Name ____ Period _______

GEOMETRY Chapter 4 Congruent Triangles

Section 4.1 Triangles and Angles

GOAL 1: Classifying Triangles

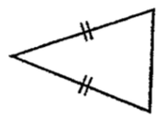
A **triangle** is a figure formed by ____.
A triangle can be classified by its sides and by its angles.

Classification by Sides

Equilateral Triangle

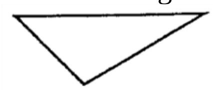
3 congruent sides

Isosceles Triangle



At least 2 congruent sides

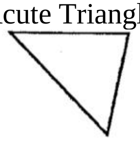
Scalene Triangle



No congruent sides

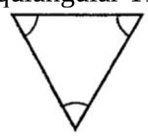
Classification by Angles

Acute Triangle



3 acute angles

Equiangular Triangle

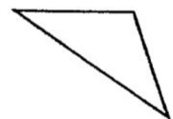


3 congruent angles

Right Triangle

1 right angle

Obtuse Triangle

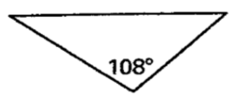


1 obtuse angle

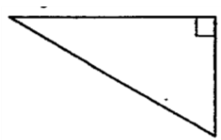
Note: An equiangular triangle is also acute.

Ex. 1 Classify the triangle by its angles and by its sides.

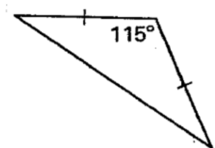
a.



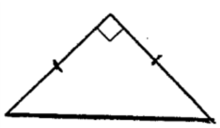
b.



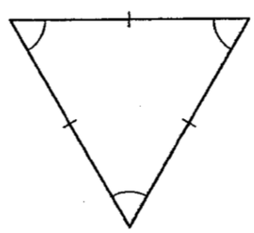
c.



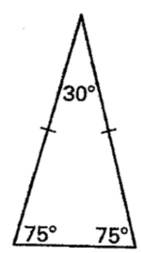
d.



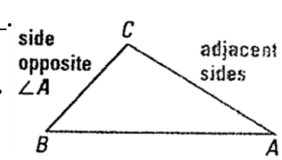
e.



f.



A **vertex** of a triangle is ____.

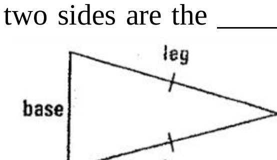


In a triangle, ____ are **adjacent sides.**

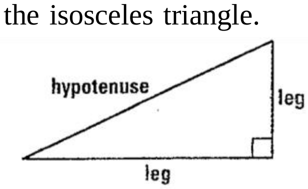
In ΔABC, $\overline{CA}$ and $\overline{BA}$ are adjacent sides. The third side, $\overline{BC}$, is the side opposite $\angle A$.

Right and Isosceles Triangles

The sides of right triangles and isosceles triangles have special names. In a right triangle, the sides that form the right angles are the ___________ of the right triangle. The side opposite the right angle is the _______________ of the triangle. An isosceles triangle can have three congruent sides, in which case it is equilateral. When an isosceles triangle has only two congruent sides, then these two sides are the ________ of the isosceles triangle. The third side is the ________ of the isosceles triangle.

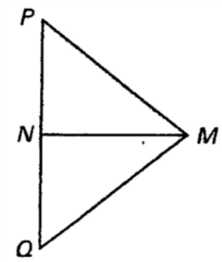


Isosceles triangle



Right triangle

Ex. 2 In the figure, $\overline{MN} \perp \overline{QP}$ and $\overline{MP} \cong \overline{MQ}$. Complete the sentence.



a. Name the legs of isosceles triangle ΔPMQ.

b. Name the base of isosceles triangle ΔPMQ.

c. Name the hypotenuse of right triangle ΔPNM.

d. Name the legs of right triangle ΔPNM.

e. Name the acute angles of right triangle ΔQNM.

Ex. 3 Classify the sentence with always, sometimes, or never

a. An isosceles triangle is ________________________ a right triangle.

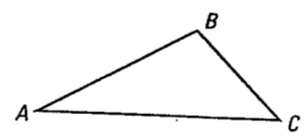
b. An obtuse triangle is ____________________ a right triangle.

c. A right triangle is _______________________ an equilateral triangle.

d. A right triangle is _______________________ an isosceles triangle.

