
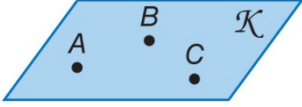
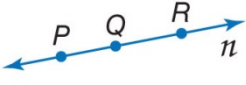
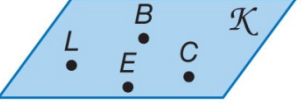
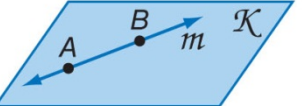
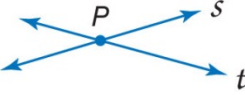
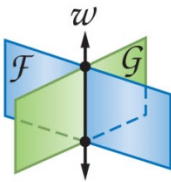


# **GEOMETRY**

## **CHAPTER 2**

### **Reasoning and Proof**

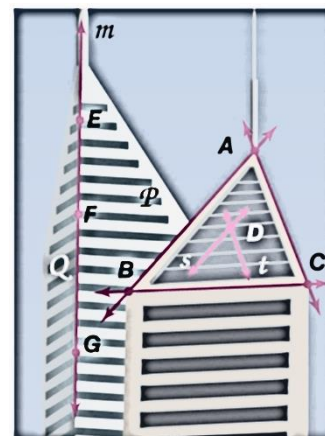
| Postulates Points, Lines, and Planes   |   |  |
|--|---|--|
| Words  |   | Example  |
| 2.1 Through any two points, there is exactly one line.   |  | Line $n$ is the only line through points $P$ and $R$ .   |
| 2.2 Through any three noncollinear points, there is exactly one plane.                             |  | Plane $\mathcal{K}$ is the only plane through noncollinear points $A$ , $B$ , and $C$ .  |
| 2.3 A line contains at least two points.   |  | Line $n$ contains points $P$ , $Q$ , and $R$ .   |
| 2.4 A plane contains at least three noncollinear points.   |  | Plane $\mathcal{K}$ contains noncollinear points $L$ , $B$ , $C$ , and $E$ .   |
| 2.5 If two points lie in a plane, then the entire line containing those points lies in that plane. |  | Points $A$ and $B$ lie in plane $\mathcal{K}$ , and line $m$ contains points $A$ and $B$ , so line $m$ is in plane $\mathcal{K}$ . |

| KeyConcept Intersections of Lines and Planes                              |   |  |
|---|---|--|
| Words   |   | Example  |
| 2.6 If two lines intersect, then their intersection is exactly one point. |  | Lines $s$ and $t$ intersect at point $P$ .                     |
| 2.7 If two planes intersect, then their intersection is a line.           |  | Planes $\mathcal{F}$ and $\mathcal{G}$ intersect in line $w$ . |

**Example 1:** Explain how the picture illustrates that the statement is true. Then state the postulate that can be used to show the statement is true.

a) Points  $F$  and  $G$  lie in plane  $Q$  and on line  $m$ . Line  $m$  lies entirely in plane  $Q$ .

b) Points  $A$  and  $C$  determine a line.



You can use postulates to explain your reasoning when analyzing statements.

**Example 2:** Determine whether the following statement is *always*, *sometimes*, or *never* true. Explain.

a) If plane  $T$  contains  $\overline{EF}$  and  $\overline{EF}$  contains point  $G$ , then plane  $T$  contains point  $G$ .

b)  $\overline{GH}$  contains three noncollinear points.

To prove a conjecture, you use deductive reasoning to move from a hypothesis to the conclusion of the conjecture you are trying to prove. This is done by writing a **proof**, which is a logical argument in which each statement you make is supported by a statement that is accepted as true.

**Sample Proof:** Basketball

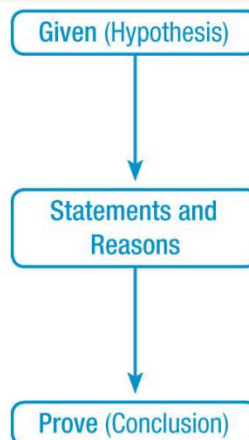
Given: Student A in this class is a basketball player.

Prove: This high school has a basketball team.

| Statements | Reasons |
|------------|---------|
| 1.         | 1.      |
| 2.         | 2.      |
| 3.         | 3.      |
| 4.         | 4.      |
| 5.         | 5.      |
| 6.         | 6.      |

### KeyConcept The Proof Process

- Step 1** List the given information and, if possible, draw a diagram to illustrate this information.
- Step 2** State the theorem or conjecture to be proven.
- Step 3** Create a **deductive argument** by forming a logical chain of statements linking the given to what you are trying to prove.
- Step 4** Justify each statement with a reason. Reasons include definitions, algebraic properties, postulates, and theorems.
- Step 5** State what it is that you have proven.



One method of proving statements and conjectures, a **paragraph proof**, involves writing a paragraph to explain why a conjecture for a given situation is true. Paragraph proofs are also called **informal proofs**, although the term *informal* is not meant to imply that this form of proof is any less valid than any other type of proof.

**Example 3:** Given  $\overline{AC}$  intersects  $\overline{CD}$ , write a paragraph proof to show that  $A$ ,  $C$ , and  $D$  determine a plane.

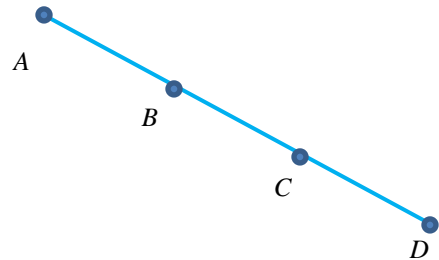
Once a statement or conjecture has been proven, it is called a **theorem**, and it can be used as a reason to justify statements in other proofs.

### Theorem 2.1 Midpoint Theorem

If  $M$  is the midpoint of  $\overline{AB}$ , then  $\overline{AM} \cong \overline{MB}$ .



**Example 4:** Point  $B$  is the midpoint of  $\overline{AC}$ . Point  $C$  is the midpoint of  $\overline{BD}$ . Prove that  $\overline{AB} \cong \overline{CD}$ .



