forms a rectangle. **11.** Construct $\overline{DF}$ and $\overline{EG}$. Slope of $\overline{DF} = 0$, so $\overline{DF}$ is horizontal. Slope of $\overline{EG}$ is undefined, so $\overline{EG}$ is vertical.

6A: Graphic Organizer

1. Quadrilaterals **2.** Answers may vary. Sample: classifying quadrilaterals; properties of parallelograms; proving that a quadrilateral is a parallelogram; and special parallelograms
3. Check students' work.

6B: Reading Comprehension

1. $\overline{QT} \cong \overline{SR}, \overline{QR} \cong \overline{ST}, \overline{QT} \parallel \overline{RS}, \overline{QR} \parallel \overline{TV}$
2. No, it cannot be proven that $\triangle QTV \cong \triangle SRU$ because with the given information, only one side and one angle of the two triangles can be proven to be congruent. Another side or angle is needed. If it were given that $QUSV$ is a parallelogram, then the proof could be made. **3.** All four sides are congruent.
4. Yes. Since $\overline{EG} \cong \overline{EG}$ by the Reflexive Property, $\triangle EFG \cong \triangle EHG$ by SSS. **5.** b

6C: Reading/Writing Math Symbols

1. $\overline{AH}, \overline{CH}$, or $\overline{BH}$ **2.** $\overline{DG}, \overline{FG}$, or $\overline{EG}$ **3.** G
4. $\overline{DE}$ **5.** rhombus **6.** rectangle **7.** square **8.** isosceles trapezoid

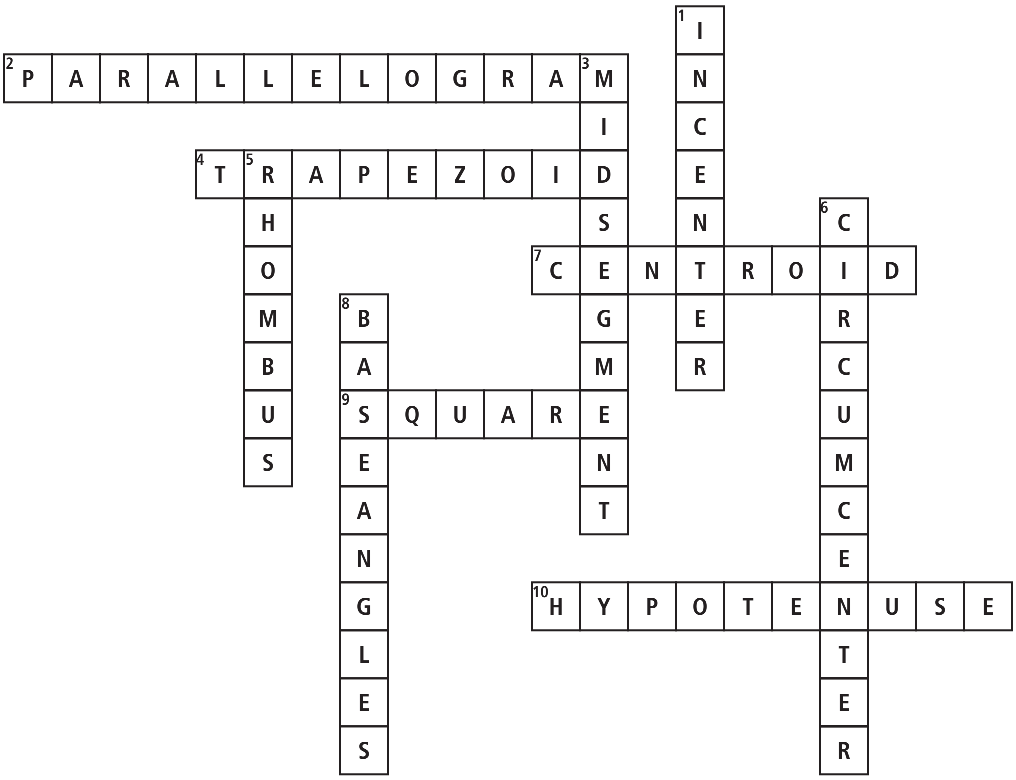
6D: Visual Vocabulary Practice/High-Use Adademic Words

1. solve **2.** deduce **3.** equivalent **4.** indirect **5.** equal
6. analysis **7.** identify **8.** convert **9.** common

6E: Visual Vocabulary Check

Consecutive angles: Angles of a polygon that share a common side.
Kite: A quadrilateral with two pairs of congruent adjacent sides and no opposite sides congruent.
Parallelogram: A quadrilateral with two pairs of parallel sides.
Rhombus: A parallelogram with four congruent sides.
Trapezoid: A quadrilateral with exactly one pair of parallel sides.