GOAL 2: Using Angle Measures of Triangle

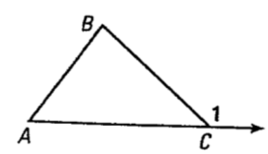
When the sides of a triangle are extended, other angles are formed. The three original angles are the _______________________. The angles that are adjacent to the interior angles are the ____________________. It is common to show only one exterior angle at each vertex.



Theorem 4.1 Triangle Sum Theorem

The sum of the measures of the interior angles of a triangle is 180°.

$$m\angle A + m\angle B + m\angle C = 180°$$



Theorem 4.2 Exterior Angle Theorem

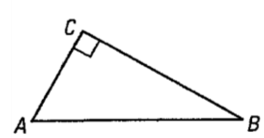
The measure of an exterior angles of a triangle is equal to the sum of the measures of the two nonadjacent interior angles.

$$m\angle 1 = m\angle A + m\angle B$$

Corollary to the Triangle Sum Theorem

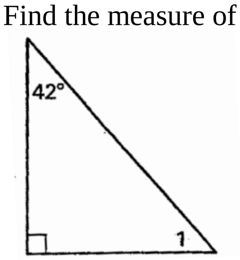
The acute angles of a right triangle are complementary.

$$m\angle A + m\angle B = 90°$$

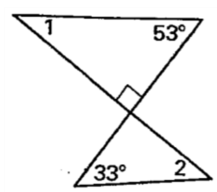


Ex. 4 Find the measure of the numbered angles.

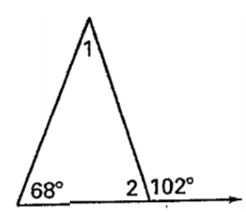
a.



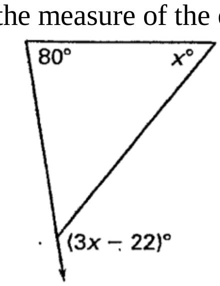
b.



c.



Ex. 5 Find the measure of the exterior angle shown.

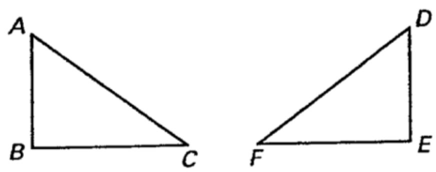


Section 4.2 Congruence and Triangles

GOAL 1: Identifying Congruent Figures

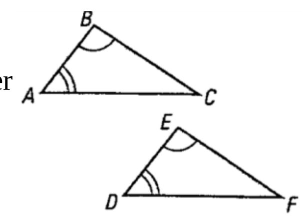
Two geometric figures are congruent if they have exactly the same _____________ and _______________.
When two figures are congruent, there is a correspondence between their angles and sides such that **corresponding angles** are congruent and **corresponding sides** are congruent.
There is more than one way to write a congruence statement, but it is important to list the corresponding angles in the same order.

Ex. 1 Given $\Delta ABC \cong \Delta DEF$. Name three pairs or congruent sides, and three pairs of congruent angles.



Theorem 4.3 Third Angles Theorem

If two angles of one triangle are congruent to two angles of another triangle, then the third angles are also congruent.

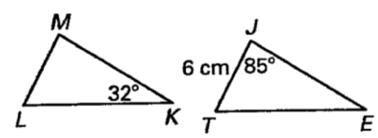


Ex. 2 In the diagram, $\Delta MKL \cong \Delta JET$. Complete the statement.

a. $\angle L \cong$ ____________ b. $\overline{MK} \cong$ ___________

c. $m\angle M =$ _________ d. $m\angle\ \ =$ ____________

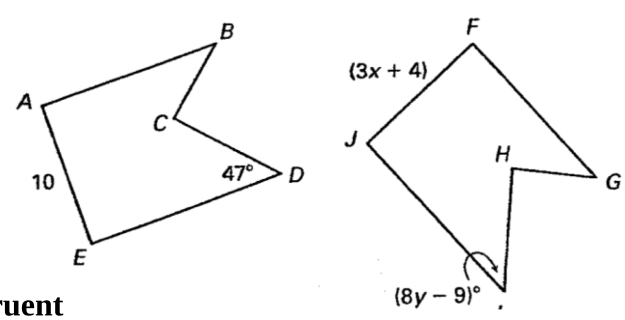
e. $ML =$ ____________ f. $\Delta ETJ \cong$ __________



Ex. 3 In the diagram, $ABCDE \cong FGHIJ$

a. Find the value of x.