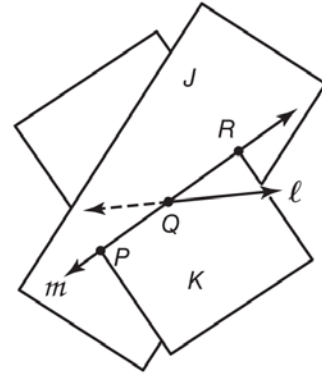
| Statements                                | Reasons             |
|---|---------------------|
| 1. $B$ is the midpoint of $\overline{AC}$ | 1.                  |
| 2.  | 2. Midpoint Theorem |
| 3.  | 3. Given            |
| 4. $\overline{BC} \cong \overline{CD}$    | 4.                  |
| 5.  | 5.                  |



For numbers 1 and 2, explain how the figure illustrates that each statement is true. Then state the postulate that can be used to show each statement is true.

1. The planes  $J$  and  $K$  intersect at line  $m$ .

2. The lines  $l$  and  $m$  intersect at point  $Q$ .



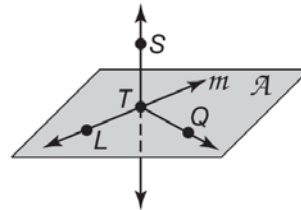
For numbers 3 and 4, determine whether the following statements are *always*, *sometimes*, or *never* true. Explain.

3. The intersection of two planes contains at least two points.

4. If three planes have a point in common, then they have a whole line in common.

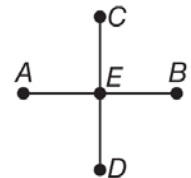
For numbers 5 and 6, state the postulate that can be used to show that each statement is true. In the figure, line  $m$  and  $\overline{TQ}$  lie in plane  $\mathcal{A}$ .

5. Points  $L$ ,  $T$  and line  $m$  lie in the same plane.



6. Line  $m$  and  $\overline{ST}$  intersect at  $T$ .

7. In the figure,  $E$  is the midpoint of  $\overline{AB}$  and  $\overline{CD}$ , and  $AB = CD$ . Write a paragraph proof to prove that  $\overline{AE} \cong \overline{ED}$ .

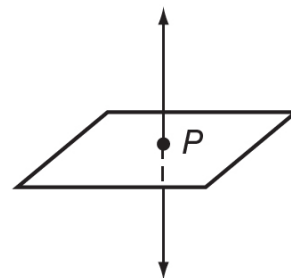


8. Noel and Kirk are building a new roof. They wanted a roof with two sloping planes that meet along a curved arch. Is this possible?

9. An airline company wants to provide service to San Francisco, Los Angeles, Chicago, Dallas, Washington D.C., and New York City. The company's CEO draws lines between each pair of cities in the list on a map. No three of the cities are collinear. How many lines did the CEO draw?

10. A sailor spots a whale through her binoculars. She wonders how far away the whale is, but the whale does not show up on the radar system. She sees another boat in the distance and radios the captain asking him to spot the whale and record its direction. Explain how this added information could enable the sailor to pinpoint the location of the whale. Under what circumstance would this idea fail?

11. Carson claims that a line can intersect a plane at only one point and draws this picture to show his reasoning. Zoe thinks it is possible for a line to intersect a plane at more than one point. Who is correct? Explain.



12. A small company has 16 employees. The owner of the company became concerned that the employees did not know each other very well. He decided to make a picture of the friendships in the company. He placed 16 points on a sheet of paper in such a way that no 3 were collinear. Each point represented a different employee. He then asked each employee who their friends were and connected two points with a line segment if they represented friends.

a) What is the maximum number of line segments that can be drawn between pairs among the 16 points?

b) When the owner finished the picture, he found that his company was split into two groups, one with 10 people and the other with 6. The people within a group were all friends, but nobody from one group was a friend of anybody from the other group. How many line segments were there?

## Geometry

### Section 2.7 Notes: Proving Segment Relationships

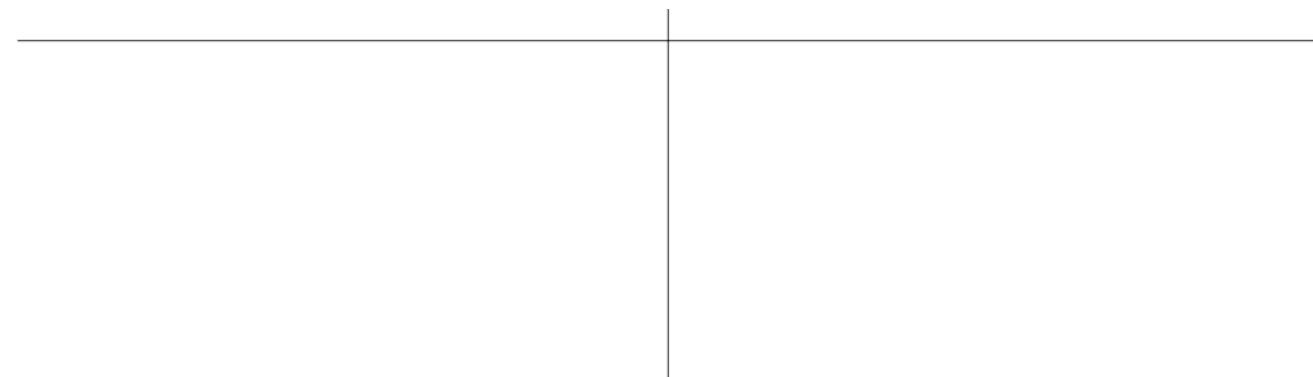
Let's refresh our memories about properties of real numbers before we start talking Geometry:

#### KeyConcept Properties of Real Numbers

The following properties are true for any real numbers  $a$ ,  $b$ , and  $c$ .