6F: Vocabulary Review Puzzle



Chapter 7

Practice 7-1

1. 1 : 278 **2.** 18 ft by 10 ft **3.** 18 ft by 16 ft **4.** 10 ft by 3 ft **5.** true **6.** false **7.** true **8.** false **9.** true **10.** true **11.** 12 **12.** 12 **13.** 33 **14.** ±8 **15.** 6 **16.** $\frac{5}{2}$ **17.** 14 : 5 **18.** 12 : 7 **19.** $\frac{8}{3}$ **20.** $\frac{7}{13}$

Guided Problem Solving 7-1

1. ratios **2.** $\frac{42}{42{,}000{,}000}$ or $\frac{1}{1{,}000{,}000}$ **3.** the denominator **4.** $\frac{x}{29{,}000} = \frac{1}{1{,}000{,}000}$ **5.** Cross-Product Property **6.** 0.029 **7.** 0.348 **8.** yes **9.** 21.912

Practice 7-2

1. $\triangle ABC \sim \triangle XYZ$, with similarity ratio 2 : 1
2. Not similar; corresponding sides are not proportional.
3. Not similar; corresponding angles are not congruent.
4. $\triangle ABC \sim \triangle KMN$, with similarity ratio 4 : 7
5. $\angle I$ **6.** $\angle O$ **7.** NO **8.** LO **9.** 3.96 ft **10.** 3.75 cm **11.** $\frac{2}{3}$ **12.** 53 **13.** $7\frac{1}{2}$ **14.** $4\frac{1}{2}$ **15.** 37

Guided Problem Solving 7-2

1. equal **2.** $\frac{6.14}{2.61}$ **3.** 2.61; 6.14 **4.** 19.3662; 19.2182 **5.** no **6.** no **7.** 2.3706; 2.3525 **8.** Since the quotients are not equal, the ratios are not equal, and the bills are not similar rectangles. **9.** 4.045

Practice 7-3

1. $\angle AXB \cong \angle RXQ$ because vertical angles are ≅. $\angle A \cong \angle R$ (Given). Therefore $\triangle AXB \sim \triangle RXQ$ by the AA ~ Postulate. **2.** Because $\frac{MP}{LW} = \frac{PX}{WA} = \frac{XM}{AL} = \frac{3}{4}$, $\triangle MPX \sim \triangle LWA$ by the SSS ~ Theorem. **3.** $\angle QMP \cong \angle AMB$ because vertical $\angle$s are ≅. Then, because $\frac{QM}{AM} = \frac{PM}{BM} = \frac{2}{1}$, $\triangle QMP \sim \triangle AMB$ by the SAS ~ Theorem. **4.** Because $AX = BX$ and $CX = RX$, $\frac{AX}{CX} = \frac{BX}{RX}$. $\angle AXB \cong \angle CXR$ because vertical angles are ≅. Therefore $\triangle AXB \sim \triangle CXR$ by the SAS ~ Theorem. **5.** $\frac{15}{2}$ **6.** $\frac{48}{7}$ **7.** $\frac{20}{3}$ **8.** 36 **9.** 33 ft

Guided Problem Solving 7-3

1. no; N/A **2.** yes; $\overline{WT}$, $\overline{RS}$ **3.** It is a trapezoid. **4.** They are congruent. **5.** They are parallel. **6.** They are congruent. **7.** $\triangle RSZ$ and $\triangle TWZ$ **8.** AA~ or Angle-Angle Similarity Postulate **9.** No; there is only one pair of congruent angles. **10.** yes; parallelogram, rhombus, rectangle, and square

Practice 7-4

1. 16 **2.** 8 **3.** $10\sqrt{2}$ **4.** $6\sqrt{5}$ **5.** h **6.** y **7.** a **8.** c **9.** $\frac{9}{2}$ **10.** $x = 6; y = 6\sqrt{3}$ **11.** $x = 4\sqrt{5}; y = \sqrt{55}$ **12.** $2\sqrt{15}$ in.