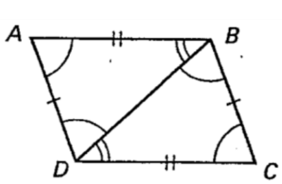
b. Find the value of y.



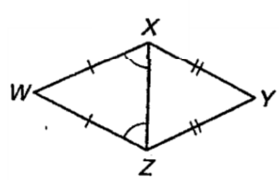
GOAL 2: Proving Triangles are Congruent

Ex. 4 Identify any figures that can be proved congruent. Explain your reasoning. For those that can be proved congruent, write a congruence statement. (You need 3 pairs of congruent angles and 3 pairs of congruent sides.)

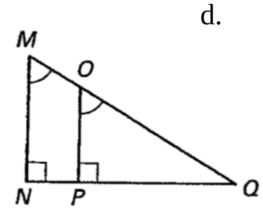
a.



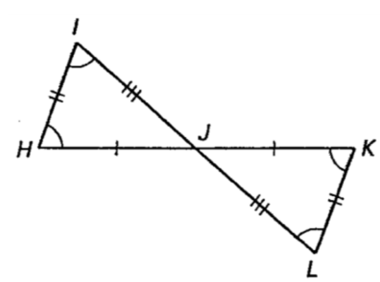
b.



c.



d.

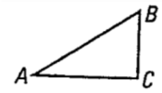


In the next couple of sections, you will learn more efficient ways of proving that triangles are congruent. The properties below will be useful in such proofs.

Theorem 4.4 Properties of Congruent Triangles

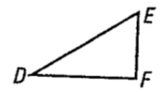
REFLEXIVE PROPERTY OF CONGRUENT TRIANGLES

Every triangle is congruent to itself.



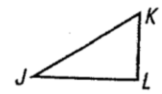
SYMMETRIC PROPERTY OF CONGRUENT TRIANGLES

If $\Delta ABC \cong \Delta DEF$, then $\Delta\text{DEF} \cong \Delta\text{ABC}$.



TRANSITIVE PROPERTY OF CONGRUENT TRIANGLES

If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta JKL$, then $\Delta ABC \cong \Delta JKL$.



Section 4.3 Proving Triangles are Congruent: SSS and SAS

GOAL 1: SSS and SAS Congruent Postulates

How much do you need to know about two triangles to prove that they are congruent? In this lesson and the next, you will learn that you do not need all six of the pieces of information that the triangles are congruent.

Postulate 19 Side-Side-Side (SSS) Congruence Postulate

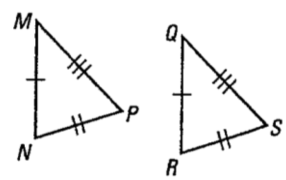
If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

If $\overline{MN} \cong \overline{QR}$,

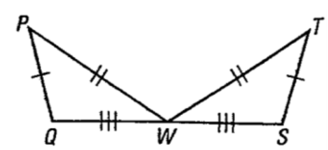
$\overline{MN} \cong \overline{QR}$, and

$\overline{MN} \cong \overline{QR}$,

Then $\Delta MNP \cong \Delta QRS$.



Ex. 1 Prove that $\Delta PQW \cong \Delta TSW$.

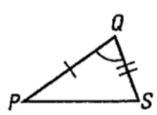


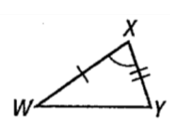
You can construct a triangle that is congruent to a given triangle.

The postulate next is another shortcut that uses two sides and the angle that is *included* between the sides.

Postulate 20 Side-Angle-Side (SAS) Congruence Postulate

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.





If $\overline{PQ} \cong \overline{WX}$,

$\angle Q \cong \angle X$, and

$\overline{QS} \cong \overline{XY}$,

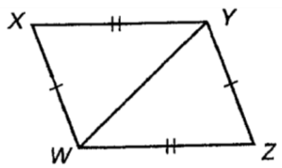
then $\Delta PQS \cong \Delta WXY$.