|                                     |   |
|-------------------------------------|---|
| Addition Property of Equality       | If $a = b$ , then $a + c = b + c$ .   |
| Subtraction Property of Equality    | If $a = b$ , then $a - c = b - c$ .   |
| Multiplication Property of Equality | If $a = b$ , then $a \cdot c = b \cdot c$ .                                 |
| Division Property of Equality       | If $a = b$ and $c \neq 0$ , then, $\frac{a}{c} = \frac{b}{c}$ .             |
| Reflexive Property of Equality      | $a = a$   |
| Symmetric Property of Equality      | If $a = b$ , then $b = a$ .   |
| Transitive Property of Equality     | If $a = b$ and $b = c$ , then $a = c$ .                                     |
| Substitution Property of Equality   | If $a = b$ , then $a$ may be replaced by $b$ in any equation or expression. |
| Distributive Property               | $a(b + c) = ab + ac$  |

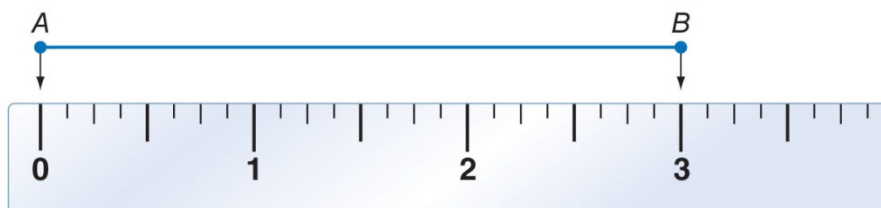
**Example 1:** Use the above properties to justify each step when solving the following equation:  $2(5 - 3a) - 4(a + 7) = 92$ .



#### Postulate 2.8 Ruler Postulate

**Words** The points on any line or line segment can be put into one-to-one correspondence with real numbers.

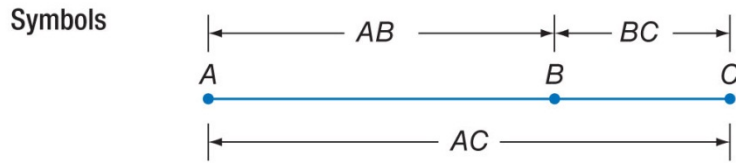
**Symbols** Given any two points  $A$  and  $B$  on a line, if  $A$  corresponds to zero, then  $B$  corresponds to a positive real number.



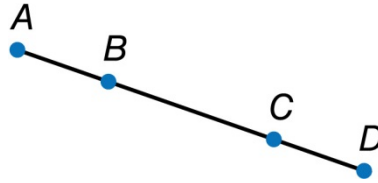


## Postulate 2.9 Segment Addition Postulate

**Words** If  $A$ ,  $B$ , and  $C$  are collinear, then point  $B$  is between  $A$  and  $C$  if and only if  $AB + BC = AC$ .



**Example 2:** Prove that if  $\overline{AB} \cong \overline{CD}$ , then  $\overline{AC} \cong \overline{BD}$ .



| Statements                             | Reasons |
|--|---------|
| 1. $\overline{AB} \cong \overline{CD}$ | 1.      |
| 2. $AB = CD$                           | 2.      |
| 3. $AB + BC = AC$                      | 3.      |
| 4. $CD + BC = AC$                      | 4.      |
| 5. $CD + BC = BD$                      | 5.      |
| 6. $AC = BD$                           | 6.      |
| 7.                                     | 7.      |

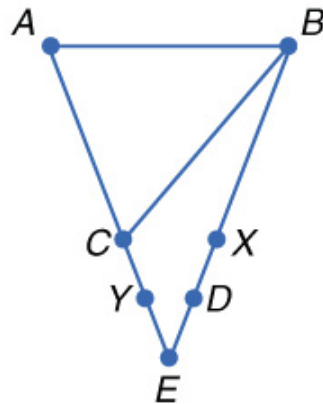
**Example 3:** Prove the following

Given:  $AC = AB$

$AB = BX$

$CY = XD$

Prove:  $AY = BD$



| Statements        | Reasons                |
|-------------------|------------------------|
| 1.                | 1.                     |
| 2. $AB = BX$      | 2.                     |
| 3.                | 3. Transitive Property |
| 4. $CY = XD$      | 4.                     |
| 5. $AC + CY = AY$ | 5.                     |
| 6. $BX + CY = AY$ | 6.                     |
| 7. $BX + XD = AY$ | 7.                     |
| 8. $BX + XD = BD$ | 8.                     |
| 9.                | 9.                     |

## Theorem 2.2 Properties of Segment Congruence

Reflexive Property of Congruence

$$\overline{AB} \cong \overline{AB}$$

Symmetric Property of Congruence

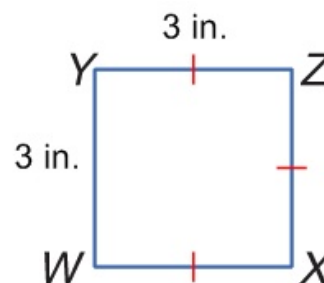
$$\text{If } \overline{AB} \cong \overline{CD}, \text{ then } \overline{CD} \cong \overline{AB}.$$

Transitive Property of Congruence

$$\text{If } \overline{AB} \cong \overline{CD} \text{ and } \overline{CD} \cong \overline{EF}, \text{ then } \overline{AB} \cong \overline{EF}.$$

**Example 4:** Jamie is designing a badge for her club. The length of the top edge of the badge is equal to the length of the left edge of the badge. The top edge of the badge is congruent to the right edge of the badge, and the right edge of the badge is congruent to the bottom edge of the badge. Prove that the bottom edge of the badge is congruent to the left edge of the badge.