Guided Problem Solving 7-4

1. $\triangle ABC, \triangle ACD, \triangle BCD$ **2.** 1 **3.** 1; 1 **4.** 1 **5.** 1 **6.** 2; 1 **7.** 2 **8.** $\sqrt{2}$ **9.** $\sqrt{2}$ **10.** yes **11.** no (This would require the Pythagorean Theorem.)

Practice 7-5

1. BE **2.** BC **3.** JD **4.** BE **5.** $\frac{16}{3}$ **6.** 4 **7.** $x = \frac{25}{9}$; $y = 4$ **8.** $\frac{15}{4}$ **9.** $x = 6; y = 6$ **10.** 2 **11.** 10

Guided Problem Solving 7-5

1. parallel **2.** CE; BD **3.** 6; 15 **4.** 90 **5.** yes **6.** yes **7.** The sides would not be parallel. **8.** They are similar triangles.

7A: Graphic Organizer

1. Similarity **2.** Answers may vary. Sample: ratios and proportions; similar polygons; proving triangles similar; and similarity in right triangles **3.** Check students' work.

7B: Reading Comprehension

1. $\triangle DHB \sim \triangle ACB$ **2.** AA similarity postulate The triangles have two similar angles. **3.** 1 : 2 **4.** 1 : 2 **5.** $\frac{DB}{AB} = \frac{HB}{CB}$ **6.** 250 ft **7.** a

7C: Reading/Writing Math Symbols

1. no **2.** no **3.** yes **4.** no **5.** yes **6.** no **7.** yes, AAS **8.** yes, Hypotenuse-Leg Theorem **9.** yes, SAS or ASA or AAS **10.** not possible

7D: Visual Vocabulary Practice

1. Angle-Angle Similarity Postulate **2.** golden ratio **3.** Side-Side-Side Similarity Theorem **4.** geometric mean **5.** scale **6.** Cross-Product Property **7.** golden rectangle **8.** simplest radical form **9.** Side-Angle-Side Similarity Theorem

7E: Vocabulary Check

Similarity ratio: The ratio of lengths of corresponding sides of similar polygons.
Cross-Product Property: The product of the extremes of a proportion is equal to the product of the means.
Ratio: A comparison of two quantities by division.
Golden rectangle: A rectangle that can be divided into a square and a rectangle that is similar to the original rectangle.
Scale: The ratio of any length in a scale drawing to the corresponding actual length.

7F: Vocabulary Review

1. L **2.** E **3.** I **4.** A **5.** J **6.** K **7.** C **8.** D **9.** G **10.** O **11.** N **12.** M **13.** H **14.** B **15.** F

Chapter 8

Practice 8-1

1. $\sqrt{51}$ **2.** $2\sqrt{65}$ **3.** $2\sqrt{21}$ **4.** $18\sqrt{2}$ **5.** 46 in. **6.** 78 ft **7.** 279 cm **8.** 19 m **9.** acute **10.** obtuse **11.** right

Guided Problem Solving 8-1

1. the sum of the lengths of the sides **2.** Pythagorean Theorem **3.** 7 cm **4.** 4 cm × 3 cm **5.** $c^2 = 4^2 + 3^2$ **6.** 5 **7.** 12 cm **8.** perimeter of rectangle = 14 cm; yes **9.** Answers will vary; example: Draw a 4 cm × 3 cm grid, copy the given figure, measure the lengths with a ruler, add them together. **10.** 20 cm