Ex. 2 For each triangle, name the included angle between the pair of sides given.

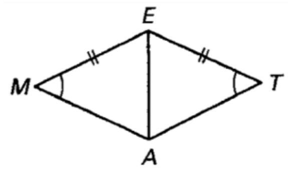
1. $\Delta MAT : \overline{MT}$ and $\overline{TA}$
2. $\Delta CAD : \overline{\text{CA}}$ and $\overline{DC}$
3. $\Delta PSC : \overline{CS}$and $\overline{PS}$
4. $\Delta WDG : \overline{DG}$and $\overline{GW}$

Ex. 3 Decide whether enough in formation is given to prove that the triangles are congruent. If there is enough information, state the congruence postulate you would use.

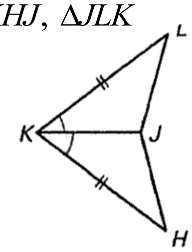
5. $\triangle XYZ$, $\triangle ZWY$



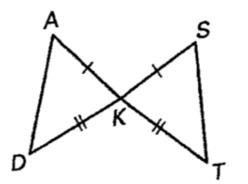
6. $\triangle MAE$, $\triangle TAE$



7. $\triangle KHJ$, $\triangle JLK$



8. $\triangle DKA$, $\triangle TKS$

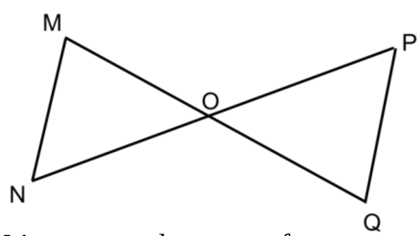


Ex. 4 Complete the proof by supplying the statement or reason.

Given: O is the midpoint of $\overline{MQ}$

O is the midpoint of $\overline{NP}$

Prove: $\triangle MON \cong \triangle QOP$



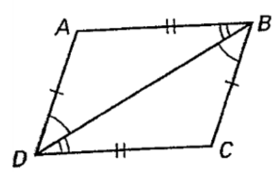
Statements	Reasons
1. O is the midpoint of $\overline{MQ}$.	1.
2. $\overline{MO} \cong \overline{QO}$	2.
3. O is the midpoint of $\overline{NP}$.	3.
4. $\overline{NO} \cong \overline{PO}$	4.
5. $\angle MON \cong \angle QOP$	5.
6. $\triangle MON \cong \triangle QOP$	6.

Ex. 5 Write a two-column proof.

Given: $\angle ABD \cong \angle CDB$, $\angle ADB \cong \angle CBD$,

$\overline{AD} \cong \overline{BC}$, $\overline{AB} \cong \overline{DC}$

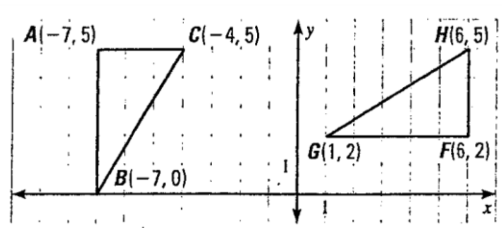
Prove: $\triangle ABD \cong \triangle CDB$



GOAL 2: Modeling a Real-Life Situation

Because of their rigidity, triangles are often used as supporting framework in construction. Supports that form triangles make any structure, whether it be simple or complex, more stable.

*** Ex. 7** Use the Distance Formula and the SSS Congruence Postulate to show that $\triangle ABC \cong \triangle FGH$.



Section 4.4 Proving Triangles are Congruent: ASA and AAS

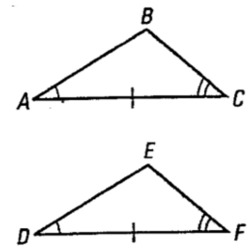
GOAL 1: Using the ASA and AAS Congruence Methods

Two additional ways to prove two triangles are congruent are listed below.

Postulate 21 Angle-Side-Angle (ASA) Congruence Postulate

If two sides and the included side of one triangle of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

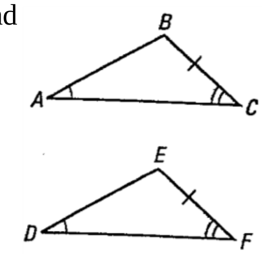
If $\angle A \cong \angle D$,
$\overline{AC} \cong \overline{DF}$, and
$\angle C \cong \angle F$,
Then $\Delta ABC \cong \Delta DEF$.