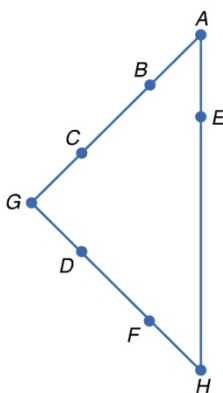
Given:  $WY = YZ$   
 $\overline{YZ} \cong \overline{XZ}$   
 $\overline{XZ} \cong \overline{WX}$   
 Prove:  $\overline{WY} \cong \overline{WX}$



| Statements | Reasons                             |
|------------|-------------------------------------|
| 1.         | 1. Given                            |
| 2.         | 2. Definition of congruent segments |
| 3.         | 3. Given                            |
| 4.         | 4. Transitive Property              |
| 5.         | 5. Given                            |
| 6.         | 6.                                  |

**Example 5:** Prove the following.

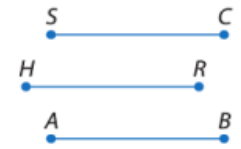
Given:  $\overline{GD} \cong \overline{BC}$   
 $\overline{BC} \cong \overline{FH}$   
 $\overline{FH} \cong \overline{AE}$   
 Prove:  $\overline{AE} \cong \overline{GD}$



| Statements                             | Reasons                |
|--|------------------------|
| 1.                                     | 1. Given               |
| 2.                                     | 2. Given               |
| 3.                                     | 3. Transitive Property |
| 4.                                     | 4. Given               |
| 5.                                     | 5. Transitive          |
| 6. $\overline{AE} \cong \overline{GD}$ | 6.                     |

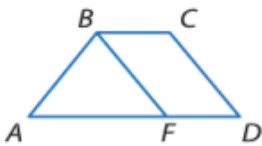


1. If  $\overline{SC} \cong \overline{HR}$  and  $\overline{HR} \cong \overline{AB}$ , then  $\overline{SC} \cong \overline{AB}$ .



| Statements | Reasons  |
|------------|----------|
| 1.         | 1. Given |
| 2.         | 2.       |
| 3.         | 3.       |

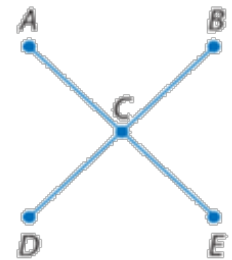
2. In the diagram,  $\overline{AB} \cong \overline{CD}$  and  $\overline{CD} \cong \overline{BF}$ . Examine the conclusions made by Leslie and Shantice. Is either of them correct? Explain how you know.



*Leslie*  
Since  $\overline{AB} \cong \overline{CD}$  and  $\overline{CD} \cong \overline{BF}$ , then  $\overline{AB} \cong \overline{BF}$   
by the Transitive Property of Congruence

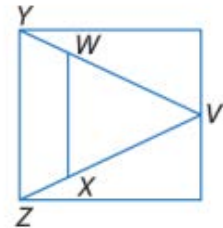
*Shantice*  
Since  $\overline{AB} \cong \overline{CD}$  and  $\overline{CD} \cong \overline{BF}$ , then  $\overline{AB} \cong \overline{BF}$  by the Reflexive Property of Congruence.

3. Given:  $C$  is the midpoint of  $\overline{AE}$ .  
 $C$  is the midpoint of  $\overline{BD}$ .  
 $AE = BD$   
Prove:  $\overline{AC} \cong \overline{CD}$



| Statements             | Reasons                              |
|------------------------|--------------------------------------|
| 1.                     | 1. Given                             |
| 2. $AC = CE$           | 2.                                   |
| 3.                     | 3. Given                             |
| 4.                     | 4. Definition of midpoint            |
| 5. $AE = BD$           | 5.                                   |
| 6.                     | 6. Segment Addition Postulate        |
| 7.                     | 7. Segment Addition Postulate        |
| 8. $AC + CE = BC + CD$ | 8.                                   |
| 9. $AC + AC = CD + CD$ | 9.                                   |
| 10.                    | 10. Simplify                         |
| 11.                    | 11. Division Property                |
| 12.                    | 12. Definition of congruent segments |

4. If  $\overline{VZ} \cong \overline{VY}$  and  $\overline{WY} \cong \overline{XZ}$ , then  $\overline{VW} \cong \overline{VX}$



| Statements  | Reasons                             |
|---|-------------------------------------|
| 1.  | 1. Given                            |
| 2.  | 2. Definition of congruent segments |
| 3. $\overline{WY} \cong \overline{XZ}$                        | 3.                                  |
| 4. $WY = XZ$  | 4.                                  |
| 5. $VZ = VX + XZ$   | 5.                                  |
| 6. $VY = \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$ | 6.                                  |
| 7. $VX + XZ = VW + WY$  | 7.                                  |
| 8. $VX + \underline{\hspace{1cm}} = VW + WY$                  | 8. Substitution                     |
| 9.  | 9. Subtraction Property             |
| 10. $VW = VX$   | 10.                                 |
| 11.   | 11.                                 |

5. If  $B$  is the midpoint of  $\overline{AC}$ ,  $D$  is the midpoint of  $\overline{CE}$ , and  $\overline{AB} \cong \overline{DE}$ , then  $AE = 4AB$ .



| Statements   | Reasons                       |
|--|-------------------------------|
| 1.   | 1. Given                      |
| 2.   | 2. Definition of midpoint     |
| 3. $D$ is the midpoint of $\overline{CE}$  | 3.                            |
| 4. $CD = DE$   | 4.                            |
| 5.   | 5. Given                      |
| 6. $AB = DE$   | 6.                            |
| 7. $AB = CE$   | 7.                            |
| 8. $AC = \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$  | 8. Segment Addition Postulate |
| 9. $CE = \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$  | 9.                            |
| 10. $AE = \underline{\hspace{1cm}} + CE$   | 10.                           |
| 11. $AE = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$ | 11. Substitution              |
| 12. $AE = AB + AB + AB + AB$   | 12.                           |
| 13.  | 13. Simplify                  |

1. Given:  $\overline{EF} = \overline{EF}$   
Prove:  $\overline{EF} \cong \overline{EF}$

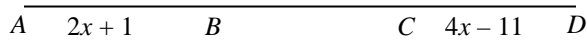
| Statements                         | Reasons |
|------------------------------------|---------|
| 1. $\overline{EF} = \overline{EF}$ | 1.      |
| 2.                                 | 2.      |

2. Given:  $\overline{AB} \cong \overline{JK}$ ,  $\overline{JK} \cong \overline{ST}$   
Prove:  $\overline{AB} \cong \overline{ST}$



| Statements | Reasons |
|------------|---------|
| 1.         | 1.      |
| 2.         | 2.      |
| 3.         | 3.      |