Practice 8-2

1. $x = 2; y = \sqrt{3}$ **2.** $8\sqrt{2}$ **3.** $14\sqrt{2}$ **4.** 2 **5.** $x = 15; y = 15\sqrt{3}$ **6.** $3\sqrt{2}$ **7.** 42 cm **8.** 10.4 ft, 12 ft **9.** $a = 4; b = 3$ **10.** $p = 4\sqrt{3}; q = 4\sqrt{3}; r = 8; s = 4\sqrt{6}$

Guided Problem Solving 8-2

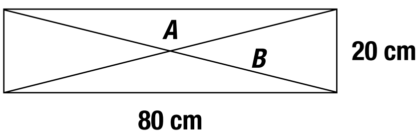
1. 30°-60°-90° triangle **2.** l **3.** h **4.** $\sqrt{3}$ **5.** $\frac{24}{\sqrt{3}}$ or $8\sqrt{3}$ **6.** 2 **7.** $\frac{48}{\sqrt{3}}$ or $16\sqrt{3}$ **8.** 28 ft **9.** 0.28 min **10.** yes **11.** 34 ft

Practice 8-3

1. $\tan E = \frac{3}{4}; \tan F = \frac{4}{3}$ **2.** $\tan E = \frac{2}{5}; \tan F = \frac{5}{2}$ **3.** 12.4 **4.** 31.0° **5.** 7.1 **6.** 6.4 **7.** 26.6 **8.** 71.6 **9.** 39 **10.** 72 **11.** 39 **12.** 54

Guided Problem Solving 8-3

1.



2. 180 **3.** $m\angle A = 2m\angle X$ **4.** 90 **5.** base: 40 cm, height: 10 cm **6.** 4 **7.** 4 **8.** 76 **9.** 152 **10.** 28 **11.** yes **12.** 46

Practice 8-4

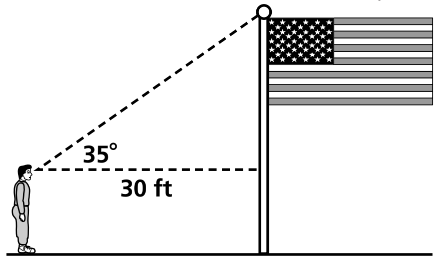
1. $\sin P = \frac{2\sqrt{10}}{7}; \cos P = \frac{3}{7}$ **2.** $\sin P = \frac{4}{5}; \cos P = \frac{3}{5}$ **3.** $\sin P = \frac{\sqrt{11}}{6}; \cos P = \frac{5}{6}$ **4.** $\sin P = \frac{15}{17}; \cos P = \frac{8}{17}$ **5.** 64 **6.** 11.0 **7.** 7.0 **8.** 7.8 **9.** 53 **10.** 6.6 **11.** 11.0 **12.** 11.5

Guided Problem Solving 8-4

1. The sides are parallel. **2.** sine **3.** $\sin 30° = \frac{w}{6}$ **4.** 3.0 **5.** yes **6.** cosine **7.** $\cos x° = \frac{3}{4}$ **8.** 41 **9.** Answers may vary. Sample: $\cos 60° \stackrel{?}{=} \frac{3}{6}; \sin 49° \stackrel{?}{=} \frac{3}{4}$ **10.** 5.2, 2.6

Practice 8-5

1a. angle of depression from the plane to the person
1b. angle of elevation from the person to the plane
1c. angle of depression from the person to the sailboat
1d. angle of elevation from the sailboat to the person
2. 116.6 ft **3.** 84.8 ft **4.** 46.7 ft **5.** 31.2 yd
6a.



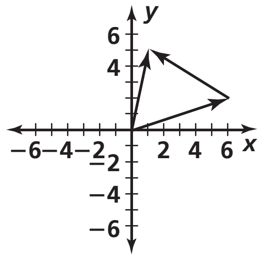
6b. 26 ft

Guided Problem Solving 8-5

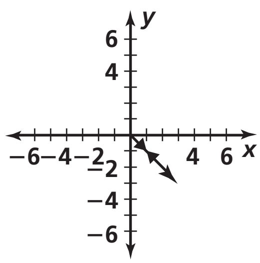
1. $\angle e = 1, \angle d = \angle 4$ **2.** congruent **3.** $m\angle e = m\angle d$ **4.** $7x - 5 = 4(x + 7)$ **5.** 11 **6.** 72 **7.** 72 **8.** yes **9.** 44, 44