Theorem 4.5 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of a second triangle, then the two triangles are congruent.

If $\angle A \cong \angle D$,
$\angle C \cong \angle F$, and
$\overline{BC} \cong \overline{EF}$,
Then $\Delta ABC \cong \Delta DEF$.

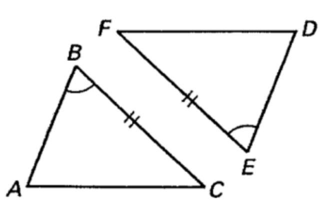


Ex. 1 For triangle ΔMAT, name the included side between the pair of given angles.

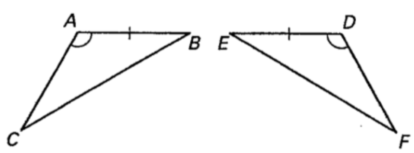
a. $\angle A$ and $\angle T$ b. $\angle T$ and $\angle M$

Ex. 2 State the third congruence that must be given to prove that $\Delta ABC \cong \Delta DEF$.

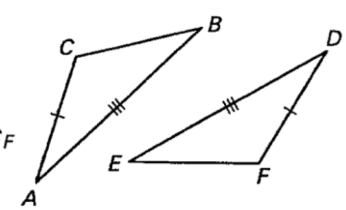
1. ASA Congruence Postulate



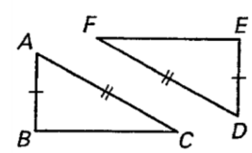
2. AAS Congruence Postulat



3. SSS Congruence Postulate

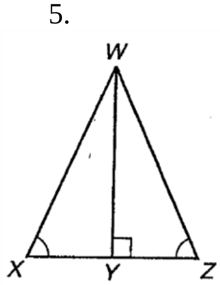


4. SAS Congruence Postulate

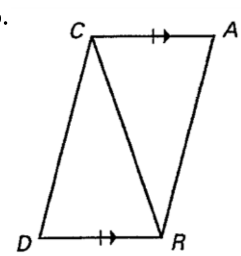


Ex. 3 Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use. Explain your reasoning.

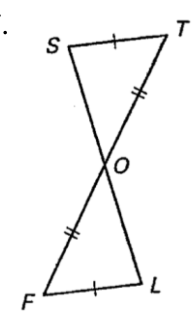
5.



6.

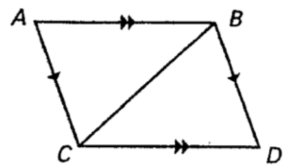


7.



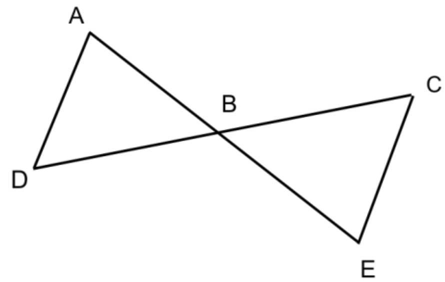
Ex. 4 Write a two-column proof.

Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AC} \parallel \overline{BD}$
Prove: $\Delta ABC \cong \Delta DCB$



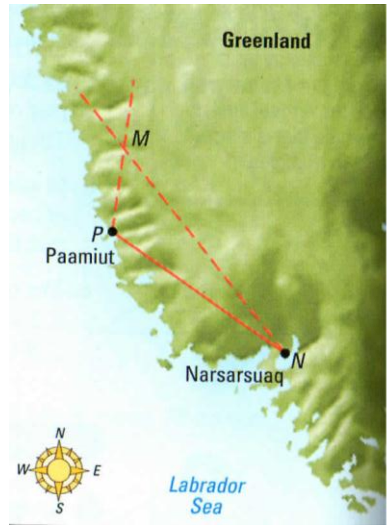
Ex. 5 Write a two-column proof.