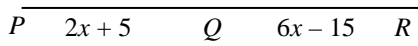
3. Given:  $\overline{AB} \cong \overline{BC}$   
 $\overline{CD} \cong \overline{BC}$   
Prove:  $x = 6$



| Statements                             | Reasons |
|--|---------|
| 1.                                     | 1.      |
| 2.                                     | 2.      |
| 3. $\overline{AB} \cong \overline{CD}$ | 3.      |
| 4. $AB = CD$                           | 4.      |
| 5. $2x + 1 = 4x - 11$                  | 5.      |
| 6.                                     | 6.      |
| 7.                                     | 7.      |
| 8.                                     | 8.      |

4. Given:  $PR = 46$

Prove:  $x = 7$



| Statements                 | Reasons |
|----------------------------|---------|
| 1.                         | 1.      |
| 2. $PQ + QR = PR$          | 2.      |
| 3. $2x + 5 + 6x - 15 = 46$ | 3.      |
| 4.                         | 4.      |
| 5.                         | 5.      |
| 6.                         | 6.      |

5. Given:  $\overline{ST} \cong \overline{SR}$   
 $\overline{QR} \cong \overline{SR}$

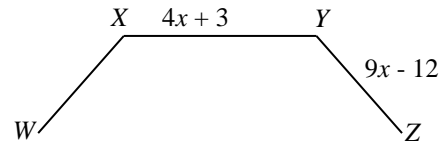
Prove:  $x = 1$



| Statements             | Reasons                |
|------------------------|------------------------|
| 1.                     | 1.                     |
| 2.                     | 2.                     |
| 3.                     | 3. Transitive Property |
| 4. $ST = QR$           | 4.                     |
| 5. $x + 4 = 5(3x - 2)$ | 5.                     |
| 6.                     | 6.                     |
| 7.                     | 7.                     |
| 8.                     | 8.                     |
| 9.                     | 9.                     |

6. Given:  $\overline{XY} \cong \overline{WX}$   
 $\overline{YZ} \cong \overline{WX}$

Prove:  $x = 3$



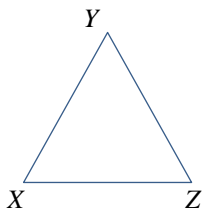
| Statements | Reasons                           |
|------------|-----------------------------------|
| 1.         | 1.                                |
| 2.         | 2.                                |
| 3.         | 3. Transitive                     |
| 4.         | 4. Definition of $\cong$ segments |
| 5.         | 5. Substitution                   |
| 6.         | 6.                                |
| 7.         | 7.                                |
| 8.         | 8.                                |

7. Given:  $XY = 8$

$XZ = 8$

$\overline{XY} \cong \overline{ZY}$

Prove:  $\overline{XZ} \cong \overline{ZY}$



| Statements                             | Reasons                           |
|--|-----------------------------------|
| 1.                                     | 1.                                |
| 2.                                     | 2.                                |
| 3. $XY = XZ$                           | 3.                                |
| 4.                                     | 4. Definition of $\cong$ segments |
| 5. $\overline{XY} \cong \overline{ZY}$ | 5.                                |
| 6.                                     | 6.                                |



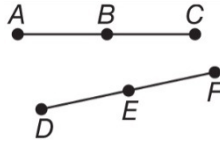


1. Given:  $\overline{AB} \cong \overline{DE}$

$B$  is the midpoint of  $\overline{AC}$ .

$E$  is the midpoint of  $\overline{DF}$ .

Prove:  $\overline{BC} \cong \overline{EF}$



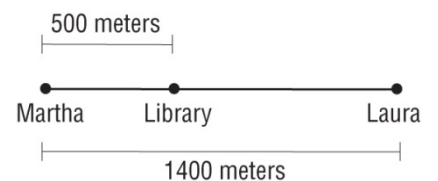
| Statements   | Reasons                   |
|--------------|---------------------------|
| 1.           | 1. Given                  |
| 2. $AB = DE$ | 2.                        |
| 3.           | 3. Given                  |
| 4.           | 4. Definition of Midpoint |
| 5.           | 5. Given                  |
| 6.           | 6. Definition of Midpoint |
| 7. $DE = BC$ | 7.                        |
| 8. $BC = EF$ | 8.                        |
| 9.           | 9.                        |

2. Refer to the figure. DeAnne knows that the distance from Grayson to Apex is the same as the distance from Redding to Pine Bluff. Prove that the distance from Grayson to Redding is equal to the distance from Apex to Pine Bluff.



3. Maria is 11 inches shorter than her sister Nancy. Brad is 11 inches shorter than his brother Chad. If Maria is shorter than Brad, how do the heights of Nancy and Chad compare? What if Maria and Brad are the same height?

4. Martha and Laura live 1400 meters apart. A library is opened between them and is 500 meters from Martha. How far is the library from Laura?



5. Byron works in a lumber yard. His boss just cut a dozen planks and asked Byron to double check that they are all the same length. The planks were numbered 1 through 12. Byron took out plank number 1 and checked that the other planks are all the same length as plank 1. He concluded that they must all be the same length. Explain how you know plank 7 and plank 10 are the same length even though they were never directly compared to each other?

6. Karla, John, and Mandy live in three houses that are on the same line. John lives between Karla and Mandy. Karla and Mandy live a mile apart. Is it possible for John to be a mile from both Karla and Mandy?

7. Five lights,  $A$ ,  $B$ ,  $C$ ,  $D$ , and  $E$ , are lined up in a row. The middle light is the midpoint of the second and fourth light and also the midpoint of the first and last light.

a) Draw a figure to illustrate the situation.

b) Complete this proof.

Given:  $C$  is the midpoint of  $\overline{BD}$  and  $\overline{AE}$ .

