

HW #1

- Read Ch 1
- **Ch 1 Review Problems pp. 36-38 #1-30 - due FRIDAY 11/6**
- Start working on Geometry badge on Khan Academy; make sure you've added me as a coach using code listed on brewermath.com!

Quiz #1

- Vocab
- Fill in the blank proofs

HW #2

- Read Ch 2 & Ch 3
- **Ch 2 Review Problems pp. 71-74**
- **Ch 3 Review Problems pp. 124-128**
- Khan Academy exercises:
 - > "Introduction to Euclidean Geometry"
 - > "Angles and intersecting lines"

2.2 – Definitions

When we define a word in mathematics, the word and its definition are understood to have the exact same meaning.

For example, if we define a “triangle” as “a polygon that has three sides,” then both of the following statements are true:

“If a figure is a triangle, then it is a polygon that has three sides.”

“If a figure is a polygon that has three sides, then it is a triangle.”

For statement “If a, then b.” $a \rightarrow b$

Its converse is “If b, then a.” $b \rightarrow a$

In general, if a statement is true, then its converse is not necessarily true.

If a statement is a definition, then its converse is always true, and the two statements can be combined into an “if an only if” statement.

iff $a \leftrightarrow b$

“A figure is a triangle if and only if it is a polygon that has three sides.”

True statement: If you are an astronaut, you are not more than six feet tall. *IF a, then b*

Hypothesis: *you are an astronaut*

Conclusion: *you are not more than six feet tall*

6. Write the converse of the statement. *IF b, then a.*

If you are not six feet tall, then you are an astronaut.

7. Is the converse true?

no

8. Does it have the same meaning as the original statement? *no*

If you are more than six feet tall, then you are not an astronaut.
contrapositive *IF not b, then not a*

Definition: You have arachibutyrophobia iff you have the fear of peanutbutter sticking to the roof of your mouth.

If the definition were represented in symbols as $a \leftrightarrow b$ and "a" represents "arachibutyrophobia," what does

14. \leftrightarrow represent?

if and only if

15. "b" represent?

fear of peanut butter ... mouth

In words, " $a \rightarrow b$ " for this definition is "If you have arachibutyrophobia, then you are afraid of peanut butter sticking to the roof of your mouth."

16. Write in words, " $b \rightarrow a$ " for this definition.

If you are afraid ... , then you have ... phobia .

17. Is this sentence necessarily true?

(1) If it is your birthday, then you get some presents.

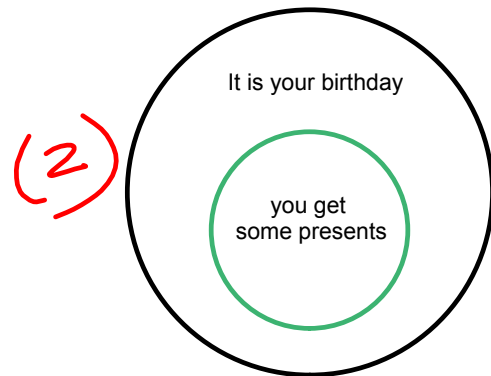
(2) Only if it is your birthday, do you get some presents.

If A, then B

24. Is the first sentence true for you?

25. Is the second sentence true for you?

27. Which sentence does this Euler Diagram illustrate?



2.3 – Direct Proof

A syllogism is an argument of the form

$a \rightarrow b$

$b \rightarrow c$

Therefore, $a \rightarrow c$.

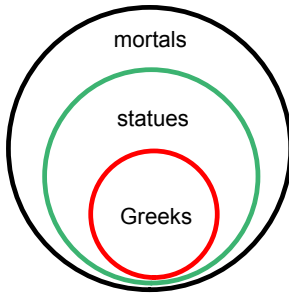
A syllogism is an example of a direct proof.

The statements $a \rightarrow b$ and $b \rightarrow c$ are called the premises of the argument.

$a \rightarrow c$ is called the conclusion of the argument, and is often considered to be a theorem.

A theorem is a statement that is proved by reasoning deductively from already accepted statements.

Syllogisms were discussed by the Greek philosopher Aristotle in the fourth century BC. Write the syllogism illustrated by the following Euler diagram.



If someone is Greek then they are a statue.

If someone is a statue then they are a mortal.

Therefore if someone is Greek, then they are a mortal.