Practice 8-6

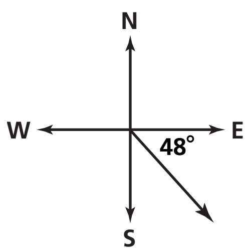
1. $\langle 46.0, 46.0 \rangle$ **2.** $\langle 89.2, -80.3 \rangle$ **3.** 38.6 mi/h; 31.2º north of east **4.** 134.5 m; 42.0º south of west **5.** 55º north of east **6.** 33º west of north **7a.** $\langle 1, 5 \rangle$
7b.



8a. $\langle 1, -1 \rangle$ **8b.**



9. Sample:



Guided Problem Solving 8-6

1. Check students' work. **2.** $\frac{x}{100}$; $\frac{y}{100}$
3. 100 cos 30°; 100 sin 30° **4.** 86.6; 50 **5.** $\langle 86.6, 50 \rangle$
6. $\langle 86.6, -50 \rangle$ **7.** $\langle 173.2, 0 \rangle$ **8.** 173; due east **9.** yes
10. 100; due east

8A: Graphic Organizer

1. Right Triangles and Trigonometry **2.** Answers may vary. Sample: the Pythagorean Theorem; special right triangles; the tangent ratio; sine and cosine ratios; angles of elevation and depression; vectors **3.** Check students' work.

8B: Reading Comprehension

1. A **2.** J **3.** B **4.** J **5.** B **6.** B **7.** b

8C: Reading/Writing Math Symbols

1. F **2.** G **3.** D **4.** A **5.** C **6.** B **7.** H **8.** E
9. $\sin^{-1} A = \frac{5}{12}$ **10.** $\triangle ABC \sim \triangle XYZ$ **11.** $m\angle A \approx 52°$
12. $\tan Z = \frac{7}{24}$

8D: Visual Vocabulary Practice

1. 30°-60°-90° triangle **2.** inverse of tangent **3.** congruent sides **4.** tangent **5.** Pythagorean Theorem **6.** hypotenuse **7.** 45°-45°-90° triangle **8.** Pythagorean triple **9.** obtuse triangle

8E: Vocabulary Check

Obtuse triangle: A triangle with one angle whose measure is between 90 and 180.
Isosceles triangle: A triangle that has at least two congruent sides.
Hypotenuse: The side opposite the right angle in a right triangle.
Right triangle: A triangle that contains one right angle.
Pythagorean triple: A set of three nonzero whole numbers a, b, and c that satisfy the equation $a^2 + b^2 = c^2$.

8F: Vocabulary Review Puzzle

J	A	T	N	A	T	L	U	S	E	R	Y	G	D	H	P	G
H	S	N	L	P	U	R	F	S	I	M	E	E	M	Q	O	I
I	Y	H	G	E	A	K	U	R	R	E	L	O	F	L	S	B
T	N	T	N	L	O	N	Q	G	M	A	M	M	D	M	T	P
B	K	V	I	X	E	X	X	E	C	S	Q	E	A	L	V	Y
D	W	M	E	T	D	O	T	S	B	U	N	T	P	X	L	T
Z	I	T	O	R	N	I	F	A	D	R	Y	R	L	P	A	H
S	R	P	S	Z	S	E	O	E	E	E	J	I	I	M	T	A
H	Y	M	C	O	N	E	D	C	L	M	Z	C	R	D	E	G
H	E	D	P	I	L	U	T	I	N	E	B	M	J	C	V	O
R	F	P	S	U	T	A	B	A	F	N	V	E	C	T	O	R
Y	O	O	R	I	N	I	V	G	N	T	M	A	D	D	K	E
B	C	O	N	G	R	U	E	N	T	G	H	N	T	B	R	A
A	N	G	L	E	O	F	D	E	P	R	E	S	S	I	O	N
R	A	E	U	A	R	A	X	L	L	Y	T	N	E	J	O	B
M	C	O	O	V	Z	L	P	R	O	P	O	R	T	I	O	N