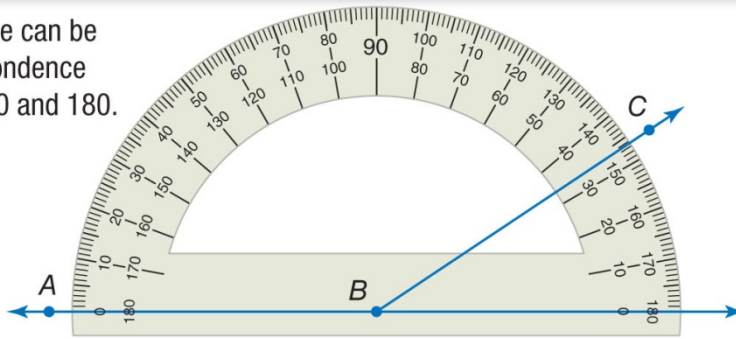
Prove:  $AB = DE$

| Statement                                   | Reason                        |
|---|-------------------------------|
| 1. $C$ is the midpoint of $\overline{BD}$ . | 1. Given                      |
| 2. $BC = CD$                                | 2.                            |
| 3. $C$ is the midpoint of $\overline{AE}$ . | 3.                            |
| 4.  | 4. Definition of Midpoint     |
| 5.  | 5. Segment Addition Postulate |
| 6. $CE = CD + DE$                           | 6.                            |
| 7. $AB = AC - BC$                           | 7.                            |
| 8.  | 8. Substitution Property      |
| 9. $DE = CE - CD$                           | 9.                            |
| 10.   | 10.                           |

### Postulate 2.10 Protractor Postulate

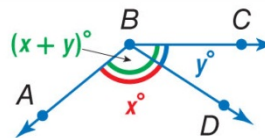
**Words** Given any angle, the measure can be put into one-to-one correspondence with real numbers between 0 and 180.

**Example** If  $\overrightarrow{BA}$  is placed along the protractor at  $0^\circ$ , then the measure of  $\angle ABC$  corresponds to a positive real number.



### Postulate 2.11 Angle Addition Postulate

$D$  is in the interior of  $\angle ABC$  if and only if  
 $m\angle ABD + m\angle DBC = m\angle ABC$ .

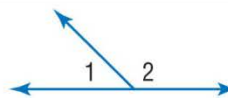


**Example 1:** Using a protractor, a construction worker measures that the angle a beam makes with a ceiling is  $42^\circ$ . What is the measure of the acute angle the beam makes with the wall?

### Theorems

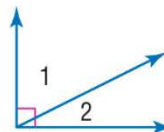
**2.3 Supplement Theorem** If two angles form a linear pair, then they are supplementary angles.

**Example**  $m\angle 1 + m\angle 2 = 180$



**2.4 Complement Theorem** If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

**Example**  $m\angle 1 + m\angle 2 = 90$



**Example 2:** At 4 o'clock, the angle between the hour and minute hands of a clock is  $120^\circ$ . When the second hand bisects the angle between the hour and minute hands, what are the measures of the angles between the minute and second hands and between the second and hour hands?

## Theorem 2.5 Properties of Angle Congruence

### Reflexive Property of Congruence

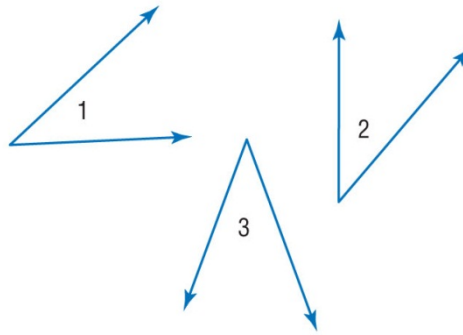
$$\angle 1 \cong \angle 1$$

### Symmetric Property of Congruence

If  $\angle 1 \cong \angle 2$ , then  $\angle 2 \cong \angle 1$ .

### Transitive Property of Congruence

If  $\angle 1 \cong \angle 2$  and  $\angle 2 \cong \angle 3$ , then  $\angle 1 \cong \angle 3$ .



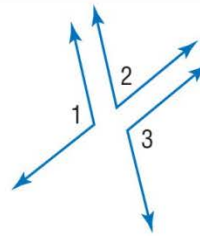
## Theorems

### 2.6 Congruent Supplements Theorem

Angles supplementary to the same angle or to congruent angles are congruent.

**Abbreviation**  $\sphericalangle$  suppl. to same  $\sphericalangle$  or  $\cong \sphericalangle$  are  $\cong$ .

**Example** If  $m\angle 1 + m\angle 2 = 180$  and  $m\angle 2 + m\angle 3 = 180$ , then  $\angle 1 \cong \angle 3$ .

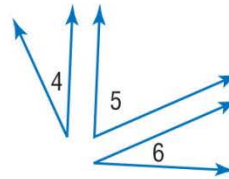


### 2.7 Congruent Complements Theorem

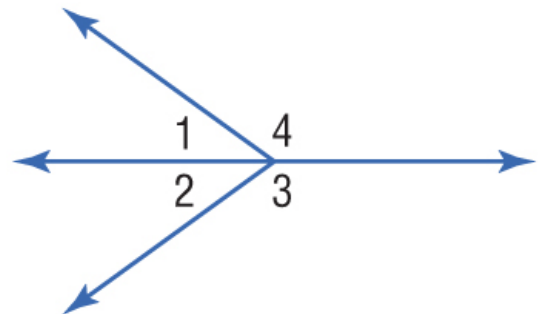
Angles complementary to the same angle or to congruent angles are congruent.

**Abbreviation**  $\sphericalangle$  compl. to same  $\sphericalangle$  or  $\cong \sphericalangle$  are  $\cong$ .

**Example** If  $m\angle 4 + m\angle 5 = 90$  and  $m\angle 5 + m\angle 6 = 90$ , then  $\angle 4 \cong \angle 6$ .



**Example 3:** In the figure,  $\angle 1$  and  $\angle 4$  form a linear pair, and  $m\angle 3 + m\angle 1 = 180^\circ$ . Prove that  $\angle 3$  and  $\angle 4$  are congruent.



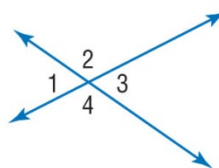
| Statements                                     | Reasons |
|--|---------|
| 1.   | 1.      |
| 2. $\angle 1$ and $\angle 4$ are supplementary | 2.      |
| 3. $m\angle 3 + m\angle 1 = 180^\circ$         | 3.      |
| 4. $\angle 3$ and $\angle 1$ are supplementary | 4.      |
| 5.   | 5.      |

### Theorem 2.8 Vertical Angles Theorem

If two angles are vertical angles, then they are congruent.