6. If the premises of a syllogism are true, does it follow that its conclusion must be true?

yes

7. If the premises of a syllogism are false, does it follow that its conclusion must be false?

no

“Admit one ridiculous premise and the rest follows.” – Aristotle

- (1) If you live at the South Pole, you live in the Antarctic. $a \rightarrow b$
- (2) If you live in the Antarctic, you live where it is cold. $b \rightarrow c$
- (3) If you live where it is cold, you see a lot of penguins. $c \rightarrow d$
- c Therefore, if you live at the South Pole, you see a lot of penguins. $\therefore a \rightarrow d$

What part of the second premise matches

8. the conclusion of the first premise? you live in the antarctic -

9. the hypothesis of the third premise? you live where it's cold

10. Starting with $a \rightarrow b$ to represent the first premise, represent the entire argument in symbols.

11. Which premise is ridiculous (false)? (3)

12. What does the fact that one premise is false indicate about the conclusion of the argument?

may be true or false

If Captain Spaulding is in the jungle, there are too many cheetahs.

If there are too many cheetahs, Captain Spaulding can't play cards.

13. What conclusion follows from these premises?

Therefore, if Captain Spaulding is in the jungle, then he can't play cards.

14. If the two premises are true, does it follow that the conclusion must be true?

yes

Write in the missing statements for the following proofs.

20. *Theorem*: If two hungry vultures took an airplane, they would be told that there is a limit of two carrion per passenger.

Proof:

If two hungry vultures took an airplane, they would want to take along some food.

> If ^{two} vultures want to take along some food, then they will try to carry on six dead raccoons

If they tried to carry on six dead raccoons, the flight attendant would object.

> If the flight attendant objects, they will tell the vultures there is a limit of two carrion per passenger

$a \rightarrow b$

$b \rightarrow c$

$c \rightarrow d$

$d \rightarrow e$

Therefore, if two vultures ...
 ... carrion per passenger $a \rightarrow e$

21. *Theorem:* If a group of chess players checked into a hotel, the manager would say “I can’t stand chess nuts boasting in an open foyer.”

Proof:

> If a group of chess players checks into a hotel then they will stand in the lobby bragging about their tournament victories

If they stood in the lobby bragging about their tournament victories, the manager would ask them to leave.

> If the manager asks them to leave, then they will ask why

If they asked why, the manager would say “I can’t stand chess nuts boasting in an open foyer.”

Therefore if a group..., then .. open foyer

2.4 – Indirect Proof

In an indirect proof, an assumption is made at the beginning that leads to a contradiction. The contradiction indicates that the assumption is false and the desired conclusion is true.

Direct versus Indirect proof of the theorem “If a, then d.”

Direct Proof:

If a, then b.

If b, then c.

If c, then d.

Therefore, if a, then d.

Indirect Proof:

Suppose not d is true.

If not d, then e.

If e, then f,

And so on until we come to a contradiction.

Therefore, not d is false; so d is true.

List the assumption with which an indirect proof of each of the following statements would begin.

Example: If a tailor wants to make a coat last, he makes the pants first.

Answer: Suppose that he does not make the pants first.

4. If a teacher is cross-eyed, he has no control over his pupils.

Suppose that he does have control over his pupils...

5. If a proof is indirect, then it leads to a contradiction.

Suppose it does not lead to a contradiction

In a book written in the 13th century on the shape of the earth, the author reasoned:

“If the earth were flat, the stars would rise at the same time for everyone, which they do not.”

11. What is the author trying to prove?

The earth is not flat.

12. With what assumption does the author begin?

The earth is flat

13. What is the contradiction?

The stars rise at the same time for everyone

14. What does the contradiction prove about the author’s beginning assumption?

It's false & therefore, the earth is not flat

Write the missing statements in the indirect proof:

16. The ammonia molecule consists of three hydrogen atoms bonded to a nitrogen atom as shown in this figure.

The fact that chemists have found that each bond angle is 107° can be used to prove the following theorem.

Theorem: The atoms of an ammonia molecule are not coplanar.

Proof:

> *Suppose the atoms are coplanar.*

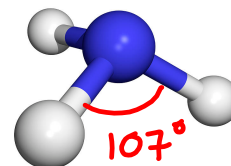
If the atoms are coplanar, then the sum of the three bond angles is 360° .

If the sum of the three bond angles is 360° , then each angle is 120° .

> *This is a contradiction to fact that the bond angles are 107°*

Therefore, our assumption is false and

> *the atoms are NOT coplanar.*



19. A particular puzzle involves separating a set of twelve weights into two sets so that one set will exactly balance the other on a scale with two pans.



Consider this argument:

If a puzzle of this type has a solution, then the weights of the two sets will be equal.

If the weights of the two sets are equal, then each set will weigh half the total weight.

What conclusion follows from these two premises?

If the puzzle has a solution, then each set of weights will weigh half the total weight.

20. Write in the missing statements in the indirect proof about this puzzle:

Theorem: If the sum of all of the weights is odd, then there is no solution.

Proof:

> Suppose there is a solution.

If there is a solution, let the weights in one set add up to S .

If the weights in each set add up to S , then the weights in both sets add up to $S+S=2S$, an even number.

> This is a contradiction to sum of the weights being odd

Therefore, our assumption is false and

> there is not a solution.