Given: B is the midpoint of $\overline{AE}$.
B is the midpoint of $\overline{CD}$.
Prove: $\Delta ABD \cong \Delta EBC$



GOAL 2: Using Congruence Postulates and Theorems

Ex. 7 On December 9, 1997, an extremely bright meteor lit up the sky above Greenland. Scientists attempted to find meteorite fragments by collecting data from eyewitnesses who had seen the meteor pass through the sky. As shown, the scientists were able to describe sighlines from observers in different towns. One sightline was from observers in Paamiut (town P) and another was from observers in Narsarsuaq (Town N). Assuming the sightlines were accurate, did the scientists have enough information to locate any meteorite fragments? Note: The scientists fooling for the meteorite searched over 1150 square miles of rough, icy terrain without finding any meteorite fragments.

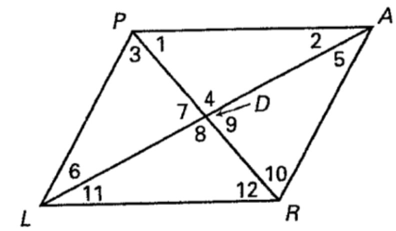


Section 4.5 Using Congruent Triangles

GOAL 1 Planning a Proof

Knowing that all pairs of corresponding parts of congruent triangles are congruent can help you reach conclusions about congruent figures. You can use the fact that corresponding parts of congruent triangles are congruent (CPCTC).

Ex. 1 Use the diagram to answer the following.



a. If $\Delta PDA \cong \Delta RDL$, then $\angle 1$ corresponds to $\angle$ ______

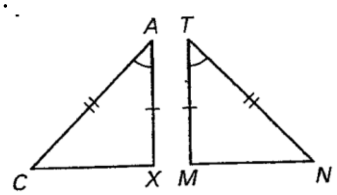
b. If $\Delta PRA \cong \Delta RPL$, then $\angle 1$ corresponds to $\angle$ ______

c. If $\Delta PDL \cong \Delta RDA$, then name 3 pair of corresponding angles.

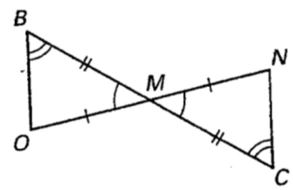
d. If $\Delta PDA \cong \Delta RDL$, then name 3 pair of corresponding angles.

Ex. 2 Use the marked diagram to state the method used to prove the triangles congruent. Name the additional corresponding parts that could then be concluded to be congruent.

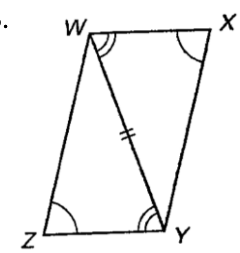
1.



2.

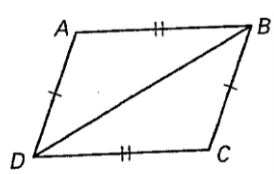


3.



Ex. 3 Complete the proof by supplying the reasons.

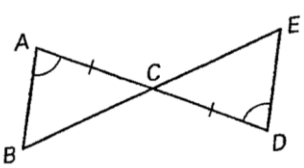
Given: $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$
Prove: $\angle A \cong \angle C$



Statements	Reasons
1. $\overline{AB} \cong \overline{DC}$	**1.**
2. $\overline{AD} \cong \overline{BC}$	**2.**
3. $\overline{BD} \cong \overline{BD}$	**3.**
4. $\Delta ABD \cong \Delta CDB$	**4.**
5. $\angle A \cong \angle C$	**5.**

Ex. 4 Write a two-column proof.

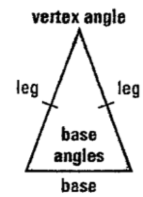
Given: $\overline{AC} \cong \overline{DC}$, $\angle A \cong \angle D$
Prove: $\angle B \cong \angle E$



Section 4.6 Isosceles, Equilateral, and Right Triangles

GOAL 1: Using Properties of Isosceles Triangles

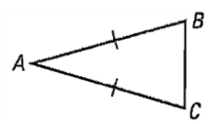
In Lesson 4.1, you learned that a triangle is isosceles if it has at least two congruent sides. If it has exactly two congruent sides, then they are the legs of the triangle and the noncongruent side is the base. The two angles adjacent to the base are the ________________________. The ________________________________ ________________________________ is the vertex angle.



Theorem 4.6 Base Angles Theorem

If two sides of a triangle are congruent, then the angles opposite them are congruent.