Chapter 9

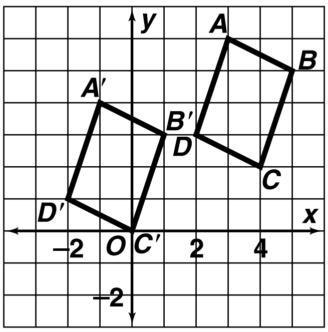
Practice 9-1

1. No; the triangles are not the same size. **2.** Yes; the ovals are the same shape and size. **3a.** $\angle C'$ and $\angle F'$ **3b.** $\overline{CD}$ and $\overline{C'D'}$, $\overline{DE}$ and $\overline{D'E'}$, $\overline{EF}$ and $\overline{E'F'}$, $\overline{CF}$ and $\overline{C'F'}$ **4.** $(x, y) \rightarrow (x - 2, y - 4)$ **5.** $(x, y) \rightarrow (x + 4, y - 2)$ **6.** $(x, y) \rightarrow (x + 2, y + 2)$ **7.** $W'(-2, 2), X'(-1, 4), Y'(3, 3), Z'(2, 1)$ **8.** $J'(-5, 0), K'(-3, 4), L'(-3, -2)$ **9.** $(x, y) \rightarrow (x + 13, y - 13)$ **10.** $(x, y) \rightarrow (x, y)$ $(x + 3, y + 3)$ **11a.** $P'(-3, -1)$ **11b.** $P'(0, 8)$, $N'(-5, 2), Q'(2, 3)$

Guided Problem Solving 9-1

1. the four vertices of a preimage and one of the vertices of the image **2.** Graph the image and preimage. **3.** $C(4, 2)$ and $C'(0, 0)$ **4.** $x = 4, y = 2, x + a = 0, y + b = 0$ **5.** $a = -4$; $b = -2$; $(x, y) \rightarrow (x - 4, y - 2)$ **6.** $A'(-1, 4), B'(1, 3), D'(-2, 1)$

7.

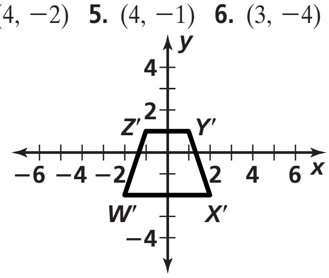


8. yes **9.** $(x, y) \rightarrow (x + 1, y - 3)$, $A'(4, 3)$, $B'(6, 2)$, $D'(3, 0)$

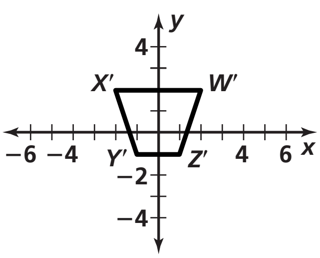
Practice 9-2

1. $(-3, -2)$ **2.** $(-2, -3)$ **3.** $(-1, -4)$ **4.** $(4, -2)$ **5.** $(4, -1)$ **6.** $(3, -4)$

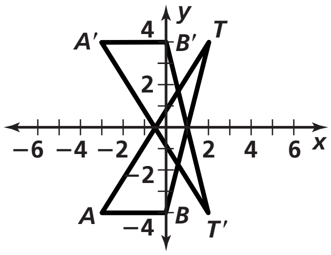
7a.



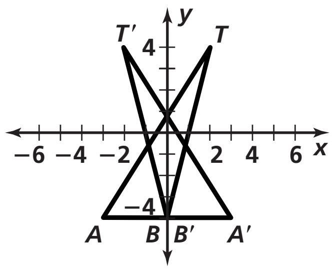
7b.



8.



9.

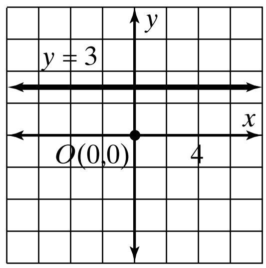


10. $(-6, 4)$

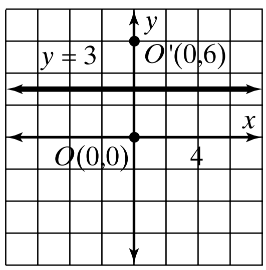
Guided Problem Solving 9-2

1. a point at the origin, and two reflection lines
2. A reflection is an isometry in which a figure and its image have opposite orientations. **3.** the image after two successive reflections

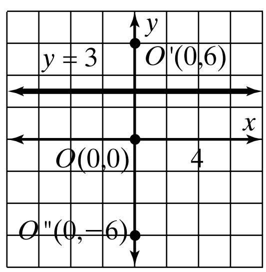
4.