**Abbreviation** *Vert.  $\sphericalangle$  are  $\cong$ .*

**Example**  $\angle 1 \cong \angle 3$  and  $\angle 2 \cong \angle 4$



**Example 4:** If  $\angle 1$  and  $\angle 2$  are vertical angles and  $m\angle 1 = (d - 32)^\circ$  and  $m\angle 2 = (175 - 2d)^\circ$ , find  $m\angle 1$  and  $m\angle 2$ . Justify each step.

| Statements                   | Reasons              |
|------------------------------|----------------------|
| 1.                           | 1.                   |
| 2. $\angle 1 \cong \angle 2$ | 2.                   |
| 3. $m\angle 1 = m\angle 2$   | 3.                   |
| 4. $d - 32 = 175 - 2d$       | 4.                   |
| 5.                           | 5. Addition Property |
| 6.                           | 6.                   |
| 7.                           | 7.                   |
| 8.                           | 8.                   |

### Theorems Right Angle Theorems

| Theorem  | Example |
|--|---------|
| <p><b>2.9</b> Perpendicular lines intersect to form four right angles.</p> <p><b>Example</b> If <math>\overrightarrow{AC} \perp \overrightarrow{DB}</math>, then <math>\angle 1</math>, <math>\angle 2</math>, <math>\angle 3</math>, and <math>\angle 4</math> are rt. <math>\sphericalangle</math>.</p>                    |         |
| <p><b>2.10</b> All right angles are congruent.</p> <p><b>Example</b> If <math>\angle 1</math>, <math>\angle 2</math>, <math>\angle 3</math>, and <math>\angle 4</math> are rt. <math>\sphericalangle</math>, then <math>\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4</math>.</p>                                    |         |
| <p><b>2.11</b> Perpendicular lines form congruent adjacent angles.</p> <p><b>Example</b> If <math>\overrightarrow{AC} \perp \overrightarrow{DB}</math>, then <math>\angle 1 \cong \angle 2</math>, <math>\angle 2 \cong \angle 4</math>, <math>\angle 3 \cong \angle 4</math>, and <math>\angle 1 \cong \angle 3</math>.</p> |         |
| <p><b>2.12</b> If two angles are congruent and supplementary, then each angle is a right angle.</p> <p><b>Example</b> If <math>\angle 5 \cong \angle 6</math> and <math>\angle 5</math> is suppl. to <math>\angle 6</math>, then <math>\angle 5</math> and <math>\angle 6</math> are rt. <math>\sphericalangle</math>.</p>   |         |
| <p><b>2.13</b> If two congruent angles form a linear pair, then they are right angles.</p> <p><b>Example</b> If <math>\angle 7</math> and <math>\angle 8</math> form a linear pair, then <math>\angle 7</math> and <math>\angle 8</math> are rt. <math>\sphericalangle</math>.</p>   |         |

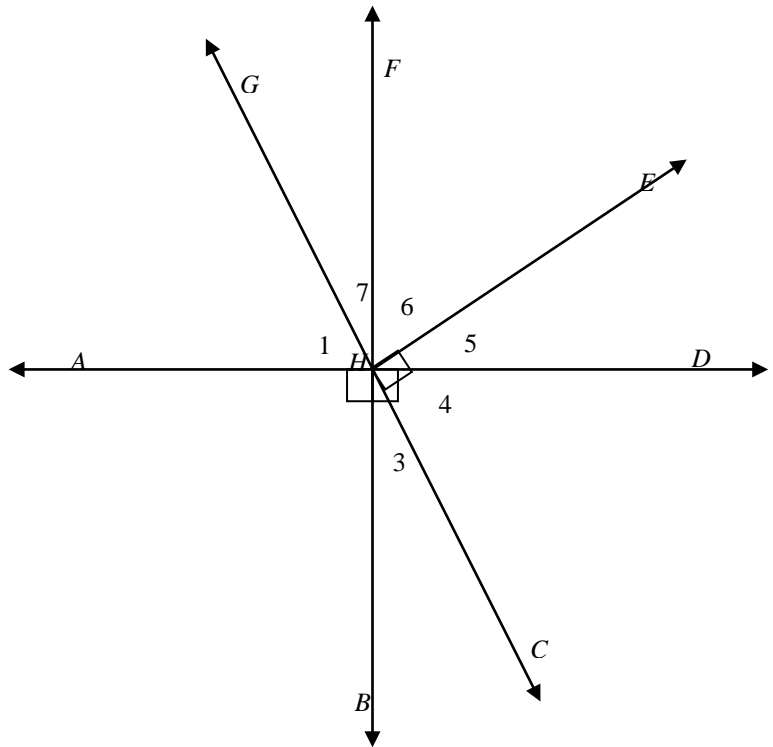


1. Given:  $\angle A \cong \angle B$   
Prove:  $\angle B \cong \angle A$

| Statements | Reasons |
|------------|---------|
| 1.         | 1.      |
| 2.         | 2.      |

For numbers 2 – 7, complete the statement given that  $m\angle EHC = m\angle DHB = m\angle AHB = 90^\circ$

2. If  $m\angle 7 = 28^\circ$ , then  $m\angle 3 =$  \_\_\_\_\_
3. If  $m\angle EHB = 121^\circ$ , then  $m\angle 7 =$  \_\_\_\_\_
4. If  $m\angle 3 = 34^\circ$ , then  $m\angle 5 =$  \_\_\_\_\_
5. If  $m\angle GHB = 158^\circ$ , then  $m\angle FHC =$  \_\_\_\_\_
6. If  $m\angle 7 = 31^\circ$ , then  $m\angle 6 =$  \_\_\_\_\_
7. If  $m\angle GHD = 119^\circ$ , then  $m\angle 4 =$  \_\_\_\_\_

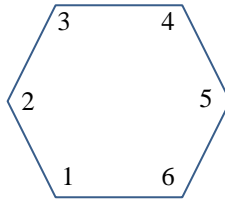


8. Make a sketch using the given information. Then, state all of the pairs of congruent angles.

$\angle 1$  and  $\angle 2$  are a linear pair.  $\angle 2$  and  $\angle 3$  are a linear pair.  $\angle 3$  and  $\angle 4$  are a linear pair.