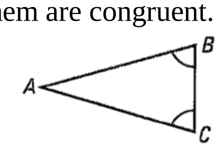
If $\overline{AB} \cong \overline{AC}$, then $\angle B \cong \angle C$.



Theorem 4.7 Converse of the Base Angles Theorem

If two angles of a triangle are congruent, then the sides opposite them are congruent.

If $\angle B \cong \angle C$, then $\overline{AB} \cong \overline{AC}$.

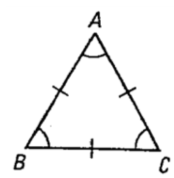


Corollary to Theorem 4.6

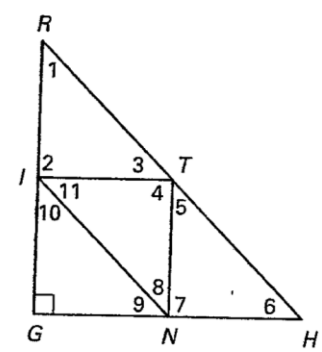
If a triangle is equilateral, then it is equiangular.

Corollary to Theorem 4.7

If a triangle is equiangular, then it is equilateral.



Ex. 1 Use the diagram to the right to answer the following.



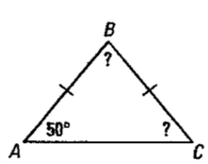
a. If $\overline{RI} \cong \overline{IT}$, what angles are congruent?

b. If $\overline{TN} \cong \overline{IT}$, what angles are congruent?

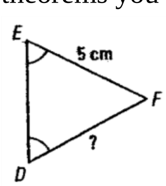
c. If $\angle 1 \cong \angle 6$, what segments are congruent?

Ex. 2 Find the unknown measure(s). Tell what theorems you used.

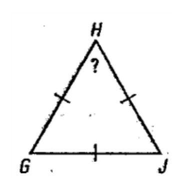
a.



b.

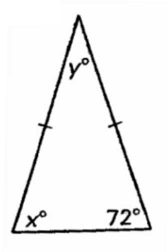


c.

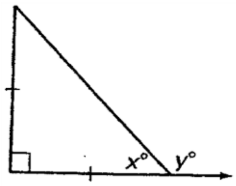


Ex. 3 Solve for *x* and *y*.

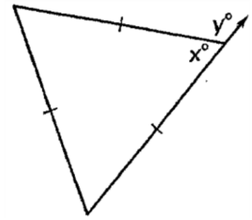
a.



b.

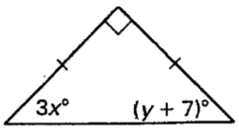


c.

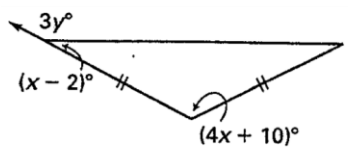


Ex. 4 Solve for *x* and *y*.

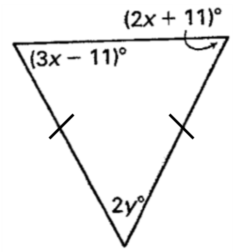
a.



b.



c.

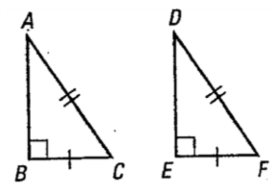


GOAL 2: Using Properties of Right Triangles

You have learned 4 ways to prove that triangles are congruent**: SSS, SAS, ASA, AAS.**
The Hypotenuse-Leg Congruence Theorem can be used to prove that two *right* triangles are congruent.

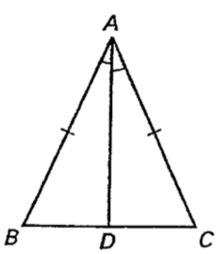
Theorem 4.8 Hypotenuse-Leg (HL) Congruence Theorem
If the hypotenuse and a leg of a right triangle are congruent to the hypotenuse and a leg of a second right triangle, then the two triangles are congruent.

If $\overline{BC} \cong \overline{EF}$ and $\overline{AC} \cong \overline{DF}$, then $\Delta ABC \cong \Delta DEF$.

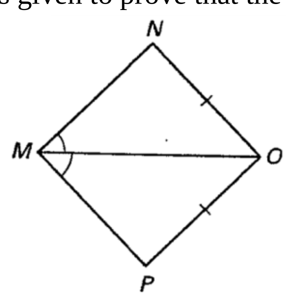


Ex. 5 Decide whether enough information is given to prove that the triangles are congruent. Explain.