5.



6.

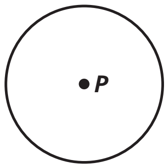


7. $(0, -6)$ **8.** Yes. The x-coordinate remains 0 throughout.
9. $O'(0, 0)$ and $O'(0, 6)$

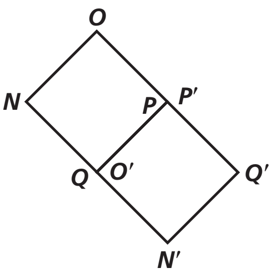
Practice 9-3

1. I **2.** I **3.** I **4.** $\overline{GH}$ **5.** G **6.** $\overline{ST}$

7.



8.



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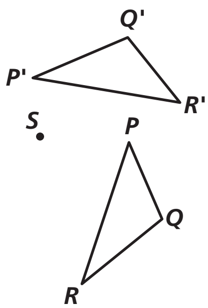
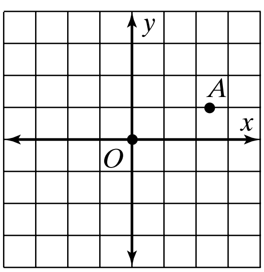


Guided Problem Solving 9-3

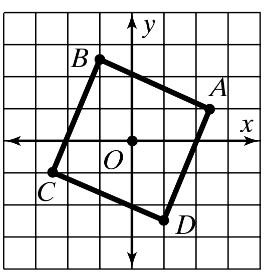
1. The coordinates of point A, and three rotation transformations. It is assumed that the rotations are counterclockwise. **2.** Parallelogram, rhombus, square

3.



slope of $\overline{A} = \frac{2}{5}$

4.

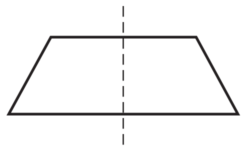


5. slope of $\overline{OB} = -\frac{5}{2}$; slope of $\overline{OC} = \frac{2}{5}$; slope of $\overline{OD} = -\frac{5}{2}$; the slopes of perpendicular line segments are negative reciprocals. **6.** square **7.** yes **8.** $B(2, 7)$, $C(7, -2)$, $D(-2, -7)$

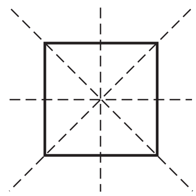
Practice 9-4

1. The helmet has reflectional symmetry. **2.** The teapot has reflectional symmetry. **3.** The hat has both rotational and reflectional symmetry.

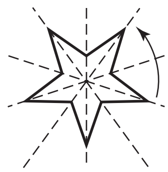
4.



5.

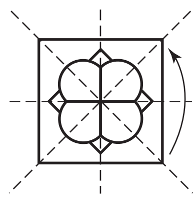


6.



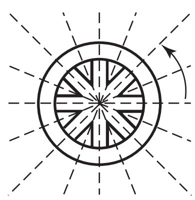
line symmetry and 72° rotational symmetry

7.



line symmetry and 90° rotational symmetry

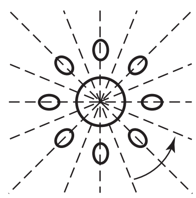
8.



line symmetry and 45° rotational symmetry

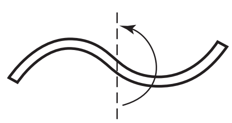
9. BOOK line symmetry

10.



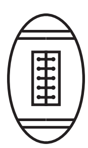
line symmetry and 45° rotational symmetry

11.



180° rotational symmetry

12.