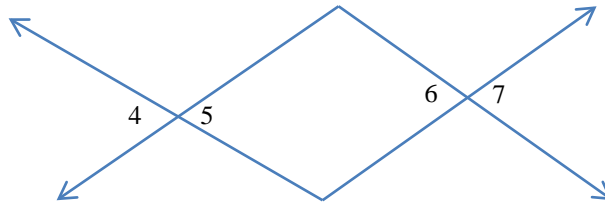


9. Given:  $m\angle 3 = 120^\circ$   
 $\angle 1 \cong \angle 4$   
 $\angle 3 \cong \angle 4$   
 Prove:  $m\angle 1 = 120^\circ$



| Statements                   | Reasons |
|------------------------------|---------|
| 1. $m\angle 3 = 120^\circ$   | 1.      |
| 2. $\angle 1 \cong \angle 4$ | 2.      |
| 3. $\angle 3 \cong \angle 4$ | 3.      |
| 4.                           | 4.      |
| 5. $m\angle 1 = m\angle 3$   | 5.      |
| 6.                           | 6.      |

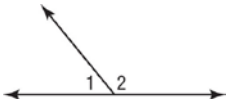
10. Given:  $\angle 5 \cong \angle 6$   
 Prove:  $\angle 4 \cong \angle 7$



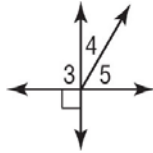
| Statements                   | Reasons                            |
|------------------------------|------------------------------------|
| 1. $\angle 5 \cong \angle 6$ | 1.                                 |
| 2. $\angle 4 \cong \angle 5$ | 2. Vertical $\angle$ s are $\cong$ |
| 3. $\angle 4 \cong \angle 6$ | 3.                                 |
| 4. $\angle 6 \cong \angle 7$ | 4.                                 |
| 5.                           | 5.                                 |

For numbers 1 – 3, find the measure of each numbered angle and name the theorems that justify your work.

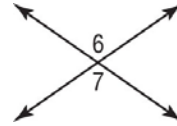
1.  $m\angle 1 = (x + 10)^\circ$   
 $m\angle 2 = (3x + 18)^\circ$



2.  $m\angle 4 = (2x - 5)^\circ$   
 $m\angle 5 = (4x - 13)^\circ$



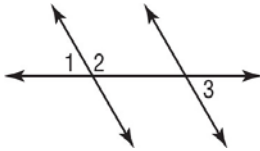
3.  $m\angle 6 = (7x - 24)^\circ$   
 $m\angle 7 = (5x + 14)^\circ$



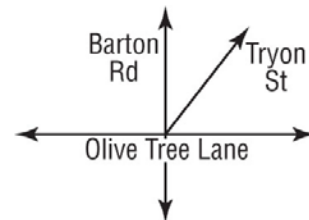
4. Write a two-column proof.

Given:  $\angle 1$  and  $\angle 2$  form a linear pair.  
 $\angle 2$  and  $\angle 3$  are supplementary.

Prove:  $\angle 1 \cong \angle 3$



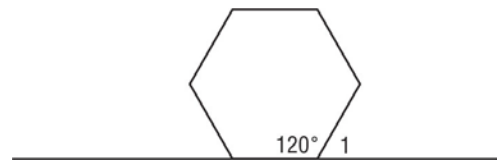
5. Refer to the figure. Barton Road and Olive Tree Lane form a right angle at their intersection. Tryon Street forms a  $57^\circ$  angle with Olive Tree Lane. What is the measure of the acute angle Tryon Street forms with Barton Road?



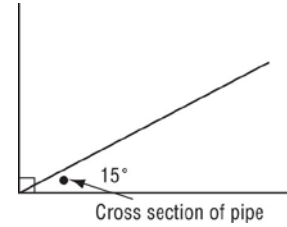
6. For a school project, students are making a giant icosahedron, which is a large solid with many identical triangular faces. John is assigned quality control. He must make sure that the measures of all the angles in all the triangles are the same as each other. He does this by using a precut template and comparing the corner angles of every triangle to the template. How does this assure that the angles in all the triangles will be congruent to each other?

7. If you look straight ahead at a scenic point, you can see a waterfall. If you turn your head  $25^\circ$  to the left, you will see a famous mountain peak. If you turn your head  $35^\circ$  more to the left, you will see another waterfall. If you are looking straight ahead, through how many degrees must you turn your head to the left in order to see the second waterfall?

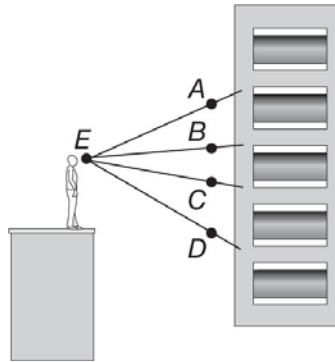
8. A tube with a hexagonal cross section is placed on the floor.  
 What is the measure of  $\angle 1$  in the figure given that the angle at one corner of the hexagon is  $120^\circ$ ?



9. Students are painting their rectangular classroom ceiling. They want to paint a line that intersects one of the corners as shown in the figure. They want the painted line to make a  $15^\circ$  angle with one edge of the ceiling. Unfortunately, between the line and the edge there is a water pipe making it difficult to measure the angle. They decide to measure the angle to the other edge. Given that the corner is a right angle, what is the measure of the other angle?



10. Clyde looks at a building from point  $E$ .  $\angle AEC$  has the same measure as  $\angle BED$ .



a) The measure of  $\angle AEC$  is equal to the sum of the measures of  $\angle AEB$  and what other angle?

b) The measure of  $\angle BED$  is equal to the sum of the measures of  $\angle CED$  and what other angle?

c) Is it true that  $m\angle AEB$  is equal to  $m\angle CED$ ?