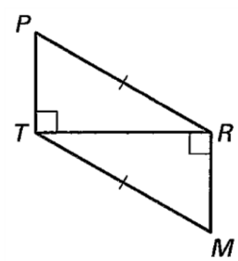
a.



b,

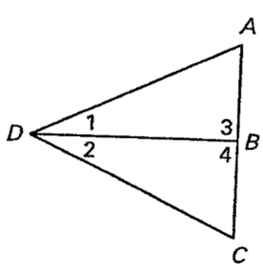


c.



Ex. 6 Write a two-column proof.

Given: $\overline{BD}$ **bisects** $\angle ADC$**.**
$\overline{DB} \perp \overline{AC}$
Prove: ΔADC is isosceles

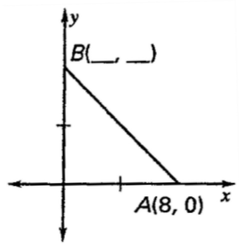


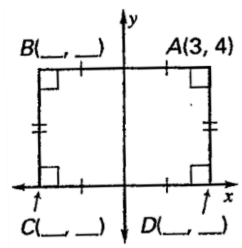
Section 4.7

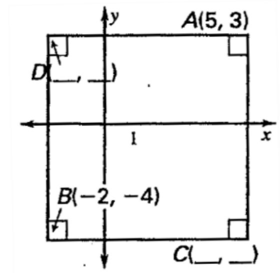
GOAL 1: Placing Figures in A Coordinate Plane

So far, you have studied two-column proofs, paragraph proofs, and flow proofs. A ______________________ involves placing geometric figures in a coordinate plane. Then you can use the Distance Formula and The Midpoint Formula, as well as postulates and theorems, to prove statements about figures.

Ex. 1 Find the missing coordinates.

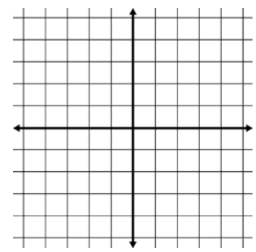
1.


2.


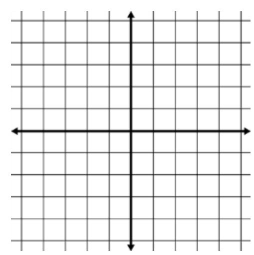
3.


Ex. 2 Place the figure in a coordinate plane. Label the vertices and give the coordinates of each vertex. Find any given information.

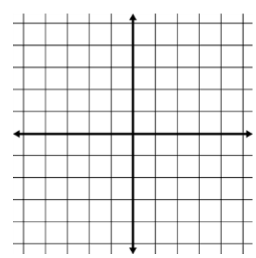
a. An 4-unit by 6-unit rectangle with one vertex at (0, 0).



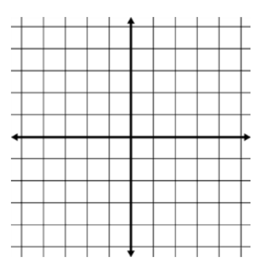
b. A 3-unit by 5-unit rectangle with one vertex at (0, − 4).



c. A right isosceles triangle with legs of 6 units; find the length of the hypotenuse.

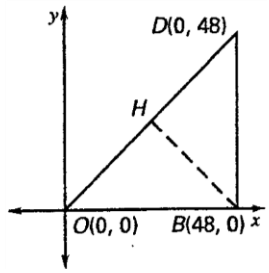


d. A rectangle with a length of 32 units and a width of 18 units; find the length of a diagonal.

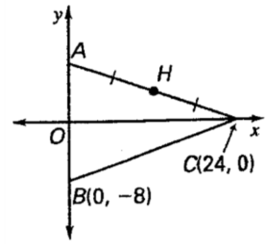


Ex. 3 Use the given information and diagram to find the coordinates of *H.*

a. $\Delta OBH \cong \Delta BDH$



b. $\Delta OBC \cong \Delta OAC$



Ex. 4 Write a coordinate proof.

Given: $\overline{OM} \perp \overline{LN}$
Prove: $\overline{OM}$ bisects $\angle LON$