13. COOK

14. H
O
A
X

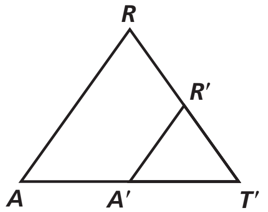
Guided Problem Solving 9-4

1. the coordinates of one vertex of a figure that is symmetric about the *y*-axis **2.** Line symmetry is the type of symmetry for which there is a reflection that maps a figure onto itself. **3.** the coordinates of another vertex of the figure **4.** images (and preimages) **5.** reflection across the *y*-axis **6.** $(-3, 4)$ **7.** Yes **8.** $(-6, 7)$

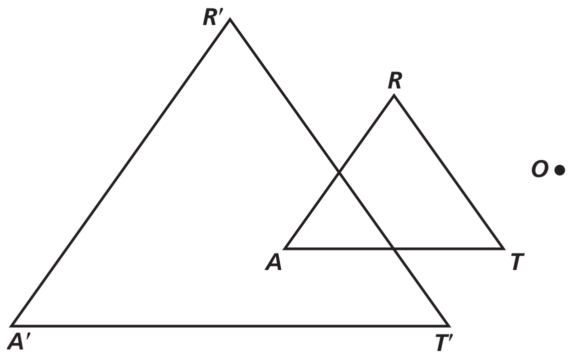
Practice 9-5

1. $\frac{5}{3}$ **2.** $\frac{1}{2}$ **3.** 2 **4.** yes **5.** no **6.** no

7.



8.



9. $P'(-12, -12), Q'(-6, 0), R'(0, -6)$ **10.** $P'(-2, 1), Q'(-1, 0), R'(0, 1)$

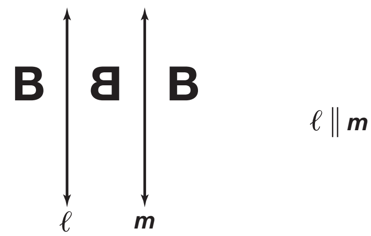
Guided Problem Solving 9-5

1. A description of a square projected onto a screen by an overhead projector, including the square's area and the scale factor in relation to the square on the transparency. **2.** The scale factor of a dilation is the number that describes the size change from an original figure to its image. **3.** the area of the square on the transparency **4.** smaller; The scale factor $16 > 1$, so the dilation is an enlargement. **5.** $\frac{1}{16}$; $\frac{1}{16}$ **6.** $\frac{1}{256}$; Being $\frac{1}{16}$ as high and $\frac{1}{16}$ as wide, the square on the tranparency has $\frac{1}{16} \times \frac{1}{16} = \frac{1}{256}$ times the area. **7.** $\frac{3}{256}$ ft^2 **8.** The shape does not matter. Regardless of the shape, the figure is being enlarged by a factor of 16 in two directions, so that the screen image has $16 \times 16 = 256$ as large an area as the figure on the transparency. **9.** 2520 ft^2

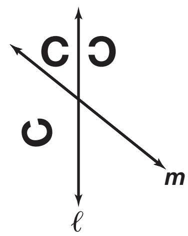
Practice 9-6

1. I. D II. C III. B IV. A

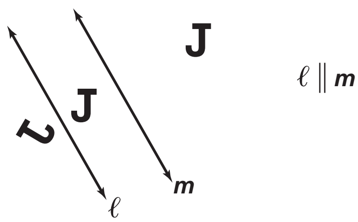
2.



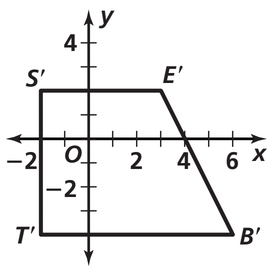
3.



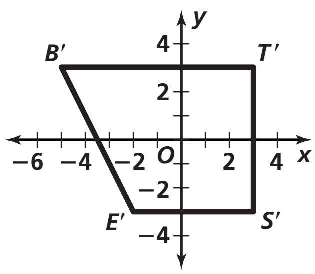
4.



5.



6.



7. reflection **8.** rotation **9.** glide reflection **10.** translation

Guided Problem Solving 9-6

1. assorted triangles and a set of coordinate axes **2.** a transformation **3.** the transformation that maps one